CIVIL ENGINEERING

For

UPSC Engineering Services Examination, GATE,

State Engineering Service Examination & Public Sector Examination.

(BHEL, NTPC, NHPC, DRDO, SAIL, HAL, BSNL, BPCL, NPCL, etc.)

OPEN CHANNEL FLOW

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CONTENT

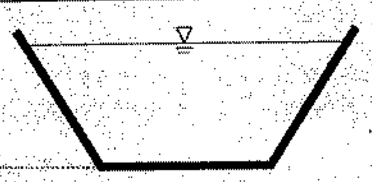
CHAPTER

1

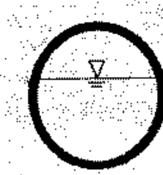
Introduction

INTRODUCTION

- An open channel is a natural or a man made structure in which liquid flows with a free surface at atmospheric pressure.
- For example, flow in rivers, streams, flow in sanitary and storm sewers flowing partially full.



A trapezoidal shaped open channel



A circular shaped open channel

TYPES OF CHANNELS

Prismatic and Non-Prismatic Channels

- A channel in which the cross sectional shape, size and the bed slope are constant is termed as Prismatic channel.
- All natural channels generally have varying cross section and consequently are known as Non-prismatic channels.
- Most of the man made channel are prismatic channels over long stretches. Rectangle, trapezoid, triangle and circle are commonly used shapes in man made channels.

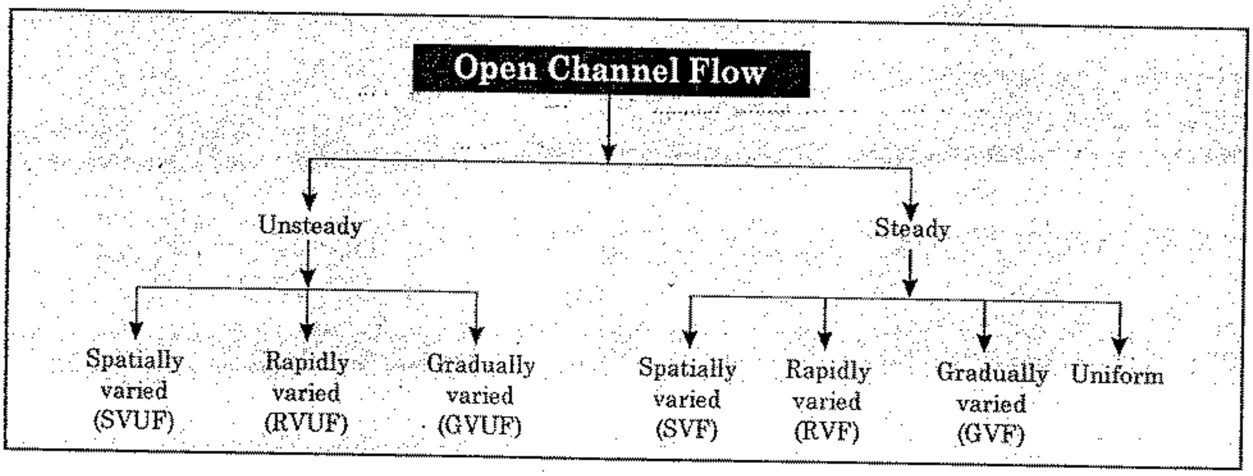
Rigid and Mobile Boundary Channels

- Rigid channels are those in which the boundary is not deformable. The shape and roughness factor is not a function of flow parameter.
- In other words, in Rigid channels, the flow velocity and shear stress distribution will be such that no major scouring, erosion or deposition will take place in the channel and the channel geometry and roughness are essentially constant with respect to time.
- For example, lined canals and non-erodible unlined canals.
- In Rigid channels only depth of flow may vary with space and time depending on the nature of flow. Hence these channels have one (1) degree of freedom.
- Mobile channels are those in which the boundaries undergo deformation due to the continous process

- In mobile channels, the resistance to flow, quantity of sediment transported and channel geometry all depends on interaction of flow with channel boundaries.
- In mobile channels, depth, bed width, bed slope and layout changes with space and time. Hence, these channels have four (4) degree of freedom.
- In mobile channels, flow carries considerable amount of sediment through suspension and in contact with the bed.

Note: In open channel flow, we study only Rigid boundary channel.

TYPES OF OPEN CHANNEL FLOWS



Steady and Unsteady flows

- A steady flow occurs when the flow properties, such as the depth or discharge at a section do not change with time.
- If the depth or discharge changes with time, the flow is termed unsteady flow.
- Flood flows in rivers and rapidly varying surges in canals are some examples of unsteady flow.

Uniform and Non-uniform flows

- If the flow properties, such as depth of flow, in an open channel remains constant along the length of the channel, the flow is said to be uniform.
- A flow in which the flow properties vary along the channel is termed as non-uniform flow.
- A prismatic channel carrying a certain discharge with a constant velocity is an example of uniform flow.
- As an unsteady uniform flow is practically impossible, the term uniform flow is used form *steady uniform flow.*
- In uniform flow, the gravity force on the flowing liquid balances the frictional resistances between the flowing fluid and inside surface of the channel. In case of non-uniform flow, the friction and gravity force are not in balance.

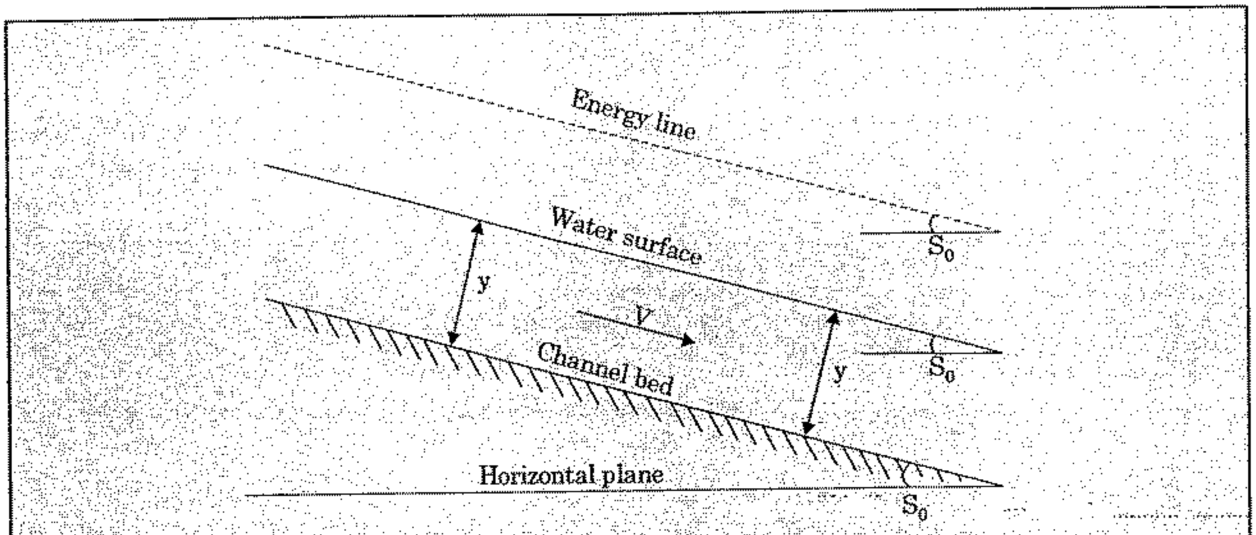
  Property of Uniform flow

  (1) Bed slope = Energy line slope

  (2) Depth of flow = constant

  This constant depth of flow is called as normal depth. In other words we can say that uniform flow occurs at normal depth.

  (3) Average velocity of flow =



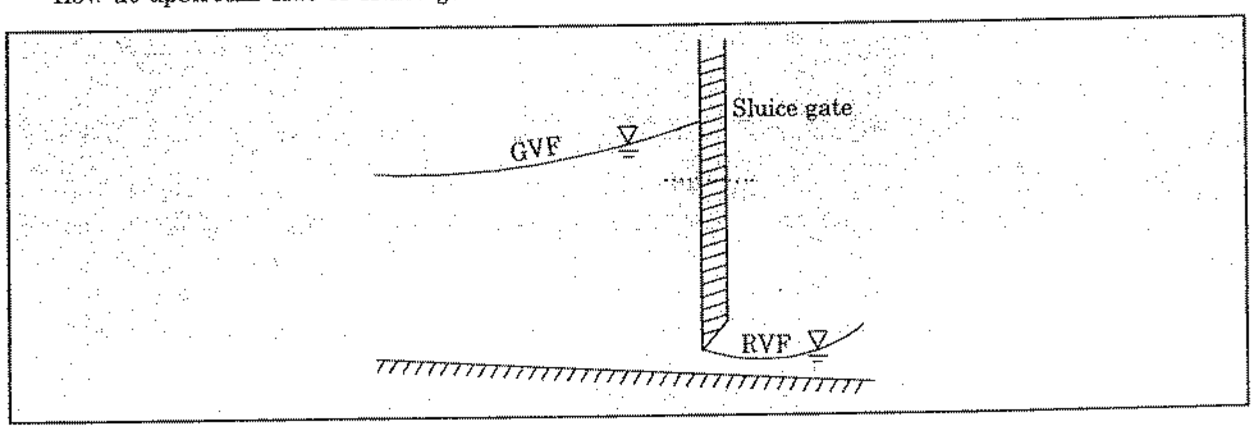
- Under uniform flow, Bed slope = Energy line slope = Water surface slope.

Gradually Varied and Rapidly Varied flows

- Flow in non-prismatic channel and flow with varying velocities in a prismatic channel are examples of Varied flow.
- As a uniform varied flow is impossible, the term varied flow is used for Non-uniform flow.
- The non-uniform flow is further classified as Gradually Varied Flow (G V F) and Rapidly Varied Flow (R V F)

Note: Varied flow assumes that no flow is externally added to or taken out of channel system. i.e The volume of water in a known time interval is conserved in the channel system.

- If depth of flow changes gradually over a long distance along the length of channel such that curvature of free surface is mild, then flow is called as Gradually varied flow (GVF). For example flow at upstream side of sluice gate.



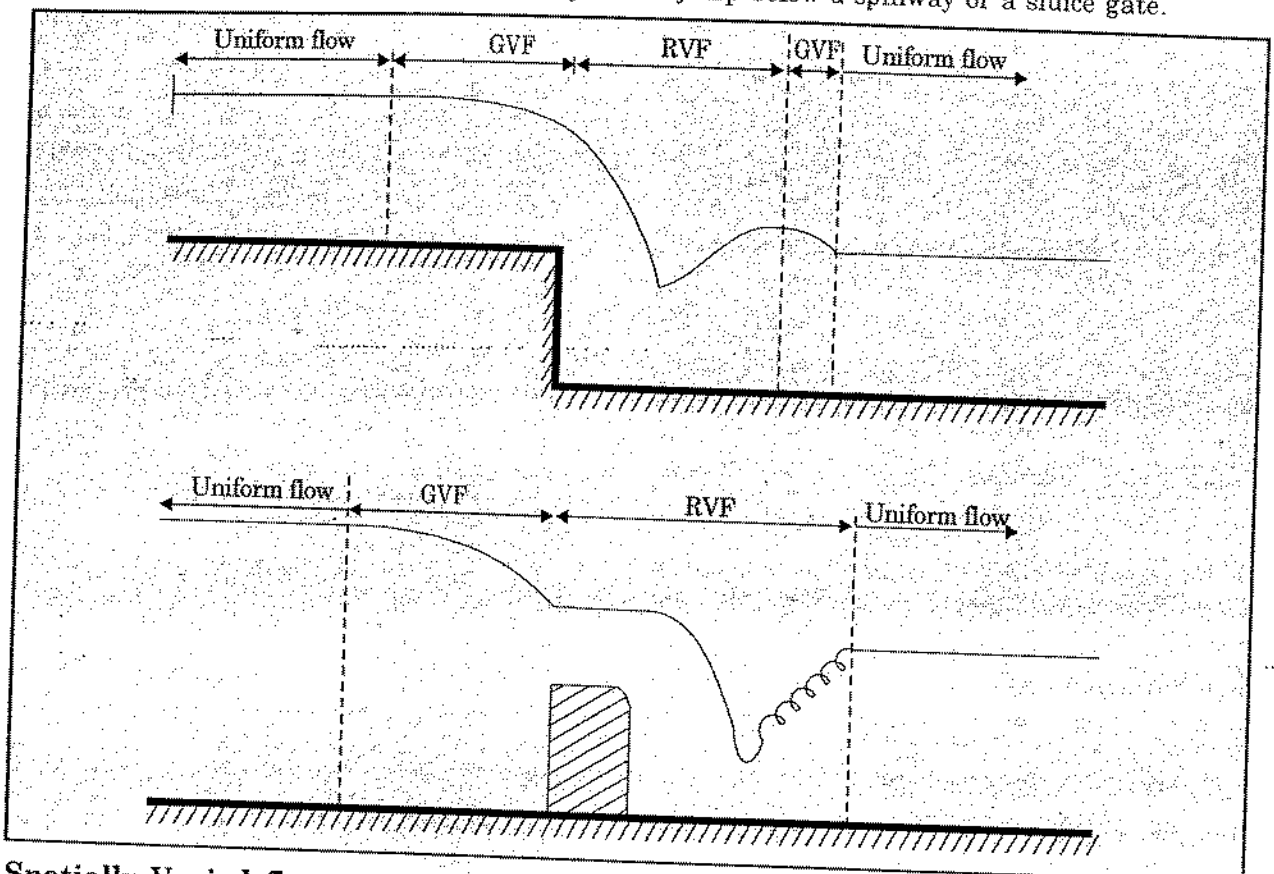
- In GVF, the loss of energy is mainly due to boundary friction.
- In GVF, the pressure distribution in vertical direction is taken as hydrostatic.
- If the curvature in a varied flow is large and the depth changes appreciably over short length, such flow is called Rapidly varied flow.
- For example, flow at downstream side of a sluice gate is a Rapidly varied flow.
- In RVF, frictional resistance are insignificant.
- GVF and RVF are further classified as steady and unsteady flow.

(a) **Gradually varied unsteady flow :** Passage of flood wave in a river.

(b) **Gradually varied steady flow :** Backing up of water in a stream due to dam.

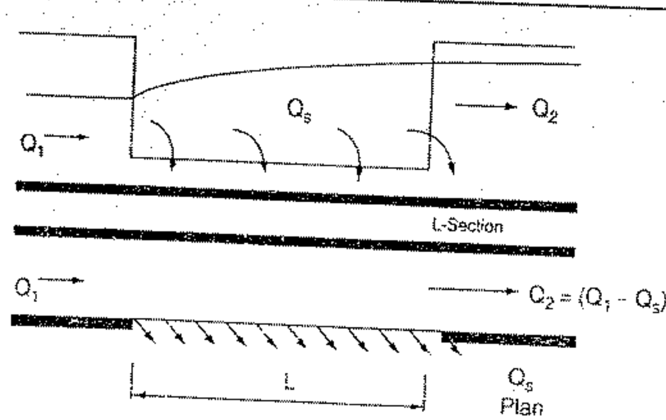
(c) **Rapidly varied unsteady flow :** A surge moving up a canal breaking of wave on the shore.

(d) **Rapidly varied steady flow :** A hydraulic jump below a spillway or a sluice gate.



Spatially Varied flow

- If some flow is added or subtracted from the system, the resulting varied flow is a Spatially varied flow (SVF).
- SVF can be steady or unsteady. The flow over a side weir, flow over a bottom rack is steady SVF, where as surface run off due to rainfall is an unsteady SVF.



Flow over a side weir

Laminar and Turbulent flow

- We know that Reynold's Number R_e is used to demarcate between laminar and turbulent flow

$$R_e = \frac{vR}{\nu}$$

Where,

R_e = Reynold's Number

v = Flow velocity.

R = Hydraulic radius

ν = Kinematic viscosity

- In an open channel flow if $R_e < 500$ flow is laminar where as if $R_e > 2000$, flow is turbulent flow.
- Generally the flow in open channel is a turbulent flow.

Critical, Sub Critical and Super Critical flow

- In open channel, flow is under gravity therefore gravity force and inertia force both plays the role, Hence Froude no. is more important than reynold no.
- In an open channel flow Froude number (F_r) is used to differentiate between the subcritical, critical and super critical flow.

$$F_r = \frac{v}{\sqrt{gL_c}}$$

Where,

F_r = Froude number

g = acceleration due to gravity

L_c = Characteristic length

- Characteristic length (L_c) for any cross sectional shape is expressed as

$$L_c = \frac{\text{Area of flow}}{\text{Top width of flow}}$$

- Characteristic length for a rectangular cross sectional area (A) and having top width of flow B is

$$L_c = \frac{A}{T} = \frac{B \times y}{B} = y$$

- At critical flow condition in Rectangular channel depth of flow is called critical depth (y_c) and velocity of flow is called critical velocity (v_c) and Froude number is equal to unity (1).

$$F_r = \frac{v_c}{\sqrt{\ldots}} = 1$$

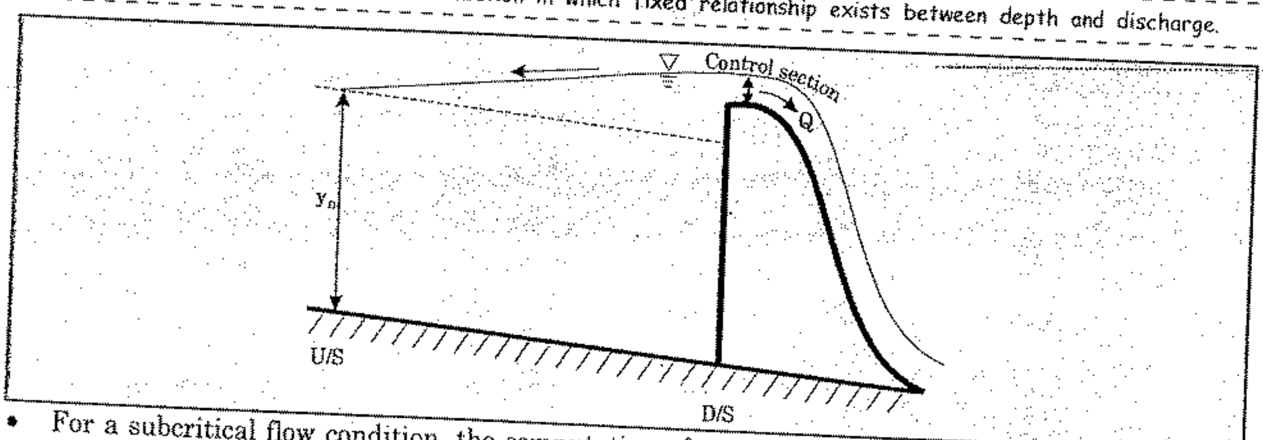
- If $F_r < 1$; Flow is called sub critical or streaming or tranqril.
- If $F_r = 1$; Flow is called critical. At critical flow conditon specific energy is minimum.
- If $F_r > 1$; Flow is called super critical or shooting flow or torrential flow called rapid flow.

Further,

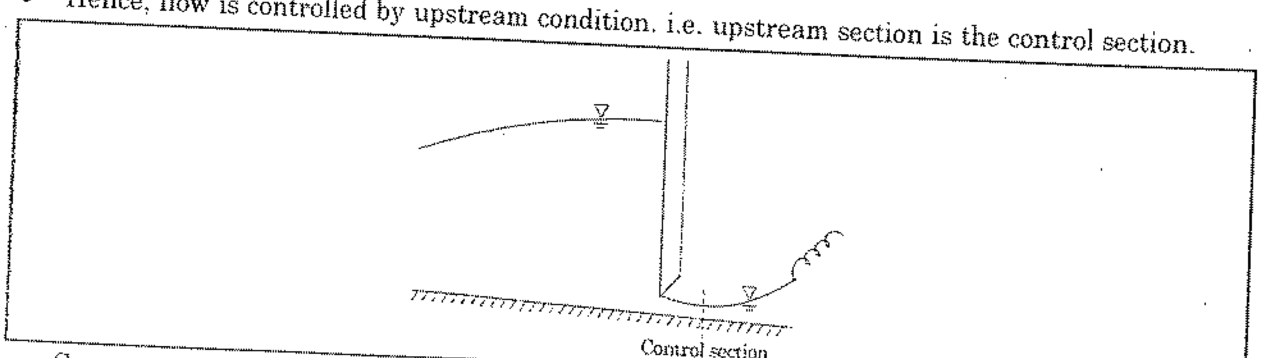
Type of flow	Depth of flow	Velocity of flow	Froude number
Subcritical	$y > y_c$	$v < v_c$	$F_r < 1$
Critical	$y = y_c$	$v = v_c$	$F_r = 1$
Super critical	$y < y_c$	$v > v_c$	$F_r > 1$

- Denominator of Froude number $(\sqrt{gL_c})$ represents the speed at which the disturbance wave travels in still water condition. It is called as celerity C_0.
- At low flow velocity i.e. $F_r < 1$, a small disturbance produced in the stream will travel in upstream direction with a velocity of $(C_0 - v)$ w.r.t. stationary observer on the ground. Where $C_0 = \sqrt{gL_c}$ and v = velocity of stream.
- We can observe that in subcritical flow conditions ($F_r < 1$) upstream conditions are affected by downstream condition. Therefore for a subcritical flow condition, downstream section is a control section.

Note: *Control section is defined as a section in which fixed relationship exists between depth and discharge.*



- For a subcritical flow condition, the computation of water surface profile must start from down stream location and proceed upstream location.
- At high flow velocity i.e. $F_r > 1$, a small disturbance can not travel upstream, as the velocity C_0 being less than v, the disturbance wave is washed downstream with a velocity equal to $v - C_0$ w.r.t to stationary observer on the ground.
- Hence, flow is controlled by upstream condition. i.e. upstream section is the control section.

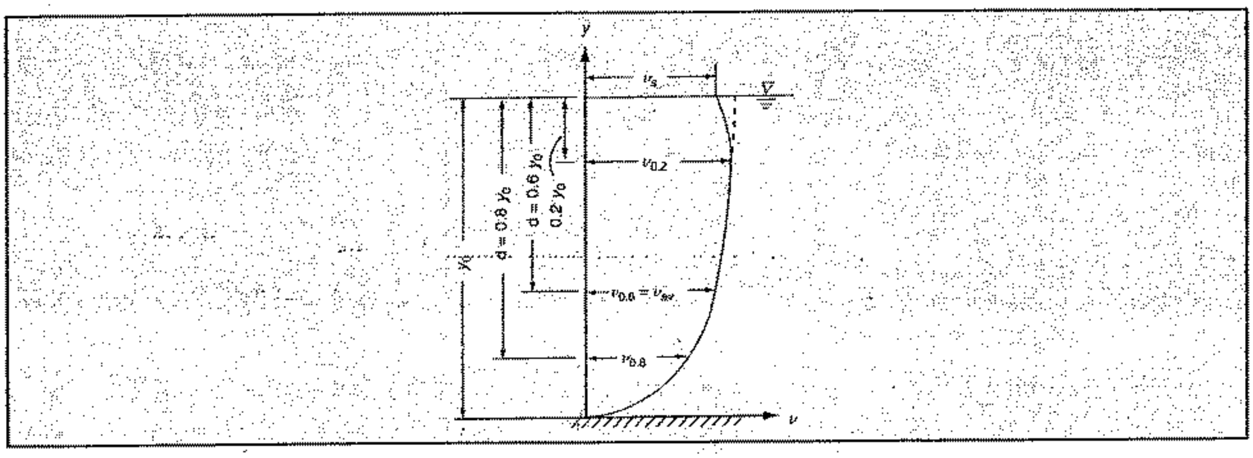


- Computation of water surface profile must start f

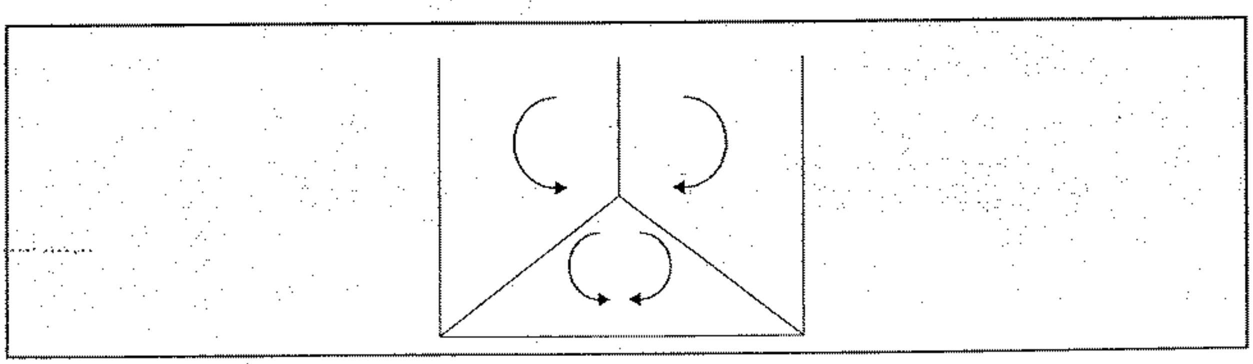
Note: In case of critical condition ($F_r = 1$) $C_0 = v_0$, i.e. velocity of travel of wave $|v - C_0| = 0$, hence the disturbance wave will not travel at all.

VELOCITY DISTRIBUTION

- A typical velocity profile at a section in a plane normal to the direction of flow is as shown below. This profile can be roughly described by a logarithmic distribution or a power law distribution.



- Velocity is zero at the boundaries and gradually increases with the distance from the boundary.
- It has been observed that maximum velocity of flow occurs at a certain distance below the free surface. This reduction is due to the production of secondary current which is a function of aspect ratio (ratio of depth to width.)
- The presence of corners and boundaries in open channel causes the velocity vectors of flow to have components not only in longitudinal and lateral direction but also normal direction of flow.
- This flow in normal direction is secondary flow and produces secondary current.



- From the field observations, it has been observed that average velocity of flow occurs at a depth 0.6 y_0 from the free surface, where y_0 = depth of flow.

$$v_{avg} = v_{0.6} \quad \text{(Less reliable)}$$

Also,
$$v_{avg} = \frac{v_{0.2} + v_{0.8}}{2} \quad \text{(More reliable)}$$

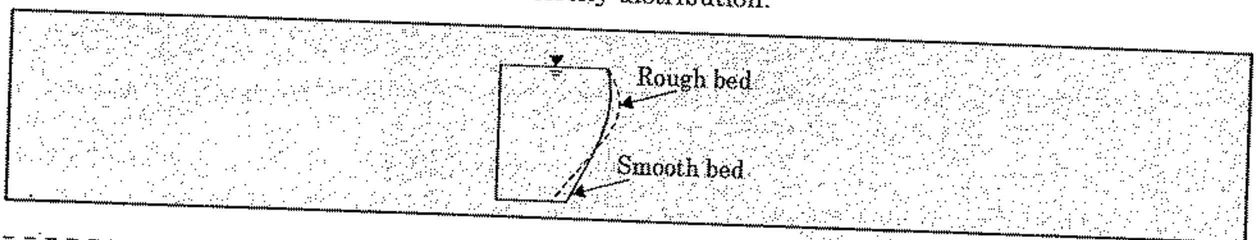
- The surface velocity v_s is related to the average velocity v_{avg} as

$$v_{avg} = Kv_s$$

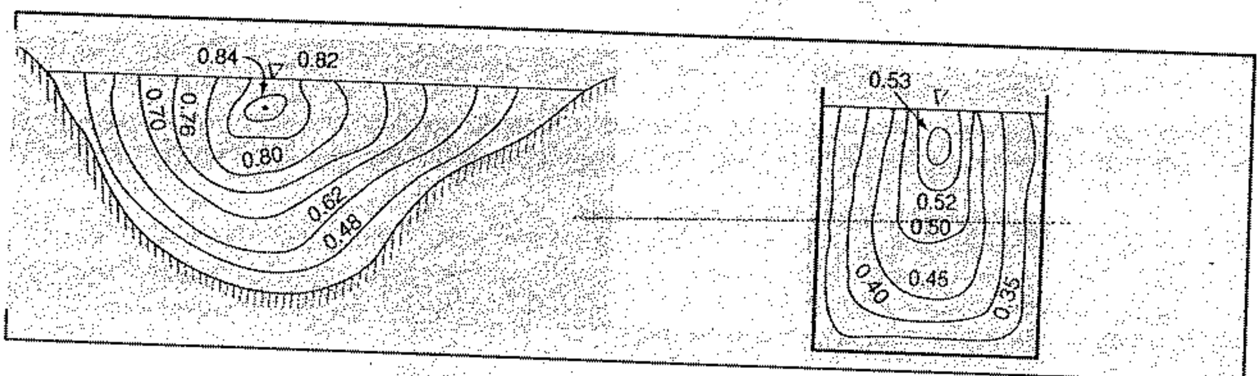
Where,
K = coefficient with a value between 0.8 – 0.95.

SALIENT POINTS

- The velocity distribution in a channel section depends on factors as shape of the section, the roughness of the channel and the pressure of bends.
- In a broad, rapid and shallow stream or in a very smooth channel, the maximum velocity may often be found very near to the free surface.
- The roughness of the channel will cause the curvature of the vertical velocity distribution curve to increase.
- Surface wind has very little effect on velocity distribution.

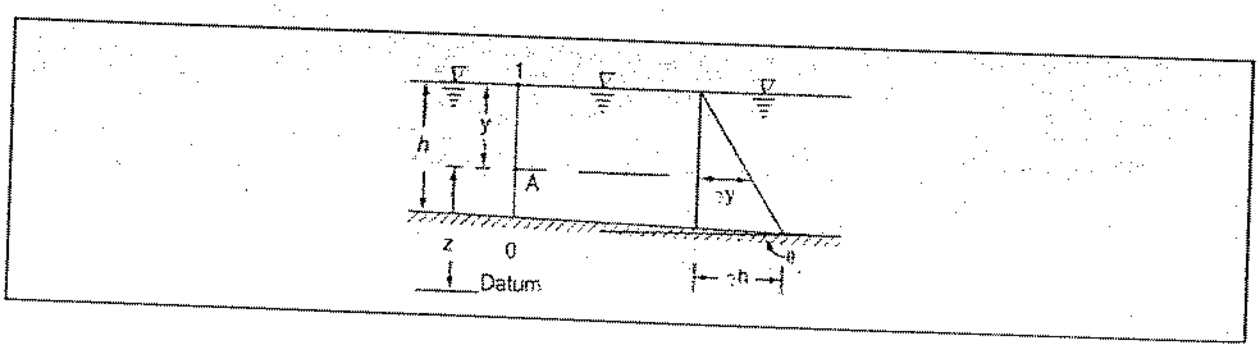


Note: *Contours of equal velocity are called isovels.*



PRESSURE DISTRIBUTION

Channels with Small Slope



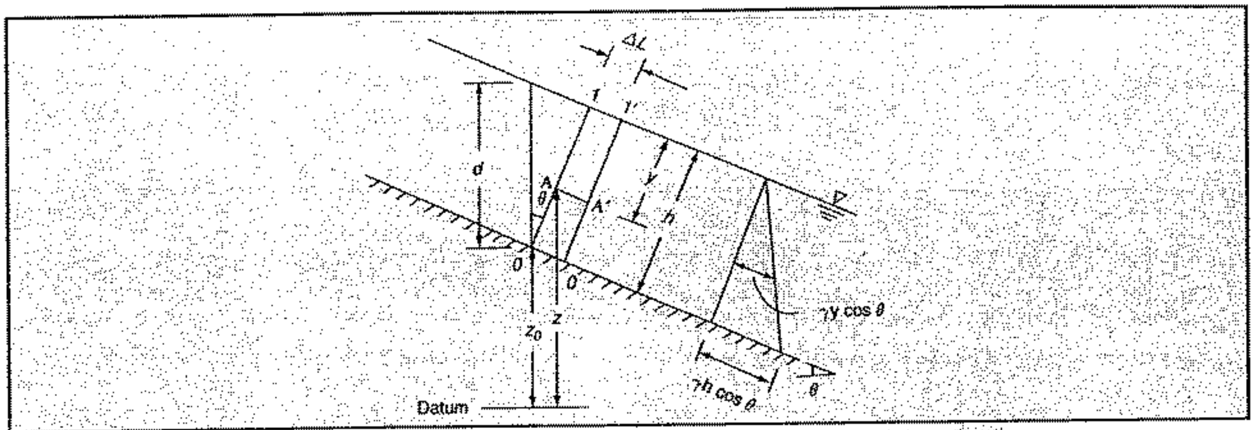
- In a channel with mild/small bed slope, vertical section is practically same as the normal section.
- Therefore, the piezometric head at any point in the channel will be equal to the water surface elevations.

$$P = \gamma y$$

where, y is elevation below water surface.

- Hydraulic grade line will coincide with th

Channels with Large Slope



- In a channel with steep/large bed slope, vertical section is larger than the normal section.
- Therefore, the piezometric head at any point in the channel will be equal to the cosine component of normal depth at any section.

$$p = \gamma\, y \cos\theta$$

Where,
y = normal depth at any section
θ = angle between vertical and normal section.

- Hydraulic gradient line will lie below the water surface.

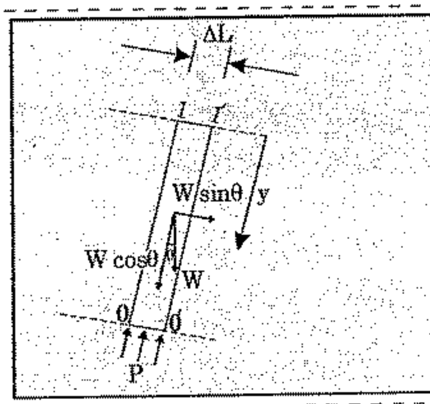
Note: Considering unit width inside

$$\Sigma F_y = 0$$

$$P.\Delta L.1 = W\cos\theta$$

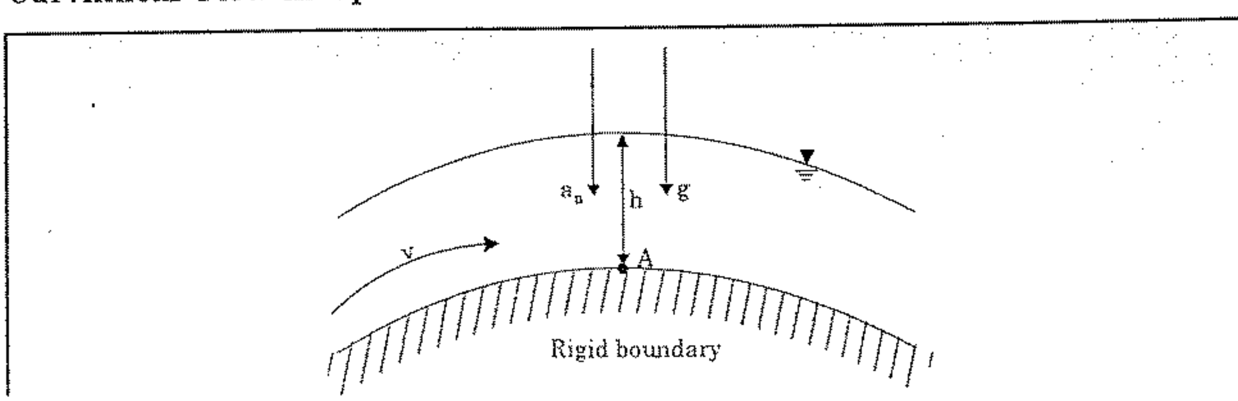
$$P.\Delta L = \Delta L.y.\gamma.\cos\theta$$

$$P = \gamma\, y\cos\theta$$



PRESSURE DISTRIBUTION IN CURVILINEAR FLOWS

Curvilinear Flow in Upward Convex Surface



- We know that , Pressure at Point A

$$\boxed{P_A = \rho\, g_{eff}\, h}$$

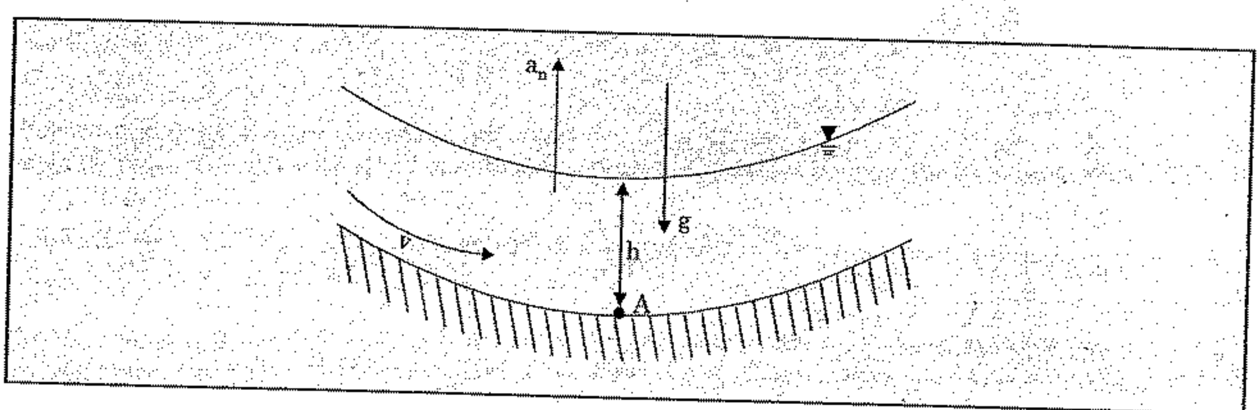
Where,

$g_{eff} = (g - a_n)$

a_n = normal acceleration

- Pressure at point A (i.e bottom most point) is less than hydrostatic pressure.

Curvilinear flow in Upward Concave Surface



- We know that

$$\boxed{P_A = \rho\, g_{eff}\, h}$$

Where,

$g_{eff} = (g + a_n)$

- Pressure at point A is more than the hydrostatic pressure.

CONTINUITY EQUATION IN OPEN CHANNEL FLOW

- Continuity equation is based on law of conservation of mass.

Steady State flow

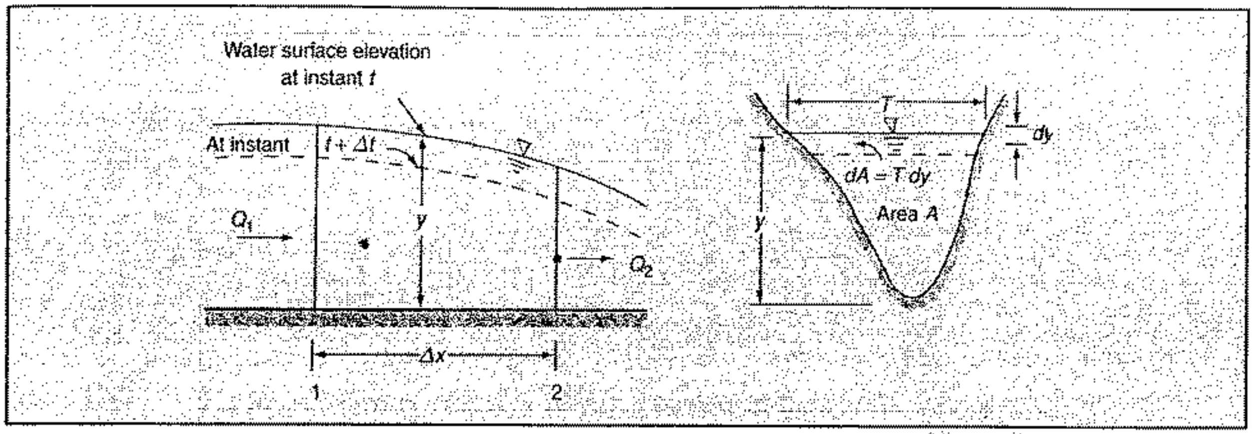
- In a steady state flow of incompressible fluid, the continuity equation states that the volumetric rate of flow (discharge) across various section must be the same.

$$Q = vA = v_1A_1 = v_2A_2$$

Unsteady State flow

- In unsteady state flow of incompressible fluid, the continuity equation states that net discharge geting out of control volume should be equal to depletion of storage in control volume.

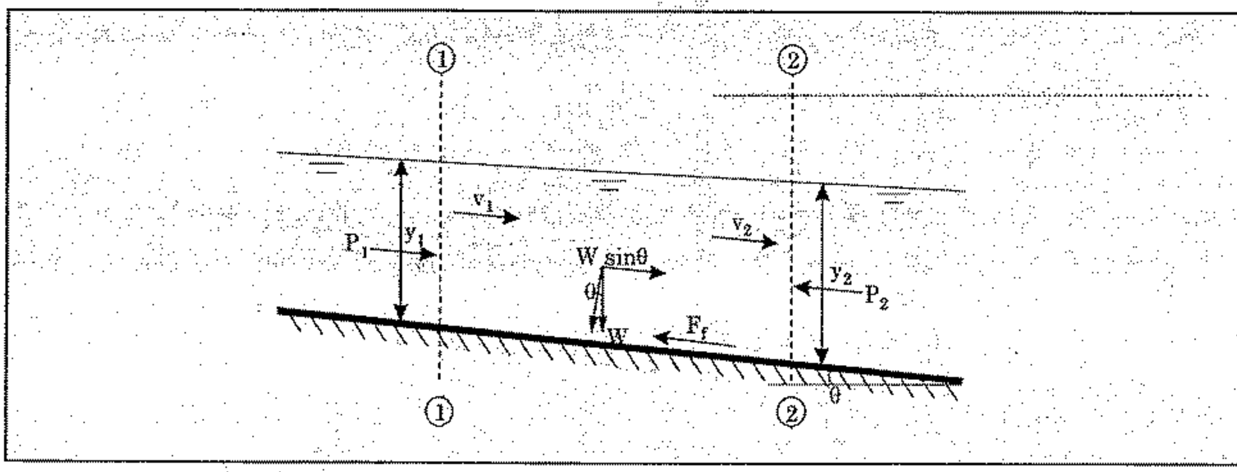
$$\boxed{\frac{\partial Q}{\partial x} = -T\frac{\partial y}{\partial t}}$$



MOMENTUM EQUATION IN OPEN CHANNEL FLOW

- Momentum is a vector quantity. The momentum equation commonly used in open channel flow problems is the linear-momentum equation.
- According to Newton's second law of motion, the rate of change of momentum is equal to the resultant of all the external force acting on the body.

Steady flow



- In steady state, the rate of change of momentum in a given direction will be equal to the net flux of momentum in that direction.
- Expression for momentum change per unit time between two sections of channel can be written as:

$$M_2 - M_1 = P_1 - P_2 + W\sin\theta - F_f$$

$$\rho Q v_1 - \rho Q v_2 = \gamma A_1 \bar{y}_1 - \gamma A_2 \bar{y}_2 + W\sin\theta - F_f$$

$$\rho A_1 v_1^2 - \rho A_2 v_2^2 = \gamma A_1 \bar{y}_1 - \gamma A_2 \bar{y}_2 + W\sin\theta - F_f$$

Where,

M_1 & M_2 = momentum per unit time at section 1 & 2 respectively

P_1 & P_2 = Pressure force acting at section 1 & 2 respectively

F_f = Total external force of friction or resistance acting along the surface of contact between the water and the channel.

W = Weight of water enclosed between two sections.

θ = bed slope

- For parallel or gradually varied flow the values of P in the momentum equation may be computed by assuming a hydrostatic distribution of pressure.
- For curvilinear or rapidly varied flow, the value of P must be corrected for curvature effect of the streamlines of the flow.

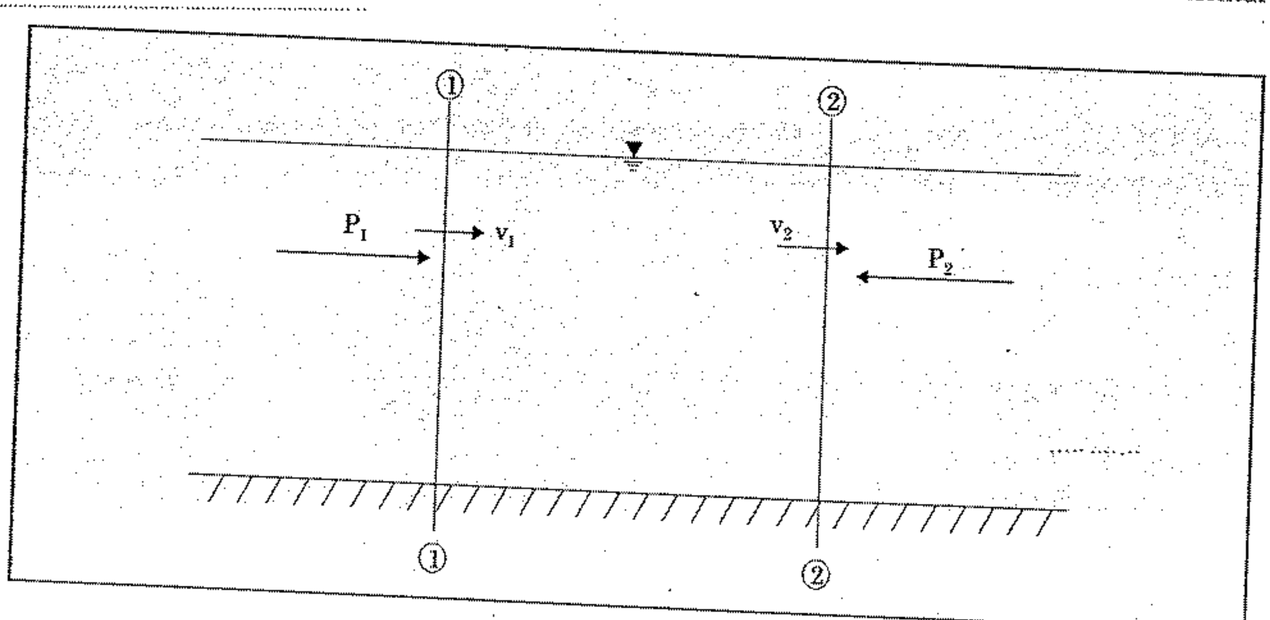
Note: The momentum equation has certain advantages in application to problem involving high internal energy changes such as problem of hydraulic jump. If energy equation is applied to such problems, the unknown internal energy loss is indeterminate. Therefore, momentum equation is applied to these problems since it deals only with external forces.

Un-steady flow

- In unsteady flow, momentum equation states that the algebric sum of all the external forces in a given direction on a fluid mass is equal to the net change of linear momentum in that direction plus the time rate of increase of momentum in that direction within control volume.
- Therefore in unsteady flow, the linear momentum equation will have an additional term over and above the steady flow equation to include rate of change of momentum in control volume.

SPECIFIC FORCE

- Specific force is the sum of pressure force and momentum flux per unit "unit weight" of the fluid at a section.
- The steady state momentum equation takes a simple form, if the channel is assumed to be horizontal and frictionless.



$$M_2 - M_1 = P_1 - P_2$$

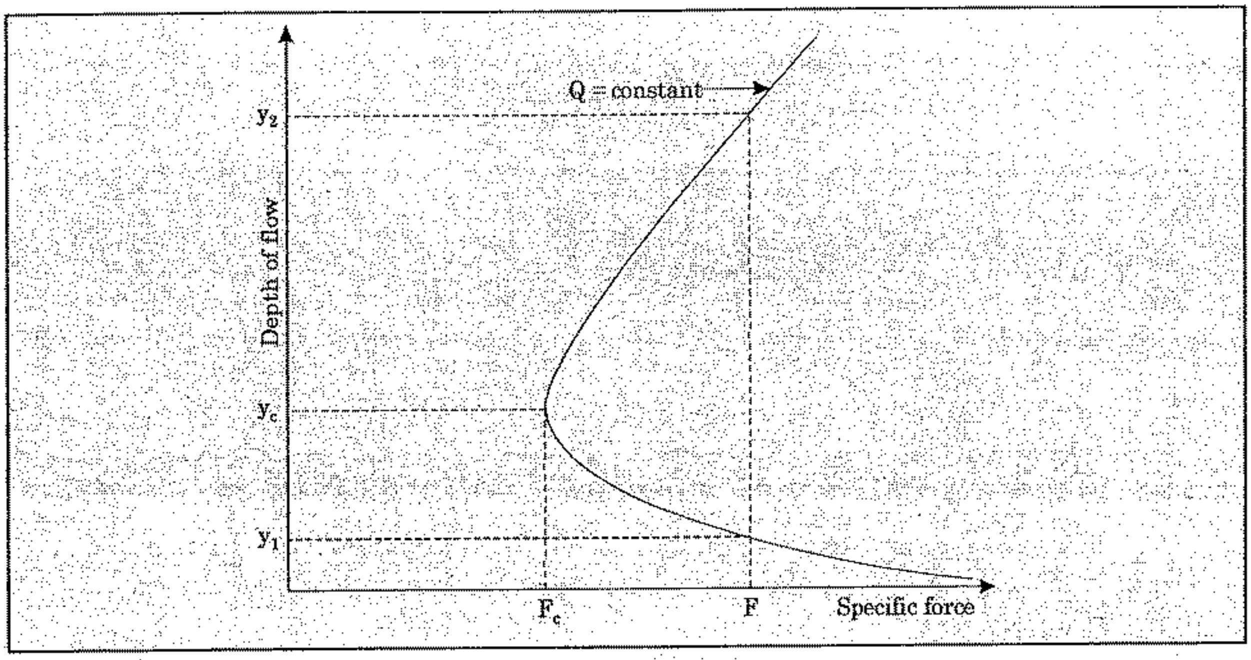
(as channel is assumed frictionless $F_f = 0$ and bed is horizontal $w \sin\theta = 0$)

$$M_1 + P_1 = M_2 + P_2$$

$$\frac{P_1 + M_1}{\gamma} = \frac{P_2 + M_2}{\gamma}$$

$$\boxed{\frac{P + M}{\gamma} = \text{Specific Force}}$$

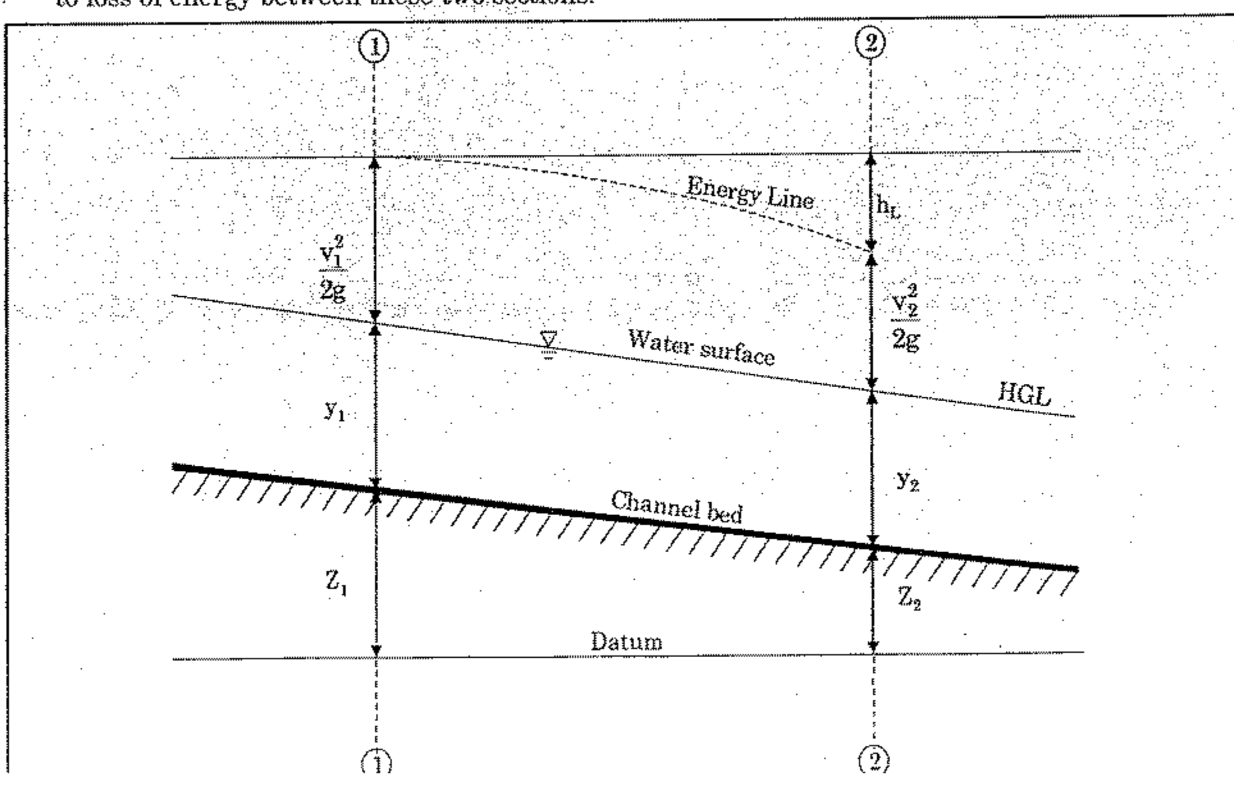
- Specific force for horizontal frictionless channel will be constant.



- The specific force for a given discharge is minimum when the flow is critical ($F_r = 1.0$). At a value of $F > F_c$, there are two depths of flow y_1 and y_2 which yield the same specific force at the given discharge. These depths are called conjugate depths or sequent depths.

ENERGY EQUATION IN OPEN CHANNEL FLOW

- According to energy equation, the difference between the total energy between two sections is equal to loss of energy between these two sections.



We know that,

Total energy at any section = Pressure energy + Kinetic energy + Elevation energy

$$\boxed{\frac{\text{Total energy}}{\text{weight}} = \text{Total energy head} = \frac{P}{\gamma} + \frac{v^2}{2g} + Z}$$

Applying energy equation between section (1) & (2)

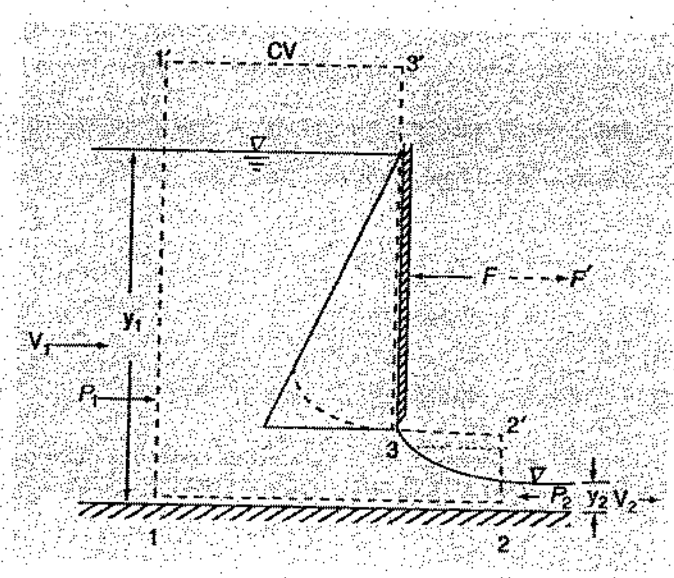
$$\boxed{\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + Z_2 + h_L}$$

h_L = Energy loss between section 1 & 2

Note: The term $\left(\frac{P}{\gamma} + Z\right)$ represents elevation of hydraulic gradient line (HGL) above datum, it is also called Piezometric head.

Example 1

Estimate the force on a sluice gate as shown in the figure below



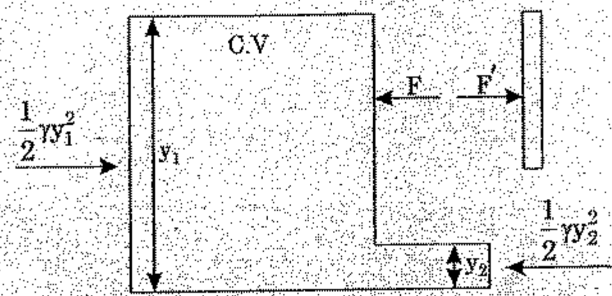
Sol. Consider a unit width of channel. The force exerted by the gate on fluid is F. An equal and opposite force exerted by the fluid on the gate is F'. Let Discharge per unit width across CV is q.

A control volume is selected between section 1 and 2 and a hydrostatic pressure distribution on the either end.

$$P_1 - P_2 - F = M_2 - M_1$$

From momentum equation

$$\frac{1}{2}\gamma y_1^2 - \frac{1}{2}\gamma y_2^2 - F = \rho q(v_2 - v_1)$$



$$\frac{1}{2}\gamma\left[y_1^2-y_2^2\right]-F = \rho q(v_2-v_1) \quad \text{... (1)}$$

From continuity equation

$$q = y_1 v_1 = y_2 v_2$$

$$\therefore \quad v_1 = \frac{q}{y_1}, v_2 = \frac{q}{y_2}$$

Substituting, $v_1 = \frac{q}{y_1}$ and $v_2 = \frac{q}{y_2}$ in equation (1)

$$\frac{1}{2}\gamma[y_1^2-y_2^2]-F = \rho q\left(\frac{q}{y_2}-\frac{q}{y_1}\right)$$

$$F = \frac{1}{2}\gamma\left[y_1^2-y_2^2\right]-\rho q^2\left[\frac{1}{y_2}-\frac{1}{y_1}\right]$$

$$F = \frac{1}{2}\gamma\left(\frac{y_1-y_2}{y_1\ y_2}\right)\left[y_1y_2(y_1+y_2)-\frac{2q^2}{g}\right] \quad \text{... (2)}$$

If the loss of energy between section 1 and 2 is assumed to be negligible, then from the energy equation

$$y_1+\frac{v_1^2}{2g} = y_2+\frac{v_2^2}{2g} \qquad \left[v=\frac{q}{y}\right]$$

$$y_1+\frac{q^2}{2gy_1^2} = y_2+\frac{q^2}{2gy_2^2}$$

$$(y_1-y_2) = \frac{q^2}{2g}\left[\frac{1}{y_2^2}-\frac{1}{y_1^2}\right]$$

$$\frac{q^2}{2g} = \frac{y_1^2\ y_2^2}{(y_1+y_2)}$$

$$\frac{2q^2}{g} = \frac{4y_1^2\ y_2^2}{(y_1+y_2)}$$

Substituting, $\frac{2q^2}{g} = \frac{4y_1^2\ y_2^2}{(y_1+y_2)}$ in equation (2)

$$F = \frac{1}{2}\gamma\left(\frac{y_1 - y_2}{y_1 y_2}\right)\left[y_1 y_2 (y_1 + y_2) - \frac{4y_1^2 y_2^2}{(y_1 + y_2)}\right]$$

$$F = \frac{1}{2}\gamma\frac{(y_1 - y_2)}{y_1 y_2} y_1 y_2\left[(y_1 + y_2) - \frac{4y_1 y_2}{y_1 + y_2}\right]$$

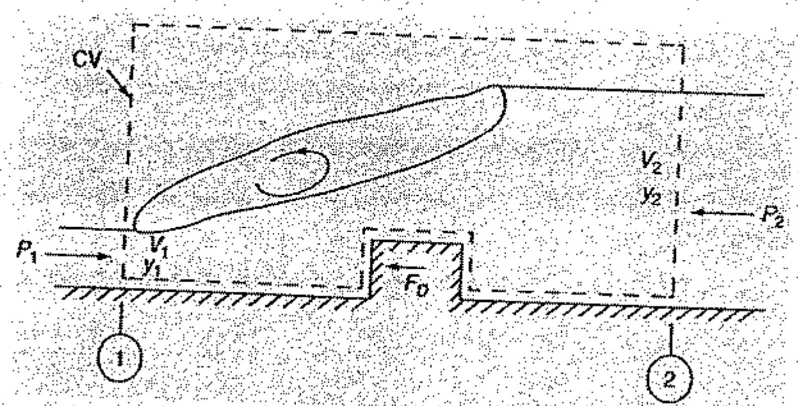
$$F = \frac{1}{2}\gamma(y_1 - y_2)\left[\frac{(y_1 + y_2)^2 - 4y_1 y_2}{y_1 + y_2}\right]$$

$$\boxed{F = \frac{1}{2}\gamma\frac{(y_1 - y_2)^3}{y_1 + y_2}}\ *$$

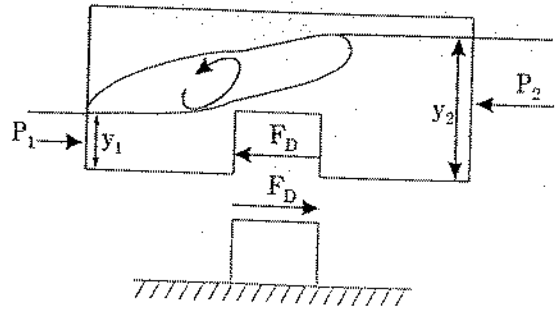
* *This is a standard result and need to be remembered.*

Example 2

Figure below shows a hydraulic jump in an apron aided by a two dimensional block on the apron. Obtain an expression for the drag force per unit length of the block.



Sol. Consider a control volume surrounding the figure as shown in the figure. A unit width of apron is considered. Let the drag force on the block is equal to F_D acting in upstream direction. Assume a frictionless, horizontal channel and hydrostatic pressure distribution at section 1 and 2.



From momentum equation

$$P_1 - P_2 - F_D = M_2 - M_1$$

$$\frac{1}{2}\gamma y_1^2 - \frac{1}{2}\gamma y_2^2 - F_D = \rho q(v_2 - v_1) \quad \ldots (1)$$

From continuity equation

$$q = v_1 y_1 = v_2 y_2$$

$$v_1 = \frac{q}{y_1}, \quad v_2 = \frac{q}{y_2}$$

Substituting v_1 and v_2 in equation (1)

$$\frac{1}{2}\gamma y_1^2 - \frac{1}{2}\gamma y_2^2 - F_D = \rho q^2\left[\frac{1}{y_2} - \frac{1}{y_1}\right]$$

$$F_D = \frac{\gamma}{2}\left[\left(y_1^2 - y_2^2\right) - \frac{2q^2}{g}\left(\frac{y_1 - y_2}{y_1 y_2}\right)\right]$$

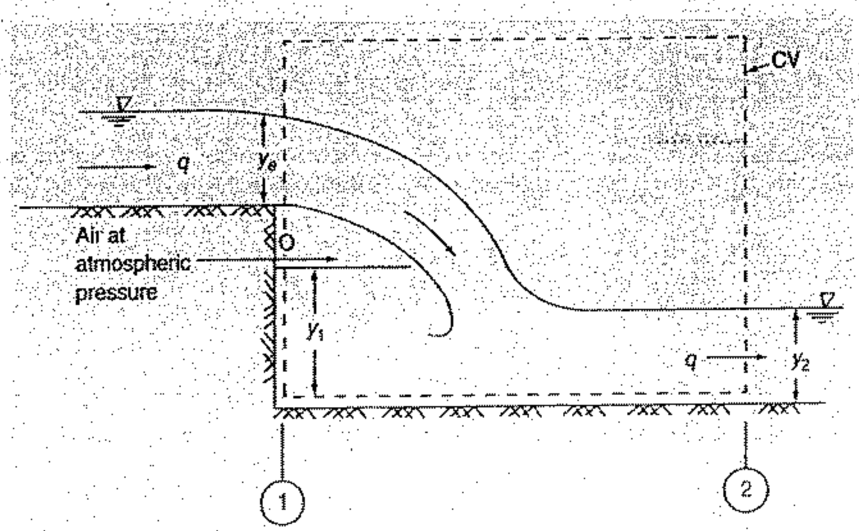
Example 3

Figure below, shows a free overfall at the end of a horizontal, rectangular and frictionless prismatic channel. The space below the lower nappe is fully ventilated. It can be assumed that the water leaves the brink horizontally at a brink depth of y_e. Considering the control volume shown in the figure, show that the back-up depth of water y_1 below the nappe is given by

$$\left[\frac{y_1}{y_2}\right]^2 = 1 + 2F_2^2\left[1 - \frac{y_2}{y_e}\right]$$

where, $F_2 = \dfrac{q}{y_2\sqrt{g y_2}}$

and q = discharge per unit width of the channel.



Sol. Consider a control volume CV as shown in the figure above. Let the discharge per unit width be q. Pressure distribution throughout y_e depth is atmospheric because the flow sheet after going

Applying Momentum equation.

$$P_1 - P_2 = M_2 - M_1$$

$$\frac{1}{2}\gamma y_1^2 - \frac{1}{2}\gamma y_2^2 = \rho q v_2 - \rho q v_1$$

$$\frac{1}{2}\gamma y_1^2 - \frac{1}{2}\gamma y_2^2 = \rho q\left(\frac{q}{y_2}\right) - \rho q\left(\frac{q}{y_e}\right)$$

$$\frac{\gamma}{2}\left[y_1^2 - y_2^2\right] = \rho q^2\left(\frac{1}{y_2} - \frac{1}{y_e}\right)$$

$$\frac{\gamma}{2}\left[\frac{y_1^2}{y_2^2} - 1\right] = \frac{\rho q^2}{y_2^2}\left(\frac{1}{y_2} - \frac{1}{y_e}\right)$$

$$\left(\frac{y_1}{y_2}\right)^2 = \frac{2\rho q^2}{\gamma y_2^3}\left(1 - \frac{y_2}{y_e}\right) + 1$$

$$\left(\frac{y_1}{y_2}\right)^2 = \frac{2q^2}{g y_2^3}\left(1 - \frac{y_2}{y_e}\right) + 1$$

$$\left(\frac{y_1}{y_2}\right)^2 = \frac{2v_2^2}{g y_2}\left(\frac{1 - y_2}{y_e}\right) + 1 \qquad \left[\because F_2 = \frac{v_2}{\sqrt{g y_2}}\right]$$

$$\left(\frac{y_1}{y_2}\right)^2 = 1 + 2F_2^2\left(1 - \frac{y_2}{y_e}\right)$$

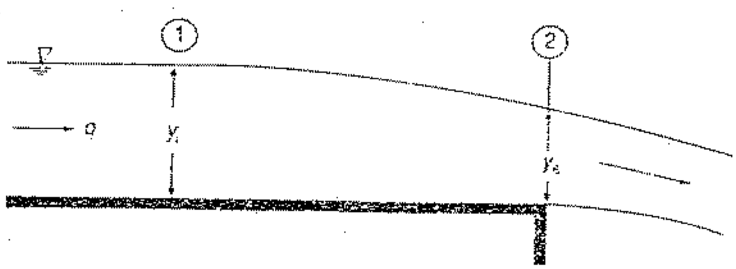
Example 4

Figure below shows a a free overfall in a horizontal frictionless rectangular channel. Assuming the flow to be horizontal at Section 1 and the pressure at the brink of Section 2 to be atmospheric throughout the depth, show that

$$\frac{y_e}{y_0} = \frac{2F_0^2}{\left(2F_0^2 + 1\right)}$$

where,
$$F_0^2 = \frac{q^2}{g y_0^3}$$

and
$$q = \text{discharge per unit width.}$$



Sol. Applying Momentum equation between section 1 and 2

$$P_1 - P_2 = M_2 - M_1$$

$$P_2 = 0$$

$$\frac{\gamma y_0^2}{2} - 0 = \rho q(v_2 - v_1)$$

$$\frac{\gamma y_0^2}{2} = \rho q\left(\frac{q}{y_e} - \frac{q}{y_0}\right)$$

$$y_0^2 = \frac{2\rho q^2}{\gamma}\left(\frac{1}{y_e} - \frac{1}{y_0}\right)$$

$$\frac{2\rho q^2}{\gamma y_0^3}\left(\frac{y_0}{y_e} - 1\right) = 1$$

$$\frac{2q^2}{g y_0^3}\left(\frac{y_0}{y_e} - 1\right) = 1$$

$$2F_0^2\left(\frac{y_0}{y_e} - 1\right) = 1$$

$$\frac{y_0}{y_e} = \frac{1}{2F_0^2} + 1$$

$$\frac{y_0}{y_e} = \frac{1 + 2F_0^2}{2F_0^2}$$

$$\frac{y_e}{y_0} = \frac{2F_0^2}{1 + 2F_0^2}$$

OBJECTIVE QUESTIONS

1. The ripples formed on the water surface by dropping a stone in open channel indicate the type of flow. The flow will be sub-critical when
   (a) the ripples are swept away downstream
   (b) the ripples travel sideways only
   (c) ripple are not formed
   (d) the ripples move in upstream and downstream directions

2. Match **List-I** (Practical case) with **List-II** (Type of flow) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Canal flow	1. Steady and non-uniform (Gradually varied) flow
B. River flow in alluvial reach during rising flood	2. Steady and non-uniform (Rapidly varied) flow
C. Flow in river upstream of a weir during winter season	3. Steady uniform flow
D. Flow downstream of an overflow spillway	4. Unsteady and non-uniform (Gradually varied) flow

Codes:

	A	B	C	D
(a)	1	2	3	4
(b)	3	4	1	2
(c)	3	2	1	4
(d)	1	4	3	2

3. A person standing on the bank of a canal drops a stone on the water surface. He notices that the disturbance on the water surface is not travelling upstream. This is because the flow in the canal is
   (a) sub-critical
   (b) super-critical
   (c) steady
   (d) uniform

4. Under which of the following conditions steady non-uniform flow in open channels occurs?
   (a) When for a constant discharge the liquid depth in the channel varies along its length
   (b) When a constant discharge flows at the constant depth
   (c) When a constant discharge flows in a channel laid at a fixed slope
   (d) When the discharge and the depth both vary along the channel length

5. When the depth of flow changes gradually over a length of the channel, then the flow will be termed as
   (a) Rapidly varied flow
   (b) Critical flow
   (c) Gradually varied flow
   (d) Uniform flow

6. Non-uniform flow may caused by
   (a) The change in width, depth, bed slope etc. of the channel
   (b) An obstruction, across a channel of uniform width
   (c) None of the above
   (d) Both (a) and (b)

7. The phenomenon occuring in an open channel when a rapidly flowing stream abruptly changes to slowly flowing stream causing a distinct rise of liquid surface, is
   (a) Uniform flow
   (b) Critical discharge
   (c) Hydraulic jump
   (d) None of the above

8. The channel whose boundary is not deformable is known as
   (a) Rigid channel
   (b) Prismatic channel
   (c) Mobile channel
   (d) Boundary channel

9. Steady flow in an open channel exists when the
   (a) flow is uniform
   (b) channel is frictionless
   (c) depth does not change with time
   (d) channel bed is not curved

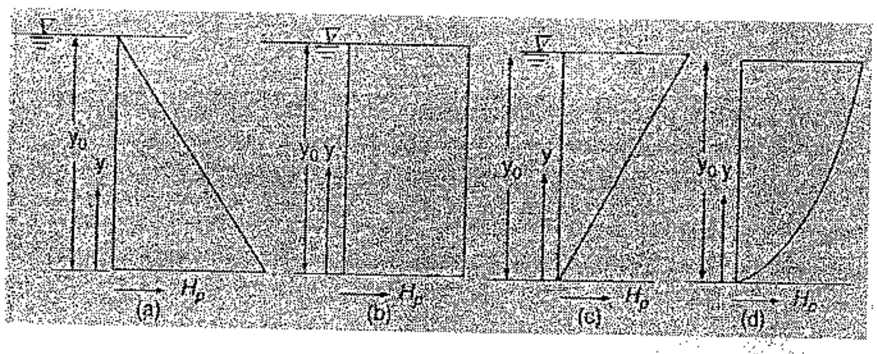
10. In a steady spatially-varied flow in a prismatic open channel, the
   (a) depth does not change along the channel length
   (b) discharge is constant along its length
   (c) discharge varies along the length of channel
   (d) discharge varies with respect to time

11. A flood wave while passing down a river section protected by embankments, spills over the embankment at certain locations. The flow is classified as
   (a) steady GVF
   (b) steady SVF
   (c) unsteady RVF
   (d) unsteady SVF

12. In the uniform flow in a channel of small bed slope, the hydraulic grade line
   (a) coincides with the the bed
   (b) is considerably below the free surface
   (c) is considerably above the free surface
   (d) coincides with the free surface

13. A uniform flow takes place in a steep channel of large slope. The hydraulic gradient line
   (a) coincides with the bed
   (b) essentially coincides with the free surface
   (c) is above the free surface
   (d) is below the free surface

14. A channel with very small value of logitudinal slope S_0 has its water surface parallel to its bed. With the channle bed as the datum, the variation of the piezometric head H_p with distance above the bed y in this channel can be represented by the following:



15. The velocity and depth of flow in a 3.0 m wide rectangular channel are 2.0m/s and 2.5 m, respectively. If the channel has its width enlarged to 3.5 m at a section, the discharge past that section is
(a) 10.0 m^3/s
(b) 20.0 m^3/s
(c) 15.0 m^3/s
(d) 17.5 m^3/s

16. For an open channel flow to take place between two sections,
(a) the channel bed must always slope in the direction of the flow
(b) the upstream depth must be larger than th downstream depth
(c) the unpstream momentum must be larger than the downstream momentum
(d) the total energy at the upstream end must be larger than the total energy at the down-stream section.

