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CIVIL ENGINEERING

For

UPSC Engineering Services Examination, GATE,

State Engineering Service Examination & Public Sector Examination.

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PART-I

1

Limit State Method

INTRODUCTION

The philosophy of the limit state method of design represents a definite advancement over the traditional design philosophies. Unlike working stress method, which is based on calculations on service load conditions alone, and unlike ultimate load method, which is based on calculations on ultimate load conditions alone, limit state method aims for a comprehensive and rational solution to the design problem, by considering *safety* at ultimate loads and *serviceability* at working loads.

The limit state philosophy uses a partial safety factor format which attempts to provide adequate safety at ultimate loads as well as adequate serviceability at service loads by considering all possible 'limit state'.

WHAT IS LIMIT STATE METHOD

- The acceptable limit for the safety and serviceability requirement of a structure or structural element before failure occurs is called limit state.
- In this structure is so designed that it carry the loads with sufficient degree of safety and serviceability and structure will not become unfit for use for which it is to be designed.

Two main limit state is

Limit state of collapse

To satisfy this limit state, the strength must be adquate to carry the loads.

This includes

- Flexure
- Compression
- Torsion
- Shear

Limit state of serviceability

To satisfy this limit state, deflection, cracking and vibration must not be excessive.

- Excessive deflection can reduce the efficiency of the structure and must be avoided.
- Cracking causes ingress of water

CHARACTERSTIC STRENGTH OF MATERIALS

The strength of material below which not more than 5% of the test results are expected to fall is known as the characteristic strength of the material and ...

$$f_{ck} = f_m - 1.65\,\delta$$

$$\delta = \sqrt{\frac{\Sigma(X-\bar{X})^2}{n-1}}$$

f_m = Mean of strength

δ = Standered devation

f_{ck} = Characteristic strength of material

Table 1.1: Specified characteristic compressive strength of concrete at 28 days

Grade designation	Specified characteristic compressive strength at 28 days N/mm²
M10	10
M15	15
M20	20
M25	25
M30	30
M35	35
M40	40

Note:

1. In the designation of a concrete mix, letter M refers to the mix and the number to the specified characteristic compressive strength of 15-cm cube at 28 days, expressed in N/mm².
2. M5 and M7.5 grades of concrete may be used for lean concrete bases and simple foundation for masonry walls. These mixes need not be designed.
3. Grade of concrete lower than M20 shall not be used in reinforced concrete.
4. For sea water grade of concrete lower than M30 shall not be used in reinforced concrete.

CHARACTERISTIC LOAD

Value of load which has a 95% probability of not being exceeded during the life of the structure is known as characteristic load and is denoted by F

$F = F_m + 1.65\,\delta$

F_m = mean value of load

$(f_m - 1.65\,\delta)$ and $(F_m + 1.65\delta)$ are two important limit within which "probability of lying test result" is maximum. These limit called confidence limit.

PARTIAL SAFETY OF FACTOR

It is called partial safety of factor because it is applied for both one for load and one for materials.

γ_{ms} for concrete = 1.5

γ_{ms} for steel = 1.15

Table 1.2: Load combinations.

S. No.	Discription	Collapse	Servicability
1.	D.L + L.L	1.5	1
2.	D.L + (W.L) or (E.L) combination		
	(i) for Normal case		
	D.L + W.L(or E.L)	1.5	1
	(ii) For checking stability against overturning/stress reversal		
	D.L + W.L (or E.L)	0.9	1
3.	(D.L) + L.L + W.L(or E.L) combination		
	D.L	1.2	1
	L.L	1.2	0.8
	W.L (or E.L)	1.2	0.8

Design Values are obtained when partial safety factore are applied to characteristic load and materials.

1. **Materials:** The design strength of the materials f_d is given by

$$f_d = \frac{f_{ck}}{\gamma_{ms}}$$

f_d = characteristic strength of the Material

γ_{ms} = partial safety of factor for mateiral

2. **Loads:** The design load F_d is given by

$$F_d = F\gamma_f$$

F = Characteristic load

γ_f = Partial safety of factor for load

Table 1.3: Indian standard specifications for loads.

Characteristic Loads	Indian Standard Specification
Dead loads	IS : 857 (Part 1) : 1987
Imposed loads	IS : 857 (Part 2) : 1987
Wind loads	IS : 857 (Part 3) : 1987
Seismic loads	IS : 1893 : 1984

ASSUMPTION LIMIT STATE OF COLLAPSE: FLEXURE

1. **Plane Section before Bending Remains Plane even after the Bending**

This assumption mean that strain at any point on the cross-section is directly proportional to its distance from its neutral axis, it mean strain diagram is linear.

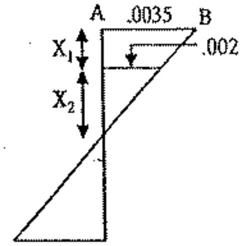


Fig. 1.1

2. **The maximum strain in concrete at the outermost compression fibre is taken as 0.0035 in bending.**

   The maximum strain in concrete in compression will be in fibre AB, (Fig. (1.1)) i.e., in the topmost fibre, and its value will be limited to 0.0035 according to this assumption. Concrete has very low ductility and so it crushes in compression at such low strain only.

3. **For design purpose, the compressive strength of concrete shall be assumed to be 0.67 times the charatristic strengtb and partial factor of saftey γ_{ms} = 1.5 shall be applied in addition to this.**

   It may be noted that for the design of flexural members the characteristic strength of concrete is taken as 0.67 f_{ck} instead of f_{ck}. This is on account for the fact that concrete in actual structure has a strength of 0.67 f_{ck}, under laboratory condition strength is f_{ck}.

   The variation of strain-stress curve shall be parabolic upto 0.002 strain and, thereafter, the stress remains constant upto the maximum permissible strain of 0.0035. At limit state, this is an idealised curve for concrete in compression and is valid for all grades of concrete irrespective of percentage of tensile reinforcement.

   Design compressive stress in concrete may be taken as f_d, where

$$f_d = \frac{0.67 f_{ck}}{\gamma_{ms}} = \frac{0.67 f_{ck}}{1.5} = 0.45 f_{ck}$$

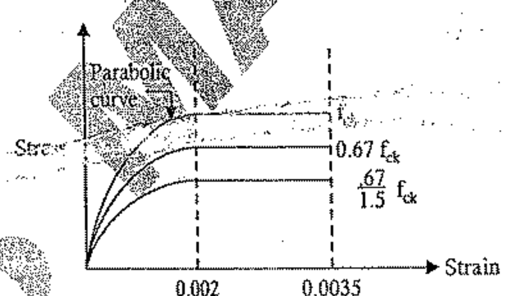


Fig. 1.2

4. **The tensile strength of concrete is ignored.**

   The cracked concrete does not contribute towards enhancement [illegible] resistance of the section except providing bond between concrete and steel for developing tensile strain and thus stress in steel.

5. **For design purpose partial factor of safety for steel γ_{ms} = 1.15 and the stress in steel are derived from stress-strain curve.**

   A nominal strain-stress curve for mild steel bars from test may be obtained a shown in Fig.1.4(a). The strain is obtained by dividing change in length by actual length; whereas stress is determined as load divided by original cross sectional area*. This strain-stress curve is simplified to a curve as shown in Fig. 1.3(a). Similarly the strain-stress curve for cold worked deformed bars of Grade Fe 415 and Fe 500 which are, generally, in use in India is as shown in Fig. 1.4(b). This is idealized to a curve of Fig. 1.3(b). For all types of steel, modulus of elasticity, $E = 2 \times 10^5$ N/mm^2 and strain can be infinite at f_y. ***The strain-stress relationship for steel in tension and in compression is assumed to be the same.***

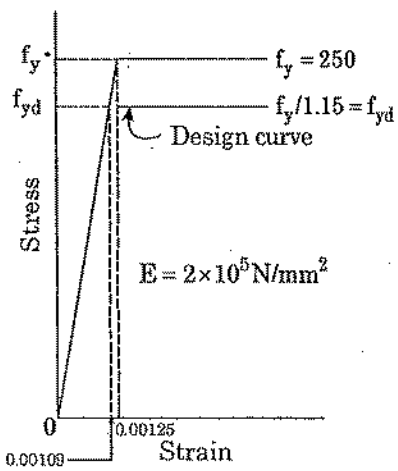


Fig. 1.3(a): Strain-stress curve for mild steel bars (grade Fe 250).

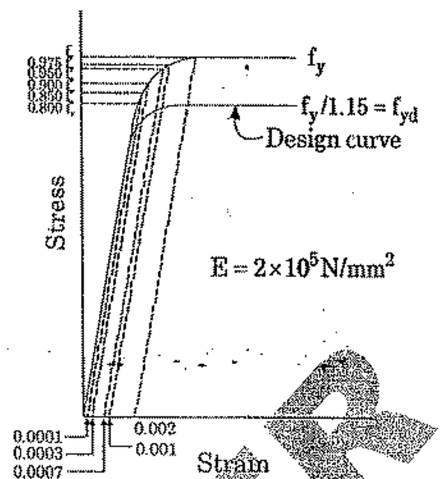


Fig. 1.3(b): Strain-stress curve for cold worked deformed bars (grade Fe 415 and Fe 500).

*Original cross section area is defined as the area of cross section at zero stress.

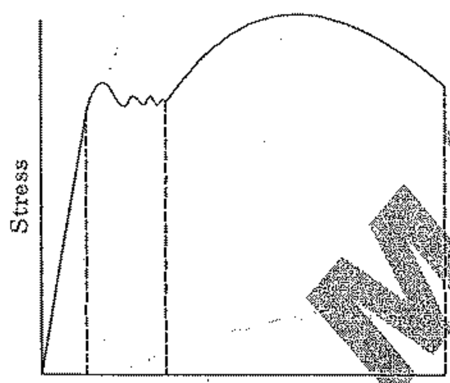


Fig. 1.4(a): Nominal strain-stress curve for mild steel bars.

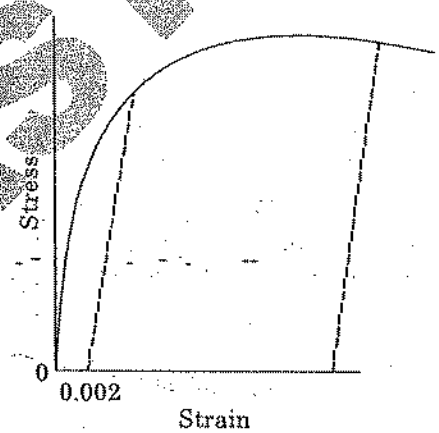


Fig. 1.4(b): Nominal strain-stress curve for cold worked deformed bars.

Note:

$$\text{Design stress for steel} = \frac{\text{Permessible stress}}{\text{Partial safety factor}} = \frac{f_y}{1.15} = 0.87 f_y$$

$$\text{for Fe 250} = 0.87 \times 250 = 257.0 \text{ N/mm}^2$$

$$\text{Design stress for concrete} = \frac{\text{Permissible stress}}{\gamma_{mc}} = \frac{f_{ck}}{1.5} = 0.67 f_{ck}$$

Total strain at yield (ϵ_y) for mild steel will be $\frac{f_y}{E_s} = \frac{250}{2\times10^5} = 0.00125$; whereas total strain at **design** yield stress, $\epsilon_{yd} = \frac{f_y}{\gamma_m E_s} = \frac{250}{1.15\times2\times10^5} = 0.00109$. In a similar manner total strains ϵ' and ϵ'_d, at any stress, say 0.9 f_y and 0.9 f_{yd}, for cold-worked deformed bars of grade Fe 415 can be evaluated as under :

$$\epsilon' = \frac{0.9 f_y}{E_s} + 0.0003 = \frac{0.9\times415}{2\times10^5} + 0.0003 = 0.0022$$

where 0.0003 is the inelastic strain at 0.9 f_y (figure 1.3b) and

$$\epsilon'_d = \frac{0.9 f_y}{\gamma_m E_s} + 0.0003 = \frac{0.9\times415}{1.15\times2\times10^5} + 0.0003 = 0.00192$$

Note: In a flexural member, where compression reinforcement is provided, the strain at the c.g. of compression reinforcement is calculated from strain diagram and the corresponding stress is determined as explained above. For ready reference, typical points on **design** strain-stress curves for all types of steel bars are given in Table 1.4.

Table 1.4: Typical points on the design strain-stress curves.

Stress level	*For f_y = 250 MPa*		*For f_y = 415 MPa*		*For f_y = 500 MPa*	
	Strain	*Stress*	*Strain*	*Stress*	*Strain*	*Stress*
0.80 f_{yd}	0.00087	173.9	0.00144	288.7	0.00174	347.8
0.85 f_{yd}	0.00093	184.8	0.00163	306.7	0.00195	369.6
0.90 f_{yd}	0.00098	195.7	0.00192	324.8	0.00226	391.3
0.95 f_{yd}	0.00104	206.5	0.00241	342.8	0.00277	413.0
0.975 f_{yd}	0.00106	212.0	0.00276	351.8	0.00312	423.9
1.0 f_{yd}	0.00109	217.4	0.00380	360.9	0.00417	434.8

6. Maximum strain in Tension reinforcement in the section at failure shall not be less than $\frac{f_y}{1.15E_S} + 0.002$.

The above value of strains for different grades of steel are as follows:

(i) for Fe 250, $\epsilon_{st} \nless \frac{250}{1.15\times2\times10^5} + 0.002$ or $\epsilon_{st} \nless 0.0031$

(ii) for Fe 415, $\epsilon_{st} \nless \frac{415}{1.15\times2\times10^5} + 0.002$ or $\epsilon_{st} \nless 0.0038$ and

(iii) for Fe 500, $\epsilon_{st} \nless \frac{500}{1.15\times2\times10^5} + 0.002$ or $\epsilon_{st} \nless 0.0042$

The minimum strains at collapse ensures that the design stress in tensile steel at collapse

Elaborating this assumption further, if the limiting values of strains, i.e., a maximum value of strain 0.0035 in concrete and a minimum value of strain 0.0031 in steel are taken; the limiting value of $x_u(x_{u,\,lim})$ can be calculated from similarity of Ds of strain diagram (Fig. 1.5) as

$$\frac{0.0035}{0.0031} = \frac{x_{u,\text{lim}}}{d - x_{u,\text{lim}}} \quad \text{or} \quad x_{u,\,\text{lim}} = 0.53\ \text{d} \quad \text{(for Fe 250)}$$

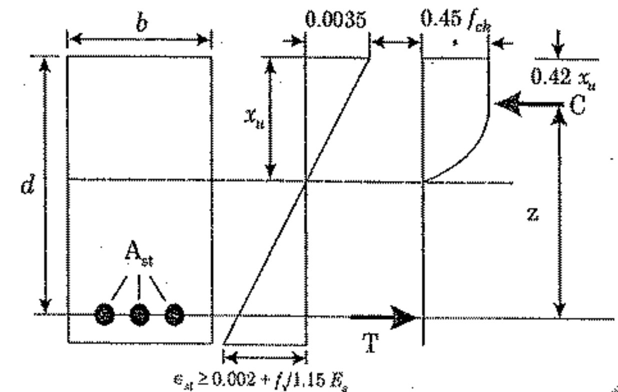


Fig 1.5: Strain and stress diagram for a cross-section

In non-dimensional form, $\frac{x_{u,\text{lim}}}{d} = 0.53$

$x_{u,\,lim}$ is the same as the maximum value of x_u, because the strain in concrete is the maximum, $\epsilon_{cu,\,max}$; whereas the strain in steel is the minimum, $\epsilon_{stu,\,max}$ Hence,

$$\frac{x_{u,\max}}{d} = \frac{x_{u,\text{lim}}}{d} = 0.53 \text{ for mild steel (Fe 250)}$$

Thus, if for mild steel, $\frac{x_{u,\max}}{d}$ exceeds 0.53, it will mean that the steel has not yielded. In other words, the strain in steel at collapse is less than 0.0031 violating the assumption (vi). It is clear, therefore, that if $\frac{x_{u,\max}}{d} > 0.53$, the analysis cannot be proceeded further unless the effective depth, d, of the beam is increased to limit the value of to $\frac{x_u}{d}$ to 0.53.

This assumption restricts the depth of neutral axis.