17. The total energy head for an open channel flow is written with usual notaion as $H = z + y + V^2/2g$. In this each of the terms represent
(a) energy in Kg m/kg mass of fluid
(b) energy in N m/N of fluid
(c) power in kW/kg mass of fluid
(d) energy in N m/ mass of fluid

18. Piezometric head is the sum of
(a) pressure head, datum head and velocity head
(b) datum head and velocity head
(c) pressure head and velocity head
(d) pressure head and datum head

19. The difference between total head line and piezometric head line represents
(a) the velocity head
(b) the pressure head
(c) the elevation of the bed of the channel
(d) the depth of flow

20. The momentum equation in x-direction as $\Sigma F_x = \rho Q_1(V_{x2} - V_{x1})$ has the assumption that the flow is
    (a) steady
    (b) uniform
    (c) unsteady
    (d) frictionless

21. Normally in a stream the ratio of the surface velocity at a location to the average velocity in the vertical through that location
    (a) is greater than 1.0
    (b) will be between 0.8 and 0.95
    (c) is less than or greater than unity depending on the type of flow
    (d) is equal to 0.6

22. If a sluice gate produces a change in the depth of water from 3.0 m to 0.6 m, then the force on the gate is about
    (a) 9.5 kN/m (b) 19.0 kN/m
    (c) 38.0 kN/m (d) 76.0 kN/m

23. The specific force is constant
    (a) in all frictionless channels irrespective of the magnitude of the longitudinal slope
    (b) in horizontal, frictionless channels of any shape
    (c) in all horizontal channels of any shape
    (d) in any open channels

ANSWERS

1.	(d)	2.	(b)	3.	(b)	4.	(a)	5.	(c)	6.	(d)	7.	(c)
8.	(a)	9.	(c)	10.	(c)	11.	(d)	12.	(d)	13.	(d)	14.	(b)
15.	(c)	16.	(d)	17.	(b)	18.	(d)	19.	(a)	20.	(a)	21.	(a)
22.	(b)	23.	(b)										

HINT

1. For a subcritical flow disturbance will travel upstream and down stream also.
3. For a supercritical flow disturbance will not travel upstream as the velocity of disturbance wave will be less than the velocity of flow and hence washed off.
4. For a steady state condition discharge should be constant and for non-uniform flow condition depth of flow should vary along the channel length.
5. For a Gradually varied flow depth of flow varies gradually along the channel length.
6. Change in width, depth and bed slope will cause change in depth of flow. Also a obstruction (*eg.*

7. When a rapidly flowing stream abruptly changes to a slowly flowing stream it results into a local phenomenon called Hydraulic jump.

8. For rigid channel, channel boundaries are non deformable.

9. In case of steady state flow, flow properties does not change with time.

10. For a steady state spatially varied flow discharge varies along the length of channel.

12. In the uniform flow in a channel of small bed slope HGL coincides with the free surface.

13. In the uniform flow in a channel of steep slope HGL is below the free surface.

14. For a channel will small bed slope, piezometric head is constant through out the cross section.

15. Discharge will be constant, from continuity equation

$$Q = A_1V_1 = A_2\, V_2$$

$$Q = 3 \times 2.5 \times 2.0 = 15\text{m}^3/\text{sec}$$

16. Flow always takes place from higher total energy to lower total energy.

17. These energy are in the form of energy per unit weight.

18. Piezometric head is a sum of datum head + pressure head.

19. Velocity head $= \left(\frac{P}{\gamma} + \frac{v^2}{2g} + Z\right) - \left(\frac{P}{\gamma} + Z\right)$

21. $v_{avg} = kv_s$

where, $k = 0.8 - 0.95$

$$\frac{v_s}{v_{avg}} = \frac{1}{k} > 1.0$$

22. $F = \frac{1}{2}\gamma\frac{(y_1 - y_2)^3}{y_1 + y_2} = \frac{1}{2} \times 9.81 \times \frac{(3 - 0.6)^3}{3 + 0.6} = 18.835 \text{ kN/m}$

23. For a horizontal frictionless channel of any shape specific force is constant.

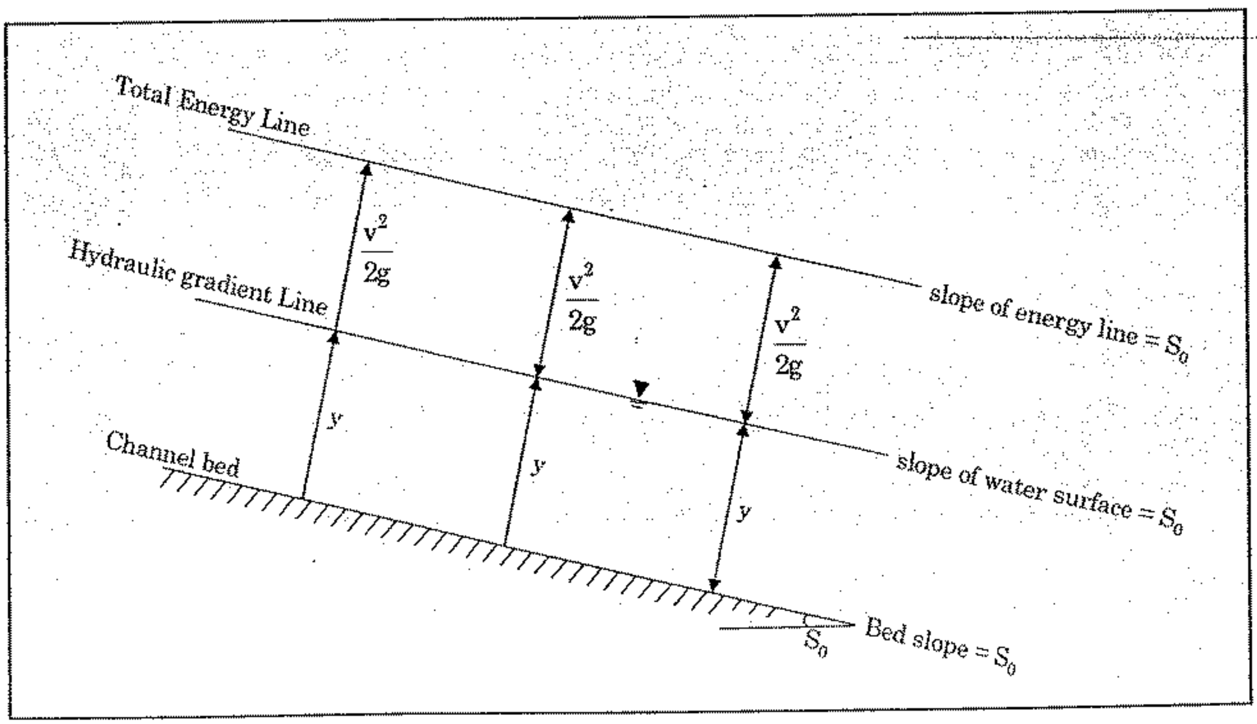
CHAPTER

2

Uniform Flow

INTRODUCTION

- A flow in open channel is said to be uniform flow if its properties remains constant with respect to distance. i.e depth of flow, area of cross section and velocity of flow remains constant along the channel.
- This constant depth of flow in uniform flow is called Normal depth.
- As the depth of flow and velocity at every section are constant therefore the channel bed slope, water surface slope and energy line slope will all be same.



- A prismatic channel carrying certain discharge with constant velocity is an example of uniform flow.
- In uniform flow, the frictional resistance acting between the fluid and channel boundary are balanced by the gravity forces.

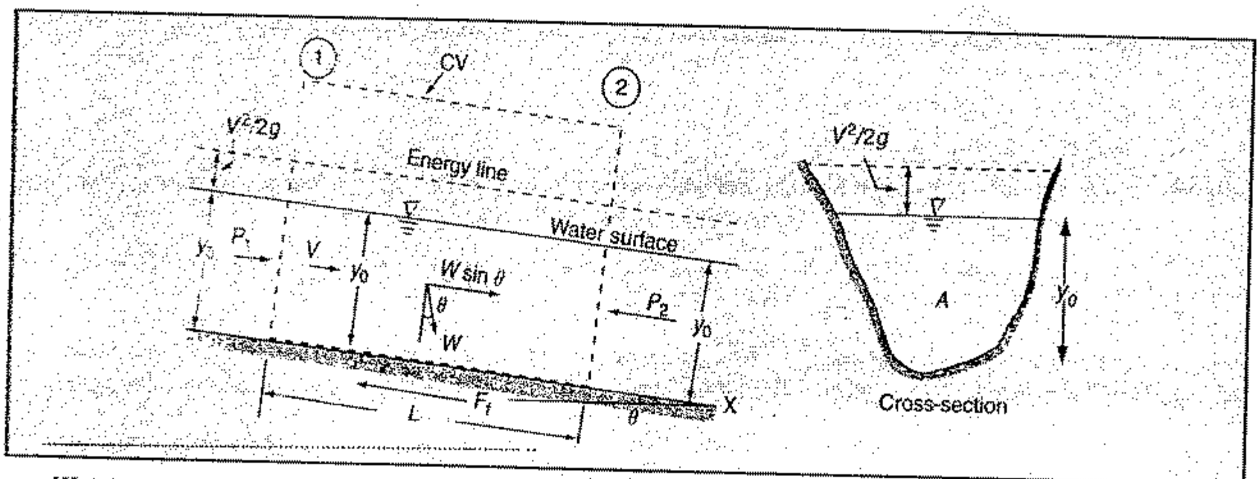
Note: In any type of flow the depth of flow corresponding to the slope of bed will be the Normal depth.

VELOCITY MEASUREMENT

- In uniform flow since the velocity of flow does not change along the length of the channel, acceleration is zero. Hence, the sum of the components of all the external forces in the direction of flow must be equal to zero.
- For practical purposes, the flow in a natural channel may be assumed uniform under normal conditions, i.e., if there are no flood flows or markedly varied flow caused by channel irregularities.

Chezy's Equation

Momentum Equation for Uniform flow.



- Writing momentum equation between section 1 & 2, which are L-distance apart.

$$\boxed{P_1 + W\sin\theta - F_f - P_2 = M_2 - M_1}$$

- As flow is uniform, we know that,

$$P_1 = P_2 \text{ and } M_1 = M_2$$

Also,

$$W = \gamma A\ L\sin\theta \text{ and } F_f = \tau_0 PL$$

Where,

A = Cross section area of channel (constant)

γ = Unit weight of water

L = Length between two section (1) & (2)

τ_0 = Average shear stress on the wetted perimeter.

P = Wetted perimeter of the channel section.

- Now as $P_1 = P_2$ & $M_1 = M_2$, momentum equation can be written as:

$$\gamma AL\ \text{Sin}\theta - \tau_0\ PL = 0$$

$$\boxed{\gamma AL\ \text{Sin}\theta = \tau_0\ PL}$$

- It should be noted that from the above equation in uniform flow, frictional forces are balanced by gravity forces. For small value of q i.e. for mild slope channel ($\theta \to 0°$) $\sin\theta \approx \tan\theta$ and we know that, $\tan\theta = S_0$ (bed slope).

$$\gamma AL\ S_0 = \tau_0\ PL$$

$$\tau_0 = \gamma \frac{A}{P} S_0$$

As, $\frac{A}{P} = R$, Hydraulic radius of channel section

$$\boxed{\tau_0 = \gamma R S_0}$$

- τ_0 is the average shear stress on the wetted area under uniform flow condition.
- From the various experiments it has been observed that wall shear stress is proportional to the dynamic pressure, $\left(\frac{\rho v^2}{2}\right)$ and is independent of viscosity.

$$\tau_0 \propto \frac{\rho v^2}{2}$$

$$\tau_0 = K\rho \frac{v^2}{2}$$

Where,

K = Constant dependent on the roughness of the channel.

ρ = Density of water

v = Velocity of flow

$$\therefore \quad K\rho \frac{v^2}{2} = \gamma RS$$

$$v^2 = \frac{2\gamma}{\rho K} RS \qquad \left[\text{As, } \frac{2\gamma}{\rho K} = \text{Constant}\right]$$

$$\boxed{v = C\sqrt{RS}}$$

- The constant $C = \sqrt{\frac{2\gamma}{\rho K}}$ is termed as Chezy's coefficient and the above equation is called as Chezy's equation i.e. $v = C\sqrt{RS}$
  where,
  - v = Average velocity
  - R = Hydraulic radius of the channel section
  - S = Bed slope of the channel
  - C = Constant, depends on the nature of flow and nature of surface.

Note: In uniform flow, we know that slope of bed = slope of energy line

Manning's Equation

- In 1889, Robert Manning proposed a formula for determination of average velocity of flow, it is the most widely used formula for uniform flow computations in open channels.

$$\boxed{v = \frac{1}{n} R^{2/3} S_0^{1/2}}$$

where,

v = Average velocity of flow.

n = Rough ness coefficient which is a function of the nature of boundary surface, popularly knows as Manning's n.

R = Hydraulic radius of channel section

S = Slope of energy line.

Surface Characteristics	Range of n
Lined Canals	
Cement plaster	0.011
Untreated gunite	0.016
Wood, planed	0.012
Wood, unplaned	0.013
Concrete, trowled	0.012
Concrete, wood forms, unfinished	0.015
Rubble in cement	0.020
Asphalt, smooth	0.013
Asphalt, rough	0.016
Natural Channels	
Gravel beds, straight	0.025
Gravel beds plus large boulders	0.040
Earth, straight with some grass	0.026
Earth, winding, no vegetation	0.030
Earth, winding with vegetation	0.050

Note: There are many empirical formulae presented to estimate Manning's n. In these empirical formulae n is related to bed particle size.

Strickler Formula $n = \frac{d_{50}^{1/6}}{21.10}$; where d_{50} (meter) is the particle size in which 50 percent of the bed material is finer.

Meyer's Formula $n = \frac{d_{90}^{1/6}}{26}$; where d_{90} (meter) is the particle size in which 90 percent of the bed material is finer. This equation is useful in predicting n in mountain stream paved with course gravel and cobbles.

RELATION BETWEEN CHEZY'S CONSTANT AND FRICTION FACTOR

- We know that

$$v = C\sqrt{RS}$$

- Hence S is the slope of energy line, for uniform flow in open channel

$$\text{Slope of Energy Line} = S = \frac{h_f}{L}$$

- From the Darcy's weisbach equation

$$h_f = f\frac{L}{D}\frac{v^2}{2g}$$

Where,
h_f = head loss due to friction in a pipe of diameter D and length l.
f = Darcy's weisbach friction factor.

Note: The above formula of head loss was mainly given for pipe flow, but an open channel flow can be related to the pipe flow at atmospheric pressure.

For a circular pipe of diameter D

$$R = \frac{A}{P} = \frac{\frac{\pi}{4}D^2}{\pi D}$$

$$R = \frac{D}{4}$$

$$\therefore \quad h_f = f\frac{L}{4R}\frac{v^2}{2g}$$

$$\therefore \quad v = \sqrt{\frac{8g}{f}}\sqrt{R}\sqrt{\frac{h_f}{L}} \quad \text{...(A)}$$

From Chezy's Equation

$$v = C\sqrt{R}\sqrt{\frac{h_f}{L}} \quad \text{...(B)}$$

By comparing both the above equation (A) & (B) we can write that

$$C = \sqrt{\frac{8g}{f}}$$

- Also comparing Mannings formula with Chezy's equation

$$v = \frac{1}{n}R^{2/3}S^{1/2} = C\sqrt{RS}$$

$$C = \frac{1}{n}R^{1/6}$$

$$C = \sqrt{\frac{8g}{f}}$$

$$\frac{1}{n}R^{1/6} = \sqrt{\frac{8g}{f}}$$

$$f = \left(\frac{n^2}{R^{1/3}}\right)(8g)$$

Note: *Mannings formula is most widely used formula for practical purpose but it does not give better results in situations where Reynold number effect is predominant.*

ECONOMICAL CHANNEL SECTION

- A section is said to be economical when its construction cost is minimum for a given discharge.
- A section is said to be most efficient, if for a given cross sectional area, the discharge carying capacity is maximum.
- As we know that, the highest component of total construction cost in a channel section is cost of lining and if the perimeter is kept minimum the cost of lining will be min., hence it will be most economical section.
- For a maximum discharge for a given area A, perimeter should be minimum As we can observe that Q is inversly proportional to $P^{2/3}$.

  From Manning's Equations

$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

$$Q = \frac{1}{n} A \left(\frac{A}{P}\right)^{2/3} S^{1/2}$$

$$Q = \frac{1}{n} \frac{A^{5/3}}{P^{2/3}} S^{1/2}$$

$$\boxed{Q \propto \frac{1}{P^{2/3}}}$$

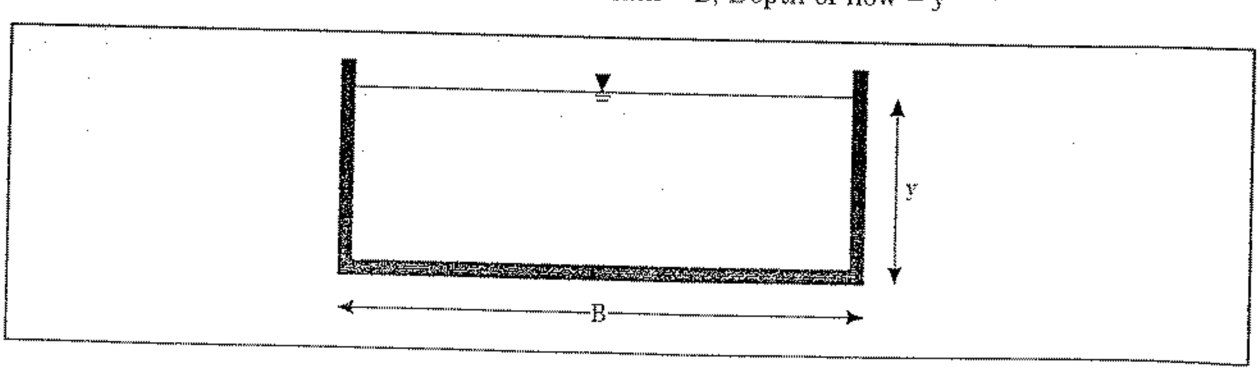
- Also if perimeter is minimum resistance offered by the perimeter will be minimum hence discharge will be maximum.
- Therefore for most economical and most efficient channel section perimeter should be minimum.

Note: *Out of all the channel sections, semicircular shape has the least amount of perimeter for a given area.*

- Relationship between various geometric elements to form an efficient channel section can be obtained as follows.

Most Efficient Rectangular Section

- Consider a rectangular channel section of Width = B, Depth of flow = y



$$\text{Area, } A = B \times y$$

$$P = \frac{A}{y} + 2y \quad \text{(Area = Constant)}$$

We know that channel will be most efficient if for a given area A, P is minimum.

$$\therefore \quad \boxed{\frac{dP}{dy} = 0}$$

$$\frac{dP}{dy} = \frac{-A}{y^2} + 2$$

$$\frac{-A}{y^2} + 2 = 0$$

$$\frac{-A}{y^2} = -2$$

$$A = 2y^2$$

$$B \times y = 2y^2$$

$$\boxed{y = \frac{B}{2}}$$

Also, Hydraulic Radius,

$$R = \frac{A}{P}$$

$$R = \frac{B \times y}{B + 2y} \quad [2y = B]$$

$$R = \frac{B \times y}{B + B}$$

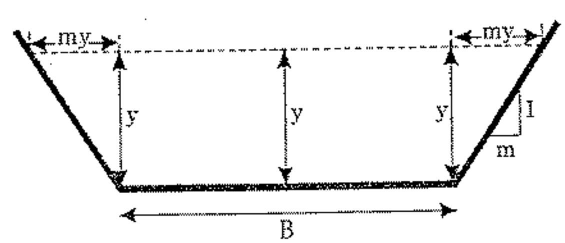
$$\boxed{R = \frac{y}{2}}$$

- For most efficient rectangular channel section depth of flow y = B/2 and Hydraulic radius $R = \frac{y}{2}$.

Most Efficient Trapezoidal Section

Case I : When side slope is fixed

Consider a channel section of bottom width = B, Depth of flow = y, Side slope = 1 : m (V : H)



Note: As we don't have complete freedom in design, the design will not be most efficient. Hence in this case our aim will be, to find out the condition for efficient channel section

There are 3 variables y, B, m but as m is fixed, the number of variable left out are two, y and B.

$$\text{Area, } A = \frac{1}{2}(B + B + 2my)y$$

$$A = \frac{1}{2}(2B + 2my)y$$

$$\boxed{A = (B + my)y}$$

Top flow width, $\boxed{T = B + 2ym}$

Perimeter, $\boxed{P = B + 2y\sqrt{m^2 + 1}}$

Substituting, $B = \frac{A}{y} - my$

$$\boxed{P = \frac{A}{y} - my + 2y\sqrt{m^2 + 1}} \quad \text{... (i) (A \& m are constant)}$$

We know that, for efficient channel section of a given Area A, perimeter should be min.

$$\therefore \quad \frac{dP}{dy} = 0$$

$$\frac{dP}{dy} = \frac{-A}{y^2} - m + 2\sqrt{m^2 + 1}$$

$$\frac{-A}{y^2} - m + 2\sqrt{m^2 + 1} = 0$$

$$\boxed{\frac{A}{y^2} + m = 2\sqrt{m^2 + 1}} \quad \text{... (ii)}$$

Multiplying y on both sides

$$\frac{A}{y} + my = 2y\sqrt{m^2 + 1}$$

Substituting, $\frac{A}{y} = B + my$

$$B + 2\,my = 2y\sqrt{m^2 + 1}$$

$$B + 2\,my = T$$

$\therefore$ $$\boxed{T = 2y\sqrt{m^2+1}}$$

$$\frac{T}{2} = y\sqrt{m^2+1}$$

We know that, $y\sqrt{m^2+1}$ = length of side slope.

$$\boxed{\frac{T}{2} = \text{length of side slope}}$$ **First condition.**

- For, efficient trapezoidal channel section with fixed side slope half of top width should be equal to length of side slope.

Also, Hydraulic Radius, $R = \frac{A}{P}$

$$R = \frac{(B+my)y}{B+2y\sqrt{m^2+1}}$$

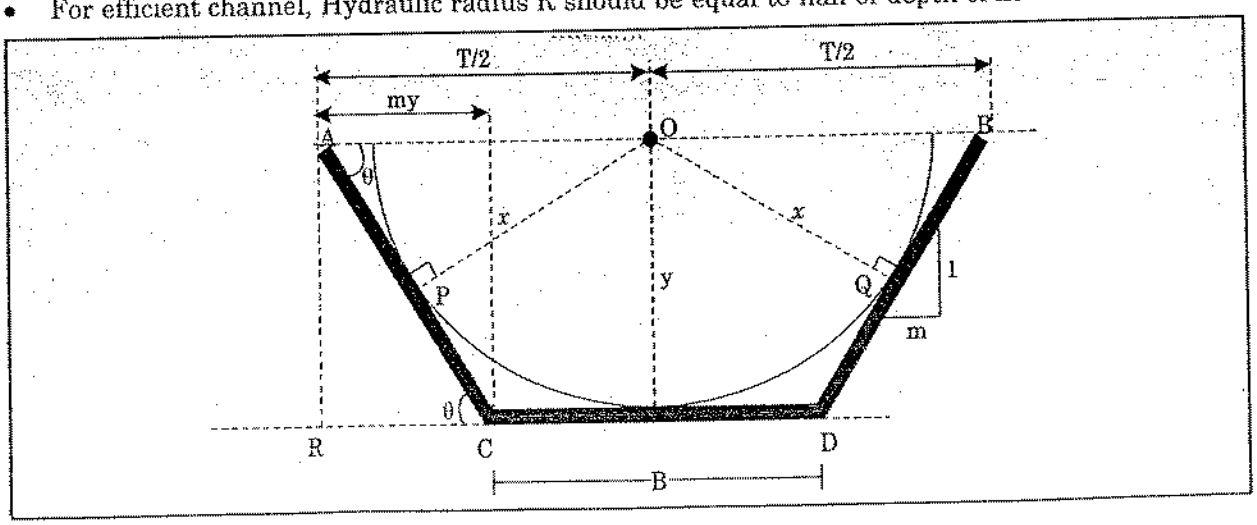
$$R = \frac{(B+my)y}{B+T}$$

$$R = \frac{(B+my)y}{B+B+2my}$$

$$R = \frac{(B+my)y}{2(B+my)}$$

$$\boxed{R = \frac{y}{2}}$$ **Second condition.**

- For efficient channel, Hydraulic radius R should be equal to half of depth of flow.



Further,

In Δ AOP, $$\sin\theta = \frac{OP}{AO} = \frac{x}{T/2}$$

In Δ ACR, $$\sin\theta = \frac{AR}{AC} = \frac{y}{y\sqrt{1+m^2}}$$

By comparing both the above equation

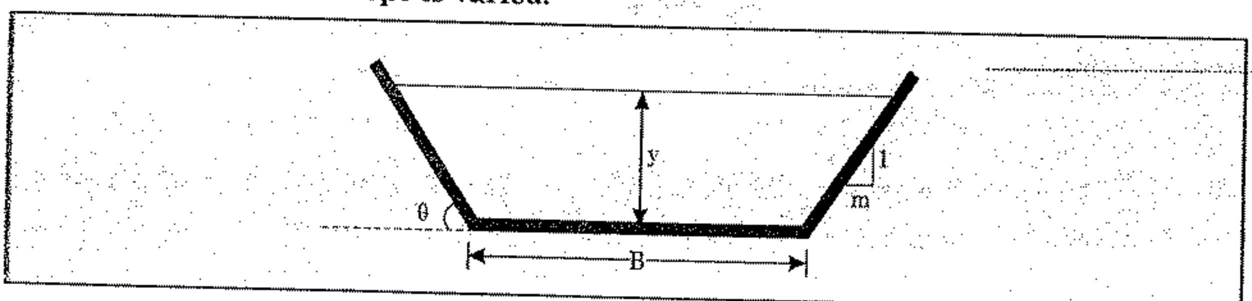
$$\frac{x}{T/2} = \frac{y}{y\sqrt{1+m^2}}$$

$$\frac{x}{y\sqrt{1+m^2}} = \frac{y}{y\sqrt{1+m^2}}$$

$\boxed{x = y}$ **Third Condition.**

- For efficient Trapezoidal channel section a circle of radius y shall be inscribed in the trapezoidal section.
- For the economical trapezoidal channel section having depth of flow = y and fixed side slope 1 : m, all the above 3 conditions should be satisfied.

Case II : When the side slope is varied.



Consider a channel section of width B, flow depth y, and varying side slope of 1 : m (V : H)

$$\text{Area, } A = (B + my)\,y$$

$$\text{Perimeter, } P = B + 2y\sqrt{1+m^2}$$

$$P = \frac{A}{y} - my + 2y\sqrt{m^2+1}$$

We know that for a given area if perimeter is to be minimum then,

$$\frac{dP}{dy} = 0$$

$$A = y^2[2\sqrt{m^2+1} - m] \qquad \text{(From equation B)}$$

Put this in above equation of perimeter

$$P = 2y\sqrt{m^2+1} - my - my + 2y\sqrt{m^2+1}$$

$$\boxed{P = 4y\sqrt{m^2+1} - 2my}$$

For economical channel section with varying side slope.

$$\frac{dP}{dm} = 0$$

$$\frac{dP}{dm} = \frac{4y \times 2m}{2\sqrt{m^2+1}} - 2y$$

$$\frac{8my}{2\sqrt{m^2+1}} - 2y = 0$$

$$2my = y\sqrt{m^2+1}$$

$$2m = \sqrt{m^2+1}$$

$$4m^2 = m^2 + 1$$

$$3m^2 = 1$$

$$m^2 = \frac{1}{3}$$

$$m = \frac{1}{\sqrt{3}} \qquad \left[\tan\theta = \frac{1}{m}\right]$$

$$\tan\theta = \sqrt{3}$$

$$\theta = \tan^{-1}\left(\sqrt{3}\right)$$

$\boxed{\theta = 60^\circ}$ **4th condition**

For economical trapezoidal section with varying side slope, the value of side slope should be $1 : \sqrt{3}$ (1 : m).

Conclusion

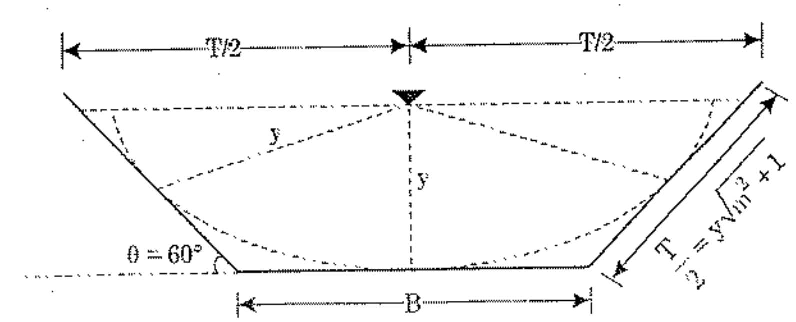
For most economical trapezoidal section following conditions should be satisfied.

(1) $\frac{T}{2}$ = Length of side slope

(2) $R = \frac{y}{2}$

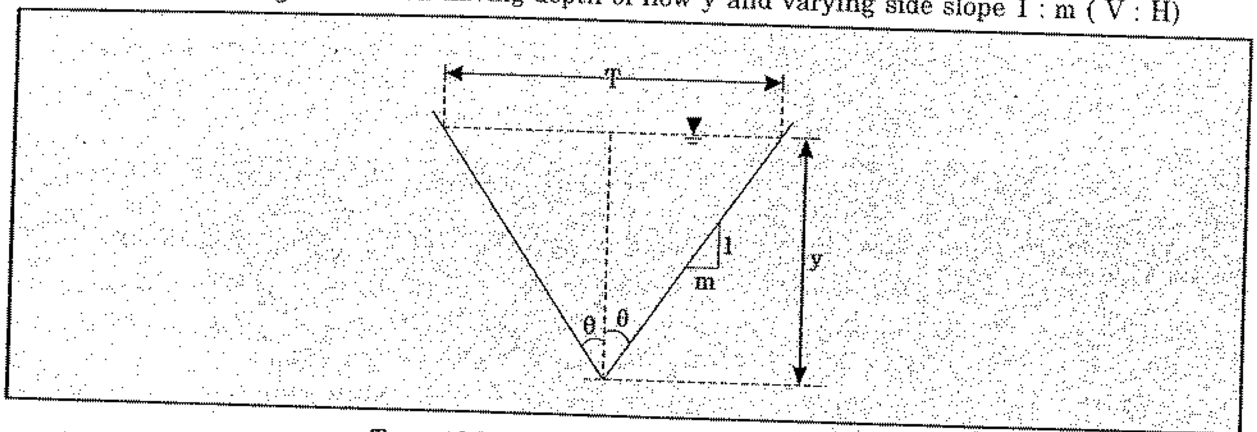
(3) A circle of radius y (normal flow depth) should be inscribed in trapezoidal section.

(4) $\theta = 60^\circ$



Most Efficient Triangular Section

Consider a triangular section having depth of flow y and varying side slope 1 : m (V : H)



$$\text{Top width, } T = 2\,my$$

$$\text{Area, } A = \frac{1}{2} \times T.y$$

$$A = \frac{1}{2} \times 2m.y.y$$

$$A = my^2$$

$$\text{Perimeter, } P = 2y\sqrt{m^2+1}$$

For most efficient economical section we know that Perimeter should be minimum.

$$\frac{dP}{dm} = 0$$

$$P = 2y\sqrt{m^2+1}$$

$$y = \sqrt{\frac{A}{m}} \qquad \text{(as area A = constant)}$$

$$P = 2\sqrt{m^2+1}\sqrt{\frac{A}{m}}$$

$$P = 2\sqrt{\frac{m^2+1}{m}}\sqrt{A}$$

$$P = 2\sqrt{A}\sqrt{m+\frac{1}{m}}$$

$$\frac{dP}{dm} = 2\sqrt{A}\,\frac{1}{2\sqrt{m+\frac{1}{m}}} \times \left(1-\frac{1}{m^2}\right)$$

$$2\sqrt{A}\,\frac{1}{2\sqrt{m+\frac{1}{m}}} \times \left(1-\frac{1}{m^2}\right) = 0$$

Solution of above equation will be.

$$\therefore \quad m+\frac{1}{m} = 0$$

$$\therefore \quad m-\frac{1}{m} = 0$$

$$m = \pm 1$$

$$\tan\theta = \frac{1}{m} = 1$$

$$\theta = \tan^{-1}(1)$$

$$\theta = 45°$$

Also, ... Hydraulic radius, $R = \frac{A}{P}$

$$R = \frac{my^2}{2y\sqrt{m^2+1}}$$

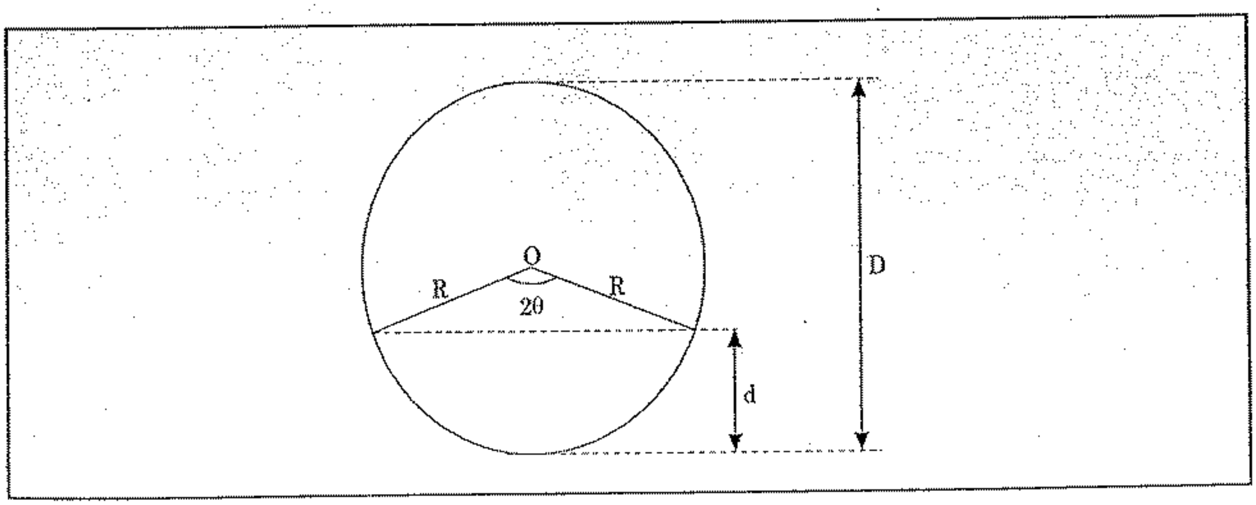
for, m = 1 *i.e.*, $\theta = 45°$

$$\boxed{R = \frac{y}{2\sqrt{2}}}$$

For most efficient triangular section of depth of flow y and varying side slope 1 : m, side slope 1 : m = 1 : 1 and Hydraulic radius $R = \frac{y}{2\sqrt{2}}$.

Most Effective Circular Section

Consider a circular section of radius R, and depth of flow = d



- In a circular section the shape of flow cross section varies with the depth of flow.
- Due to the divergence and convergence, the depth required to achieve maximum discharge and maximum velocity will be different.

Condition for Maximum Discharge

$$\text{Area of flow, } A = \frac{2\theta}{2\pi} \times \pi R^2 - \frac{1}{2} \times 2R\sin\theta \times R\cos\theta$$

$$A = \theta R^2 - \frac{R^2}{2}\sin 2\theta$$

$$\boxed{A = \frac{R^2}{2}(2\theta - \sin 2\theta)}$$

Wetted Perimeter, $\boxed{P = R \times 2\theta}$

From Manning's equation,

$$Q = \frac{1}{n} \times A \times R^{2/3} S^{1/2}$$

$$Q = \frac{1}{n} \times A \times \frac{A^{2/3}}{P^{2/3}} S^{1/2}$$

$$Q = \frac{1}{n}\frac{A^{5/3}}{P^{2/3}} S^{1/2}$$

For maximum discharge condition $\frac{dQ}{d\theta} = 0$ (Assuming m and s to be constant)

$$Q = f(A, p)$$

$$\therefore \quad \frac{dQ}{d\theta} = \frac{d}{d\theta}\left(\frac{A^{5/3}}{P^{2/3}}\right) = 0$$

$$\frac{d}{d\theta}\left(\frac{A^5}{P^2}\right)^{1/3} = 0$$

$$5A^4P^{-2}\frac{dP}{d\theta} - 2A^5P^{-3}\frac{dP}{d\theta} = 0$$

$$5P\frac{dA}{d\theta} - 2A\frac{dP}{d\theta} = 0$$

$$\because \quad \frac{dA}{d\theta} = R^2(1 - \cos 2\theta) \qquad \left[A = \frac{R^2}{2}(2\theta - \sin 2\theta)\right]$$

$$\because \quad \frac{dP}{d\theta} = 2R \qquad [P = 2R\theta]$$

Substituting the values of $\frac{dA}{d\theta}$ and $\frac{dP}{d\theta}$

$$\therefore \; 5 \times (2R\theta) \times R^2(1 - \cos 2\theta) - 2 \times \frac{R^2}{2}(2\theta - \sin 2\theta) \times 2R = 0$$

$$10\,R^3\theta\,(1-\cos2\theta) = 2R^3\,(2\theta-\sin2\theta)$$
$$10\theta-10\theta\cos2\theta = 4\theta-2\sin2\theta$$
$$6\theta-10\theta\cos2\theta+2\sin2\theta = 0$$

Solving By Hit and trial

$$\theta = 2.636 \text{ rad or } 2\theta = 302°\,22'$$

$$d = R-R\cos\theta$$
$$d = R\,(1-\cos\theta)$$
$$\frac{d}{R} = (1-\cos\theta)$$
$$\frac{d}{R} = 1.876$$
$$\frac{d}{D} = 0.938$$

Condition for maximum velocity

$$A = \frac{R^2}{2}(2\theta-\sin2\theta)$$
$$P = 2\,R\,\theta$$

We know that

$$v = \frac{1}{n}R^{2/3}S^{1/2}$$
$$v = \frac{1}{n}\frac{A^{2/3}}{P^{2/3}}S^{1/2}$$

For maximum velocity $\frac{dv}{d\theta}=0$ (Assuming that n and S are constant)

$$v = f\,(A, P)$$

$$\therefore \quad \frac{dv}{d\theta} = \frac{d}{d\theta}\left(\frac{A^{2/3}}{P^{2/3}}\right)$$

$$\frac{d}{d\theta}\left(\frac{A}{P}\right)^{2/3} = 0$$

$$P\frac{dA}{d\theta}-A\frac{dP}{d\theta} = 0$$

$$\because \quad \frac{dA}{d\theta} = R^2\,(1-\cos2\theta) \qquad \left[A = \frac{R^2}{2}(2\theta-\sin2\theta)\right]$$

$$\because \quad \frac{dP}{d\theta} = 2\,R \qquad [P = 2R\theta]$$

$$2R\theta\times R^2\,(1-\cos2\theta)-\frac{R^2}{2}(2\theta-\sin2\theta)\times2R = 0$$

$$2R^3\theta\,(1-\cos 2\theta) = R^3\,(2\theta - \sin 2\theta)$$
$$2\theta - 2\theta\cos 2\theta = 2\theta - \sin 2\theta$$
$$-2\theta\cos 2\theta + \sin 2\theta = 0$$

Solving by hit and trail,

$$2\theta = 257°\ 27'\ 56''$$

Depth of water for maximum velocity

$$d = R + R\cos(180° - \theta)$$
$$d = R + R\cos\left(180° - \frac{257°27'56''}{2}\right)$$
$$d = R + 0.626\ R$$
$$d = 1.626\ R$$
$$\frac{d}{R} = 1.626$$
$$\frac{d}{D} = 0.81$$

Note: From Chezy's Equation

For condition for maximum discharge

$$2\theta = 308° \text{ or } \frac{d}{D} = 0.95$$

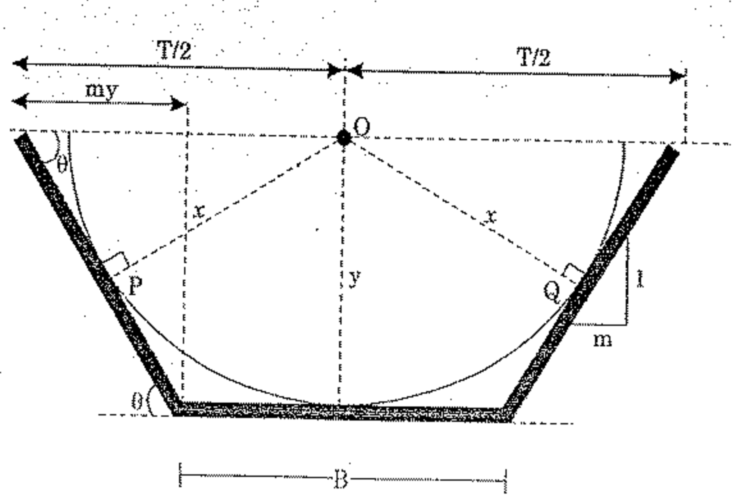
For condition for maximum velocity

$$2\theta = 257°\ 27' \text{ or } \frac{d}{D} = 0.81$$

Example 1

Show that hydraulically most efficient trapezoidal section is half of a regular hexagon.

Sol. For hydraulically most efficient trapezoidal section we know that



(1) $\frac{T}{2} = y\sqrt{1+m^2}$

(2) $R = \frac{y}{2}$

(3) $m = \frac{1}{\sqrt{3}}$ or $\theta = 60°$

(4) $x = y$ i.e., a semicircle of radius y is inscribed in the trapezoidal section.

If L is the length of sloping side

$$L = \sqrt{y^2 + m^2y^2}$$

$$L = y\sqrt{1+m^2} \qquad \left[m = \frac{1}{\sqrt{3}}\right]$$

$$L = \frac{2y}{\sqrt{3}} \qquad \text{...(i)}$$

Also,

$$\frac{T}{2} = L$$

$$\frac{B+2my}{2} = \frac{2y}{\sqrt{3}}$$

$$B = \frac{4y}{\sqrt{3}} - \frac{2}{\sqrt{3}}y$$

$$B = \frac{2}{\sqrt{3}}y \qquad \text{...(ii)}$$

$\therefore$ From (i) & (ii), we have $\boxed{B = L}$

We can observe that length of sloping sides is equal to the bottom width of the trapezoidal with side slope $\theta = 60°$, which is a property of hexagon. Therefore the hydraulically most efficient trapezoidal section is half of a regular hexagon.

Example 2

A lined channel of trapezoidal section has one side vertical and the other side having a slope 1H : 1V. The channel has to deliver $8 m^3/sec$ when laid on a slope of 0.0002. What would be the dimensions of the efficient section which requires minimum lining? Also calculate the corresponding mean velocity if Manning's n is 0.015.

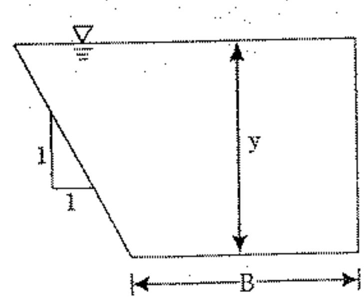
Sol. Given,

$Q = 8\ m^3/sec$

$S = 0.0002$

$n = 0.015$

$1 : m = 1 : 1$



$$\text{Area, } A = \frac{1}{2}(B + B + my)y$$

$$A = \frac{1}{2}(2B + my)y$$

$$A = \frac{1}{2}(2B+y)y$$

$$\text{Perimeter, } P = B+y+y\sqrt{1+m^2}$$

$$P = B+y\left(1+\sqrt{2}\right)$$

$$\therefore \quad B = \frac{A}{y}-\frac{y}{2} \quad \text{... (i)}$$

$$P = \frac{A}{y}-\frac{y}{2}+y\left(1+\sqrt{2}\right)$$

We know that for efficient channel section

$$\frac{dP}{dy} = 0$$

$$\frac{-A}{y^2}-\frac{1}{2}+(1+\sqrt{2}) = 0$$

$$\therefore \quad \frac{-A}{y^2}+1.914 = 0$$

$$\frac{A}{y^2} = 1.914$$

$$\frac{A}{y} = 1.914y \quad \text{...(ii)}$$

Put the value to (ii) in (i), we get

$$B + \frac{y}{2} = 1.914y$$

$$B = 1.414y$$

$$\text{Area, } A = \frac{1}{2}(2B+y)y$$

$$A = \frac{1}{2}(2\times 1.414y+y)y$$

$$A = 1.914\ y^2$$

$$\text{Perimeter, } P = B+y\left(1+\sqrt{2}\right)$$

$$P = 1.414y+\left(1+\sqrt{2}\right)y$$

$$P = 3.828y$$

$$\text{Hydraulic Radius, } R = \frac{A}{P}$$

$$R = \frac{1.914y^2}{3.828y}$$

$$R = \frac{y}{2}$$

From Manning's equation

$$Q = \frac{1}{n} AR^{2/3} (S_0)^{1/2}$$

$$8 = \frac{1}{0.015} \times 1.914y^2 \times \left(\frac{y}{2}\right)^{2/3} \times (0.0002)^{1/2}$$

$$y^{8/3} = \frac{8 \times 0.015 \times 2^{2/3}}{(0.0002)^{1/2} \times 1.914}$$

$$y^{8/3} = 7.037$$

$$y = 2.079\text{m.}$$

$$B = 1.414y = 2.939\text{m}$$

Channel dimensions will be, $B = 2.939$

$y = 2.079$

$$\text{Mean flow velocity} = \frac{Q}{A} = \frac{8}{1.914 \times 2.079^2}$$

$$v = 0.967 \text{ m/sec}$$

Example 3

What should be the cross section of most efficient trapezoidal shape for a concrete lined canal to carry a discharge of 250 m^3/sec. The channel slope is 0.0004 and the Manning's n is 0.014. Use the side slopes as 1 : 1

Sol. Given:

$$Q = 250 \text{ m}^3\text{/sec}, \quad n = 0.014$$

$$S = 0.0004$$

$$\text{Side slope} = 1 : m = 1 : 1$$

For most efficient trapezoidal channel cross section.

$$\text{Hydraulic Radius, } R = \frac{y}{2}$$

$$\text{Top width, } T = 2y\sqrt{1+m^2}$$

$$B + 2my = 2y\sqrt{1+m^2}$$

$$B = 2\sqrt{2}y - 2y$$

$$B = 0.828\ y$$

$$\text{Area, } A = (B + my)\ y$$

$$= (0.828y + y)\ y$$

$$= 1.828\ y^2$$

From mannings equation $$Q = \frac{1}{n} AR^{2/3}S^{1/2}$$

$$250 = \frac{1}{0.014} \times 1.828\ y^2 \times \left(\frac{y}{2}\right)^{2/3} \times (0.0004)^{1/2}$$

$$\frac{250 \times 0.014 \times 2^{2/3}}{1.828 \times (0.0004)^{1/2}} = y^{8/3}$$

$$y^{8/3} = 151.967$$

$$y = 6.579 \approx 6.580$$

Width of channel bottom, $B = 0.828\ y = 5.448$ m

Depth of channel, $y = 6.580$ m

Area of cross section, $A = 1.828\ y^2 = 79.146\ m^2$

Perimeter $P = B + 2y\sqrt{1+m^2} = 24.059$ m

Top width $T = B + 2my = 18.608$

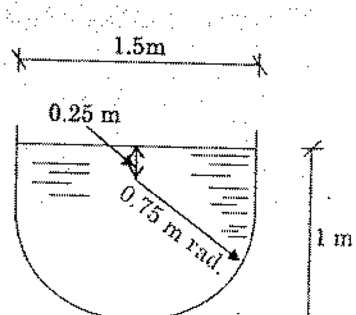
Example 4

A channel has vertical walls 1.5 m apart and semicircular invert. If the centerline depth is 1 m and the bed slope is 1 in 2500, what would be the value of "C" in the chezy formula is the discharge is 0.60 m^3/s?

Sol. Given,

$Q = 0.60\ m^3/sec$

$S = 1$ in 2500



$$\text{Area, } A = (1.5 \times 0.25) + \frac{\pi}{2} \times 0.75^2$$

$$= 1.259\ m^2$$

$$\text{Perimeter, } P = 2 \times 0.25 + \pi \times 0.75$$
$$= 2.856 \text{ m}$$

$$\text{Hydraulic Radius, } R = \frac{A}{P} = \frac{1.259}{2.856} = 0.441 \text{ m}$$

$$\text{According to Chezy's equation, } Q = CA\sqrt{RS}$$

$$0.60 = C \times 1.259 \times \sqrt{\frac{0.441 \times 1}{2500}}$$

$$C = 35.882$$

Example 5

A trapezoidal channel with one side vertical and the other sloping at two horizontal to one vertical carries a discharge of a $28 m^3/sec$ at a mean velocity of 1.5m/sec. Determine the longitudinal slope and the channel dimensions for the best hydraulic efficiency, if Manning's $n = 0.014$.

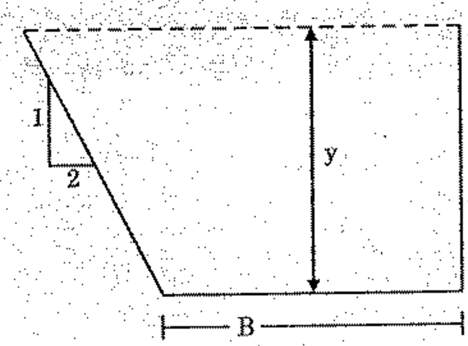
Sol. Given,

$Q = 28 m^3/sec$

$v = 1.5$ m/sec

$n = 0.014$

$1 : m = 1 : 2$



$$\text{Area, } A = \frac{Q}{v} = \frac{28}{1.5} = 18.667 \text{ m}^2$$

$$\text{Area, } A = \frac{1}{2}(2B + my)y$$

$$A = \frac{1}{2}(2B + 2y)y$$

$$A = (B + y)\,y$$

$$\text{Perimeter, } P = B + y + y\sqrt{1 + m^2}$$

$$P = B + y + y\sqrt{5}$$

$$P = B + (1 + \sqrt{5})\,y$$

For best hydraulically efficient channel, we know that $\frac{dP}{dy} = 0$.

$$P = B + (1 + \sqrt{5})y$$

$$B = \frac{A}{y} - y$$

$$P = \frac{A}{y} - y + y + \sqrt{5}\,y$$

$$P = \frac{A}{y} + \sqrt{5}\,y$$

$$\frac{dP}{dy} = -\frac{A}{y^2} + \sqrt{5}$$

$$\frac{A}{y^2} = \sqrt{5} \qquad [A = 18.667]$$

$$y^2 = \frac{18.667}{\sqrt{5}}$$

$$y = 2.889 \text{ m}$$

$$B = \frac{A}{y} - y = \frac{18.667}{2.889} - 2.889 = 3.572$$

$$\text{Hydraulic radius, } R = \frac{A}{P} = \frac{18.667}{B + (1 + \sqrt{5})y} = 1.445$$

$$\text{We know that, } v = \frac{1}{n} R^{2/3} S^{1/2}$$

$$S^{1/2} = \frac{v \times n}{R^{2/3}} = \frac{1.5 \times 0.014}{(1.445)^{2/3}} = 0.0164$$

$$S = 0.0164^2$$

$$S = 0.00027$$

Example 6

A trapezoidal channel has side slope of 1 : 1. It is required to discharge 10m³/sec of water with a bed slope of 1 in 1000. If unlined, the value of chezy's constant C is 45. If lined with concrete C is 60, the cost per cubic meter of excavation is 4 times the cost of lining per square meter. The channel to be most efficient one, find whether the lined canal or the unlined canal will be cheaper? What will be the dimensions of economical canal.

Sol.

Given, $Q = 10\text{m}^3/\text{sec},\ S = \frac{1}{1000},\ 1 : m = 1:1\ (V : H)$

We know that for efficient trapazoidal channel

$$\text{Hydraulic radius, } R = \frac{y}{2} \quad ...(1)$$

$$\text{Area, } A = (B + my)\,y = (B + y)y \quad [m = 1]$$

$$\text{Perimeter, } P = B + 2y\sqrt{1+m^2} = B + 2\sqrt{2}\,y.$$

From Equation (1),

$$R = \frac{A}{P} = \frac{(B+y)y}{B+2\sqrt{2}y} = \frac{y}{2}$$

$$B = 2\left(\sqrt{2}-1\right)y$$

$$B = 0.828\,y$$

$$\therefore A = 1.828y^2$$

$$P = 3.656\,y$$

From Chezy's equation we know that $Q = A.C\sqrt{RS}$

For unlined canal, $C = 45$

$$10 = 1.828\,y^2 \times 45 \times \sqrt{\frac{y}{2} \times \frac{1}{1000}}$$

$$y^{5/2} = \frac{10\sqrt{2000}}{45 \times 1.828}$$

$$y = 1.968$$

Volume of excavation for unit meter length $= 1.828\,y^2 \times 1 = 7.080\ m^3$

Let the cost of 1 square meter of lining be Rs. x then cost of $1m^3$ of earth work is $4x$.

$$\text{Cost of excavation} = 4x \times 7.080 = \text{Rs. } 28.320x$$

For lined canal, $C = 60$

$$10 = 1.828\,y^2 \times 60\ \sqrt{\frac{y}{2} \times \frac{1}{1000}}$$

$$y^{5/2} = \frac{10 \times \sqrt{2000}}{1.828 \times 60}$$

$$y = 1.755$$

Volume of excavation for unit meter length $= 1.828y^2 \times 1 = 5.630$

Cost of excavation for unit meter length $= \text{Rs. } 4x \times 5.630$

$= \text{Rs. } 22.521x$

Area of lining for unit meter length $= P \times 1$

$= 3.656 \times y \times 1$

$= 3.656 \times 1.755$

Cost of lining for 1 meter length $= \text{Rs. } 6.416x$

Total cost $= \text{Rs. } 22.521x + \text{Rs. } 6.416\,x$

$= \text{Rs. } 28.937\,x$

As we can observe that unlined canal is more cheaper hence it will be most economical channel its dimensions are

$$y = 1.968 \text{ m}$$

$$B = 1.626 \text{ m}$$

Example 7

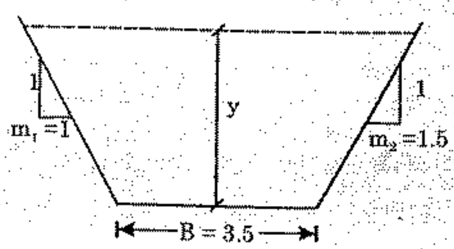
A trapezoidal channel with bottom width 3.5 m and side slopes 1H : 1V on the left and 1.5 H : 1V on the right, with n = 0.016, and a bed slope of 2.6 in 10,000, carries a discharge of 8 m^3/sec. Determine the normal depth and the average shear stress on the channel bed.

Sol. Given,

$Q = 8 \text{ m}^3/\text{sec}$

$S = \dfrac{2.6}{10000}$

$n = 0.016$



From Manning's equation

$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

$$\text{Area, } A = \frac{1}{2}(B + B + 1.5\,y + y)y$$

$$A = (B + 1.25y)y$$

$$\text{Perimeter, } P = B + y\left(\sqrt{1+1.5^2} + \sqrt{1+1^2}\right)$$

$$P = B + 3.217y$$

$$\text{Hydraulic Radius, } R = \frac{A}{P} = \frac{(B+1.25y)y}{B+3.217y} = \frac{(3.5+1.25y)y}{3.5+3.217y}$$

$$A = (3.5 + 1.25\,y)y$$

$$P = 3.5 + 3.217y$$

Substituting A & P in the Manning's equation

$$8 = \frac{1}{0.016}(3.5 + 1.25\,y)y\left(\frac{(3.5+1.25y)y}{3.5+3.217y}\right)^{2/3}\left(\frac{2.6}{10,000}\right)^{1/2}$$

$$(3.5 + 1.25y)y\left(\frac{(3.5+1.25y)y}{3.5+3.217y}\right)^{2/3} = 8\times0.016\times\left(\frac{10,000}{2.6}\right)^{1/2}$$

Solving by hit and trial method

$$y = 1.505$$

Average shear stress on the bottom of bed $= \gamma\ RS$

$$\tau = 1000 \times 9.81 \times \frac{(3.5+1.25\times1.505)\times1.505}{3.5+3.217\times1.505}\times\frac{2.6}{10000}$$

$$\tau_0 = 2.434\ N/m^2$$

Example 8

In a wide rectangular channel if the normal depth of flow is increased by 20%, determine the corresponding percentage discharge. Use mannings equation. (Take finite increment and find out the result)

Sol. We know that, from Manning's equation

$$Q = \frac{1}{n}\ AR^{2/3}S^{1/2}$$

In a wide rectangular channel, Hydraulic Radius, $R = y$'

due wide rectangular channel $R = \left(\frac{By}{B+2y}\right)$ if $B >> 2y$, $R \approx y$

$$\therefore \quad Q = \frac{1}{n}(B.\ y)\ (y)^{2/3}\ S^{1/2}$$

$$Q = \frac{1}{n}\ B\ S^{1/2}y^{5/3}$$

$$Q = Ky^{5/3} \qquad \dots (1)$$

$$\frac{dQ}{dy} = \frac{5}{3}Ky^{2/3} \qquad \dots (2)$$

for large increments we should work on finite increment method.

$$\frac{dQ}{Q} = \frac{5}{3}\ \frac{dy}{y}$$

$$\text{If, } \frac{dy}{y} = 20\%$$

$$\frac{dQ}{Q} = \frac{5}{3} \times 20\% = \frac{100\%}{3}$$

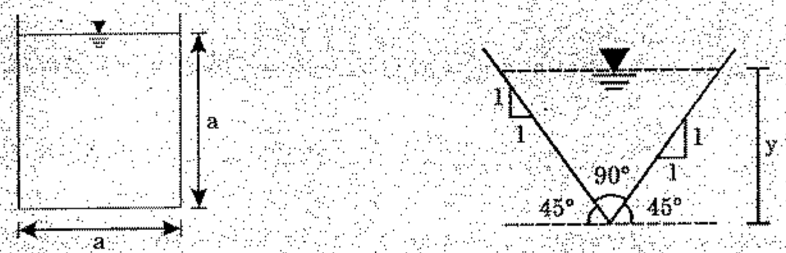
$$dQ = 33.33\% \text{ of } Q$$

Example 9

A triangular channel of apex angle 90° and a rectangular channel of the same material have the same bed slope. If the rectangular channel has same depth of flow equal to width of channel and flow areas in both channels are same, find the ratio of discharges in rectangular and triangular channels respectively. Use mannings roughness equation for estimation of velocity.

Sol. We know that from Manning's equation $Q = \frac{1}{n} AR^{2/3}S^{1/2}$

$$\text{As, } Q \propto \frac{1}{P^{2/3}}$$



$$\frac{Q_{rectangle}}{Q_{Triangle}} = \left(\frac{P_{Triangle}}{P_{Rectangle}}\right)^{2/3}$$

$$\text{Area of flow in rectangular channel} = a^2$$

$$\text{Area of flow in triangular channel} = my^2 = y^2 \qquad [m = 1]$$

As both the flow areas are equal

$$a^2 = y^2$$

$$a = y$$

$$\text{Perimeter of rectangle} = 3a$$

$$\text{Perimeter of triangle} = 2y\sqrt{1+m^2} = 2\sqrt{2}a$$

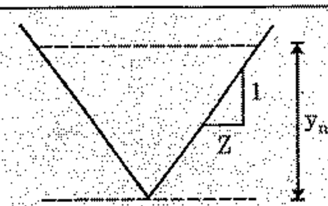
$$\text{Ratio of discharges} = \frac{Q_{Rectangle}}{Q_{Triangle}} = \left(\frac{2\sqrt{2}a}{3a}\right)^{2/3} = \left(\frac{2\sqrt{2}}{3}\right)^{2/3} = 0.961$$

Example 10

Show that normal depth of flow in a triangular channel having side slope Z : 1 as H : V is given by

$$y_n = 1.189 \left[\frac{Q \cdot n}{\sqrt{S_0}}\right]^{3/8} \left(\frac{Z^2+1}{Z^5}\right)^{1/8}$$

Sol. From Manning's equation



$$Q = \frac{1}{n} AR^{2/3}S_0^{1/2}$$

$$\frac{Qn}{\sqrt{S_o}} = AR^{2/3} \quad \text{... (1)}$$

$$\text{Area, } A = Z\, y_n^2$$

$$\text{Perimeter, } P = 2y_n\sqrt{1+Z^2}$$

$$\text{Hydraulic Radius, } R = \frac{A}{P} = \frac{Zy_n^2}{2y_n\sqrt{1+Z^2}}$$

$$R = \frac{Zy_n}{2\sqrt{1+Z^2}}$$

Substituting R in equation (1)

$$Z\,y_n^2\left(\frac{Zy_n}{2\sqrt{1+Z^2}}\right)^{2/3} = \frac{Qn}{\sqrt{S_o}}$$

$$\frac{Z^{5/3}y_n^{8/3}}{2^{2/3}\left(1+Z^2\right)^{1/3}} = \frac{Qn}{\sqrt{S_o}}$$

$$y_n^{8/3} = 2^{2/3}\left(\frac{1+Z^2}{Z^5}\right)^{1/3}\left(\frac{Qn}{\sqrt{S_o}}\right)$$

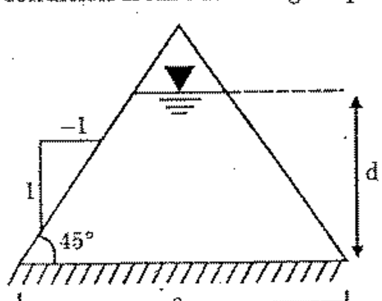
$$y_n = 2^{1/4}\left(\frac{Z^2+1}{Z^5}\right)^{1/8}\left(\frac{Qn}{\sqrt{S_o}}\right)^{3/8}$$

$$y_n = 1.189\left(\frac{Z^2+1}{Z^5}\right)^{1/8}\left(\frac{Qn}{\sqrt{S_o}}\right)^{3/8}$$

Example 11

Water flows in a channel of the shapes of isosceles triangle of bed width 'a' and sides making an angle of 45° with bed. Determine the relation between depth of flow 'd' and bed width 'a' for maximum velocity condition and for maximum discharge condition. Use Manning's formula and note that 'd' is less than '0.5a'.

Sol. For maximum discharge condition from Manning's equation



$$Q = \frac{1}{n} AR^{2/3}S^{1/2}$$

$$A = (a - md)\, d = (a - d)\, d$$

$$P = a + 2d\sqrt{1+m^2} = a + 2\sqrt{2}\, d$$

As n and s are constant, and for maximum discharge $\frac{dQ}{dd} = 0$

$$\frac{dQ}{dd} = \frac{d}{dd}\left(\frac{A^{5/3}}{P^{2/3}}\right) = 0$$

$$\frac{5}{3}A^{2/3}P^{-2/3}\frac{dA}{dd} - \frac{2}{3}A^{5/3}P^{-5/3}\frac{dP}{dd} = 0$$

$$5P\frac{dA}{dd} - 2A\frac{dP}{dd} = 0$$

$$5(a + 2\sqrt{2}\, d)(a - 2d) - 2(a - d)\, d\, 2\sqrt{2} = 0$$

$$5(a^2 - 2ad + 2\sqrt{2}\, da - 4\sqrt{2}\, d^2) - 4\sqrt{2}\, ad + 4\sqrt{2}\, d^2 = 0$$

$$5a^2 - 10\, ad + 10\sqrt{2}\, ad - 20\sqrt{2}\, d^2 - 4\sqrt{2}\, ad + 4\sqrt{2}\, d^2 = 0$$

$$22.627\, d^2 + 1.515\, ad - 5a^2 = 0$$

$d = 0.438\, a,\ -0.505\, a$

$\therefore$ $d = 0.438\, a$ maximum discharge will occur

For maximum velocity condition, $v = \frac{1}{n} R^{2/3}S^{1/2}$

$$\frac{dv}{dd} = \frac{d}{dd}\left(\frac{A}{P}\right)^{2/3} = 0$$

$$P\frac{dA}{dd} - A\frac{dP}{dd} = 0$$

$$(a + 2\sqrt{2}\, d)(a - 2d) - (a - d)d\, 2\sqrt{2} = 0$$

$$a^2 - 2ad + 2\sqrt{2}\, ad - 4\sqrt{2}\, d^2 - 2\sqrt{2}\, ad + 2\sqrt{2}\, d^2 = 0$$

$$2\sqrt{2}\, d^2 + 2ad - a^2 = 0$$

$d = 0.338a,\ -1.045a$

$\therefore$ $d = 0.338a$, maximum velocity will occur

Example 12

Determine the dimension of an economical trapezoidal section of an open channel with side slopes 2H : 1V laid at a slope of 1 in 1600 to carry a discharge of 36 m³/sec assuming Chezy's coefficient C = 50.