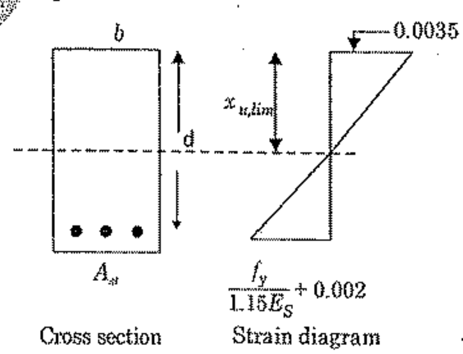


Fig. 1.6

From strain diagram

$$\frac{0.0035}{x_{u,\lim}} = \frac{\frac{f_y}{1.15E_s}+0.002}{d-x_{u,\lim}}$$

$$\frac{d-x_{u,\lim}}{x_{u,\lim}} = \frac{\frac{f_y}{1.15\,E_s}+0.002}{0.0035}$$

$$\frac{d}{x_{u,\lim}}-1 = \frac{\frac{f_y}{1.15\,E_s}+0.002}{0.0035}$$

$$\frac{d}{x_{u,\lim}} = \frac{\frac{f_y}{1.15E_s}+0.002}{0.0035}+1 = \frac{\frac{f_y}{1.15E_s}+0.002+0.0035}{0.0035}$$

$$= \frac{\frac{f_y}{1.15E_s}+0.0055}{0.0035}$$

$$\frac{x_{u,\lim}}{d} = \frac{0.0035}{\frac{f_y}{1.15E_s}+0.0055}$$

Put $E_s = 2 \times 10^5$ N/mm²

$$= \frac{0.0035}{\frac{0.87f_y+0.0055\times E_s}{E_s}}$$

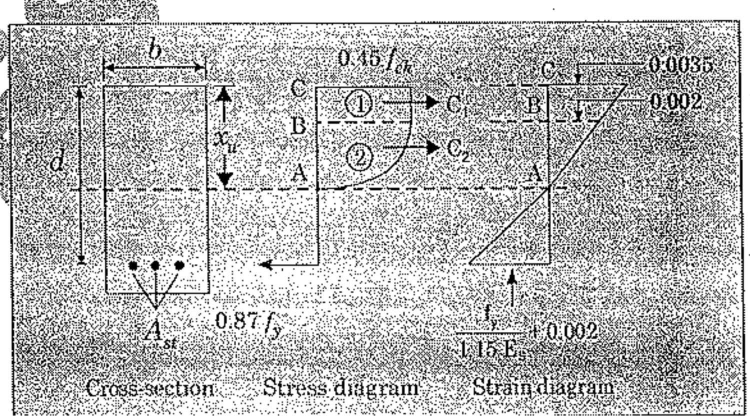
$$\boxed{\frac{x_{u,\lim}}{d} = \frac{0.0035\times E_s}{0.87f_y+.0055E_s} = \frac{700}{0.87f_y+1100}}$$

$\frac{x_{u,\lim}}{d}$ for 250 = 0.53

415 = 0.48

500 = 0.46

ANALYSIS AND STUDY OF STRESS BLOCK (IES)



From strain diagram (similar triangle)

$$\frac{0.0035}{AC} = \frac{0.002}{AB}$$

$$AB = \frac{0.002}{0.0035} AC$$

$$= \frac{2}{3.5} x_u$$

$$\boxed{AB = \frac{4}{7} x_u}$$

$$BC = AC - AB$$

$$= x_u - \frac{4}{7} x_u$$

$$\boxed{BC = \frac{3}{7} x_u}$$

Compression force of rectangular portion = Area of stress diagrame × Width of the section

$$C_1 = 0.45 f_{ck} \cdot \frac{3}{7} x_u . b$$

$$\boxed{C_1 = 0.193\, f_{ck} x_u . b}$$

This force will act at a distant $\boxed{y_1 = \frac{3}{14} x_u}$ from the top

Compression force of parabolic portion = Area of stress dia × Width of the section

$$= \frac{2}{3} \text{ base} \times \text{Height} \times \text{Width of the section}$$

$$= \frac{2}{3} \times 0.45\, f_{ck} \cdot \frac{4}{7} x_u b$$

$$\boxed{C_2 = 0.171\, f_{ck} . x_u . b}$$

This will act at a distant

$$y_2 = \left(\frac{3}{7} x_u + \frac{3}{8} \times \frac{4}{7} x_u \right)$$

$$= \frac{3}{7} x_u + \frac{12}{56} x_u$$

$$= \frac{24\, x_u + 12 x_u}{56}$$

$$= \frac{36}{56} x_u$$

$$\boxed{y_2 = \frac{9}{14} x_u} \text{ from top}$$

This combined compressive force will act at a $\bar{y}$ dist from top

$$\text{C.G. of total force } \bar{y} = \frac{C_1 y_1 + C_2 y_2}{C_1 + C_2}$$

$$\frac{0.193 f_{ck} x_u . b \times \frac{3}{14} x_u + 0.171 . f_{ck} x_u . \frac{9}{14} x_u}{0.193 f_{ck} x_u . b + 0.171 f_{ck} x_u . b}$$

$$\bar{y} = 0.42\, x_u$$

∴ Lever arm distant between centroid of compressive force to centroid of tension

$$\text{force } z = d - \bar{y}$$

$$\boxed{z = d - 0.42 x_u}$$

Total compressive force

$$C = C_1 + C_2$$

$$= 0.193\, f_{ck} x_u . b + 0.171\, f_{ck}\, x_u . b$$

$$\boxed{C = 0.36 f_{ck} x_u . b}$$

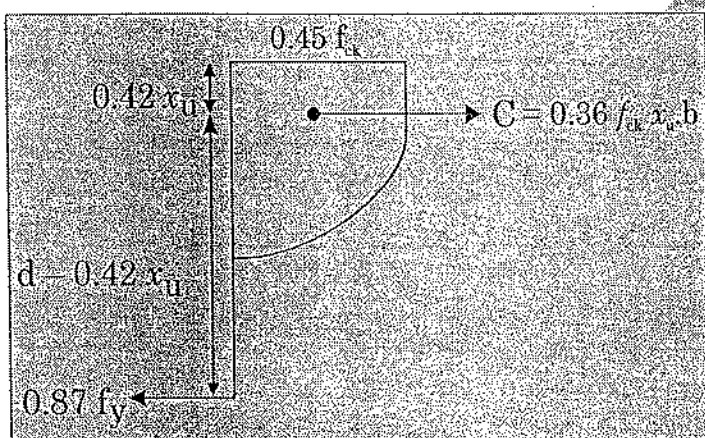


Fig 1.8: Stress block diagram.

ANALYSIS AND DESIGN

Three types of problems may arise while analysing and designing beams of rectangular cross-section without compression reinforcemtn (singly reinforced section).

(a) To determine moment of resistance (M_u) when cross-section of a beam is known;

(b) To determine steel area in tension when *concrete* cross-section and applied moment are known; and

(c) To design a cross-section for a bending moment.

(a) To Determine Moment of Resistance (M_u) When Cross-section of a Beam is Known

First of all the depth of neutral axis (x_u) is calculated assuming that, on application of bending moment M_u, the strain in outermost compression fibre of concrete has reached a value of 0.0035 and strain in tensile steel is not less than $0.002 + \frac{f_y}{1.15 E_s}$. In other words, the maximum stress in concrete at the top most fibre

is 0.45 f_{ck} and the stress in tensile steel is f_y ... (Figure)

From equilibrium of horizontal forces on a section

$$\text{Total compressive force (C)} = \text{Total tensile force (T)}$$

$$\text{or} \quad 0.36 f_{ck}\, x_u b = \frac{f_y}{1.15} A_{st} = 0.87 f_y A_{st}$$

$$\text{or} \quad x_u = \frac{0.87\, f_y A_{st}}{0.36 f_{ck} b} \qquad \text{---(1)}$$

For analysis purposes, it is convenient to express x_u in non-dimensional form as follows:

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b d} \qquad \text{---(2)}$$

(i) If $\frac{x_u}{d} = \frac{x_{u,\text{lim}}}{d}$, (the limiting value) the limiting value of moment of resistance may then be deducted as given below: Referring Figure below.

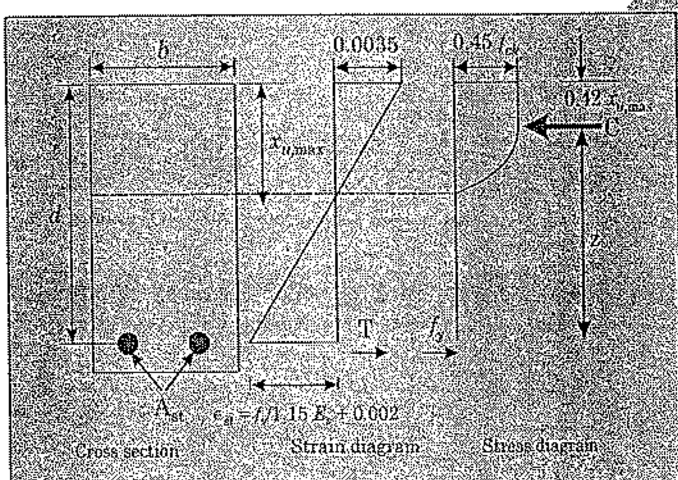


Fig. 1.8(a)

$$M_{u,\text{lim}} = \text{Max. compressive force} \times \text{Lever arm}$$

$$= 0.36 f_{ck} x_{u,\text{lim}}\, b(d - 0.42\, x_{u,\text{lim}})$$

$$= 0.36 \frac{x_{u,\text{lim}}}{d}\left(1 - 0.42 \frac{x_{u,\text{lim}}}{d}\right) b d^2 f_{ck}$$

Alternatively,

$$M_{u,\text{lim}} = \text{Maximum tensile force} \times \text{Lever arm}$$

$$= \frac{f_y}{1.15} A_{st} \times (d - 0.42\, x_{u,\text{lim}})$$

$$= 0.87\, f_y\, A_{st} \times d\left(1 - 0.42 \frac{x_{u,\text{lim}}}{d}\right)$$

Substituting $x_u = x_{u,\text{lim}}$ in equation (2)

$$\frac{x_{u,\lim}}{d} = \frac{0.87\, f_y\, A_{st}}{0.36\, f_{ck} bd}$$

$$\therefore \quad M_{u,\lim} = 0.87\, f_y A_{st}\, d\left(1 - \frac{0.42 \times 0.87\, f_y A_{st}}{0.36\, f_{ck} bd}\right)$$

$$= 0.87\, f_y\, A_{st}\, d\left(1 - \frac{A_{st} f_y}{bd f_{ck}}\right)$$

(ii) But if $\frac{x_u}{d} < \frac{x_{u,\lim}}{d}$

Moment of resistance, M_u, for the given section will be less than $M_{u,\lim}$ which can be calculated as follows: Referring Figure

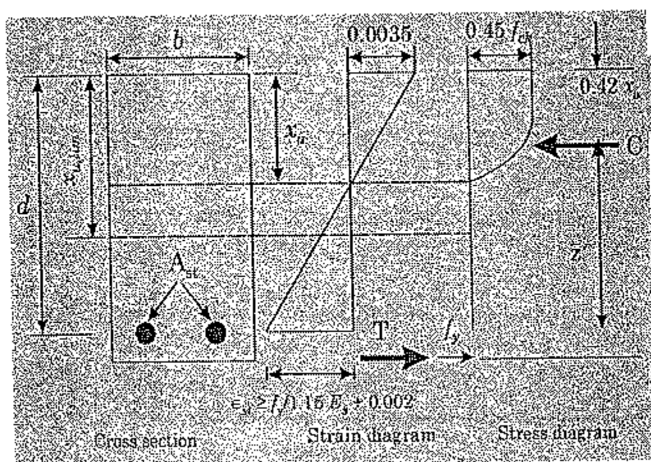


Fig 1.8(b): Strain-stress diagram for cross-section when $x_u/d < x_{u,\lim}/d$.

$$M_u = \text{Compressive force} \times \text{Lever arm} = C \times z$$

$$= 0.36\, f_{ck}\, x_u b(d - 0.42\, x_u)$$

$$= 0.36\, f_{ck}\, x_u\, bd\left(1 - 0.42\frac{x_u}{d}\right)$$

Substituting above $\frac{x_u}{d}$ from (Eq. 2)

$$M_u = 0.36\, f_{ck}\, x_u bd\left(1 - 0.42 \times \frac{0.87\, f_y\, A_{st}}{0.36\, f_{ck}\, bd}\right)$$

$$= 0.36\frac{x_u}{d}\left(1 - \frac{A_{st} f_y}{bd f_{ck}}\right) bd^2 f_{ck}.$$

Alternatively,

$$M_u = T \times z$$

$$= \frac{f_y}{1.15} A_{st}(d - 0.42\, x_u)$$

$$= 0.87\, f_y\, A_{st} d\left(1 - 0.42\frac{x_u}{d}\right)$$

Substituting $\frac{x_u}{d}$ from (Equation 2)

$$M_u = 0.87 f_y A_{st} d\left(1 - \frac{A_{st} f_y}{bd f_{ck}}\right)$$

(iii) If $\frac{x_u}{d} > \frac{x_{u,\lim}}{d}$ i.e. the section is over-reinforced. The section may be changed as it violates the assumption that the maximum strain in tensile reinforcement shall not be less than $\frac{f_y}{1.15 E_s} + 0.002$ (Figure below)

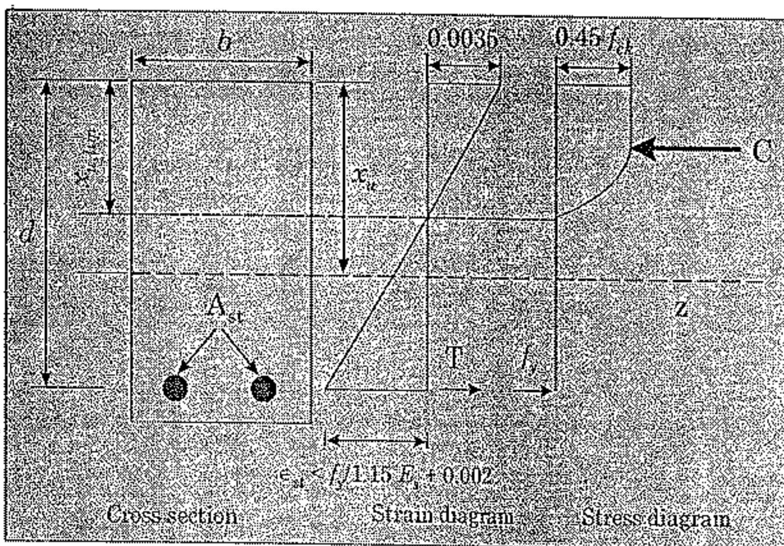


Fig 1.8(c): Strain-stress diagrams for cross-section when $x_u/d > x_{u,\lim}/d$.

(b) To Determine Steel Area in Tension When Concrete Cross-section and Applied Moment are known.

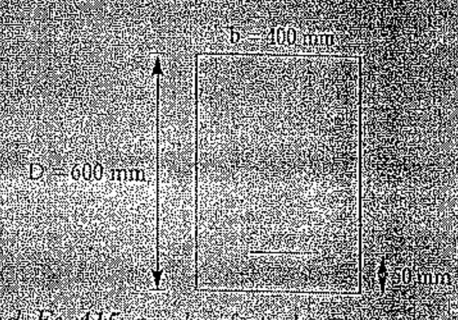
$$M_u = M_{u,\text{applied}},$$

$$M_u = 0.87 f_y A_{st} d\left(1 - \frac{A_{st} f_y}{bd f_{ck}}\right);$$

A_{st} is calculated.

Example 1

Calculate the moment of resistance of the section?



Use M 20 Grade of concrete and Fe 415 grade of steel.

For case 1: 4 nos. 20 mm ϕ

For case 2: 8 nos. 25 mm ϕ

For case 3: Calculate M.R. of limiting section and also area of steel required. Use effective cover 50 mm.

Sol: (1) ***Case I:*** Use 4 nos. 20 mm ϕ

$$\text{Effective depth} = 600 - 50 = 550 \text{ mm}$$

$$\text{Area of steel} = 4 \times \frac{\pi}{4} \times 20^2 = 1256.63 \text{ mm}^2$$

Actual depth of N.A.

$$C = T$$

$$0.36 f_{ck} x_u b = 0.87 f_y A_{st}$$

$$0.36 \times 20 \times x_u 400 = 0.87 \times 415 \times 1256.63$$

$$x_u = 157.53 \text{ mm}$$

Limiting depth of N.A.

$$\frac{x_{u\,lim}}{d} = \left(\frac{700}{0.87 f_y + 1100}\right)$$

$$x_{ulim} = \left(\frac{700}{0.87 \times 415 + 1100}\right) 550$$

$$x_{u,lim} = 263.51 \text{ mm}$$

$x_u < x_{ulim}$ section is under reinforced section.

$$M_u = 0.36 f_{ck} x_u b (d - 0.42 x_u)$$

$$= 0.36 \times 20 \times 157.53 \times 400(550 - 0.42 \times 157.53)$$

$$= 219.51 \text{ kN-m}$$

OR

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$= 0.87 \times 415 \times 4 \frac{\pi}{4} \times 20^2 (550 - 0.42 \times 157.53)$$

$$M_u = 219.52 \text{ kN-m}$$

(2) ***Case II:*** 8 nos. 25 mm ϕ

$$A_{st} = 8 \times \frac{\pi}{4} \times 25^2$$

$$A_{st} = 3927 \text{ mm}^2$$

Actual depth of N.A.

$$C = T$$

$$0.36 f_{ck} x_u b = 0.87 f_y A_{st}$$

$$0.36 \times 20\, x_u 400 = 0.87 \times 415 \times 3927$$

$$x_u = 492.30 \text{ mm}$$

Limiting depth of N.A. $\frac{x_{u,lim}}{d} = 0.48$

$$x_{u,lim} = 0.48 \times 550 = 264 \text{ mm}$$

$x_u > x_{u,lim}$ over rinforced section so take

$$x_u = x_{u,lim}$$

$$M_{u,lim} = 0.36 f_{ck} x_{u,lim} b(d - 0.42 x_{u,lim})$$

$$= 0.36 \times 20 \times 0.48 \times 550 \times 400(d - 0.42 \times 0.48 \times 550)$$

$$M_{u,lim} = 333.87 \text{ kN-m}$$

(3) $$M_{u,lim} = 333.87 \text{ kN-m}$$

and $$A_{st,lim} = \frac{M_{u,lim}}{0.87 f_y (d - 0.42 x_{u,lim})}$$

$$= \frac{345.87 \times 10^6}{0.87 \times 415(550 - 0.42 \times 0.48 \times 550)}$$

$$A_t = 2105 \text{ mm}^2$$

OR

from

$$C = T$$

$$0.36 f_{ck} x_{u,lim} b = 0.87 f_y A_{st,lim}$$

$$A_{st,lim} = \frac{0.36 \times f_{ck} x_{u,lim} b}{0.87 f_y}$$

$$= \frac{0.36 \times 20 \times 0.48 \times 550 \times 400}{0.87 \times 415}$$

$$A_{st,lim} = 2105 \text{ mm}^2$$

Example 2

Determine moment of resistance of a R.C. beam having a rectangular cross-section 300 mm wide and 500 mm deep reinforced with 5 nos. 20 mm φ as tensile reinforcement. The characteristic strength of concrete is 15 MPa and that for steel is 250 MPa.

Sol: Assuming clear cover of 25 mm and taking dia. of bar 20 mm, $d = 500 - 25 - 10 = 465$ mm (from Fig. 1.9).

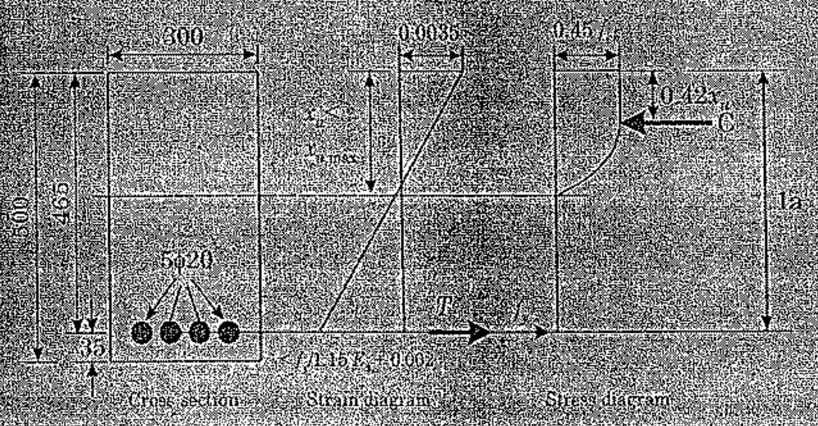


Fig 1.9: Strain stress diagrams for the cross section.

Let the N.A. be at x_u from top fibre, then $\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} bd}$

$$\text{or} \quad x_u = \frac{0.87 \times 250 \times 5 \times 314}{0.36 \times 15 \times 300} = 210.79$$

The limiting value of $x_{u,lim}$ for Fe 250 is given by

$$\frac{x_{u,lim}}{d} = 0.53$$

$$\text{or} \quad x_{u,lim} = 0.53 \times 465 = 246.45 > 210.79$$

Shows that the section is under reinforced. Hence O.K.

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{b d f_{ck}}\right)$$

$$= 0.87 \times 250 \times 5 \times 314 \times 465 \left(1 - \frac{5 \times 314 \times 250}{300 \times 465 \times 15}\right)$$

$$= \mathbf{129\ kNm}$$

Example 3

A reinforced concrete beam of size b = 400, D = 800 mm effective cover 50 mm has to be designed for a M = 200 kN-m (working moment) using M 20 grade of concrete and Fe 415 steel, calculate area of steel required? (State whether the section is under reinforced/over reinforced or balanced section).