Sol. Given,

1: m (V : H) = 1 : 2

$$S = \frac{1}{1600}$$

$$Q = 36 \text{ m}^3/\text{sec}$$

$$C = 50$$

$$\text{Area, } A = (B + my)\,y$$

$$A = (B + 2y)\,y \qquad \text{... (i)}$$

$$\text{Perimeter, } P = B + 2y\sqrt{1+m^2}$$

$$P = B + 2\sqrt{5}\,y$$

Also,

$$B = \frac{A}{y} - 2y \qquad \text{[from (i)]}$$

$$P = \frac{A}{y} - 2y + 2\sqrt{5}\,y$$

We know that for economical channel section $\frac{dP}{dy} = 0$

$$\frac{dP}{dy} = -\frac{A}{y^2} - 2 + 2\sqrt{5}$$

$$\frac{A}{y^2} = 2\sqrt{5} - 2 \qquad \left[\frac{A}{y} = B + 2y\right]$$

$$\frac{B}{y} + 2 = 2\sqrt{5} - 2$$

$$B = \left(2\sqrt{5} - 4\right)y$$

$$B = 0.472\,y$$

$$Q = AC\sqrt{RS}$$

$$36 = (0.472\,y + 2y)\,y \times 50\sqrt{\frac{(0.472y + 2y)y}{0.472y + 2\sqrt{5}y} \times \frac{1}{1600}}$$

$$36 = 2.472\,y^2 \times 50\sqrt{\frac{2.472y^2}{4.944y} \times \frac{1}{1600}}$$

$$36 = 2.185y^{5/2}$$

$$y^{5/2} = 16.476$$

$$y = 3.067$$

$$B = 1.448$$

Example 13

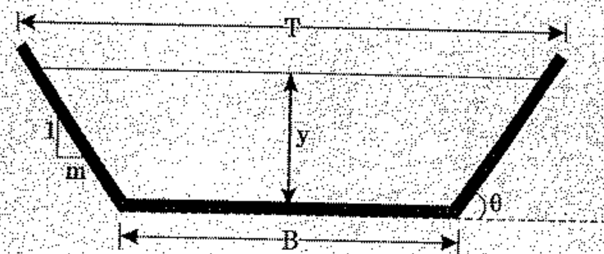
Design a hydraulically efficient trapezoidal channel for conveying 100m³/sec discharge, Manning's n = 0.015, bed slope 1 in 5000, add a free board of 10% of the depth of flow. Compute the froud number and sketch the design cross section.

Sol. Given

Q = 100m³/sec

n = 0.015

S = 1 in 5000



For hydraulically efficient trapezoidal section

We know that,

(1) $\frac{T}{2} = y\sqrt{1+m^2}$

(2) $R = \frac{y}{2}$

(3) $\theta = 60°$

(4) Section inscribe a circle of radius of depth of flow.

$$\text{Area, } A = (B + my)y \qquad [\because \; B = y\sqrt{1+m^2}\,y]$$

$$A = \left(y\sqrt{1+m^2} + my\right)y \qquad \left(m = \frac{1}{\sqrt{3}}\right)$$

$$A = \left(\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)y^2$$

$$A = \sqrt{3}y^2$$

For Manning's equation

$$Q = \frac{1}{n}AR^{2/3}S^{1/2}$$

$$100 = \frac{1}{0.015}\times\sqrt{3}y^2\times\left(\frac{y}{2}\right)^{2/3}\left(\frac{1}{5000}\right)^{1/2}$$

$$y^{8/3} = \frac{100\times0.015\times2^{2/3}\times(5000)^{1/2}}{\sqrt{3}}$$

$$y = 5.564\text{m}$$

$$B = \frac{2y}{\sqrt{3}} = 6.424\text{m}$$

$$v = \frac{Q}{A} = \frac{100}{\sqrt{3}y^2} = \frac{100}{\sqrt{3}\times5.564^2} = 1.865 \text{ m/sec}$$

$$A = \sqrt{3}y^2 = \sqrt{3}\times5.564^2 \ldots$$

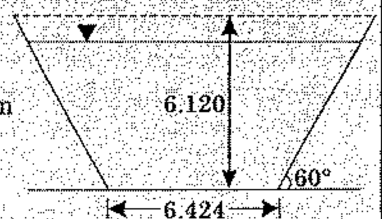
$$T = 2y\sqrt{1+m^2} = 2\times5.564\sqrt{1+\left(\frac{1}{\sqrt{3}}\right)^2} = 12.843$$

$$F_r = \frac{1.865}{\sqrt{9.81\times\frac{53.621}{12.849}}} = 0.291$$

As 10% free board has to be provided, hence channel dimensions will be

$$y' = 1.1y = 1.1 \times 5.564 = 6.120\text{m}$$

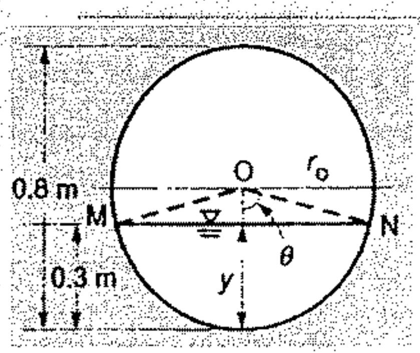
$$B = 6.424\text{m}$$



Example 14

A circular drainage pipe 0.80 m in diameter conveys a discharge at a depth of 0.30 m. If the pipe is laid on a slope of 1 in 900, estimate the discharge. Manning's n = 0.015.

Sol.



$$D = 0.80 \text{ m}$$

$$y = 0.30 \text{ m}$$

Area of flow section, A = Area of sector OMN – Area of triangle OMN

$$A = \frac{1}{2}r_0^2 \,.\, 2\theta - \frac{1}{2}(2r_0 \sin\theta)r_0\cos\theta$$

$$A = \frac{D^2}{8}(2\theta - \sin 2\theta)$$

Also,

$$\cos\theta = \frac{\left(\frac{D}{2} - y\right)}{\left(\frac{D}{2}\right)} = 1 - \frac{2y}{D}$$

Hence,
$$2\theta = 2\cos^{-1}\left(1-\frac{2y}{D}\right)$$

$$= 2\cos^{-1}\left(1-\frac{2\times 0.3}{0.8}\right) = 151.044^\circ$$

$$= \left(\frac{151.044^\circ \times \pi}{180^\circ}\right) = 2.636 \text{ rad.}$$

$$\sin 2\theta = 0.4841$$

$$\text{Area,} \quad A = \frac{(0.8)^2}{8}(2.636-0.4841)$$

$$= 0.1722 \text{ m}^2$$

$$\text{Wetted perimeter, } P = \frac{1}{2}D(2\theta)$$

$$= \frac{0.8}{2}\times 2.636 = 1.055 \text{ m}$$

$$\text{Hydraulic radius } R = A/P = \frac{0.1722}{1.055} = 0.1633 \text{ m}$$

By Manning's formula

$$Q = \frac{1}{n}AR^{2/3}S_0^{1/2}$$

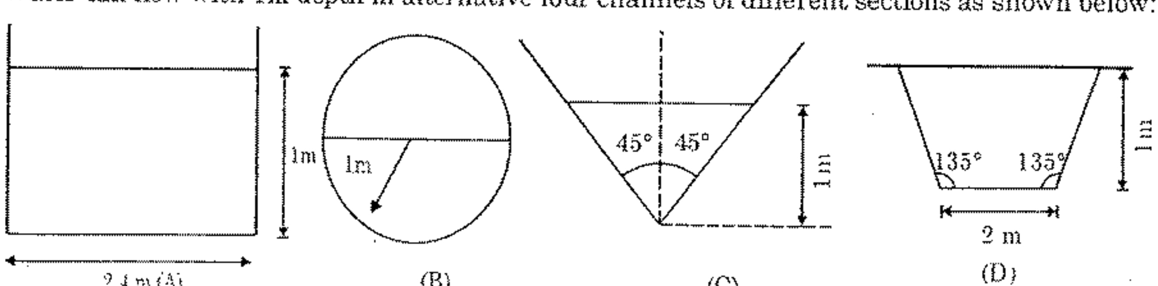
$$= \frac{1}{0.015}\times(0.1722)(0.1633)^{2/3}(1/900)^{1/2}$$

$$= 0.1143 \text{ m}^3/\text{s} = \textbf{114.3 L/s}$$

OBJECTIVE QUESTIONS

1. A channel bed slope 0.0009 carries a discharge of 30 m^3/s when the depth of flow is 1.0 m. What is the discharge carried by an exactly similar channel at the same depth of flow if the slope is decreased to 0.0001?
   (a) 10 m^3/s (b) 15 m^3/s
   (c) 60 m^3/s (d) 90 m^3/s
2. Which one of the following pairs is NOT correctly matched (b = bottom width, y = depth of flow, θ = side slope with vertical)?
   (a) For least perimeter of rectangular canal section ... b = 2y
   (b) For least perimeter of trapezoidal canal section ... b = 2y (sec θ – tan θ)
   (c) For critical flow through rectangular canal ... v = gy
   (d) For critical flow through trapezoidal canal ... $V = \sqrt{\dfrac{gy(b + y\tan\theta)}{b + 2y\tan\theta}}$
3. A rectangular open channel carries a discharge of 15 m^3/s when the depth of flow is 1.5 m and the bed slope is 1 : 1440. What will be the discharge through the channel at the same depth if the slope would have been 1 : 1000?
   (a) 21.6 m^3/s (b) 18 m^3/s
   (c) 14.4 m^3/s (d) 12.5 m^3/s
4. For a hydraulic efficient rectangular section, the ratio of width to normal depth is
   (a) 0.5 (b) 1.0
   (c) 2.0 (d) $2\sqrt{3}$
5. A rigid boundary rectangular channel having a bed slope of $\frac{1}{800}$ has its width and depth of flow equal to 2 m and 1 m respectively. If the flow is uniform and the value of Chezy's constant is 60, the discharge through the channel is
   (a) 1.0 m^3/s (b) 1.5 m^3/s
   (c) 2.0 m^3/s (d) 3.0 m^3/s
6. Consider the following statements regarding the conditions to be satisfied for the maximum discharge through a trapezoidal channel section with side slope 1: n, bed width b, flow depth d and having a fixed bed slope:
   1. Sloping sides should have an angle of 30° with vertical.
   2. Hydraulic mean depth equals half the flow depth.
   3. Length of sloping sides should be equal to twice the bottom width.

   Which of these statements are correct?
   (a) 1, 2 and 3 (b) 1 and 2
   (c) 2 and 3 (d) 1 and 3
7. Water can flow with 1m depth in alternative four channels of different sections as shown below:



Which one of the following sequences shows their hydraulic radii, arranged in descending order?

(a) D-C-B-A (b) D-A-B-C

(c) A-B-C-D (d) A-B-D-C

8. What is the normal depth in a wide rectangular channel carrying 0.5 m^3/s discharge at a bed slope of 0.0004 and Manning's n = 0.01?

(a) 0.13 m (b) 0.32 m

(c) 0.43 m (d) 0.50 m

9. When discussing most efficient section of flow into open channels, what is the perimeter p as a proportion of depth of flow h (i.e., P/h) for (i) a triangular section, and (ii) a trapezoidal section, respectively?

(a) 2.25, 2.83 (b) 2.25, 3.15

(c) 2.83, 3.15 (d) 2.83, 3.46

10. Consider the following statements :

1. Hydraulically most efficient channel section for an open channel flow will carry maximum discharge for a given area of cross section.
2. For a given cross sectional area hydraulic radius is maximum when the wetted perimeter is minimum.

Which of these statements is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

11. The maximum velocity through a circular channel takes place when depth of flow is equal to

(a) 0.95 times the diameter (b) 0.5 times the diameter

(c) 0.81 times the diameter (d) 0.3 times the diameter

12. Given that g = acceleration due to gravity and R = hydraulic mean depth, the Darcy- Weisbach frction factor is related to Manning's rugosity coefficient 'n' as

(a) $\frac{8gn^2}{R^{1/3}}$ (b) $\frac{gn^2}{8R^{1/3}}$

(c) $\frac{64ng}{R^{1/3}}$ (d) $\frac{R^{1/3}}{8gn^2}$

13. For a circular channel (with r_0 as the radius of the channel) to be efficient,

(a) the half subtended angle at the centre with respect to the water level must be 151° 10`

(b) depth of flow must be 2.00 r_0

(c) depth for maximum velocity must be 1.62 r_0

(d) the half subtended angle at the centre with respect to the water level must be 90°.

14. Normal depth of flow computed using Manning's equation in a wide rectangular open channel of given slope and roughness and carrying a discharge of q m^3/s per meter width is proportional to

(a) q (b) $q^{0.5}$

(c) $q^{0.6}$ (d) $q^{0.67}$

15. In the most efficient trapezoidal section, which of the following is NOT true?
   (a) The top width is twice the length of sloping side.
   (b) The hydraulic mean depth is half the depth of flow.
   (c) The shape is half of regular hexagon.
   (d) The depth must be equal to top width.

16. A very wide channel in laboratory is meant to deliver uniform flow rate of 2.0 m^3/s per metre width, assuming Manning's n = 0.02. Which one of the following combinations will not give uniform flow?
   (a) Flow depth 0.5 m, bed slope 1/62
   (b) Flow depth 1.0 m, bed slope 1/625
   (c) Flow depth 1.5 m, bed slope 1/2400
   (d) Flow depth 2.0 m, bed slope 1/4000

17. For a pipe of radius, r, flowing half full under the action of gravity, the hydraulic depth is
   (a) r
   (b) $\frac{\pi r}{4}$
   (c) $\frac{r}{2}$
   (d) 0.379r

18. A triangular irrigation lined canal carries a discharge of 25 m^3/sec at bed slope = $\frac{1}{6000}$. If the side slopes of the canal are 1 : 1 and Manning's coefficient is 0.018, the central depth of flow is equal to
   (a) 2.98 m
   (b) 3.62 m
   (c) 4.91 m
   (d) 5.61 m

19. A very wide rectangular channel is designed to carry a discharge of 5 m^3/s per meter width. The design is based on the Manning's equation with the roughness coefficient obtained from the grain size using Strickler's equation and results in a normal depth of 1.0 m. By mistake, however, the engineer used the grain diameter in mm in the Strickler's equation instead of in meter. What should be the correct normal depth?
   (a) 0.32 m
   (b) 0.50 m
   (c) 2.00 m
   (d) 3.20 m

Statement for Linked Answer Questions 20 and 21:

A rectangular open channel needs to be designed to carry a flow of 2.0 m^3/s under uniform flow conditions. The Manning's roughness coefficient is 0.018. The channel should be such that the flow depth is equal to half the width, and the Froude number is equal to 0.5.

20. The bed slope of the channel to be provided is
   (a) 0.0012
   (b) 0.0021
   (c) 0.0025
   (d) 0.0052

21. Keeping the width, flow depth and roughness the same, if the bed slope of the above channel is doubled, the average boundary shear stress under uniform flow conditions is
   (a) 5.6 N/m^2
   (b) 10.8 N/m^2
   (c) 12.3 N/m^2
   (d) 17.2 N/m^2

22. In a non-prismatic channel
(a) unsteady flow is not possible
(b) uniform flow is not possible
(c) the flow is always uniform
(d) the flow is not possible

23. In a uniform open channel flow
(a) the total energy remains constant along the channel
(b) the total energy line either rises or falls along the channel depending on the state of the flow
(c) the specific energy decreases along the channel
(d) the line representing the total energy is parallel to the bed of the channel

24. Uniform flow in an open channel exists when the flow is steady and the channel is
(a) prismatic
(b) non-prismatic and the depth of the flow is constant along the channel
(c) prismatic and the depth of the flow is constant along the channel
(d) frictionless

25. Uniform flow is not possible if the
(a) friction is large
(b) fluid is an oil
(c) $S_0 \leq 0$
(d) $S_0 > 0$

26. A rectangular channel of longitudinal slope 0.002 has a width of 0.80 m and carries an oil (rel density = 0.80) at a depth of 0.40 m in uniform flow mode. The average shear stress on the channel boundary in pascals is
(a) 3.14
(b) 6.28
(c) 3.93
(d) 0.01256

27. A triangular channel with a side slope of 1.5 horizontal; 1 vertical is laid on slope of 0.005. The shear stress in N/m^2 on the boundary for a depth of flow of 1.5 m is
(a) 3. 12
(b) 10.8
(c) 30.6
(d) 548

28. The dimensions of the Chezy coefficient C are
(a) $L^2 T^{-1}$
(b) $LT^{-1/2}$
(c) $M^0L^0T^0$
(d) $L^{1/2} T^{-1}$

29. The dimensions of of Manning's n are
(a) $L^{1/6}$
(b) $L^{1/2} T^{-1}$
(c) $L^{-1/3} T$
(d) $L^{-1/3} T^{-1}$

30. The dimensions of the Darcy - Weisbach coefficient f are
(a) $L^{1/6}$
(b) LT^{-1}
(c) $L^{-1/2} T^{-1}$
(d) $M^0L^0T^0$

31. If the bed particle size d_{50} of a natural stream is 2.0mm, then by Strickler formula, the Manning's n for the channel is about
(a) 0.017
(b) 0.023
(c) 0.013
(d) 0.044

32. The Manning's n for a smooth, clean, unlined, sufficiently weathered earthen channel is about
(a) 0.012
(b) 0.20
(c) 0.02

33. An open channel carries water with a velocity of 0.605 m/s. If the average bed shear stress is 1.0 N/m^2, the Chezy coefficient C is equal to

(a) 500 (b) 60
(c) 6.0 (d) 30

34. A trapezoidal channel had a 10 per cent increase in the roughness coefficient over years of use. This would represent, corresponding to the same stage as at the beginning, a change in discharge of

(a) + 10% (b) –10%
(c) 11% (d) –9.1%

35. A triangular section is hydraulically-efficient when the vertex angle θ is

(a) 90° (b) 120°
(c) 60° (d) 30°

36. In a gradually varied unsteady, open-channel flow dQ/dx = 0.10 If the top width of the channel is 10.0 m, $\partial A/\partial t$ is

(a) 0.1 (b) 0.01
(c) – 0.1 (d) – 0.01

Consider the following statements :

Of these statements

(a) both A and R are true and R is the correct explanation of A
(b) both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

37. **Assertion (A) :** For a hydraulically efficient channel, the hydraulic radius is equal to half the depth of flow.
Reason (R) : A hydraulically efficient channel has the minimum perimeter for a given area of flow.

38. **Assertion (A) :** An economical channel section gives maximum discharge for a given cross-sectional area.
Reason (R) : An economical channel section has smooth surface for reduced friction.

39. **Assertion (A) :** For steady uniform flow through a canal, the water surface will be parallel to the canal bed.
Reason (R) : The following are the equations for steady state uniform flow:

$$\frac{dy}{dt} = 0 \text{ and } \frac{dy}{ds} = 0$$

ANSWERS

1.	(a)	2.	(c)	3.	(b)	4.	(c)	5.	(d)	6.	(b)	7.	(b)
8.	(c)	9.	(d)	10.	(c)	11.	(c)	12.	(a)	13.	(a)	14.	(c)
15.	(d)	16.	(d)	17.	(c)	18.	(c)	19.	(b)	20.	(b)	21.	(d)
22.	(b)	23.	(d)	24.	(c)	25.	(c)	26.	(a)	27.	(c)	28.	(d)
29.	(c)	30.	(d)	31.	(a)	32.	(c)	33.	(b)	34.	(d)	35.	(a)
36.	(c)	37.	(d)	38.	(c)	39.	(a)						

HINT

1. As, $Q \propto \sqrt{S}$

$$\frac{Q_2}{Q_1} = \sqrt{\frac{S_2}{S_1}}$$

$$Q_2 = Q_1\sqrt{\frac{S_2}{S_1}}$$

$$Q_2 = 30\sqrt{\frac{0.0001}{0.0009}}$$

$$Q_2 = 10 \text{ m}^3/\text{sec.}$$

2. In trapazoidal section, for least perimeter

$$B + 2my = 2y\sqrt{1+m^2}$$

$$\text{if, } \tan\theta = m$$

$$B + 2 \times \tan\theta \times y = 2y\sqrt{1+\tan^2\theta}$$

$$B = 2y(\sec\theta - \tan\theta)$$

In rectangular section, for critical flow

$$v = \sqrt{gy}$$

In trapezoidal section, for critical flow

$$v = \sqrt{g \times \frac{A}{T}}$$

$$v = \sqrt{\frac{g(b+\tan\theta y)y}{b+2y\tan\theta}} \qquad \text{(if, } m = \tan\theta\text{)}$$

3. As, $Q \propto \sqrt{S}$

$$\frac{Q_2}{Q_1} = \sqrt{\frac{S_2}{S_1}}$$

$$Q_2 = Q_1\sqrt{\frac{S_2}{S_1}}$$

$$Q_2 = 15\sqrt{\frac{1400}{1000}} = 17.748 \simeq 18$$

4.
$$y = \frac{B}{2}$$
$$\frac{B}{y} = 2$$

5.
$$Q = CA\sqrt{RS}$$
$$Q = 60\times2\times1\sqrt{\frac{1}{2}\times\frac{1}{800}}$$
$$Q = 3 \text{ m}^3\text{/sec}$$

7.

Section	Hydraulic Radius
Rectangular	$R = \frac{A}{P} = \frac{2.4\times1}{4.4} = 0.545$
Circular	$R = \frac{A}{P} = \frac{\pi R^2}{2\pi R} = \frac{R}{2} = 0.5$
Triangle	$R = \frac{A}{P} = \frac{\frac{1}{2}\times2\times1}{2\sqrt{1+1}} = 0.354$
Trapezoidal	$R = \frac{A}{P} = \frac{(2+1\times1)\times1}{2+2\sqrt{2}} = \frac{3}{2+2\sqrt{2}} = 0.621$

8. In wide rectangular channel R = y

$$Q = \frac{1}{n}AR^{2/3}S^{1/2}$$
$$Q\times n = B\times y\times y^{2/3}S^{1/2}$$
$$y = \left(\frac{Q\times n}{B\times S^{1/2}}\right)^{3/5}$$
$$y = \left(\frac{0.5\times0.01}{\sqrt{0.0004}}\right)^{3/5} = 0.435\text{m}$$

9.

Section	Minimum Peremeter (P)	(P/h)
Triangular	2.83y	2.83
Trapezoidal	3.46y	3.46

10. $R \propto \frac{1}{P}$

14.
$$Q = \frac{1}{n} A \times R^{2/3} S^{1/2}$$

$$Q = \frac{S^{1/2}}{n} B \times y\, y^{2/3}$$

$S^{1/2}$ and n are constant

$$\frac{Q}{B} \propto y^{5/3}$$

$$q^{0.6} \propto y$$

16.
$$Q = \frac{1}{n} B \times y (y)^{2/3} S^{1/2}$$

$$2 \times 0.02 = y^{5/3} S^{1/2}$$

17.
$$R = \frac{A}{P} = \frac{\frac{\pi r^2}{2}}{\pi r} = \frac{r}{2}$$

18. For 1 : m = 1 : 1 triangular channel $R = \frac{y}{2\sqrt{2}}$

$$Q = \frac{1}{n} \times A \times R^{2/3} S^{1/2}$$

$$25 = \frac{1}{0.018} y^2 \frac{y^{2/3}}{(2\sqrt{2}))^{2/3}} \left(\frac{1}{6000}\right)^{1/2}$$

$$y^{8/3} = 25 \times 0.018 \times 2 \times (6000)^{1/2}$$

$$y = 4.91 \text{ m}$$

19. From Manning's formula

$$Q = \frac{1}{n} A \times R^{2/3} S^{1/2}$$

$$q = \frac{1}{n} y^{5/3} S^{1/2}$$

$$n \propto y^{5/3}$$

From Strickler formula $n \propto d_{50}^{1/6}$

$\therefore$
$$y^{5/3} \propto d_{50}^{1/6}$$

$$y \propto d_{50}^{1/10}$$

$$y_2 = y_1\left(\frac{(d_{50})_2}{(d_{50})_1}\right)^{1/10}$$

$$y = 1\times\left(\frac{0.001\times x}{x}\right)^{1/10}$$

$$y = 0.501$$

20. For a rectangular channel when y = B/2 channel is most efficient one. Hence R = y/2

$$F_r = \frac{v}{\sqrt{gy}}$$

$$Q = B\times y\sqrt{gy}\times F_r$$

$$Q = 2y^2\sqrt{g}y^{1/2}0.5$$

$$2 = 2\sqrt{9.81}\times 0.5\times y^{5/2}$$

$$y = 0.836$$

$$Q = \frac{1}{n}A\times R^{2/3}S^{1/2}$$

$$Q = \frac{1}{n}2y^2\left(\frac{y}{2}\right)^{2/3}S^{1/2}$$

$$\frac{2\times 0.018\times 2^{2/3}}{2\times 0.836^{8/3}} = S^{1/2}$$

$$S = 0.0021$$

21.

$$\tau_0 = \rho gRS$$

$$\tau \propto S$$

$$\tau = 1000\times 9.81\times\frac{0.836}{2}\times 0.0021 = 8.611$$

$$\tau_1 = \tau_0\frac{S_2}{S_1}$$

$$\tau_1 = 2\times\tau_0 = 17.222\ N/m^2$$

22. In a non prismatic channel, uniform flow is not possible.

23. For a uniform flow bed slope = water surface slope = energy line slope.

24. In a prismatic channel when the depth of flow is constant flow is said to be steady uniform flow.

25. For a adverse slope channel uniform flow is not possible.

26.

$$\tau = \rho gRS$$

$$\tau = 1000\times 0.8\times 9.81\times\frac{0.4}{2}\times 0.002$$

$$\tau = 3.139\ N/m^2 \simeq 3.14Pa$$

27.
$$R = \frac{A}{P} = \frac{my^2}{2y\sqrt{1+m^2}} = \frac{1.5\times1.5}{2\times\sqrt{1+1.5^2}} = 0.624$$
$$\tau = \rho gRS = 1000 \times 9.81 \times 0.624 \times 0.005 = 30.609$$

31.
$$n = \frac{d_{50}^{1/6}}{21.6} = \frac{(0.002)^{1/6}}{21.6} = 0.0164 \simeq 0.017$$

33.
$$\tau = \rho gRS$$
$$1 = 9810\ RS$$
$$RS = \frac{1}{9810}$$
$$v = C\sqrt{RS}$$
$$C = 0.605\times\sqrt{9810}$$
$$C = 59.92$$

34.
$$Q \propto \frac{1}{n}$$
$$Q = \frac{k}{n}$$
$$\frac{dQ}{dn} = \frac{-k}{n^2}$$
$$dQ = \frac{-1}{n}Qdn$$
$$dQ = \frac{-10\%}{1.1}Q$$
$$dQ = -9.09\%$$

36.
$$\frac{dQ}{dx} = -T\frac{dy}{dt}$$
$$\frac{dQ}{dx} = \frac{dA}{dy}\times\frac{dy}{dt}$$
$$\frac{dQ}{dx} = \frac{-dA}{dy}$$
$$\frac{dA}{dy} = -0.1$$

37. A hydraulic efficient channel has the minimum perimeter for a given area of flow. But the hydraulic radius depends on the type of cross section.

38. It is not necessary that in a channel section the lined surface ...

CHAPTER

3

Energy Depth Relationship

SPECIFIC ENERGY

- Specific energy is the total energy at a section w.r.t the channel bed as datum.
- Specific energy is expressed in terms of flow depth and velocity head.
- When the channel slope is small, specific energy is given by

$$E = y + \frac{v^2}{2g}$$

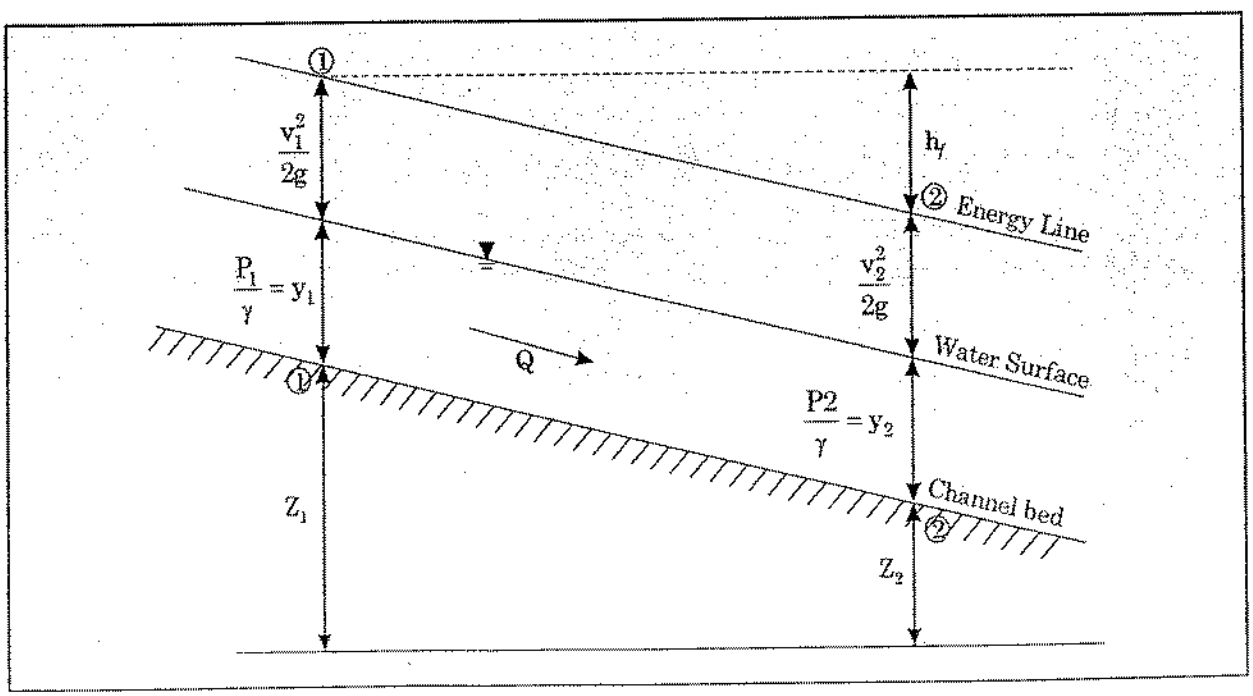
(Assuming K.E. correction factor as unity. Because channel flow will always be turbulent flow and for turbulent flow K.E. correction factor is approximately unity)

- When the channel slope is large, specific energy is given by

$$E = y\cos\theta + \frac{v^2}{2g}$$

Where,

θ = Bed slope of channel bottom.



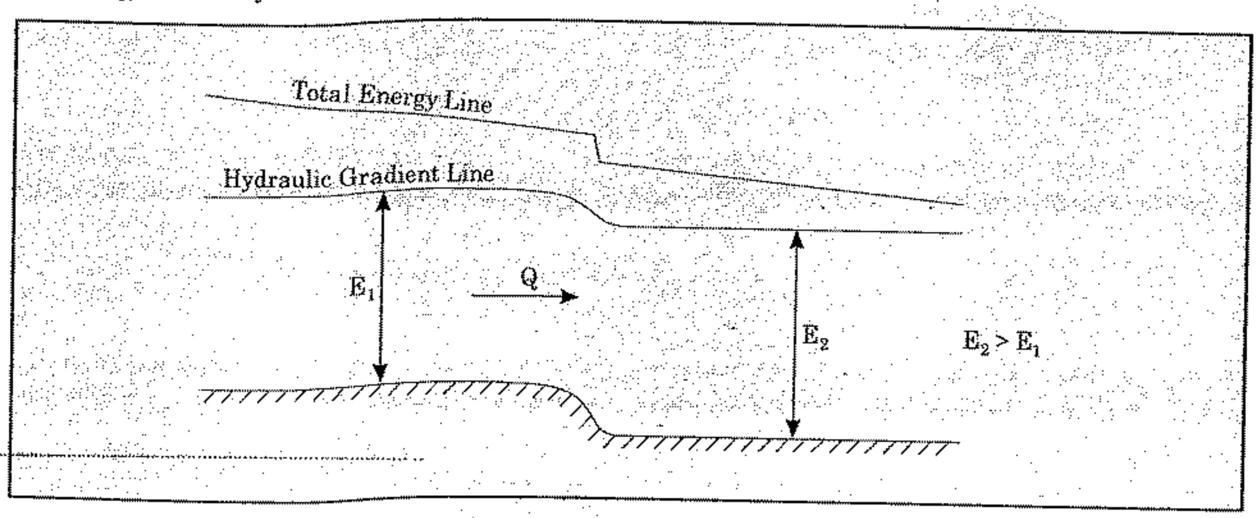
- Specific Energy at section 1-1

$$E_1 = y_1 + \frac{v_1^2}{2g}$$

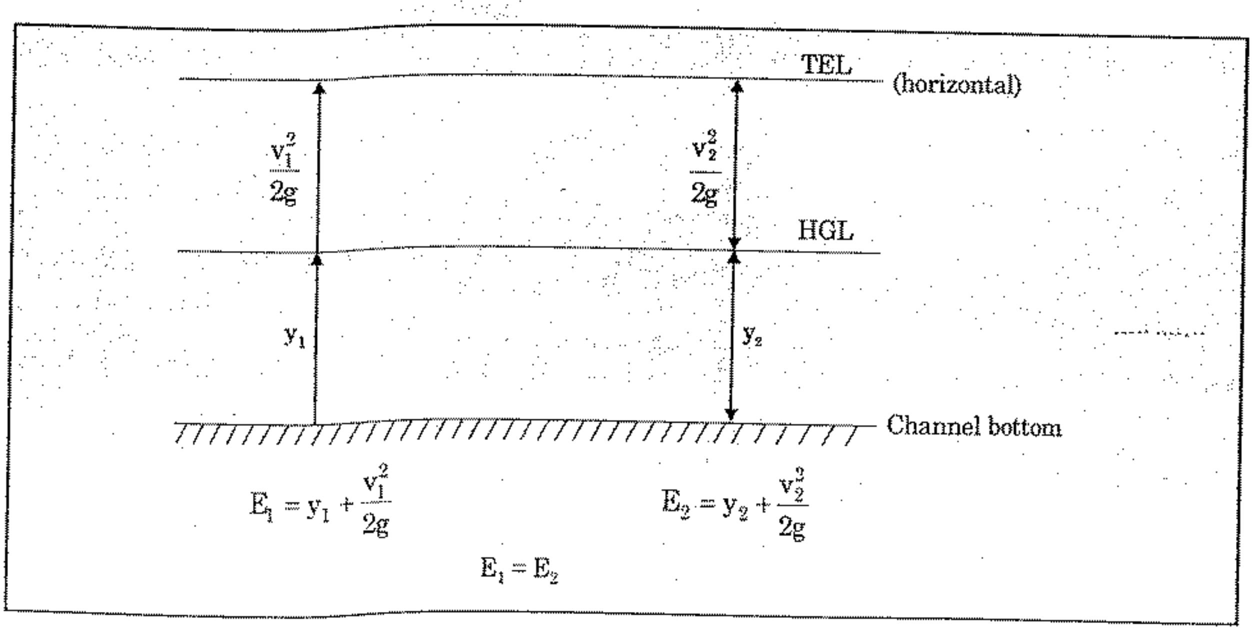
- Specific Energy at section 2 - 2

$$E_2 = y_2 + \frac{v_2^2}{2g}$$

- For uniform flow specific energy will be constant.
- For varied flow specific energy may either increase or decrease in the direction of flow. But total energy will always decrease in the direction of flow.

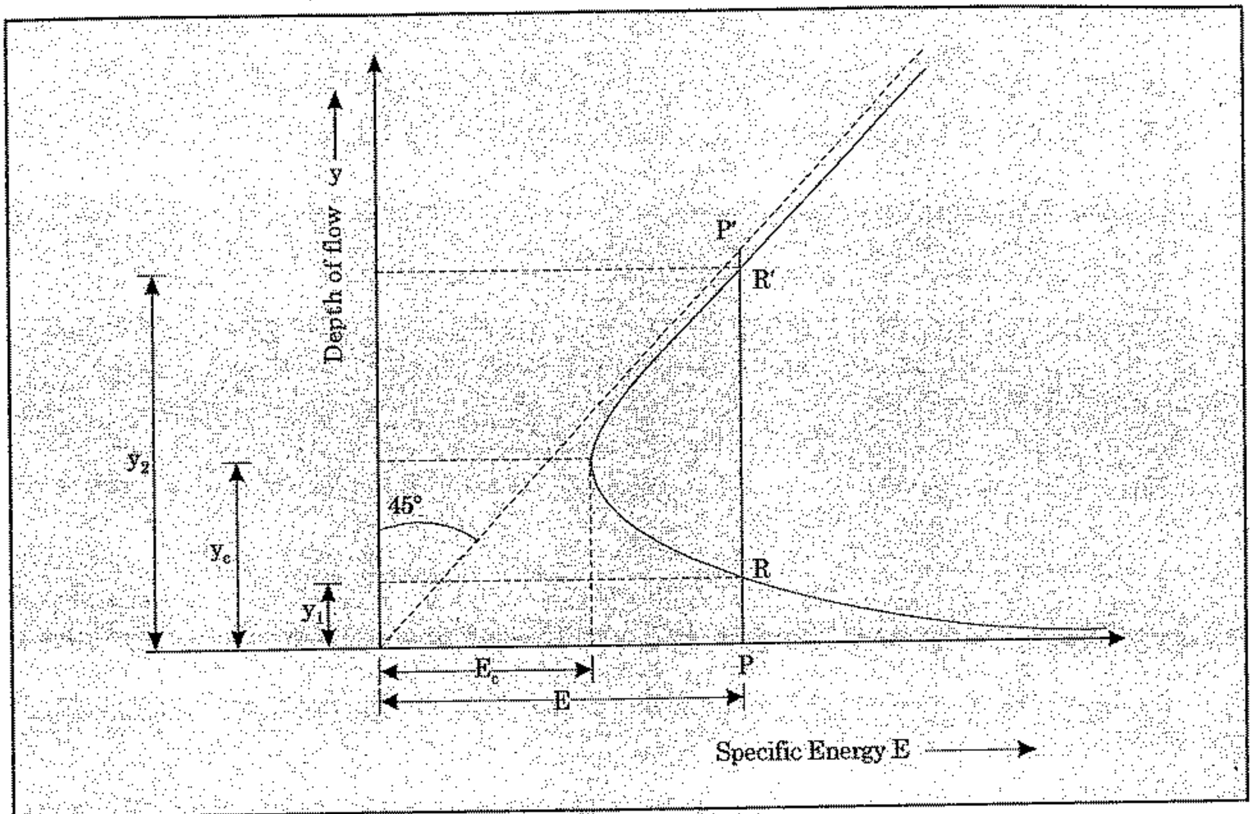


- For frictionless and horizontal channel specific energy will be constant.



Relationship between specific energy and depth of flow

- When the depth of flow is plotted against the specific energy for a given channel section and discharge, a specific curve is obtained.



- For a given specific energy E, there are two possible depth y_1 (PR) and y_2 (PR'), these are called the **Alternate depth**.
- As shown in the above figure for specific energy E there are two depths of flow.

  If, Depth of flow, y_1 = PR

  Then, Velocity head = RP'

  Also,

  If, Depth of flow, y_2 = PR'

  Then, Velocity head = R'P'
- Minimum specific energy (E_c) for a particular discharge Q corresponds to the critical state of flow. Hence, at the critical state of flow, the two alternate depths apparently becomes one, which is known as critical depth (y_c).

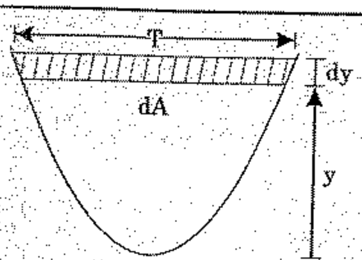
$$E = y + \frac{v^2}{2g}$$

$$\text{As, } v = \frac{Q}{A}$$

$$E = y + \frac{Q^2}{2gA^2}$$

- For E to be minimum at constant Q,

$$\frac{dE}{dy} = 0$$



$$\therefore \quad \frac{dE}{dy} = 1+\frac{Q^2}{2g}\frac{d}{dy}\left(A^{-2}\right)$$

$$\frac{dE}{dy} = 1+\frac{Q^2}{2g}\left(\frac{-2}{A^3}\right)\frac{dA}{dy}$$

dA can be written as Tdy. Therefore $\frac{dA}{dy} = T$, and the hydraulic depth of flow $y = \frac{A}{T}$

$$\therefore \quad \frac{dE}{dy} = 1-\frac{Q^2T}{gA^3}$$

$$1-\frac{Q^2T}{gA^3} = 0$$

$$\boxed{\frac{Q^2T}{gA^3} = 1}$$

$$\frac{\left(\frac{Q^2}{A^2}\right)\times T}{gA} = 1$$

$$\frac{v^2T}{gA} = 1$$

$$\frac{v}{\sqrt{g\frac{A}{T}}} = 1$$

- For a rectangular channel, we know that $\frac{A}{T} = y$

$$\frac{v}{\sqrt{gy}} = 1 \qquad \left[F_r = \frac{v}{\sqrt{gy}} = 1\right]$$

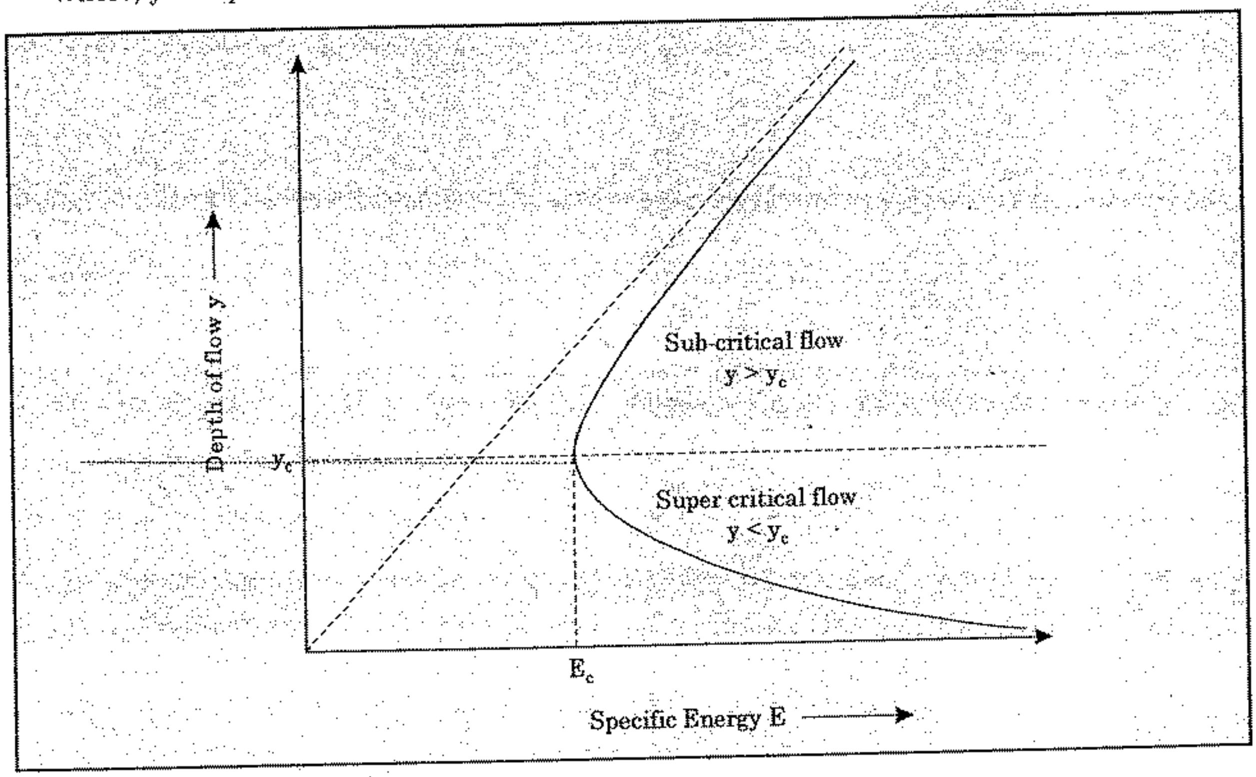
Thus, when the specific energy is minimum for a given discharge, flow will be critical flow and depth of flow will be called as critical depth y_c and velocity of flow is called as critical velocity v_c.

- When the depth of flow is greater than the critical depth, the velocity of flow is less than the critical velocity for the given constant discharge and hence flow is subcritical.

- When the depth of flow is less than the critical depth, the flow is supercritical.

Type of flow	Depth of flow condition	Froude Number $F_r = \frac{v}{\sqrt{gy}}$
Subcritical	$y > y_c$	$F_r < 1$
Critical	$y = y_c$	$F_r = 1$
Super critical	$y < y_c$	$F_r > 1$

Where, y is depth of flow

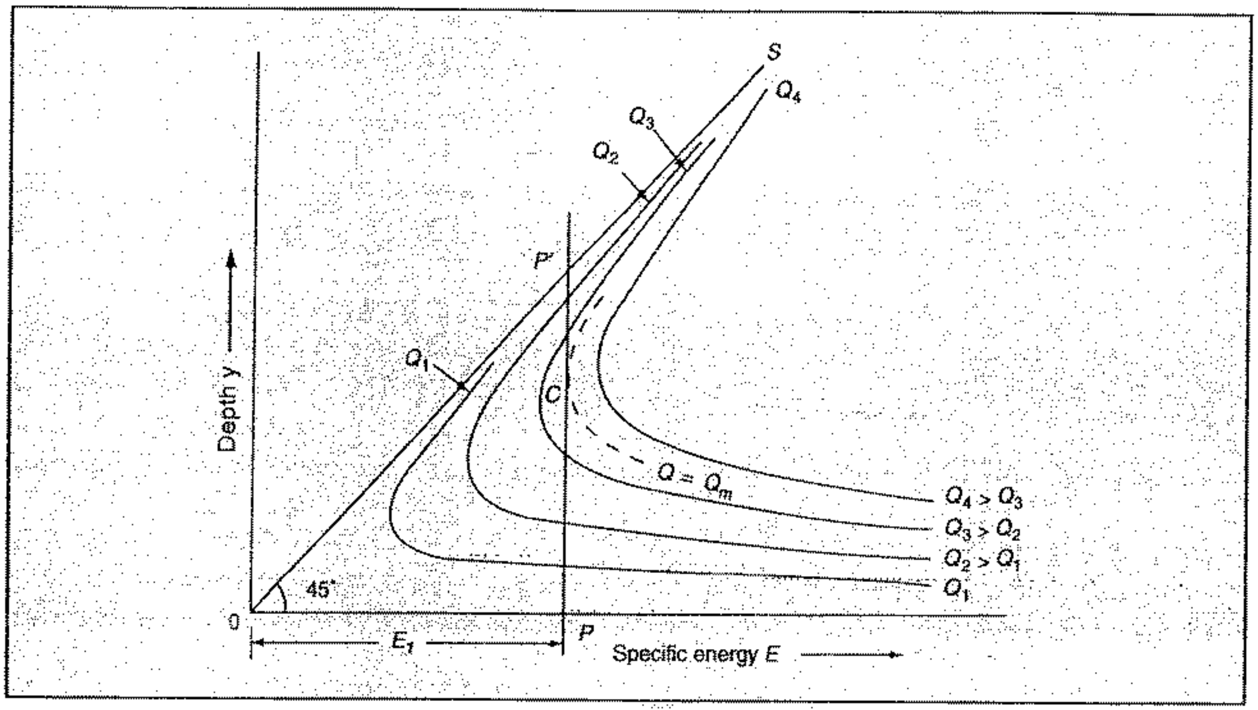


- A flow at or near the critical state is unstable. This is because a minor change in specific energy at or close to the critical state will cause a major change in depth.
- As the curve is almost vertical near the critical depth, a slight change in energy would change depth to a much greater alternate depth corresponding to specific energy after the change.
- It is also observed that, when the flow is near the critical state, the water surface appears unstable and wavy. Such phenomena are generally caused by the minor changes in energy due to variations in channel roughness, cross section, slope or deposits of sediment.

Note: For non prismatic channel, the channel section varies along the length of the channel and hence, the specific energy differs from section to section.

Relationship Between Discharge and Specific Energy

- In the previous section we have derived the condition for critical flow for constant discharge.
- Specific energy plot can be drawn for different constant discharges also.



- In the above figure $Q_1 < Q_2 < Q_4$, and is constant along the E vs y plot.
- Consider a section PP' in this plot.
- It is observed that different Q gives different intercepts and the difference between alternate depth decreases as the value of Q increases.
- Draw the specific energy curve for $Q = Q_m$ such that $Q_3 < Q_m < Q_4$. This curve will be just tangential to PP' at Point C.
- Specific energy curve for $Q = Q_m$ ($Q_3 < Q_m < Q_4$), represents the maximum value of discharge that can be passed in the channel while keeping the specific energy at a constant value E_1.
- Specific energy curve for $Q > Q_m$ will have no intercept on the line PP' and hence there will be no depth at which such discharge can be passed in the channel with the given specific energy.

$$E = y + \frac{Q^2}{2gA^2}$$

$$\therefore \quad Q = A\sqrt{2g(E-y)}$$

- Condition for maximum discharge for a given specific energy at a channel section (i.e E & A = Constant) can be found out by

$$\frac{dQ}{dy} = 0$$

$$\therefore \quad Q = A\sqrt{2g} \times \sqrt{E-y}$$

For maximum Q at a constant specific energy E,

$$\frac{dQ}{dy} = 0$$

$$\therefore \quad \frac{dQ}{dy} = \sqrt{2g}\left[\frac{dA}{dy}\times\sqrt{E-y}-\frac{A}{2\sqrt{E-y}}\right]$$

$$\frac{dA}{dy}\times\sqrt{E-y}-\frac{A}{2\sqrt{E-y}} = 0$$

$$\frac{dA}{dy}(2(E-y)) = A$$

We know that, $\frac{dA}{dy} = T$ and $E-y=\frac{v^2}{2g}$

$$T\,[2\,(E-y)] = A$$

$$T\times 2\times\frac{v^2}{2g} = A$$

$$\frac{Tv^2}{g} = A \qquad \left[v=\frac{Q}{A}\right]$$

$$\frac{TQ^2}{gA^2} = A$$

$$\boxed{\frac{Q^2T}{gA^3}=1}$$

- $\frac{Q^2T}{gA^3}=1$ corresponds to critical flow condition therefore critical flow conditon corresponds to the condition for maximum discharge in a channel for a fixed specific energy.

Note: $\frac{Q^2T}{gA^3}=1$, is the condition such that

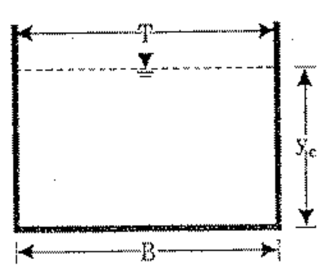
(a) For a given discharge, specific energy to be min.

(b) For a given specific energy, discharge to be maximum.

CALCULATION OF CRITICAL DEPTH

Rectangular Channel Section

Consider a rectangular channel section of width = B and having constant discharge Q under a having critical depth of flow $y = y_c$.



$$\text{Area of flow, } A = B\,y_c$$
$$\text{Top width, } T = B$$

For critical flow condition

$$\frac{Q^2T}{gA^3} = 1$$

$$\frac{Q^2B}{gB^3y_c^3} = 1$$

$$y_c^3 = \frac{Q^2}{B^2g}$$

Let discharge per unit width (B), $q = \frac{Q}{B}$

$$\therefore \quad y_c^3 = \frac{q^2}{g}$$

$$\boxed{y_c = \left(\frac{q^2}{g}\right)^{1/3}}$$

Specific energy at critical depth y_c, will be

$$E_c = y_c + \frac{v_c^2}{2g}$$

$$E_c = y_c + \frac{\left(\frac{q}{y_c}\right)^2}{2g}$$

$$E_c = y_c + \frac{q^2}{2g\,y_c^2} \qquad \left[\frac{q^2}{g} = y_c^3\right]$$

$$E_c = y_c + \frac{y_c^3}{2y_c^2}$$

$$E_c = y_c + \frac{y_c}{2}$$

$$\boxed{E_c = \frac{3}{2}y_c} \text{ For Rectangular section}$$

∴ Note that specific energy at critical depth of flow is independent of width of channel section.

- Froude number for rectangular channel will be

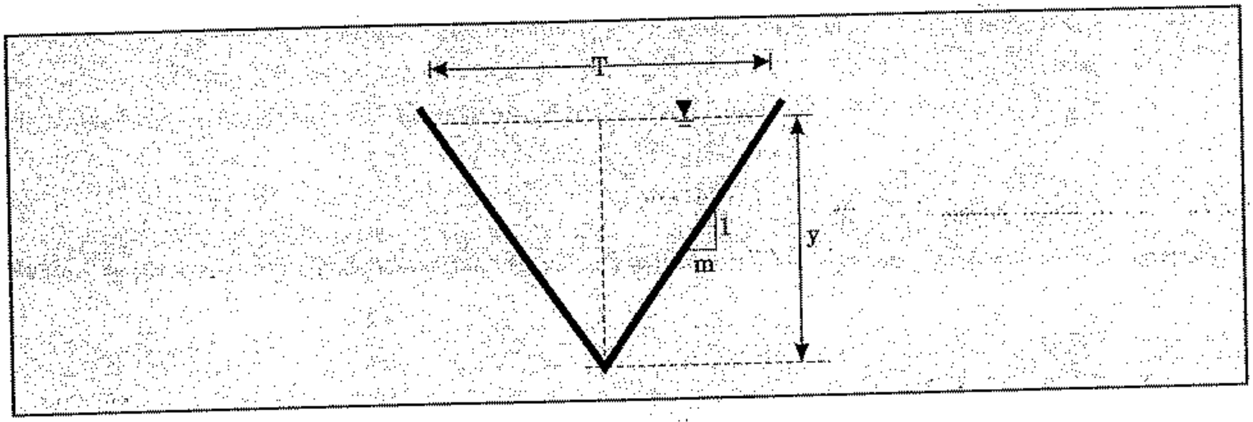
$$F_r = \frac{v}{\sqrt{g \times \frac{A}{T}}} \qquad \left[\frac{A}{T} = y\right]$$

$$\therefore \quad \boxed{F_r = \frac{v}{\sqrt{gy}}}$$

$$\therefore \quad \text{For critical flow } \frac{v}{\sqrt{gy}} = 1$$

Triangular Channel section

Consider a triangular channel section having constant discharge Q under a critical depth of flow y = y_c and side slope 1 : m (V : H)



$$\text{Area of flow, } A = my_c^2$$

$$\text{Top width, } T = 2my_c$$

For critical flow,

$$\frac{Q^2T}{gA^3} = 1$$

$$\frac{Q^2 2my_c}{gm^3y_c^6} = 1$$

$$\frac{2Q^2}{gm^2} = y_c^5$$

$$\boxed{y_c = \left(\frac{2Q^2}{gm^2}\right)^{1/5}}$$

Specific energy at critical depth y_c, will be

$$E_c = y_c + \frac{v_c^2}{2g}$$

$$E_c = y_c + \frac{Q^2}{2gA^2}$$

$$E_c = y_c + \frac{\frac{A^3}{T}}{2A^2}$$

$$E_c = y_c + \frac{A}{2T}$$

$$E_c = y_c + \frac{my_c^2}{2 \times 2my_c} \qquad \left[E_c = y_c + \frac{A_c}{2T_c}\right]$$

$$E_c = y_c + \frac{y_c}{4}$$

$$\boxed{E_c = 1.25y_c} \text{ for triangular section}$$

Froude number for Triangular section will be

$$F_r = \frac{v}{\sqrt{g\frac{A}{T}}}$$

$$F_r = \frac{v}{\sqrt{g\frac{y_c}{2}}}$$

$$\boxed{F_r = \frac{\sqrt{2}\ v}{\sqrt{g\,y_c}}} \text{ for triangular section.}$$

For critical flow, $\frac{\sqrt{2}\ v}{\sqrt{g\ y_c}} = 1$

Note: General formula for E_c

$$\boxed{E_c = y_c + \frac{A_c}{2T_c}}$$

RELATION BETWEEN DISCHARGE & DEPTH OF FLOW

- For a rectangular channel the specific energy equation is

$$E = y + \frac{v^2}{2g}$$

$$E = y + \frac{Q^2}{2A^2g} \qquad [Q = vA]$$

$$E = y + \frac{(q \times B)^2}{2g(By)^2} \qquad [A = B \times y]$$

Where,

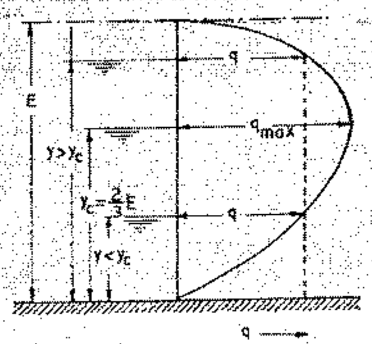
$q = \frac{Q}{B}$, discharge per unit width

$$E = y + \frac{q^2}{2gy^2}$$

$$\boxed{q = \sqrt{2g(E-y)y^2}} \quad ...(A)$$

- From the previous discussion in this chapter we know that at critical flow condition discharge per unit width is maximum.

  Using Equation A, a plot between q and y can be prepared for a given specific energy E. This curve is called as Discharge diagram.



- It is observed that there are two possible depth for a certain value of q, as long as q is less than a certain value.
- At maximum discharge the two depths become one, equal to the critical depth y_c.
- If $y < y_c$, supercritical flow.
- If $y > y_c$, subcritical flow.

SECTION FACTOR Z

- Section factor, z is a function of depth y for a given channel geometery.

$$\boxed{z = A\sqrt{\frac{A}{T}}}$$

At Critical flow condition $y = y_c$

$$\therefore \quad Z_c = A_c\sqrt{\frac{A_c}{T_c}}$$

As $$\frac{Q^2T}{gA^3} = 1, \text{ for critical flow}$$

$$\frac{A^3}{T} = \frac{Q^2}{g}$$

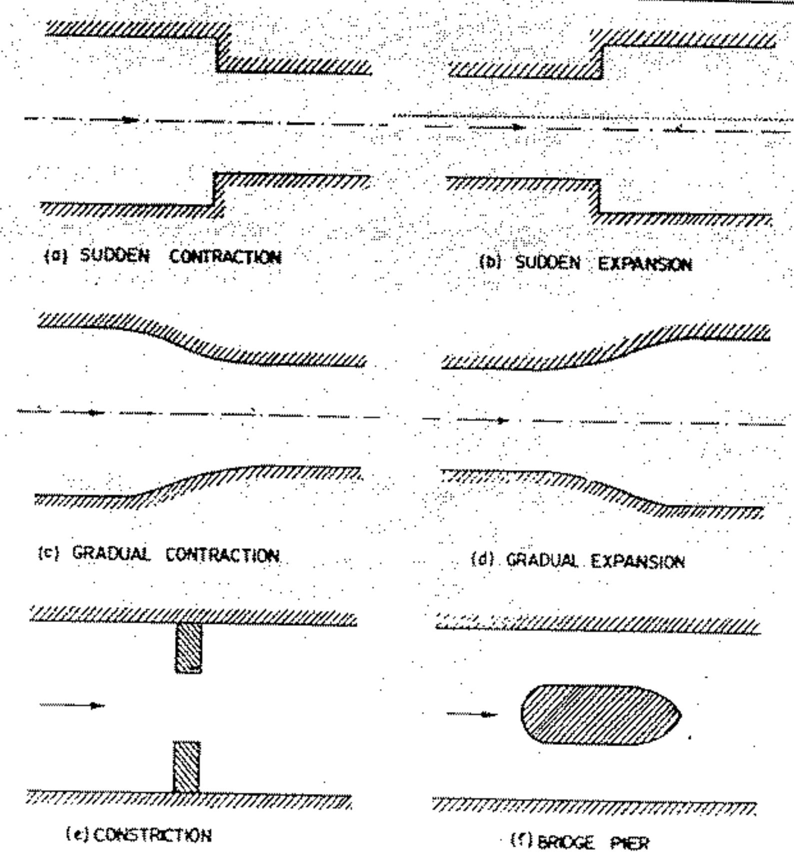
$$z_c = \frac{Q_c}{\sqrt{g}}$$

$$\boxed{Q_c = z_c\sqrt{g}}$$

- Q_c is the discharge that would make the depth flow y critical, and is known as critical discharge.

CHANNEL TRANSITION

- A transition is the portion of a channel with varying cross section, which connects one uniform flow channel to another uniform flow channel.
- This variation of a channel section may be caused either by reducing or increasing the width, or by raising or lowering the bottom of the channel.
- Some use of channel transition are metering of flow, dissipation of energy, reduction or increase of velocities, change in channel section or alignment with minimum of energy dissipation.
- When the change in cross sectional dimensions of the channel occurs in a relatively short length then the transition is termed as sudden transition.
- When the change of cross sectional area of the channel takes place gradually in a relatively long length of channel then the transition is termed as gradual transition.



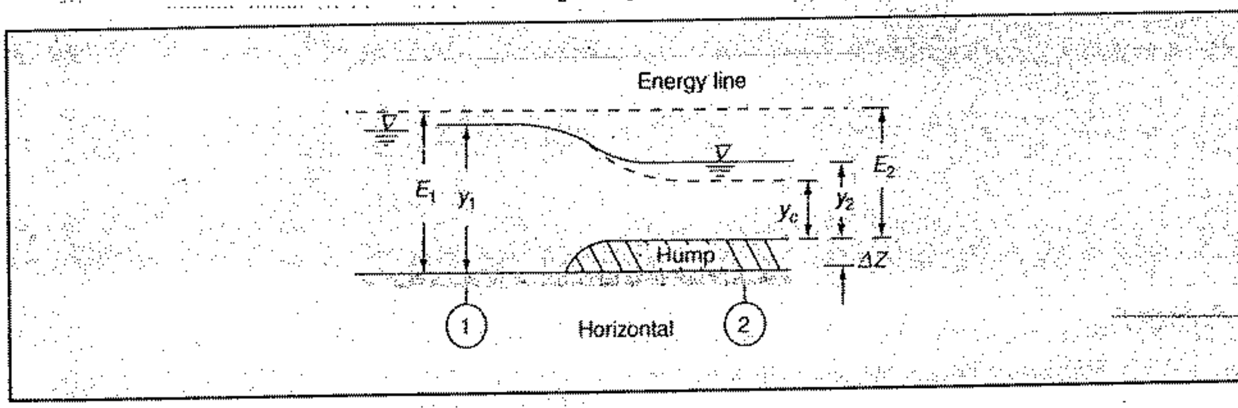
- The concept of specific energy and critical depth are extremly useful in analysis of problems assosiated with channel transitions.

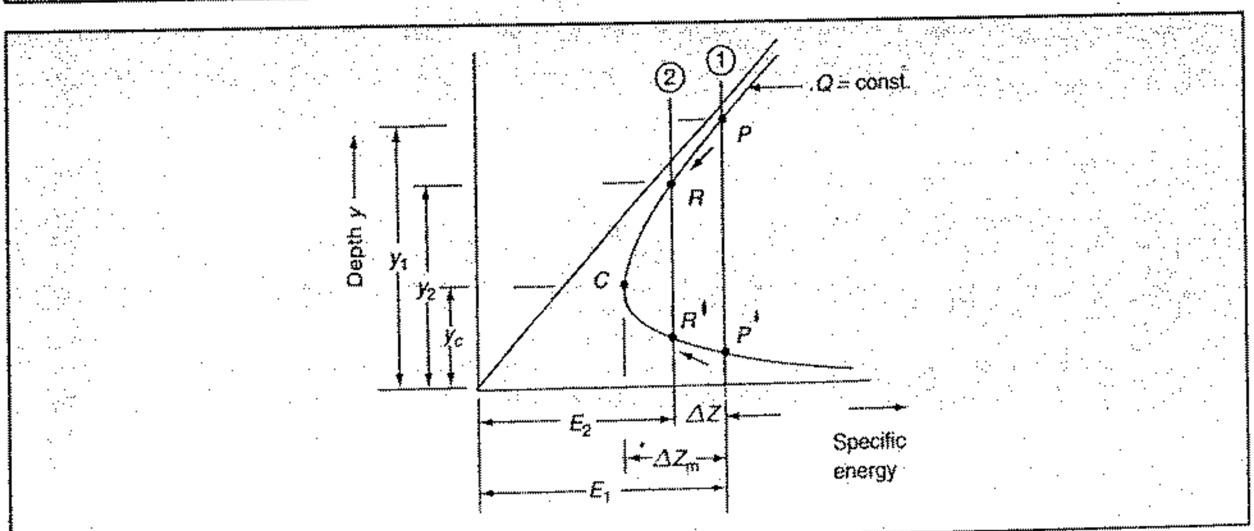
- While analyzing channel transitions following assumptions are made.
  1. Channel bed is horizontal and frictionless i.e. T.E.L will be parallel to the channel bed.
  2. There is no loss of energy takes place between any two section in channel.

Transitions with raised bottom in a rectangular channel.

- Consider a horizontal frictionless channel of rectangular cross section having uniform width B and provided with a rise in the bottom of the channel which is frequently known as hump.
- As the width of channel is not changing, the discharge per unit width will be same at different sections for a total discharge Q flowing through the channel.
- Consider two sections (1) & (2) at some distance l in the channel.
- Since the specific energy is measured w.r.t channel bed as datum, a rise in the bottom of channel causes a decrease in specific energy. Therefore, specific energy E_2 at the hump will be less than the specific energy E_1 at section1, by an amount equal to the height of hump ΔZ.

$$E_2 = E_1 - \Delta Z$$





Case I : Subcritical flow at section 1 – Free surface drops down at hump

- It can be observed from the specific energy V/S Depth of flow plot that if the flow is in subcritical state, a decrease in specific energy is associated with a decrease in depth of flow and increase in velocity.
- Therefore at hump, depth of flow decreases and the velocity head increases.
- Let the depth of flow at section 1 be y_1 and depth of flow at section 2 be y_2.

- If the height of hump at section 2, increases further the depth of flow and specific energy at section 2 decreases.
- But for a constant discharge Q this reduction in specific energy is limited to the critical depth y_c. At this point the height of hump will be maximum, say $\Delta Z = \Delta Z_{max}$, $y = y_c$ = critical depth and $E_2 = E_c$.

$$E_2 = E_1 - \Delta Z_{max} = E_c$$

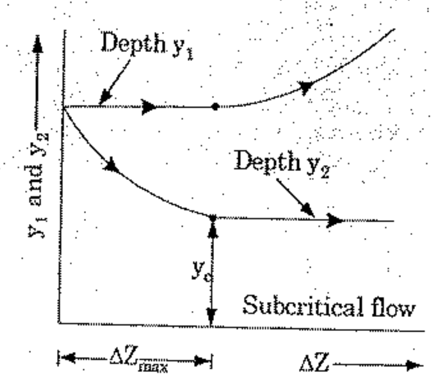
- Now a question arises that, what happens when $\Delta Z > Z_{max}$?
- If the height of the hump is increased further such that $\Delta Z > \Delta Z_{max}$, then in order to pass the same discharge the specific energy will have to be increased other wise flow is not possible with given conditions and the flow in the section channel is said to be choked.
- In this case for subcritical flow, the approaching the hump, the required increase in the specific energy is provided with the increase in depth of flow at section 1.
- If the specific energy is held constant with the height of hump more than the ΔZ_{max} value, the discharge is decreased until the given specific energy is equal to the minimum specific energy corresponding to the new discharge

$$E_1' - \Delta Z = E_2 = E_c$$

$$E_c = y_c + \frac{Q^2}{2gB^2y_c^2}$$

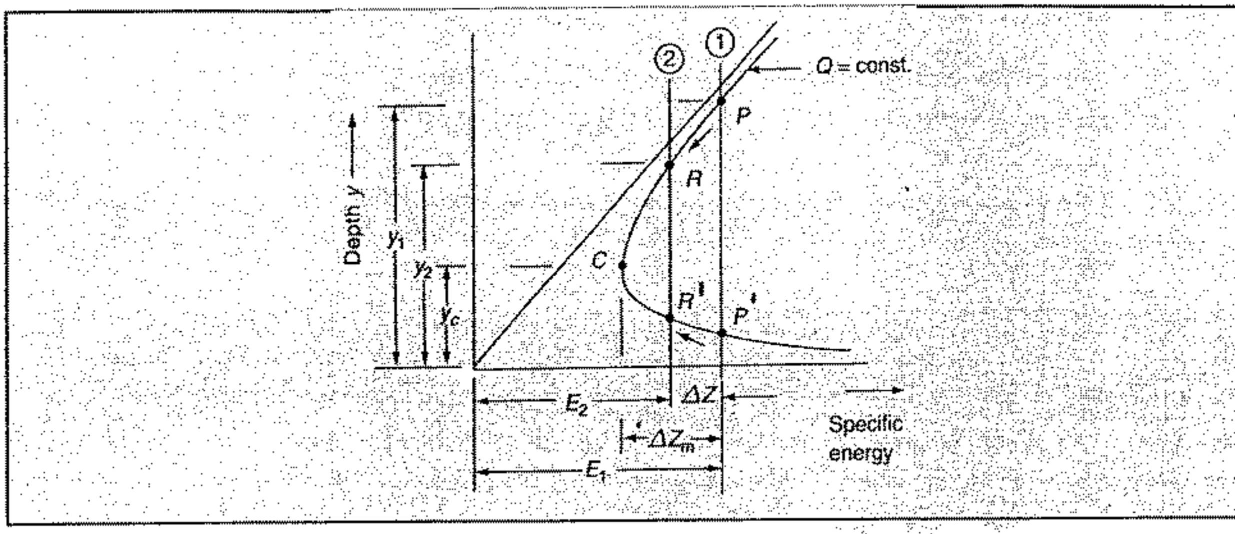
Where E_1' new specific energy at section 1.

- Summarising all the various sequences for subcritical flow, when $0 < \Delta Z < \Delta Z_{max}$ the upstream water level remains stationary at y_1 while the depth of flow at section 2 decreases with y reaching a minimum value of y_c at $\Delta Z = \Delta Z_{max}$. With further increase in the value of ΔZ, i.e $\Delta Z > \Delta Z_{max}$, y_2 will continue to remain at y_c and y_1 will increase to y_1', to have higher specific energy E_1'.



Case II : Super critical flow at section 1–free surface rise at hump

- If the flow at section 1 is in supercritical state, the depth of flow increases at the hump.
- This increase in depth of flow can be explained from the specific energy V/S depth of flow, plot.



- It is observed that for a super critical flow, a decrease in specific energy is associated with an increase in the depth of flow and decrease in velocity head.
- Let the depth of flow at section 1 be y_1 and depth of flow at section 2 be the y_2.
- If for a constant discharge, the height of hump is increased at section 2, the depth of flow increases corresponding to decreasing specific energy at section 2.
- As discussed earlier this reduction in specific energy is limited to the critical depth y_c and at this point height of hump will be maximum, say equal to ΔZ_{max}, $y = y_c$ = critical depth and $E_2 = E_c$

$$E_2 = E_1 - \Delta Z_{max} = E_c$$

- As observed in the case of subcritical flow, when $\Delta Z > \Delta Z_{max}$, flow in the channel got choked, then in order to pass the same discharge specific energy was increased at section 1.
- In case of supercritical flow, approaching the hump, the required increase in the specific energy is provided with the decrease in depth of flow at section 1.

Note: In case of subcritical flow, the required increase in specific energy was provided with the increase in depth of flow at section 1.

- Similarly like subcritical flow, if the specific energy is held constant with $\Delta Z > \Delta Z_{max}$, the choking condition arrives and discharge is decreased until the given specific energy is equal to the minimum specific energy corresponding to the new discharge.

$$E_1' - \Delta Z = E_2 = E_c$$

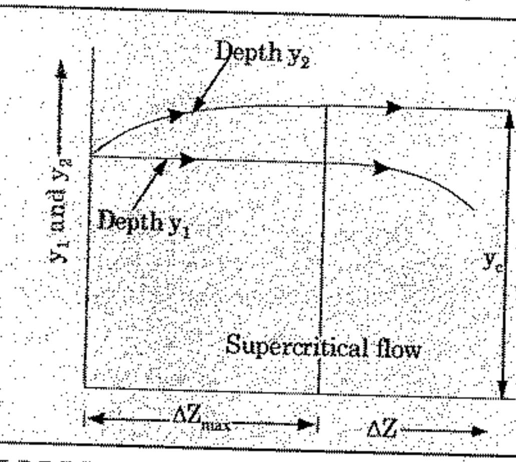
$$E_c = y_c + \frac{Q^2}{2gB^2y_c^2}$$

Where,

E_1' = New specific energy at section 1.

- Summarising all the various sequence for supercritical flow, when $0 < \Delta Z < \Delta Z_{max}$ depth of flow y_1 at section 1, is constant while the depth of flow y_2 at section 2 increase upto the critical depth y_c.

- Further for $\Delta Z > \Delta Z_{max}$, the depth fo flow over hump $y_2 = y_c$ will remain constant and the upstream depth y_1 decreases to y_1', to have a higher specific energy E_1'.



Note: Flow over broad crest wier is a critical flow.

Size of Maximum Hump for Critical flow

- Relation between specific energies at a section upstream of hump (E_1) and at the section on the hump (E_2) is given as

$$E_1 = E_2 + \Delta Z$$

- Let ΔZ_{max} be the height of hump that will cause the critical flow at hump without changing the upstream specific energy.
- Note that any value of $\Delta Z > \Delta Z_{max}$ will also cause the critical flow over the hump but the upstream specific energy will get change.
- Therefore ΔZ_{max} is called as the maximum height of hump required to cause the critical flow over the hump without changing the upstream flow conditions
- Expression for ΔZ_{max} is given as below.

$$\Delta Z = \Delta Z_{max}$$

$$E_2 = E_c = \frac{3}{2} y_c$$

$$E_1 = E_2 + \Delta Z_{max}$$

$$E_1 = \frac{3}{2} y_c + \Delta Z_{max}$$

$$\boxed{\Delta Z_{max} = E_1 - \frac{3}{2} y_c}$$

Energy Loss due to Hump

- If there is a energy loss taking place between upstream and down stream sections equal to h_L.
- We can consider a new hump of height ($\Delta Z + h_L$) in place of original hump of ΔZ and proceed the calculation.
- New specific energy equation will be

$$\boxed{E_1 = E_2 + \Delta Z + h_L}$$

Transition with reduction of width in a rectangular channel

- Consider a friction less horizontal channel of width B_1 carying a discharge Q at a depth of flow y_1. At section 2 the channel width is been reduced to B_2 by a smooth transition.
- As there are no energy losses involved and since bed elevations are same at section 1 and section 2, the specific energy at both the section is same.
- As the total discharge Q remains same between section 1 and 2. But discharge per unit width changes between section 1 and section 2.

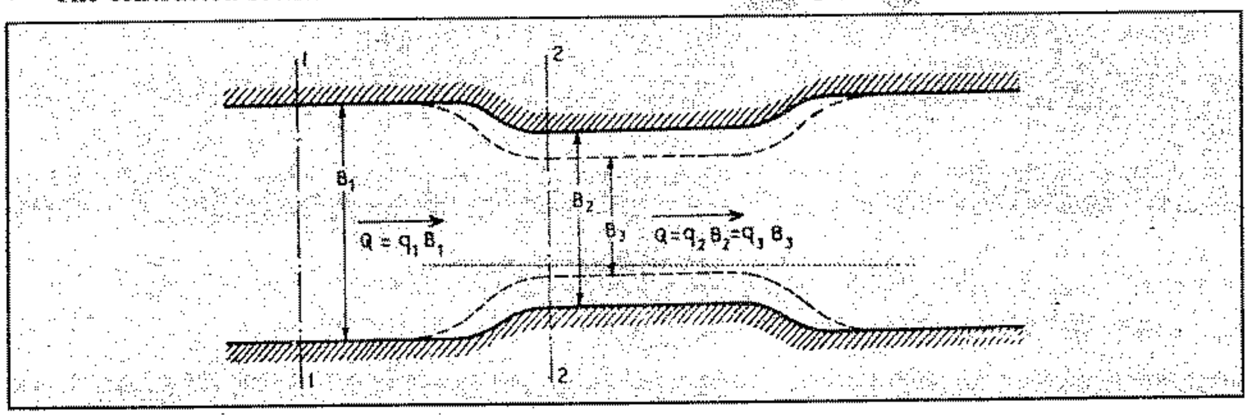
Discharge per unit width at section 1 ($B = B_1$), $q_1 = \frac{Q}{B_1}$

Discharge per unit width at section 2 ($B = B_2$), $q_2 = \frac{Q}{B_2}$

$\because$ Q = Constant and $B_1 > B_2$

$\therefore$ $q_1 < q_2$

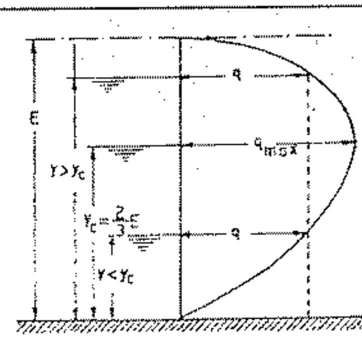
- The contracted section 2 – 2 is called as throat section or simply throat.



- Further if width of section is reduced to B_3, the discharge per unit width is increased to $q_3 = \frac{Q}{B_3}$

Case I : Subcritical flow at section 1 - free surface rise at section 2.

- As the approaching flow at section 1 is in subcritical state and depth of flow y_1, discharge per unit width $q_1 = \frac{Q}{B_1}$.
- At section 2, width of section gets reduced to B_2 and hence discharge per unit width increase to $q_2 > q_1$.



- It is observed that for subcritical state of flow if discharge per unit width of flow increases, the depth of flow will decrease.
- Therefore now new depth of flow at section 2 will be $y_2 < y_1$.
- This depth of flow decrease upto a certain minimum value y_c corresponding to which discharge per unit width is maximum equal to q_{max} and width of throat equal to B_{min}.
- We know that further increase in q beyond q_{max} is not possible for the same value of specific energy E.
- In case of subcritical flow, when the width of section is reduced further $B_3 < B_{min}$, then $q_3 > q_{max}$
- To pass this discharge, $q_3 > q_{max}$ from section 2, there will be rise in the free surface on the section 1 with new depth of flow equals to $y_1' > y_1$ so that new specific energy $E_1' > E_1$ is formed, which will be sufficient to cause critical flow at section 2.
- It should be noted that although flow at section 2 will be critical but depth of flow will not be equal to y_c. A new critical depth of flow y_c' will be formed such that $y_c' > y_c$.
- We know that specific energy at section 1 = E_1' and $E_1' > E_1$

  As no loss in energy between section 1 & 2.

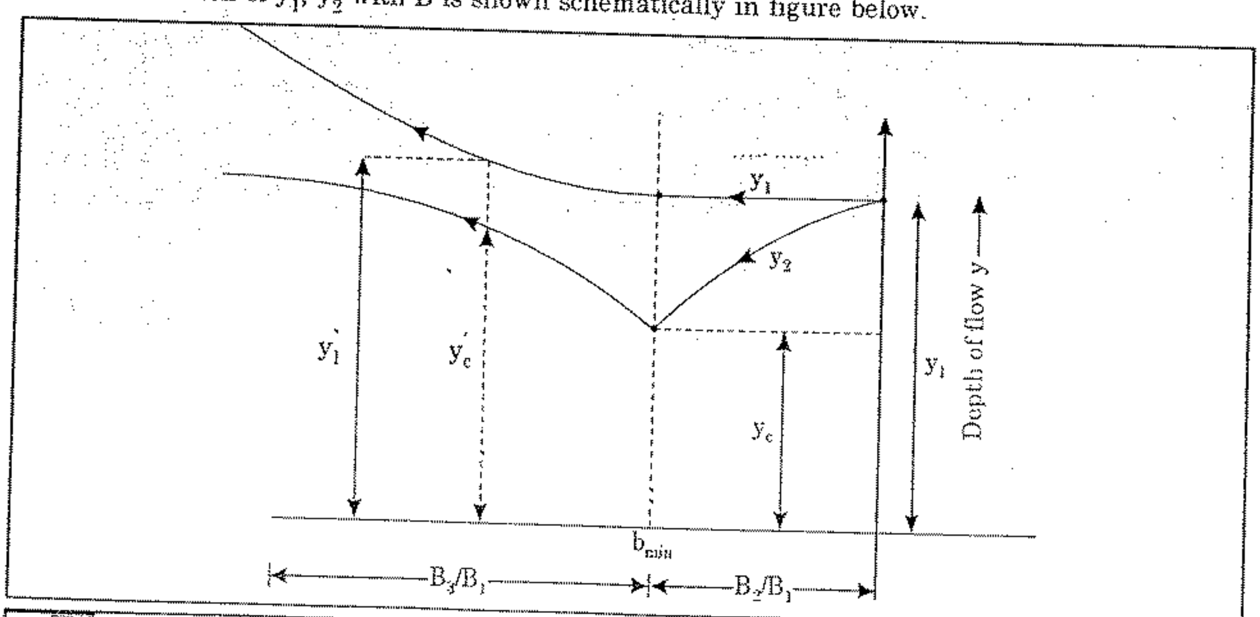
$$E_1' = E_c'$$

  E_c' = new specific energy at section 2.

  As $E_1' > E_1$, therefore $E_c' > E_c$

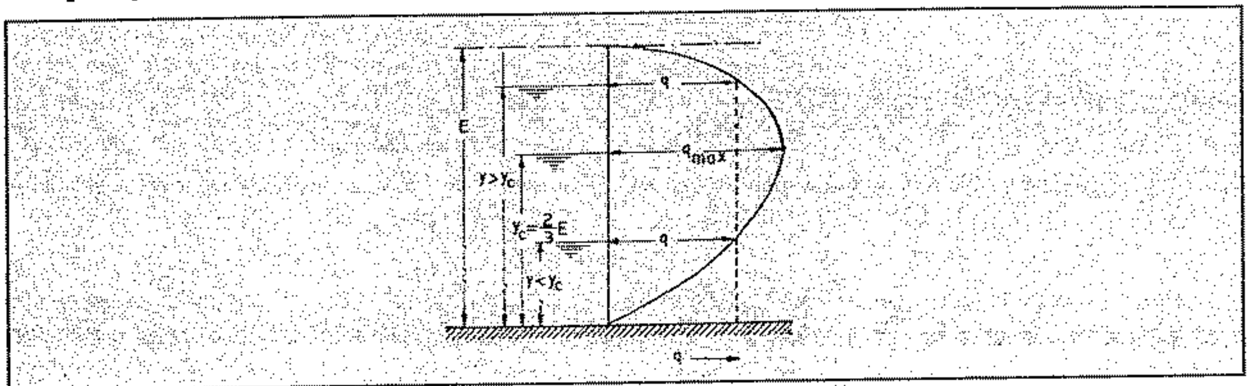
  $E_c' = \frac{2}{3} y_c'$

  Therefore, $y_c' > y_c$
- Here by we can conclude that in subcritical state of flow if width of the throat section $b_3 < b_{min}$ there will be critical flow at section 2 such that new critical depth of flow y_c' will be greater than previous critical depth of flow y_c. At section 1 also new depth of flow $y_1' > y_1$ (previous depth of flow for $q = q_{max}$).
- The variation of y_1, y_2 with B is shown schematically in figure below.



Case II : Supercritical flow at section 2 - free surface rise at section 2

- As the approaching flow at section 1 is in subcritical state having depth of flow y_1 and discharge per unit width $q_1 = \frac{Q}{B_1}$.
- Now, if at section 2 width of section gets reduced to B_2, discharge per unit width increase to $q_2 > q_1$.



- It is observed that for super critical flow if discharge per unit width of flow increases, the depth of flow will increase.
- Therefore, now new depth of flow at section 2 will be $y_2 > y_1$.
- This depth of flow increases upto y_c corresponding to which discharge per unit width is maximum equal to q_{max} and width of throat equal to B_{min}.
- We know that, further increase in q beyond q_{max} is not possible for same value of specific energy E.
- In case of supercritical flow, when the width of section is reduced further to $B_3 < B_{min}$ then $q_3 > q_{max}$.
- To pass this discharge $q_3 > q_{max}$ from section 2, there will be fall in free surface on the section 1 with new depth of flow equal to $y_1'(y_1' < y_1)$, so that new specific energy $E_1' > E_1$ is formed, which will be sufficient to cause critical flow at section 2.
- Similarly like subcritical flow case y_c will not be constant. A new critical depth $y_c' > y_c$ will be formed.

  As new specific energy at section 1, $E_1' > E_1$

  As no loss in energy between section 1 & 2, $E_1' = E_c'$

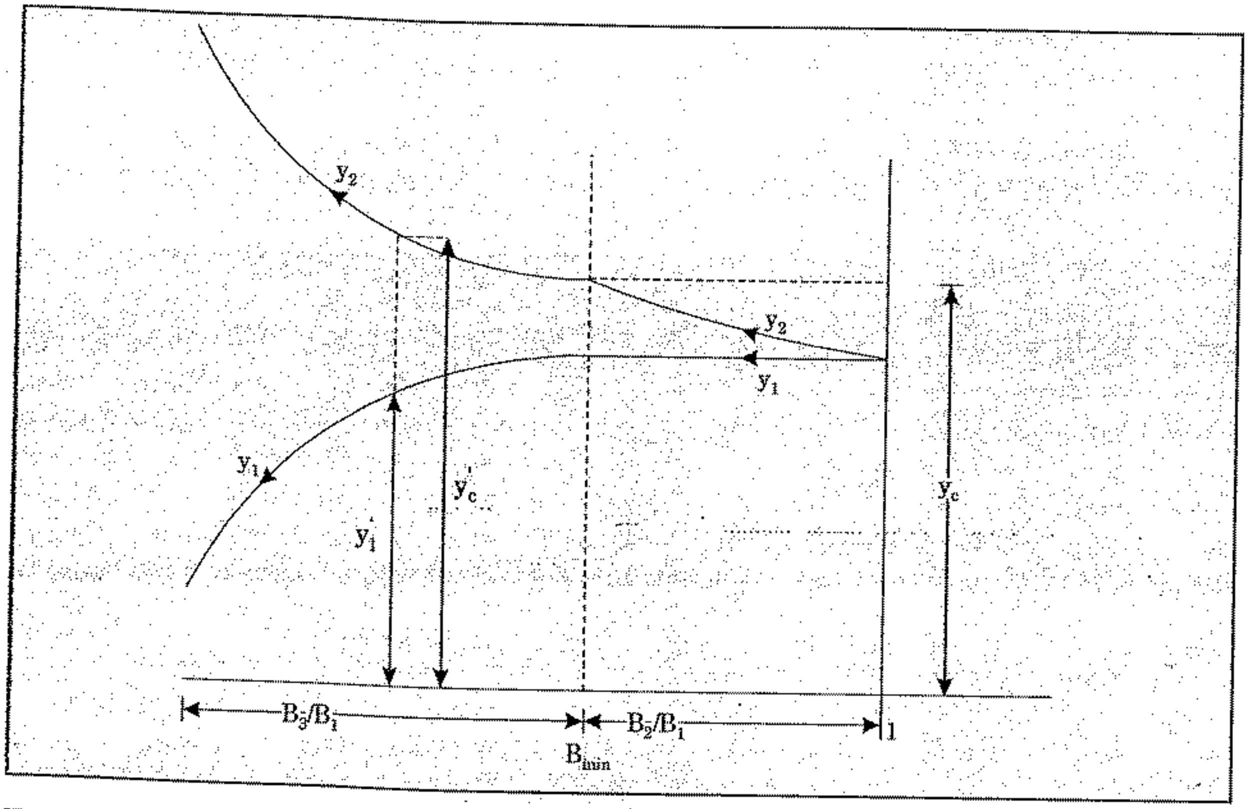
  As, $E_1' > E_1, E_c' > E_c$

  $$E_c' = \frac{2}{3}y_c'$$

  $$\therefore \quad y_c' > y_c$$

- Here by we conclude that in supercritical state of flow if width of throat section $b_3 < b_{min}$ there will be critical flow at section 2 such that $y_c' > y_c$. At section new critical depth, $y_1' < y_1$ will be formed.

- The variation of y_1, y_2 with width B is shown schematically in figure below.



IMPORTANT POINTS ABOUT CRITICAL FLOW

The various characteristics of the critical state of flow through a channel section.

1. Specific energy is minimum for a given discharge.
2. Discharge is maximum for a given specific energy.
3. Velocity head is equal to half of the hydraulic depth in a rectangular channel of small slope.
4. Froude number is equal to unity.
5. Specific force is minimum for a given discharge.
6. Discharge is maximum for a specific force.

We know that, specific force, $F = \dfrac{P+M}{\gamma}$

$$F = \frac{\gamma A\bar{y} + \rho Q v}{\gamma}$$

$$F = A\bar{y} + \frac{Q^2}{gA} \qquad \left[v = \frac{Q}{A}\right]$$

For a minimum value of F, $\dfrac{dF}{dy} = 0$

$$\frac{dF}{dy} = \frac{-Q^2}{gA^2}\frac{dA}{dy} + \frac{d(A\bar{y})}{dy} = 0$$

For the change dy in the depth, the corresponding change $(A\bar{y})$ in the static moment of the water area about the free water surface is

$$d(A\bar{y}) = \left[A(\bar{y}+dy)+(Tdy)\frac{dy}{2}\right]-\bar{y}A$$

lgnoring the differential of higher order i.e, $(dy)^2 = 0$

$$d(A\bar{y}) = A \times dy$$

$$\therefore \quad \frac{dF}{dy} = \frac{-Q^2}{gA^2}\frac{dA}{dy}+A = 0$$

$$\frac{Q^2T}{gA^2} = A$$

$$\boxed{\frac{Q^2T}{gA^3} = 1}$$

This is the condition of critical state of flow. Therefore the depth of minimum specific force is critical depth.

Similarly, it can be shown that for maximum Q at given F, flow condition is critical.

METERING FLUMES

- Critical depth of flow may be obtained at certain sections in an open channel where the channel bottom is raised by the construction of a hump or the channel is constricted by reducing its width.
- At critical state of flow the relationship between the depth of factors, it provides a theoretical basis for the measurement of discharge in open channels.
- Therefore, various devices for flow measurement are based on the principle of critical flow.

VENTURI FLUME

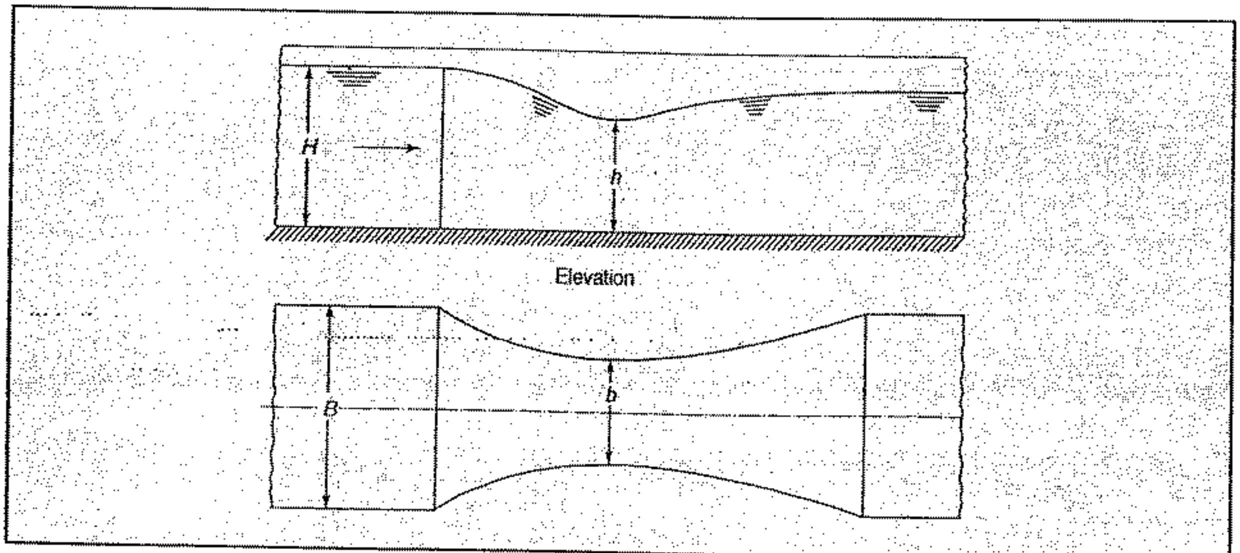
- A venturi flume is a structure in a channel which has a contracted section called throat, downstream of which follows a flared transition section designed to restore the stream to its original width.
- At the throat section there will be a drop in the water surface may be related to discharge.
- Velocity of flow at the throat is always less than critical velocity and hence the discharge passing through it will be a function of the difference between the depths of flow upstream of the entrance section and at the throat.
- As the velocity of flow at the throat is less than critical velocity, standing wave or hydraulic jump will not be formed at any section in the venturi flume.
- The discharge Q flowing through the channel can be calculated by measuring the depths of flow at the entrace and the throat of the flume.

$$\boxed{Q = k\frac{Aa\sqrt{2g}}{\sqrt{A^2-a^2}}\left(\sqrt{H-h}\right)}$$

$$Q = k\frac{BH \times bh \times \sqrt{2g}}{\sqrt{(BH)^2 - (bh)^2}}\left(\sqrt{H-h}\right)$$

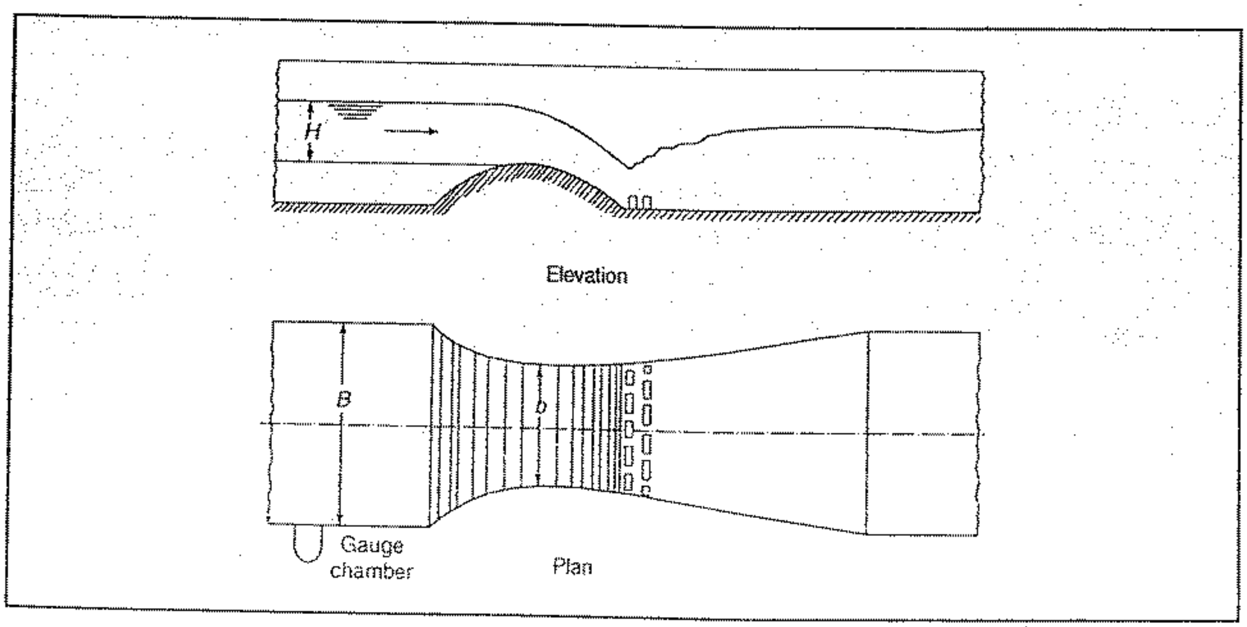
Where,

k = discharge coefficient, determined by calibration through the entire range of head.



STANDING WAVE FLUME OR CRITICAL DEPTH FLUME

- Standing wave flume is a structure in a channel which has a narrowed throat having a hump at the bottom, which acts as a broad-crested weir.
- Downstream of the throat section is followed by a flared transition section designed to restore the stream to its original width.



- For any discharge flowing in a channel, the velocity of flow at the throat of the flume is greater than critical velocity. Such that a standing wave or hydraulic jump is invariably formed at or near the downstream end of the raised floor.

- The velocity of flow at the throat is more than critical velocity, the depth of flow at a section upstream of the entrance of the flume remains unaffected by variations in the downstream depth until the downstream depth of submergence becomes greater than about 0.7 of the usptream depth.
- In other words as long as the downstream depth of flow is kept below the limiting value just mentioned above, the discharge passing through a standing wave flume will be function of only the depth of flow H (above the raised floor) at a section well upstream of the entrance section.
- To determine the discharge flowing in the channel, only the depth of flow H is required to be measured and the discharge Q is given as :

$$Q = CbH^{3/2}$$

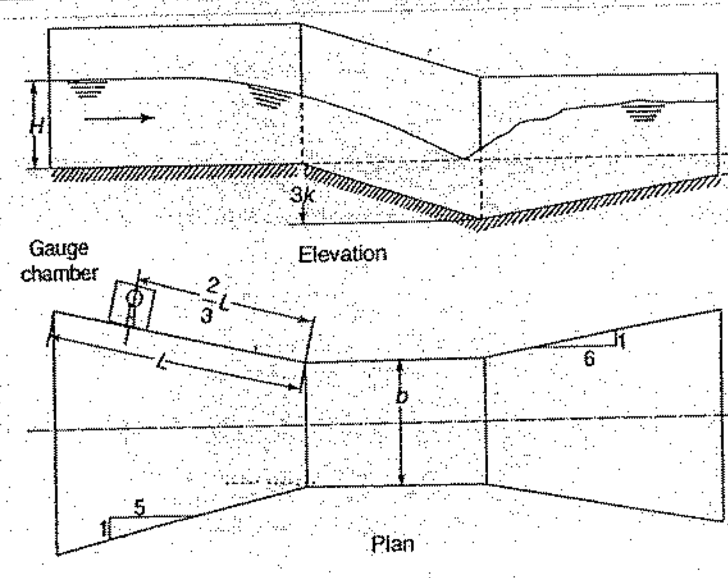
where, C is the discharge coefficient of the flume = 1.706, but an exact value of C for any flume may be determined by calibration.

PARSHALL FLUME

- The converging section of the flume has a level floor, the throat section has a downward sloping floor, while the floor in the diverging section slopes upwards.
- This flume has no raised floor as in the case of an ordinary standing wave flume, but the upward sloping floor of the diverging section facilitates the formation of standing wave in this portion of the flume. The depth–discharge relationship

$$Q = CbH^{n}$$

Where, C and n are the constants of flume determined by calibration.



Example 1

A trapezoidal channel has a bottom width of 6.0 m and side slopes of 1:1. The depth of flow is 1.5m as a discharge of 15 m^3/sec. Determine the specific energy.

Sol. Given,

B =6.0; y = 1.5 m; 1 : m = 1 : 1 (v : H) ; Q = 15 m^3/sec

Area, A = (B + my)y

$$A = (6 + 1.5)\,1.5 = 11.250 \text{ m}^2$$

$$\text{Velocity, } v = \frac{Q}{A} = \frac{15}{11.250} = 1.333 \text{ m/sec}$$

$$\text{Specific energy, } E = y + \frac{v^2}{2g} = 1.5 + \frac{1.333^2}{2\times 9.81} = 1.591\text{m}$$

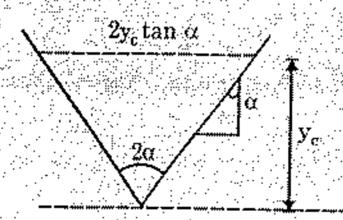
Example 2

Prove that the critical depth, y_c in, V–shaped channel of central angle 2α is given by: $y_c = \sqrt[5]{\frac{2Q^2}{g(\tan\alpha)^2}}$. Where Q is the discharge and g is acceleration due to gravity.

Sol. Let critical depth, $y = y_c$

Top width, $T = 2y_c \tan\alpha$

Area, $A_c = y_c^2 \tan\alpha$



$$\text{As critical flow, } \frac{Q^2 T}{gA^3} = 1$$

$$\frac{Q^2}{g} = \frac{A^3}{T}$$

$$\frac{Q^2}{g} = \frac{y_c^6 \tan^3\alpha}{2y_c \tan\alpha}$$

$$\frac{Q^2}{g} = \frac{y_c^5 \tan^2\alpha}{2}$$

$$y_c^5 = \frac{2Q^2}{g\tan^2\alpha} \Rightarrow y_c = \left(\frac{2Q^2}{g\tan^2\alpha}\right)^{1/5}$$

Example 3

If y_1 and y_2 are alternate depth in a rectangular channel show that $y_c^3 = \frac{2y_1^2 y_2^2}{(y_1 + y_2)}$, where y_c = critical depth and hence the Specific energy, $E = \frac{y_1^2 + y_1 y_2 + y_2^2}{(y_1 + y_2)}$

Sol. Let a rectangular channel of width B, discharge Q m^3/ sec. Specific energy E. We know that specific energy E for a certain discharge can occur at two depth y_1 and y_2 called as alternate depth.

Energy equation, $E_1 = y_1 + \frac{v^2}{2g}$

$$E_1 = y_1 + \frac{\frac{Q^2}{B^2 . y_1^2}}{2g}$$

$$E_1 = y_1 + \frac{q^2}{2g\,y_1^2} \qquad \left[\frac{q^2}{g} = y_c^3\right]$$

$$E_1 = y_1 + \frac{y_c^3}{2y_1^2} \qquad \text{...(1)}$$

Similarly for other depth y_2, $E_1 = y_2 + \frac{y_c^3}{2y_2^2}$ ...(2)

Comparing (1) and (2) $E_1 = E_2$

$$y_1 + \frac{y_c^3}{2y_1^2} = y_2 + \frac{y_c^3}{2y_2^2}$$

$$(y_1 - y_2) = \frac{y_c^3}{2y_2^2} - \frac{y_c^3}{2y_1^2}$$

$$(y_1 - y_2) = y_c^3\,\frac{\left(y_1^2 - y_2^2\right)}{2y_1^2\,y_2^2}$$

$$y_c^3 = \frac{2y_1^2\,y_2^2(y_1 - y_2)}{(y_1 - y_2)(y_1 + y_2)}$$

$$\boxed{y_c^3 = \frac{2y_1^2\,y_2^2}{y_1 + y_2}}^{*} \qquad \text{....(3)}$$

From equation (1)

$$E = y_1 + \frac{y_c^3}{2y_1^2}$$

Substituting value of y_c^3 in above equation

$$E = y_1 + \frac{2y_1^2\,y_2^2}{2y_1^2\,(y_1 + y_2)}$$

$$E = y_1 + \frac{y_2^2}{(y_1 + y_2)}$$

$$\boxed{E = \frac{y_1^2 + y_1 y_2 + y_2^2}{y_1 + y_2}}^{*}$$

*These are standard result and hence need to be remembered.

Example 4

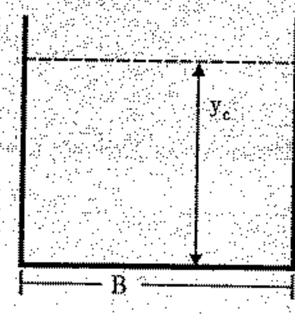
A rectangular channel is to carry a certain discharge at a critical depth. If the section is to have a minimum perimeter show that $y_c = \frac{3}{4}B$ where y_c is critical depth of flow and B is the width of the channel.

Sol. Given that flow is critical, hence

$$Q = B \cdot y_c \cdot v_c$$

$$Q = B \cdot y_c \sqrt{gy_c} \qquad \left[v_c = \sqrt{gy_c}\right]$$

$$B = \frac{Q}{y_c\sqrt{gy_c}}$$



$$\text{Perimeter, } P = B + 2y_c$$

For minimum perimeter we know that, $\frac{dP}{dy_c} = 0$

$$P = \frac{Q}{\sqrt{g}\, y_c^{3/2}} + 2y_c$$

$$\frac{dP}{dy_c} = -\frac{3}{2}\frac{Q}{\sqrt{g}}\, y_c^{-5/2} + 2$$

$$-\frac{3}{2}\frac{Q}{\sqrt{g}}\, y_c^{-5/2} + 2 = 0 \qquad \left[\frac{Q}{\sqrt{g}} = B \cdot y_c^{3/2}\right]$$

$$-\frac{3}{2} B y_c^{3/2} \cdot y_c^{-5/2} = -2$$

$$\frac{3}{4} B\, y_c^{-1} = 1$$

$$y_c = \frac{3}{4} B$$

Example 5

In a rectangular channel 3.5 m wide laid at a slope of 0.0036 uniform flow occurs at a depth of 2m. Find how high can the hump be raised without causing afflux? If the upstream depth is to be raised to 2.4 m what should be the height of hump? Assume Mannings n = 0.015

Sol. Given,

$B = 3.5\text{m}$; $y = 2.0\text{m}$; $S = 0.0036$; $n = 0.015$

$$\text{Area, } A = B \cdot y = 3.5 \times 2 = 7\text{m}^2$$

$$\text{Perimeter, } P = B + 2y = 3.5 + 2 \times 2 = 7.5 \text{ m}$$

$$\text{Hydraulic Radius, } R = \frac{A}{P} = \frac{7}{7.5} = 0.933\text{m}$$

$$\text{From Mannings equation, } Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

$$Q = \frac{1}{0.015} \times 7 \times (0.933)^{2/3} (0.0036)^{1/2}$$

$$Q = 26.735 \text{ m}^3/\text{sec}$$

$$v = \frac{Q}{A} = \frac{26.735}{7} = 3.819 \text{ m/sec}$$

$$\text{Specific energy, } E = y + \frac{v^2}{2g}$$

$$E = 2.0 + \frac{3.819^2}{2 \times 9.81}$$

$$E = 2.743 \text{ m}$$

We know that for the maximum height of hump without causing the change in upstream flow conditions, flow over hump has to be critical flow.

$$\text{Critical depth of flow, } y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$y_c = \left(\frac{(26.735/3.5)^2}{9.81}\right)^{1/3} = 1.812 \text{ m}$$

As $y_c < y$, Flow is subcritical.

Specific energy at critical depth of flow $E_c = \frac{3}{2} y_c = 1.5 \times 1.812 = 2.718$ m

Applying energy equation between upstream and over hump, assuming no loss of energy.

$$E = \Delta Z_{max} + E_c$$

$$2.743 = \Delta Z_{max} + 2.718 \text{ m}$$

$$\Delta Z_{max} = 0.025 \text{ m}$$

When the hump height is increased further $\Delta Z > \Delta Z_{max}$ the depth of flow over hump will be critical and upstream depth of flow is 2.4 m (given) for same discharge Q. Now for the new height of hump ΔZ.

$$E' = E_c + \Delta Z$$

$$y' + \frac{v^2}{2g} = E_c + \Delta Z$$

$$2.4+\frac{(Q/B\times y')^2}{2g} = 2.718 + \Delta Z$$

$$\Delta Z = 2.4 - 2.718 + \frac{(26.735/3.5\times 2.4)^2}{2\times 9.81}$$

$$\Delta Z = 0.198\text{m} \approx 0.200\text{m}$$

Example 6

In a rectangular channel 4m wide and with a bed slope of 0.0036 uniform flow occurs at depth of 2.0 m. Find how high can hump be raised without causing afflux. If the upstream depth flow is to be raised to 2.5 m what should be the height of hump? Assume Mannings n = 0.015

Sol. Given,

$B = 4\text{m};\ y = 2.0\text{m};\ S = 0.0036;\ n = 0.015$

$$\text{Area, } A = B.y = 4 \times 2 = 8\text{m}^2$$

$$\text{Perimeter, } P = B + 2y = 4 + 2 \times 2 = 8\text{m}$$

$$\text{Hydraulic radius, } R = \frac{A}{P} = \frac{8}{8} = 1$$

$$\text{From Mannings equation, } Q = \frac{1}{n} A \times R^{2/3} S^{1/2}$$

$$Q = \frac{1}{0.015} 8\times 1^{2/3}(0.0036)^{1/2} = 32\ \text{m}^3/\text{sec}$$

$$v = \frac{Q}{A} = \frac{32}{8} = 4\text{m/sec}$$

$$\text{Specific energy at upstream section, } E = y + \frac{v^2}{2g}$$

$$= 2.0 + \frac{4^2}{2\times 9.81}$$

$$= 2.815\ \text{m}$$

We know that for maximum height of hump without causing afflux, flow over hump will be critical flow.

$$\text{Critical depth of flow over hump, } y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{(32/4)^2}{9.81}\right)^{1/3}$$

$$y_c = 1.868\ \text{m}$$

$$\text{Specific energy for critical depth of flow, } E_c = \frac{3}{2} y_c$$

$$E_c = 1.5 \times 1.868 = 2.802 \text{ m}$$

For maximum size of hump without afflux condition.

$$E = E_c + \Delta Z_{max}$$

$$\Delta Z_{max} = 2.815 - 2.802 = 0.013 \text{ m}$$

Let height of hump be ΔZ when upstream depth is raised to 2.5 m for same amount of discharge 32 m^3/sec. Still flow over hump will be critical fow then,

$$y' + \frac{v'^2}{2g} = E_c + \Delta Z$$

$$2.5 + \frac{(Q/B \times y')^2}{2g} = 2.802 + \Delta Z$$

$$2.5 + \frac{(32/4 \times 2.5)^2}{2 \times 9.81} = 2.802 + \Delta Z$$

$$\Delta Z = 2.5 + \frac{(3.2)^2}{2 \times 9.81} - 2.802$$

$$\Delta Z = 0.2199 \text{ m} \simeq 0.220 \text{m}$$

Example 7

A rectangular channel 2.4 m wide carries uniform flow of 7 cummecs at a depth of 1.5 m. If there is a local rise of 0.15 m in bed level, calculate the change of water surface elevation. What can be the maximum rise in the bed elevation such that the upstream depth is not affected.

Sol. Given,

$B = 2.4$; $y = 1.5$m; $Q = 7$ m^3/sec

$$\text{Area, } A = b \cdot y = 2.4 \times 1.5 = 3.600 \text{ m}^2$$

$$\text{Velocity, } v = \frac{Q}{A} = \frac{7}{3.6} = 1.944 \text{ m/sec}$$

$$\text{Specific energy, } E = y + \frac{v^2}{2g} = 1.5 + \frac{1.944^2}{2 \times 9.81} = 1.693 \text{ m}$$

$$\text{Critical depth, } y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{(7/2.4)^2}{9.81}\right)^{1/3} = 0.954 \text{ m}$$

$$\text{Specific energy at critical depth, } E_c = \frac{3}{2} y_c = 1.431 \text{ m}$$

As the upstream bed level is raised by 0.15 m specific energy will decrease.

$$\text{New specific energy, } E' = E - 0.15 = 1.693 - 0.15$$

$$E = 1.543$$

As $E > E_c$ upstream depth of flow will not get changed.

For maximum rise of bed level without changing the upstream depth of flow.

$$\Delta Z_{max} = E - E_c$$

$$\Delta Z_{max} = 1.693 - 1.431$$

$$\Delta Z_{max} = 0.262\text{m}$$

Example 8

A rectangular channel is 3.0 m wide and carries a discharge of 15.0 m³/sec at a depth of 2.0m. At a certain section of the channel it is proposed to reduce the width to 2.0m and alter the bed elevation by ΔZ to obtain critical flow at the contracted section without altering the upstream depth. What should be the value of ΔZ.

Sol. Given,

$B = 3.5$ m; $y = 2.0$ m; $Q = 15.0$ m³/sec; $B' = 2.0$ m

$$\text{Area of flow, } A = 3 \times 2 = 6 \text{ m}^2$$

$$\text{Velocity, } v = \frac{Q}{A} = \frac{15}{6} = 2.5 \text{ m/sec}$$

$$\text{Specific energy, } E = y + \frac{v^2}{2g}$$

$$E = 2 + \frac{2.5^2}{2 \times 9.81}$$

$$E = 2.319 \text{ m}$$

After the contraction of width to 2.0m

$$q = \frac{Q}{B'} = \frac{15}{2} = 7.5 \text{ m}^3\text{/sec/m}$$

Due to the hump and contracted section at down stream side, a critical flow is caused. Assuming no loss of energy and applying energy equation for constant discharge Q

$$E = \Delta Z + y_c' + \frac{v'^2}{2g}$$

$$E = y_c' + \frac{\frac{Q^2}{B'^2 y_c'^2}}{2g} + \Delta Z$$

We know that y_c' will be corresponding to q'

$$y_c' = \left(\frac{q'^2}{g}\right)^{1/3}$$

$$y_c' = \left(\frac{7.5^2}{9.81}\right)^{1/3}$$

$$y_c = 1.789 \text{ m}$$

$$\text{Now, } E = \Delta Z + 1.789 + \frac{\frac{15^2}{2^2 \times 1.789^2}}{2 \times 9.81}$$

$$2.319 = 1.789 + 0.896$$

$$\Delta Z = 2.319 - 1.789 - 0.896$$

$$\Delta Z = -0.366$$

Hence the channel bed should be lowered by 0.366 m to get the critical flow at contracted section.

Example 9

A rectangular channel has 5.0 m width and 3.0 m water depth. If the bed slope of the channel is 1 in 1200, find (i) minimum width of throat, (ii) maximum height of the hump to produce critical depth, without changing the water level at the entrance. Consider manning is n = 0.02.

Sol. Given,

B = 5.0 m ; S = 1 in 1200; y = 3.0 m; n = 0.02

(i) **Minimum width of throat**

For a rectangular channel

$$\text{Area, } A = B \cdot y = 5 \times 3 = 15 \text{ m}^2$$

$$\text{Perimeter, } P = B + 2y = 5 + 2 \times 3 = 11\text{m}$$

$$\text{Hydraulic radius, } R = \frac{A}{P} = \frac{15}{11} = 1.363 \text{ m}$$

$$\text{From manning's equation, } Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

$$Q = \frac{1}{0.02} \times 15 \times (1.363)^{2/3} \times \left(\frac{1}{1200}\right)^{1/2} = 26.615 \text{ m}^3\text{/sec}$$

$$\text{Velocity of flow, } v = \frac{Q}{A} = \frac{26.615}{15} = 1.774$$

$$\text{Specific energy, } E = y + \frac{v^2}{2g}$$

$$E = 3 + \frac{1.774^2}{2 \times 9.81} = 3.160\text{m}$$

When channel section is contracted to minimum width and for constant discharge Q, the flow over contracted section will be critical flow and under assumption that no energy loss has taken place.

$$E = E_c$$

$$\text{We know that, } E_c = \frac{3}{2} y_c$$

$$y_c = \frac{2}{3} \times 3.160 = 2.107 \text{ m}$$

For critical flow condition, $\frac{Q^2 T}{gA^3} = 1$

$$\frac{Q^2 B_{min}}{g B_{min}^3 y_c^3} = 1$$

$$B_{min}^2 = \frac{Q^2}{g y_c^3}$$

$$B_{min} = \left(\frac{26.615^2}{9.81 \times 2.107^3}\right)^{1/2} = 2.778 \text{ m}$$

(ii) Maximum height of hump.

In case of maximum height of hump without changing the upstream depth of flow, the flow over the hump will be critical flow.

$$\therefore \quad y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{\left(\frac{26.615}{5}\right)^2}{9.81}\right)^{1/3} = 1.424 \text{ m}$$

Specific energy of critical, $E_c = \frac{3}{2} y_c$

$$\text{Critical depth of flow} = \frac{3}{2} \times 1.424 = 2.136 \text{ m}$$

Assuming no loss of energy, $E = E_c + \Delta Z_{max}$

We know that, $E = 3.160$

$$3.160 = 2.136 + \Delta Z_{max}$$

$$\Delta Z_{max} = 1.024 \text{ m}$$

Example 10

At what depths may a flow of 1 m³/sec occur in a rectangular channel 2 m wide if the specific energy is 0.05 m? What would be the corresponding channel bed slopes required to sustain uniform flow if Mannings roughness n = 0.015? Also find the minimum specific energy required to carry this discharge.

Assume g = 10.0m²/sec

Ans. Given,

$Q = 1\text{m}^3/\text{sec}$; $E = 0.50\text{m}$; $B = 2\text{m}$

Let the height of depth of flow will be y then from energy equation

$$E = y + \frac{v^2}{2g}$$

$$0.5 = y + \frac{Q^2}{2gB^2y^2}$$

$$0.5 = y + \frac{1^2}{2\times10.00\times2^2\times y^2}$$

$$0.5 = y + \frac{0.0125}{y^2}$$

$$0.5y^2 - y^3 - 0.0125 = 0$$

$$\therefore \quad y^3 - 0.5y^2 + 0.0125 = 0$$

By solving the above cubic equation we get,

$$y = -0.1397,\ 0.433,\ 0.206$$

Subcritical depth of flow will be, $y_2 = 0.433$ m

Supercritical depth of flow will be, $y_1 = 0.206$ m

Negative depth of flow is discarded as it is not possible.

Cbannel bed slope required to have supercritical uniform flow

$$\text{Area, } A_1 = B \times y_1 = 2 \times 0.206 = 0.412\text{m}^2$$

$$\text{Perimeter, } P_1 = B + 2y_1 = 2 + 2 \times 0.206 = 2.412$$

$$\text{Hydraulic Radius, } R_1 = \frac{A_1}{P_1} = \frac{0.412}{2.412} = 0.175$$

For Manning's equation,

$$Q = \frac{1}{n}AR^{2/3}S^{1/2}$$

$$1 = \frac{1}{0.015}\times0.412\times(0.175)^{2/3}S^{1/2}$$

$$S^{1/2} = \frac{0.015}{0.412\times(0.175)^{2/3}} = 0.116$$

$$S = 0.0135$$

Channel bed slope required to bave sub-critical uniform flow

$$\text{Area, } A_2 = B \times y_2 = 2 \times 0.433 = 0.866\ \text{m}^2$$

$$\text{Perimeter, } P_2 = B + 2y_2 = 2 + 2 \times 0.433 = 2.866\text{m}$$

$$\text{Hydraulic Radius, } R_2 = \frac{A_2}{P_2} = \frac{0.866}{2.866} = 0.302\text{m}$$

From Manning's equation

$$Q = \frac{1}{n}AR^{2/3}S^{1/2}$$

$$1 = \frac{1}{0.015}\times0.866\times(.302)^{2/3}S^{1/2}$$

$$S^{1/2} = \frac{0.015}{0.866\times(0.302)^{2/3}}$$

$$S = 0.0014$$

Minimum specific energy required for this discharge Q will be specific energy at critical depth of flow.

$$E_c = \frac{3}{2}\ y_c$$

$$E_c = \frac{3}{2}\left(\frac{q^2}{g}\right)^{1/3}$$

$$E_c = \frac{3}{2}\left(\frac{(1/2)^2}{10}\right)^{1/3}$$

$$E_c = 0.438 \text{ m}$$

Example 11

State the essential condition to be full filled for generating uniform flow in an open channel. A rectangular channel of 6 m widtb carries uniform flow at a normal depth of 3m. If the flow is subcritical or supercritical, if it carries a discharge of 60 m³/sec? If the channel is smoothly contracted to 3m width, what will be the change in water level between the upstream normal section and the contracted section? Will there be any change in the water depth upstream before and after contraction? If so, by what amount.

Sol. Given,

$Q = 60\ m^3/sec;\ B = 6\ m;\ y = 3\ m$

Critical depth of flow, $y_c = \left(\frac{q^2}{g}\right)^{1/3}$

$$y_c = \left(\frac{(60/6)^2}{9.81}\right)^{1/3} = 2.168 \text{ m}$$

As critical depth of flow is less than the normal depth of flow, flow is subcritical flow.

We know that at critical depth of flow specific energy will be minimum and discharge per unit width will be maximum.

$$\therefore \quad E_c = \frac{3}{2}\ y_c = \left(\frac{q_{max}^2}{g}\right)^{1/3}$$

$$\left(\frac{3}{2}\times 2.168\right)^3 = \frac{q_{max}^2}{9.81}$$

$$q_{max} = \sqrt{\left(\frac{3}{2}\times 2.168\right)^3 \times 9.81}$$

$$q_{max} = 18.368\ m^3/sec$$

For maximum value of q_{max}, width will be min i.e.

$$B_{min} = \frac{Q}{q_{max}} = \frac{60}{18.368} = 3.267m$$

If the width is contracted to 3.267 m from 6m flow will be critical flow. But as the width is contracted to 3m ie $B' < B_{min,}$ flow will still be critical flow at the contracted section and the specific energy at this critical depth of flow will also increase as $q' > q_{max}$.

To pass this q' ($q' > q_{max}$) specific energy at upstream side shall also increase and hence depth of flow will also increase as in case of subcritical flow.

depth of flow for $q'\left(\frac{60}{3}\right)$

$$\text{Critical depth, } y_c' = \left(\frac{q'^2}{g}\right)^{1/3}$$

$$y_c' = \left(\frac{(60/3)^2}{9.81}\right)^{1/3} = 3.442\ m$$

Specific energy for newCritical depth of flow,

$$E' = \frac{3}{2}\ y_c' = 5.163$$

Assuming no loss of energy between upstream and downstream and let the new depth of flow at upstream will be y' and new flow velocity v'

$$E = E_c$$

$$E = y' + \frac{v'^2}{2g}$$

$$5.163 = y' + \frac{Q^2/B^2y'^2}{2g}$$

$$y' + \frac{60^2}{6^2 \times y'^2 \times 2 \times 9.81} = 5.163$$

$$y' + \frac{5.097}{y'^2} = 5.163$$

$$y'^3 - 5.163\ y'^2 + 5.097 = 0$$

$$\therefore \quad y' = 4.955,\ 1.123,\ -0.916$$

New depth of flow at upstream will be $y' = 4.955$

Upstream depth of flow will increase, $\Delta y = y' - y = 4.955 - 3 = 1.955$ m

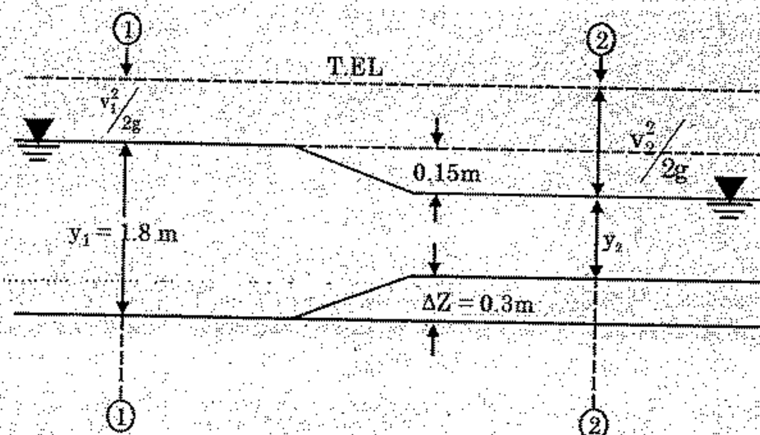
Note : *$y' = 1.123$ is not accepted because $y' < y_c'$ it will be for supercritical flow and $y' = -0.915$ can not happen as depth of flow can not be negative.*

Example 12

A 3.6 m wide rectangular channel caries water to a depth of 1.8m. In order to measure the discharge, the channel width is reduced to 2.4 m and a hump of 0.3m height is provided in the bottom calculate the discharge if water surface in the contracted section drops by 0.15 m. Assume no losses.

Sol. Given,

B = 3.6 m; y = 1.8 m; B′ = 2.4 m



As there is no loss of energy between section 1 & 2, writing energy equation

$$\frac{v_1^2}{2g} + y_1 = \Delta Z + y_2 + \frac{v_2^2}{2g}$$

$$\text{Here, } y_1 = \Delta Z + y_2 + 0.15$$

$$y_2 = y_1 - \Delta Z - 0.15 = 1.8 - 0.3 - 0.15 = 1.35 \text{ m}$$

$$\text{Also, } \frac{v_1^2}{2g} + 0.15 = \frac{v_2^2}{2g}$$

$$\frac{v_2^2}{2g} - \frac{v_1^2}{2g} = 0.15$$

$$\frac{\frac{Q^2}{(1.35\times 2.4)^2}}{2g} - \frac{\frac{Q^2}{(1.8\times 3.6)^2}}{2g} = 0.15$$

$$\frac{Q^2}{205.963} - \frac{Q^2}{823.852} = 0.15$$

$$Q^2 = 0.15 \times \left[\frac{823.852\times 205.963}{823.852 - 205.963}\right]$$

$$Q = 6.418 \text{ m}^3/\text{sec}$$

Example 13

A trapezoidal channel of base width 6 m and slopes of 2 horizontal to 1 vertical carries a flow of 60 cumecs as a depth of 2.5 m. There is a smooth transition to a rectangular section 6m wide accompanied by a gradual lowering of the channel bed by 0.6 m. (i) Find the depth of water in rectangular cross section and change in water surface level (ii) In case the drop in water surface level is to be restricted to 0.3m, what is the amount by which the bed must be lowered? (Assume no losses).

Sol. Flow in trapezoidal section

$$\text{Area, } A_1 = (B_1 + my_1) = (6 + 2 \times 2.5)\ 2.5$$

$$= 27.5 \text{ m}^2$$

$$\text{Top width, } T = B + 2my = 6 + 2 \times 2 \times 2.5 = 16 \text{ m}$$

$$\text{Velocity, } v_1 = \frac{Q}{A_1} = \frac{60}{27.5} = 2.182 \text{ m/sec}$$

$$\text{specific energy, } E_1 = y_1 + \frac{v_1^2}{2g} = 2.5 + \frac{2.182^2}{2 \times 9.81} = 2.743 \text{ m}$$

$$F_1 = \frac{v_1}{\sqrt{g\frac{A_1}{T_1}}} = \frac{2.182}{\sqrt{9.81 \times \frac{27.5}{16}}} = 0.531 < 1, \text{ Flow is subcritical flow.}$$

Flow in rectangular channel let the

depth of flow = y_2

Width of rectangular channel = 6m

Assuming there is no energy loss and writing the energy equation between these two section.

$$E_1 = E_2 - \Delta Z$$

$$E_1 = y_2 + \frac{v_2^2}{2g} - \Delta Z$$

$$2.743 = y_2 - 0.6 + \frac{Q^2}{(6 \times y_2)^2 \times 2 \times 9.81}$$

$$y_2 + \frac{60^2}{36 \times 2 \times 9.81 \times y_2^2} = 3.343$$

$$y_2 + \frac{5.097}{y_2^2} = 3.343$$

$$y_2^3 - 3.343\ y_2^2 + 5.097 = 0$$

$$y_2 = -1.074,\ 2.573,\ 1.844$$

Example 14

A trapezoidal channel with a base of 6m and side slope of 2 horizontal to 1 vertical conveys water at 17 m^3/sec with a depth of 1.5 m. Is the flow suitation sub or super critical?

Sol. Given,

$B = 6$ m; $1 : m = 1 : 2$ (v : H); $Q = 17 m^3/sec$; $y = 1.5$ m

$$\text{Area, } A = (B + my)\,y = (6 + 2 \times 1.5) \times 1.5 = 13.5$$

$$\text{Velocity, } v = \frac{Q}{A} = \frac{17}{13.5} = 1.259$$

$$\text{Top width, } T = B + 2my = 6 + 2 \times 2 \times 1.5 = 12 \text{ m}$$

$$F = \frac{v}{\sqrt{g\frac{A}{T}}} = \frac{1.259}{\sqrt{9.81 \times \frac{13.5}{12}}} = 0.379$$

As Froude number is $F < 1$ flow is subcritical flow.

Example 15

A rectangular channel 2.4 m wide carries uniform flow of water at a rate of 7 cumec/sec at a depth of 1.5m. If there is a local rise of 15 cm in the bed, calculate the change in water level. What is the maximum rise in bed that will be permissible so that there is no change in the upstream depth of flow?

Sol. Given,

$B = 2.4$; $Q = 7 m^3/sec$; $y = 1.5$ m; $\Delta Z = 0.15$ m

$$\text{Area, } A = 2.4 \times 1.5 = 3.6 m^2$$

$$\text{Velocity, } v = \frac{Q}{A} = \frac{7}{3.6} = 1.944 m^2$$

$$\text{Specific energy, } E = y + \frac{v^2}{2g} = 1.5 + \frac{1.944^2}{2 \times 9.81} = 1.693 \text{ m}$$

$$\text{Critical depth of flow, } y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{(7/2.4)^2}{9.81}\right)^{1/3} = 0.954$$

As $y > y_c$ flow is subcritical

$$\text{Specific energy at critical depth of flow, } E_c = \frac{3}{2} y_c$$

$$E_c = 1.5 \times 0.954 = 1.430 m$$

As bed is raised by 0.15 m on the down stream energy equation will be

$$E' = E - \Delta Z$$

$$E' = 1.693 - 0.15 = 1.543 \text{ m}$$

$E' > E_c$, hence no rise in water level will take place on upstream side.

For subcritical flow, maximum rise in bed level without raising the upstream depth of flow.

$$\Delta Z_{max} = E - E_c = 1.693 - 1.430$$
$$= 0.263 \text{ m}$$

Example 16

Find the critical depth for a discharge of 4 m³/sec for flow in right angle triangular channel.

Sol. We know that critical depth of flow in a triangular channel is

$$y_c = \left(\frac{2Q^2}{gm^2}\right)^{1/5}$$

For a right angular channel, m = 1

$$\therefore \quad y_c = \left(\frac{2\times 4^2}{9.81\times 1^2}\right)^{1/5} = 1.267 \text{ m}$$

Example 17

A rectangular channel 15 m wide has a normal depth of 0.8 m. The discharge carried is 10m³/sec what is the alternate depth?

Sol. We know that there are two depth of flow for a specific energy E as constant discharge Q. Let y_1 and y_2 be the alternate depth of flow.

$$y_1 + \frac{v_1^2}{2g} = y_2 + \frac{v_2^2}{2g}$$

Given,

B = 15 m; y_1 = 0.8 m; Q = 10 m³/sec

$$v_1 = \frac{Q}{By_1} = \frac{10}{15\times 0.8} = 0.833$$

$$v_2 = \frac{Q}{By_2} = \frac{10}{15\times y_2}$$

$$0.8 + \frac{0.833^2}{2\times 9.81} = y_2 + \frac{10^2}{15^2\times 9.81\times 2\times y_2^2}$$

$$0.835 = y_2 + \frac{0.023}{y_2^2}$$

$$y_2^3 - 0.835\ y_2^2 + 0.023 = 0$$

$$y_2 = -0.153,\ 0.799,\ 0.189$$

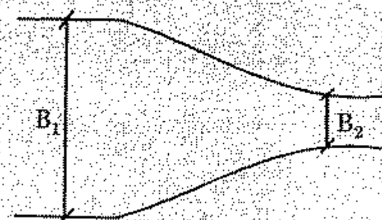
Aternate depth for y_1 = 0.8 m will be y_2 = 0.189.

Example 18

A rectangular channel having 30 m width carries a discharge of 200 cumec as a normal depth of 4m. It is to be smoothly contaracted at the bridge site so that Froude number of flow at the contracted rectangular section under the bridge is 0.5. Assuming no loss in energy find the clear waterway and the depth of flow under the bridge.

Sol. Given,

$B_1 = 30$ m; $Q = 200 m^3/sec$; $y_1 = 4m$



$$\text{Area, } A_1 = B_1 \times y_1 = 30 \times 4 = 120 \text{ m}^2$$

$$\text{Velocity, } v_1 = \frac{Q}{A_1} = \frac{200}{120} = 1.667 \text{ m/sec}$$

$$\text{Froude number, } F_1 = \frac{v_1}{\sqrt{gy_1}} = \frac{1.667}{\sqrt{9.81 \times 4}} = 0.266$$

As $F_1 < 1$ flow is super critical.

As contracted section let width = B_2, depth of flow = y_2 and velocity = v_2

Assuming no loss of energy between two sections, write energy equation

$$E_1 = E_2$$

$$y_1 + \frac{v_1^2}{2g} = y_2 + \frac{v_2^2}{2g}$$

$$4 + \frac{1.667^2}{2 \times 9.81} = y_2 + \frac{v_2^2}{2g}$$

$$Fr_2 = \frac{v_2}{\sqrt{gy_2}}$$

$$Fr_2 = 0.5$$

$$v_2 = 0.5\sqrt{gy_2}$$

$$4.142 = y_2 + \frac{0.5^2}{2} y_2$$

$$4.142 = 1.125 y_2$$

$$y_2 = 3.682\text{m}$$

$$v_2 = 0.5\sqrt{9.81\times3.682} = 3.005$$

Applying contunity equation

$$A_1 v_1 = A_2 v_2$$

$$200 = B_2 \times y_2 \times v_2$$

$$B_2 = \frac{200}{3.682\times3.005}$$

$$B_2 = 18.076 \text{ m}$$

Example 19

A trapezoidal channel of base width 6 m and slopes of 2 horizontal to 1 vertical carries a flow of 60 cumecs at a depth of 2.5 m. There is a smooth transition to a rectangular section 6m wide. (i) if the channel bed is horizontal determine whether the upstream flow is possible as specified, if not determine the upstream depth of flow. (ii) Determine the minimum amount by which the bed must be lowered for the upstream flow to be possible as specified. (iii) Determine the depth of flow in rectangular section and the change in the water surface level if the transition is accompanied by a gradual lowering of channel bed by 0.6 m. (iv) Determine the amount by which the bed must be lowered if the depth in water surface is to be restricted to 0.3m. Assume no losses.

Sol. Given,

$Q = 60$ m^3/sec

Upstream section	**Downstream section**
Trapaezoidal section	Rectangular section
$B_1 = 6$ m	$B_2 = 6$m
$1 : m_1 = 1V : 2H$	
$y_1 = 2.5$	

(i) At upstream section

$$\text{Area, } A_1 = (B_1 + m_1 y_1) y_1 = (6 + 2 \times 2.5)\,2.5$$

$$A_1 = 27.5\text{m}^2$$

$$\text{Topwidth, } T_1 = B_1 + 2m_1 y_1 = 6 + 2 \times 2 \times 2.5 = 16 \text{ m}$$

$$\text{Velocity, } v_1 = \frac{Q}{A_1} = \frac{60}{27.5} = 2.182\text{m/sec}$$

$$\text{Specific energy, } E_1 = y_1 + \frac{v_1^2}{2g}$$

$$E_1 = 2.5 + \frac{2.182^2}{2\times9.81} = 2.743\text{m}$$

$$\text{Froude number, } F_1 = \frac{v_1}{\sqrt{g\frac{A}{T}}} = \frac{2.182}{\sqrt{9.81\times\frac{27.5}{16}}} = 0.531(<1)$$

Flow at trapaezoidal section is sub critical flow. We know that the flow at upstream section will be possible if specific energy at trapaezoidal section (E_1) will be greater than or equal to specific energy at rectangular section under critical flow condition (E_c).

$$\boxed{E_1 \geq E_{c\ \text{rectangular}}}$$

Specific energy of rectangular section under critical flow condition

$$E_{c\,(\text{rectangular})} = \frac{3}{2} y_c$$

$$= \frac{3}{2}\left(\frac{q^2}{g}\right)^{1/3} = \frac{3}{2} \times \left(\frac{\left(\frac{60}{6}\right)^2}{9.81}\right)^{1/3}$$

$$= 2.168 \times \frac{3}{2} = 3.252 \text{ m}$$

As $E_1 < E_{c(\text{rectangular})}$, upstream flow will not be possible and hence it is a choking condition. Now for a possible flow condition we have to increase the upstream specific energy atleast equal to the specific energy at rectangular section under critical flow condition. Let y_1' be the new depth of flow at upstream section then,

$$\text{Area, } A_1' = (B + 2y_1')y_1' = (6 + 2y_1')y_1'$$

$$\text{Velocity, } v_1' = \frac{Q}{A_1'} = \frac{60}{(6 + 2y_1')y_1'}$$

$$\text{Specific energy } E_1' = y_1' + \frac{v_1'^2}{2g}$$

$$E_1' = y_1' + \frac{Q^2}{A_1'^2 \times 2g}$$

$$E_1' = y_1' + \frac{60^2}{2 \times 9.81 \times (6 + 2y_1')^2 y_1'^2}$$

Assuming no loss of energy and writing energy equation

$$E_1' = E_c\,(\text{rectangular})$$

$$y_1' + \frac{45.872}{(3 + y_1')^2 y_1'^2} = 3.252 \text{ m}$$

Solving by hit and trial $y_1' = 3.127$ m

Flow will be possible if upstream depth of flow will become equal to $y_1 = 3.127$ m.