Sol. *Given:*

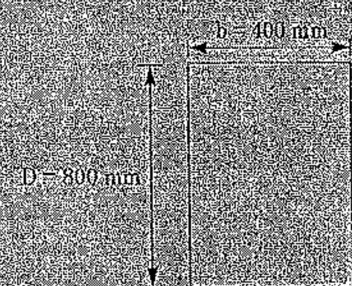


Fig. 1.10

$d' = 50$ mm (effective cover)

M 20 and Fe 415 grade of steel

$$\text{Effective depth, } d = 800 - 50 = 750 \text{ mm}$$

$$\text{B.M.} = 200 \text{ kN-m}$$

First check whether the section in under reinforced or over reinforced

$$M_{u,lim} = 0.36 f_{ck} x_{u,lim} b(d - 0.42 x_{u,lim})$$

$$= 0.36 \times 20 \times 0.48 \times 750 \times 400(750 - 0.42 \times 0.48 \times 750)$$

$$M_{u,lim} = 620.84 \text{ kN-m}$$

$$M_u = 1.5 \times 200 = 300 \text{ kN-m}$$

$M_u < M_{u,lim}$ so the section is under reinforced

$$M_u = 0.36 f_{ck} x_u b(d - 0.42 x_u)$$

$$300 \times 10^6 = 0.36 \times 20 x_u 400(750 - 0.42 x_u)$$

$$300 \times 10^6 = 2.16 \times 10^6 x_u - 1209.6 x_u^2$$

$$x_u = 151.79 \text{ mm}$$

$$300 \times 10^6 = 0.87 f_y A_{st}(d - 0.42\, x_u)$$

$$300 \times 10^6 = 0.87 \times 415\, A_{st}(750 - 0.42 \times 151.79)$$

$$A_{st} = 1210.8 \text{ mm}^2$$

or $\quad C = T$

$$0.36 f_{ck} x_u b = 0.87 f_y A_{st}$$

$$A_{st} = \frac{0.36 \times 20 \times 151.79 \times 400}{0.87 \times 415}$$

$$= 1210.8 \text{ mm}^2$$

Example 4

For a given concrete cross-section of rectangular RC beam, $b \times d = 250 \times 415$, determine the tensile steel area to resist an applied moment of 70 kNm. Take M 20 concrete and Fe 415 grade steel.

Sol: This type of problem can be solved by this method:

$$M_u = 0.87 f_y A_{st} d\left(1 - \frac{A_{st} f_y}{b d f_{ck}}\right)$$

$$70 \times 10^6 = 0.87 \times 415 \times A_{st} \times 415\left(1 - \frac{A_{st} \times 415}{250 \times 415 \times 20}\right)$$

$$467.18 = A_{st} - 2 \times 10^{-4} A_{st}^2$$

or $A_{st}^2 - 5 \times 10^3 A_{st} + 233.59 \times 10^4 = 0$

$$\text{or} \quad A_{st} = \frac{5 \times 10^3 \pm \sqrt{25 \times 10^6 - 4 \times 1 \times 233.5 \times 10^4}}{2}$$

$$= \frac{5 \times 10^3 \pm 3.95727 \times 10^3}{2}$$

$$= 4478.5 \text{ mm}^2 \text{ or } 521.5 \text{ mm}^2 \; (A_{st} = 521.5 \text{ mm}^2 \text{ is applicable})$$

Check:

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87 \times 415 \times 521.5}{0.36 \times 20 \times 250} = 104.6 \text{ mm}$$

for $f_y = 415$ MPa

$$x_{u,\lim} = 0.48 \times 415 = 199.2 > 104.6$$

With $A_{st} = 521.5$ mm², the section is under-reinforced.

$$A_{st} = \mathbf{521.5 \text{ mm}^2}$$

Example 5

A RC beam of width 150 mm and overall depth 300 mm is reinforced with 3 nos. of 16 ϕ bars. Effective cover to the reinforcement is 50 mm. Cube strength of concrete used is 200 kg/cm², yield stress of steel is 2100 kg/cm². Calculate ultimate moment of resistance.

Sol:

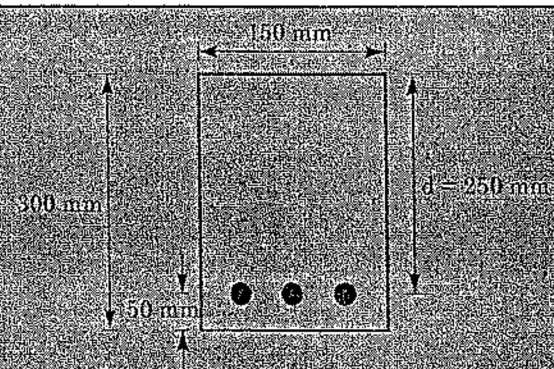


Fig. 1.11

$$b = 150 \text{ mm}$$

$$f_y = 2100 \text{ kg/cm}^2 = 210 \text{ MPa}$$

$$f_{ck} = 200 \text{ kg/cm}^2 = 20 \text{ MPa}$$

$$A_{st} = 3 \times \frac{\pi}{4} \times 16^2 = 603.18 \text{ mm}^2$$

Effective cover = 50 mm

$$D = 300 \text{ mm}$$

$$d = 300 - 50 = 250 \text{ mm}$$

(i) Calculating limiting depth of neutral axis

$$x_{u,lim} = \left[\frac{700}{0.87 f_y + 1100}\right] \times 250$$

$$= 0.5457 \times 250$$

$$= 136.43 \text{ mm}$$

(ii) Calculating actual depth of neutral axis

Total compressive force = Total tensile force

$$\Rightarrow \quad 0.36 f_{ck} \, b \, x_u = 0.87 f_y A_{st}$$

$$\Rightarrow \quad x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$\Rightarrow \quad x_u = \frac{0.87 \times 210 \times 603.18}{0.36 \times 20 \times 150}$$

$$\Rightarrow \quad x_u = 102.04 \text{ mm}$$

$x_u < x_{u,lim}$. Hence section is underreinforced.

$$M_u = 0.36 f_{ck} \, b \, x_u \, (d - 0.42 \, x_u)$$

$$= 0.36 \times 20 \times 150 \times 102.04 \, (250 - 0.42 \times 102.04)$$

$$M_u = 22.83 \text{ kNm}$$

Example 6 Part (a)

Calculate ultimate moment of resistance of an RCC beam b = 230 mm, D = 550 mm

Sol:

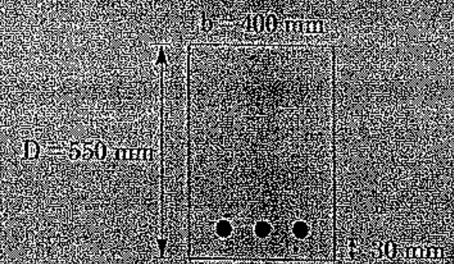


Fig. 1.12

$$\text{Effective depth} = 550 - 30 - \frac{20}{2}$$

$$= 510 \text{ mm}$$

Actual depth of N.A.

$$C = T$$

$$0.36 f_{ck} x_u b = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 \times 415 \times 4 \times \frac{\pi}{4} \times 20^2}{0.36 \times 20 \times 230}$$

$$x_u = 273.97 \text{ mm}$$

Limiting depth of N.A. for Fe 415

$$\frac{x_{u,\lim}}{d} = 0.48$$

$$x_{u,\lim} = 0.48 \times 510 = 244.8 \text{ m}$$

$x_u > x_{u,\lim}$, over reinforced but take $x_u = x_{u,\lim}$

$$M_{u,\lim} = 0.36 f_{ck}\, x_{u,\lim}\, b(d - 0.42\, x_{u,\lim})$$

$$= 0.36 \times 20 \times 244.8 \times 230\,(510 - 0.42 \times 244.8)$$

$$M_{u,\lim} = 165.06 \text{ kN-m}$$

Example 6: Part (b)

Also, determine intensity of super imposed load excluding of self weight, the beam can carry over a simple support span of 5 m?

Sol:

$$M_{u,\lim} = 165.06 \text{ kN-m}$$

$$\frac{M_{u,\lim}}{1.5} = \frac{wl^2}{8} = \frac{w \times 5^2}{8}$$

$$\frac{165.06 \times 10^6}{1.5} = \frac{w \times 5^2}{8}$$

$$w = 35.21 \text{ kN/m}$$

$$\text{Self } wt = b \times D \times \gamma$$

$$= 0.230 \times 0.550 \times 25 = 3.16 \text{ kN/m}$$

Net intensity of superimposed load = 35.21 − 3.16 = 32.05 kN/m

Example 7

Prove that limiting moment of resistence 'M_u' of single (under) reinforced rectangular beam section using the stress block parameter of code 456 is

$$M_u = 0.87 f_y\left(\frac{p_t}{100}\right)\left[1 - 1.005\frac{f_y}{f_{ck}}\left(\frac{p_t}{100}\right)\right]bd^2$$

$$p_t = \frac{A_{st}}{bd} \times 100 \qquad d = 0.416\, x_u$$

Sol. From stress block

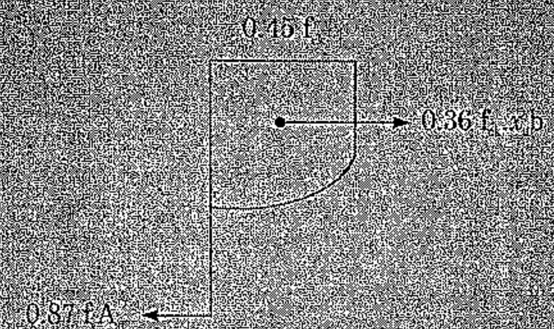


Fig. 1.13

$$M_u = 0.87 f_y A_{st}(d - 0.416 x_u)$$

$$M_u = 0.87 f_y \frac{A_{st}}{bd}(d - 0.416 x_u) bd \qquad \text{...(i)}$$

$$C = T$$

$$0.36 f_{ck} x_u b = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} \quad \text{put in eq. (i)}$$

$$M_u = \frac{0.87 f_y A_{st}}{bd}\left(d - \frac{0.416 \times 0.87 f_y A_{st}\, d}{0.36 f_{ck} b\, d}\right) bd \qquad \text{...(ii)}$$

We know that $\qquad p_t = \dfrac{A_{st}}{bd} \times 100$

$\dfrac{A_{st}}{bd} = \dfrac{p_t}{100}$ put this value in eq. (ii), we get

$$\boxed{M_u = 0.87 f_y\left(\frac{p_t}{100}\right)\left[1 - 1.005\frac{f_y}{f_{ck}}\left(\frac{p_t}{100}\right)\right]bd^2}$$

Example 8

A beam is S.S. over an effective span of 7.5 m. load over the beam is 30 kN/m (excluding self weight). Design the beam using LSM, use M 20 and Fe 415 grade of steel? Keep b = 350 mm

Sol.

$$\text{Load} = 30 \text{ kN/m}$$

$$\text{Assume } D = 750 \text{ mm} \left(\frac{\text{span}}{10}\right) \text{thumb rule}$$

$$\text{Dead load} = b \times D \times \gamma$$

$$\text{Dead load} = 0.350 \times 0.750 \times 25 = 6.5 \text{ kN/m}$$

$$\text{Total load, } w = 30 + 6.5 = 36.5 \text{ kN/m}$$

$$\text{Factored load} = 1.5 \times 36.50 = 54.75 \text{ kN}$$

$$\text{B.M.} = \frac{wl^2}{8} = \frac{54.75 \times 7.5^2}{8}$$

$$M_u = 384.96 \text{ kN-m}$$

Now depth required is

$$M_{u\,lim} = 0.36 f_{ck} x_{u\,lim}\, b(d - 0.42\, x_{u\,lim})$$

$$384.96 \times 10^6 = 0.36 \times 20 \times 0.48\, d\, 350\, (d - 0.42 \times 0.48 \times d)$$

$$384.96 \times 10^6 = 965.74\, d^2$$

$$d = \sqrt{\frac{384.96 \times 10^6}{965.74}}$$

$$d = 631.35 \text{ mm}$$

Use effective cover = 30 mm

$$D = 631.35 + 30 = 661.35 \text{ mm} \approx 700 \text{ mm} < D_{\text{provided}}\ (750 \text{ mm})$$

$$d = 700 - 30 = 670 \text{ mm}$$

$$\text{Area of steel } M_u = 0.87 f_y A_{st} (d - 0.42\, x_u)$$

$$= 0.87 f_y A_{st} \left[d - \left(\frac{0.42 \times 0.87 f_y A_{st}}{0.36 f_{ck} b}\right)\right]$$

$$384.96 \times 10^6 = 0.87 \times 415\, A_{st} \left[670 - \left(\frac{0.42 \times 0.87 \times 415 A_{st}}{0.36 \times 20 \times 350}\right)\right]$$

$$384.96 \times 10^6 = 241.9 \times 10^3\, A_{st} - 21.72\, A_{st}^2$$

$$A_{st} = 1923.66 \text{ mm}^2$$

(v)

$$\frac{A_{st,min}}{bd} = \frac{0.85}{f_y}$$

$$A_{st\,min} = \frac{0.85 \times bd}{f_y}$$

$$= \frac{0.85 \times 350 \times 670}{415} = 480.30 \text{ mm}^2$$

$$A_{st} > A_{st\,min} \quad \text{O.K.}$$

Provide 4 nos. 25 mm and 1 nos. 12 mm dia bar

$$A_{st\,provided} = 4 \times \frac{\pi}{4} \times 25^2 + 1 \times \frac{\pi}{4} \times 12^2 = 2076.59 \text{ mm}^2$$

$$\frac{A_s}{bd} \times 100 = \frac{2076.59}{350 \times 670} \times 100$$

$$= 0.88\% < 4\% \text{ maximum of tension reinforcement} \therefore \text{O.K.}$$

Example 9

An RC beam of overall rectangular dimensions 300 × 600 mm is reinforced with 4 bars of 25 mm dia with an effective cover to the centre of reinforcements being 50 mm. Effective span of the beam is 6 m. Calculate its ultimate moment capacity. Find the safe load (working). If the load factor is 1.5, yield stress f_y of steel is 2500 kg/cm², cube strength of concrete f_{ck} is 200 kg/cm².

Sol. ***Using limit state method***

$$f_{ck} = 200 \text{ kg/cm}^2 = 20 \text{ N/mm}^2$$

$$f_y = 2500 \text{ kg/cm}^2 = 250 \text{ N/mm}^2$$

$$l_{eff} = 6 \text{ m}$$

$$\text{Load factor} = 1.5$$

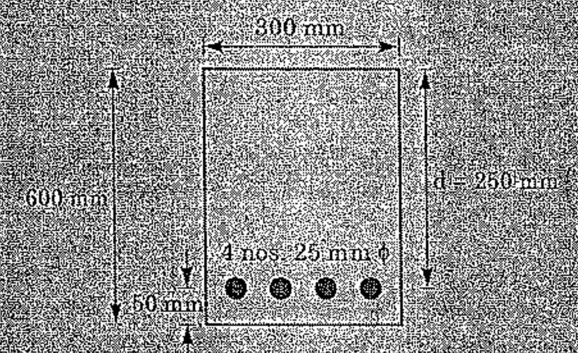


Fig. 1.14

$$A_{st} = 4 \times \frac{\pi}{4} \times 25^2 = 1963.49 \text{ mm}^2$$

$$d = 600 - 50 = 550 \text{ mm}$$

(i) Calculation of limiting depth of N.A.

$$x_{u,lim} = 0.05\,d \qquad [\because \text{for Fe 250 } x_{u,lim} = 0.53\,d]$$

$$= 0.53 \times (550)$$

$$= 291.5 \text{ mm}$$

(ii) Calculation of actual depth of neutral axis

Equating total compressive force = total tensile force

$$C = T$$

$$\Rightarrow \quad 0.36 f_{ck}\, b\, x_u = 0.87\, f_y\, A_{st}$$

$$\Rightarrow \quad x_u = \frac{0.87\, f_y A_{st}}{0.36 f_{ck}\, b\, x_u}$$

$$0.87 \times 250 \times 1963.49$$

$$x_u = 197.71 \text{ mm}$$

$x_u < x_{u,lim}$ section is under reinforced

Therefore
$$M_u = 0.36 f_{ck}\, b\, x_u\, (d - 0.42\, x_u)$$
$$= 0.36 \times 20 \times 300 \times 197.71\,(550 - 0.42 \times 197.71)$$
$$M_u = 199.42 \text{ kNm}$$

$$\text{This ultimate moment capacity} = \frac{w_u l_{eff}^2}{8}$$

$$\Rightarrow \quad \frac{w_u \times 6^2}{8} = 199.42$$

$$\Rightarrow \quad w_u = 44.32 \text{ kN}$$

But
$$\text{Load factor} = \frac{\text{Ultimate load}}{\text{Working load}}$$

$$\Rightarrow \quad \text{Working load} = \frac{\text{Ultimate load}}{\text{Load factor}}$$

$$w = \frac{44.32}{1.5}$$
$$w = 29.54 \text{ kN}$$

Example 10

Design a beam of effective span 6 mm to support a total design load of 12 kN/m including self weight of the beam using LSM of design. Width of the beam is limited to 250 mm. Load factor for live load and dead load is 1.5. M 15/Fe 415 are to be used.

Sol: (i) Load calculations

$$\text{L.L including self weight} = 12 \text{ kN/m}$$
$$w_u = 1.5 \times 12 = 18 \text{ kN/m}$$

(ii) Maximum bending moment

$$M_{u,\max} = \frac{w_u l_{eff}^2}{8} = \frac{18 \times 6^2}{8} = 81 \text{ kNm}$$

(iii) Assume $D = \dfrac{\text{Span}}{10} = \dfrac{6000}{10} = 600$ mm

(iv) Equation max. bending moment with MR equation for balanced case i.e.

$$M_u = M_{u,lim} = 0.36 f_{ck}\, b\, x_{u,lim}\,(d - 0.42\, x_{u,lim})$$
$$x_{u,lim} = 0.48\, d \text{ [for Fe 415]}$$

$\Rightarrow \; 0.36 \times 15 \times 250 \times 0.48\, d\,(d - 0.42 \times 0.48\, d) = 81 \times 10^6$

$\Rightarrow \; 0.36 \times 15 \times 250 \times 0.48\, d^2\,(1 - 0.2016) = 81 \times 10^6$

$$\Rightarrow \quad d = 395.68 \text{ mm}$$

Use nominal cover for beam is 25 mm and maximum 20 mm dia bar

$$D = 395.68 + 25 + \frac{20}{2} = 430.68 \text{ mm}$$

take $D = 450$ mm $< D_{provided}$ (600 mm) O.K.

$$d = 450 - 25 - \frac{20}{2} = 415 \text{ mm}$$

(4) Calculating area of steel

$$M_{u,lim} = 0.87 f_y A_{st} (d - 0.42 x_{u,lim})$$

$$81 \times 10^6 = 0.87 \times 415 A_{st} (415 - 0.42 \times 0.48 \times 415)$$

$$A_{st} = 677.09 \text{ mm}^2$$

(v) $\frac{A_{st\,min}}{bd} = \frac{0.85}{f_y}$

$$A_{st\,min} = \frac{0.85\,bd}{f_y} = \frac{0.85 \times 250 \times 415}{415} = 212.5 \text{ mm}^2$$

A_{st} (677.09 mm²) > $A_{st\,min}$ (212.5 mm²) O.K.