(ii) Let ΔZ be the height of increase in bed at the downstream section for the possibility of flow when there is a critical flow at downstream section and upstream depth of flow is 2.5m, writing energy equation

$$E_1 = E_c - \Delta Z$$
$$\Delta Z = E_c - E_1$$
$$\Delta Z = 3.252 - 2.743$$
$$\Delta Z = 0.509 \text{ m}$$

(iii) Transition is accompanied with lowering of bed then specific energy at the rectangular section will increase by the amount by which bed is lowered.

LetΔZ_{min} be the minimum depth by which bed shall be lowered to avoid choking as upstream section

$$E_1 = E_c - \Delta Z_{min}$$
$$\Delta Z_{min} = E_c - E1$$
$$\Delta Z_{min} = 3.252 - 2.743 = 0.509 \text{ m}$$

But in our case bed is lowered by $\Delta Z = 0.60$ m hence no choking will be at upstream section.

Let new specific energy at rectangular section = E_2, new depth of flow = y_2, velocity of flow v_2.

Writing energy equation

$$E_2 = E_1 + \Delta Z$$
$$E_2 = 2.743 + 0.6$$
$$E_2 = 3.343 \text{ m}$$

$$y_2 + \frac{v_2^2}{2g} = 3.342$$

$$y_2 + \frac{Q^2}{A_2^2 \times 2g} = 3.342$$

$$y_2 + \frac{60^2}{(6 \times y_2)^2 \times 2 \times 9.81} = 3.342$$

$$y_2 + \frac{5.097}{y_2^2} = 3.342$$

$$y_2^3 - 3.342 y_2^2 + 5.097 = 0$$

Solving above equation

$$y_1 = 1.846,\ 2.571,\ -1.074$$

As the upstream flow is subcritical new depth of flow at rectangular section will be $y_2' = 2.571$

Change in water surface level

$$= (2.5 + 0.6) - 2.571 = 0.529$$

(iv) Let y be the depth of flow and ΔZ be the required amount by which channel bed shall be lowered at rectangular section such that change in water level is 0.3 m only, then

$$2.5 + \Delta Z = y + 0.3$$

$$\Delta Z = y - 2.2$$

Writing energy equation, $E_1 = E_2 - \Delta Z$

$$2.743 = y + \frac{Q^2}{2 \times g \times B_2^2 \times y^2} - y + 2.2$$

$$0.543 = \frac{60^2}{2 \times 9.81 \times 6^2 \times y^2}$$

$$y^2 = 9.386$$

$$y = 3.064 \text{ m}$$

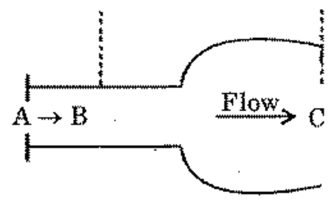
$$\Delta Z = 3.064 - 2.2$$

$$\Delta Z = 0.864 \text{ m}$$

If the channel bed is lowered by $\Delta z = 0.864$ m the drop in water level is restricted by 0.3 m only.

OBJECTIVE QUESTIONS

1. The given figure shows a subcritical open channel flow, expansion (BC in the figure) of rectangular cross-section. With respect to the water level in the flume (A in the figure), water level in the expansion BC will



(a) fall
(b) not change
(c) rise
(d) first rise and then fall

2. Consider the following statements in regard to the critical flow:

1. Specific energy is maximum for a given discharge.
2. Specific force is maximum for a given discharge.
3. Discharge is maximum for a given specific force.
4. Discharge is maximum for a given specific energy.

Which of these statements are correct?

(a) 1, 2 3 and 4
(b) 1 and 2
(c) 2 and 3
(d) 3 and 4

3. For a smooth hump in a sub-critical flow to function as a broad crested weir, the height DZ of the hump above the bed must satisfy which one of the following?

(a) $\Delta Z \geq (E_1 - y_c)$
(b) $\Delta Z \geq (E_1 - E_c)$
(c) $\Delta Z \leq (E_1 - y_c)$
(d) $\Delta Z \leq (E_1 - E_c)$

(E_1 = Specific head upstream of the hump,

E_c = Specific head at the critical depth y_c)

(Neglect friction effects)

4. Consider the following statements in respect of specific energy of flow in an open channel of fixed width:

1. There is only one specific energy curve for a given channel.
2. Alternate depths are the depths of flow at which the specific energy is the same.
3. Critcal flow occurs when the specific energy is minimum.

Which of these statements are correct?

(a) 1 and 2
(b) 1 and 3
(c) 2 and 3
(d) 1,2 and 3

5. If the Froude number of flow in a rectangular channel at a depth of flow of y_0 is F_0, then what is y_c/y_0 equal to?

(a) $F_0^{1/3}$
(b) $F_0^{2/3}$
(c) $F_0^{3/2}$
(d) $F_0^{-1/2}$

6. In a wide rectangular channel if the normal depth is increased by 20%, then what is the approximate increase in discharge?

(a) 25%
(b) 30%
(c) 35%
(d) 40%

7. A right-angled triangular channel symmetrical in section about the vertical carries a discharge of 5 m^3/s with a velocity of 1.25 m/s. What is the approximate value of the Froude number of the flow?

(a) 0.3
(b) 0.4
(c) 0.5
(d) 0.6

8. A rectangular channel is 6m wide and discharges 30 m^3/sec. The upstream depth is 2.0m, acceleration due to gravity is 10 ms^{-2}. Then, what is the specific energy (approximate)?

(a) 2.5
(b) 0.3
(c) 2.3
(d) None of the above

9. If the Froude number of flow in an open channel is more than 1.0, then the flow is said to be

(a) critical
(b) shooting
(c) streaming
(d) transitional

10. Which one of the following conditions is a typical characteristic of critical flow?

(a) $\frac{Q^2T}{gA^3}=1$
(b) $\frac{QT^2}{gA^2}=1$
(c) $\frac{Q^2}{gA^2}=1$
(d) $\frac{Q^2T^2}{gA^2}=1$

11. Which one of the following statements is not correct?

(a) Specific energy is the total energy above the floor of an open channel
(b) For a given specific energy, two depths exist and these are called alternate depths
(c) Velocity of flow is critical at maximum specific energy
(d) Critical velocity occurs at Froude number = 1

12. While defining Froude number applicable to channels of any shape, the length parameter used is the

(a) hydraulic radius
(b) wetted perimeter
(c) ratio of area to top width
(d) depth of flow

13. Match List-I with List-II and select the correct answer using the codes given below the lists :

List-I	List-II
A. Specific force	1. Head loss due to friction
B. Specific energy	2. Rapidly varied flow
C. hydraulic jump	3. Alternate depths
D. Darcy-Weisbach equation	4. Conjugate depths

Codes :

	A	B	C	D
(a)	4	3	2	1
(b)	4	3	1	2
(c)	3	4	2	1
(d)	3	4	1	2

14. A rectangular channel of 2.5 m width and 2m depth of water carries a flow of $10 m^3/s$. The specific energy for the flow is given by

(a) 1.18 m (b) 3.4 m
(c) 2.0 m (d) 2.2 m

15. Water flows in a channel at a Froude number greater than 1. If the channel is contracted, then in the contracted section

(a) both y and v increase (b) both y and v decreases
(c) y increases and v decreases (d) y decreases and v increases

16. Consider the following statements:

A constant discharge flows in a channel across which a hump of moderate height has been constructed. Then, along the flow direction

1. depth of flow reduces
2. specific energy remains unchanged
3. froude number reduces
4. critical depth remains unchanged

Which of these statements is/are correct?

(a) 1 only (b) 1 and 4
(c) 2 and 3 (d) 2, 3 and 4

17. In a triangular channel, the critical depth is 1.4 m. The corresponding least possible specific energy is

(a) 1.65 m (b) 1.75
(c) 2.1 m (d) 2.4 m,

18. In a triangular channel, the critical depth for a discharge of $1.2 m^2/s$ is 1.3 m. The critical depth for a discharge of $38.4 m^2/s$ through the same channel would be

(a) 2.6 m (b) 3.4 m
(c) 5.2 m (d) 6.2 m

19. A rectangular channel carries uniform flow with a normal depth of 0.6 m and Froude number of 2.0. For this flow the critical depth is

(a) $0.6 \times (2)^{4/3}$ m
(b) $0.6^2 \times (2)^{1/3}$ m
(c) $0.6^{1/3} \times (2)$ m
(d) $0.6 \times (2)^{2/3}$ m

20. It has been observed for critical depth y_{cr} in a rectangular channel carrying a constant discharge that:

1. Specific energy is maximum
2. Froude number = 1
3. Specific energy = 1.33 y_{cr}
4. Specific force is minimum

Which of the above observations are correct?

(a) 1, 2 and 3
(b) 2, 3 and 4
(c) 1 and 4
(d) 2 and 4

21. Consider the following statements in respect of the critical depth of flow in a prismatic rectangular channel?

1. For known specific energy, the discharge is minimum.
2. For known discharge, the specific energy is minimum

Which of these statements is/are correct?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

22. An open channel carrying super critical flow is provided with a smooth expansion along the direction of flow. When no other considerations interfere, then the water surface

(a) before transition will rise
(b) after transition will rise
(c) before transition will drop
(d) after transition will drop

23. In a wide rectangular channel with uniform flow the specific energy is 1.08 m. What is the velocity at critical flow? Given $\sqrt{2g} = 4.43$

(a) 0.981 m/s
(b) 4.430 m/s
(c) 2.658 m/s
(d) 0.360 m/s

24. Critical depth at a section of a rectangular channel is 1.5 m. The specific energy at that section is

(a) 0.754 m
(b) 1.0 m
(c) 1.5 m
(d) 2.25 m

25. A triangular open channel has a vertex angle of 90° and carries flow at a critical depth of 0.30 m. The discharge in the channel is

(a) 0.08 m^3/s
(b) 0.11 m^3/s
(c) 0.15 m^3/s
(a) 0.2 m^3/s

Common data for Q. 26, 27, 28

A rectangular channel 6.0m wide carries as discharge of 16.0 m^3/sec under uniform flow condition with normal depth of 1.60 m. Manning 'n' is 0.015.

26. The longitudinal slope of the channel is

(a) 0.000585
(b) 0.000485
(c) 0.000385
(d) 0.000285

27. A hump is to be provided on the channel bed. The maximum height of the hump without affecting the upstream flow condition is

(a) 0.50 m (b) 0.40 m.
(c) 0.30 m (d) 0.20 m

28. The channel width is to be contracted. The minimum width to which the channel can be contracted without affecting the upstream flow condition is

(a) 3.0 m (b) 3.8 m
(c) 4.1 m (d) 4.5 m

29. The term alternate depths is used in open channel flow to denote the depths

(a) having the same kinetic energy for a given discharge
(b) having the same specific force for a given discharge
(c) having the same specific energy for a given discharge
(d) having the same total energy for a given discharge

30. In a rectangular channel, the alternate depths are 1.0 m and 2.0 m respectively. The specific energy head in m is

(a) 3.38 (b) 1.33
(c) 2.33 (d) 3.0

31. A rectangular channel carries a certain flow for which the alternate depths are found to be 3.0 m and 1.0 m. The critical depth in m for this flow is

(a) 2.65 (b) 1.65
(c) 0.65 (d) 1.33

32. For a triangular channel of side slopes m horizontal ; 1 vertical, the Froude number is given by F =

(a) $\frac{m}{\sqrt{gy}}$ (b) $\frac{v}{\sqrt{2gy}}$

(c) $\frac{v\sqrt{2}}{gy}$ (d) $\frac{v}{\sqrt{gy}}$

33. A triangular channel has a vertex angle of 90° and carries a discharge of 1.90 m^3/s at a depth of 0.8 m. The Froude number of the flow is

(a) 0.68 (b) 1.06
(c) 0.75 (d) 1.50

34. A triangular channel of apex angle of 120° carries a discharge of 1573 l/s. The critical depth in m is

(a) 0.600 (b) 0.700
(c) 0.800 (d) 0.632

35. A triangular channel of apex angle of 60° has a critical depth of 0.25 m. The discharge in l/s is

(a) 60 (b) 640
(c) 160 (d) 40

36. For a given open channel carrying a certain discharge the critical depth depends on
    (a) the geometry of the channel
    (b) the roughness of the channel
    (c) the viscosity of water
    (d) the longitudianl slope of the channel

37. In a triangular channel the value of E_c/y_c is
    (a) 1.25
    (b) 2.5
    (c) 3.33
    (d) 1.5

38. Supercritical flow at Froude number of $F_0 = 2.0$ occurs at a depth of 0.63 m in a rectangular channel. The critical depth in m is
    (a) 0.857
    (b) 0.735
    (c) 1.000
    (d) 0.500

39. In a rectangular channel with subcritical flow the height of a hump to be built to cause subcritical flow over it was calculated by neglecting energy losses. If, after building the hump, it is found that the energy losses in the transition are appreciable, the effect of this hump on the flow will be
    (a) to make the flow over the hump subcritical
    (b) to make the flow over the hump supercritical
    (c) to cause the depth of flow upstream of the hump to raise
    (d) to lower the upstream water surface

40. Flow happens at a critical depth of 0.5 m in a rectangular channel of 4 m width. What is the value of discharge?
    (a) 5.4 m^3/s
    (b) 5.1 m^3/s
    (c) 4.9 m^3/s
    (d) 4.4 m^3/s

41. What is the discharge corresponding to a critical depth of 1.20 m in a 3.0 m wide rectangular channel?
    (a) 4.12 m^3/s
    (b) 4.94 m^3/s
    (c) 8.24 m^3/s
    (d) 12.35 m^3/s

42. Match **List-I** (Flow section type) with **List-II** (Critical discharge is proportional to) where y is the depth of flow and select the correct answer using the codes given below the lists:

List-I	**List-II**
A. Shallow parabolic	1. $y(z^{3/2})$
B. Triangular	2. $y^{3/2}$
C. Rectangular	3. $y^{5/2}$
D. Trapezoidal	4. y^2

Codes:

	A	B	C	D
(a)	2	3	4	1
(b)	4	1	2	3
(c)	2	1	4	3
(d)	4	3	2	1

Consider the following statements:
Of these statements

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

43. **Assertion (A):** At the critical state of flow, the specific force is a minimum for the given discharge.
Reason (R): For a minimum value of specific force, the first derivative of force with respect to depth should be unity.

44. **Assertion (A):** The vertical co-ordinate of the apex of the Q-curve is $\frac{2}{3}$ E.
Reason (R): The horizontal co-ordinate of the apex of E-curve is 1.5 times $\left(\frac{q^2}{g}\right)^{1/3}$.

45. **Assertion (A):** Total energy of flow decreases in the direction of flow.
Reason (R): The specific energy may decrease, increase or remain constant.

46. **Assertion (A):** A minor change in specific energy at or close to critical state will cause a major change in depth.
Reason (R): A critical state of flow is characterised by its Froude number being equal to unity.

47. **Assertion (A)** : For any discharge flow will be critical in a wide rectangular channel whose bed slope is 1 in C^2/g.
Reason (R) : The critical depth of flow through a wide rectangular channel is $(q^2/g)^{1/3}$.

48. **Assertion (A)** : For a given specific energy in a prismatic channel, the critical state of flow corresponds to maximum discharge.
]**Reason (R)** : Gradient of discharge w.r.t. depth $\frac{dQ}{dy}$ is maximum when depth is equal to critical depth which comprises critical state of flow.

49. **Assertion (A):** A channel carrying steady flow which is subjected to raising of its bottom and reduction of its width at a d/s reach, generally results in raising of water surface on upstream.
Reason (R): Raising of channel bed d/s results in impounding of water u/s and hence the raising of u/s water surface.

ANSWERS

1. (c)	2. (d)	3. (d)	4. (c)	5. (b)	6. (b)	7. (b)	8. (c)	9. (b)	10. (a)
11. (c)	12. (c)	13. (a)	14. (d)	15. (c)	16. (b)	17. (b)	18. (c)	19. (d)	20. (d)
21. (b)	22. (d)	23. (c)	24. (d)	25. (b)	26. (a)	27. (b)	28. (c)	29. (c)	30. (c)
31. (b)	32. (c)	33. (d)	34. (b)	35. (d)	36. (a)	37. (a)	38. (c)	39. (c)	40. (d)
41. (d)	42. (d)	43. (a)	44. (b)	45. (b)	46. (b)	47. (a)	48. (a)	49. (a)	

HINT

1. As the flow is subcritical in the given channel and there is a expansion in channel section at section C, hence discharge per unit width reduces and therefore depth of flow rises (as in case of subcritical flow).

2. We know that at critical flow condition.
   - Specific energy is minimum for a given discharge.
   - Specific force is minimum for a given discharge.
   - Discharge is maximum for a given specific force.
   - Discharge is maximum for a given specific energy.

3. For subcritical flow condition, maximum permissible height of hump without changing upstream flow condition

$$E_1 \geq E_c + \Delta Z$$

$$(E_1 - E_c) \geq \Delta Z$$

4. For non- prismatic channel, the channel section varies along the length of the channel and hence, the specific energy differs from section to section

5.

$$F_0 = \frac{v}{\sqrt{gy_0}}$$

$$F_0^2 = \frac{v^2}{gy_0}$$

$$F_0^2 = \frac{q^2}{gy_0^3}$$

$$y_0^3 = \frac{q^2}{gF_0^2}$$

Similarly,

$$\frac{v_c}{\sqrt{gy_c}} = 1$$

$$\frac{q_c^3}{g} = y_c^3$$

$$\frac{y_c^3}{y_0^3} = F_0^2$$

$$\frac{y_c}{y_0} = F_0^{2/3}$$

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Of these statements

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Reason (R): Raising of channel bed d/s results in impounding of water u/s and hence the raising of u/s water surface.

ANSWERS

1. (c)	**2.** (d)	**3.** (d)	**4.** (c)	**5.** (b)	**6.** (b)	**7.** (b)	**8.** (c)	**9.** (b)	**10.** (a)
11. (c)	**12.** (c)	**13.** (a)	**14.** (d)	**15.** (c)	**16.** (b)	**17.** (b)	**18.** (c)	**19.** (d)	**20.** (d)
21. (b)	**22.** (d)	**23.** (c)	**24.** (d)	**25.** (b)	**26.** (a)	**27.** (b)	**28.** (c)	**29.** (c)	**30.** (c)
31. (b)	**32.** (c)	**33.** (d)	**34.** (b)	**35.** (d)	**36.** (a)	**37.** (a)	**38.** (c)	**39.** (c)	**40.** (d)
41. (d)	**42.** (d)	**43.** (a)	**44.** (b)	**45.** (b)	**46.** (b)	**47.** (a)	**48.** (a)	**49.** (a)	

HINT

1. As the flow is subcritical in the given channel and there is a expansion in channel section at section C, hence discharge per unit width reduces and therefore depth of flow rises (as in case of subcritical flow).

2. We know that at critical flow condition.
   * Specific energy is minimum for a given discharge.
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5. $$F_0 = \frac{v}{\sqrt{gy_0}}$$

$$F_0^2 = \frac{v^2}{gy_0}$$

$$F_0^2 = \frac{q^2}{gy_0^3}$$

$$y_0^3 = \frac{q^2}{gF_0^2}$$

Similarly,

$$\frac{v_c}{\sqrt{gy_c}} = 1$$

$$\frac{q_c^3}{g} = y_c^3$$

$$\frac{y_c^3}{y_0^3} = F_0^2$$

$$\frac{y_c}{y_0} = F_0^{2/3}$$

6. From Chezy's Equation

$$Q = CA\sqrt{RS}$$

For wide rectangular channel, $R = y$

$$Q = CB\, y^{3/2}\, S^{1/2}$$

$$\frac{dQ}{Q} = \frac{3}{2} \times \frac{1}{y} dy \times 100$$

$$= \frac{3}{2} \times \frac{0.2y}{y} \times 100 = 30\%$$

7.

$$F_r = \frac{v}{\sqrt{g\frac{A}{T}}}$$

$$A = \frac{Q}{v} = \frac{5}{1.25} = 4m^2$$

$\because$ $A = my^2 = 4$, where $m = 1$

$\therefore$ $y = 2$

$\because$ $T = 2my = 2 \times 1 \times 2 = 4$

$$F_r = \frac{1.25}{\sqrt{9.81 \times \frac{4}{4}}} = 0.4$$

8.

$$E = y + \frac{v^2}{2g}$$

$$E = y + \frac{Q^2}{2B^2 \times y^2 g}$$

$$E = 2 + \frac{30^2}{2 \times 6^2 \times 2^2 \times 10} = 2.3$$

9. Flow velocity higher than the critical is said to be rapid flow, shooting flow, or supercritical flow, while flow at velocity lower than the critical is said to be tranquil flow, streaming flow, or sub critical flow.

11. Velocity of flow is critical at minimum specific energy.

12. $F_r = \dfrac{v}{\sqrt{g\frac{A}{T}}}$

dimensions of $\frac{A}{T} = M^0L^1T^0$ i.e. a length parameter.

14. B = 2.5, y = 2, Q = 10m³/sec

$$E = y + \frac{Q^2}{2B^2y^2g} = 2 + \frac{10^2}{2\times 2.5^2 \times 2^2 \times 9.81} = 2.20\text{m}$$

15. For a super critical flow, when discharge per unit width is increased then depth of flow increases and velocity decreases for a constant discharge.

16. When a hump is constructed on the d/s side of a channel then, specific energy reduces and hence the depth of flow decreases. But critical depth of flow will remain unchanged.

17.

$$E_c = 1.25\, y_c$$

$$E_c = 1.25 \times 1.4 = 1.75 \text{ m}$$

18. For triangular channel

$$y_c = \left(\frac{2Q^2}{gm^2}\right)^{1/5}$$

$$\therefore \quad \frac{y_{c_2}}{y_{c_1}} = \left(\frac{Q_2^2}{Q_1^2}\right)^{1/5}$$

$$y_{c_2} = 1.3\left(\frac{38.4}{1.2}\right)^{2/5} = 5.2$$

19.

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$F_r = \frac{v}{\sqrt{gy}}$$

$$v = 2.0\sqrt{gy}$$

$$q = \frac{Q}{B} = \frac{v\times B\times y}{B} = v \times y$$

$$y_c = \left(\frac{\left(2.0\sqrt{gy}\right)^2 . y^2}{g}\right)^{1/3}$$

$$y_c = \left(\frac{2.0^2 gy^3}{g}\right)^{1/3}$$

$$y_c = 2.0^{2/3}\, 0.6$$

20. At critical flow in rectangular channel

- Specific energy is minimum
- Froud number = 1
- Specific energy = $1.5y_{cr}$
- Specific force is minimum

21. At critical flow in prismatic rectangular channel

- For a given discharge specific energy is minimum.
- For a given specific energy discharge is maximum.

22. From discharge diagram, in super critical flow when discharge per unit width decreases depth of flow decreases.

23. Assuming no loss of energy.

$$E = E_c$$

$$1.08 = y_c + \frac{v_c^2}{2g}$$

$$1.08 = \frac{2}{3}\times 1.08 + \frac{v_c^2}{2g}$$

$$\sqrt{\frac{1}{3}\times 1.08}\times\sqrt{2g} = v_c$$

$$v_c = 4.43\times 1.08\times\frac{1}{3} = 2.658 \text{ m/sec}$$

24.

$$E_c = \frac{3}{2}y_c = 1.5 \times 1.5 = 2.25 \text{ m}$$

25. For a 90° channel, m = 1

$$y_c = \left(\frac{2Q^2}{g}\right)^{1/5}$$

$$\sqrt{\frac{y_c^5\times 9.81}{2}} = Q$$

$$Q = \sqrt{\frac{0.3^5\times 9.81}{2}} = 0.11 \text{ m}^3\text{/sec}$$

26. From Manning's Equation,

$$Q = \frac{1}{n}A\times R^{2/3}S^{1/2}$$

$$16 = \frac{1}{0.015}\times(6\times1.60)\left(\frac{6\times1.60}{6+2\times1.60}\right)^{2/3} S^{1/2}$$

$$S = 0.000585$$

27.

$$V = \frac{Q}{A} = \frac{16}{6\times1.60} = 1.667$$

$$E = y+\frac{v^2}{2g} = 1.6+\frac{1.667^2}{2\times9.81} = 1.742$$

$$E_c = \frac{3}{2}y_c = 1.5\times\left(\frac{(16/6)^2}{9.81}\right)^{1/3} = 1.347$$

Maximum height of hump,

$$\Delta Z_{max} = E_1 - E_c = 1.742 - 1.347 = 0.395$$

28.

$$E = E_c = \frac{3}{2}y_c$$

$$1.742\times\frac{2}{3} = y_c$$

$$1.742\times\frac{2}{3} = \left(\frac{q^2}{g}\right)^{1/3}$$

$$1.742\times\frac{2}{3} = \left(\frac{(Q/B_{min})^2}{9.81}\right)^{1/3}$$

$$1.742\times\frac{2}{3} = \left(\frac{162/B_{min^2}}{9.81}\right)^{1/3}$$

$$B_{min} = 4.082 = 4.1\text{m}$$

29. The term alternate depth is used in open channel flow to denote the depths having the same specific energy for a given discharge.

30.

$$E = \frac{y_1^2+y_1y_2+y_2^2}{(y_1+y_2)} = \frac{1^2+1\times2+2^2}{1+2} = \frac{1+2+4}{3} = \frac{7}{3} = 2.33$$

31.

$$y_c^3 = \frac{2y_1^2y_2^2}{(y_1+y_2)}$$

$$y_c = \left(\frac{2\times1^2\times3^2}{3+1}\right)^{1/3} = \left(\frac{18}{4}\right)^{1/3} = 1.65$$

32.

$$F_r = \frac{\sqrt{2}v}{\sqrt{gy}}$$

33.

$$F_r = \frac{v}{\sqrt{g\frac{A}{T}}}$$

$$v = \frac{Q}{A} = \frac{Q}{my^2} = \frac{1.90}{0.8^2} = 2.969 \text{ m/sec}$$

$$\frac{A}{T} = \frac{my^2}{2my} = \frac{0.8}{2} = 0.4$$

$$F_r = \frac{2.969}{\sqrt{9.81\times0.4}} = 1.498$$

34.

$$Q = 1.573 \text{ m}^3\text{/sec}$$

$$y_c = \left(\frac{2Q^2}{gm^2}\right)^{1/5}$$

$$m = \sqrt{3}$$

$$y_c = \left(\frac{2\times1.573^2}{9.81\times\left(\sqrt{3}\right)^2}\right)^{1/5} = 0.700$$

35.

$$y_c = \left(\frac{2Q^2}{gm^2}\right)^{1/5}$$

$$m = \frac{1}{\sqrt{3}}$$

$$0.25 = \left(\frac{2Q^2}{9.81\times\left(\frac{1}{\sqrt{3}}\right)^2}\right)^{1/5}$$

$$Q = 0.040 \text{ m}^3\text{/sec} = 40 \ l\text{/sec}$$

36. Critical depth for a given open channel depends on the geometry of channel and discharge $\frac{Q^2}{gA^2} = \left(\frac{A}{T}\right)$

37.
$$E_c = 1.25y_c$$
$$\frac{E_c}{y_c} = 1.25$$

38.
$$F_r = \frac{v}{\sqrt{gy}} = 2$$
$$v = 2\sqrt{gy}$$
$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{(v \times y)^2}{g}\right)^{1/3} = \left(\frac{4 \times g \times y \times y^2}{g}\right)^{1/3}$$
$$y_c = (4)^{1/3}\, y = 1.00$$

39. If there is a energy loss taking place then we consider height of hump as $(\Delta Z + h_L)$. But as the energy losses are neglected. The upstream depth of flow will increase.

40.
$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$
$$q = \left(y_c^3 g\right)^{1/2}$$
$$Q = q \times B = B \times \left(y_c^3 \times g\right)^{1/2} = 4 \times (0.5^3 \times 9.81)^{1/2} = 4.4\text{m}^3\text{/sec}$$

41.
$$Q = B(y_c^3 \times g)^{1/2} = 3(1.20^3 \times 9.81)^{1/2} = 12.35\text{m}^3\text{/sec}$$

42.

Channel section		
Rectangular	→	$y^{3/2} \propto q$
Triangular	→	$y^{5/2} \propto Q$
Shallow Parabolic	→	$y_c = \left(\frac{27}{32}\frac{Q^2}{gc^2}\right)^{1/4} \Rightarrow y_c^2 \propto Q$

44. The condition of the apex of E-curve is. $E_c = \frac{3}{2}y_c = 1.5\left(\frac{q^2}{g}\right)^{1/3}$

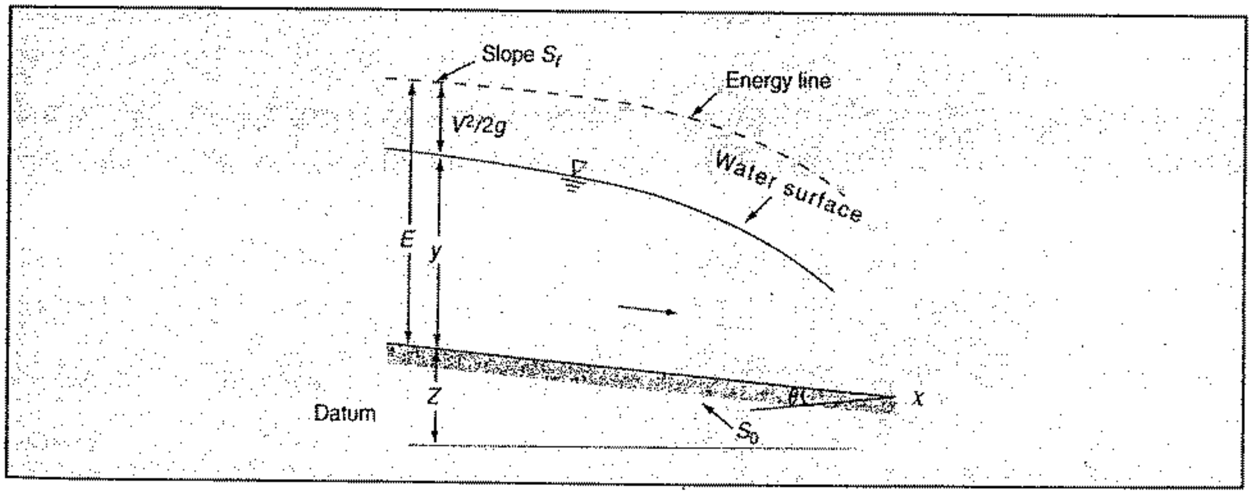
CHAPTER

4

Gradually Varied Flow

INTRODUCTION

- The gradually varied flow (GVF) is defined as steady non-uniform flow, where depth of flow varies gradually from section to section along the length of the channel.
- The back water produced by a dam or weir across a river and the drawdown produced at a sudden drop in channel are examples of G.V.F.
- In GVF, losses are negligible, the curvature of streamlines is negligible and the loss of energy is essentially due to boundary friction.
- In GVF Bed slope (S_0), water surface slope (S_w) and Energy slope (S_f) will all differ from each other.



- Almost all major hydraulic engineering activities in free surface flow involve the computation of GVF profiles.

ASSUMPTIONS IN GVF

(1) Pressure distribution is hydrostatic because curvature of stream lines is small.

(2) Resistance to flow is given by Manning's or Chezey's equation with the slope taken as slope of energy line.

$$\text{Manning's Equation, } V = \frac{1}{n} R^{2/3} (S_f)^{1/2}$$

$$\text{Chezey's Equation, } V = C\sqrt{RS}$$

(3) Channel bed slope is small i.e HGL will lie at free surface.

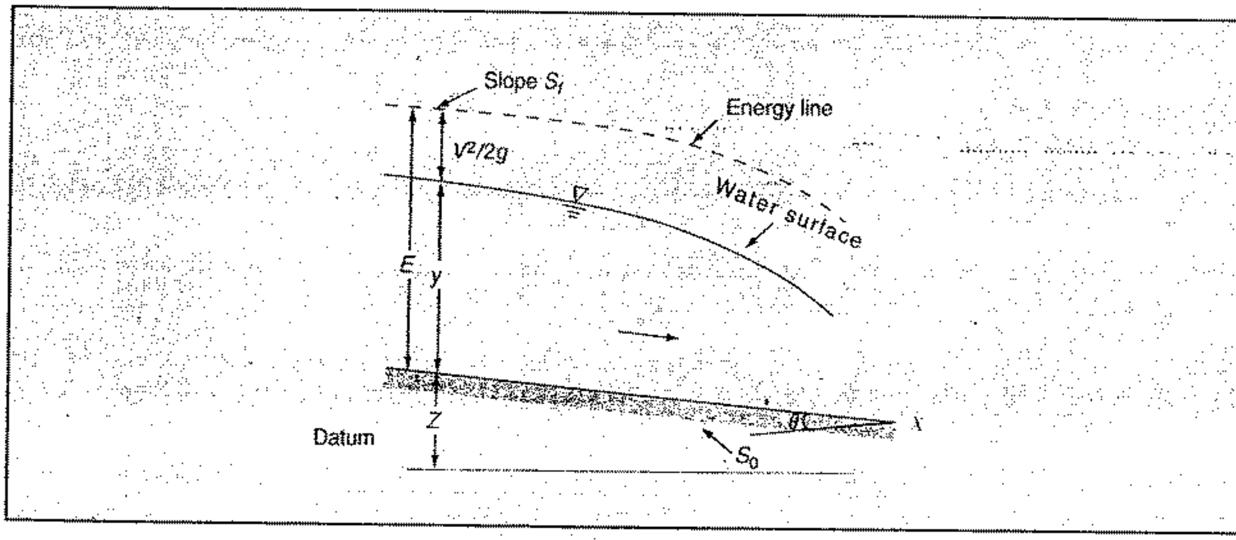
(4) There is no air entrainment.

(5) Velocity distribution is invarient i.e. kinetic energy correction factor, $\propto =$ Constant

(6) Resistance coefficients (C & n) are constant with the depth.

(7) Channel is prismatic i.e. the channel has a constant alingment and its shape remains same over the reach of the channel under consideration.

DIFFERENTIAL EQUATION FOR GVF



- Consider the total energy H of a gradually varied flow in a channel of small slope.

$$H = Z + E = Z + y + \frac{v^2}{2g}$$

Where,

E = Specific energy.

- Since water surface generally varies in the longitudinal (x) direction, the depth of flow and total energy are functions of x.

Differentiating the above equation w.r.t x

$$\frac{dH}{dx} = \frac{dz}{dx} + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{v^2}{2g}\right) \qquad ...(i)$$

1. $\frac{dH}{dx}$ represents the slope of total energy line (S_f) and the slope of energy line decreases in the direction of motion. It is common to consider the slope of the decreasing line as positive.

$$\frac{dH}{dx} = -S_f$$

2. $\frac{dz}{dx}$ represents the S_0 slope of bed.

$$\frac{dz}{dx} = -S_0$$

3. $\frac{dy}{dx}$ represents the water surface slope relative to the bottom of the channel.

4. $\frac{d}{dx}\left(\frac{v^2}{2g}\right)$ represents the slope of velocity head. Which can also be written as

$$\frac{d}{dx}\left(\frac{v^2}{2g}\right) = \frac{d}{dy}\left(\frac{Q^2}{2gA^2}\right)\frac{dy}{dx}$$

$$\frac{d}{dx}\left(\frac{v^2}{2g}\right) = \frac{-Q^2}{2gA^3}\frac{dA}{dy}\frac{dy}{dx} \qquad \left[\frac{dA}{dy} = T\right]$$

$$\frac{d}{dx}\left(\frac{v^2}{2g}\right) = \frac{-Q^2T}{gA^3}\frac{dy}{dx}$$

Now equation 1 can be written as

$$-S_f = -S_0 + \frac{dy}{dx} - \frac{Q^2T}{gA^3}\frac{dy}{dx}$$

$$S_0 - S_f = \left(1 - \frac{Q^2T}{gA^3}\right)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \frac{Q^2T}{gA^3}}$$

The above equation is known as Dynamic equation of GVF

$$\text{Also, } \frac{Q^2T}{gA^3} = F_r^2 \qquad \left[F_r = \frac{v}{\sqrt{g\frac{A}{T}}}\right]$$

$$\therefore \quad \frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^2}$$

DIFFERENTIAL ENERGY EQUATION

We know that, $H = E + Z$

Differentiating write to x

$$\frac{dH}{dx} = \frac{dE}{dx} + \frac{dz}{dx}$$

$$-S_f = -S_0 + \frac{dE}{dx}$$

$$\boxed{\frac{dE}{dx} = S_0 - S_f}$$

CLASSIFICATION OF FLOW PROFILES

- In a given channel y_n and y_c are two fixed depth if Q, n and S_0 are fixed.

  Where,

  y_n = normal depth of flow

  y_c = critical depth of flow

- Critical depth of flow depends on the shape of cross section and discharge of the channel.

$$\boxed{\frac{Q^2}{gA^2} = y_c}$$

- There are 3 relations between y_n and y_c.

  (1) $y_n > y_c$ Mild slope

  (2) $y_n = y_c$ Critical slope

  (3) $y_n < y_c$ Steep slope

- There are 2 relations when y_n does not exists.

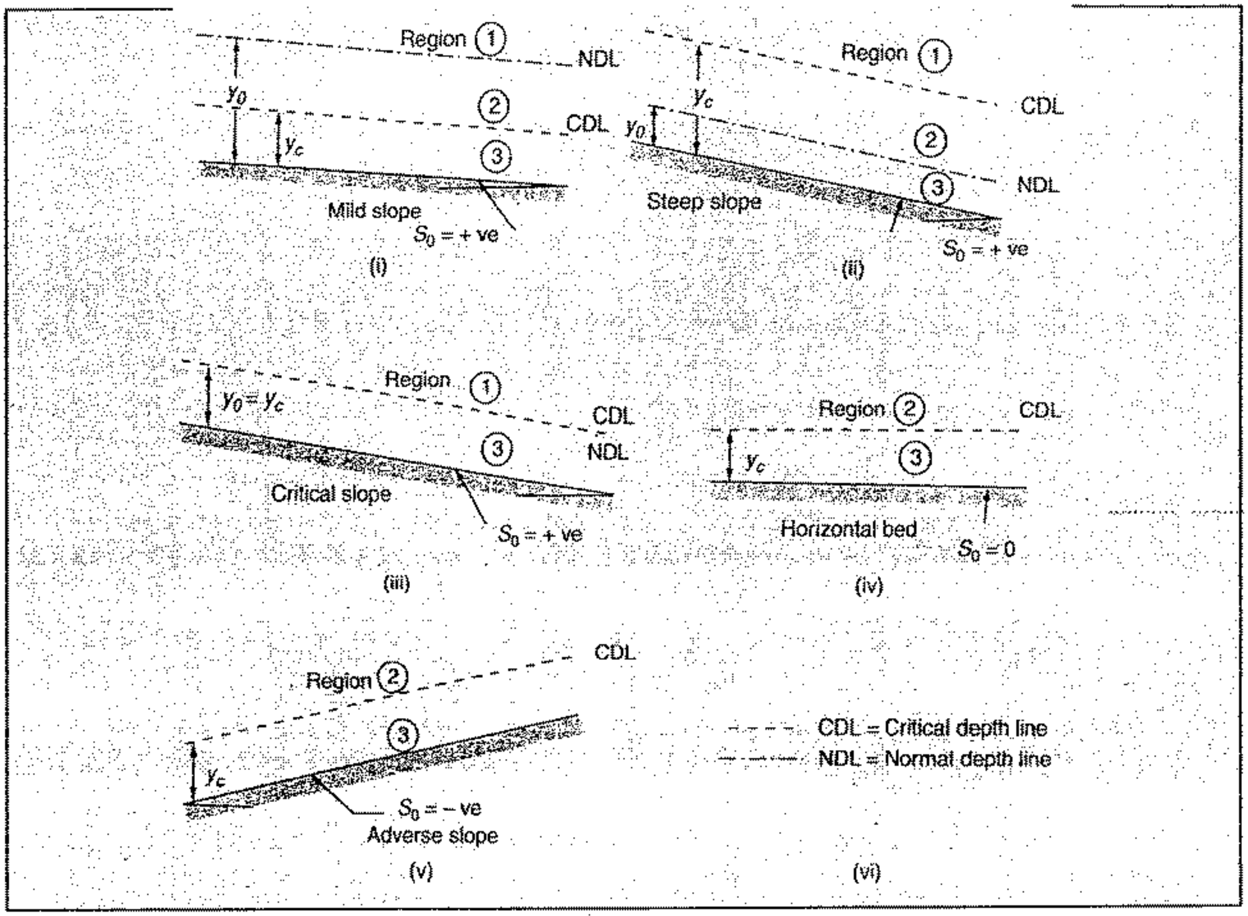
  (1) $S_0 < 0$ Adverse slope

  (2) $S_0 = 0$ Horizontal slope.

  Based on above relations channel are classified into five cateogries.

Sl. No.	*Channel category*	*Symbol*	*Characteristic condition*	*Remark*
1.	Mild slope	M	$y_0 > y_c$	Subcritical flow at normal depth
2.	Steep slope	S	$y_c > y_0$	Supercritical flow at normal depth
3.	Critical slope	C	$y_c = y_0$	Critical flow at normal depth
4.	Horizontal bed	H	$S_0 = 0$	Cannot sustain uniform flow
5.	Adverse slope	A	$S_0 < 0$	Cannot sustain uniform flow

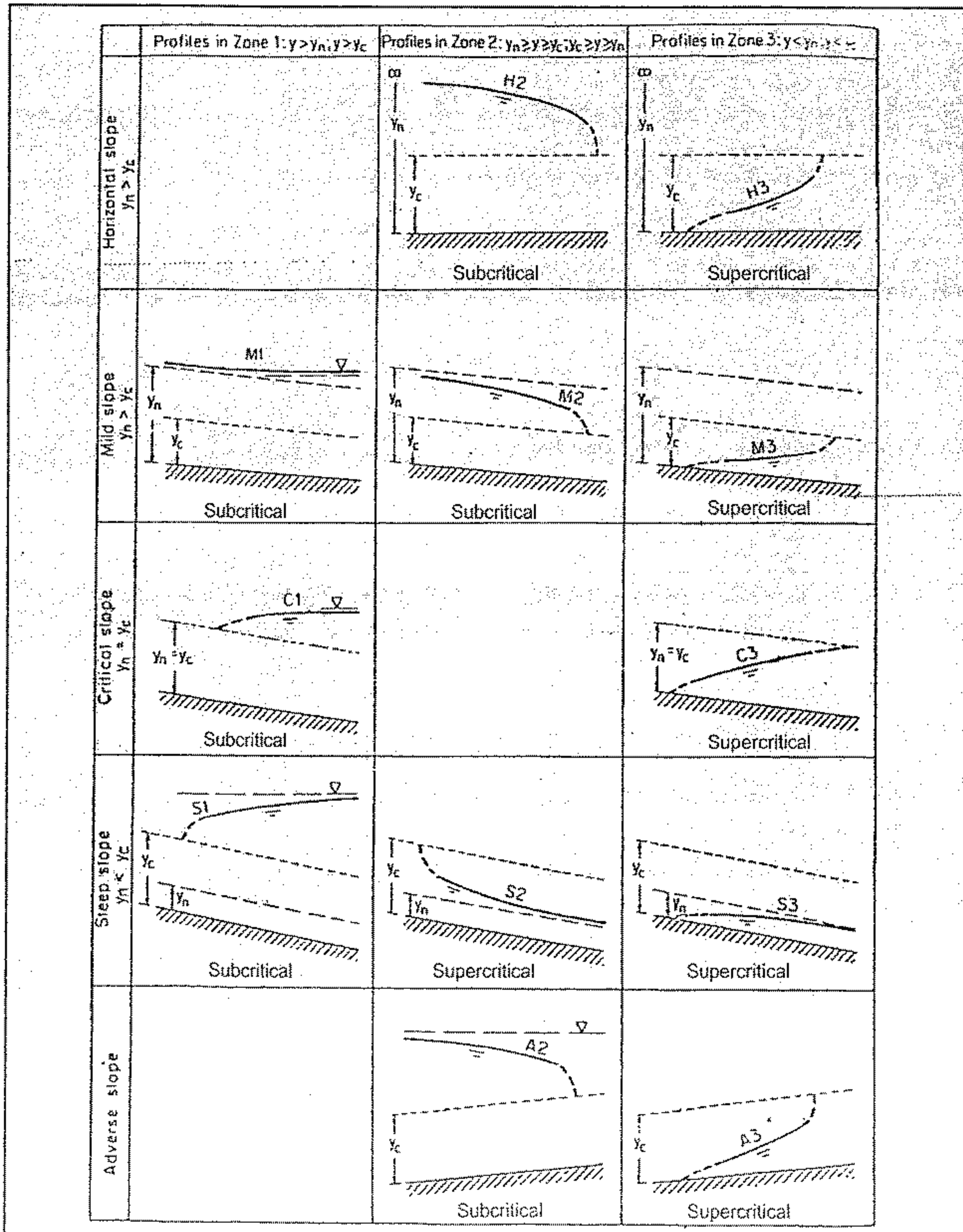
- For each of the five cateogries, longitudinal sections can be drawn as lines representing the critical depth and normal depth (if exist)



- Classification of flow profile is done based on the slope of channel and the regions of flow. Hence, we can have 12 different types of flow profiles.

Channel	Region	Condition	Type
Mild slope	1	$y > y_0 > y_c$	M_1
	2	$y_0 > y > y_c$	M_2
	3	$y_0 > y_c > y$	M_3
Steep slope	1	$y > y_c > y_0$	S_1
	2	$y_c > y > y_0$	S_2
	3	$y_c > y_0 > y$	S_3
Critical slope	1	$y > y_0 = y_c$	C_1
	3	$y < y_0 = y_c$	C_3
Horizontal bed	2	$y > y_c$	H_2
	3	$y > y_c$	H_3
Adverse slope	2	$y > y_c$	A_2
	3	$y > y_c$	A_3

- Classification of flow profile is important because it gives an understanding of how the flow depth varies in the channel.
- Qualitative observation of various types of flow profile can be made and profile can be sketched without performing any calculation.



Important Points for Qualitative Calculations

(1) As depth of flow decreases, head loss increases

we know that,

$$h_L \propto v \propto \frac{1}{y}$$

(2) If Energy line is parallel to the channel bed, i.e. slope of energy line is equal to the bed slope, flow will takes place at normal depth.

$$S_f = S_0 \Rightarrow y = y_n$$

(3) If slope of energy line is greater than the bed slope, flow will take place at depth less then the normal depth.

$$S_f > S_0 \Rightarrow y < y_n$$

(4) Similarly if slope of energy line is less than the bed slope, flow will take place at depth greater than normal depth

$$S_f < S_0 \Rightarrow y > y_n$$

Summary

1. $(S_0 - S_f) < 0$ if $y < y_n$
2. $(S_0 - S_f) > 0$ if $y > y_n$
3. $(S_0 - S_f) < 0$ if $S_0 = 0$ (i.e. horizontal slope)
4. $(S_0 - S_f) < 0$ if $S_0 < 0$ (i.e. adverse slope)
5. $F_r > 1$ if $y < y_c$ (Super critical flow)
6. $F_r < 1$ if $y > y_c$ (Sub critical flow)
7. $\frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^2}$
8. Depth of flow will increase in Region I and Region III

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^2} = +\text{ve, if, } \left.\begin{array}{l}(a)\ y > y_n, y > y_c \\ (b)\ y < y_n, y < y_c\end{array}\right\} \text{Region } 1 \,\&\, 3$$

9. Depth of flow will decrease in Region II

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^2} = -\text{ve, if, } \left.\begin{array}{l}(a)\ y_c > y > y_n \\ (b)\ y_n > y > y_c\end{array}\right\} \text{Region } 2$$

10. Surface profile will approach normal depth asymtotically if depth of flow approaches to normal depth of flow.

if $y \to y_n$

then $S_f \to S_0$

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^2} = 0$$

11. Surface profile will approach the critical depth line vertically if depth of flow approaches to critical depth vertically.

if $y \to y_c$

then $f_r \to 1$

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^2} = \frac{S_0 - S_f}{0} = \infty$$

Note: Under this situation, flow profile will have very steep slope and hence curvature of flow profile will large and it can not be treated as GVF. Due to large curvature, normal acceleration can not be assumed to be zero and hydrostatic condition will not be valid. Thus over here assumption of hydrostatic condition will be neglected.

12. Surface profile becomes horizontal as flow depth becomes large.

if, $y \to \infty$

then $S_f \to 0,$

$F_r \to 0$

$$\frac{dy}{dx} = S_0$$

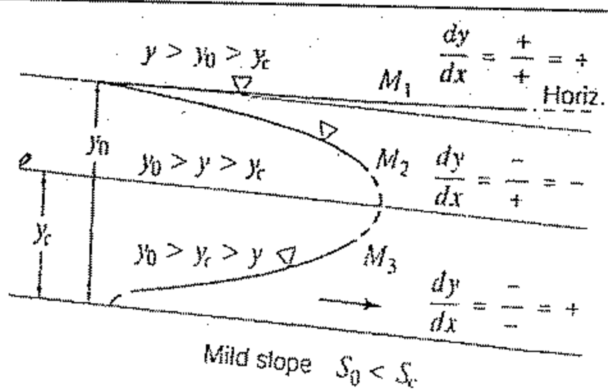
13. Surface profile meets the bed vertically if depth of flow reduces to zero.

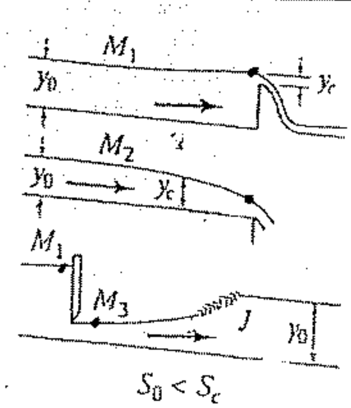
if, $y \to 0,$

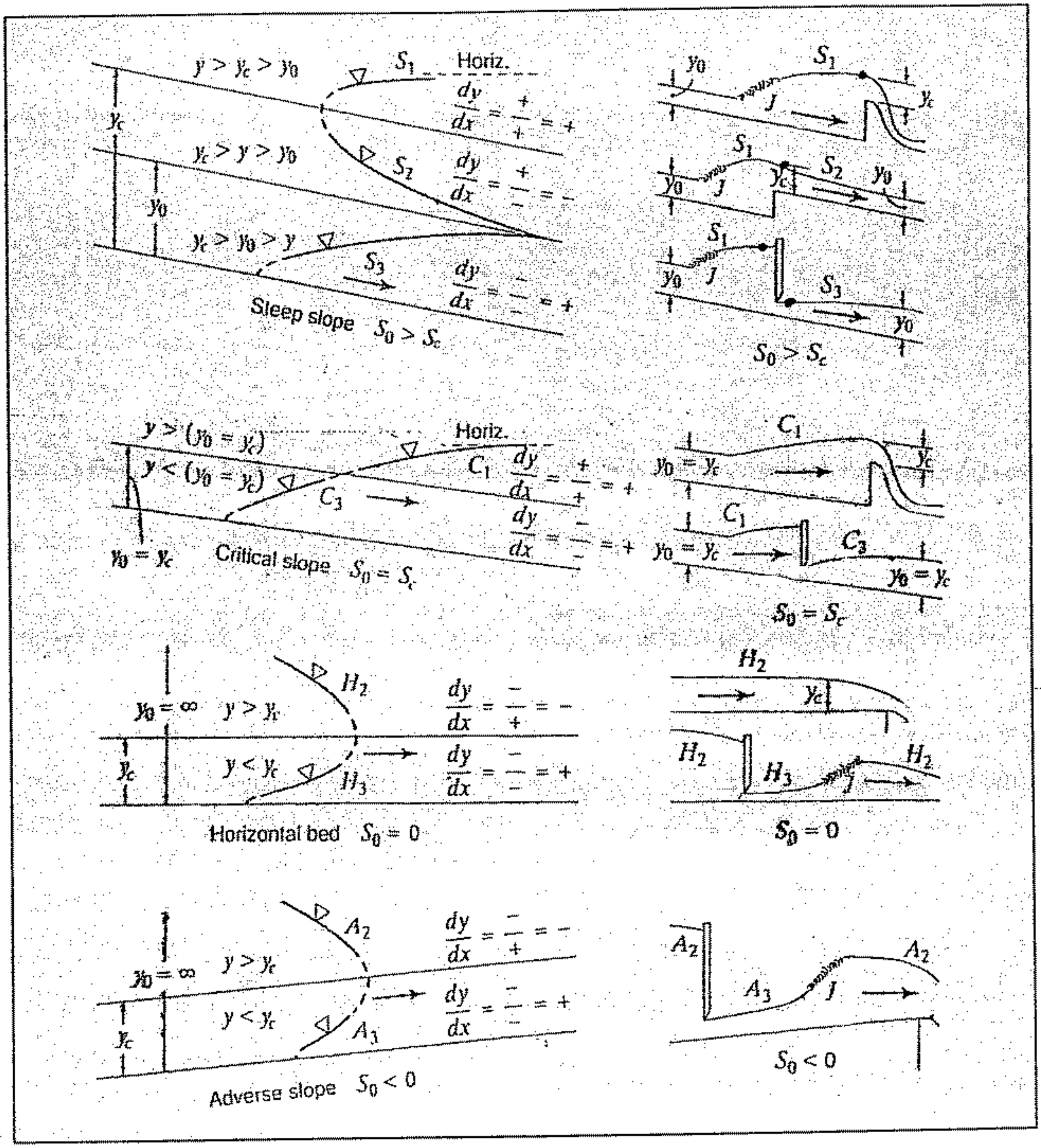
$F_r \to 0$

$$\frac{dy}{dx} = \infty$$

Note: It can be shown that $\frac{dy}{dx} = \frac{gy^3(S_0 y^{10/3} - q^2 n^2)}{y^{10/3}(gy^3 - q^2)}$, from the above relation we can observe that, as $y \to 0$, $\frac{dy}{dx} \to \infty$, i.e. water surface profile becomes vertical as the flow depth tends to zero.

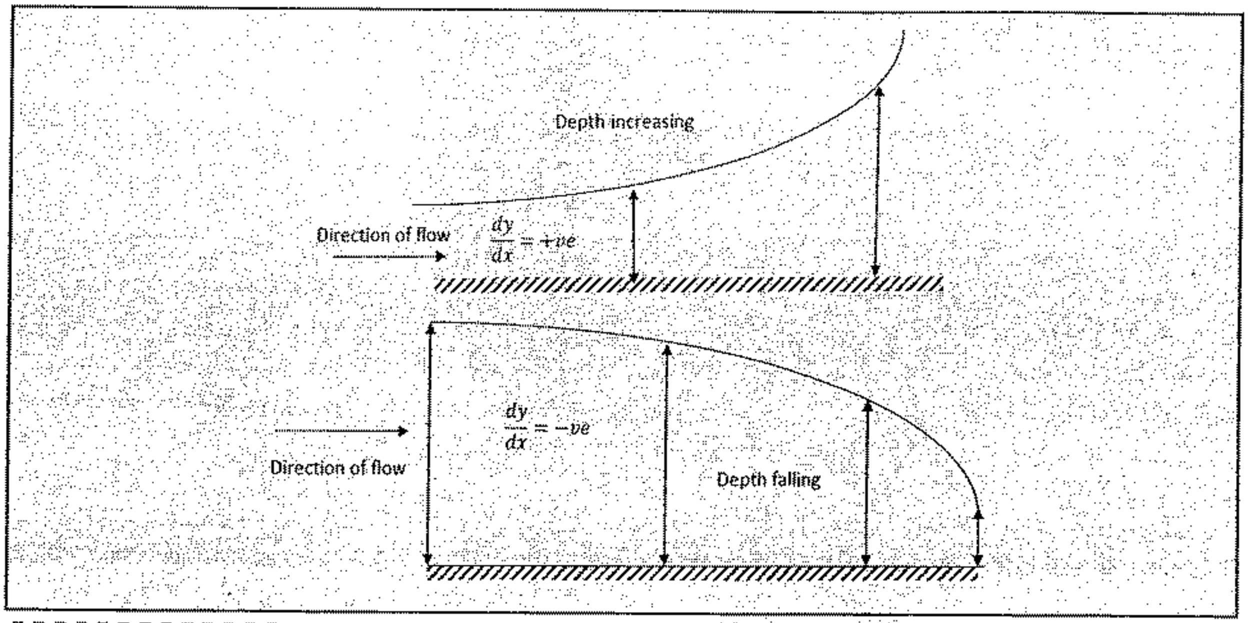






BACK WATER AND DRAWDOWN CURVE

- If the depth of flow increases in the direction of flow, i.e. $\frac{dy}{dx}$ = +ve then the dynamic equation of GVF will represent a Back water curve.
- If the depth of flow decreases in the direction of flow, i.e. $\frac{dy}{dx}$ = –ve then the dynamic equation of curve will represent a drawdown curve.

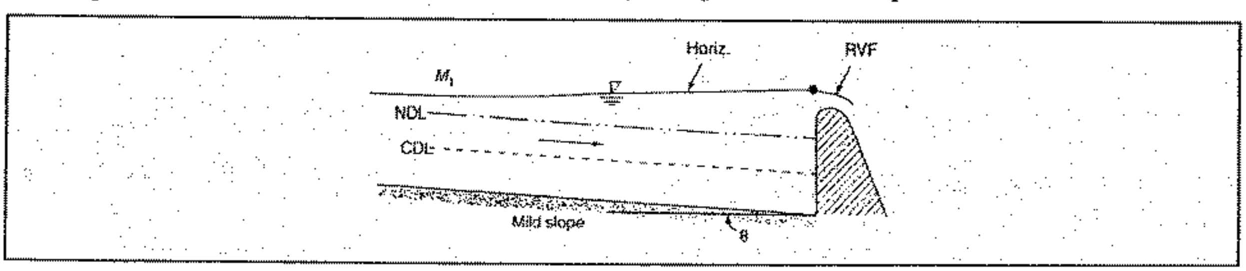


Note: Depth of flow increases because of obstructions or because slope is insufficient to maintain the rate of flow. Flow region in (1) & (3) are back water curve where as flow in region (2) are drawdown curves.

FEATURES OF FLOW PROFILE

M_1 Profile

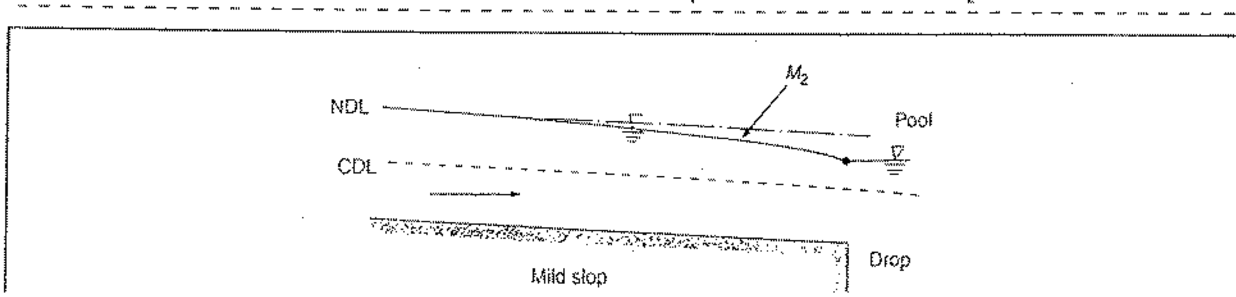
- M_1 curve is the most common of all GVF profiles.
- When a subcritical flow gets obstructed by structures such as weirs , dams, control structures and natural features produce M_1 back water curves.
- M_1 curves extend to several kilometers before joining the normal depth of flow.



M_2 Profile

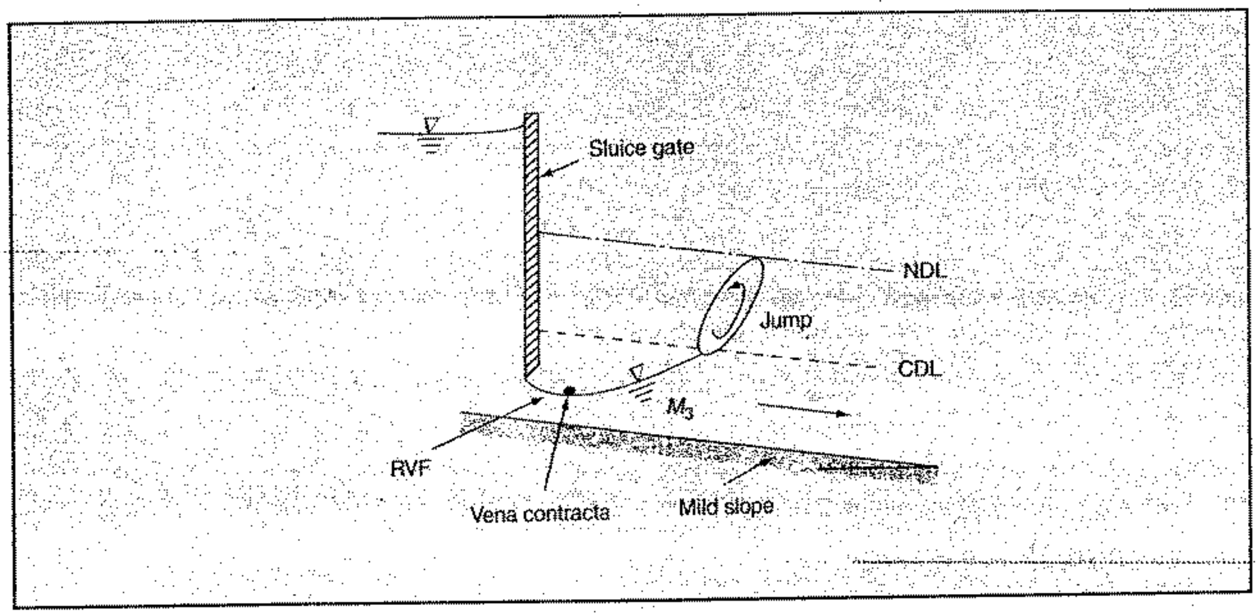
- M_2 profile occurs when a subcritical flow experiences a sudden drop in the channel bed for example at constriction type transitions and at canal outlet into pools.

Note: If the pool level is above NDL at the end of channel profile is M1 otherwise M_2.



M_3 Profile

- M_3 Profile occurs when a supercritical stream enters a mild-slope channel. For example flow leading from a spillway or a sluice gate to a mild slope.
- Begining of M_3 curve is usually followed by a small streach of RVF and down stream is terminated by a Hudraulic jump.
- In comparision to M_1 & M_2, M_3 curve is of relatively short in length.

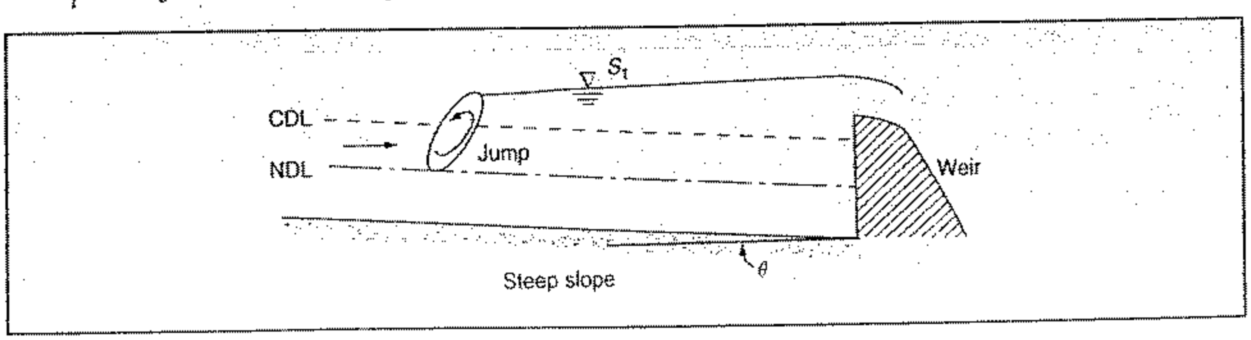


Note: Flow when changes from super critical to subcritical it is accomparied by Hydraullic jump.

Hydraullic jump on Mild slope will be repelled by M_3 curve.

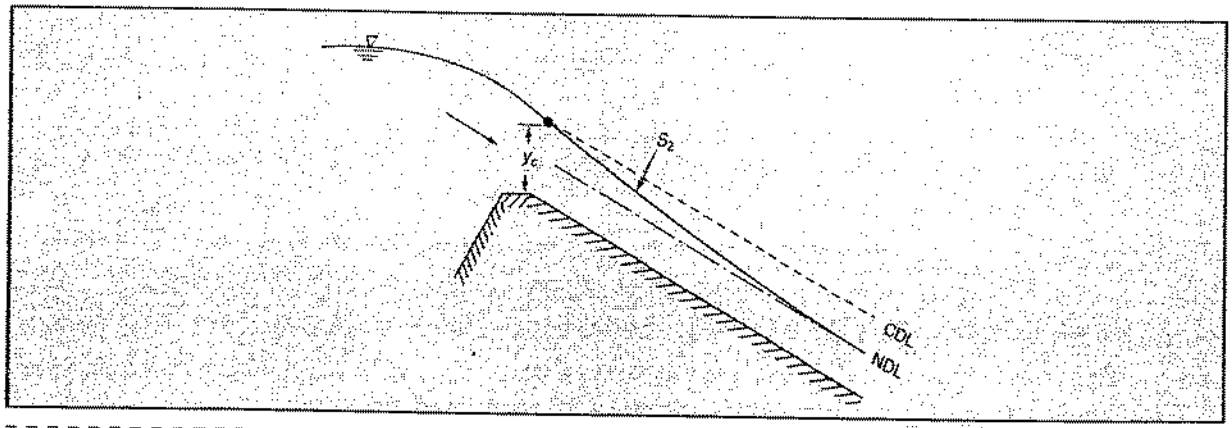
S_1 Profile

- S_1 curve is produced when a supercritical flow over a steep channel is terminated by a deep pool created by an obstruction, such as weir or dam.
- During S_1 curve flow changes from supercritical flow to subcritical flow through a Hydraulic jump. i.e. S_1 curve is preceeded by Hydraulic jump.
- S_1 curve joins downstream pool with an horizontal asymptote curve.



S_2 Profile

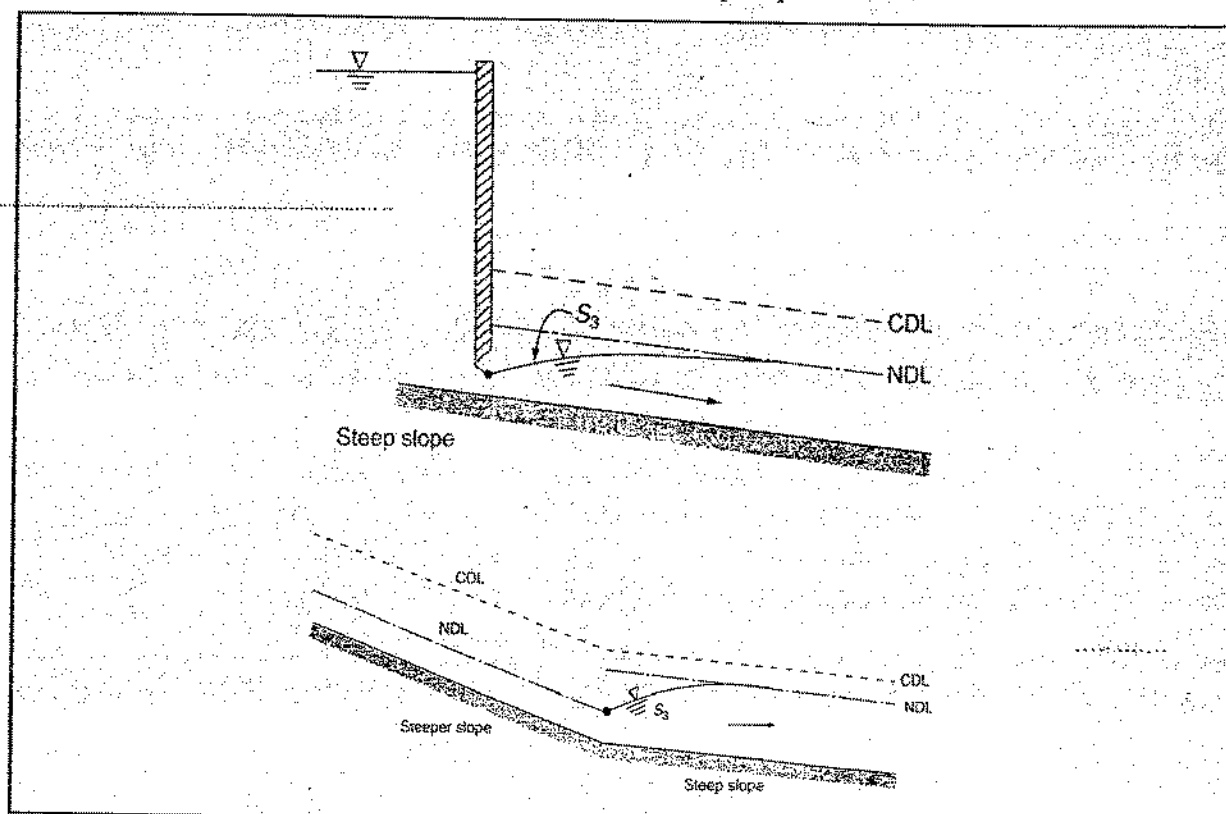
- S_2 curve occurs at the entrance region of step channel leading from a reservior and at a break of grade from mild slope to steep slope.
- S_2 curves are generally short in length



Note: Depth at the entrance region will be critical depth.

S_3 Profile

- S_3 curves results when a free flow from a sluice gate enters a steep slope on downstream side. Or when a flow exister from a steeper slope to a less steep slope.



Note: A, C & H profiles froms rarely.

CONTROL SECTION

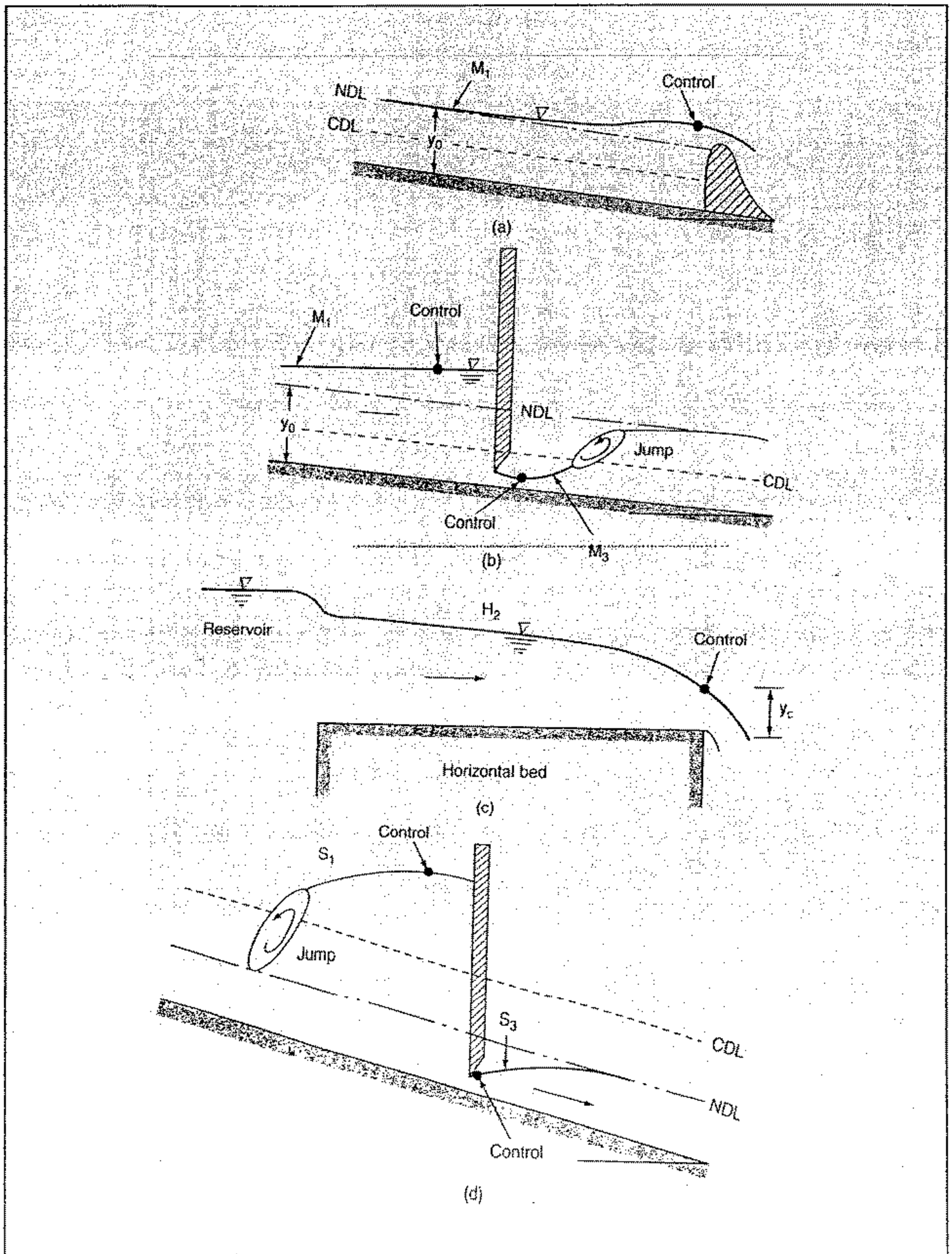
- A control section is defined as a section in which fixed relationship exists between depth of flow and discharge

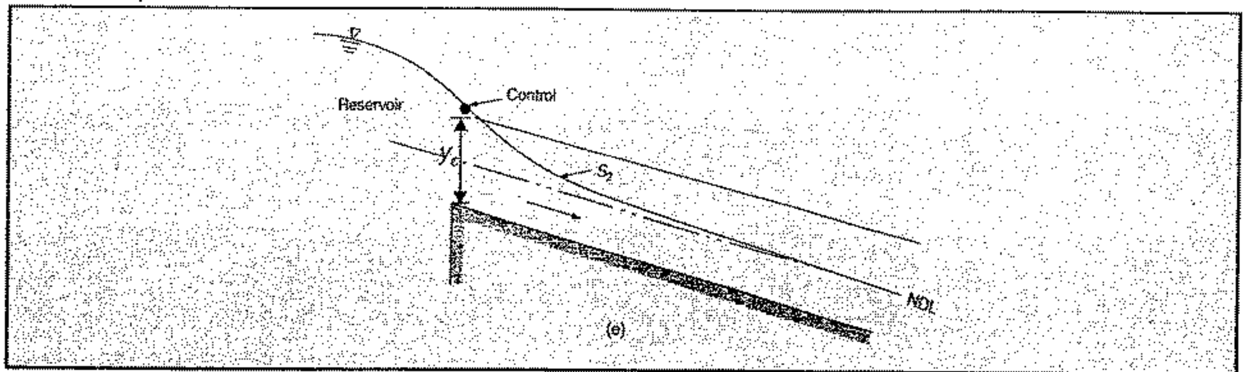
  eg., Wire, Spillway, Sluice gate.

- Control section is a point where calculation of GVF profile starts. It proceeds upstream in case of subcritical flow and d/s for super critical flow.

Hence, Super critical flow have u/s contro.

Sub-critical flow have d/s contro.

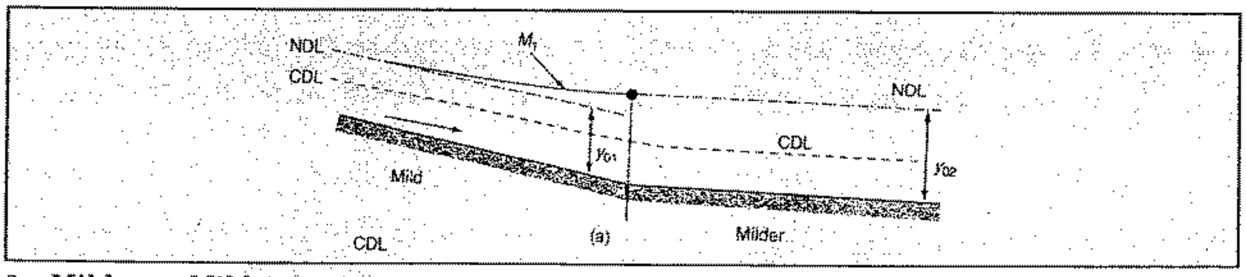




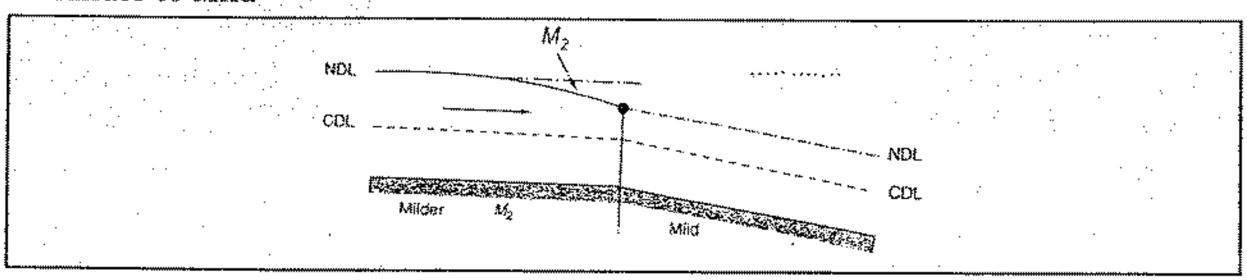
BREAK IN GRADE

- When two channel section have different bed slope the section is called break in grade.
- Following points should be noted for the qualitative analysis of flow profiles
  - (a) For sub-critical flow → Control section is downstream.
  - (b) For supercritical flow → Control section is upstream.
  - (c) Draw CDL and NDL for various slopes.
  - (d) CDL is at a constant height above the channel bed in both slopes as y_c is independent of slope of channel.
  - (e) NDL for Milder slope will higher than the NDL for Mild slope.
  - (f) Match NDL to NDL for various types of flow.

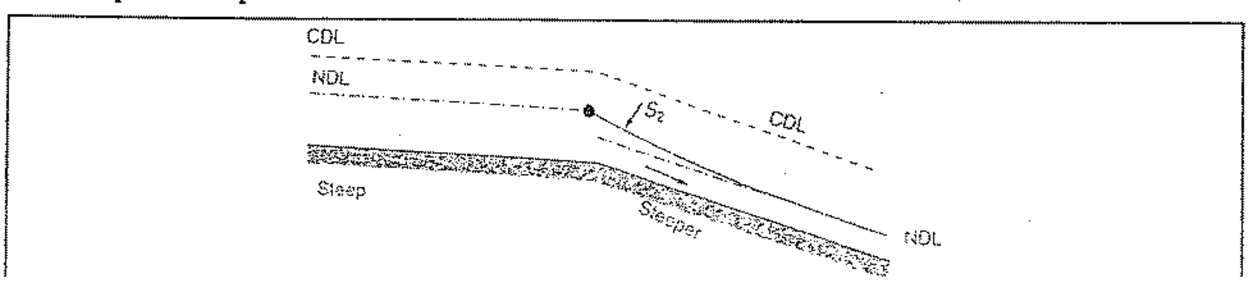
1. Mild to Milder



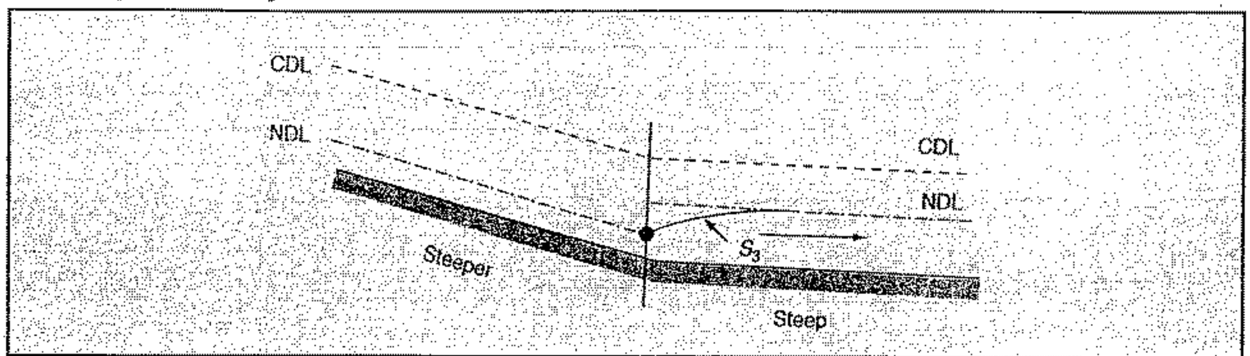
2. Milder to Mild



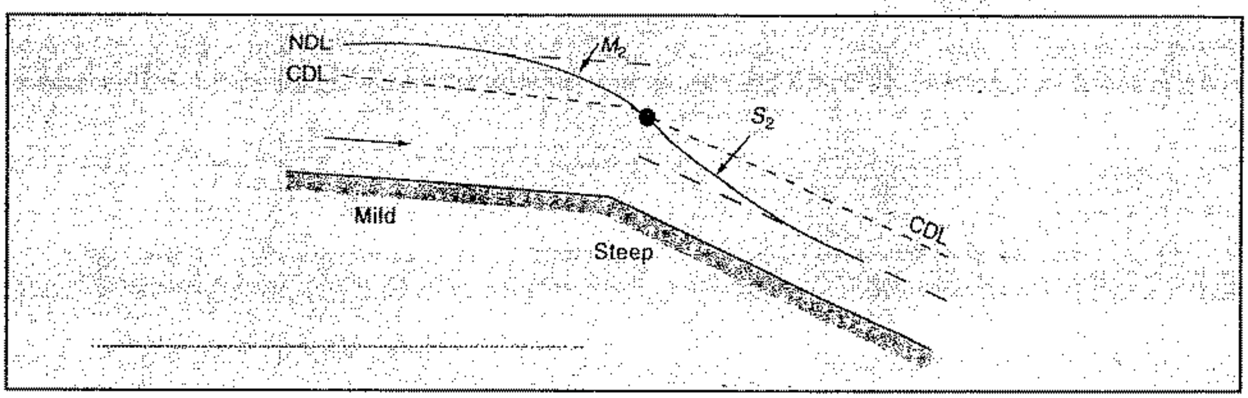
3. Steep to Steeper



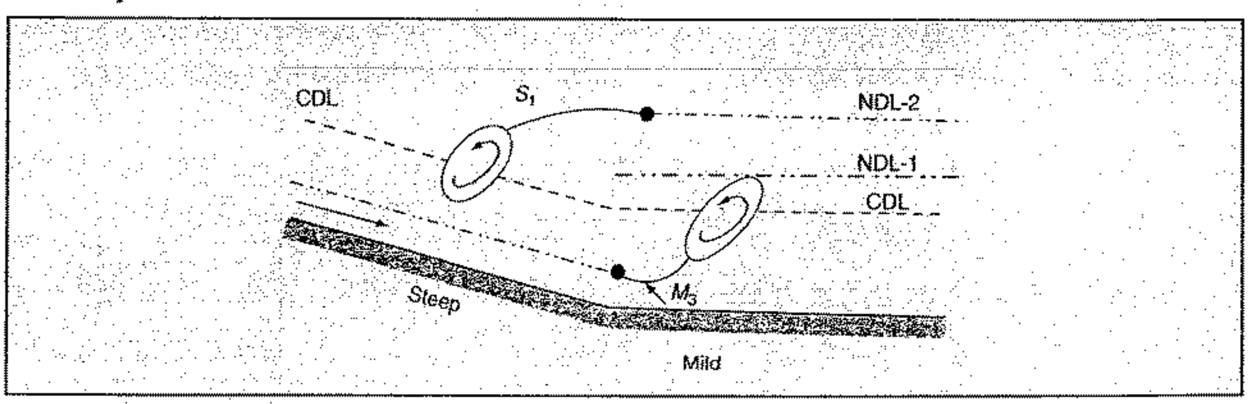
4. Steeper to Steep



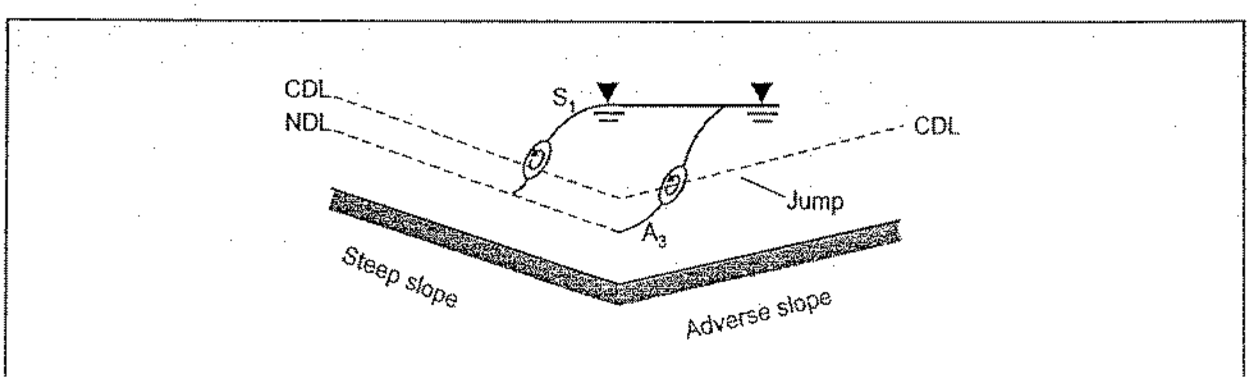
5. Mild to Steep



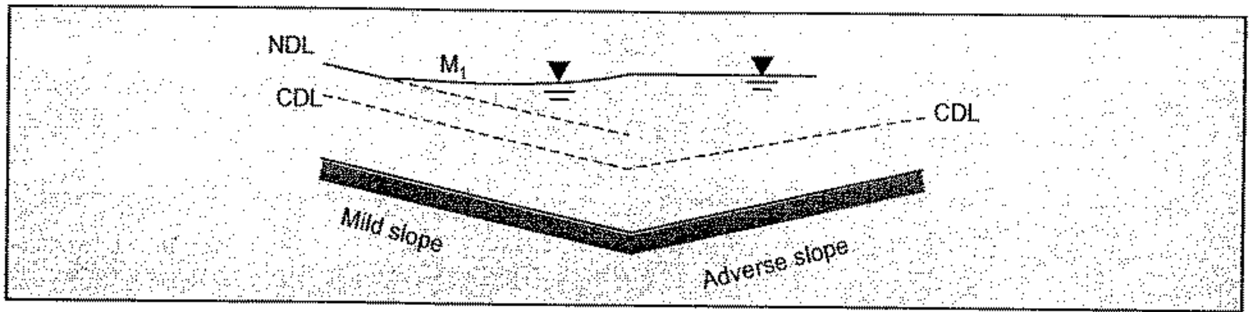
6. Steep to Mild



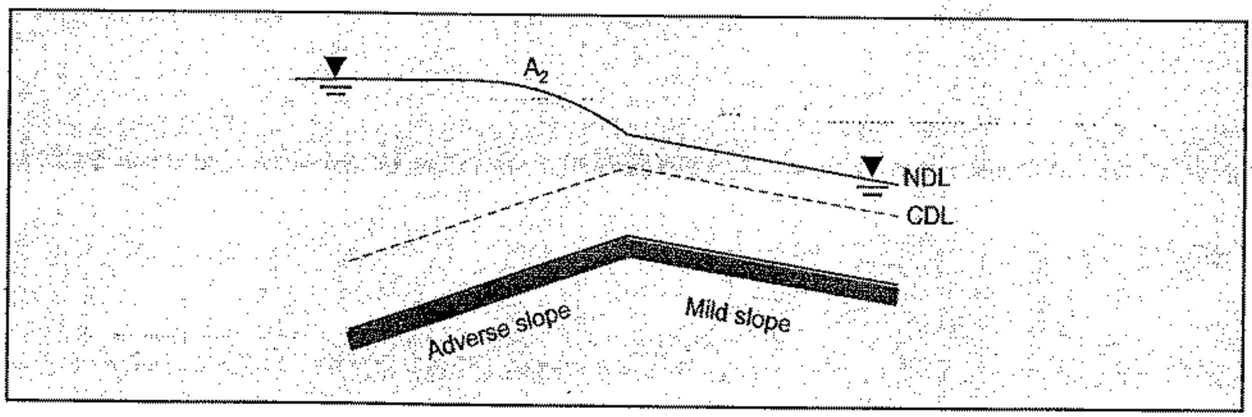
7. Steep to Adverse



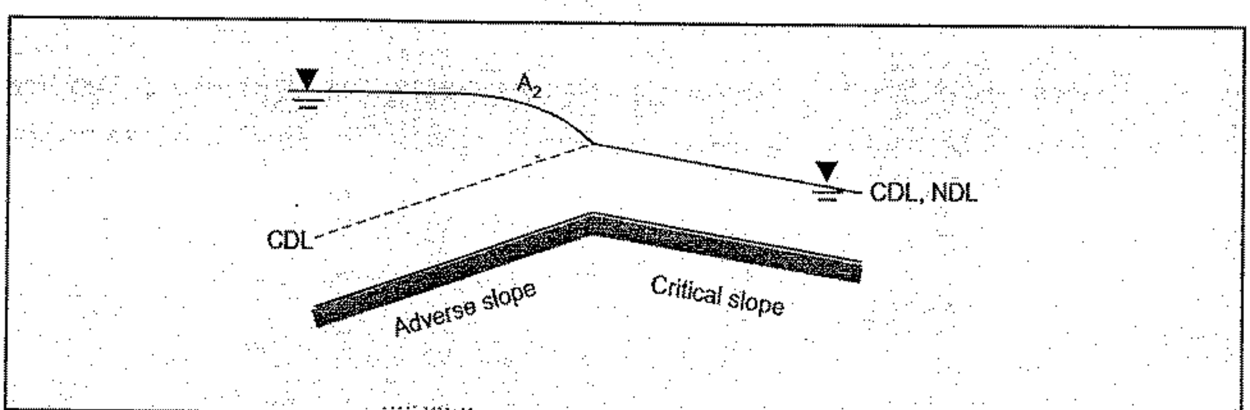
8. Mild to Adverse



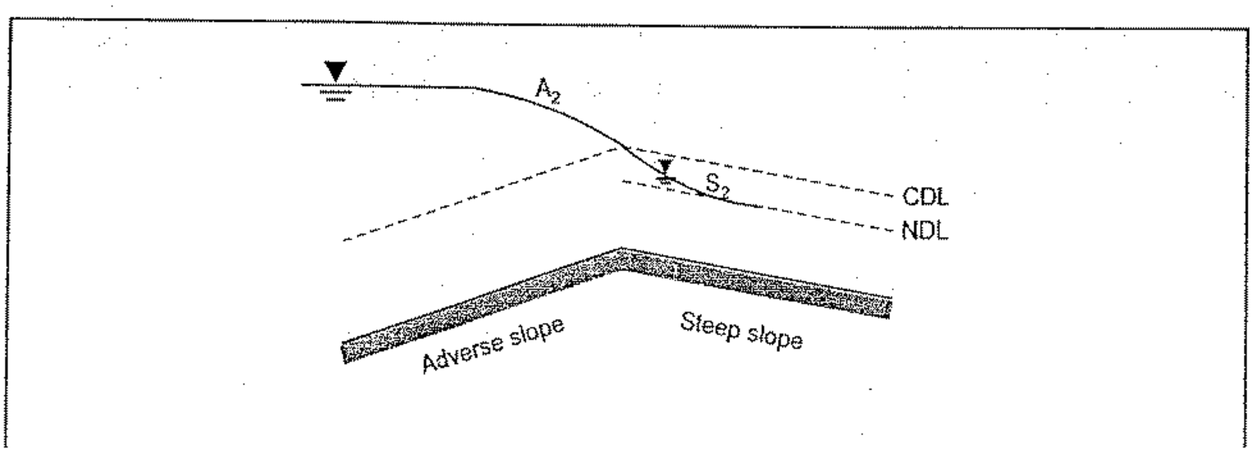
9. Adverse to Mild



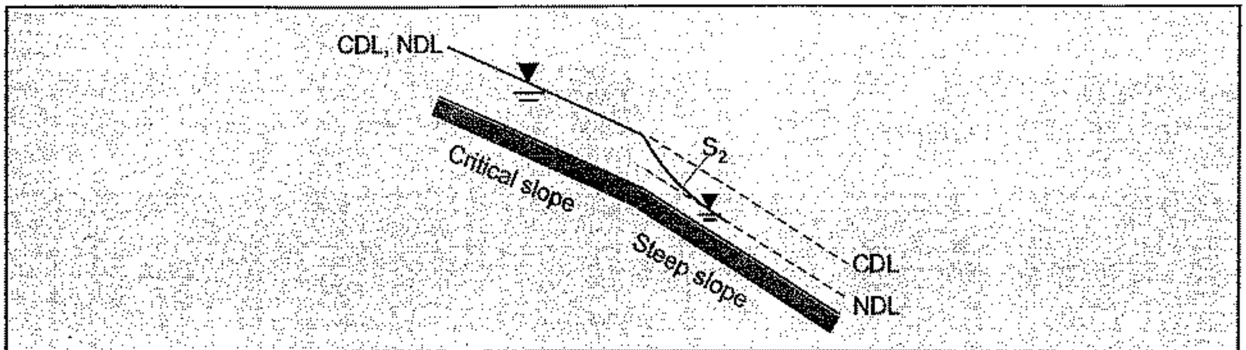
10. Adverse to Critical



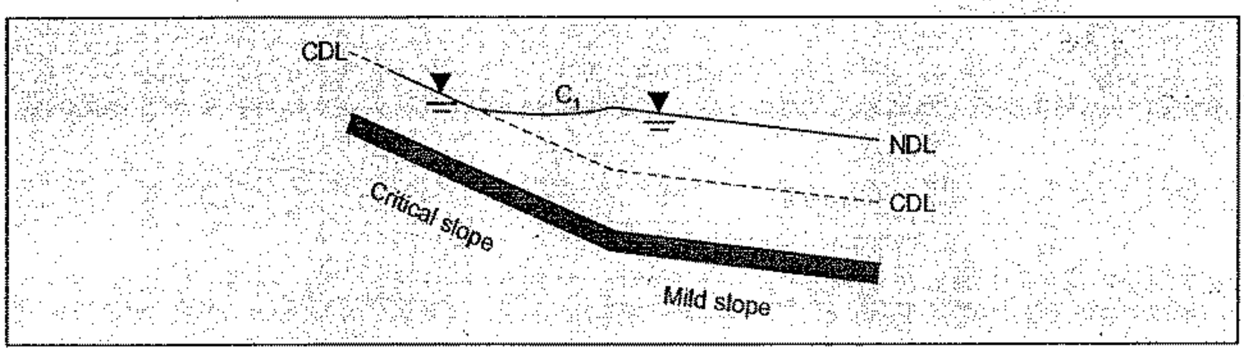
11. Adverse to Steep



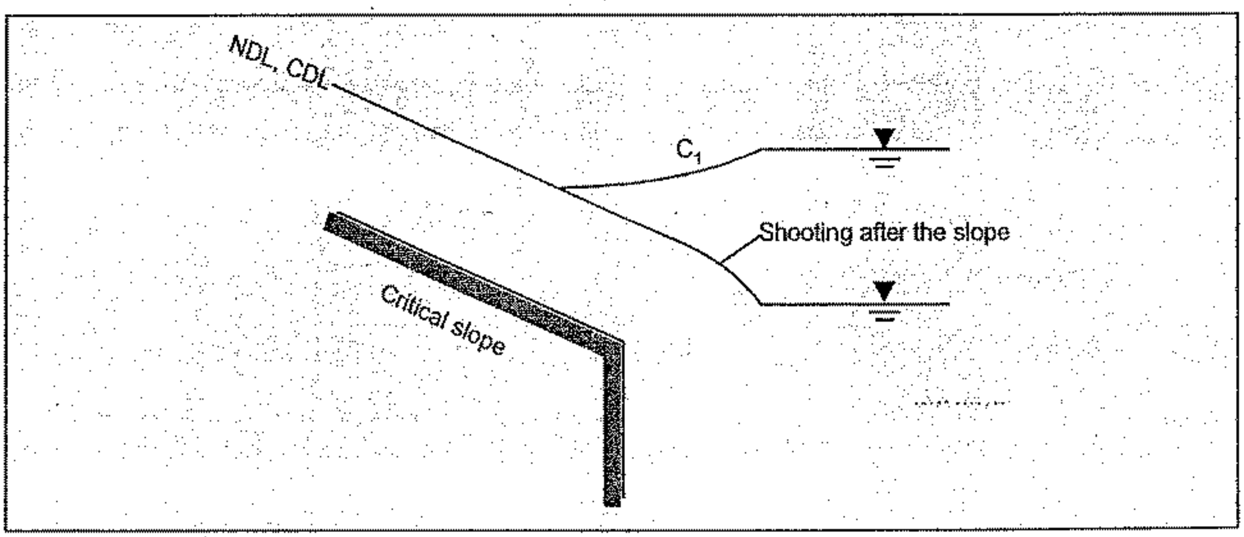
12. Critical to Steep



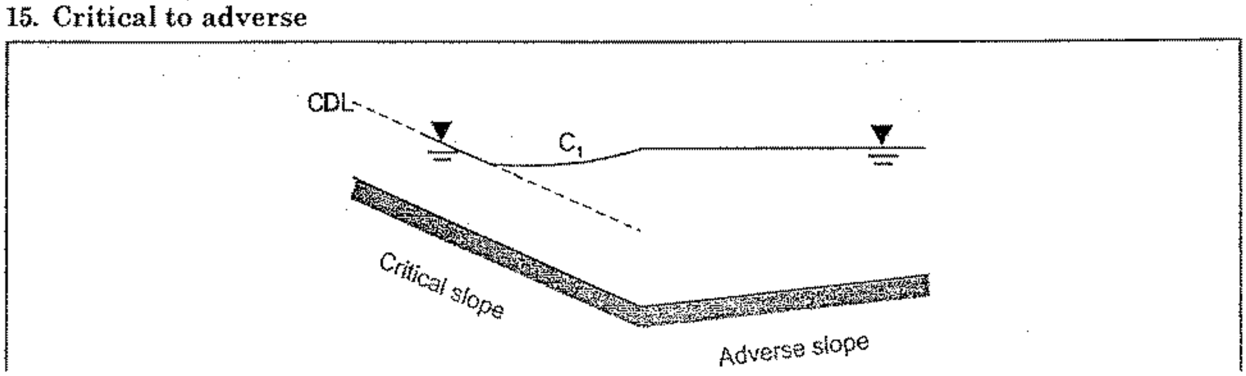
13. Critical to Mild



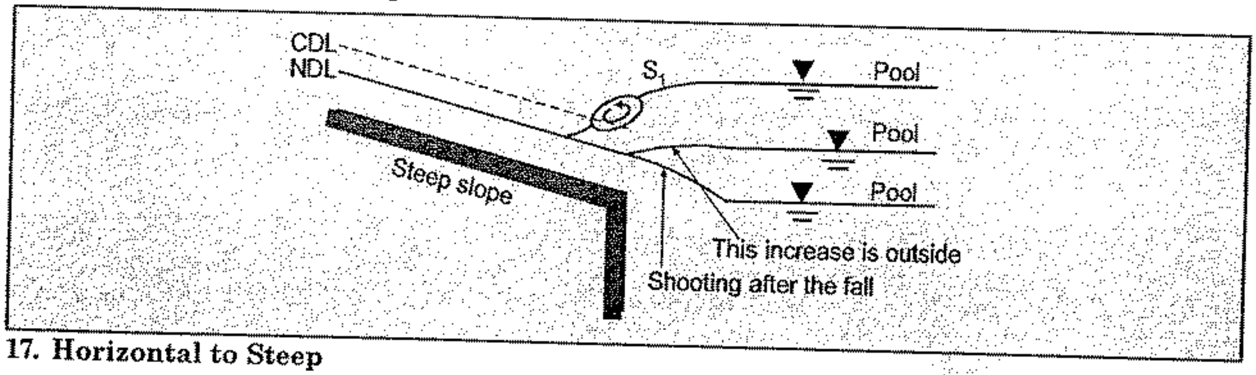
14. Sudden end of critical slope



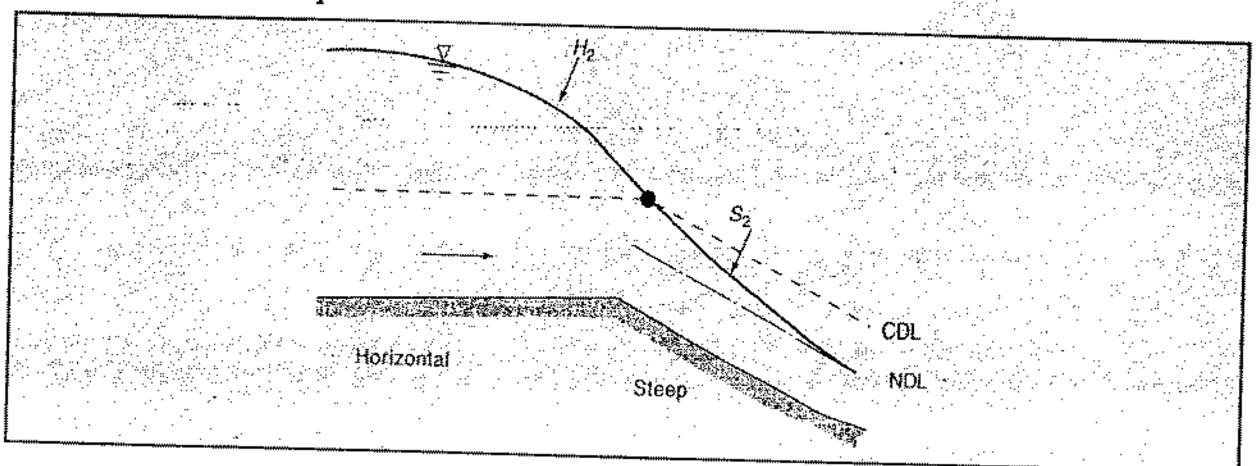
15. Critical to adverse



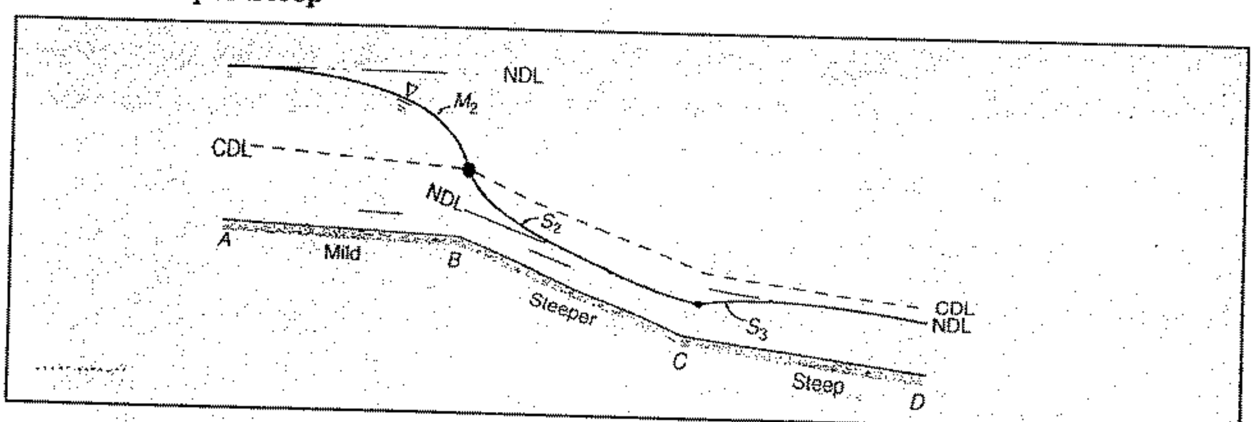
16. Sudden end of steep slope



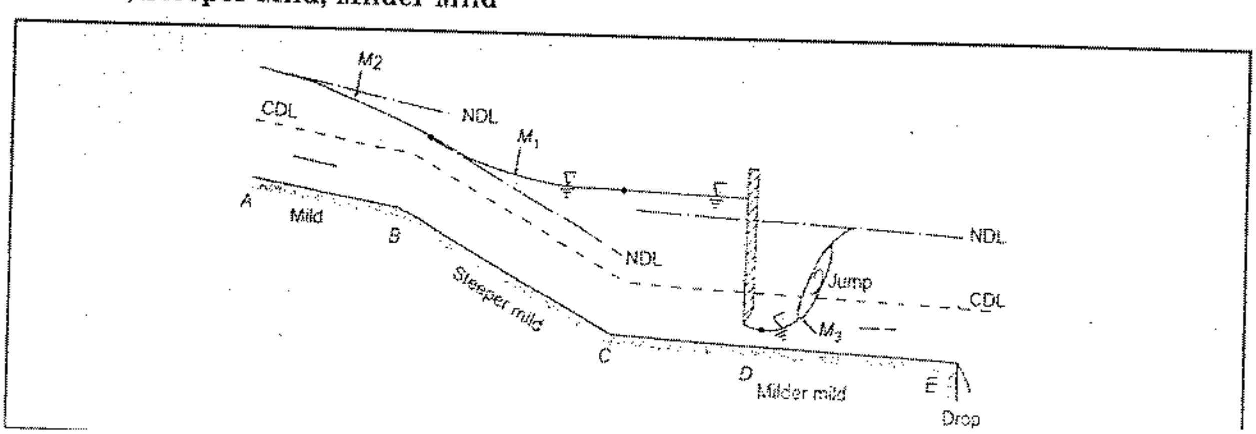
17. Horizontal to Steep



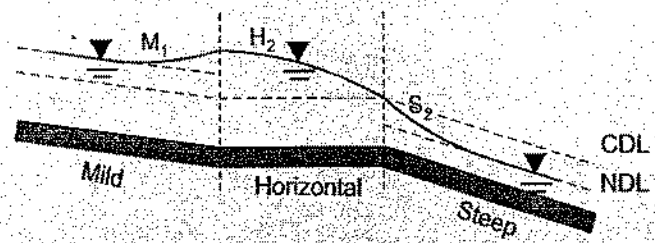
18. Mild Steeper Steep



19. Mild, Steeper Mild, Milder Mild



20. Mild, Horizontal, Steep



Example 1

Water flows from a lake into a steep rectangular channel 3 m wide. Determine the lake level above the channel bed at the outfall if discharge is 40 m^3/sec.

Sol. As we know that depth of flow at outfall will be critical depth and hence that will be depth of lake above the channel bed.

Given,

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$= \left(\frac{\left(40/3\right)^2}{9.81}\right)^{1/3} = 2.627 \text{ m}$$

Example 2

Show that for wide rectangular channel the bed slope 'S_o' is mild or steep according to so being less than or greater than $\frac{n^2 g^{10/9}}{q^{2/9}}$

Sol. We know that

$$v = \frac{1}{n} R^{2/3} S^{1/2}$$

$$S^{1/2} = \frac{vn}{R^{2/3}}$$

$$S = \frac{v^2 n^2}{R^{4/3}}$$

If $S = S_c$, critical bed slope then depth of flow taking place will be critical depth only i.e. $y = y_c$.

Further in wide recatangular channel ie. B >> y

$$R = \frac{B.y}{B+y} \cong \frac{B.y}{B} = y \; (B >> y)$$

$$S_c = \frac{n^2 v_c^2}{y_c^{4/3}}$$

We know that as critical flow in rectangular channel

$$v_c = \sqrt{g y_c}$$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$S_c = \frac{n^2 g y}{y_c^{4/3}} = \frac{n^2 g}{y_c^{1/3}} = \frac{n^2 g}{\left(\frac{q^2}{g}\right)^{1/9}}$$

$$S_c = \frac{n^2 g . g^{1/9}}{q^{2/9}} = \frac{n^2 . g^{10/9}}{q^{2/9}}$$

Example 3

A concrete lined trapezoidal irrigation canal has a bottom width of 10m, side slope 1H : 1V and longitudianl slope of 0.0005. If the channel is several kilometers long, what is the depth of flow near the downstream end for a flow of 60m³/sec under a free fall condition.

Sol. Given,

$Q = 60 \text{ m}^3/\text{sec}$; $B = 10$ m; $1 : m = 1V : 1H$; $S = 0.0005$

$$\text{Area, } A = (B + my)y = (10 + y_c)y_c$$

We know that under free fall condition depth of flow at free fall location will be the critical flow.

$$\frac{Q^2 T}{g A^3} = 1$$

$$\left[\frac{A}{T} = y_c\right]$$

$$\frac{Q^2}{g A^2} = y_c$$

$$y_c = \frac{60^2}{9.81 \times (10 + y_c)^2 y_c^2}$$

$$(10 + y_c)^2 y_c^3 = \frac{60^2}{9.81}$$

Solving by hit and trial method

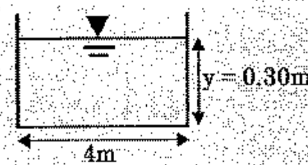
$$y_c = 1.412 \text{ m}$$

Example 4

A rectangular channel with bottom width of 4.0m and a bottom slope of 0.0008 has a discharge of 1.50 m^3/sec. In a gradually varied flow in this channel the depth at a certain location is found to be 0.30m. Assuming n = 0.016, determine the type of GVF profile.

Sol. Given,

$S_0 = 0.008$; $Q = 1.50$ m^3/sec; $n = 0.016$; $B = 4$ m



From Manning's Equation

$$Q = \frac{1}{n} A \times R^{2/3} S^{1/2}$$

$$\text{Area, } A = B \times y_n = 4 \times y_n$$

$$\text{Perimeter, } P = B + 2y_n = 4 + 2y_n$$

$$\text{Hydraulic Radius, } R = \frac{A}{P} = \frac{B \times y_n}{B + 2y_n} = \frac{4 \times y_n}{4 + 2y_n}$$

$$1.5 = \frac{1}{0.016} \times (4 \times y_n) \left(\frac{4 \times y_n}{4 + 2y_n}\right)^{2/3} \sqrt{0.0008}$$

Solving by hit and trial method.

$y_n = 0.426$m, which is normal depth of flow.

$$\text{Critical depth, } y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$y_c = \left(\frac{(Q/B)^2}{g}\right)^{1/3}$$

$$y_c = \left(\frac{(1.5/4.0)^2}{9.81}\right)^{1/3} = 0.243 \text{ m.}$$

As $y_n > y_c$, the channel slope is mild slope.

Also given, y = 0.30m is such that

$$y_n > y > y_c$$

Therefore profile is M_2 profile.

Example 5

A rectangular channel of 4.0 m width has a manning's coefficient of 0.025. for a discharge of 6.0 m^3/sec in this channel, identify the possible GVF profiles produced in the following break in grades.

(a) $S_{01} = 0.0004$ to $S_{02} = 0.015$

(b) $S_{01} = 0.005$ to $S_{02} = 0.004$

Sol. Given,

$B = 4.0m;\ n = 0.025;\ Q = 6.0\ m^3/sec$

Let the normal depth of flow be y

$$\text{Area, } A = B \times y = 4 \times y$$

$$\text{Perimeter, } P = B + 2y = 4 + 2y$$

$$\text{Hydraulic Radius, } R = \frac{A}{P} = \frac{4 \times y}{4 + 2y}$$

$$\text{Critical depth of flow, } y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{(6/4)^2}{9.81}\right)^{1/3} = 0.612\ m$$

(a) From Manning's Equation

$$Q = \frac{1}{n} AR^{2/3} S^{1/2}$$

At $S = S_{01} = 0.004$, Let normal depth of flow be y_1

$$6.0 = \frac{1}{0.025}(4 \times y_1)\left(\frac{4 \times y_1}{4 + 2y_1}\right)^{2/3} \sqrt{.0004}$$

Solving by hit and trial

$$y_1 = 1.906\ m.$$

As $y_1 > y_c$, slope is Mild slope.

At $S = S_{02} = 0.015$ Let Normal depth of flow be y_2

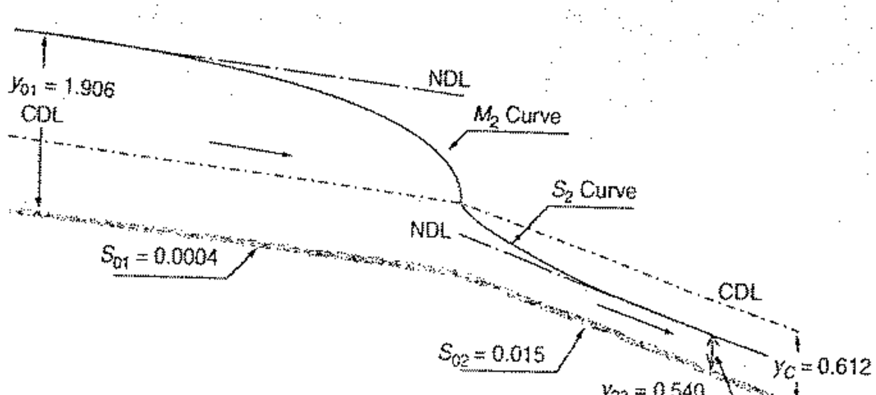
$$6.0 = \frac{1}{0.025}(4 \times y_2)\left(\frac{4 \times y_2}{4 + 2y_2}\right)^{2/3} \sqrt{0.015}$$

Solving by hit and trial

$$y_2 = 0.540\ m.$$

As $y_2 < y_c$, slope is steep slope.

We can observe that type of change in grade is mild to steep and hence GVF profiles are plotted below.



(b) From Manning's equation

$$Q = \frac{1}{n} A \times R^{2/3} S^{1/2}$$

At $S = S_{01} = 0.005$, Let normal depth of flow be y_1

$$6.0 = \frac{1}{0.025}(4 \times y_1)\left(\frac{4 \times y_1}{4 + 2y_1}\right)^{2/3} \sqrt{0.005}$$

Solving by hit and trial

$$y_1 = 0.780m.$$

As $y_1 > y_c$, slope is Mild

At $S = S_{02} = 0.004$, Let depth of flow be y_2

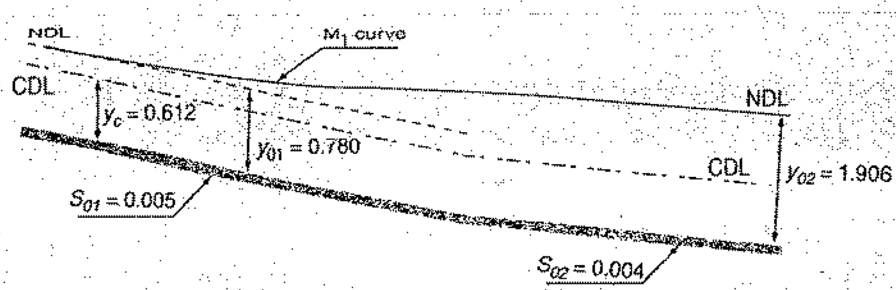
$$6.0 = \frac{1}{0.025}(4 \times y_2)\left(\frac{4 \times y_2}{4 + 2y_2}\right)^{2/3} \sqrt{0.004}$$

Solving by hit and trial

$$y_2 = 1.906$$

As $y_2 > y_c$, slope is mild but $y_1 < y_2$ hence slope is Milder S_{02}.

We can observe that type of change in grade is Mild to milder and hence GVF profiles are plotted below.



Example 6

Prove that if Q_n represents normal discharge at a particular depth 'y' and Q_c represents the critical discharge at same depth 'y' then.

$$\frac{dy}{dx} = S_0\left[\frac{1-(Q/Q_n)^2}{1-(Q/Q_c)^2}\right]$$

Sol. We know that,

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^2}$$

$$\frac{dy}{dx} = \frac{S_0\left(1 - \frac{S_f}{S_0}\right)}{1 - F_r^2} \quad ...(i)$$

From manning's Equation

$$Q = \frac{1}{n} A \times R^{2/3} S^{1/2}$$

If depth of flow is y_1 then $Q = k\sqrt{S}$

For depth y, normal discharge, $Q = k\sqrt{S_0}$

But in GVF, for depth y, $Q = k\sqrt{S_f}$

$$\therefore \quad \left(\frac{Q}{Q_n}\right)^2 = \frac{S_f}{S_0} \qquad \text{...(ii)}$$

Also,

$$F_r^2 = \frac{v^2}{\left(\sqrt{\frac{gA}{T}}\right)^2}$$

$$F_r^2 = \frac{v^2}{g\frac{A}{T}} \qquad \text{...(iii)}$$

$$F_r^2 = \frac{Q^2 T}{gA^3}$$

For crtical flow condition $F_r = 1$ & $Q = Q_c$

$$\therefore \quad \frac{Q_c^2 T}{gA^3} = 1$$

$$\frac{gA^3}{T} = Q_c^2 \qquad \text{...(iv)}$$

Put the value of (iv) in (iii) we get,

$$\therefore \quad F_r^2 = \left(\frac{Q}{Q_c}\right)^2$$

Put value of F_r and $\frac{S_f}{S_0}$ in equation (1)

$$\frac{dy}{dx} = S_0\left[\frac{1-\left(\frac{Q}{Q_n}\right)^2}{1-\left(\frac{Q}{Q_c}\right)^2}\right]$$

Example 7

A trapezoidal channel has three reaches A, B and C connected in series with the following properties.

Reach	Bed width (B)	Side slope (m)	Bed slope (S_0)	Mannings Coefficeint
A	4.0m	1.0	0.0004	0.015
B	4.0 m	1.0	0.009	0.012
C	4.0 m	1.0	0.004	0.015

For a discharge of 22.5 m³/sec through this channel, sketch the resulting water -surface profiles.

Sol. From Manning equation we know that

$$Q = \frac{1}{n} A \times R^{2/3} S^{1/2}$$

$$\text{Area, } A = (B + my)\,y$$

$$\text{Perimeter, } P = B + 2y\sqrt{1+m^2}$$

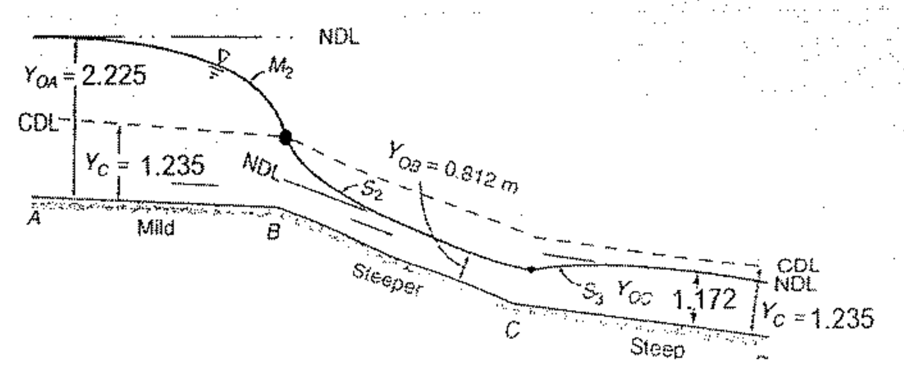
$$\text{Hydraulic Radius, } R = \frac{A}{P} = \frac{(B++my)y}{B+2y\sqrt{1+m^2}}$$

$$Q = \frac{1}{n}(B+my)y\left(\frac{(B+my)y}{B+2y\sqrt{1+m^2}}\right)^{2/3} S^{1/2}$$

$$y_c = \frac{Q^2}{gA^2} = \frac{Q^2}{g(B+my_c)^2 y_c^2} \qquad \left[\because \frac{A}{T} = y_c\right]$$

By substituting values of Q, B, m, S, and n we can calculate the normal depth of flow y

Reach	B(m)	m	S	n	Q(m³/sec)	y_n (m)	y_c (m)	Classification of reach
A	4.0	1.0	0.004	0.015	22.5	2.225	1.235	Mild slope
B	4.0	1.0	0.009	0.012	22.5	0.812	1.235	Steep Slope
C	4.0	1.0	0.004	0.015	22.5	1.172	1.235	Steep slope



Example 8

A wide rectangular channel carries a discharge intensity of 4.0m³/sec per meter width. The longitudinal slope of the channel is 0.0005. Determine the Gradually varied flow (GVF) profile produced by a sudden drop in the bed of channel. Assume Manning's n = 0.025.

Sol. Given,

$q = 4.0\ m^3/s/m;\ S = 0.0005;\ n = 0.025$

From Manning equation

$$Q = \frac{1}{n} A \times R^{2/3} S^{1/2}$$

For wide rectangular channel Hydraulic Radius, $R = y_n$

$$Q = \frac{1}{n}(B \times y_n) \times (y_n)^{2/3} S^{1/2}$$

$$\frac{Q}{B} = \frac{1}{n} y_n^{5/3} S^{1/2}$$

$$4.0 = \frac{1}{0.025} y_n^{5/3} (0.0005)^{1/2}$$

$$y_n^{5/3} = 4.472$$

$$y_n = 2.456\text{m}$$

A sudden drop in the channel bed will cause depth of flow to be critical, hence

$$\text{Critical depth of flow, } y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$y_c = \left(\frac{4^2}{9.81}\right)^{1/3}$$

$$y_c = 1.177\ \text{m}$$

As $y_n > y_c$, slope is Mild slope and flow profile will be M_2.

GRADUALY VARIED FLOW COMPUTATION

- Almost all major hydraulic engineering activities in free-surface flow involve the computation of GVF profiles.
- The various available procedure for computing GVF profiles can be classified as
  1. Direct integration — Chow's Method
  2. Numerical Method — Direct step Method
  3. Graphical Method.

DIRECT STEP METHOD

- This method is the simplest and suitable for prismatic channels.

  From differential energy equation of GVF

$$\frac{dE}{dx} = S_0 - S_f$$

Writing above equation in finite difference form

$$\frac{\Delta E}{\Delta x} = S_0 - \bar{S}_f$$

Where,

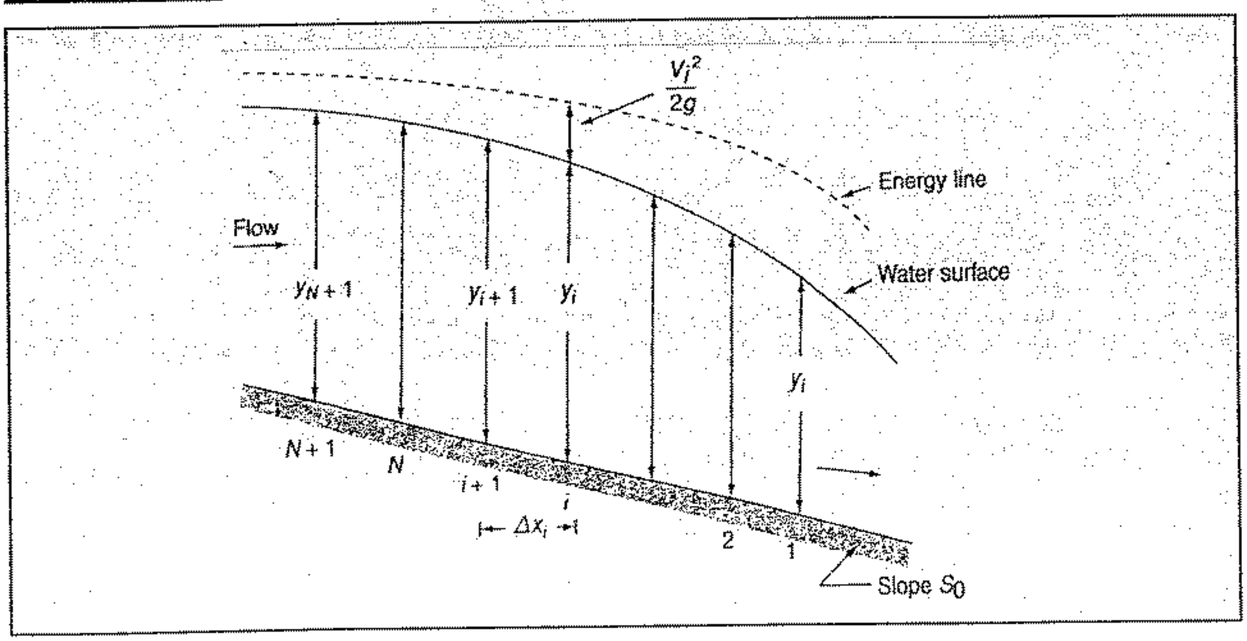
$\bar{S}_f$ = average friction slope in reach Δx

$$\therefore \quad \Delta x = \frac{\Delta E}{S_0 - \bar{S}_f}$$

Between section (1) and (2)

$$x_2 - x_1 = \Delta x = \frac{E_2 - E_1}{S_0 - \frac{1}{2}(S_{f_1} + S_{f_2})}$$

PROCEDURE



- Let it be required to find the water surface profile between two section 1 and (N + 1), where the depths are y_1 and y_{N+1} respectively.
- Channel reach is divided into N parts of known depth, i.e values of y_i, i = 1, N are N are known.
- Our requirement is to find distance Δx_i between y_i and y_{i+1}.

- Now between two sections i and i + 1

$$\Delta E = \Delta\left(y+\frac{v^2}{2g}\right) = \Delta\left(y+\frac{Q^2}{2gA^2}\right)$$

$$\Delta E = E_{i+1} - E_i$$

$$\Delta E = \left[y_{i+1} - \frac{Q^2}{2gA_{i+1}^2}\right] - \left[y_i + \frac{Q^2}{2gA_i^2}\right]$$

$$\bar{S}_f = \frac{1}{2}\left(\bar{S}_{f_{i+1}} + S_{f_i}\right) \qquad \left[S_f = \frac{n^2Q^2}{A^2E^{4/3}}\right]$$

$$\bar{S}_f = \frac{n^2Q^2}{2}\left[\frac{1}{A_{i+1}^2R_{i+1}^{4/3}} + \frac{1}{A_i^2R_i^{4/3}}\right]$$

Substituting values of $E_{i+1} - E_i$ and $\bar{S}_f$ in the equation $\Delta x_i = \frac{E_{i+1} - E_i}{S_0 - S_f}$, we can calculate the value of Δx_i.

- The sequential evalution of Δx_i starting from $i = 1$ to N_1 will given the distance between the N sections and hence the GVF profile.

Calculations of Direct step methods are done in tobulur form as shown below.

y (m)	A (m^2)	R (m)	v (m/s)	E (m)	ΔE	S_f	$\bar{S}_f$	$S_0 - \bar{S}_f$	Δx
y_1	A_1	R_1	v_1	E_1		S_{f1}			
					$E_2 - E_1$		$\bar{S}_{f_1} = \frac{S_{f_1} + S_{f_2}}{2}$	$S_0 - \bar{S}_{f_1}$	Δx_1
y_2	A_2	R_2	v_2	E_2		S_{f2}			
					$E_3 - E_2$		$\bar{S}_{f_2} = \frac{S_{f_2} + S_{f_3}}{2}$	$S_0 - \bar{S}_{f_2}$	Δx_2
y_3	A_3	R_3	v_3	E_3		S_{f3}			
					$E_4 - E_3$		$\bar{S}_{f_3} = \frac{S_{f_3} + S_{f_4}}{2}$	$S_0 - \bar{S}_{f_3}$	Δx_3
y_4	A_4	R_4	v_4	E_4		S_{f4}			
					$E_5 - E_4$		$\bar{S}_{f_4} = \frac{S_{f_4} + S_{f_5}}{2}$	$S_0 - \bar{S}_{f_4}$	Δx_4
y_5	A_5	R_5	v_5	E_5		S_{f5}			

The required length of GVF will be $\Delta x = \Delta x_1 + \Delta x_2 + \Delta x_3 + \Delta x_4$

Example 9

A 10.0 wide rectangular channel with bottom slope of 0.00016 carries a discharge of 22.92 m^3/sec at a normal depth of 0.8m. The depth immediately upstream of dam is 10.0m. Compute the length of the surface profile between 10.0m and 6.0m using Chow's or step method. Take step of 2.0m and assume M = 3.0 and N = 3.33 and Manning's n = 0.015. The length of surface profile by Chow's method between two depths is given by

$$x_2 - x_1 = \frac{y_n}{S_0}\left[(u_2 - u_1) - \{F(u_2, N) - F(u_1, N)\} + \left(\frac{y_c}{y_n}\right)^M \times \frac{J}{N}\{F(v_2, J) - F(v_1, J)\}\right]$$

u	F(u, N)	v	F(v, J)
5	0.01	9	0.027
4	0.017	8	0.031
3	0.034	7	0.038
		6	0.048
		5	0.062
		4	0.087

Sol. Direct step Method

Given,

y_n = 2.0m; B = 10.0m; Q = 22.92m^3/sec; y_c = 0.8m; n = 0.015; S_0 = 0.00016

$$\text{Velocity, } v = \frac{Q}{B \times y}$$

$$\text{Discharge per unit width, } q = \frac{Q}{B} = \frac{22.92}{10} = 2.292 \text{m}^3/\text{s/m}$$

$$\text{Energy Line Slope, } S_f = \frac{v^2 n^2}{R^{4/3}}$$

$$\bar{S}_f = \frac{S_{f_1} + S_{f_2}}{2}$$

y (m)	v (m/sec)	E (m)	ΔE (m)	S_f	$\bar{S}_f$	Δx (m)	L (m)
10	0.2292	10.0027		2.3738×10^{-6}			
			1.9985		3.2502×10^{-6}	12749.62	12749.62
8	0.2865	8.0042		4.1267×10^{-6}			
			1.9968		6.3716×10^{-6}	12997.60	25747.22
6	0.382	6.0074		8.6166×10^{-6}			

Example 10

In a gradually varied flow in a rectangular channel of bottom width 3.0m the discharge is $8m^3/s$ and the depth of flow changes from 1.4 m at section M to 1.05 m at section N. Calculate the average energy slope between these two section. Assume n = 0.018

Sol. In gradually varied flow the Manning's formula is written for any section as

$$V = \frac{1}{n}R^{2/3}S_f^{1/2}$$

where, S_f = energy slope at that section.

Hence,
$$S_f = \frac{n^2V^2}{R^{4/3}}$$

Average energy slope between two section

$$\bar{S}_f = \frac{S_{f1}+S_{f2}}{2}$$

The calculations are shown in Table below

Property	Section M	Section N
A	$3.0\times 1.4 = 4.2\ m^2$	$3.0 \times 1.05 = 3.15\ m^2$
P	$3.0 + (2 \times 1.4) = 5.8$ m	$3.0 + (2 \times 1.05) = 5.10$ m
R	$4.2/5.8 = 0.274$ m	$3.15/5.10 = 0.6176$ m
V	$8.0/4.2 = 1.9048$ m/s	$8.0/3.15 = 2.5397$ m/s
S_f	$\frac{(0.018)^2\times(1.9048)^2}{(0.724)^{4/3}} = 1.8077 \times 10^{-3}$	$\frac{(0.018)^2\times(2.5397)^2}{(0.6176)^{4/3}} = 3.973 \times 10^{-3}$

$$\bar{S}_f = \frac{1.8077\times10^{-3}+3.9730\times10^{-3}}{2} = 2.890 \times 10^{-3}$$

Example 11

A sluice gate discharge a stream of depth 0.15 m at the vena contracta. The channel can be taken as a wide rectangular horizontal channel and discharge intensity is $1.40\ m^3/sm$. If a hydraulic jump is formed at a depth of 0.25 m estimate the distance from the toe of the jump to the vena contracta. Take two steps and use Direct step method. (Manning's n = 0.015)

Sol. Consider two steps with depths of 0.15, 0.22 m and 0.25 m forming the ends of the reaches.

The distance of the water surface in the (sttep) Δx is obtained as $\Delta x = \frac{\Delta E}{S_0 - S_f}$.

$q = 1.40\ m^3/s/m$, n = 0.015.

Assume channel to be wide rectangular channel with bed slope be horizontal i.e. $S_0 = 0$

$$\text{Velocity, } v = \frac{q}{y}$$

For wide Rectangular Channel

Hydraulic Radius, R = y

$$\text{Specific energy, } E = y + \frac{v^2}{2g}$$

$$\text{Energy Slope, } S_f = \frac{v^2 n^2}{R^{4/3}}$$

The calculation are done in the table below.

y	V	E	ΔE	S_f	$\bar{S}_f$	$S_0 - \bar{S}_f$	Δx	x
0.15	9.33	4.590		0.2459				0
			2.306					
0.22	6.36	2.284		0.0686	0.1573	–0.1573	–14.7	–14.7
			0.436					
0.25	5.60	1.848		0.0448	0.0567	–0.0563	–7.7	–22.4

The distance between the two depths 0.15 m and 0.25 m is found as 22.4m.

OBJECTIVE QUESTIONS

1. Identify the incorrect statement
   The possible GVF profiles in
   (a) mild slope channels are M_1,M_2 and M_3
   (b) adverse slope channel s are A_2 and A_3
   (c) horizontal channels are H_1 and H_3
   (d) critical slope channels are C_1 and C_3

2. The following types of GVF profiles do not exist
   (a) C_2, H_2, A_1
   (b) A_2, H_1, C_2
   (c) H_1, A_1, C_2
   (d) C_1, A_1, H_1

3. The total number of possible types of GVF profiles are
   (a) 9
   (b) 11
   (c) 12
   (d) 15

4. dy/dx is negative in the following GVF profiles
   (a) M_1, S_2, A_2
   (b) M_2, A_2, S_3
   (c) A_3, A_2, M_2
   (d) M_2, A_2, H_2, S_2

5. If in a GVF dy/dx is positive, then dE/dx is
   (a) always positive
   (b) negative for an adverse slope
   (c) negative if $y > y_c$
   (d) positive if $y > y_c$

6. In a channel the gradient of the specific energy dE/dx is equal to
   (a) $S_0 - S_f$
   (b) $S_f - S_0$
   (c) $S_0 - S_f - \frac{dy}{dx}$
   (d) $S_0 (1- F^2)$

7. The M_3 profile is indicated by the following inequality between the various depths:
   (a) $y_0 > y_c > y$
   (b) $y > y_0 > y_c$
   (c) $y_c > y_0 > y$
   (d) $y > y_c > y_0$

8. A long prismatic channel ends in an abrupt drop. If the flow in the channel far upstream of the drop is subcritical, the resulting GVF profile
   (a) starts from the critical depth at the drop and joins the normal depth asymptotically
   (b) lies wholly below the critical depth line
   (c) lies wholly above the normal depth line
   (d) lies partly below and partly above the critical depth line

9. When there is a break in grade due to a mild slope A changing into a mider slope B, the GVF profile produced is
   (a) M_3 curve on B
   (b) M_2 curve on B
   (c) M_1 curve on B
   (d) M_1 curve on A

10. In a channel the bed slope changes from a mild slope to a steep slope. The resulting GVF profiles are
   (a) (M_1, S_2)
   (b) (M_1, S_3)
   (c) (M[illegible], S[illegible])
   (d) (M[illegible], S[illegible])

11. The flow will be in the supercritical state in the following types of GVF profiles

(a) All S curves
(b) M_2
(c) A_3, M_3, S_2
(d) S_2, M_2, S_3

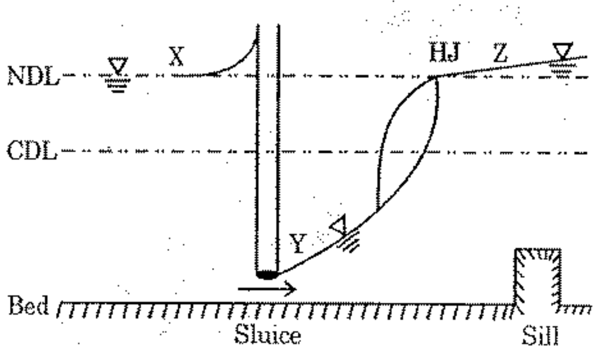
12. Match **List-I** (Hydraulic structure) with **List-II** (Water surface profile at the structure) and select the correct answer using the codes given below the lists:

List-I	List-II
A. On a flat topped broad-crested weir	1. M_1
B. Immediately below a sluice gate on a level apron	2. M_2
C. Behind the weir on an alluvial river	3. S_1
D. In a chute spillway	4. S_2
	5. H_2
	6. H_3

Codes:

	A	B	C	D
(a)	5	6	1	4
(b)	4	6	2	1
(c)	2	5	6	4
(d)	5	4	1	3

13. A sluice gate opening in a canal is shown in the given figure. The shapes of water surface profiles at X, Y and Z will be respectively (NDL = Normal Depth Line, CDL = Critical Depth Line, HJ = Hydraulic Jump)



(a) M_1, M_3 and M_1
(b) M_2, M_3 and M_2
(c) S_1, S_3 and S_2
(d) H_2, S_3 and S_1

14. Water surface profiles that are asymptotic at one end and terminated at the other end would include

(a) H_2 and S_2
(b) H_3 and S_2
(c) M_2 and H_2
(d) M_2 and H_3

15. A hydraulic jump is always needed in case of

(a) an A_2 profile followed by an A_3 profile
(b) an A_3 profile followed by an A_2 profile
(c) an H_2 profile followed by an M_2 profile
(d) an M profile followed by an M profile

16. Match **List-I** (Slope) with **List-II** (Description) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Mild	1. is when the flow is at Froude number one along the channel
B. Adverse	2. is an example of non-sustaining slope
C. Limit	3. always sustains uniform subcritical flow
D. Critical	4. is one which has smallest critical slope for a given channel shape and roughness

Codes:

	A	B	C	D
(a)	3	2	4	1
(b)	1	4	2	3
(c)	3	4	2	1
(d)	1	2	4	3

17. Consider the following statements which relate to different types of water surface profiles. Curve types conform to usual classifications y_c and y_0 (critical and normal depths):

1. Type-3 curves lie between y_c and y_0
2. All curves where $y_c < y_0$ are unaffected by upstream disturbances.
3. All curves where $y_c > y_0$ are influenced by downstream disturbance.
4. All curves approaching the 'y' line approach it asymptotically except for C curve where $y_0 = y_c$.