Provide 2 nos. 20 mm dia and one nos. 12 mm dia.

$$\text{Actual area of steel provided} = 2 \times \frac{\pi}{4} \times 20^2 + \frac{\pi}{4} \times 12^2$$

$$= 741.41 \text{ mm}^2$$

% of steel $= \frac{741.41}{250 \times 415} \times 100 = 0.71\% < 4\%$ maximum of tension reinforcement.

Example 11

A rectangular RC beam of concrete grade M 15 is 200 mm wide and 350 mm deep (effective depth). It is provided with 4 nos. of 12 mm diameter HYSD steel bars as tension reinforcement. Determine the MR of the beam by LSM.

Sol.

$$b = 200 \text{ mm}$$

$$d = 350 \text{ mm}$$

$$A_{st} = 4 \times \frac{\pi}{4} \times 12^2 = 144\pi = 452.39 \text{ mm}^2$$

M 15 grade concrete, HYSD bars means Fe 415

(i) Finding actual neutral axis by equaing compressive and tensile forces.

$$C = T$$

$$0.36 f_{ck} b x_u = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 415 \times 452.39}{0.36 \times 15 \times 200}$$

$$x_u = 151.24 \text{ mm}$$

(ii) Limiting depth of neutral axis $x_{u,lim} = 0.48\,d$

$$= 0.48 \times 350$$

$$= 168 \text{ mm}$$

$\therefore x_u < x_{u,lim}$ section is under reinforced

(iii) Moment of resistance of under reinforced section

$$MR = 0.36 f_{ck} b x_u (d - 0.42 x_u)$$

$$= 0.36 \times 15 \times 200 \times 151.24 (350 - 0.42 \times 151.24)$$

$$= MR = 46.79 \text{ kNm}$$

Example 12

Use LSM to design a RC rectangular beam having an effective SS span of 6 m. The beam is required to support live service and super imposed (dead) loads of 14 kN/m and 9.5 kN/m respectively. The materials to be used are M 20 concrete and HYSD bars of grade Fe 415. Unit weight of concrete 25 kN/m³. Adopt $\frac{d}{b}$ ratio as 2. For the given materials $P_{t,lim} = 0.955\%$.

Sol.

(i) Given values

$$f_{ck} = 20 \text{ N/mm}^2 \quad LL = 14 \text{ kN/m}$$

$$f_y = 415 \text{ N/mm}^2 \quad DL = 9.5 \text{ kN/m}$$

$$\frac{d}{b} = 2 \quad \text{effective span, } l = 6 \text{ m}$$

(ii) Assuming, $d = \frac{1}{10} \times 6000 = 600$ mm

$$b = \frac{600}{2} = 300 \text{ mm}$$

(iii) Load calculations

Live load = 14 kN/m

Super imposed load = 9.5 kN/m

Self weight of the beam = 0.3 × 0.6 × 1 × 25 = 4.5 kN/m

Total load w = 28 kN/m (14 + 9.5 + 4.5)

Factored load, w_u = 1.5 × w = 1.5 × 28 = 42 kN/m

(iv) Maximum bending moment

$$BM_{max} = \frac{w_u l^2}{8} = \frac{42 \times 6^2}{8} = 189 \text{ kNm}$$

(v) Equating BM_{max} with MR of the section (balanced)

$$\frac{d}{b} = 2 \Rightarrow b = \frac{d}{2} \quad \text{and} \quad x_{u,lim} = 0.48\, d$$

$$BM_{max} = MR = 0.36 f_{ck} b\, x_{u,lim} (d - 0.42\, x_{u,lim})$$

$$\Rightarrow \quad 0.36 \times 20 \times \frac{d}{2} \times 0.48d\,(d - 0.42 \times 0.48 \times d) = 189 \times 10^6$$

$$\Rightarrow \quad d^2(d - 0.2016d) = 109.375 \times 10^6$$

$$\Rightarrow \quad d^3 = 136.99 \times 10^6$$

$$\Rightarrow \quad d = 5.1550 \times 10^2 = 515.50 \text{ mm}$$

$$\text{Adopt } d = 515.50$$

∴ Using moment cover for beam is 25 mm and 20 mm dia bar

$$D = 515.50 + 25 + \frac{20}{2} = 550.5 \text{ adopt } D = 560 \text{ mm} < 600 \text{ mm}$$

Now,
$$d = 560 - 35 = 525 \text{ mm}$$

$$b = \frac{d}{2} = \frac{525}{2} = 262.5 \text{ mm} \simeq 265 \text{ mm}$$

(vi) Area of steel

$$A_{st} = \frac{MR}{0.87 f_y (d - 0.42 x_{u,lim})}$$

$$= \frac{189 \times 10^6}{0.87 \times 415 \times (525 - 0.42 \times 0.48 \times 525)}$$

$$A_{st} = 1248.86 \text{ mm}^2 > A_{st,min} \ (284.95 \text{ mm}^2)$$

$$P_t = \frac{A_{st}}{bd} \times 100 = \frac{1248.86 \text{ mm}^2}{265 \times 525} \times 100$$

$$= 0.897\% < 0.955\% \text{ O.K.}$$

Example 13

Determine the lever arm for section shown in figure if effective cover = 40 mm, and $f_{ck} = 20\ N/mm^2$, $f_y = 250\ N/mm^2$ and for $f_{ck} = 25\ N/mm^2$ and $f_y = 415\ N/mm^2$.

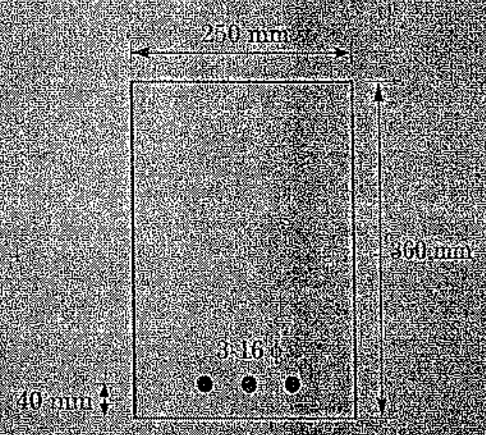


Fig. 1.15

Sol:

$$C = 0.36 f_{ck} x_u b$$
$$C = 0.36 \times 20\ x_u\ 250$$
$$C = 1800\ x_u \text{ N}$$
$$T = 0.87 f_y A_{st}$$
$$T = 0.87 \times 250 \times 3 \times \frac{\pi}{4} \times 16^2$$
$$T = 131100 \text{ N}$$

$$C = T$$
$$1800\, x_u = 131100$$
$$x_u = 72.8 \text{ mm}$$

Critical depth of N.A.

$$\frac{x_{u,lim}}{d} = 0.53 \text{ for Fe 250}$$
$$x_{u,lim} = 0.53 \times 360 = 190.8 \text{ mm}$$

$x_u < x_{u,lim}$ underreinforced section

Liver Arm = distance between centroid of compressive force to tensile force

$$\text{Liver arm} = d - 0.42\, x_u$$
$$= 360 - 0.42 \times 72.8 = 329.4 \text{ mm}$$

(ii) $C = 0.36\, f_{ck}\, x_u b$

$$= 0.36 \times 25\, x_u\, 250 = 2250\, x_u \text{ N}$$
$$T = 0.87\, f_y\, A_{st} = 0.87 \times 415 \times 3\frac{\pi}{4} \times 201$$
$$= 217700 \text{ N}$$

for actual depth of N.A.

$$C = T$$
$$2250\, x_u = 217700$$
$$x_u = 96.76 \text{ mm}$$
$$x_{u,lim} = 0.48 \text{ d (for Fe 415)}$$
$$x_{u,lim} = 0.48 \times 360 = 172.8 \text{ mm}$$

$x_u < x_{u,lim}$ under reinforced section

$$\text{Liver arm} = d - 0.42\, x_u$$
$$= 360 - 0.42 \times 96.76$$
$$\text{Liver arm} = 319.36 \text{ mm}$$

Example 14

Design a rectangular beam to resist a bending moment equal to 45 kNm using (1) M 15 and mild steel?

Sol: Let us assume ratio of overall depth to breadth of the beam equal to 2.

For a balanced design

$$\text{Factosed B.M.} = 1.5 \times 45 = 67.5 \text{ kNm}$$
$$M_{u,lim} = 0.36\, f_{ck}\, x_{u,lim}\, b\,(d - 0.42\, x_{u,lim})$$

For Fe 250 steel, $x_{u,lim} = 0.53\, d$

$$M_{u,lim} = 0.36 \times 15 \times 0.53\, d\, b\,(d - 0.42 \times 0.53\, d)$$
$$\text{Since } \frac{D}{b} = 2 \text{ or } \frac{d}{b} = 2 \text{ or } b = \frac{d}{2}$$
$$67.5 \times 10^6 = 1.11\, d^3$$
$$d = 394 \text{ mm and } b = 200 \text{ mm}$$
$$D = 394 + 25 + \frac{20}{2} = 429 \text{ mm}$$
$$\text{take } D = 450 \text{ mm}, b = 225 \text{ mm}$$

$$d = 450 - 25 - \frac{20}{2} = 415 \text{ mm}$$

$$M_{u,lim} = 0.87 f_y A_{st}(d - 0.42\, x_{u,lim})$$

$$67.5 \times 10^6 = 0.87 \times 250\, A_{st}(415 - 0.42 \times 0.53 \times 415)$$

$$A_{st} = 961.94 \text{ mm}^2 \simeq 962 \text{ mm}^2$$

$$A_{st,min} = \frac{0.85\, bd}{f_y} = \frac{0.85 \times 225 \times 415}{250} = 317.48 \text{ mm}^2$$

$$A_{st} > A_{st,min} \quad \text{O.K.}$$

Provide 2 nos. 20 mm dia bar and 2 nos. 16 mm dia.

Actual area of steel is provided = 1030.44 mm² > 962 mm² O.K.

$$\text{Percentage of steel} = \frac{1030.44}{225 \times 415} \times 100 = 1.10\% < 4\% \text{ maximum}$$

DOUBLY REINFORCED SECTION

When ever the size of beam is restricted and beam has to bear higher value of B.M. than the moment of resistance of the balance section of given. Beam, then to resist this higher moment doubly reinforced section can be used.

$$\boxed{B.M. > M.R._{\text{balanced}}}$$

In doubly rainforced concrete beam reinforcement provided on both side tension side and compression side.

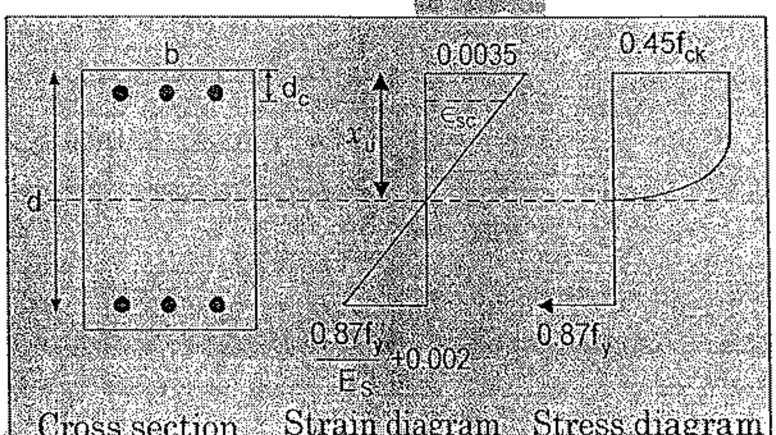


Fig. 1.16

Stress in tension side = $0.87\, f_y$.

Stress in steel in compression side is calculated based on strain at the level of compression steel

$$\frac{0.0035}{x_u} = \frac{\epsilon_{sc}}{x_u - d_c}$$

$$\boxed{\epsilon_{sc} = .0035 \times \frac{(x_u - d_c)}{x_u}}$$

Analysis: Find

1. **Limiting depth of N.A.**

$$\frac{x_{u,\lim}}{d} = \left(\frac{700}{0.87 f_y + 1100}\right)$$

$$x_{u,\lim} = 0.53\, d \text{ for Fe 250}$$

$$x_{u,\lim} = 0.48\, d \text{ for Fe 415}$$

$$x_{u,\lim} = 0.46\, d \text{ for Fe 500}$$

2. **Actual Depth of N.A.**

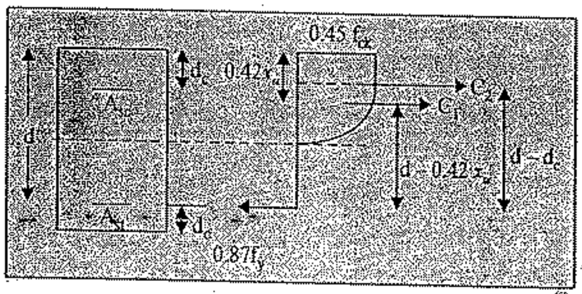


Fig. 1.17

where,

C_1 = Combined compression force of stress block

C_2 = Compression force at the level of steel at compression side.

$$C = T$$

Total compressive force = Total tension force

$$C_1 + C_2 = T$$

$$0.36\, f_{ck} x_u . b + A_{st} . f_{sc} - A_{sc} . 0.45\, f_{ck} = 0.87\, f_y A_{st}$$

$$0.36\, f_{ck} x_u . b + A_{sc} (f_{sc} - 0.45\, f_{ck}) = 0.87\, f_y A_{st}$$

3. **M.R. (Moment of Resistance)**

M.R. of compression side $= C_1 LA_1 + C_2 LA_2$

where LA_1 and LA_2 is lever arm

$$\text{M.R.} = 0.36 f_{ck} . b . x_u (d - 0.42 x_u) + A_{sc} (f_{sc} - 0.45\, f_{ck}) (d - d_c)$$

Design Formulae

Beam is converted in rectangular section with tension reinforcement A_{st1}, for balanced condition giving moment of resistance M_1 + an auxiliary section reinforced with compression reinforcemtn A_{sc}, tensile reinforcement A_{st2} giving moment of resistance M_2.

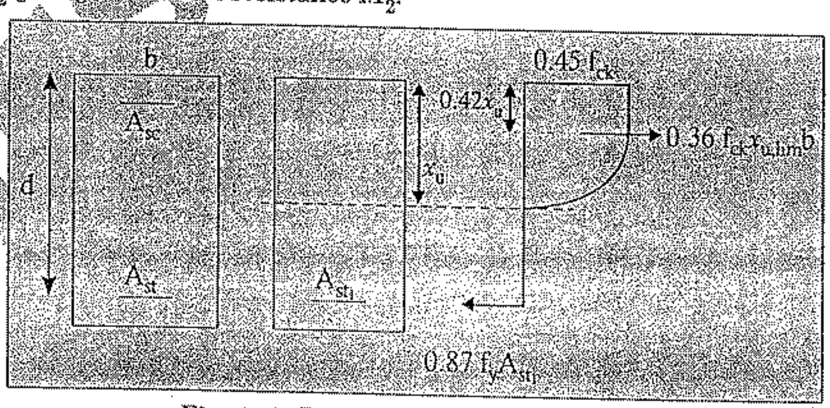


Fig. 1.18: For moment of resistance (M₁)

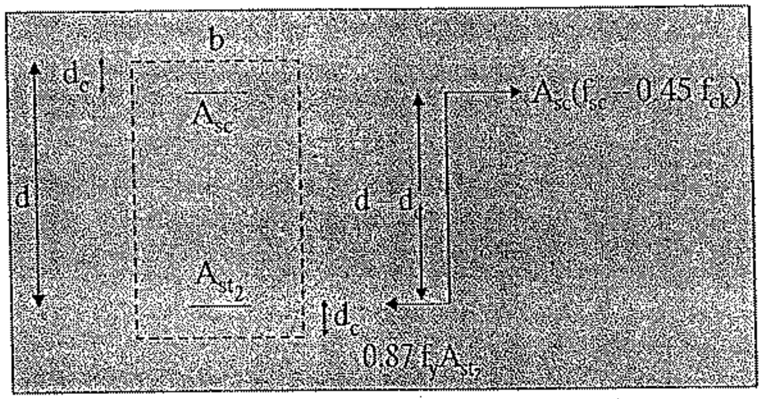


Fig. 1.19. For moment of resistance (M_2).

$$M = M_1 + M_2$$

$M_1 = 0.36\, f_{ck}\, x_{u,\lim}\; b\,(d - 0.42\, x_{u,\lim})$ for compression side

$M_1 = 0.87\, f_y A_{st_1}\,(d - 0.42\, x_{u,\lim})$ for tension side

$M_2 = 0.87\, f_y A_{st_2}\,(d - d_c)$

For A_{st_1} equate moment of resistence from compression side to tension side (Figure 1.18).

$$0.36 f_{ck} x_{u,\lim} b(d - 0.42\, x_{u,\lim}) = 0.87\, f_y A_{st1}(d - 0.42\, x_{u,\lim})$$

$$\boxed{A_{st_1} = \frac{M_1}{0.87 f_y (d - 0.42 x_{u\,\lim})}}$$

Then; $M_2 = M - M_1$ and also,

$$M_2 = 0.87\, f_y A_{st_2}(d - d_c)$$

$$\boxed{A_{st_2} = \frac{M_2}{0.87 f_y (d - d_c)}}$$

for A_{sc} equate compression force (C) to tensile force (T) in figure 1.19

$$C = T$$

$$A_{sc}\,(f_{sc} - 0.45\, f_{ck}) = 0.87\, f_y A_{st_2}$$

$$\boxed{A_{sc} = \frac{0.87 f_y A_{st_2}}{f_{sc} - 0.45 f_{ck}}}$$

$$\boxed{A_{st} = A_{st_1} + A_{st_2}}$$

From observation we get

$$A_{st_1} = \frac{M_1 = Qbd^2}{0.87 f_y (d - 0.42 x_{u,\lim})}$$

$$A_{st_2} = \frac{M_2 = M - M_1}{0.87 f_y (d - d_c)}$$

Example 15

Size of a R.C. beam is restricted to 250 × 500 mm. It carries a super imposed load of 25 kN/m over a span of 6 m using LSM find area of steel?

Use M 20/Fe 415 grade of steel effective cover use = 40 mm.

Stress		*Strain*
$0.8 f_{yd}$	→	0.00144
$0.85 f_{yd}$	→	0.00163
$0.90 f_{yd}$	→	0.00192
$0.95 f_{yd}$	→	0.00241
$0.975 f_{yd}$	→	0.00276
$1.00 f_{yd}$	→	0.00380

Sol.