Which of these statements are correct?

(a) 1, 2, 3 and 4
(b) 1, 3 and 4
(c) 2, 3 and 4
(d) 1 and 2

18. Match **List-I** (Type of curve) with **List-II** (Flow condition) and select the correct answer using the codes given below the lists :

List-I	List-II
A. M_1	1. Slope upward in the direction of flow
B. H_3	2. Back water profile
C. S_2	3. Hydraulic jump occurs
D. A_2	4. Hydraulic drop occurs

Codes :

	A	B	C	D
(a)	1	3	4	2
(b)	2	4	3	1
(c)	1	4	3	2
(d)	2	3	4	1

19. In the step methods (both direct and standard), the computations must

(a) proceed downstream in subcritical flow
(b) proceed upstream in subcritical flow
(c) always proceed upstream
(d) always start at a control section

20. Which of the following equations are used for the derivation of the differential equation for water surface profile in open channel flow?

1. Continuity Equation 2. Energy Equation
3. Momentum Equation

Select the correct answer using the codes given below:

(a) 1, 2 and 3
(b) 1 and 3 only
(c) 1 and 2 only
(d) 2 aned 3 only

21. Which one of the following statement is not correct?

A control section in an open channel is the site

(a) where the flow quantity can be controlled
(b) at which flow is known to be critical
(c) where the discharge can be measured
(d) where the specific energy is determined

22. In connection with a gradually varied flow with notations y_0 = normal depth, y_c = critical depth ande y = depth in the gradually varied flow, match **List-I** with **List-II** and select the correct answer using the codes given below the lists :

List-I	List-II
A. $y_c > y_0 > y$	1. M_1
B. $y_0 > y > y_c$	2. S_3
C. $y > y_c > y_0$	3. M_2
D. $y > y_0 > y_c$	4. S_1

Codes :

	A	B	C	D
(a)	4	1	2	3
(b)	2	3	4	1
(c)	4	3	2	1
(d)	2	1	4	3

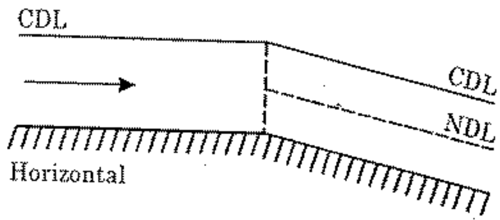
23. The following statements relate to water surface profiles in gradually varied flow of an open channel:

1. M_1 and S_1 curves meet y_0 line asymptotically and tend to the horizontal as $y \to \infty$
2. M_2 and S_2 curves meet y_0 line normally and y_c line asymptotically.
3. M_3 and S_3 curves meet y_c line normally and also channel bed normally.
4. C_1 and C_3 curves will be slightly curved as Chezy's equation is used. Otherwise, they tend to be straight lines.

Which of these statements are correct?

(a) 1 and 3
(b) 1 and 4
(c) 2 and 4
(d) 3 and 4

24. The given figure shows gradually varied flow in an open channel with a break in bed slope.



Type of water surface profiles occurring from left to right are

(a) H_3, S_3
(b) H_2, S_2
(c) H_2, M_2
(d) H_3, M_2

25. Which one of the following pairs of situations and types of water surface profiles is not correctly matched?

(a) Mild slope; flow over free overfall: M_2
(b) Mild slope; flow downstream of a sluice: M_1
(c) Critical slope; flow downstream of a sluice gate; C_3
(d) Critical slope; flow behind an overflow weir: C_1

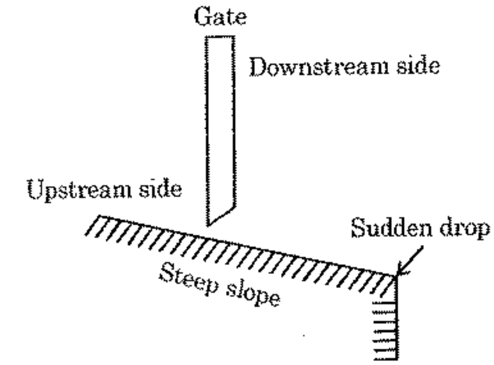
26. Match **List-I** (Surface profile) with **List-II** (Description of the profile) and select the correct answer using the codes given below the lists :

List-I	List-II
A. M_2	1. Convex upward; asymptotic horizontal at d/s end; depth increasing with d/s
B. S_3	2. Convex downward; upstream asymptotic to normal depth with depth decreasing in d/s direction
C. C_1	3. Depth increasing downstream and meeting at a angle to CDL; a curve with an inflexion point
D. A_3	4. Convex upwards and depth increasing in flow direction; asymptotic to NDL at d/s end

Codes :

	A	B	C	D
(a)	2	4	1	3
(b)	2	1	4	3
(c)	3	4	1	2
(d)	2	3	4	2

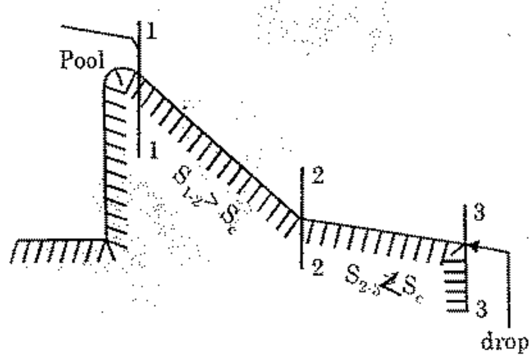
27. A sluice-gate is operated in long open channel flow with a steep slope, as shown in the figure, in such a manner that its opening is smaller than normal and critical depths.



The hydraulic jump will occur on

(a) downstream side
(b) upstream side
(c) both upstream and downstream sides
(d) neither on upstream side nor on the downstream side

28. A prismatic channel is laid with a break in bottom slope, as shown at section 2-2, so that water flows from a crest (section 1-1), in the channel to discharge over a sudden drop (section 3-3). The surface profiles along the flow along 1 to 3 will be



(a) S_2, M_3, HJ, M_2
(d) S_2, HJ, M_1, M_2
(c) M_2, M_1, HJ, S_2
(d) M_2, HJ, M_1 S_2

29. Match **List-I** with **List-II** and select the correct answer using the codes given below the lists :

	List-I		List-II
A.	Flow downstream of a sluice gate with mild bed slope of channel	1.	C_3 curve
B.	Flow upstream of an overflow weir with critical bed slope of channel	2.	M_2 curve
C.	Flow downstream of a sluice gate with critical bed slope of channel	3.	C_1 curve
D.	Flow upstream of a free overfall with mild bed slope of channel	4.	M_3 curve

Codes :

	A	B	C	D
(a)	4	3	2	1
(b)	2	1	4	3
(c)	4	3	1	2
(d)	2	1	3	4

30. Consider y_n = normal depth, y_c = critical depth and y = depth of gradually varied flow. The slope of the surface $\frac{dy}{dx}$ is positive if

1. $y > y_n$ and $y < y_c$
2. $y > y_c$ and $y < y_n$
3. $y > y_n$ and $y > y_c$
4. $y < y_n$ and $y < y_c$

Which of these conditions are correct?

(a) 1 and 2
(b) 1 and 3
(c) 2 and 3
(d) 3 and 4

Common Data for Q 31 & 32

A very wide rectangular channel carries a discharge of $8 m^3/sec/m$, channel has a bed slope of 0.004 and n = 0.015 at a certain section of the channel flow depth is 1m

31. What Gradually Varied Flow profile exists at this section?

(a) M_2
(b) M_3
(c) S_2
(d) S_3

32. At what distance from this section the flow depth will be 0.9 m? (Use the direct step method employing a single step.)

(a) 65 m downstream
(b) 50 m downstream
(c) 50 m upstream
(d) 65 m upstream

33. A channel with a mild slope is followed by a horizontal channel and then by a steep channel. What gradually varied flow profiles will occur?

(a) M_1, H_1, S_1
(b) M_2, H_2, S_2
(c) M_1, H_2, S_3
(d) M_1, H_2, S_2

34. There is a freee overfall at the end of a long open channel. For a given flow rate, the critical depth is less than the normal depth. What gradually varied flow profile will occur in the channel for this flow rate?

(a) M_1
(b) M_2
(c) M_3
(d) S_1

35. Direct step method of computation for gradually varied flow is

(a) applicable to non-prismatic channels
(b) applicable to prismatic channels
(c) applicable to both prismatic and non-prismatic channels
(d) non-applicable to both prismatic and non-prismatic channels

Consider the following statement

Of these statements:

(a) both A and R are true and R is the correct explanation of A
(b) both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

36. **Assertion (A):** A hydraulic jump cannot be expected on a long steep slope (fed by a large reservoir) when it is followed by a short reach of adverse slope termination in vertical drop into a deep and wide reservoir.

Reason (R): The terminal depth is such an adverse slope reach will be critical and the flow in the steep slope may be nearly at or at normal depth.

ANSWERS

1. (c)	**2.** (c)	**3.** (c)	**4.** (d)	**5.** (d)	**6.** (a)	**7.** (a)	**8.** (a)	**9.** (d)	**10.** (c)
11. (c)	**12.** (a)	**13.** (a)	**14.** (c)	**15.** ~~(a)~~ (b)	**16.** (a)	**17.** (c)	**18.** (d)	**19.** ~~(b)~~ (d)	**20.** (c)
21. (c)	**22.** (b)	**23.** (b)	**24.** (b)	**25.** (b)	**26.** (a)	**27.** (b)	**28.** (a)	**29.** (c)	**30.** (d)
31. (d)	**32.** (c)	**33.** (d)	**34.** (b)	**35.** (b)	**36.** (a)				

HINT

1. GVF profiles in horizontal channels in H_2, H_3

31.

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{8^2}{9.81}\right)^{1/3} = 1.868 \text{ m.}$$

$$q = \frac{1}{n} y_n^{5/3} S^{1/2}$$

$$\Rightarrow \quad y_n = 1.468 \text{ m.}$$

$$y = 1.0 \text{ m. (given)}$$

As, $y_n < y_c$ steep slope

As, $y < y_n < y_c$ flow is in region 3 $\Rightarrow S_3$

32.

$$\frac{dE}{dx} = S_0 - S_f$$

$$\frac{dE}{dx} = S_0 - \bar{S}_f$$

$$\Delta x = \frac{\Delta E}{S_0 - \bar{S}_f}$$

$$\text{Depth of mid point} = \frac{1 + 0.9}{2} = 0.95 \text{ m}$$

$$\text{at this point, } q = \frac{1}{n} y^{5/3} \left(\bar{S}_f\right)^{1/2}$$

$$\bar{S}_f = 0.0171$$

$$E_2 = y_2 + \frac{q^2}{2gy_2^2} = 1 + \frac{8^2}{2 \times 9.81 \times 1^2} = 4.26 \text{ m.}$$

$$E_1 = y_1 + \frac{q^2}{2gy_1^2} = 0.9 + \frac{8^2}{2 \times 9.81 \times 0.9^2} = 4.927 \text{ m.}$$

$$\Delta x = \frac{4.26 - 4.927}{0.004 - 0.017} = -51.31 \text{ m} \quad \{(-\text{ve}) \text{ means upstream}\}$$

36. As the adverse slope is of short reach profile will be $S_2 - A_3$ and fall in the reservior at y_c. But if it would have been of long reach then profile will be $S_2 - A_3 - H_j - A_2 -$ fall in reservior at y_c.

CHAPTER

5

Rapidly Varied Flow : Hydraulic Jump

INTRODUCTION

- In Rapidly Varied Flow (R.V.F.), a sudden change of depth occurs at a particular point of a channel and the change from one depth to another takes place in a distance of very short length.
- Streamlines in Rapidly Varied Flow (R.V.F.) have very large curvature. Most of times, an about change in curvature breaks the flow profile and this results in high state of turbulence causing considerable loss of energy.
- Hydraulic jump is best example of RVF.
- Pressure distribution is non-hydrostatic due to huge curvature of flow.

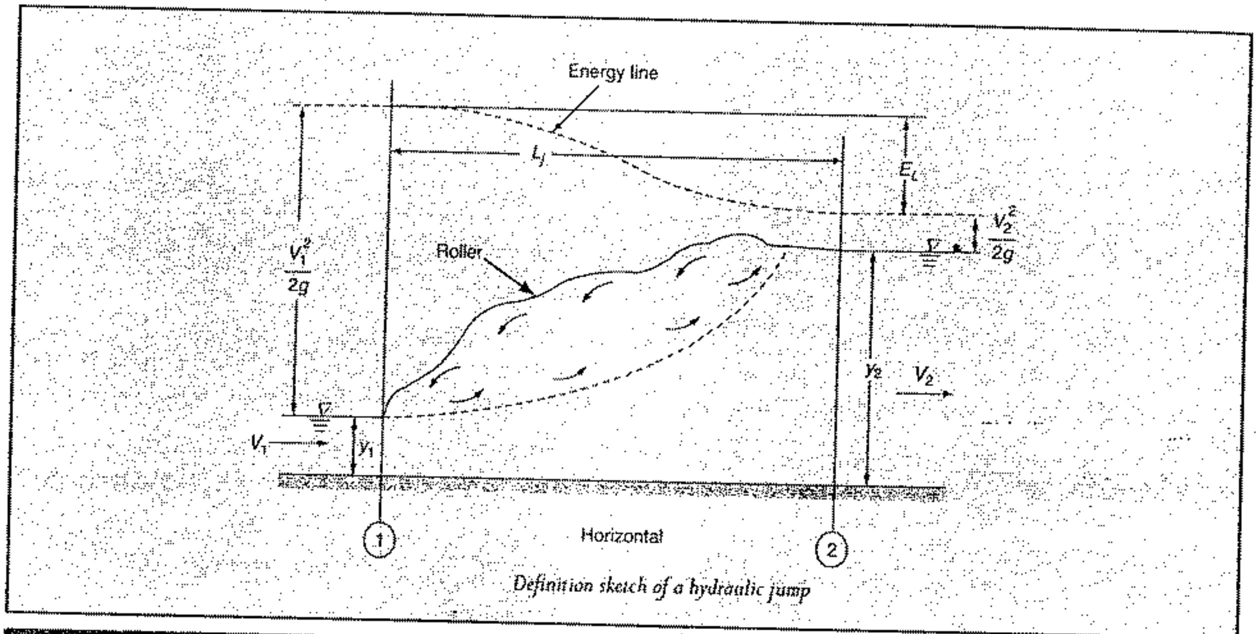
HYDRAULIC JUMP

- When the flow condition changes from supercritical ($F_r > 1$) to subcritical ($F_r < 1$), the result is an abrupt rise of water accompanied by turbulent rollers is called as hydraulic jump or standing wave.
- The change in flow regime takes place over a relatively short reach of a channel, so it is a local phenomenon, therefore, boundary friction is relatively small and in many cases insignificant.
- A hydraulic jump will form when water moving at a supercritical velocity in a relatively shallow stream strikes water having large depth and subcritical velocity. For example:
  (a) Down stream of sluice gate.
  (b) At the bottom of spillway.
  (c) When steep channel changes to flat channel.
  (d) Downstream of narrow channel.
  (e) Irrigation canal falls.

APPLICATIONS OF HYDRAULIC JUMP

1. As an energy dissipator, to dissipate the excess energy of flowing water downstream of hydraulic structures and the present scouring downstream.
2. To mix chemicals used for water purification or wastewater treatment.
3. To aerate polluted stream.
4. For desalination of sea water.
5. To raise the water level on the downstream side of a metering flume and thus maintaing high water

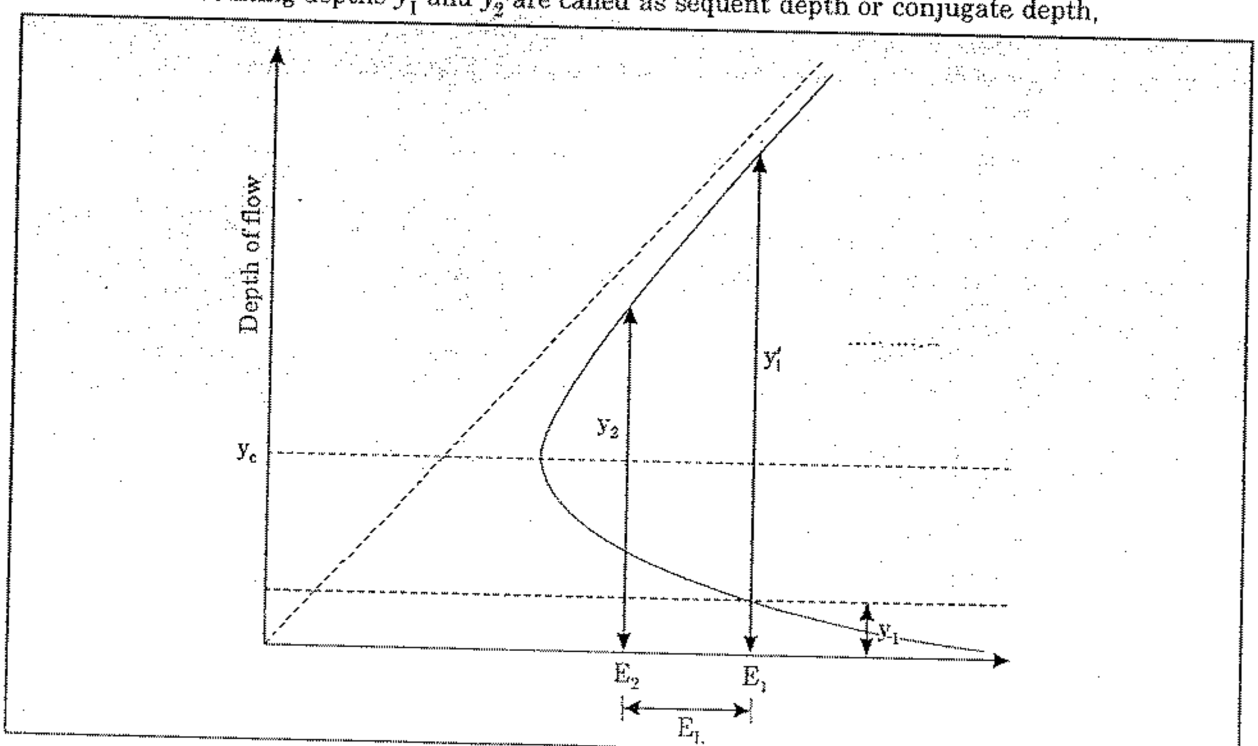
6. It increases weight on an arpon and thus reduces uplift pressure under a masonary structure by raising the water depth on the apron.



Definition sketch of a hydraulic jump

SEQUENT DEPTH OR CONJUGATE DEPTH

- During hydraulic jump the supercritical stream jumps to meet its alternate depth. But while doing so it generates considerable disturbances in the form of large scale eddies and a reverse flow roller, because of this, jump falls short of attaining its alternate depth.
- Thus the resulting depths y_1 and y_2 are called as sequent depth or conjugate depth,



Note: Alternate depth : Depth with same specific energy.
Sequent depth : Two depth of hydraulic jump with same specific Force.

Toe of Jump

- Section 1, where the incoming supercritical stream undergoes an abrupt rise in the depth forming the comencement of jump is called as Toe of jump.

End of Jump

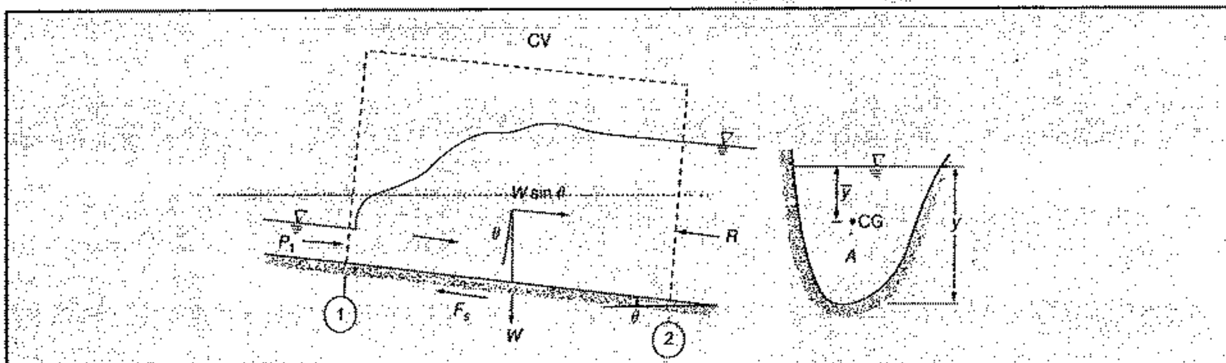
- It is the point at which the roller formation terminates and water surface is levelled. Section 2 in the figure represents the end of jump.

Length of Jump

- Distance between Toe of jump (section 1) and end of jump (section 2) is called as length of jump.

MOMENTUM EQUATION FOR THE JUMP

- Due to the appreciable energy loss associated with the hydraulic jump, and the nature of which is difficult to estimate, the application of energy equation to the channel sections before and after jump does not provide an adequate means of analysis.
- In such situation the use of the momentum equation with suitable assumptions is done.



Definition sketch for the general momentum equation

Assumptions

(i) The portion of the hydraulic jump is considered as the control volume and it is assumed that before and after the jump formation the flow is uniform and the pressure distribution is hydrostatic.

(ii) The length of the jump is small, such that the losses due to friction on the channel floor are small.

(iii) The channel bed is horizontal or the bed slope is so gentle that the weight component of the water mass comprising the jump is negligibly small.

Applying the linear momentum equation in the longitudinal direction to the control volume.

$$P_1 - P_2 + W \sin\theta - F_s = M_2 - M_1$$

where,

$P_1 = \gamma A_1 \bar{y}_1 \cos\theta$ pressure force at section 1,

$\bar{y}_1$ = depth of centeroid of the area below the water surface.

$P_2 = \gamma A_2 \bar{y}_2 \cos\theta$, pressure force at section 2,

F_s = Shear force on control surface adjacent to channel boundary

$W \sin\theta$ = Longitudinal component of weight of water contained in the control volume.

$M_2 = \rho Q v_2$, momentum in the longitudinal direction going out of the control volume.

$M_1 = \rho Q v_1$, momentum in the longitudinal direction going in the control volume.

- As per assumption (ii) ($W \sin\theta = 0$) ($\cos\theta = 1$) and (iii) ($F_S = 0$), the momentum equation becomes.

$$P_1 - P_2 = M_2 - M_1$$

$$P_1 + M_1 = P_2 + M_2$$

$$\frac{P_1 + M_1}{\gamma} = \frac{P_2 + M_2}{\gamma}$$

- Here by we observe that specific force is constant under friction less and horizontal surface. Further, pressure $= \gamma A\bar{y}$ (for $\theta \to 0°$, $\cos\theta \to 1$) and Momentum $= \rho Q v$

$$\therefore \quad \frac{\gamma A\bar{y} + \rho Q v}{\gamma} = A\bar{y} + \frac{Q^2}{Ag}$$

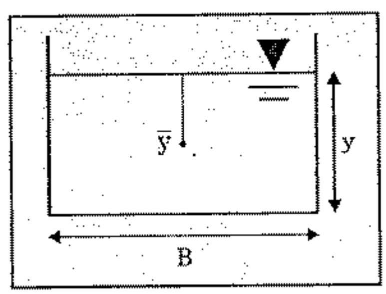
$$\boxed{A\bar{y} + \frac{Q^2}{Ag} = \text{constant}}$$

- Above formula can be used for computations of hydraulic jump in all type of horizontal bed channels.

HYDRAULIC JUMP IN HORIZONTAL RECTANGULAR CHANNEL

- Consider a horizontal, frictionless and rectangular channel of width = B, and depth of flow = y

Sequent Depth Ratio



$[q = \frac{Q}{B}]$

$$\therefore \quad A\bar{y} + \frac{Q^2}{gA} = \text{constant}$$

$$By\,\frac{y}{2} + \frac{Q^2}{gB.y} = \text{constant}$$

$$\frac{y^2}{2} + \frac{Q^2}{gB^2 y} = \text{constant}$$

$$\frac{y^2}{2} + \frac{q^2}{gy} = \text{constant}$$

Relation between Conjugate Depth

- If y_1 and y_2 are depth of flow of ends of hydraulic jump then we can write that

$$F_1 = F_2$$

$$\frac{y_1^2}{2} + \frac{q^2}{gy_1} = \frac{y_2^2}{2} + \frac{q^2}{gy_2}$$

$$\frac{q^2}{g}\left[\frac{1}{y_1} - \frac{1}{y_2}\right] = \frac{y_2^2 - y_1^2}{2}$$

$$\boxed{\frac{2q^2}{g} = y_1 y_2 (y_1 + y_2)} \quad \text{.......(A)}$$

We know that, Froude number for rectangular channel

$$F = \frac{V}{\sqrt{gy}} = \frac{Q}{By\sqrt{gy}} = \frac{q}{y\sqrt{gy}}$$

$$\boxed{F^2 = \frac{q^2}{gy^3}}$$

$$\therefore \quad \boxed{F_1^2 = \frac{q^2}{gy_1^3} \ \& \ F_2^2 = \frac{q^2}{gy_2^3}}$$

Put the value of $\left(\frac{q^2}{g}\right)$ in Equation (A)

$$2F_1^2 y_1^3 = y_1 y_2 (y_1 + y_2) \quad \text{...(i)}$$

$$2F_1^2 = \frac{y_2}{y_1^2}(y_1 + y_2) = \left(\frac{y_2}{y_1}\right) + \frac{y_2^2}{y_1^2}$$

$$\Rightarrow \quad \left(\frac{y_2}{y_1}\right)^2 + \left(\frac{y_2}{y_1}\right) - 2F_1^2 = 0 \quad \text{...(ii)}$$

Since Equation (ii) is quadratic in $\left(\frac{y_2}{y_1}\right)$

Hence, $$\frac{y_2}{y_1} = \frac{-1 \pm \sqrt{1+8F_1^2}}{2}$$

$$\therefore \quad \boxed{\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1+8F_1^2} - 1\right]}$$

$$\boxed{\text{Similarly, } \frac{y_1}{y_2} = \frac{1}{2}\left[\sqrt{1+8F_2^2} - 1\right]}$$

Note: Above formula is for rectangular, horizontal and frictionless channel and if friction is not neglected y_2 actual will be less than y_2 calculated from the above formula.

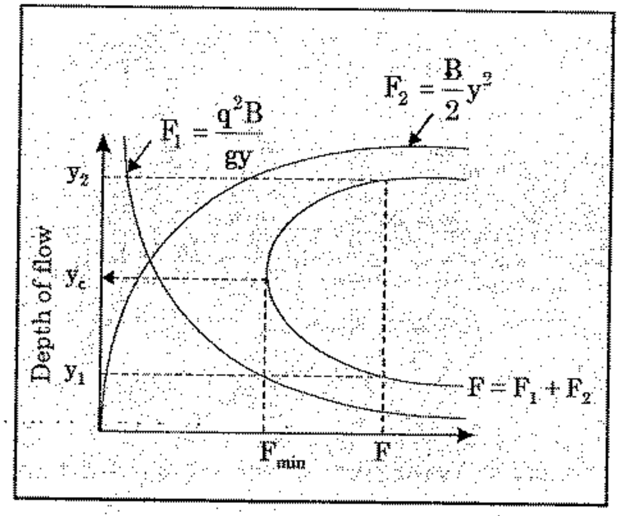
SPECIFIC FORCE CURVE FOR RECTANGULAR CHANNEL

Specific force is defined as, $F = \frac{Q^2}{Ag} + A\bar{y}$

For rectangular channel, $F = \frac{Q^2}{By \times g} + By \times \frac{y}{2}$

$$\therefore \quad = \left(\underbrace{\frac{q^2 \times B}{gy}}_{F_1} + \underbrace{\frac{By^2}{2}}_{F_2}\right)$$

Plotting, $F_1 = \frac{q^2 \times B}{gy}$ and $F_2 = \left(\frac{B}{2}y^2\right)$



Note: $y_1 < y_c$: Supercritical flow

$y_2 > y_c$: Subcritical flow

- From above graph there is one depth of flow (y_c). When specific force is minimum, In this condition, Unit discharge is found to be Maximum. It is called critical flow condition.
- For specific force other than minimum, there are two depths of flow y_1 & y_2 called conjugate depth or sequent depth.
- For critical flow condition following condition will hold true. for all type of channel.

$$\frac{Q^2}{g} = \frac{A^3}{T}$$

Energy loss

The energy loss E_L the jump is given by the energy equation application between section 1 and 2.

$$E_L = E_1 - E_2$$

$$E_L = \left(y_1 + \frac{q^2}{2gy_1^2}\right) - \left(y_2 + \frac{q^2}{2gy_2^2}\right)$$

$$E_L = (y_1 - y_2) + \frac{1}{2}\frac{q^2}{g}\left(\frac{y_2^2 - y_1^2}{y_1^2 y_2^2}\right)$$

We know that,

$$\frac{2q^2}{g} = y_1 y_2 (y_1 + y_2)$$

$$E_L = (y_1 - y_2) + \frac{y_1 y_2 (y_1 + y_2)}{4}\left(\frac{y_2^2 - y_1^2}{y_1^2 y_2^2}\right)$$

$$E_L = (y_1 - y_2) + \frac{(y_1 + y_2)(y_2^2 - y_1^2)}{4y_1 y_2}$$

$$E_L = \frac{4y_1^2y_2 - 4y_1y_2^2 + y_1y_2^2 - y_1^3 + y_2^3 - y_2y_1^2}{4y_1y_2}$$

$$E_L = \frac{y_2^3 - y_1^3 + 3y_1^2y_2 - 3y_1y_2^2}{4y_1y_2}$$

$$E_L = \frac{(y_2 - y_1)^3}{4y_1y_2} \qquad [(a-b)^3 = a^3 - b^3 - 3ab^2 - 3ba^2]$$

Relative Energy Loss

$$\text{Relative Energy Loss} = \frac{E_L}{E_1}$$

$$E_1 = y_1 + \frac{v^2}{2g} = y_1 + \frac{v^2}{B^2y^2 2g} = \left(y_1 + \frac{q^2}{2gy_1^2}\right)$$

$$F_1 = \frac{v}{\sqrt{gy_1}} = \frac{q}{y_1\sqrt{gy_1}}$$

$$F_1^2 = \frac{q^2}{gy_1^3}$$

$$E_1 = y_1 + \frac{gy_1^3F_1^2}{2gy_1^2} = \left(y_1 + \frac{F_1^2y_1}{2}\right)$$

$$\frac{E_1}{y_1} = 1 + \frac{F_1^2}{2}$$

$$\text{Also, Relative Energy loss} = \frac{(E_L/y_1)}{(E_1/y_1)} \qquad \text{....(B)}$$

$$\frac{E_1}{y_1} = 1 + \frac{F_1^2}{2} \qquad \text{....(i)}$$

$$\frac{E_L}{y_1} = \frac{\left(\frac{y_2}{y_1} - 1\right)^3}{4\left(\frac{y_2}{y_1}\right)} \qquad \text{....(ii)}$$

Substituting (i) & (ii) in equation (B)

$$\frac{E_L}{E_1} = \frac{\left(\frac{y_2}{y_1} - 1\right)^3}{4\left(\frac{y_2}{y_1}\right)\left(1 + \frac{F_1^2}{2}\right)}$$

$$\text{As, } \frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1+8F_1^2}-1\right]$$

$$\frac{E_L}{E_1} = \frac{\left(-3+\sqrt{1+8F_1^2}\right)^3}{2(2+F_1^2)(\sqrt{1+8F_1^2}-1)}$$

Efficiency of Jump

$$\eta_{jump} = \frac{E_2}{E_1}$$

$$\eta_{jump} = \frac{E_1 - \Delta E}{E_1}$$

$$\eta_{jump} = 1 - \text{Relative energy loss}$$

Height of Jump

$$H_j = y_2 - y_1$$

$$\textbf{Relative Height} = \frac{h_j}{E_1}$$

$$\textbf{Relative Initial Depth} = \frac{h_1}{E_1}$$

$$\textbf{Relative sequent depth} = \frac{y_2}{E_1}$$

Length of Jump

- Length of jump is the horizontal distance between the toe of the jump to a section where the water surface levels off after reaching the maximum depth.
- The length of jump is required while selecting the apron length and the height of the side walls of the stilling basin.

  Experimentally it is found that in horizontal channel.

$$\frac{L_j}{y_2} = f(F_1) \qquad F_1 < 5.0$$

$$\frac{L_j}{y_2} = 6.1 \qquad F_1 > 5.0$$

Note: For $F_1 < 5$ $L_j = F(F_1)$

For $F_1 > 5$ L_j = constant

- Experimentaly it has been also found out that,

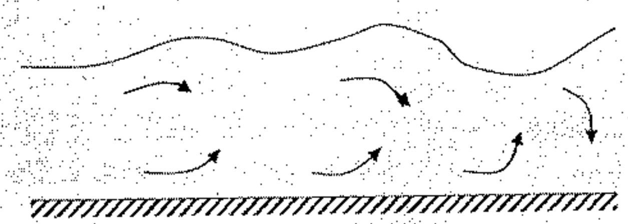
$$L_j = 6.9\,(y_2 - y_1)$$

TYPES OF JUMP

- Depending on the values of Froude number F_1 of incoming flow, there are five distinct type of hydraulic jump.
- It should be noted that the Froude number of incoming flow F_1 should be always greater than 1 to occur.

1. Undular jump $1.0 < F_1 < 1.7,\ \frac{E_L}{E_1} \cong 0$

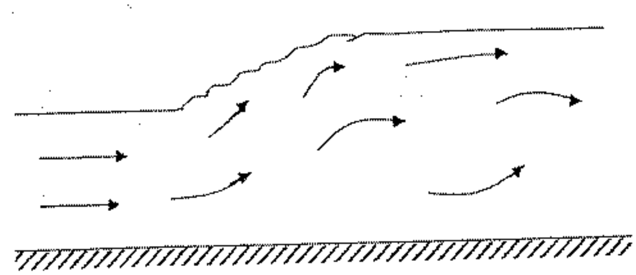
- Water surface is undulating with a very small ripples on the surface.
- Sequent depth ratio $\frac{y_2}{y_1} \cong 0.$



Undular jump

2. Weak Jump $1.7 < F_1 \leq 2.5,\ \frac{E_L}{E_1} = 5\text{-}18\%$

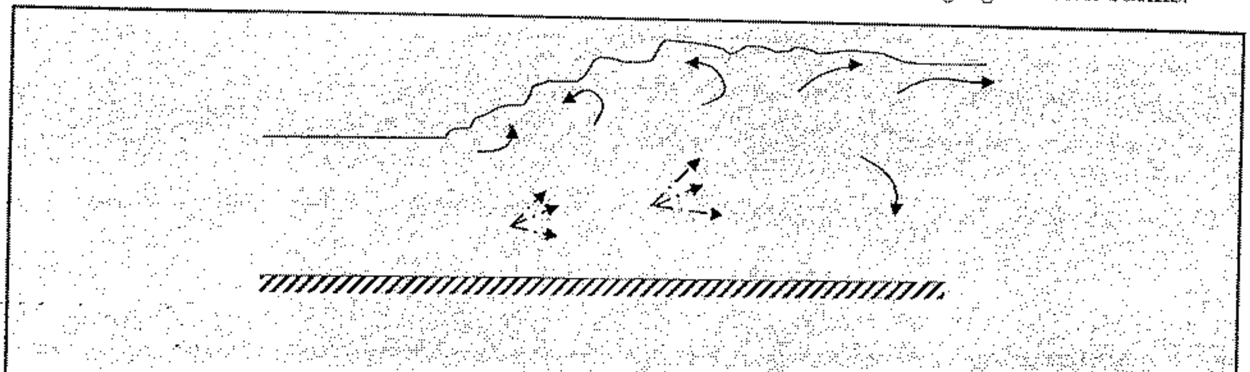
- A series of small rollers forms on the jump surface, but the downstream water surface remains smooth.



Weak jump

3. Oscillating jump $\quad 2.5 < F_1 \le 4.5, \quad \frac{E_L}{E_1} = 18\text{-}45\%$

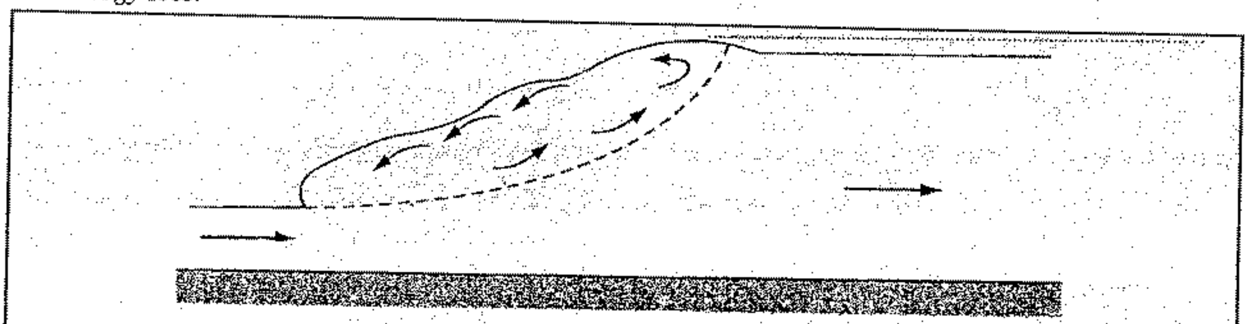
The entering jet of water oscillates in a random manner between bed and surface. These oscillations are very common in canals and can travel considerable distances and damaging earthen banks.



Oscillating jump

4. Steady Jump $\quad 4.5 < F_1 \le 9.0, \quad \frac{E_L}{E_1} = 45 - 70\%$

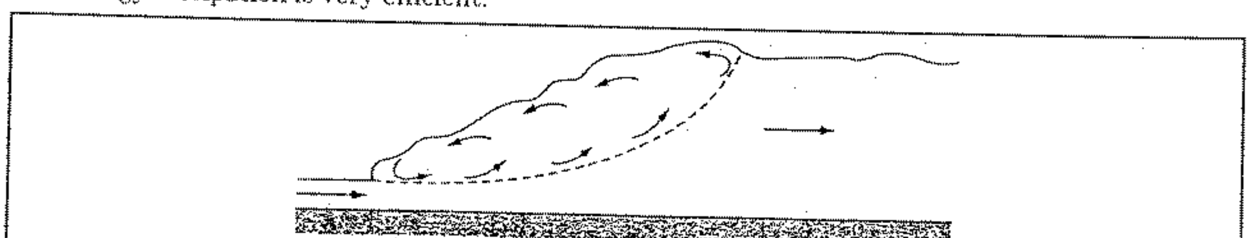
- The jump is well established, the roller and jump action is fully developed to cause appreciable energy loss.



'Steady' jump

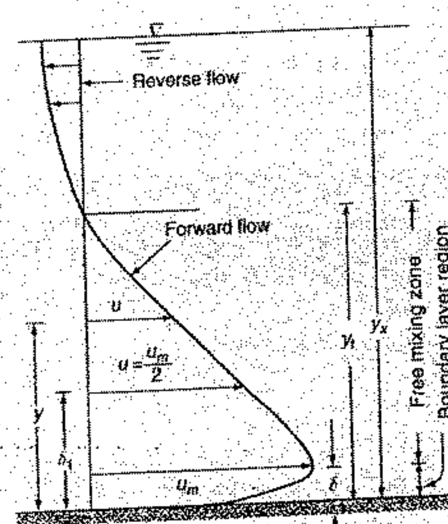
5. Strong or Choppy Jump $\quad F_1 > 9.0, \quad \frac{E_L}{E_1} \ge 70\%.$

- During this jump water surface is very rough and Choppy, which continues downstream for a long distance.
- Sequent depth ratio $\frac{y_2}{y_1}$ is quite large.
- Energy dissipation is very efficient.



Strong or Choppy jump

VELOCITY PROFILE



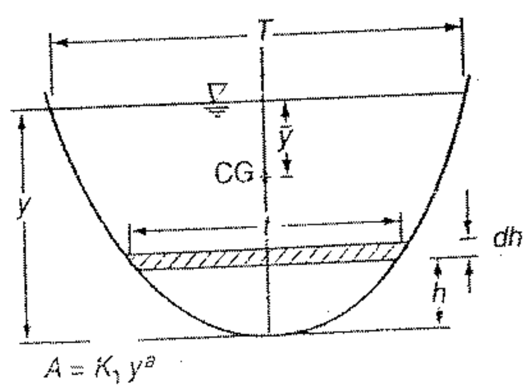
Velocity distribution in a jump

- It is observed that velocity profile has two distinct portions a forward flow in the lower main body and a negative Reverse flow at the top.
- It should be noted that in the forward flow the total volumetric rate of flow will be in excess of discharge Q entering the jump at the toe.
- Forward velocity profile has zero velocity at the bed, maximum velocity at a distance δ and then gradually decreases to zero at a height y_F above the bed.
- Region $0 < y < \delta$ is called boundary layer.
- Region $\delta < y < y_F$ is called free missing zone.

JUMP IN EXPONENTIAL CHANNEL

Exponential channels are those channels in which area of the cross-section of the channel is related to depth as below.

$$A = K_1 y^a$$



where,

A = Area of channel

K_1 and a = Characteristics constants

a = 1 for rectangular channel

a = 1.5 for parabolic channel

a = 3 for triangular channel

$$\text{Top width, } T = \frac{dA}{dy} = K_1 a y^{a-1}$$

$$\frac{A}{T} = \frac{y}{a}$$

$$\bar{y} = \frac{y}{a+1}$$

By substituting values of T, $\frac{A}{T}$ and $\bar{y}$ in the equation below we can analyze the jump.

$$A\bar{y} + \frac{Q^2}{gA} = \text{Constant}$$

JUMP ON NON-RECTANGULAR HORIZONTAL CHANNEL

- When the sidewalls of the channel are not vertical; for example, triangular or trapezoidal channel, the flow in a jump will involve lateral expansion of the stream in addition to increase in depth
- As the cross sectional areas are not linear function of the depth of flow. Difficulties in computation of sequent depth are experienced.
- Specific force can be used to find the sequent depths for a given discharge in a given horizontal channel.
- The expression for the sequent depth ratios in channels of regular shapes can be obtained by

$$\frac{P+M}{\gamma} = \text{constant}$$

$$A\bar{y} + \frac{Q^2}{gA} = \text{constant}$$

$$A_1\bar{y}_1 + \frac{Q^2}{gA_1} = A_2\bar{y}_2 + \frac{Q^2}{gA_2}$$

$$A_2\bar{y}_2 - A_1\bar{y}_1 = \frac{Q^2}{g}\left[\frac{1}{A_1} - \frac{1}{A_2}\right]$$

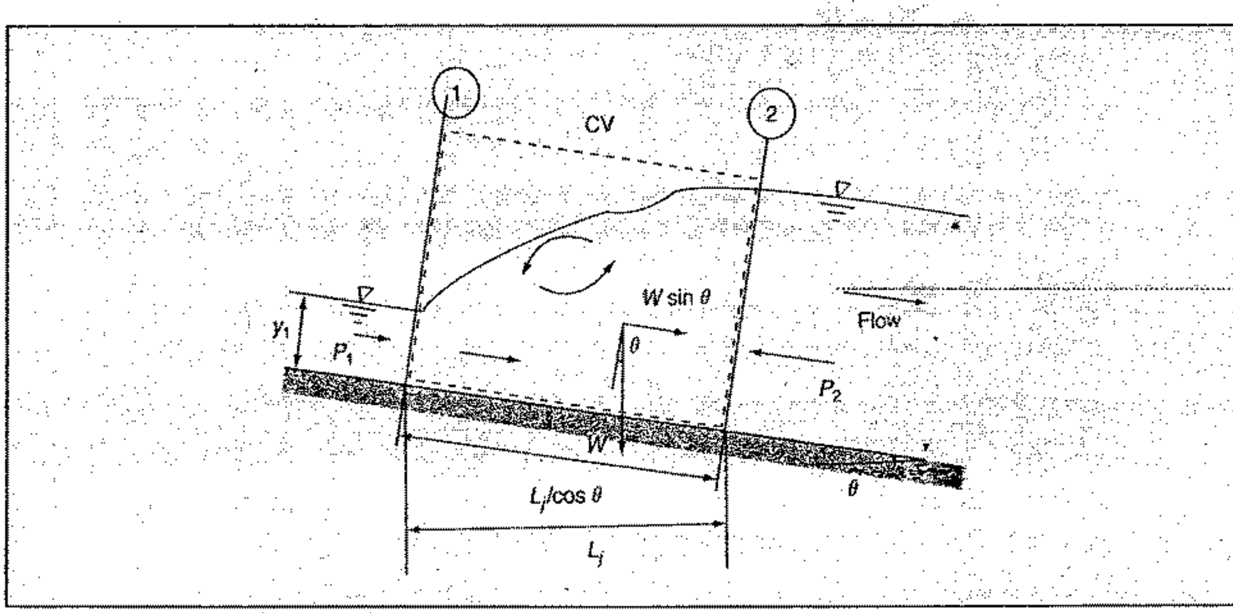
$$A_1\bar{y}_1\left(\frac{A_2\bar{y}_2}{A_1\bar{y}_1} - 1\right) = \frac{Q^2}{g}\left[\frac{A_2 - A_1}{A_1 A_2}\right]$$

$$\left(\frac{A_2\bar{y}_2}{A_1\bar{y}_1}-1\right) = \frac{Q^2}{A_1^2 g y_1}\left[\frac{A_2-A_1}{A_2}\right] \qquad \left[F_1=\frac{v_1}{\sqrt{gA_1/T_1}}\right]$$

$$\left(\frac{A_2\bar{y}_2}{A_1\bar{y}_1}-1\right)=F_1^2\left(\frac{A_1/T_1}{\bar{y}_1}\right)\left(1-\frac{A_1}{A_2}\right)$$

- Substituting the expression of A, T and $\bar{y}$ for a given channel geometry will give an equation relating the sequent depth to F_1 and other geometric parameters of channel.
- Most of the non-rectangular channel contains sequent-depth ratio such that it needs a trial and error producure to evaluate it.

JUMP ON THE SLOPING FLOOR



Analysis of hydraulic jump on sloping floor is done with the help of momentum equation.

$$P_1 - P_2 + W\sin\theta = M_2 - M_1$$

- There are two unknown relative to the number of available equations and unless some additional information is provided the solution of momentum equation is not possible.

  For example the term $W\sin\theta$ representing the longitudinal component of weight of water in jump posses a problem as an unknown quantity.
- As $W\sin\theta$ term involves the length and profile of jump, information about which can only be obtained through experimental observations.

Sequent Depth Ratio on Sloping Floor

- It is observed that on sloping floor jump requires more tail water depth than the corresponding horizontal floor jumps.
- Consider y_2 = Equivalent depth corresponding to y_1 in a sloping floor.

Then, $$y_2 = \frac{y_1}{2}\left(\sqrt{1+8F_1^2}-1\right)$$

Now,
$$\frac{y_t}{y_2} = F(Q)$$

- As the value of Q increases the value of $\frac{y_t}{y_2}$ also increase hence on sloping floor jump requires more tail water depth, than corresponding to horizontal floor jumps.

Length of Jump on Sloping Floor (L_j)

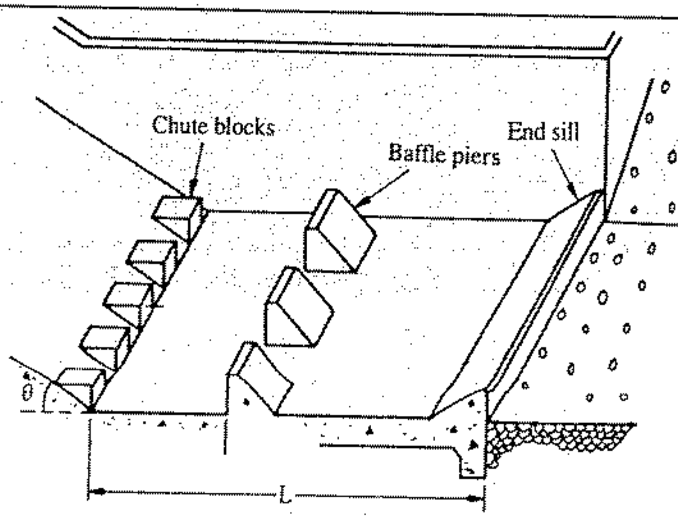
The length of jump on sloping floor is longer than the corresponding length on the horizontal floor.

Energy Loss (E_L) on Sloping Floor

It is observed that relative energy loss $\left(\frac{E_L}{E_1}\right)$ during the sloping floor decreases with the increasing value of θ.

ENERGY DISSIPATION

- Due to high energy loss occuring during the jump, has led to its adoption as a part of the energy – dissipator system below a hydraulic structure.
- Stilling basin is the downstream portion of the hydraulic structure where the energy dissipation is intentionaly allowed to occur such that out going stream can be safely discharged into channel below.
- Function of stilling basin is to provide a stable, good jump with high energy dissipation.
- Stilling basin is a fully paved section and more additional appurtenances such as baffel blocks, chute blocks and End sills.
- Due to the provisions of chut blocks, Baffel blocks. The length of jump and tail water depth required reduces as compared to unaided jump.



Functions of Appurtenances

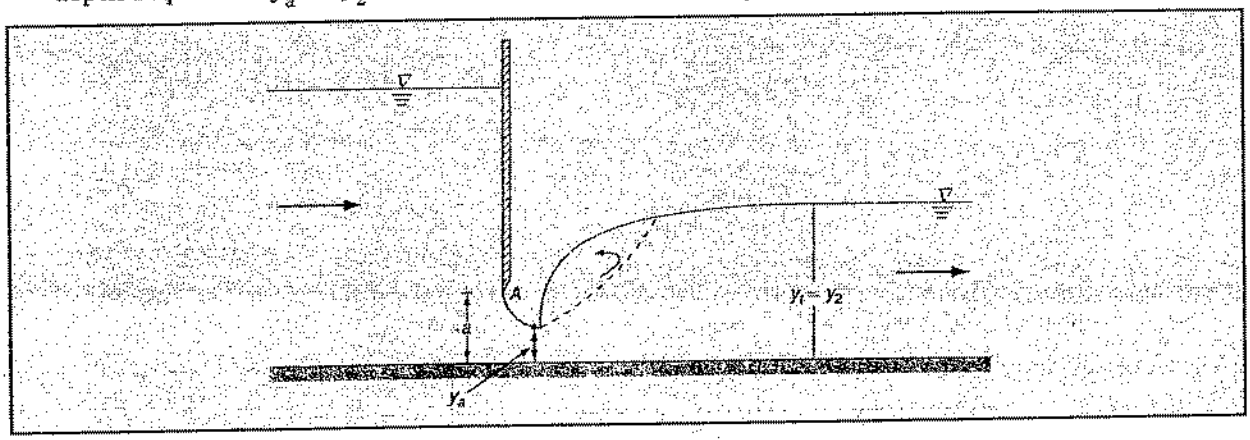
Chute Blocks → For spliting and aeration of flow.

Baffel Blocks → Provides additional resistance to flow.

End Sill → Helps in lifting the outgoing stream into a trajectory so that basin end is not

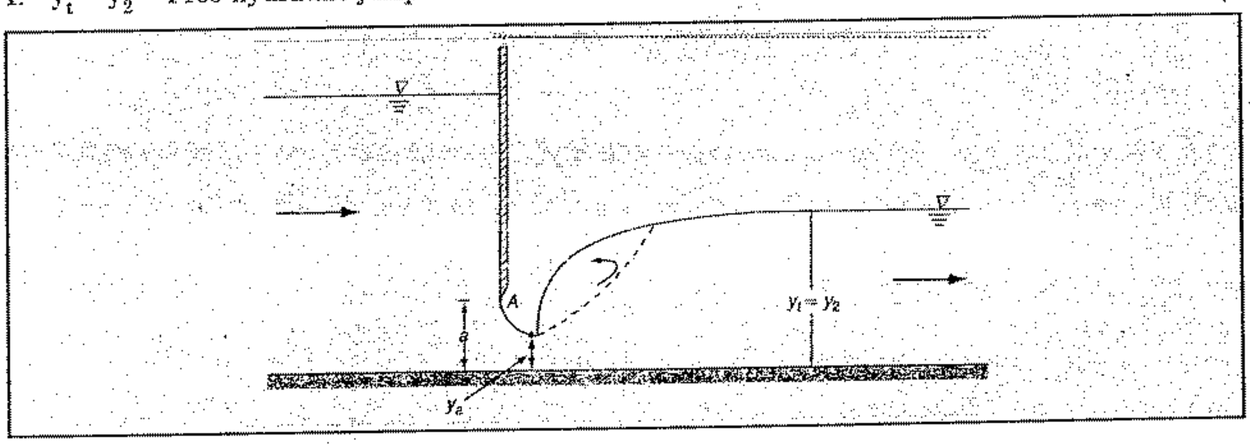
LOCATION OF JUMP

- Tail water depth plays a important role in jump formation.
- The depth downstream of a hydraulic structure, such as a sluice gate, controlled by the downstream channel or local control is called as tail water depth.
- Consider a flow from a sluice gate of opening 'a'. The depth of flow at vera contracta is y_a and the depth requent to y_a be y_2. Let the tail water depth be y_t.



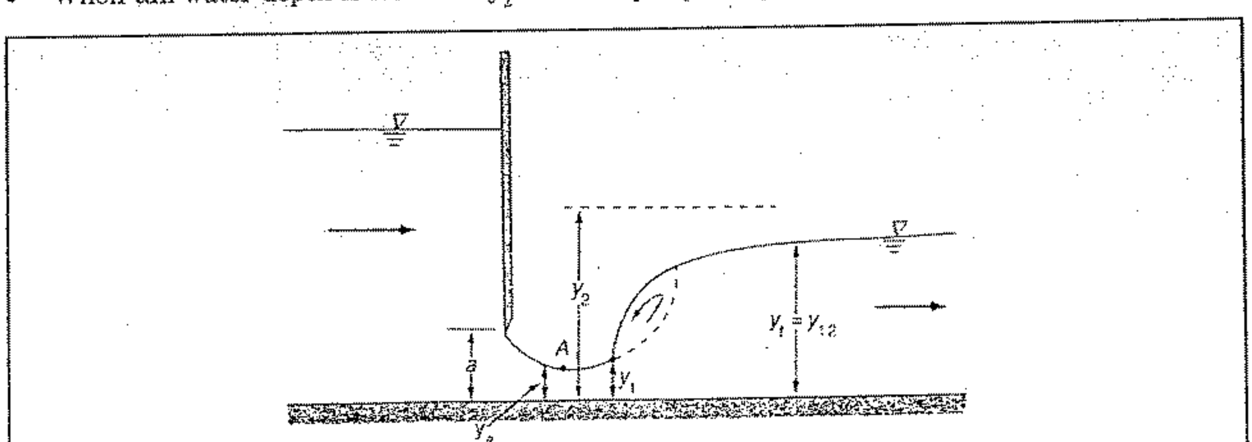
Depending on values of y_2 and y_t following types of jump are indetified.

1. $y_t = y_2$ – Free hydraulic jump will form at Vena contracta.



2. $y_t < y_2$ – Free repelled jump

- When tail water depth is less than y_2 then the jump is repelled downstream of vena contracta.

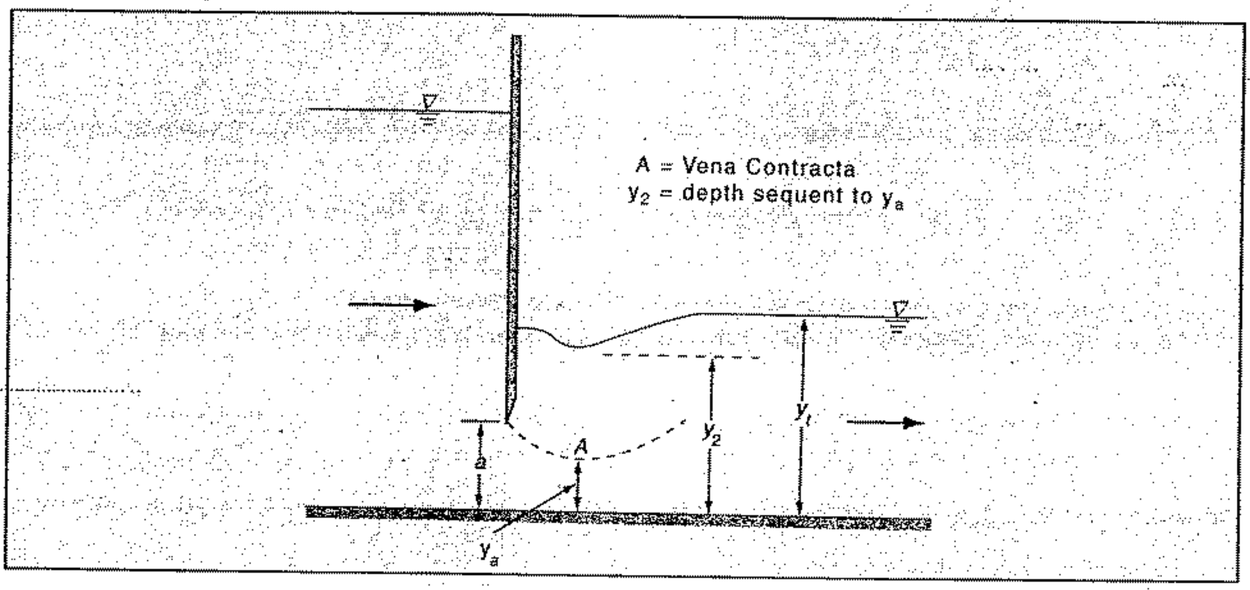


- In this case jump will form late, after vena contracta at some distance l and some depth y_1 (not y_a) and will end at depth of flow y_t.
- To find out the depth y_1 will proceed our calculation with the help of tail water depth i.e.

$$\frac{y_1}{y_t} = \frac{1}{2}\left[\sqrt{1+F_t^2}-1\right]$$

3. $y_t > y_2$ – Submerged or drowned jump.
   Energy loss in submerged jump is less

- Ratio of $\frac{y_t - y_2}{y_2}$ = S, Submergence factor As the submergence factor increases, Energy loss decreases.



Example 1

If the energy loss in a hydraulic jump in a rectangular channel is found to be 6m and pre jump Froude's number of flow $F_1 = 6$, determine y_1 and y_2.

Sol. Given,

$E_L = 6m$; $F_1 = 6$

we know that,

$$\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1+8F_1^2}-1\right]$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1+8\times6^2}-1\right]$$

$$\frac{y_2}{y_1} = 8$$

$$y_2 = 8\,y_1 \quad \text{....(I)}$$

$$\text{Further, } E_1 = \frac{(y_2 - y_1)^3}{4\, y_2\, y_1} \qquad \text{....(II)}$$

$$\text{Put the value of I in II : We get} \quad 6 = \frac{(8y_1 - y_1)^3}{4 \times 8y_1 \times y_1}$$

$$\frac{(7y_1)^3}{32\, y_1^2} = 6$$

$$y_1 = \frac{6 \times 32}{343} = 0.560 \text{ m}$$

$$y_2 = 4.478 \text{ m}$$

Example 2

An overflow spillway has its crest at an elevation of 138m and a horizontal apron at an elevation of 100m on the downstream side. Estimate the tailwater elevation required to form a hydraulic jump when the elevation of energy line just upstream of spilway crest is 140m. Take $C_d = 0.735$ and neglect energy loss due to flow over spillway.

Sol. Given,

Elevation of energy line over crest = 140 m

Elevation of top of crest = 138 m

Elevation of top of apron = 100 m

$C_d = 0.735$

$$\text{Head of water over spillway} = 140 - 138 = 2 \text{ m}$$

$$\text{Discharge per unit width over spillway, } q = \frac{2}{3} C_d \sqrt{2g}\ H^{3/2}$$

$$q = \frac{2}{3} \times 0.735\sqrt{2 \times 9.81} \times 2^{3/2} = 6.139 \text{ m}^3/\text{s}/\text{m}$$

$$\text{Specific energy over apron, } E = 140 - 100 = 40\text{m}$$

From energy equation over apron

$$E_1 = y_1 + \frac{v_1^2}{2g}$$

$$E = y_1 + \frac{q^2}{2g\,y^2}$$

$$40 = y + \frac{6.139^2}{2 \times 9.81\, y^2}$$

$$y^3 - 40y^2 + 1.921 = 0$$

$$y = 39.999,\ 0.220,\ -0.219$$

We know that flow over apron will be super critical flow, hence depth of flow, y = 0.220

$$\text{Velocity, } V_1 = \frac{q}{y_1} = \frac{6.139}{0.220} = 27.905$$

$$F_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{27.905}{\sqrt{9.81 \times 0.220}} = 18.995$$

From sequent depth ratio.

$$\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1+8F_1^2} - 1\right]$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1+8 \times 18.995^2} - 1\right]$$

$$y_2 = \frac{0.220}{2}\left[\sqrt{1+8 \times 18.995^2} - 1\right]$$

$$y_2 = 5.801$$

Required tailwater elevation = 100 + 5.801 = 105.801 m

Example 3

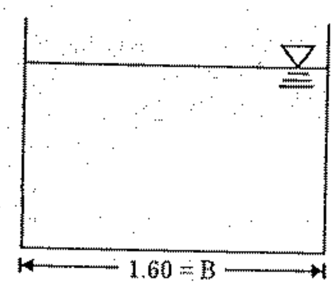
Water flows at the rate of 1000 liters/sec. along a channel of rectangular section 1.60 m in width calculate the critical depth. A hydraulic jump forms at an upstream point where the depth is 80% of the critical depth. What would be the rise in water level due to the jump formation? Also calculate horse power lost in the jump formation.

Sol. Given,

$Q = 1000\ l/s = 1 m^3/sec$

$B = 1.60$

$y_1 = 0.80\ y_c$



We know for rectangular channel

$$\text{Critical depth of flow, } y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$y_c = \left(\frac{(1/1.60)^2}{9.81}\right)^{1/3}$$

$$y_c = 0.341 \text{ m}$$

$$y_1 = 0.80\ y_c$$

$$y_c = 0.8 \times 0.341 = 0.273 \text{ m}$$

$$v_1 = \frac{q}{y_1} = \frac{1/1.6}{0.273} = 2.289 \text{ m / sec}$$

$$F_1 = \frac{v_1}{\sqrt{g\, y_1}} = \frac{2.289}{\sqrt{9.81 \times 0.273}} = 1.399$$

From sequent depth ratio

$$\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1+8F_1^2} - 1\right]$$

$$y_2 = \frac{0.273}{2}\left[\sqrt{1+8\times 1.399^2} - 1\right]$$

$$y_2 = 0.421 \text{ m}$$

$$\text{Energy lost, } E_L = \frac{(y_2 - y_1)^3}{4 y_1\, y_2} = \frac{(0.421 - 0.273)^3}{4\times 0.421 \times 0.273}$$

$$E_L = 0.007 \text{ m}$$

$$\text{Power lost in jump formation in H.P} = \gamma Q\ E_L = \frac{9.81\times 1000 \times 1 \times 0.007}{745}$$

$$= 0.092 \text{ HP/m width}$$

Example 4

An overflow spillway has its crest elevation of 125.4m and a horizontal apron at elevation of 95.00 m on the downstream side. Find the tail water elevation required to form a hydraulic jump when the upstream energy line is 127.90 m. The coefficient of discharge for the flow over a spillway can be assumed to be 0.735. The energy loss occured in this hydraulic jump.

Sol. Given,

Elevation of energy line = 127.90

Elevation of top of crest = 125.4

Elevation of top apron = 95.00

$C_d = 0.735$

$$\text{Head of water over crest} = 127.90 - 125.40 = 2.50 \text{ m}$$

$$\text{Discharge per unit width over spillway, } q = \frac{2}{3} C_d \sqrt{2g}\ H^{3/2}$$

$$q = \frac{2}{3} \times 0.735 \times \sqrt{2 \times 9.81} \times 2.5^{3/2}$$

Specific energy over the apron, E $=127.90 - 95.0 = 32.90$ m

Applying energy equation over apron, $E = y_1 + \frac{v^2}{2g} = y_1 + \frac{q^2}{2g\,y_1^2}$

$$32.90 = y_1 + \frac{8.579^2}{2\times 9.81\,y_1^2} = y_1 + \frac{3.751}{y_1^2}$$

$$y_1^3 - 32.90y_1^2 + 3.751 = 0$$

On solving above equation

$$y_1 = 32.897,\ 0.339,\ -0.336$$

We know that flow over apron at the toe of jump will be supercritical flow, hence depth of flow over apron $y_1 = 0.339$

$$\text{Velocity of flow, } V_1 = \frac{q}{y_1} = \frac{8.579}{0.339} = 25.307$$

$$\text{Froude number, } F_1 = \frac{V_1}{\sqrt{g\,y_1}} = \frac{25.307}{\sqrt{9.81\times 0.339}} = 13.877$$

From sequent depth ratio

$$\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1+8F_1^2} - 1\right]$$

$$y_2 = \frac{0.339}{2}\left[\sqrt{1+8\times 13.877^2} - 1\right]$$

$$y_2 = 6.486$$

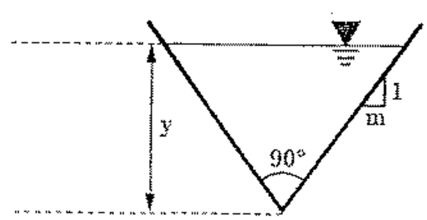
Required tailwater elevation $= 95 + 6.486 = 101.486$

Example 5

A hydraulic jump occurs in a 90° triangular channel derive an equation for discharge and the conjugate depths. If the depth before and after the jump in the channel above are 0.6 m and 1.5 m respectively. Find the Froude number before and after the jump.

Sol. Consider a triangular channel of side slope 1 : m (V : H)

Area of cross section of channel, $A = \frac{1}{2}y \times 2my = my^2$



We know that,

$$\text{Pressure force, } P = \gamma A \bar{y}$$

$$P = \gamma(m\,y^2)\frac{y}{3} = \frac{\gamma m\,y^3}{3}$$

$$\text{Momentum, } M = \rho Q\,V = \frac{\rho\,Q^2}{A} = \frac{\rho Q^2}{m\,y^2}$$

For hydraulic jump in horizontal friction less channel specific force is constant.

$$\frac{P_1 + M_1}{\gamma} = \frac{P_2 + M_2}{\gamma}$$

$$\frac{\frac{\gamma\,m\,y_1^3}{3} + \frac{\rho Q^2}{m\,y_1^2}}{\gamma} = \frac{\frac{\gamma m\,y_2^3}{3} + \frac{\rho Q^2}{m\,y_2^2}}{\gamma}$$

$$\frac{m\,y_1^3}{3} + \frac{Q^2}{mg\,y_1^2} = \frac{m\,y_2^3}{3} + \frac{Q^2}{mg\,y_2^2} \qquad [\because \gamma = \rho g]$$

$$\frac{Q^2}{mg}\left[\frac{1}{y_1^2} - \frac{1}{y_2^2}\right] = \frac{m}{3}\left[y_2^3 - y_1^3\right]$$

$$\frac{Q^2}{mg}\left(\frac{y_2^2 - y_1^2}{y_1^2\,y_2^2}\right) = \frac{m}{3}(y_2 - y_1)\left(y_1^2 + y_1 y_2 + y_1^2\right)$$

$$\boxed{Q^2 = \frac{m^2 g}{3}\;\frac{y_1^2\;y_2^2\left(y_2^2 + y_1\,y_2 + y_1^2\right)}{(y_1 + y_2)}}$$

$$Q = \left[\frac{m^2 g}{3}\;\frac{y_1^2\;y_2^2\left(y_2^2 + y_1\,y_2 + y_1^2\right)}{(y_1 + y_2)}\right]^{1/2}$$

Given,

$y_1 = 0.6, \quad y_2 = 1.5, \; m = 1$

$$Q = \left[\frac{1\times 9.81}{3} \times \frac{1.5^2 \times 0.6^2(1.5^2 + 0.6\times 1.5 + 0.6^2)}{(0.6 + 1.5)}\right]^{1/2} = 2.104 \text{ m}^3/\text{sec}$$

We know that Froude number

$$F = \frac{v}{\sqrt{g\,A/T}} = \frac{Q/A}{\sqrt{g\,A/T}}$$

$$F^2 = \frac{Q^2 T}{\ldots}$$

$$F^2 = \frac{Q^2\, 2my}{gm^3y^6} \qquad \left[T = 2my,\ \ A = my^2\right]$$

$$F = \left(\frac{2Q^2}{gm^2\, y^5}\right)^{1/2}$$

Froude number at the start of jump,

$$F_1 = \left(\frac{2Q^2}{gm^2\, y_1^5}\right)^{1/2}$$

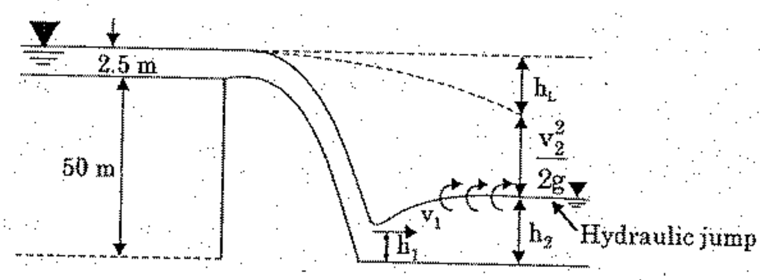
$$F_1 = \left(\frac{2\times 2.104^2}{9.8\times 1^2\times 0.6^5}\right)^{1/2} = 3.407$$

Froude number at the end of jump

$$F_2 = \left(\frac{2Q^2}{gm^2\, y_2^5}\right)^{1/2} = \left(\frac{2\times 2.104^2}{9.81\times 1^2\times 1.5^5}\right)^{1/2} = 0.345$$

Example 6

An overflow spillway as shown in figure is 50m high. At the design head of 2.5 m over the, spillway, find the sequent depth and energy loss in hydraulic jump formed on horizontal apron at the toe of the spillway. Neglect energy loss over the spillway, $C_d = 0.74$. Find also the percentage of energy loss occured in hydraulic jump. Use specific energy concept to find y_1, the supercritical depth of flow of hydraulic jump



Sol. Given,

Design Head, H = 2.5

Discharge per unit width over spillway, $q = \frac{2}{3}C_d\sqrt{2g}\ H^{3/2}$

$$q = \frac{2}{3}\times 0.74\times\sqrt{2\times 9.81}\times 2.5^{3/2}$$

$$q = 8.638\ m^3/sec/m$$

Specific energy at the apron, E = 50 + 2.5 = 52.5

Applying energy Equation at the toe of hydraulic jump

$$E = y_1 + \frac{v_1^2}{2g}$$

$$E = y_1 + \frac{q^2}{2g\,y_1^2}$$

$$52.5 = y_1 + \frac{8.638^2}{2 \times 9.81 \times y_1^2}$$

$$52.5 = y_1 + \frac{3.803}{y_1^2}$$

$$y_1^3 - 52.5y_1^2 + 3.803 = 0$$

on solving above equation

$$y_1 = 52.499,\ 0.270,\ -0.268$$

As the flow over apron before the toe of jump will be super critical flow, hence depth of flow will be $y_1 = 0.270$ m

$$\text{Velocity, } v_1 = \frac{q}{y_1} = \frac{8.638}{0.270} = 31.993$$

$$E_1 = \left(y_1 + \frac{v_1^2}{2g}\right) = \left[\left(0.270 + \frac{(31.993)^2}{2 \times 9.81}\right)\right] = 52.5$$

$$\text{Froude number, } F_1 = \frac{v_1}{\sqrt{g\,y_1}} = \frac{31.993}{\sqrt{9.81 \times 0.270}} = 19.658$$

From sequent depth ratio

$$\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1 + 8F_1^2} - 1\right]$$

$$y_2 = \frac{0.270}{2}\left[\sqrt{1 + 8 \times 19.658^2} - 1\right]$$

$$y_2 = 7.372 \text{ m}$$

$$\text{Energy loss in hydraulic jump, } E_L = \frac{(y_2 - y_1)^3}{4\,y_1\,y_2}$$

$$E_L = \frac{(7.372 - 0.270)^3}{4 \times 7.372 \times 0.270} = 44.999\text{m}$$

$$\text{Energy loss percentage} = \frac{E_L}{E} \times 100 = \frac{44.999}{52.5} \times 100 = 85.712\%$$

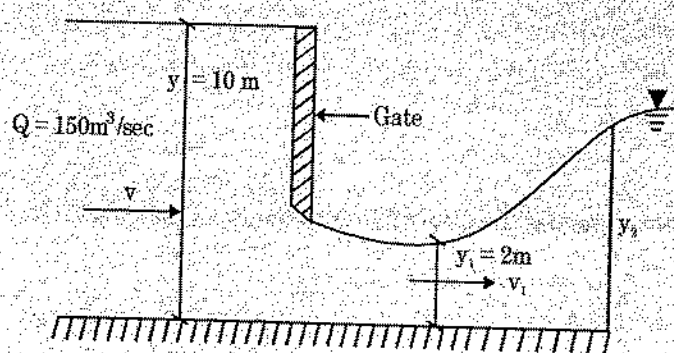
Example 7

A hydraulic jump is formed in a 5 m wide outlet at a short distance downstream of control gate. If the flow depths are 10 m and 2m in the u/s and d/s respectively of the sluice gate and the Q = 150 m³/sec, determine

(i) Flow depth downstream of the jump

(ii) Thrust on the gate

(iii) Head loss in the jump.



Sol. Let y be the depth of flow at the u/s of sluice gate, y_1 & y_2 be the sequent depths of hydraulic jump

Given,

B = 5 m; y = 10m; y_1 = 2m; Q = 150 m³/sec

Velocity of flow at the toe of jump, $V_1 = \frac{Q}{B \times y_1} = \frac{150}{5 \times 2} = 15\text{m / sec}$

$$F_1 = \frac{v_1}{\sqrt{g\,y_1}} = \frac{15}{\sqrt{9.81 \times 2}} = 3.386$$

From sequent depth ratio

$$\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1+8F_1^2} - 1\right]$$

$$y_2 = \frac{2}{2}\left[\sqrt{1+8\times 3.386} - 1\right] = 8.629$$

Flow depth downstream of jump y_2 = 8.629 m

$$\text{Thrust on gate, } F = \frac{1}{2}\gamma\frac{(y-y_1)^3}{y_1+y_2}$$

$$F = \frac{1}{2}\times 9.81\frac{(10-2)^3}{10+2} = 209.28 \text{ kN}$$

Energy loss during jump, $E_L = \frac{(y_2-y_1)^3}{4\,y_1\,y_2} = \frac{(8.629-2)^3}{4\times 8.629\times 2} = 4.220$ m

Example 8

A 5.0 m wide rectangular channel 15 m^3/sec of water with a velocity of 6 m/sec. State whether hydraulic jump is a possible. If yes compute the hight of jump and power dessipated.

Sol. Given,

$$B = 5.0 \text{ m};\ Q = 15 \text{ m}^3\text{/sec.};\ v = 6 \text{ m/sec};\ y = \frac{Q}{v \times B} = \frac{15}{6 \times 5} = 0.5\text{m}$$

$$\text{Froude number, } F = \frac{v}{\sqrt{gy}} = \frac{6}{\sqrt{9.81 \times 0.5}} = 2.709$$

As F > 1, flow is super critical and hence hydraulic jump can form.

Let $y_1 = 0.5$ m be the depth of flow before jump and y_2 be the depth of flow after jump. Froude number before jump, $F_1 = 2.709$

From sequent depth ratio

$$\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1+8F_1^2} - 1\right]$$

$$y_2 = \frac{0.5}{2}\left[\sqrt{1+8\times 2.709^2} - 1\right] = 1.682 \text{ m}$$

$$\text{Energy loss during jump, } E_L = \frac{(y_2 - y_1)^3}{4y_1\, y_2}$$

$$E_L = \frac{(1.682 - 0.5)^3}{4 \times 1.682 \times 0.5} = 0.491 \text{ m}$$

$$\text{power lost during jump} = \gamma Q\, E_L$$

$$P_L = 1000 \times 9.81 \times 15 \times 0.491$$

$$= 72236.50$$

$$W = 72.24 \text{ Kw.}$$

Example 9

Water energes from a spillway with a velocity of 15m/sec and a depth of 0.5m. Calculate the necessary subcritical depth at toe of the spillway for the occurence of a hydraulic jump. Calculate the associated energy loss.

Sol. Given,

$v_1 = 15$m/sec; $y_1 = 0.5$m

$$\therefore \quad \text{Froude number, } F_1 = \frac{v_1}{\sqrt{gy_1}} = \frac{15}{\sqrt{9.81 \times 0.5}} = 6.773$$

Let the subcritical depth of hydraulic jump is y_2.

From sequent depth ratio,

$$\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1+8F_1^2}-1\right]$$

$$y_2 = \frac{0.5}{2}\left[\sqrt{1+8\times 6.773^2}-1\right] = 4.546 \text{ m}$$

$$\text{Energy loss} = \frac{(y_2-y_1)}{4y_1\,y_2} = \frac{(4.546-0.5)^3}{4\times 4.546\times 0.5} = 7.28\text{m.}$$

Example 10

Given that unit discharge in a rectangular channel is 18m³/sec/m and the head loss across a hydraulic jump that forms in this channel is 1.1m, estimate the pre jump and post jump depths.

Sol. Let pre jump and post jump are y_1 and y_2 respectively

Given,

$q = 18$ m³/sec/m; $E_L = 1.1$ m

we know that,

$$\frac{2q^2}{g} = y_1\,y_2\,(y_1+y_2)$$

$$\frac{2q^2}{g} = y_1^3\,\frac{y_2}{y_1}\left(1+\frac{y_2}{y_1}\right) \qquad ...(1)$$

$$\text{Also, } E_L = \frac{(y_2-y_1)^3}{4y_1\,y_2} = 1.1$$

$$\frac{y_1^3\left(\frac{y_2}{y_1}-1\right)^3}{4\,y_1^2\,\frac{y_2}{y_1}} = 1.1$$

$$\frac{4.4\,\frac{y_2}{y_1}}{\left(\frac{y_2}{y_1}-1\right)^3} = y_1$$

$$\frac{\left(4.4\,\frac{y_2}{y_1}\right)^3}{\left(\frac{y_2}{y_1}-1\right)^9} = y_1^3$$

Put y_1 in equation (1)

$$\frac{\left(4.4\frac{y_2}{y_1}\right)^3}{\left(\frac{y_2}{y_1}-1\right)^9}\left(\frac{y_2}{y_1}\right)\left(1+\frac{y_2}{y_1}\right) = \frac{2q^2}{g}$$

$$\frac{(4.4)^3\left(\frac{y_2}{y_1}\right)^4\left(1+\frac{y_2}{y_1}\right)}{\left(\frac{y_2}{y_1}-1\right)^9} = 66.055$$

$$\text{Let, } \frac{y_2}{y_1} = x$$

$$\frac{x^4(1+x)}{(x-1)^9} = 0.775$$

Solving the above equation by trial and error

$$x = 2.932$$

$$\frac{y_2}{y_1} = 2.932$$

$$y_2 = 2.932\ y_1$$

Now,

$$y_1 y_2 (y_1 + y_2) = 66.055$$

$$2.932\ y_1^2 (3.932\ y_1) = 66.055$$

$$11.529\ y_1^3 = 66.055$$

$$y_1^3 = 5.730$$

$$y_1 = 1.789$$

$$\therefore\ y_2 = 2.932 \times y_1 = 5.247$$

Therefore prejump $y_1 = 1.789$ and post jump $y_2 = 5.247$

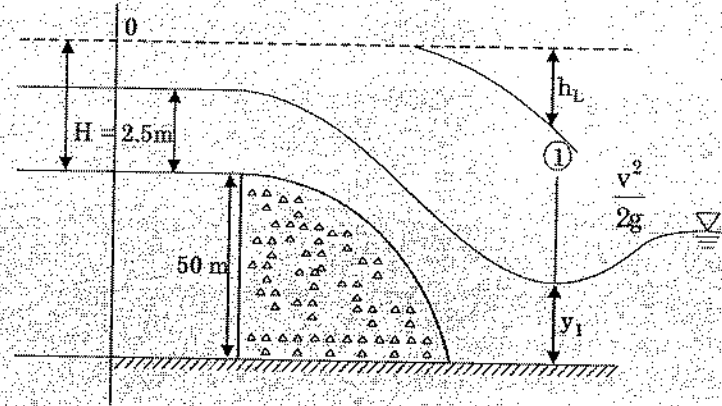
Example 11

An overflow spillway is 50m high. The design energy head is 2.5m over the spillway. Find the sequent depth and the energy loss in hydraulic jump formed on a horizontal apron at the toe of spillway. Neglect the energy loss due to the flow over the spillway face. Assume the coefficient of discharge $C_d = 0.735$ and the velocity at the toe before jump can be approximated to

$$V = \{2g\,(\text{total head})\}^{1/2}$$

Where g is acceleration due to gravity.

Sol.



Given,

Design head over spillway = 2.5 m

Height of spillway = 50 m

Total head over the Apron = 52.5 m

$$\text{Discharge per unit width over spillway, } q = \frac{2}{3}C_d\sqrt{2g}H^{3/2}$$

$$q = \frac{2}{3}\times 0.735\sqrt{2\times 9.81}\times(2.5)^{3/2} = 8.579 \text{ m}^3\text{/sec/m}$$

$$\text{Velocity at the before jump, } v_1 = \sqrt{2g(\text{T.H.})}$$

$$v_1 = \sqrt{2\times 9.81\times 52.5}$$

$$v_1 = 32.094 \text{ m/sec}$$

$$y_1 = \frac{q}{v_1} = \frac{8.579}{32.094} = 0.267$$

$$F_1 = \frac{v_1}{\sqrt{gy_1}} = \frac{32.094}{\sqrt{9.81\times 0.267}} = 19.831$$

From sequent Depth ratio

$$\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1+8F_1^2}-1\right)$$

$$y_2 = \frac{0.267}{2}\left(\sqrt{1+8\times 19.831^2}-1\right) = 7.356 \text{ m}$$

$$\text{Energy loss, } E_L = \frac{(y_2-y_1)^3}{4y_1y_2}$$

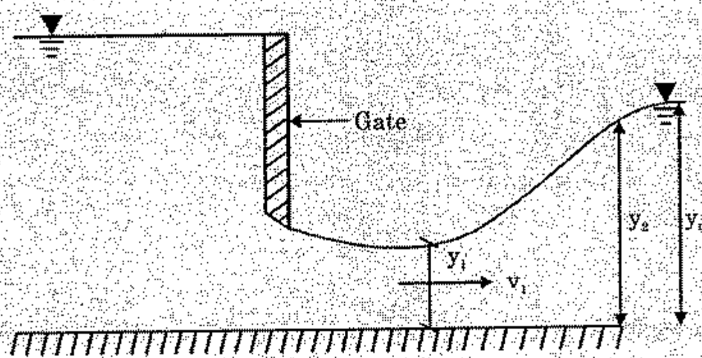
$$E_L = \frac{(7.356-0.267)^3}{4\times 0.267\times 7.356} = 45.376 \text{ m}$$

Example 12

A sluice gate is a 3 m wide rectangular horizontal channel releases a discharge of 18m³/sec. Gate opening is 0.67m and C_c is 0.6. Determine the type of hydraulic jump when tail water depth is

(a) 3.6 m (b) 5.0 m (c) 4.09 m

Sol.



Given,

Gate opening, a = 0.67 m; C_c = 0.6 m; Q = 18m³/sec; B = 3 m.

$$\text{Depth of water at toe of jump, } y_1 = C_c \times a = 0.6 \times 0.67 = 0.4 \text{ m}$$

$$\text{Velocity of water at toe of jump, } v_1 = \frac{Q}{B\,y_1}$$

$$v_1 = \frac{18}{3 \times 0.4} = 15.00 \text{ m/sec}$$

$$\text{Froude number at toe of jump, } F_1 = \frac{v_1}{\sqrt{g\,y_1}}$$

$$F_1 = \frac{15.00}{\sqrt{9.81 \times 0.4}} = 7.572$$

From Sequent depth ratio

$$\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1+8F_1^2} - 1\right)$$

$$y_2 = \frac{0.4}{2}\left(\sqrt{1+8 \times 7.572^2} - 1\right)$$

$$= 4.09 \text{ m}$$

(a) If y_t = 3.6 m (i.e $y_t < y_2$) Repelled jump.

(b) If y_t = 5.0 m (i.e $y_t > y_2$) Submerged jump.

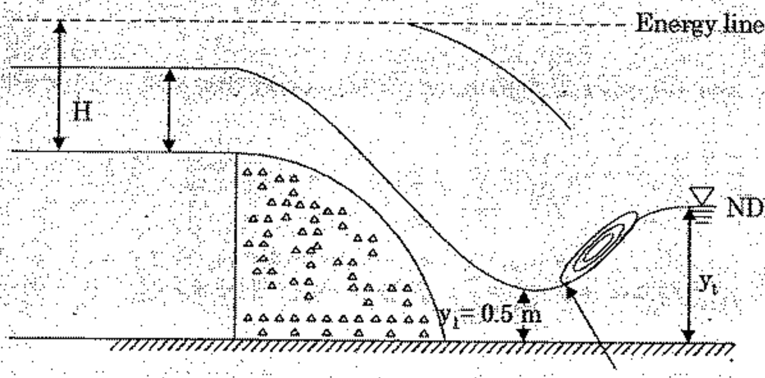
(c) If y_t = 4.09 m (i.e $y_t = y_2$) Free jump.

Example 13

Water emegers from a spillway with a velocity of 20m/sec and a depth of 0.5 m at the toe which is the min depth estimate one distance of hydraullic jump from the spillway toe. If no specific measures are taken for the formation of jump at the toe. Assume the d/s channel as having n = 0.020 and $S_0 = 0.001$ calculation may be performed by direct step method taking only one step assume channel to be wide rectangular.

Sol.

Note: *at the toe of spill way depth of flow will be min always.*



Considerting width of spillway to be wide

From Mannings equation

$$Q = \frac{1}{n} A R^{2/3} S_0^{1/2}$$

Assuming the flow over spillway similar to wide rectangular channel hence $R = y_n$

$$Q = \frac{1}{n}(B \times y_n)(y_n)^{2/3} S_0^{1/2}$$

$$\frac{Q}{B} = \frac{1}{n} y_n^{5/3} S^{1/2}$$

$$q = \frac{1}{n} y_n^{5/3} S_0^{1/2} \qquad [q = y_1 \times v]$$

$$20 \times 0.5 = \frac{1}{0.020} y_n^{5/3} \sqrt{0.001}$$

$$y_n^{5/3} = 6.325$$

$$y_n = 3.024 \text{ m}$$

Tailwater depth on down stream side will be

$$y_t = y_n = 3.024 \text{ m}$$

Let y_a be the sequent depth corresponding to intial depth

$$y_1 = 0.5$$

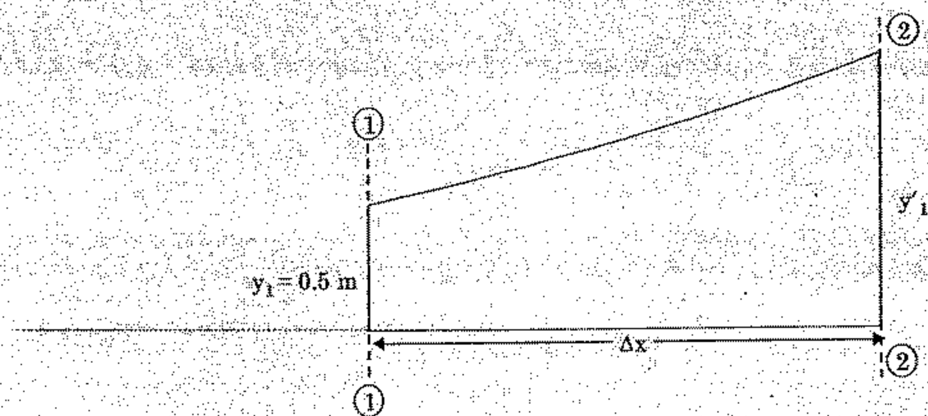
$$\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1+8F_1^2}-1\right]$$

$$F_1 = \frac{v}{\sqrt{gy_1}} = \frac{20}{\sqrt{9.81\times 0.5}} = 9.03$$

$$y_2 = \frac{0.5}{2}[\sqrt{1+8\times 9.03^2}-1]$$

$$= 6.14 \text{ m}$$

As $y_2 > y_t$ jump will be repelled jump.



Let jump will formed at a distance Δx at depth of flow y_1' with its sequent depth equal to $y_t = 3.024$ m

Therefore,

$$\frac{y_1'}{y_t} = \frac{1}{2}\left[\sqrt{1+8F_{rt}^{\ 2}}-1\right]$$

Where F_{rt} is the froude number of tail water depth

$$F_{rt} = \frac{v_t}{\sqrt{gy_t}} = \frac{20\times 0.5}{3.024\sqrt{9.81\times 3.024}} = 0.606$$

$$\frac{y_1'}{y_t} = \frac{1}{2}\left[\sqrt{1+8F_{rt}^{\ 2}}-1\right]$$

$$y_1 = \frac{3.024}{2}\left[\sqrt{1+8\times 0.606^2}-1\right]$$

$$y_1 = 1.488 \text{ m}$$

Calculation for distance of repelled jump from direct step method.

$$\frac{\Delta E}{\Delta x} = S_0 - \bar{S}_f$$

$$\Delta x = \frac{\Delta E}{S_0 - \bar{S}_f}$$

$$E_1 = y_1 + \frac{v_1^2}{2g} = 0.5 + \frac{20^2}{2 \times 9.81} = 20.887$$

$$E_2 = y_2 + \frac{v_2^2}{2g} = 1.492 + \frac{(20 \times 0.5)^2}{2 \times 9.81} = 3.782 \text{ m.}$$

We know that,

$$Q = \frac{1}{n} A R^{2/3} S_f^{1/2}$$

At normal depth of flow $y_n = 0.5$

$$(20 \times 0.5) = \frac{1}{n} \times y_n^{5/3} S_{f1}^{1/2}$$

$$S_{f1} = \frac{20 \times 0.5 \times 0.02}{0.5^{5/3}} = 0.403$$

At normal depth of flow $y_n = 1.492$

$$S_{f2} = \frac{20 \times 0.5 \times 0.02}{(1.492)^{5/3}} = 0.0106$$

$$\bar{S}_f = \frac{0.403 + 0.0106}{2} = 0.2068$$

$$\Delta x = \frac{20.887 - 3.782}{0.001 - 0.2068} = 83.11$$

OBJECTIVE QUESTIONS

1. At a hydraulic jump, the depths at the two sides are 0.4 m and 1.4 m. The head loss in the jump is nearly
   (a) 1.0 m (b) 0.9 m
   (c) 0.7 m (d) 0.45 m

2. If F_1 and F_2 are the Froude numbers of flow before and after the hydraulic jump occurring in a rectangular channel, then

   (a) $F_2^2 = \frac{F_1^2}{\left(-1+\sqrt{1+8F_1^2}\right)}$ (b) $F_2^2 = \frac{8F_1^2}{\left(-1+\sqrt{1+8F_1^2}\right)^3}$

   (c) $F_2^2 = \frac{F_1^2}{\left(-0.5+\sqrt{1+8F_1^2}\right)^3}$ (d) $F_2^2 = \frac{8F_1^2}{\left(-0.5+\sqrt{1+F_1^2}\right)^3}$

3. The Froude number of a hydraulic jump is 5.5. The jump can be classified as a/an
   (a) undular jump (b) oscillating jump
   (c) weak jump (d) steady jump

4. A hydraulic jump occurs at the toe of a spillway. The depth before jump is 0.2 m. The sequent depth is 3.2 m. What is the energy dissipated in m (approximate)?
   (a) 27 (b) 10.5
   (c) 15 (d) 42

5. The sequent depth ratio of a hydraulic jump in a rectangular channel is 16.48. What is the Froude number (approximate) at the beginning of the jump?
   (a) 9.0 (b) 12.0
   (c) 5.0 (d) 8.0

6. The type of jump that forms when initial Froude number lies between 2.5 and 4.5 is
   (a) weak jump (b) steady jump
   (c) undular jump (d) oscillating jump

7. The loss of energy in a hydraulic jump formed in a rectangular channel is given by (Symbols have the usual meanings)

   (a) $DE = \frac{(y_2-y_1)^2}{4y_1y_2}$ (b) $DE = \frac{VQ(y_2-y_1)^3}{8y_1y_2}$

   (c) $DE = \frac{VQ(y_2-y_1)^3}{75\times 4y_1y_2}$ (d) $DE = \frac{(y_2-y_1)^3}{4y_1y_2}$

8. The flow characteristics before and after a hydraulic jump are such that
   (a) specific forces are equal but specific energies are unequal
   (b) specific forces are unequal but specific energies are equal
   (c) neither specific forces nor specific energies are equal
   (d) specific forces as well as specific energies are equal

9. Which one of the following is analogous to normal shock wave?
   (a) An elementary wave in a still liquid
   (b) Hydraulic jump
   (c) Flow of liquid through an expanding nozzle
   (d) Sub-critical flow in an open-channel

10. The sequent depths in a hydraulic jump formed in a 4.0 m wide rectangular channel are 0.2 m and 1.0 m. The discharge in the channel is
    (a) 1.12 m^3/s (b) 2.17 m^3/s
    (c) 4.340 m^3/s (d) 5.0 m^3/s

11. A partially open sluice gate discharges water into a rectangular channel. The tail water depth in the channel is 3m and Froude number is $\frac{1}{2\sqrt{2}}$. If a free hydraulic jump is to be formed at downstream of the sluice gate at the vena contracta of the jet coming out from the sluice gate, the sluice gate opening should be (coefficient of contraction $C_c = 0.9$)
    (a) 0.3 m (b) 0.4 m
    (c) 0.69 m (d) 0.9 m

12. A hydraulic jump occurs when there is a brake in grade from a
    (a) mild slope to steep slope (b) steep slope to mild slope
    (c) steep slope to steeper slope (d) mild slope to milder slope

13. The initial depth of a hydraulic jump in a rectangular channel is 0.2 m and the sequent depth ratio is 10. The length of the jump is about
    (a) 4 m (b) 6m
    (c) 12 m (d) 20 m

14. Seventy per cent of the initial energy is lost in a jump taking place in a horizontal rectangular channel. The Froude number of the flow at the toe is
    (a) 4.0 (b) 9.0
    (c) 20.0 (d) 15.0

15. If y_2= sequent depth for a rectangular channel obtained by assuming horizontal frictionless channel in the momentum equation and y_{2a} = coresponding actual sequent depth measured in a horizontal rectangular channel having high friction, one should expect
    (a) $y_2 > y_{2a}$ (b) $y_2 = y_{2a}$
    (c) $y_2 < y_{2a}$ (d) $y_2 \leq y_{2a}$

16. If the length of the jump in a sloping rectangular channel = L_{js} and the corresponding length of the jump in a horizontal rectangular channel having same y_1 and F_1 is L_j, then

(a) $L_j > L_{js}$
(b) $L_{js} > L_j$
(c) $L_j = L_{js}$
(d) $L_j/L_{js} = 0.80$

17. A sluice gate discharges a flow with a depth of y_1 at the vena contracta. y_2 is the sequent depth corresponding to y_1. If the tailwater depth y_t is larger then y_2 then

(a) a repelled jump occurs
(b) a free jump occurs
(c) a submerged jump takes place
(d) no jump takes place

ANSWERS

1. (d)	2. (b)	3. (d)	4. (b)	5. (b)	6. (d)	7. (d)	8. (a)	9. (b)	10. (c)
11. (c)	12. (b)	13. (c)	14. (b)	15. (a)	16. (b)	17. (c)			

HINT

1.
$$E_L = \frac{(y_2 - y_1)^3}{4y_2y_1} = \frac{(1.4-0.4)^3}{4\times1.4\times0.4} = 0.45$$

2.
$$\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1+8F_1^2}-1\right) \quad ...(1)$$

$$\frac{y_1}{y_2} = \frac{1}{2}\left(\sqrt{1+8F_2^2}-1\right) \quad ...(2)$$

Equation (i) can be written as,

$$\text{Also, } \frac{y_1}{y_2} = \frac{2}{\left(\sqrt{1+8F_1^2}-1\right)} \quad ...(3)$$

Equating (2) & (3)

$$\frac{1}{2}\left(\sqrt{1+8F_2^2}-1\right) = \frac{2}{\left(\sqrt{1+8F_1^2}-1\right)}$$

$$\sqrt{1+8F_2^2} = \frac{4}{\left(\sqrt{1+8F_1^2}-1\right)}+1$$

$$\sqrt{1+8F_2^2} = \frac{4+\sqrt{1+8F_1^2}-1}{\left(\sqrt{1+8F_1^2}-1\right)}$$

$$1+8F_2^2 = \frac{\left(3+\sqrt{1+8F_1^2}\right)^2}{\left(\sqrt{1+8F_1^2}-1\right)^2}$$

$$1+8F_2^2 = \frac{9+1+8F_1^2+6\sqrt{1+8F_1^2}}{\left(\sqrt{1+8F_1^2}-1\right)^2}$$

$$8F_2^2 = \frac{10+8F_1^2+6\sqrt{1+8F_1^2}}{\left(\sqrt{1+8F_1^2}-1\right)^2}-1$$

$$8F_2^2 = \frac{10+8F_1^2+6\sqrt{1+8F_1^2}-\left(1+8F_1^2-2\sqrt{1+8F_1^2}+1\right)}{\left(\sqrt{1+8F_1^2}-1\right)^2}$$

$$8F_2^2 = \frac{8+8\sqrt{1+8F_1^2}}{\left(\sqrt{1+8F_1^2}-1\right)^2}$$

$$F_2^2 = \frac{1+\sqrt{1+8F_1^2}}{\left(\sqrt{1+8F_1^2}-1\right)^2}$$

$$F_2^2 = \frac{\left(1+\sqrt{1+8F_1^2}\right)}{\left(\sqrt{1+8F_1^2}-1\right)^2}\times\frac{\left(\sqrt{1+8F_1^2}-1\right)}{\left(\sqrt{1+8F_1^2}-1\right)} = \frac{1+8F_1^2-1}{\left(\sqrt{1+8F_1^2}-1\right)^3}$$

$$F_2^2 = \frac{8F_1^2}{\left(\sqrt{1+8F_1^2}-1\right)^3}$$

4.

$$E_L = \frac{(y_2-y_1)^3}{4y_1y_1}$$

$$= \frac{(3.2-0.2)^3}{4\times3.2\times0.2} = \frac{3^3}{4\times3.2\times0.2} = 10.5$$

5.

$$\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1+8F_1^2}-1\right)$$

$$16.48 = \frac{1}{2}\left(\sqrt{1+8F_1^2}-1\right)$$

$$F_1 = 12.001$$

8. For a horizontal and frictionless channel specific forces are constant but specific energies are un equal.

10.
$$y_1y_2(y_1 + y_2) = \frac{2q^2}{g}$$

$$0.2 \times 1\ (0.2 + 1) \times \frac{9.81}{2} = q^2$$

$$q = 1.085\ m^3/s/m$$

$$Q = q \times B = 1.085 \times 4 = 4.340\ m^3/Sec$$

13. $\frac{y_2}{y_1} = 10,\ y_1 = 0.2\ m,\ y_2 = 2\ m$

$$L_j = 6.9\ (y_2 - y_1)$$

$$L_j = 6.9\ (2 - 0.2) = 6.9 \times 1.8 = 12.42\ m$$

11.
$$\frac{y_1}{y_2} = \frac{1}{2}\left(\sqrt{1+8F_1^2}-1\right)$$

$$\frac{y_1}{y_2} = \frac{1}{2}\left(\sqrt{1+8\left(\frac{1}{2\sqrt{2}}\right)^2}-1\right)$$

$$y_1 = \frac{3}{2}\left(\sqrt{1+8\times\frac{1}{8}}-1\right)$$

$$= 1.5\times\left(\sqrt{2}-1\right) = 0.621$$

$$\text{Opening of sluice gate} = \frac{y_1}{c_c} = \frac{0.621}{0.9} = 0.69\ m$$

15. Due to friction some of energy will be lost and jump will not be able to reach y_2 depth and fall short to y_{2a} depth.

18. In normal shock wave, u/s flow is in supersonic state where as d/s flow to shock wave is in subsonic state which is similar to supercritical flow before hydraulic jump and subcritical flow after jump.

CHAPTER

6

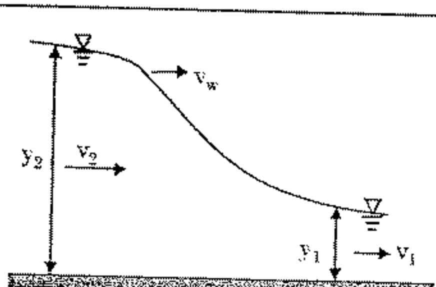
Unsteady Flow-Surge

INTRODUCTION

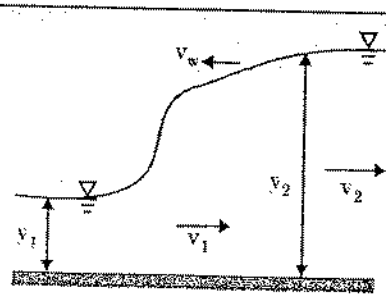
- Unsteady flow are also called as transient flow, occur in open channel when the discharge or depth or both vary with time at a section.
- Unsteady flow can be due to natural cause, planned action or accidental happenings.
- Unsteady flows can be further classified, based on curvature
  1. Gradually - varied unsteady flow (GUVF), Example : Flood flow in a stream.
  2. Radpidly - varied unsteady flow (RUVF), Example : Formation and travel of surge due to sudden closure of a sluice gate

SURGES IN OPEN CHANNEL

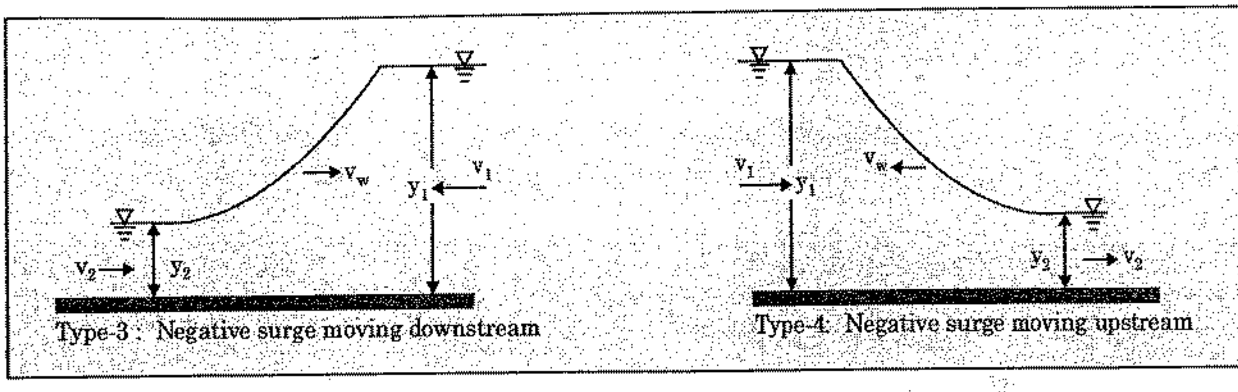
- Surges are waves and their presence in the flow field leads to a trasient or time varying flow depth and velocity at a section.
- Surges may be caused by various factors such as closure or opening of valves, gates, loading or unloading of turbines, start or stopping of pumps, failure of dams leading to flow of reservoir downstream, wind dirven circulation in lakes and other water bodies.
- Depending on the direction of movement of these waves, flow depth either may decrease or increase in the flow direction.
- A surge producing an increase in flow depth is called positive surge and one which cause a decrease in flow depth is called as negative surge.
- In other words if the wave is higher than the orignal steady flow depth, we call it as a positive surge. In negative surge, situation is just opposite.
- A surge can travel either upstream or downstream direction, thus giving rise to four basic types.



Type 1: Positive surge moving downstream



Type 2: Positive surge moving upstream



Type-3 : Negative surge moving downstream

Type-4: Negative surge moving upstream

where, v_w = Velocity of wave or surge.

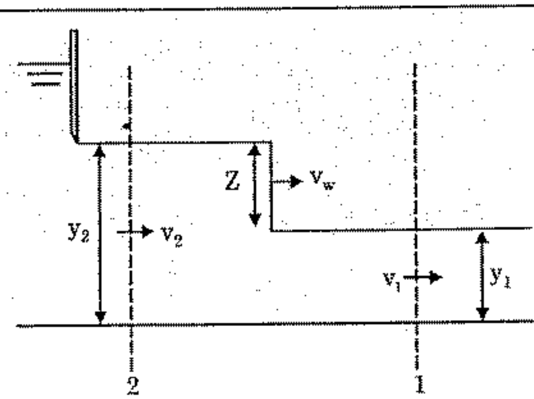
Surge type	Example
Positive surge moving down stream	Down stream to sluice gate when suddenly open.
Positive surge moving up stream	Upsteam to sluice gate when suddenly closed.
Negative surge moving upstream	Upstream to sluice gate when suddenly open.
Negative surge moving down stream	Down stream to sluice gate when suddenly closed.

ANALYSIS OF SURGES

- For the analysis of surges which is a rapidly varied unsteady flow, an equivalent steady flow situation is formed by considering the flow w.r.t the surge.

Case A – Positive surge moving downstream

- Consider a suitation as shown in figure below, flow depth at section 1 and 2 are y_1 and y_2 with velocities being v_1 and v_2. A wave of height Z (+ve surge) is moving downstream with velocity v_w.



Positive surge moving downstream

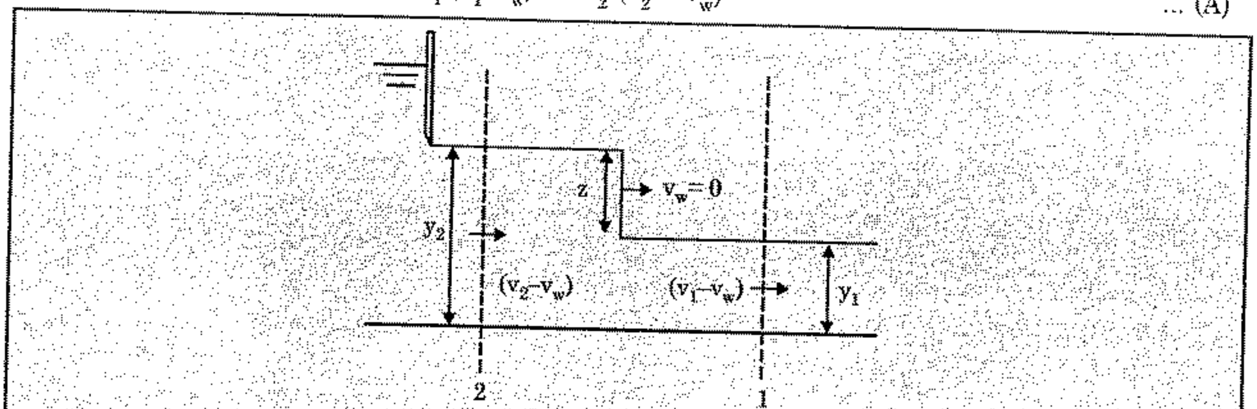
- If a negative velocity equal to $(-v_w)$ is super imposed on the flow field, the wave will appear to be stationary and flow field will convert to a steady state.

Note : Superimposing a negative velocity of $(-v_w)$ can be thought of as observing the flow field by taking surge as a reference frame.

- Now analysis of flow becomes much simplified and can be done in following steps.

Step-1 Apply continuity equation between section 1 and 2

$$A_1 (v_1 - v_w) = A_2 (v_2 - v_w) \quad \text{... (A)}$$



Simulated Steady flow

Step-2 Application of Momentum Equation

- Applying momentum equation between section 1 and 2 by neglecting frictional losses, and assuming hydrostatic pressure distribution at section 1 & 2.

$$P_1 - P_2 = M_2 - M_1$$

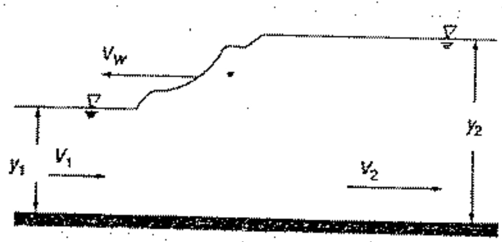
- As mass of flow entering into section 1 and 2 is same. Hence we can write either $\rho A_1 (v_1 - v_w)$ or $\rho A_2 (v_2 - v_w)$ as the mass of flow per unit time.

$$\gamma A_1 \bar{y}_1 - \gamma A_2 \bar{y}_2 = \rho A_1 (v_1 - v_w)\left[(v_2 - v_w) - (v_1 - v_w)\right] \quad \text{... (B)}$$

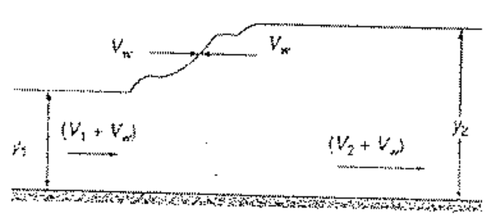
- With the help of equation A & B, y_1, y_2, Q, v_w can be found out.

Case B – Positive Surge Moving Upstream

- Consider a situation as shown in figure below, flow depth at section 1 and 2 are y_1 and y_2 with velocities being v_1 and v_2. A wave of height Z (+ve surge) is moving upstream with velocity v_w.



Positive Surge Moving Upstream



Simulated Steady Flow

Step-1 Apply continuity equation between section 1 and 2

$A_1 (v_1 + v_w) = A_2 (v_2 + v_w)$

Step-2 Application of Momentum Equation

- Applying momentum equation between section 1 and 2 by neglecting frictional losses, and assuming hydrostatic pressure distribution at section 1 & 2.

$$P_1 - P_2 = M_2 - M_1$$

$$\gamma A_1 \bar{y}_1 - \gamma A_2 \bar{y}_2 = \rho A_1 (v_1 + v_w)\left[(v_2 + v_w) - (v_1 + v_w)\right]$$

- With the help of equation A & B, y_1, y_2, Q, v_w can be found out.

CELERITY

The velocity of surge relative to the intial flow velocity in the canal is called as celerity of surge (C).

For surge moving down stream $\boxed{C = \upsilon_\omega - \upsilon_1}$

For surge moving up stream $\boxed{C = \upsilon_\omega + \upsilon_1}$

where, v_ω = velocity of surge wave

v_1 = Initial velocity of the stream

***Note :* Wave Moving Upstream** ← C

$$V_{wave\text{-}ground\ (u/s)} = V_{wave\text{-}water\ (u/s)} + V_{water\text{-}ground\ (u/s)}$$

$$V_w = C - V$$

$$C = V_w + V$$

Wave Moving Downstream C →

$$V_{wave\text{-}ground\ (d/s)} = V_{wave\text{-}water\ (d/s)} + V_{wave\text{-}ground\ (d/s)}$$

$$V_w = C + V$$

$$C = V_w - V$$

- If we put the equation of surges in momentum equation, of a rectengular channel we will get a expression

$$\boxed{C = \sqrt{\frac{1}{2} g \frac{y_2}{y_1}(y_1 + y_2)}}$$

where, y_1 and y_2 are intial and final height of flow.

- Surge height can be calculated as, $y_2 - y_1$.
- For a wave of small height $y_1 \cong y_2$ and droping suffixes from the above equation

$$\boxed{C = \sqrt{gy}}$$

- The above equation can be used to determine the celerity of stream when the wave height is negligible

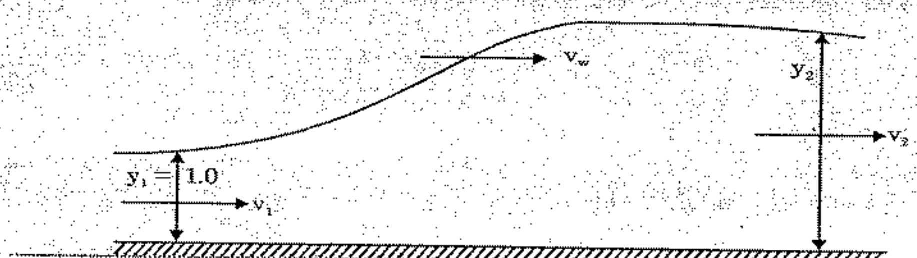
Note : A positive surge is stable and its shape is pressured. Whereas a negative surge is unstable and the shape of negative surge of various time interval will be different.
Tides in estuaries and tidal reivers causing a surge, usually called a bore, which is propagated upstream.

Example 1

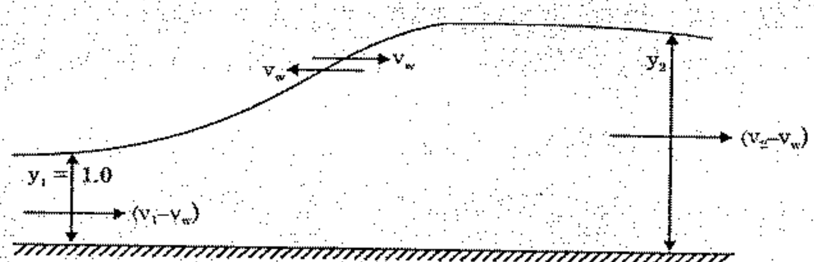
Water flows below a Sluice gate into a rectangular channel at a velocity of 0.60 m/sec and a depth of 1.0m. The discharge is suddenly increased to 3 times to its orignal value by opening the gate. Find the change in depth of flow.
Assume $g = 10.0\ m/sec^2$.

Sol.

Given: $y_1 = 1.0m$; $v_1 = 0.60 m/sec$ $g = 10.0 m/sec^2$



Positive Surge Moving Downstream



Simulated Steady Flow Condition

After opening of gate discharge has increased 3 times hence

$$v_2 y_2 = 3\ v_1\ y_1$$
$$v_2 y_2 = 3 \times 0.60 \times 1$$
$$v_2 y_2 = 1.80 m^3/sec$$

From Contunity Equation

$$(v_1 - v_w)\ y_1 = (v_2 - v_w)\ y_2$$
$$(0.60 - v_w)\ 1 = (v_2 y_2 - y_2 v_w)$$
$$v_w\ \ y_2 - v_w = 1.80 - 0.60$$
$$v_w\ \ (y_2 - 1) = 1.20$$
$$v_w = \frac{1.20}{y_2 - 1}$$

Applying Momentum Equation

$$\frac{1}{2}\gamma\left[y_1^2 - y_2^2\right] = \frac{\gamma}{g}(v_1 - v_w)(v_2 + v_w - v_1 - v_w)$$

$$\frac{9.81}{2}\left[1^2 - y_2^2\right] = \left(0.6 - \frac{1.20}{y_2 - 1}\right)\left(\frac{1.80}{y_2} - 0.60\right)$$

Solving by Hit and Trial

$$y_2 = 1.2745 \text{ m}$$

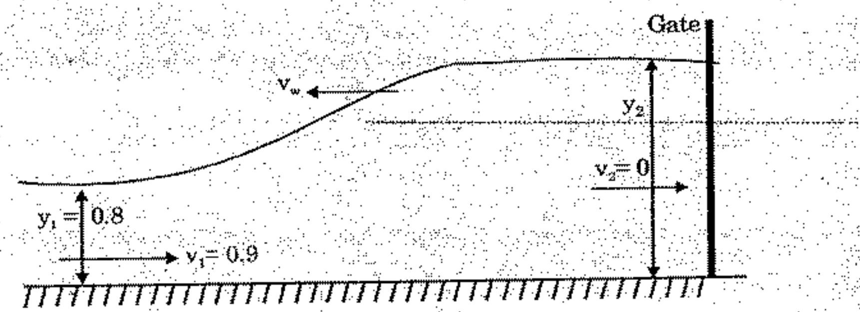
Height of Surge, $\Delta y = y_2 - y_1 = 1.2745 - 1$

$$\Delta y = 0.2745 \text{ m}$$

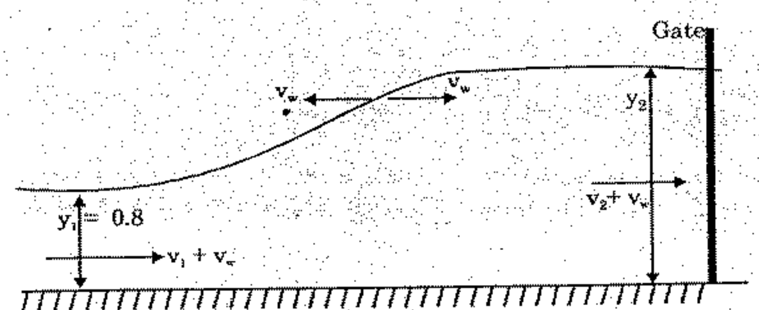
Example 2

Water flows as a depth of 0.8m of a velocity of 0.9 m/sec in wide rectangular channel in a reach immediately upstream of a control gate. The gate is suddenly closed completely. Determine the resulting surge conditions.

Sol. Given : $y_1 = 0.8;\ v_1 = 0.9;\ v_2 = 0$



Positive Surge Moving Downstream



Simulated Steady Flow Condition

Applying Contiunity Equation

$$(v_1 + v_w)\,0.8 = y_2\,(v_2 + v_w) \qquad [v_2 = 0]$$

$$(0.9 + v_w)\,0.8 = y_2\,(v_w)$$

$$0.72 + 0.8\,v_w = y_2\,v_w$$

$$v_w = \frac{0.72}{y_2 - 0.8}$$

Applying Momentum Equation

$$P_1 - P_2 = M_2 - M_1$$

$$\frac{1}{2}\gamma y_1^2 - \frac{1}{2}\gamma y_2^2 = \rho\,(v_1 + v_w)\, y_1\, (v_2 + v_w - v_1 - v_w)$$

$$\frac{1}{2}\times g\left[y_1^2 - y_2^2\right] = (0.9 + v_w)\ 0.8\ (-0.9)$$

$$-6.813\ [0.64 - y_2^2] = \left(0.9 + \frac{0.72}{y_2 - 0.8}\right)$$

$$-6.813\left[0.64 - y_2^2\right] = \left[\frac{0.9y_2}{y_2 - 0.8}\right]$$

Solving by Hit and Trial Method,

$$y_2 = 1.075$$

$$\text{Height of surge} = y_2 - y_1 = 1.075 - 0.8 = 0.275$$

$$\text{Velocity of surge } v_w = \frac{0.72}{y_2 - 0.8}$$

$$v_w = \frac{0.72}{1.075 - 0.8}$$

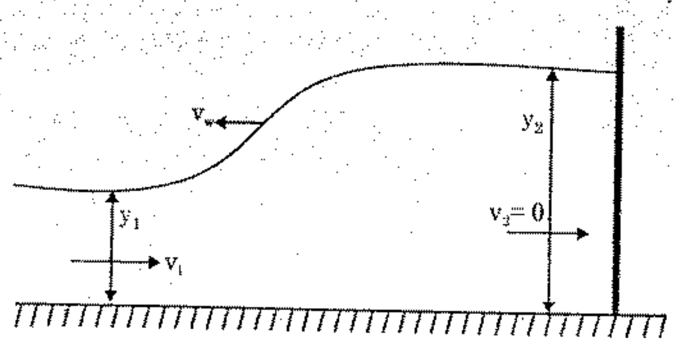
$$v_w = 2.618 \text{ m/sec}$$

Example 3

The depth and velocity of flow in a rectangular channel are 0.9 m and 1.5 m per sec respectively. If a gate at the down stream end of the channel is abruply closed, what will be the height of absolute velocity resulting surge.

Sol. Given: $y_1 = 0.9$ m; $v_1 = 1.5$ m/sec

Let v_w be the velocity of positive surge moving up stream

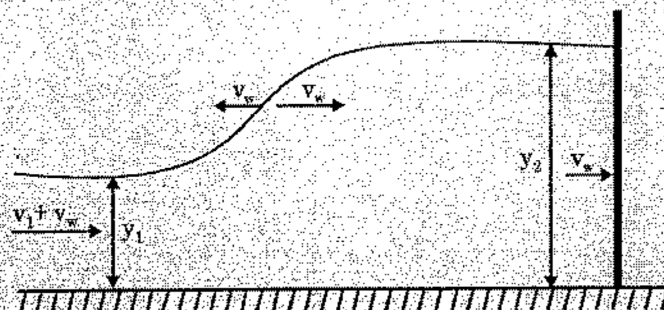


Positive Surge Moving Upstream

From contiunity equation

$$y_1\ (v_1 + v_w) = y_2\ (v_2 + v_w)$$

$$0.9\ (1.5 + v_w) = y_2\ v_w$$



Simulated steady flow

$$v_w = \frac{0.9 \times 1.5}{y_2 - 0.9}$$

$$v_w = \frac{1.350}{y_2 - 0.9}$$

Applying Momentum Equation

$$P_1 - P_2 = M_2 - M_1$$

$$\frac{1}{2}\gamma y_1^2 - \frac{1}{2}\gamma y_2^2 = \rho y_1 (v_1 + v_w)[v_w - v_1 - v_w]$$

$$\frac{1}{2}\gamma\left[y_1^2 - y_2^2\right] = \rho(v_1 + v_w)(-v_1)y_1$$

$$\frac{g}{2}\left[y_1^2 - y_2^2\right] = v_1 y_1 (v_1 + v_w)$$

$$-3.633\left[0.9^2 - y_2^2\right] = 1.5 + \frac{1.350}{y_2 - 0.9}$$

Solving by hit and trial

$$y_2 = 1.401$$

Height of absolute velocity resulting surge $= y_2 - y_1$

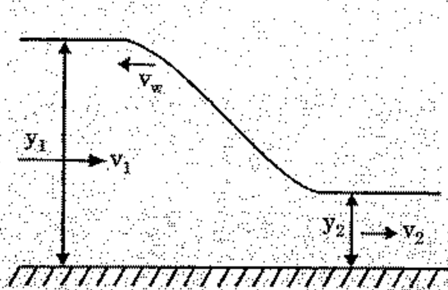
$= 1.401 - 0.9$

$= 0.501$ m

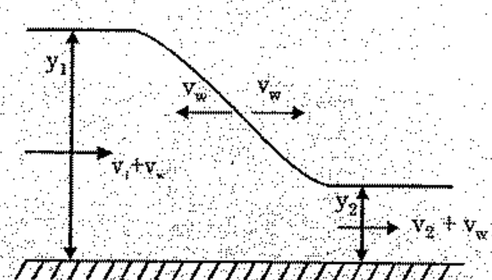
Example 4

A rectangular horizontal channel of 3m wide and 2m depth conveys water at 18 m^3/sec. If the flow rate is suddendly reduced to $\frac{2}{3}$ of its orignal value, compute the magnitude and speed of upstream ward surge?

Sol. Given: B = 3 m; y_1 = 2m; Q = 18m^3/sec



Positive Surge Moving Upstream



Simulated Steady Flow

$$v_1 = \frac{Q}{By_1} = \frac{18}{3\times 2} = 3\text{m/sec}$$

$$v_2 y_2 = \frac{2}{3}\frac{Q}{B}$$

$$v_2 = \frac{2}{3}\times\frac{18}{3\times y_2}$$

$$v_2 = \frac{4}{y_2}$$

Applying Contunity Equation

$$y_1 (v_1 + v_w) = y_2 (v_2 + v_w)$$

$$2 (3 + v_w) = y_2\left(\frac{4}{y_2} + v_w\right)$$

$$6 + 2 v_w = 4 + v_w y_2$$

$$v_w = \frac{2}{y_2 - 2}$$

Applying momentum equation

$$P_1 - P_2 = M_2 - M_1$$

$$\frac{1}{2}\gamma y_1^2 - \frac{1}{2}\gamma y_2^2 = \rho y_1 (v_1 + v_w) [v_2 - v_1]$$

$$\frac{g}{2y_1}\left[y_1^2 - y_2^2\right] = \left[3 + \frac{2}{y_2 - 2}\right]\left[\frac{4}{y_2} - 3\right]$$

$$\frac{9.81}{2\times 2}\left[4 - y_2^2\right] = \left[3 + \frac{2}{y_2 - 2}\right]\left[\frac{4}{y_2} - 3\right]$$

Solving by Hit and Trial

$$y_2 = 2.751$$

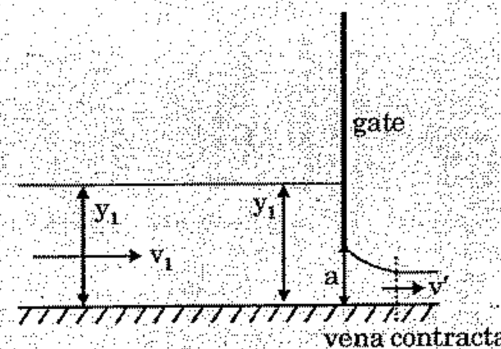
$$v_w = \frac{2}{y_2 - 2} = \frac{2}{2.751 - 2} = 2.663$$

$$\text{Height of jump} = y_2 - y_1$$

$$= 2.751 - 2$$

Example 5

Water is flowing at a depth of 1.3 m and the velocity of 0.95 m/sec when controlled by a sluice gate at a downstream location. The gate is abruptly dropped 0.14 m. Find the resulting depth on upstream side of gate. The coefficient of contraction under gate is 0.61.



Sol. Given:

$y_1 = 1.3$ m

$v_1 = 0.95$ m

We know that discharge per unit width through verna contracta will be

$$q = C_c a\sqrt{2gy_1}$$

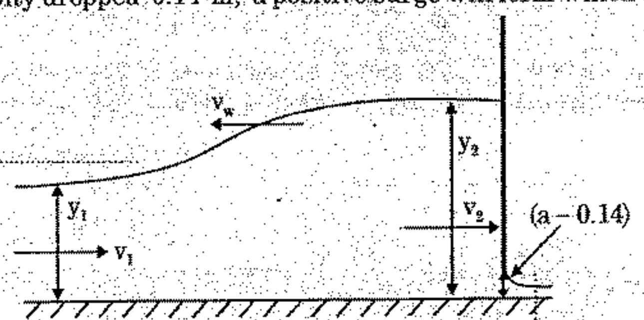
Applying contunity equation

$$v_1 y_1 = C_c a\sqrt{2gy_1}$$

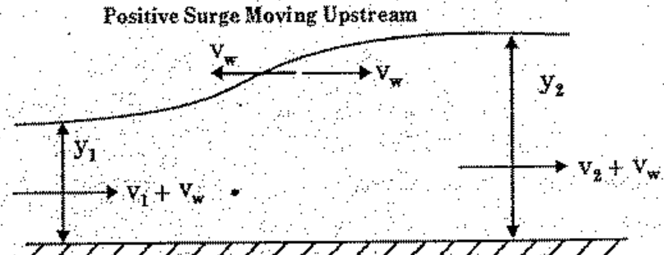
$$0.95 \times 1.3 = 0.61 \cdot a\sqrt{2\times9.81\times1.3}$$

$$a = 0.401 \text{ m}$$

As the gate is abruplty dropped 0.14 m, a positive surge will form which will travel upstream.



Positive Surge Moving Upstream



Stimulated Steady Flow

$$v_2 y_2 = C_c(a - 0.14)\sqrt{2gy_2}$$

$$v_2 y_2 = 0.61(0.401 - 0.14)\sqrt{2\times9.81y_2}$$

$$v_2 y_2 = 0.705\sqrt{y_2}$$

$$v_2 = \frac{0.705}{\sqrt{y_2}}$$

Analysis of positive surge

Applying Contunity Equation

$$(v_1 + v_w)y_1 = (v_2 + v_w)y_2$$

$$(0.95 + v_w)1.3 = (v_2 y_2 + v_w y_2)$$

$$1.235 + 1.3v_w = 0.705\sqrt{y_2} + v_w y_2$$

$$v_w = \frac{1.235 - 0.705\sqrt{y_2}}{y_2 - 1.3}$$

Applying Momentum Equation

$$\gamma A_1 \bar{y}_1 - \gamma A_2 \bar{y}_2 = \rho Q\left[(v_2 + v_w) - (v_1 + v_w)\right]$$

$$\frac{g}{2}\left(y_1^2 - y_2^2\right) = (v_1 + v_w)y_1[v_2 - v_1]\ldots$$

$$4.905\left[1.3^2 - y_2^2\right] = \left(0.95 + \frac{1.235 - 0.705\sqrt{y_2}}{y_2 - 1.3}\right)1.3\left[\frac{0.705}{\sqrt{y_2}} - 0.95\right]$$

Solving by trail and error method

$$y_2 = 1.435\text{m}$$

OBJECTIVE QUESTIONS

1. A partially open sluice gate is suddenly raised to its full opening. The resulting surge waves at the gate are
   (a) A positive wave travels towards the gate from the upstream side while a negative wave travels downstream from the gate
   (b) A positive wave travesls from the gate onto the upstream side while a positive wave travelos downstream form the gate
   (c) A negative wave travels from the gate onto the upstream side while a positive wave travels downstream for the gate
   (d) A negative wave travels towards the gate from the upstream side whereas a positive wave travels downstream from the gate
2. Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

	List-I		List-II
A.	Positive surge travelling upstream	1.	Occurs on upstream of gate that is partly closed suddenly.
B.	Positive surge travelling downstream	2.	Occurs on downstream of gate that is partly closed suddenly.
C.	Negative surge travelling upstream	3.	Occurs on upstream of gate that is opened suddenly.
D.	Negative surge travelling downstream	4.	Occurs on downstream of gate that is partly opened suddenly.

Codes :

	A	B	C	D
(a)	1	4	3	2
(b)	1	2	3	4
(c)	3	4	1	2
(d)	3	4	2	1

3. Match **List-I** with **List-II** and select the correct answer using the codes given below the lists :

	List-I		List-II
A.	Free over fall	1.	Celerity
B.	Standing wave flume	2.	Clinging nappe
C.	Surge	3.	Hydraulic jump
D.	Weir	4.	Control section

Codes :

	A	B	C	D
(a)	3	4	1	2
(b)	4	3	1	2
(c)	4	3	2	1
(d)	[illegible]	[illegible]	[illegible]	1

4. A positive surge of height 0.50 m was found to occur in a rectangular channel with a depth of 2.0m. The celerity of the surge is in m/s

(a) $\pm$ 4.43 (b) $\pm$ 2.25

(c) $\pm$ 1.25 (d) $\pm$ 5.25

5. In a wide rectangular channel, the small surface waves caused due to disturbance by a suddenly thrown heavily weighted log of wood, thrown parallel to the cross section, were seen to move at 4.2 m/s downstream and 1.4 m/s upstream (with reference to the banks). The depth of flow and the mean flow velocity are, respectively, nearly

(a) 0.2 m and 1.4 m/s (a) 0.3 m and 1.4 m/s

(a) 0.2 m and 1.5 m/s (a) 0.3 m and 1.5 m/s

6. If the depth of the flow in a channel is 1.0 m and velocity of flow is 2m/s, what is the velocity with which an elementary wave can travel upstream?

(a) 1.13 m/s (b) 2.0 m/s

(c) 3.25 m/s (d) 5.65 m/s

7. In a rectangular channel, flow happens at depth 1.35 m and velocity 2.1 m/s. What will be the absolute velocity of an elementary wave moving upstream?

(a) 1.54 m/s (b) 2.10 m/s

(c) 3.64 m/s (d) 5.74 m/s

8. A wide channel is 1m deep and has a velocity of flow, V, as 2.13 m/s. If a disturbance is caused, an elementary wave can travel upstream with a velocity of

(a) 1.00 m/s (b) 2.13 m/s

(a) 3.13 m/s (a) 5.26 m/s

9. A canal has a velocity of 2.5 m/s and a depth of flow 1.63 m. A negative wave formed due to a decrease in the discharge at an upstream control moves at this depth with a celerity of

(a) + 6.5 m/s (b) – 6.5 m/s

(c) + 1.5 m/s (d) – 4.0 m/s

Consider the following statements:

Of these statements:

(a) both A and R are true and R is the correct explanation of A

(b) both A and R are true but R is not a correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

10. **Assertion (A):** Surges can be positive or negative.

Reason (R): Negative surges may occur when a gate at the head of a channel is suddenly opened or when a gate at tail end of a channel is suddenly closed.

11. **Assertion (A) :** In a running irrigation canal, regulating gates are installed at intermediate locations for control purposes. One such regulating gate is partially closed. Such a movement gives rise to a positive surge travelling downstream and a negative surge travelling upstream of the gate.

Reason (R) : Any sudden change in the intermediate gate position suddenly changes the discharge both upstream and downstream of the gate. This sudden change in discharge gives rise to sudden changes in water elevations both upstream and downstream of the gate and develops in the form of an unsteady, rapidly varying flow which propagates in the form of a surge wave moving on either side of the gate.

ANSWERS

1. (c)	**2.** (a)	**3.** (b)	**4.** (d)	**5.** (a)	**6.** (a)	**7.** (a)	**8.** (a)	**9.** (d)	**10.** (c)
11. (a)									

HINT

4.

$$\Delta y = y_2 - y_1$$

$$0.5 = y_2 - 2.0$$

$$y_2 = 2.5$$

$$C = \sqrt{\frac{1}{2} g \frac{y_2}{y_1}(y_1 + y_2)} = \sqrt{\frac{1}{2} \times 9.81 \times \frac{2.5}{2.0}(2.5 + 2.0)} = \pm 5.25 \text{ m}$$

5.

$$v_{w_{u/s}} = v_f - C$$

$$1.4 = v_f - C \quad \text{... (1)}$$

$$v_{w_{d/s}} = v_f + C$$

$$4.2 = v_f + C \quad \text{... (2)}$$

Subtracting (1) from (2)

$$2.8 = 2C$$

$$C = 1.4 \text{ m/sec}$$

$$C = \sqrt{gy}$$

$$y = \frac{1.4^2}{9.81}$$

$$y = 0.200 \text{ m}$$

6.

$$C = \sqrt{gy}$$

$$C = \sqrt{9.81 \times 1} = 3.13$$

$$v_w = C - v = 3.13 - 2 = 1.13 \text{ m/sec}$$

7.

$$C = \sqrt{gy} = \sqrt{9.81 \times 1.35} = 3.639$$

$$v_w = C - v = 3.639 - 2.1 = 1.539 \text{ m/sec}$$

8.

$$C = \sqrt{gy} = \sqrt{9.81 \times 1} = 3.13 \text{ m/sec}$$

$$v_w = C - v = 3.13 - 2.13 = 1.0 \text{ m/sec}$$

9.

$C = \sqrt{gy} = \sqrt{9.81 \times 1.63} = 3.999$ m/sec, negative sign is for direction