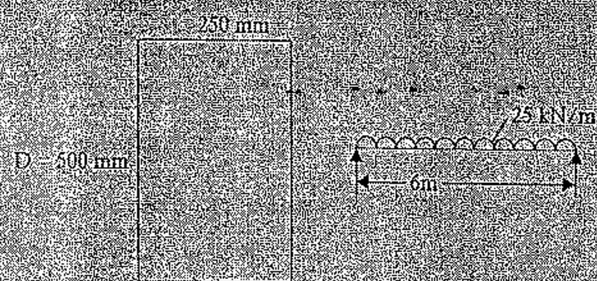


Fig. 1.20

$$d = 500 - 40 = 460 \text{ mm}$$

1. Load calculation

$$\text{Self } wt = 0.250 \times 0.50 \times 25 = 3.125 \text{ kN-m}$$

Live load = 25 kN/m

Total load = 28.125 kN/m

$$\text{B.M.} = \frac{wl^2}{8} = \frac{28.125 \times 6^2}{8} = 126.56 \text{ kN-m}$$

(M_u) Factored B.M. = 126.56 × 1.5 = 189.84 kN-m

$(M_{u,lim})$ M.R. $= 0.36 f_{ck} x_{u,lim} b(d - 0.42 x_{u,lim})$

$= 0.36 \times 20 \times 0.48 \times 460 \times 250(460 - 0.42 \times .48 \times 460)$

$(M_{u,lim})$ M.R. = 145.96 kN-m

B.M. > M.R. so doubly reinforced section is designed

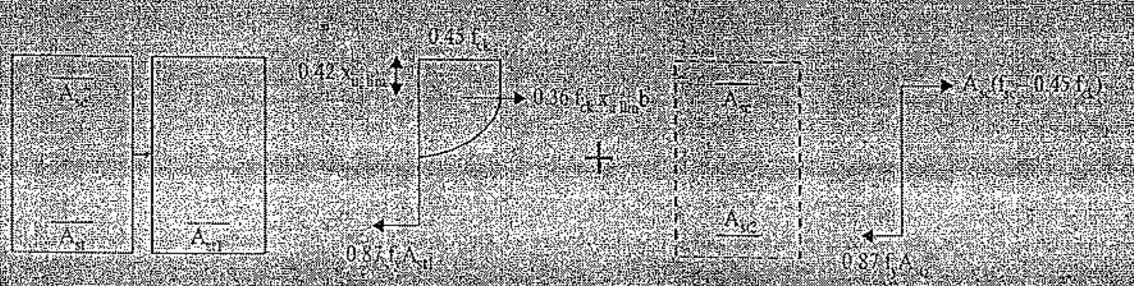


Fig. 1.21

$$M = M_1 + M_2$$
$$= 0.36\, f_{ck} x_{u,lim}\, b(d - 0.42\, x_{u,lim}) + A_{sc}(f_{sc} - 0.45\, f_{ck})$$

Equate M.R. of compression side to M.R. of Tension side

$$0.36\, f_{ck} x_{u,lim}\, b(d - 0.42\, x_{u,lim}) = 0.87 f_y A_{st_1}(d - 0.42\, x_{u,lim})$$

$$A_{st_1} = \frac{0.36 \times 20 \times 0.48 \times 460 \times 250\,(460 - 0.42 \times 48 \times 460)}{0.87 \times 415 \times (460 - 0.42 \times 48 \times 460)}$$

$$= 1100.74 \text{ mm}^2$$

$$M_2 = 189.84 - 145.96 = 43.88 \text{ kN-m}$$

Equate M.R of compression side to M.R. of Tension side of Auxiliary section

$$M - M_1 = 0.87 f_y A_{st_2}(d - d_c)$$

$$A_{st_2} = \frac{M - M_1}{0.87 \times f_y (d - d_c)}$$

$$= \frac{43.88 \times 10^6}{0.87 \times 415 \times (460 - 40)}$$

$$A_{st_2} = 289.36 \text{ mm}^2$$

For A_{sc} equate $C = T$

$$A_{sc}(f_{sc} - 0.45\, f_{ck}) = 0.87 f_y A_{st_2} \quad \text{...(1)}$$

In above equation f_{sc} is unknown. This can be found out using strain value given in question.

$$\epsilon_{sc} = \left(\frac{x_{u\,lim} - d_c}{x_{u\,lim}}\right).0035$$

$$= \left(\frac{0.48 \times 460 - 40}{0.48 \times 460}\right) \times .0035$$

$\epsilon_{sc} = 0.00285$ corresponding to this strain stress can be found out from the given value of stress, but the stress value given is to be changed in designed stress and the

$$\text{Design stress} = \frac{\text{Permissible stress}}{1.15\ (\text{Safety Factor of Steel})}$$

$$\text{Design stress for } 0.975\, f_{yd} = \frac{0.975 \times 415}{1.15} = 351.84 = 352 \text{ N/mm}^2$$

and $$\text{Design stress for } 1\, f_{yd} = \frac{1 \times 415}{1.15} = 360.86 = 361 \text{ N/mm}^2$$

f_{sc} can be find out using above value.

From similar triangle

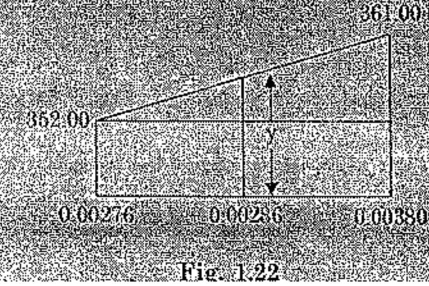


Fig. 1.22

$$\frac{361-352.00}{(0.00380-0.00276)} = \frac{y-352.0}{0.00286-0.00276}$$

$$\boxed{y = 352.77\ \text{N/mm}^2}$$

For A_{sc}

$$A_{sc}(352.77 - 0.45\times 20) = 0.87\times 415\times 289.36$$

$$A_{sc} = 303.37\ \text{mm}^2$$

$$A_{st} = A_{st_1} + A_{st_2} = 1100.74 + 289.36$$

$$A_{st} = 1390.1\ \text{mm}^2$$

Example 16

An RCC beam of size 230 mm × width 500 mm overall effective cover to compression steel and tension steel 40 mm. The compression reinforcement consists of 2 nos. 16 mm bar dia and tension steel consists of 2 nos. 25 mm diameter bars. This doubly reinforced beam is made of M 20 concrete and Fe 415 steel. Find the moment of resistance of this beam by limit state method.

For *Fe* 415 d'/d values				
d'/d	0.05	0.10	0.15	0.70
f_s (N/mm²)	355	353	342	329

Sol.

$$b = 230\ \text{mm}$$

$$d = 460\ \text{mm}$$

$$D = 500\ \text{mm}$$

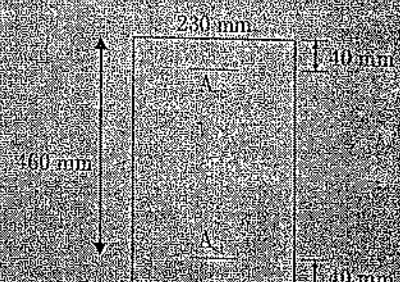


Fig. 1.23

$$A_{sc} = 2\times\frac{\pi}{4}\times 16^2$$

$$= 402.12\ \text{mm}^2$$

$$A_{st} = 2\times\frac{\pi}{4}\times 25^2$$

$$= 981.74\ \text{mm}^2$$

1. $x_{u,lim} = 0.48\,d = 0.48\times 460 = 220.8$ mm

2. Actual depth of N.A.

$$C = T$$

$$0.36 f_{ck} x_u b + (f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y A_{st}$$

$$0.36 \times 20\, x_u\, 230 + (f_{sc} - 0.45 \times 20) \times 402.12 = 0.87 \times 415 \times 981.74$$

$$1656 x_u + (f_{sc} - 0.9) \times 402.12 = 354.819 \times 10^3 \quad \text{...(1)}$$

For $\frac{d'}{d} = 0.087$, value of f_{sc} is

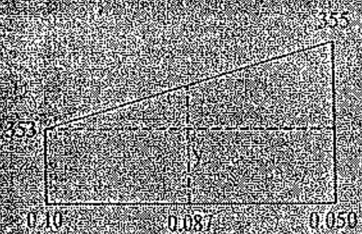


Fig. 1.24

$$\frac{355 - 353}{(0.050 - 0.10)} = \frac{y - 353}{(0.087 - 0.10)}$$

$$y = 353 + 0.52$$

$$y = 353.52 \text{ N/mm}^2$$

Put this value in equation (1)

$$1656\, x_u + (353.52 - 0.9)\, 402.12 = 354.819 \times 10^3$$

$$\boxed{x_u = 128.63 \text{mm}}$$

$x_u < x_{u,lim}$ this is under reinforced section

$$M_u = 0.36 f_{ck}\, b x_u (d - 0.42\, x_u) + A_{sc}(f_{sc} - 0.45 f_{ck})\,(d - d')$$

$$= 0.36 \times 20 \times 230 \times 128.63\,(460 - 0.42 \times 128.63) + 402.12(353.52 - 0.45 \times 20)\,(460 - 40)$$

$$\boxed{M_u = 144.66 \text{ kN-m}}$$

Example 17

A doubly reinforced section beam of over all dimensions 250 × 450 mm is reinforced with 4 bars of high strength steel of 25 dia on the tension side and with 4 bars of 18 mm dia on the compression side. Effective cover to centre of reinforcement is 50 mm. If the grade of concrete is M 20 and f_y of steel is 250 N/mm². Calculate the ultimate moment capacity of the section.

Sol:

$$b = 250$$

$$D = 450$$

$$A_{st} = 4 \times \frac{\pi}{4} \times 25^2 = 1963.49 \text{ mm}^2$$

$$A_{sc} = 4 \times \frac{\pi}{4} \times 18^2 = 1017.87 \text{ mm}^2$$

Effective cover = 50 mm

M 20/Fe 250 is used.

(i) Value of limiting depth of neutral axis

$$x_{u,lim} = 0.53\, d$$
$$= 0.53 \times 400$$
$$= 212.52 \text{ mm}$$

(ii)
$$\varepsilon_{sc} = 0.0035 \frac{(x_{u,lim} - d_c)}{x_{u,lim}}$$
$$= 0.0035 \times \frac{(212.52 - 50)}{212.52}$$
$$\varepsilon_{sc} = 0.002676 > 0.00109$$
$$f_s = 0.87\, f_y$$

(iii) Actual depth of NA

$$C = T$$
$$C_1 + C_2 = T$$
$$\Rightarrow \quad 0.36\, f_{ck}\, b\, x_u + (f_{sc} - 0.45\, f_{ck})\, A_{sc} = 0.87\, f_y\, A_{st}$$
$$\Rightarrow \quad 0.36 \times 20 \times 250 \times x_u + (f_{sc} - 0.45 \times 20) \times 1017.87 = 0.87 \times 250 \times 1963.49$$
$$\Rightarrow \quad 1800\, x_u + (f_{sc} - 9) \times 1017.87 = 708918.06 \quad \text{...(i)}$$

Put $f_{sc} = 0.87\, f_y$ in above eq. (i)

$$1800\, x_u + (0.87 \times 250 - 9) \times 1017.87 = 708918.06$$
$$x_u = 275.94 \text{ mm} > x_{u,lim} \text{ section is over reinforced}$$
$$M_{u,lim} = 0.36\, f_{ck}\, x_{u,lim}\, b(d - 0.42\, x_{u,lim}) + (f_{sc} - 0.45\, f_{ck}) A_{sc} (d - d_c)$$
$$= 0.36 \times 20 \times 212.52 \times 250\, (400 - 0.42 \times 212.52)$$
$$+ (0.87 \times 250 - 0.45 \times 20) \times 1017.87 \times (400 - 50)$$
$$M_{u,lim} = 193.14 \text{ kN m}$$

Example 18

Determine ultimate moment capacity of a doubly reinforced beam with $b = 300$ mm, $d = 600$ mm, $A_{st} = 2060$ mm², $A_{sc} = 804$ mm² and effective cover $d_c = 50$ mm for both tension and compression steels. The materials used are M 20 concrete and HYSD steel of grade Fe 415.

Sol:

(i) ***Given data:***

$$b = 300 \text{ mm}$$
$$D = 600 \text{ mm}$$
$$A_{st} = 2060 \text{ mm}^2$$
$$A_{sc} = 804 \text{ mm}^2$$
$$d_c = 50 \text{ mm}$$
$$d = D - d_c = 600 - 50 = 550 \text{ mm}$$

M 20/Fe 415

(ii) ***Limiting depth of neutral axis***

$$x_{u,lim} = 0.48\, d$$
$$= 0.48 \times 550$$

(iii) *Actual depth of neutral axis:*

Total compressive force = Total tensile force

$$0.36 f_{ck} b x_u + A_{sc}(f_{sc} - 0.45 f_{ck}) = 0.87 f_y A_{st}$$

$$\Rightarrow 0.36 \times 20 \times 300 \times x_u + 804 (f_{sc} - 0.45 \times 20) = 0.87 \times 415 \times 2060$$

$$\Rightarrow 2160 x_u + 804 (f_{sc} - 9) = 743763$$

$$\Rightarrow 2160 x_u + 804 f_{sc} - 7236 = 743763$$

$$\Rightarrow 2160 x_u + 804 f_{sc} = 736527$$

$$\Rightarrow x_u = \frac{736527 - 804 f_{sc}}{2160}$$

Trial 1:

Taking $f_{sc} = 350$ MPa

We get, $$x_u = \frac{736527 - 804 \times 350}{2160}$$

$$\Rightarrow x_u = 210.71 \text{ mm}$$

$$\text{Value of } \varepsilon_{sc} = \frac{0.0035}{x_u}(x_u - d_c)$$

$$= \frac{0.0035}{210.71}(210.71 - 50)$$

$$\varepsilon_{sc} = 0.00267$$

$$f_{sc} \text{ for } 0.00267 = \frac{(351 - 342)}{(0.00276 - 0.00241)}(0.00267\ 0.00241)$$

$$= 342 + 6.686$$

$$= 348.68 \text{ MPa}$$

Trial 2

$$x_u = \frac{736527 - 804 \times 348.68}{2160}$$

$$x_u = 211.19 \text{ mm}$$

$$\text{Value of } \varepsilon_{sc} = \frac{0.0035}{211.19}(211.19 - 50)$$

$$\varepsilon_{sc} = 0.00267$$

Hence $f_{sc} = 348.68$ MPa and $x_u = 211.19$ mm (adopted)

$\because x_u < x_{u,lim}$. Hence section is under reinforced.

(iv) *Moment of resistance*

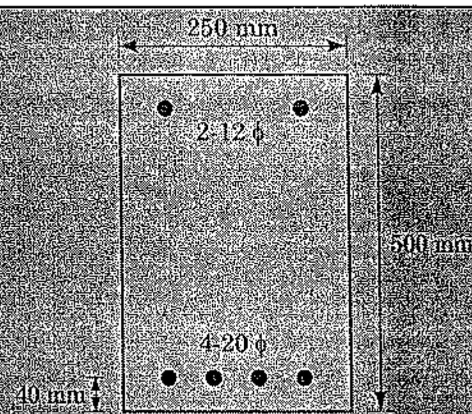
$$MR = 0.36 f_{ck} b_{xu} + (f_{sc} - 0.45 f_{ck}) A_{sc}(d - d_c) \times (d - 0.42 x_u)$$

$$= [0.36 \times 20 \times 300 \times 211.19 \times (550 - 0.42 \times 211.18)] + [(348.68 - 0.45 \times 20) \times 804 \times (550 - 50)]$$

$$= 346.98 \text{ kNm}$$

Example 19

Find the moment of resistance of a beam 25 cm by 50 cm deep if it is reinforced with 2-12 mm bars in compression zone and 4-20 mm bars in tension zone, each at an effective cover of 40 mm as shown



Section

Fig. 1.25

Sol: For M 15 mix and Fe 250 grade steel.

First trial:

Critical depth of N.A.

$$\frac{x_{u,lim}}{d} = 0.53$$

$$x_{u,lim} = 0.53 \times d$$

$$= 0.53 \times (500 - 40)$$

$$x_{u,lim} = 244 \text{ mm}$$

$$\varepsilon_{sc} = 0.0035 \frac{(244-40)}{244} = 0.00293$$

$\varepsilon_{sc} > 0.00109$ (yield strain for mild steel)

$$f_{sc} = 0.87 f_y$$

$$A_{sc} = 2 \times \frac{\pi}{4} \times 12^2 = 226 \text{ mm}^2$$

Total force of compression $\quad C = C_1 + C_2 = T$

$$0.36 f_{ck} x_u b + (f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y A_{st}$$

$$0.36 \times 15 x_u 250 + (f_{sc} - 0.45 \times 15)\, 226 = 0.87 \times 250 \times 4 \times 314$$

$$1350\, x_u + (f_{sc} - 6.75)\, 226 = 273180 \qquad \text{...(i)}$$

Put $f_{sc} = 0.87 f_y$ in eq. (i)

$$1350\, x_u + (0.87 \times 250 - 6.75)\, 226 = 273180$$

$$x_u = 167.07 \text{ mm}$$

$$x_u = 167.07 \text{ mm} < 244 \text{ mm} \quad \text{section is under reinforced}$$

$$\varepsilon_{sc} = 0.0035 \left(\frac{167.07 - 40}{167.07}\right) = 0.00266 > 0.00109$$

$$f_{sc} = 0.87 f_y$$

$$M_u = 0.36 f_{ck}\, b\, x_u (d - 0.42 x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d_c)$$

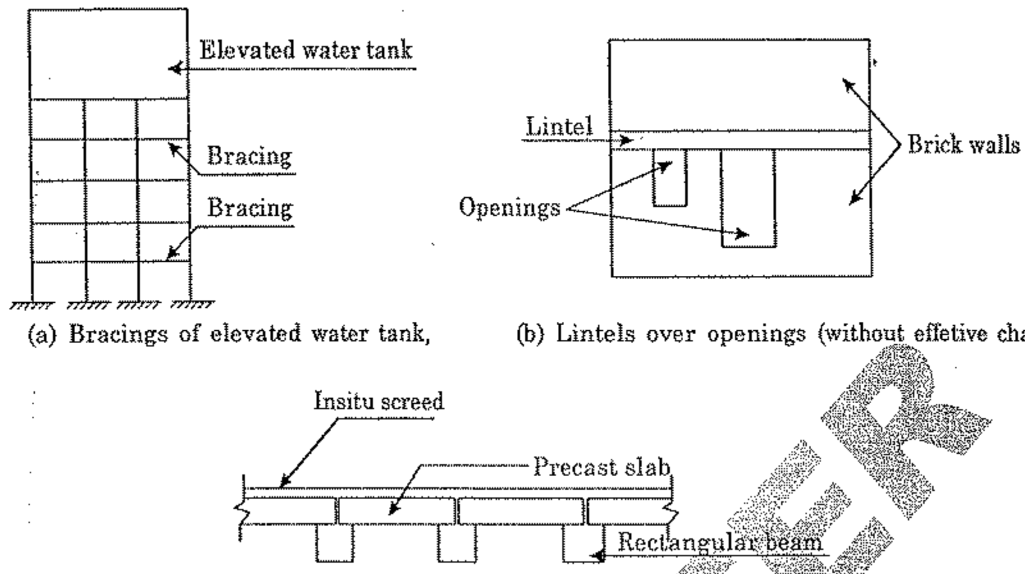
$$= 0.36 \times 15 \times 250 \times 167.07 (460 - 0.42 \times 167.07) +$$

$$(0.87 \times 250 - 0.45 \times 15) \times 226 (460 - 40)$$

$$M_u = 107.92 \text{ kN-m}$$

FLANGED BEAM

INTRODUCTION

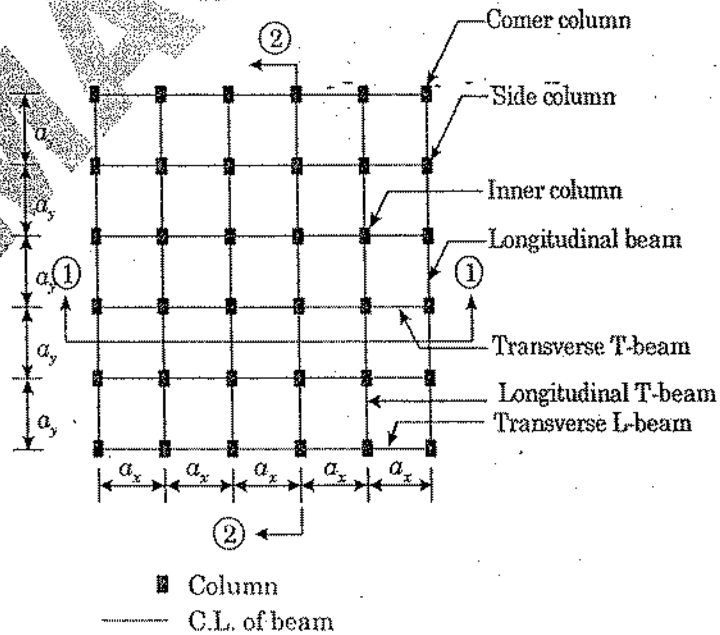


(a) Bracings of elevated water tank, (b) Lintels over openings (without effetive chajjas).

(c) Precast slab on rectangular beams.

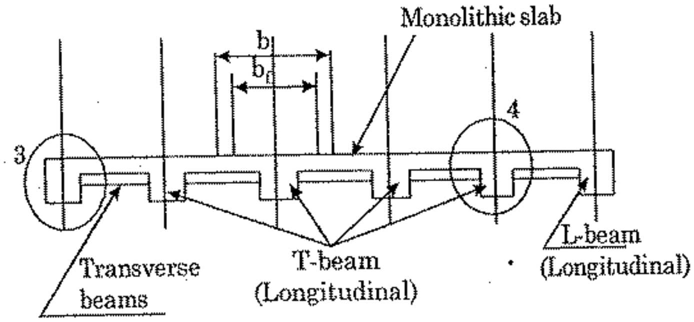
Fig. 1.26

Reinforced concrete slabs used in floors, roofs and decks are mostly cast monolithic from the bottom of the beam to the top of the slab. Such rectangular beams having slab on top are different from others having either no slab (bracings of elevated tanks, lintels etc.) or having disconnected slabs as in some pre-cast systems (Figs. 1.26 (a)-(c)). Due to monolithic casting, beams and a part of the slab act together. Under the action of positive bending moment, i.e. between the supports of a continuous beam, the slab, up to a certain width greater than the width of the beam, forms the top part of the beam. Such beams having slab on top of the rectangular rib are designated as the flanged beams-either *T* or *L* type depending on whether the slab is on both sides or on one side of the beam (Figs. 1.27(a)-(e)). Over the supports of a continuous beam, the bending moment is negative and the slab, therefore, is in tension while a part of the rectangular beam (rib) is in compression. The continuous beam at support is thus equivalent to a rectangular beam (Figs. 1.27(a), (c), (f) and (g)).



Notations: a_x = c/c distance of longitudinal beams
a_y = c/c distance of transverse beams

Fig. 1.27(a): Key plan



Natations:
b = Actual width of flange
b_f = Effective width of flange

Fig. 1.27(b): Section 1-1

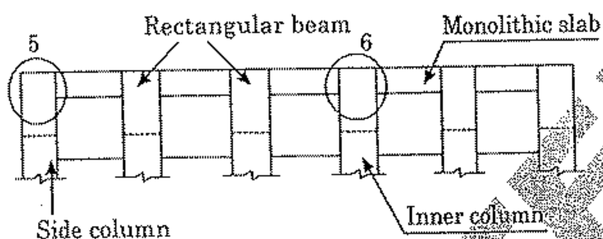
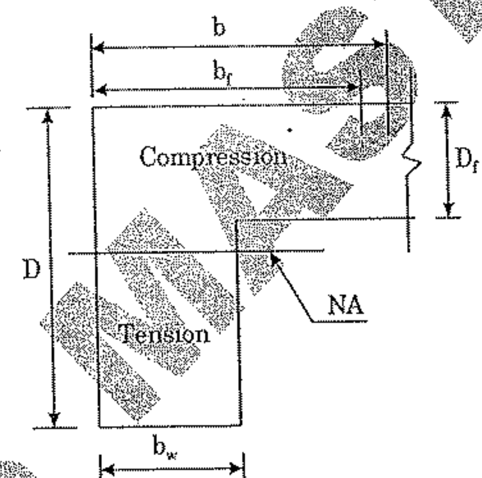
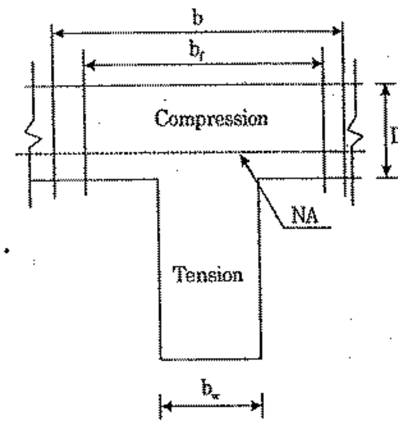


Fig. 1.27(c): Section 2-2



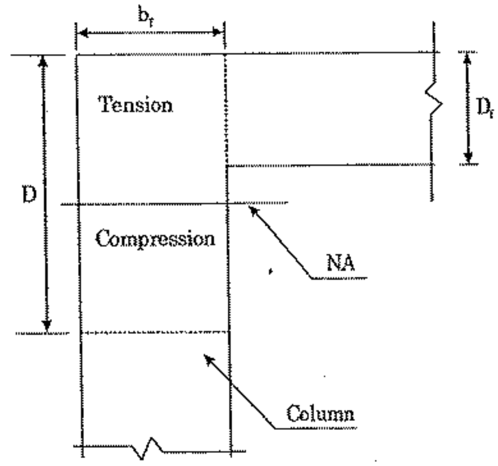
Notations:
b = Actual width of flange
b_f = Effective width of flange
b_w = Wdith of web
D_f = Depth of flange
NA = Neutral axis

Fig. 1.27(d): Detail at 3 (L-beam).



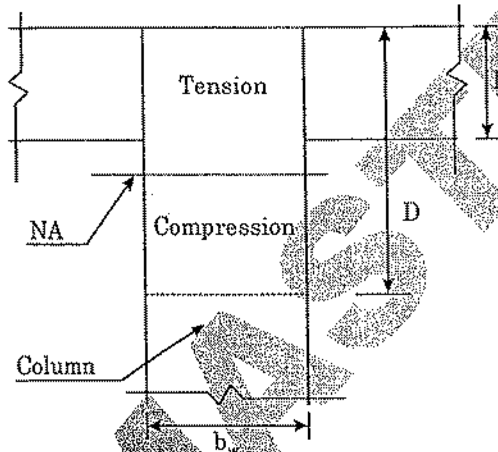
Notations:
b = Actual width of flange
b_f = Effective width of flange
b_w = Wdith of web
D_f = Depth of flange
NA = Neutral axis

Fig. 1.27(e): Detail at 4(T-beam)



Notations:
b_f = Effective width of flange
b_w = Wdith of web
D_f = Depth of flange
NA = Neutral axis

Fig. 1.27(f): Detail at 5 (rectangular beam).



Notations:
b_f = Effective width of flange
b_w = Wdith of web
D_f = Depth of flange
NA = Neutral axis

Fig. 1.27(g): Detail at 6 (rectangular beam).

The actual width of the flange is the spacing of the beam, which is the same as the distance between the middle points of the adjacent spans of the slab, as shown in Fig. 1.14(b). However, in a flanged beam, a part of the width less than the actual width, is effective to be considered as a part of the beam. This width of the slab is designated as the effective width of the flange.

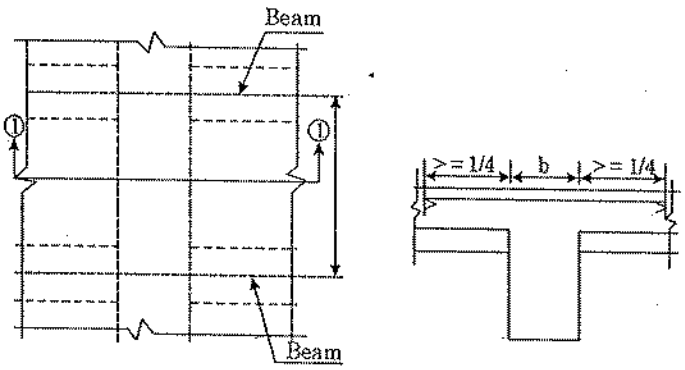
EFFECTIVE WIDTH

IS Code Requirements

The following requirements (cl. 23.1.1 of IS 456) are to be satisfied to ensure the combined action of the part of the slab and the rib (rectangular part of the beam).

(a) The slab and the rectangular beam shall be case integrally or they shall be effectively bonded in any other manner.

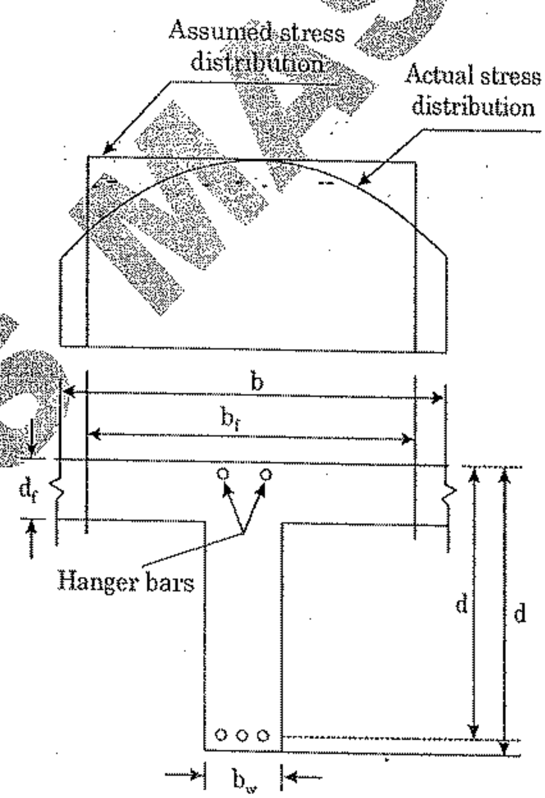
(b) Slabs must be provided with the traverse reinforcement of at least 60 per cent of the main reinforcement at the mid span of the slab if the main reinforcemtn of the slab is parallel to the traverse beam (Figs. 1.28(a) and (b)).



(a) Plan of slab-beam (b) Section 1-1

Fig. 1.28: Traverse reinforcement of flange of T-beam

- The variation of compressive stress (Fig. 1.29) along the actual width of the flange shows that the compressive stress is more in the flange just above the rib than the same at some distance away from it.
- The nature of variation is complex and, therefore, the concept of effective width has been introduced.
- The effective width is a convenient hypothetical width of the flange over which the compressive stress is assumed to be uniform to give the same compressive force as it would have been in case of the actual width with the true variation of compressive stress.



IS Code Specifications

Clasue 23.1.2 of IS 456 specifies the following effective widths of T and L beams:

(a) For T-beams, the lesser of

(i) $b_f = l_o/6 + b_w + 6\,d_f$

(ii) b_f = Actual width of the flange

(b) For isolated T-beams, the lesser of

(i) $$b_f = \frac{l_o}{(l_o/b)+4} + b_w$$

(ii) b_f = Actual width of the flange

(c) For L-beams, the lesser of

(i) $b_f = l_o/12 + b_w + 3d_f$

(ii) b_f = Actual width of the flange

(d) For isolated L-beams, the lesser of

(i) $$b_f = \frac{0.5\,l_o}{(l_o/b)+4} + b_w$$

(ii) b_f = Actual width of the flange

where b_f = effective width of the flange,

l_o = distance between points of zero moments in the beam, which is the effective span for simply supported beams and 0.7 times the effective span for continuous beams and frames,

b_w = beadth of the web,

d_f = thickness of the flange,

and b = actual width of the flange.

Analysis

Actual depth of N.A. of T beam

Case 1: **When N.A. lies is flange area. ($x_u < d_f$)**

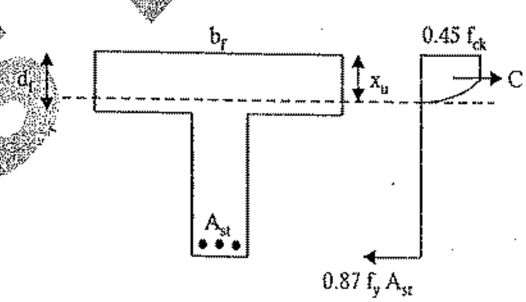


Fig. 1.30

This is the simple case of rectangular beam. For that depth of neutral axis is

$$C = T$$

$$0.36 f_{ck}\, x_u\, b_f = 0.87\, f_y\, A_{st}$$

$$\boxed{x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck}.b_f}}$$

$$M_u = C \times L.A$$

$$= 0.36\, f_{ck}\, x_u.b_f\, (d - 0.42\, x_u)$$

$$M_u = T \times L.A$$

$$M_u = 0.87\, f_y A_{st}\, (d - 0.42\, x_u)$$

where C and T is the compressive and tensile force respectively.

***Case 2:* When neutral axis lies out side the flange area ($x_u > d_f$)**

Sub case* (1): $x_u > d_f$ but $d_f < \frac{3}{7} x_u$ *(it means flange lie within the rectangular portion of stress diagram, where uniform stress is 0.45 f_{ck})

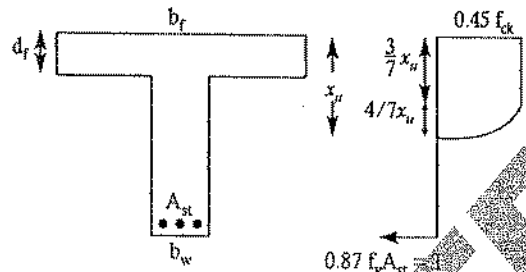


Fig. 1.30(a)

Figure 1.30(a) can be converted in two portion that shown in Fig. 1.30(b) below.

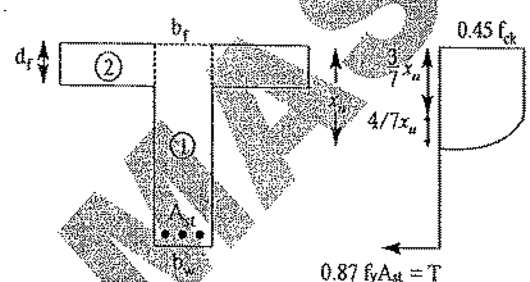
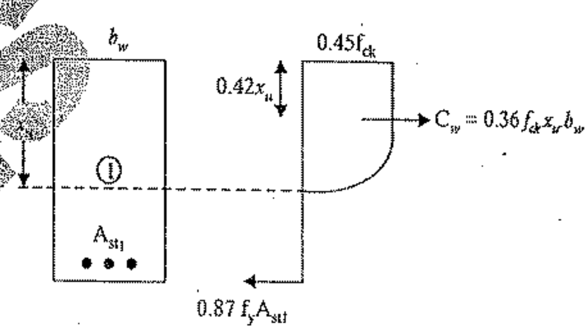


Fig. 1.30(b)

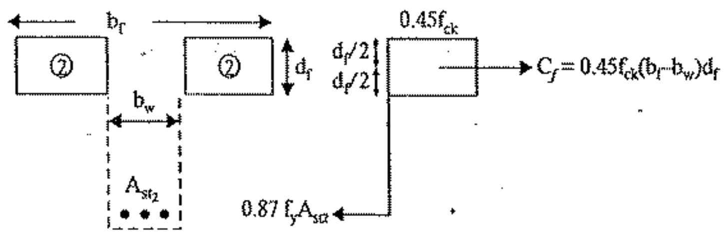


Stress diagram of rectangular portion 1 of Fig. 1.30(b)

$$C_w = \text{Compressive force in web}$$

$$C_w = 0.36\, f_{ck} x_u b_w$$

$$LA_1 = (d - 0.42\ x_u),\quad LA_1 = \text{Lever Arm}$$



Stress diagram of flange portion 2 of Fig. 1.30(b)

C_f = Compressive force in flange portion

$$C_f = 0.45\ f_{ck}\ (b_f - b_w)\ d_f$$

$$LA_2 = \left(d - \frac{d_f}{2}\right)$$

For actual depth of neutral axis

$$C_w + C_f = T$$

$$0.36\ f_{ck} x_u . b_w + 0.45 f_{ck}(b_f - b_w).d_f = 0.87\ f_y A_{st}$$

For moment of resistance

$$M_u = C_w . LA_1 + C_f . LA_2$$

$$M_u = 0.36\ f_{ck}\ b_w x_u\ (d - 0.42 x_u) + 0.45\ f_{ck}\ (b_f - b_w) d_f\ (d - d_f/2)$$

Design (for a balance section)

$$M_{u,\text{lim}} = 0.36\ f_{ck} b_w x_{u,\text{lim}} (d - 0.42\ x_{u,\text{lim}}) + 0.45\ f_{ck}\ (b_f - b_w)\ d_f \left(d - \frac{d_f}{2}\right)$$

$$M_{u,\text{lim}} = 0.87\ f_y A_{st1} (d - 0.42\ x_{u,\text{lim}}) + 0.87\ f_y A_{st2} \left(d - \frac{d_f}{2}\right)$$

Equate moment of resistance of web of compression side to moment of resistance of web of tensile side to get A_{st1}

$$0.36 f_{ck}\ b_w\ x_{u\text{lim}}\ (d - 0.42\ x_{u,\text{lim}}) = 0.87\ f_y\ A_{st1}\ (d - 0.42\ x_{u,\text{lim}})$$

$$A_{st1} = \frac{0.36\ f_{ck} b_w x_{u,\text{lim}} (d - 0.42 x_{u,\text{lim}})}{0.87\ f_y (d - 0.42 x_{u,\text{lim}})}$$

$$A_{st1} = \frac{0.36\ f_{ck} b_w x_{u,\text{lim}}}{0.87\ f_y}$$

Similarly, for A_{st2}

$$0.45 f_{ck}\ (b_f - b_w) d_f \left(d - \frac{d_f}{2}\right) = 0.87\ f_y\ A_{st2} \left(d - \frac{d_f}{2}\right)$$

$$A_{st2} = \frac{0.45\ f_{ck} (b_f - b_w) d_f}{0.87\ f_y}$$

***Sub Case (2)*: When $x_u > d_f$ and $d_f > \frac{3}{7}X_u$. In this case flange will be lie partially in rectangular and parabolic portion of stress diagram.**

Is 456 recomended an equivalent depth of flange such that stress diagrams for equivalent flange is rectangular having constant stress of 0.45 f_{ck}.

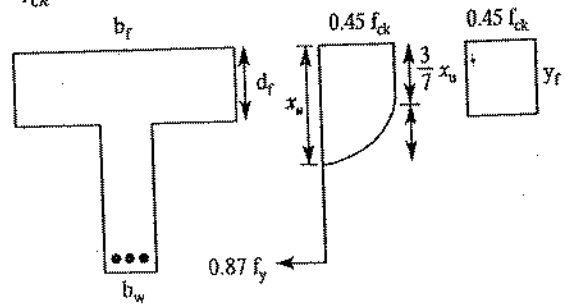


Fig. 1.31

where, $\boxed{y_f = 0.15x_u + 0.65d_f}$

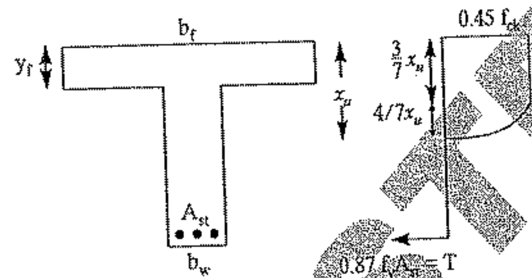


Fig. 1.32(a)

Figure 1.32(a) can be converted in two portion that shown in Fig. 1.32(b) below.

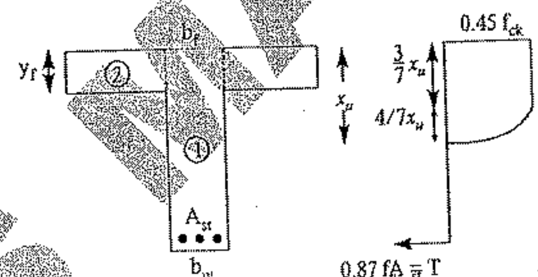
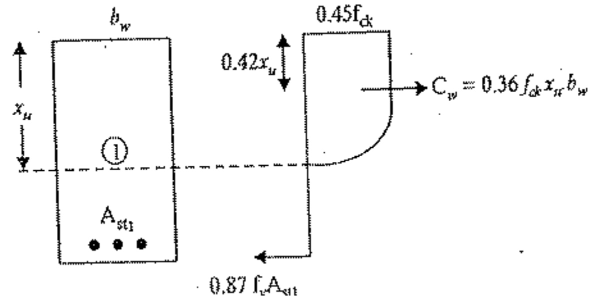


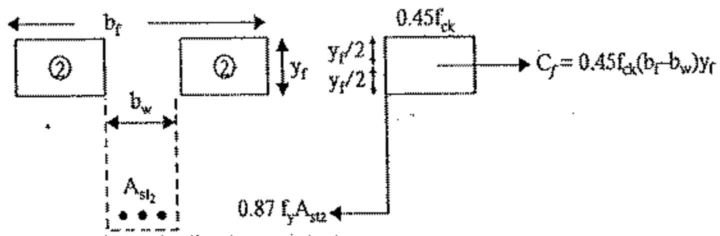
Fig. 1.32(b)



C_w = Compressive force in web

$C_w = 0.36\, f_{ck} x_u b_w$

$LA_1 = (d - 0.42\, x_u)$, $L.A_1$ = Lever Arm



Stress diagram of flange portion 2 of Fig. 1.32(b)

C_f = compressive force in flange portion

$C_f = 0.45\, f_{ck}\, (b_f - b_w)\, y_f$

$$LA_2 = \left(d - \frac{y_f}{2}\right)$$

For actual depth of neutral axis

$$C_w + C_f = T$$

$$0.36\, f_{ck} x_u . b_w + 0.45 f_{ck}(b_f - b_w).y_f = 0.87\, f_y A_{st}$$

For moment of resistance

$$M_u = C_w . LA_1 + C_f . LA_2$$

$$M_u = 0.36\, f_{ck}\, b_w\, x_u\, (d - 0.42 x_u) + 0.45\, f_{ck}\, (b_f - b_w) y_f\, (d - y_f/2)$$

Design (for a balance section)

$$M_{u,\lim} = 0.36\, f_{ck} b_w x_{u,\lim}\, (d - 0.42\, x_{u,\lim}) + 0.45\, f_{ck}\, (b_f - b_w)\, y_f \left(d - \frac{y_f}{2}\right)$$

$$M_{u,\lim} = 0.87\, f_y A_{st_1} (d - 0.42\, x_{u,\lim}) + 0.87\, f_y A_{st2} \left(d - \frac{y_f}{2}\right)$$

Equate moment of resistance of web of compression side to moment of resistance of web of tensile side to get A_{st_1}

$$0.36 f_{ck}\, b_w x_{u\lim} (d - 0.42\, x_{u,\lim}) = 0.87\, f_y\, A_{st1}\, (d - 0.42\, x_{u,\lim})$$

$$A_{st_1} = \frac{0.36\, f_{ck} b_w x_{u,\lim} (d - 0.42 x_{u,\lim})}{0.87\, f_y (d - 0.42 x_{u,\lim})}$$

$$A_{st_1} = \frac{0.36\, f_{ck} b_w x_{u,\lim}}{0.87\, f_y}$$

Similarly, for A_{st2}

$$0.45 f_{ck}\, (b_f - b_w) y_f \left(d - \frac{y_f}{2}\right) = 0.87\, f_y\, A_{st2} \left(d - \frac{y_f}{2}\right)$$

$$A_{st2} = \frac{0.45\, f_{ck} (b_f - b_w) y_f}{0.87\, f_y}$$

Numerical Problems (Analysis Type)

Example 20

Determine the moment of resistance of the T-beam. Given data: b_f = 1000 mm, d_f = 100 mm, b_w = 300 mm, cover = 50 mm, d = 450 mm and A_{st} = 1963 mm² (4-25 T). Use M 20 and Fe 415.

Sol: ***Step 1: To determine the depth of the neutral axis x_u***

Assuming x_u in the flange and equating total compressive and tensile forces from the expressions of *C* and *T* as the *T*-beam can be treated as rectangular beam of width b_f and effective depth *d*, we get

$$x_u = \frac{0.87 f_y A_{st}}{0.36 b_f f_{ck}} = \frac{0.87(415)(1963)}{0.36(1000)(20)} = 98.44 \text{ mm} < 100 \text{ mm}$$

So, the assumption of x_u in the flange is correct.

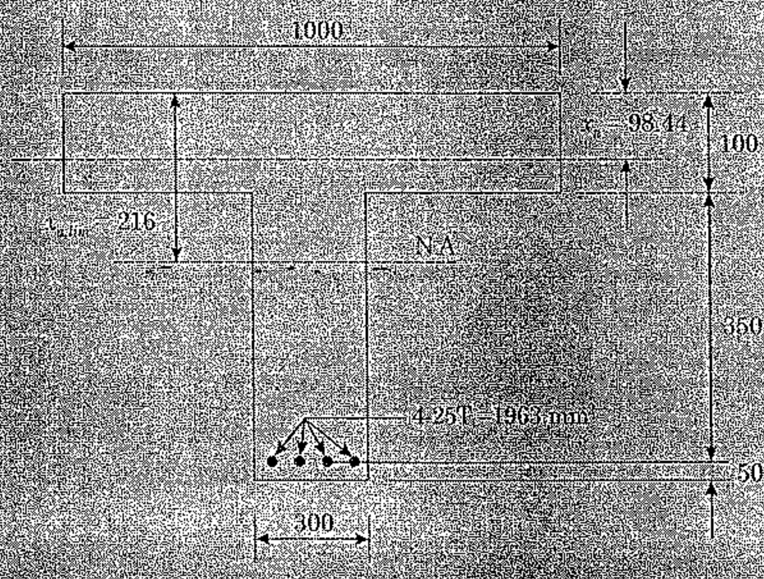
$x_{u,lim}$ for the balanced rectangular beam = 0.48 *d* = 0.48 (450) = 216 mm.

Step 2: To determine M_u

(using $b = b_f$) for M_u, we have:

$$M_u = 0.87 f_y A_{st} (d - .42 x_u)$$
$$= 0.87 (415)(1963) (450 - 0.42 \times 98.44) = 289.63 \text{ kNm}$$

This problem belongs to the case (i).



Exmaple 20 case (i)

Example 21

Determine $A_{st,lim}$ and $M_{u,lim}$ of the flanged beam. Given data are: b_f = 1000 mm, d_f = 100 mm, b_w = 300 mm, cover = 50 mm and d = 450 mm. Use M 20 and Fe 415.

Sol: *Step 1:*

For the limiting case $x_u = x_{u,lim}$ = 0.48 (450) = 216 mm > d_f. The ratio d_f/d is computed.

$$\frac{3}{7}x_{u,lim} = \frac{3}{7}\times 216 = 92.57 \text{ mm}$$

$$x_{u,lim} > d_f \text{ and } d_f > \frac{3}{7}x_{u,lim}$$

This is sub case (2) of case (ii) type of problem.

Step 2: Computations of y_f, C and T

First, we have to compute y_f

$$y_f = 0.15\,x_{u,lim} + 0.65\,d_f = 0.15\,(216) + 0.65\,(100) = 97.4 \text{ mm}$$

$$C = 0.36\,f_{ck}\,b_w\,x_{u,lim} + 0.45\,f_{ck}\,(b_f - b_w)\,y_f$$

$$= 0.36\,(20)\,(300)(216) + 0.45\,(20)(1000 - 300)\,(97.4)$$

$$C = 1{,}080.18 \text{ kN}$$

$$T = 0.87\,f_y\,A_{st} = 0.87\,(415)\,A_{st}$$

Equating C and T, we have

$$A_{st} = \frac{(1080.18)(1000)\,N}{0.87(415)\,\text{N/mm}^2} = 2{,}991.77 \text{ mm}^2$$

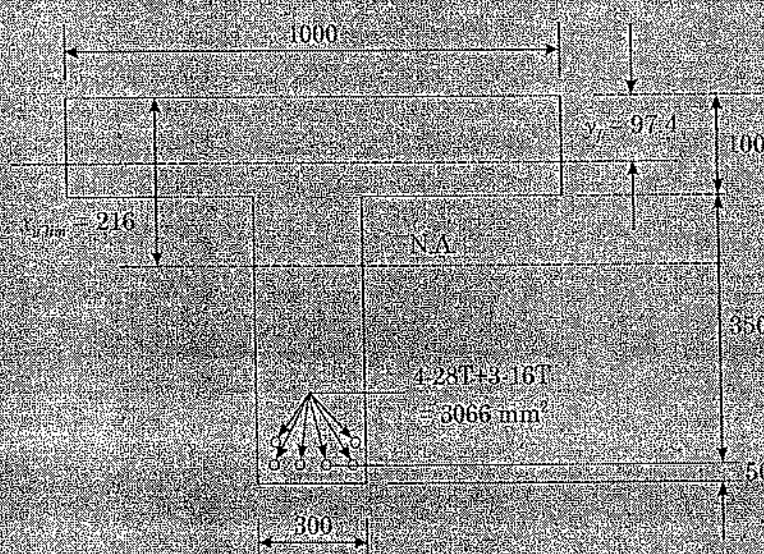
Provide 4-28 T (2463 mm²) + 3-16 T (603 mm²) = 3,066 mm²

Step 3: Computational of M_u

$$M_{u,lim} = 0.36\left(\frac{x_{u,lim}}{d}\right)\left\{1 - 0.42\left(\frac{x_{u,lim}}{d}\right)\right\}f_{ck}b_w d^2 + 0.45 f_{ck}(b_f - b_w)\,y_f\,(d - y_f/2)$$

$$= 0.36(0.48)\{1 - 0.42(0.48)\}(20)(300)(450)^2 + 0.45(20)(1000 - 300)(97.4)(450 - 97.4/2)$$

$$= 413.87 \text{ kNm}$$



Example 21 sub case (2) of case (ii)

Example 22

Determine the moment of resistance of the beam. When A_{st} = 2,591 mm² (4-25 T and 2-20 T), b_f = 1,000 mm, d_f = 100 mm, b_w = 300 mm, cover = 50 mm and d = 450 mm. Use M 20 and Fe 415. Use LSM.

Sol: ***Step 1: To determine x_u***

Assuming x_u to be in the flange and the beam is under- reinforced

$$x_u = \frac{0.87 f_y A_{st}}{0.36\, b_f f_{ck}} = \frac{0.87(415)(2591)}{0.36(1000)(20)} = 129.93 \text{ mm} > 100 \text{ mm}$$

Since $x_u > d_f$ the neutral axis is in web. Here,

$$d_f (100 \text{ mm}) > \frac{3}{7} \times 4 \left(\frac{3}{7} \times 129.93 = 55.68 \text{ mm} \right)$$

This is sub case (2) of case (ii) type of problem.

So, we have to substitute the term y_f, we get depth of N.A. from $C = T$

$$0.36 f_{ck} b_w x_u + 0.45 f_{ck} (b_f - b_w) y_f = 0.87 f_y A_{st}$$

or $\quad 0.36(20)(300)(x_u) + 0.45(20)(1000 - 300)\{0.15 x_u + 0.65(100)\}$

$$= 0.87(415)(2591)$$

or $\quad x_u = 169.398 \text{ mm} < 216 \text{ mm } (x_{u,lim} = 0.48 x_u = 216 \text{ mm})$

So, the section in under-reinforced.

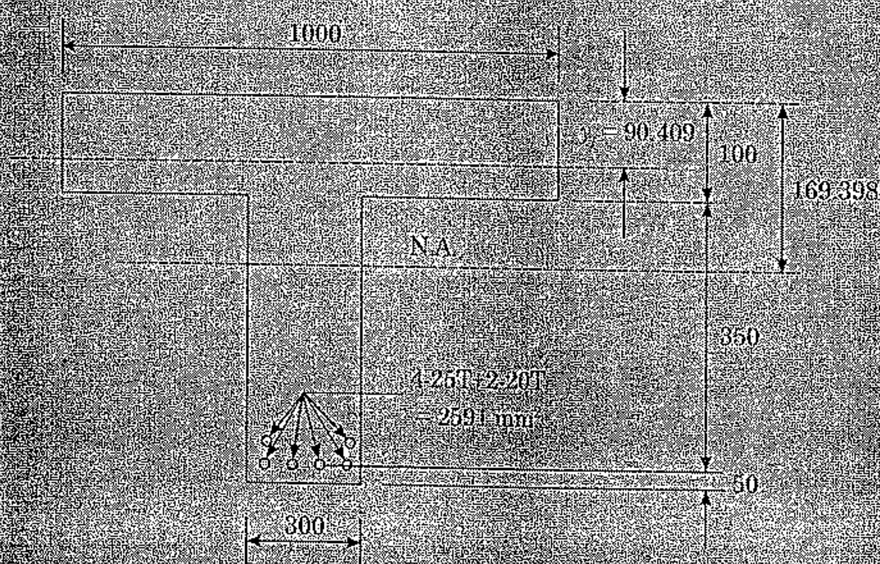
Step 2: To determine M_u

$$y_f = 0.15 x_u + 0.65 d_f = 0.15(169.398) + 0.65(100) = 90.409 \text{ mm}$$

$$M_u = 0.36(169.398/450)\{1 - 0.42(169.398/450)\}(20)(300)(450)$$

$$(450) + 0.45(20)(1000 - 300)(90.409)(450 - 90.409/2)$$

$$= 138.62 + 230.56 = 369.18 \text{ kNm.}$$



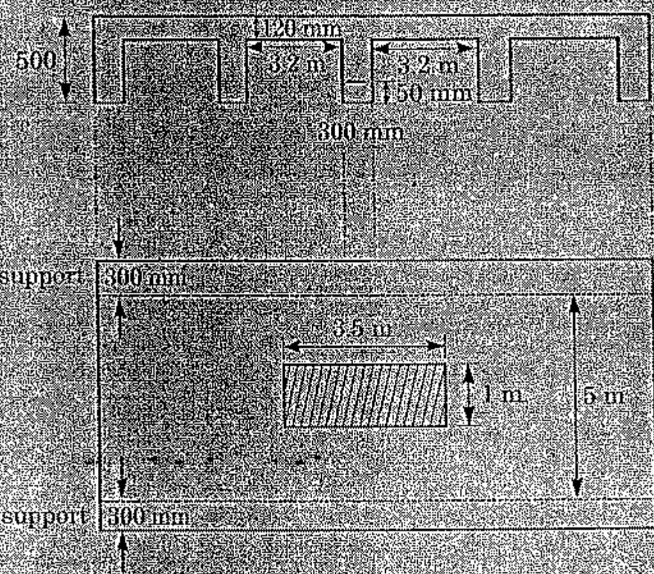
Example 22 sub case (2) of case (ii)

Example 23

A RC beam of overall dimensions 300 × 500 mm rests on a brick wall 300 mm thick clear span is 5 m. The beams are spaced at 3.5 m internals. Thickness of slab supported by the beam is 120 mm. Live load of the (on the) slab is 200 kg/m². Floor finish weight 60 kg/m². Concrete adopted is M 20 and mild steel is used for reinforcements. Effective cover to the centre of reinforcement is 50 mm. Design the reinforcements required for flexure and shear. Draw a net longitudinal section of the beam and indicate the placement of reinforcements.

Sol:

(i) Using LSM



Live load = 200 kg/m^2 = 2 kN/m^2

Floor finishing = 60 kg/m^2 = 0.6 kN/m^2

Clear spear l_0 = 5 m

l = width of support = 300 mm

d_f = 120 mm

M 20/Fe 250 are used

b_w = 300 m

Effective cover = 50 cm

D = 500 mm

$d = D - 50 = 500 - 50 = 450$ mm

Step (i) Effective span

If $l < \frac{1}{12}$ clear span (l_0), then effective span will be

$\left.\begin{matrix} l_0 + d \\ l_0 + l_s \end{matrix}\right\}$ lesser of the two

Now $300 < \frac{1}{12} \times 5000$

$\Rightarrow 300 < 416.66$

$5000 + 450 = 5450$

$5000 + 450 = 5300$

Hence l_{eff} = 5300 mm = 5.3 m

Step (2) Load calculations

Load for slab (load per miter run of beam) = load on slab per unit area × centre to centre distance between beams.

Self wt. of slab = 3.5 × 1 × 0.12 × 25 = 10.5 kN

Load for beam

$$\text{Self weight of beam} = (0.5 - 0.12) \times 0.3 \times 1 \times 25 = 2.85 \text{ kN}$$

$$\text{Floor finishing load} = 0.6 \times 3.5 \times 1 = 2.1 \text{ kN}$$

$$\therefore \text{Total load on the beam 1 m length} = 10.5 + 7 + 2.85 + 2.1 = 22.45 \text{ kN}$$

Step (3)

$$\text{Max BM} = \frac{wl_{eff}^2}{8} = \frac{22.45 \times 5.3 \times 5.3}{8} = 78.83 \text{ kNm}$$

$$\text{Max shear force } V = \frac{wl_{eff}}{2} = \frac{22.45 \times 5.3}{2} = 59.49 \text{ kNm}$$

Step (4)

Effective width of flange (b_f)

$$b_f = \frac{l_0}{6} + b_w + 6d_f$$

$$l_0 = l_{eff}$$

$$b_f = \frac{5300}{6} + 300 + (6 \times 120)$$

$$b_f = 1903.33 \text{ mm}$$

Also $\quad b_f \geq b_w + \frac{l_1}{2} + \frac{l_2}{2} \Rightarrow 1903.33 \geq 300 + \frac{3200}{3} + \frac{3200}{3} = 3500$

or $\quad b1903.33 \geq 3500$ Hence OK

By limit state method

Step 1 Load calculatings and BM and SF

$$w_u = 1.5\, w = 1.5 \times 22.45 = 33.675 \text{ kN}$$

$$\text{BM}_{max} = \frac{w_u l_{eff}^2}{8} = \frac{33.675 \times 5.3^2}{8} = 118.24 \text{ kNm}$$

$$V_u = \frac{w_u l_{eff}}{2} = \frac{33.675 \times 5.3}{2} = 89.24 \text{ kN}$$

Assuming the neutral axis lies in flange

$$M_u = 0.87\, f_y\, A_{st}\, (d - 0.42\, x_u)$$

and $\quad x_u = \frac{0.87\, f_y A_{st}}{0.36\, f_{ck}\, b_f}$

$$x_u = 0.01587\, A_{st}$$

$$M_u = 0.87\, f_y\, A_{st}\, (d - 0.42 \times 0.01587\, A_{st})$$

$$118.24 \times 10^6 = 0.87 \times 250\, A_{st}\, (450 - 0.42 \times 0.1587\, A_{st})$$

$$= 97875\, A_{st} - 1.45\, A_{st}^2$$

$$A_{st} = 1230.50 \text{ mm}^2$$

and $\quad x_u = 0.01587 \times 1230.50$

$= 19.52 \text{ mm} < 120 \text{ mm}$. N.A. lies in flange

$$x_{u,lim} = 0.53 \times 450 = 238.5 \text{ mm}$$

$x_u < x_{u,lim}$ section is under reinforced.

NA lies in the flange

$$A_{st\,min} = \frac{0.85\,bd}{f_y} = \frac{0.85\,b_w d}{f_y}$$

$$= \frac{0.85 \times 300 \times 561}{250} = 572.22 \text{ mm}^2$$

$$A_{st}(2328.50) > A_{st,min}(572.22)$$

Provide 4 nos. 28 mm dia bar (2463 mm²) 2328.50 mm²

Maximum area of tension steel $= 0.04\,b_w D$

$= 0.04 \times 300 \times 600 = 7200 \text{ mm}^2$

$> 2463.0 \text{ mm}^2$

Step (5)

$$V_u = 89.24 \text{ kN}$$

$$\tau_v = \frac{V_u}{b_w d} = \frac{89.24 \times 10^3}{300 \times 450} = 0.66 \text{ N/mm}^2$$

for τ_c, we have $\frac{A_{st} \times 100}{b_w d} = \frac{1230 \times 100}{300 \times 450} = 0.91\%$

$$\tau_c = 0.35 + \left(\frac{0.39 - 0.35}{1.00 - 0.75}\right)(0.91 - 0.75)$$

$$\tau_c = 0.3756 \text{ N/mm}^2$$

$$V_c = \tau_c b_w d = 0.3756 \times 300 \times 450 = 50.706 \text{ kN}$$

$V_u > V_c$ hence shear reinforcement is provided.

$$V_{us} = V_u - V_c = 89.24 - 20.706 = 38.53 \text{ kN}$$

Proving 2-legged 8 mm ϕ stirrups

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.53 \text{ mm}^2$$

$$\text{Spacing } s_v = \frac{A_{sv} \times d \times 0.87 f_y}{V_{us}}$$

$$= \frac{100.53 \times 450 \times 0.87 \times 250}{38.53 \times 10^3}$$

$$= 255.37 \approx 260 \text{ mm}$$

Check

$$s_v \le \frac{A_{sv} \times 0.87 f_y}{0.4\,b}$$

$$s_v \le \frac{100.53 \times 0.87 \times 250}{0.4 \times 300}$$

$$s_v \le 182 \text{ mm}$$

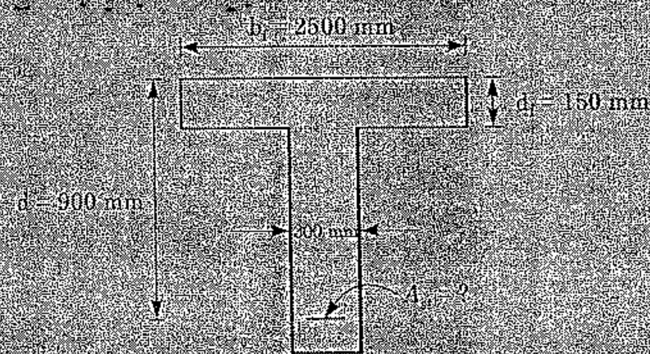
$$\text{Max. spacing} = 0.75\,d = 0.75 \times 450 = 337.7 \text{ mm}$$

Hence provide 180 mm c/c.

Example 24

A T-beam having an effective flange width of 2500 mm is required to resist an ultimate moment of 1200 kNm. Thickness of flange is 150 mm, width of the beam is 300 mm and the effective depth is 900 mm. Using M-15 grade concrete and Fe 250 grade steel. Determine the area of reinforcement requirement.

Sol:



$$M = 1200 \text{ kNm}$$

$$\text{Factor moment } M_u = 1.5 \times 1200 = 1800 \text{ kNm}$$

(i) Assuming the neutral axis in the flange area i.e. $x_u < d_f$ and calculating MR by putting $x_u = d_f = 150$ mm

$$M_1 = 0.36 f_{ck} b_f x_u (d - 0.42 x_u)$$

$$= 0.36 \times 15 \times 2500 \times 150\ (900 - 0.42 \times 150)$$

$$M_1 = 1694.92 \text{ kNm}$$

$M_1 < M_u$ (Hence our assumption is incorrect)

(ii) Now assuming the NA lies in the web area i.e. $x_u > d_f$ and $d_f < \frac{3}{7} x_u$ and calculating MR by putting

$$d_f = \frac{3}{7} x_u$$

$$\Rightarrow x_u = \frac{3}{7} d_f$$

$$= \frac{7}{3} \times 150 = 350 \text{ mm}$$

$$x_{u,lim} = 0.53\,d$$

$$= 0.53 \times 900 = 477 \text{ mm}$$

$x_u < x_{u,lim}$, Hence OK.

$M_2 = 0.36\, f_{ck}\, x_u\, (d - 0.42\, x_u) + 0.45\, f_{ck}\, (b_f - b_w)\, d_f \left(d - \frac{d_f}{2}\right)$

$= 0.36 \times 15 \times 300 \times 350 \times (900 - 0.42 \times 350) + 0.45 \times 15 \times (2500 - 300) \times 150 \times \left(900 - \frac{150}{2}\right)$

$= (0.36 \times 15 \times 300 \times 350 \times 753) + (0.45 \times 15 \times 2200 \times 150 \times 825)$

$M_2 = 2264.63$ kNm

$M_2 > M_u$. Hence our assumption is correct. It shows that $d_f > \frac{3}{7} x_u$ therefore y_f is used in the MR formula instead of d_f. Finding the position of actual neutral axis (x_u)

$$y_f = 0.15\, x_u + 0.65\, d_f$$
$$= 0.15\, x_u + 0.65 \times 150$$
$$y_f = 0.15\, x_u + 97.5$$

(iii) $MR = 0.36\, f_{ck}\, b_w x_u\, (d - 0.42\, x_u) + 0.45\, f_{ck}\, (b_f - b_w)\, y_f \left(d - \frac{y_f}{2}\right)$

$\Rightarrow 0.36 \times 15 \times 300 \times x_u (900 - 0.42\, x_u) + 0.45 \times 15\, (2500 - 300) \times (0.15\, x_u + 97.5) \left[900 - \left(\frac{0.15\, x_u + 97.5}{2}\right)\right]$

$= 1800 \times 10^6$

$\Rightarrow 900\, x_u - 0.42\, x_u^2 + 9.16\, (0.15\, x_u + 97.5)(900 - 0.075\, x_u - 48.75)$

$= 1.11 \times 10^6$

$\Rightarrow 900\, x_u - 0.42\, x_u^2 + (1.374\, x_u + 893.1)(851.25 - 0.075\ \text{kN}) = 1.11 \times 10^6$

$\Rightarrow 900\, x_u - 0.42\, x_u^2 + (1169.62\, x_u - 0.103\, x_u^2 + 760251.37 - 66.98\, x_u) = 1.11 \times 10^6$

$\Rightarrow -0.523\, x_u^2 + 2002.64\, x_u - 349748.63 = 0$

$\Rightarrow x_u = 183.43$ mm

$x_u < x_{u,lim}$. Hence O.K.

$y_f = 0.15\, x_u + 0.65\, d_f$

$= 0.15 \times 183.43 + 0.65 \times 150$

$y_f = 125.01$ mm

(iv) Find tensile reinforcement

Total compressive force = total tensile force

$0.36\, f_{ck}\, b_w\, x_u + 0.45\, f_{ck}\, (b_f - b_w) \times y_f = 0.87\, f_y\, A_{st}$

$\Rightarrow A_{st} = \frac{(0.36 \times 15 \times 300 \times 183.43) + (0.45 \times 15 \times (2500 - 300) \times 125.01)}{0.87 \times 250}$

$= 9901.40\ \text{mm}^2$

But $A_{st} \geq 0.04\, b_w d = 0.04 \times 300 \times (900 + 50) = 11400\ \text{mm}^2$

Hence O.K.

Example 25

A T-beam floor consists of 15 cm thick RC slab monolithic with 30 cm wide beams. The beams are spaced at 2.5 m centre to centre and their effective span is 6 m as shown in Fig. 7.7. If the superimposed

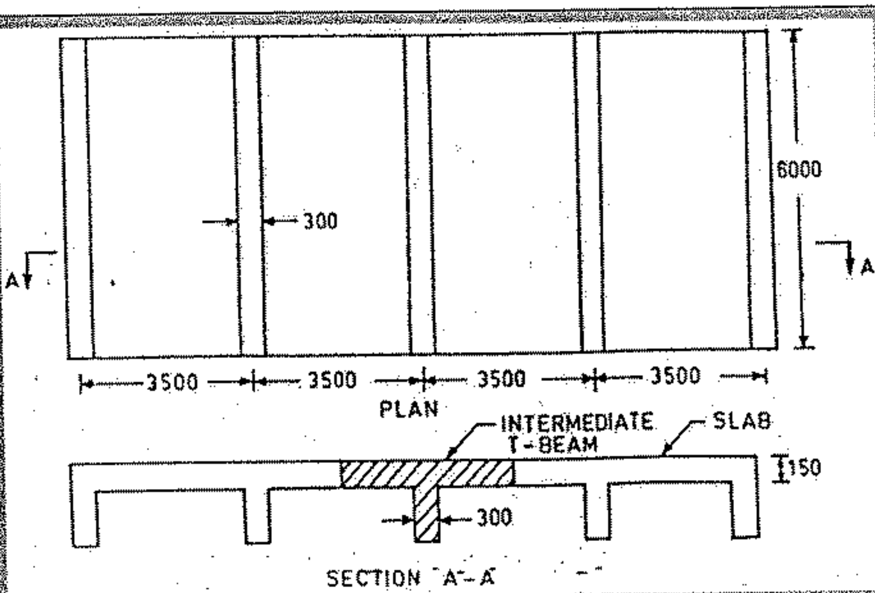


Fig.: T beam floor

Sol:

$$\text{Dead load} = 0.15 \times 25 = 3.75 \text{ kN/m}^2$$

$$\text{Super imposed load on slab} = 5 \text{ kN/m}^2$$

$$\text{Total load on slab} = 8.75 \text{ kN/m}^2$$

Load per meter run of beam = load on slab per unit area × centre to centre distance between beams

$$= 8.75 \times 3.5 = 30.6 \text{ kN/m}$$

$b_f = $ (I) $\dfrac{l_0}{6} + b_w + 6D_f = \dfrac{600}{6} + 30 + 6\times15 = 2200$ mm is minimum of (II) b (3500 mm)

So $$b_f = 2200 \text{ mm}$$

Let us adopt over all depth $$D = \frac{\text{Span}}{10} = \frac{6000}{10} = 600 \text{ mm}$$

(Use clear cover 25 mm and dia of bar is 28 mm)

$$d = 600 - 25 - \frac{28}{2} = 561 \text{ mm}$$

Dead load of web of beam = Width of web × depth of web concrete density

Depth of beam = depth of T beam − thickness of slab

$$= 561 - 150 = 411 \text{ mm}$$

$$\text{Dead load of web of beam} = 0.30 \times 0.411 \times 25 = 3.08 \text{ kN/m}$$

$$\therefore \text{Total load on beam per metre run} = 30.6 + 3.08 = 33.68 \text{ kN/m}$$

$$M_u = 1.5 \times \frac{wl^2}{8} = 1.5 \times \frac{33.68 \times 6^2}{8} = 227.34 \text{ kN/m}$$

Let us assume that neutral axis lies in flange from

$$C = T$$

$$0.36 f_{ck} x_u b_f = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 \times 250\ A_{st}}{20 \times 2200 \times 0.36} = 0.0137\ A_{st}$$

$$M_u = 0.87\ f_y\ A_{st}\ (d - 0.42\ x_u)$$

$$227.34 \times 10^6 = 0.87 \times 250\ A_{st}\ (561 - 0.42 \times 0.0137\ A_{st})$$

$$= 122.017 \times 10^3\ A_{st} - 1.25\ A_{st}^2$$

$$= 2328.50\ \text{mm}^2$$

and $x_u = 0.0137 \times 2328.50 = 31.90 < 150$ mm O.K.

$$A_{st,min} = \frac{0.85\ bd}{f_y} = 0.85\ \frac{b_w d}{f_y}$$

$$= \frac{0.85 \times 300 \times 450}{250} = 459\ \text{mm}^2$$

$A_{st} > A_{st,\ min}$ O.K.

Provide 4 nos. 20 mm dia (1256.63 mm²)

Maximum area of tension steel $= 0.04\ b_w\ D$

$= 0.04 \times 300 \times 500 = 6000\ \text{mm}^2 > 1256.63\ \text{mm}^2$ O.K.

PART-II

1

Working Stress

WORKING STRESS METHOD

- This has been the traditional method used for reinforced concrete design where it is assumed that concrete is elastic, steel and concrete act together elastically and the relationship between loads and stresses is linear right up to the collapse of the structure.
- The basis of the method is that the permissible stress for concrete and steel are not exceeded any where in the structure when it is to subjected to the worst combination of loading is applied the section are designed in accordance with the elastic theory of bending assuming that both materials obey the Hooks law.
- The elastic theory assume a linear variation of strain and stress for zero at the Neutral axis to a maximum at the extreme fibre.
- The stresses of concrete and steel in a structure designed by the working stress method are not allowed to exceed some specified values of stresses known as permissible stresses.
- The permissible stresses are determined dividing the characteristic strength f_{ck} of the material by the respective factor of safety.
- The values of the factor of safety depend on the grade of the material and the type of stress. Thus, for concrete in bending compression, the permissible stress of grade M 20 is 7 N/mm^2, which is obtained by dividing the characteristic strength f_{ck} of M 20 concrete by a number 3 and then rationalising the value to 7.
- This permissible stress is designated by σ_{cbc}, the symbol σ stands for permissible stress and the letters c, b and c mean concrete in bending compression, respectively.

PERMISSIBLE STRESSES

- In working stress method, the stresses in materials are not exceeded beyond their permissible values. The permissible stress in a material is given by

$$\text{Permissible stress} = \frac{\text{Limiting strength}}{\text{Factor of safety}}$$

- In case of steel reinforcement, the limiting strength is either the yield stress or 0.2% proof stress, as the case may be. For concrete, the limiting strength is the crushing strength in compression.

- The factor of safety in the case of tensile steel reinforcement is aproximately = 1.82. Hence, the permissible tensile stress in steel is $\sigma_{st} = 0.55\ f_y$. For concrete, the factor of safety is higher than for steel.
- This is so because concrete suffers from higher degree of variability regarding its strength and properties than steel which is produced under well controlled conditions.
- The factor of safety for flexural compressive in concrete is = 3. Thus, the permissible compressive stress in concrete in flexural compression is $\sigma_{cbc} = 0.333\ f_{ck}$.

PERMISSIBLE STRESSES IN CONCETE

- The permissible stress of concrete in direct tension is denoted by σ_{td}. The values of σ_{td} for member in direct tension for different grades of concrete are given in cl. B-2.1.1 of IS 456.
- However, for members in tension, full tension is to be taken by the reinforcement alone.
- Though full tension is taken by the reinforcement only, the actual tensile stress of concrete f_{td} in such members shall not exceed the respective permissible values of σ_{td} to prevent any crack. Table a presents the values of σ_{td} for selective grades of concrete as a ready reference.
- It may be worth nothing that the factor of safety of concrete in direct tension is from 8.5 to 9.5.
- The permissible stresses of concrete in bending compression σ_{cbc}, in direct compression σ_{cc} and the permisible stress in bond for plain bars in tension τ_{bd} are given in table 21 of IS 456 for different grades of concrete.
- However, table a presents these vaues for selective grades of concrete, as a ready reference.
- The factors of safety of concrete in bending compression, direct compression and in bond for plain bars are 3, 4 and from 25 to 35, respectively.
- For plain bars in compression, the values of bond stress are obtained by increasing the respective value in tension by 25 percent, as given in the note of table 21 of IS 456.
- For deformed bars, the values of table are to be increased by sixty per cent, as stipulated in cl. B-2.1.2 of IS 456.

Table 1.5: (Table 21 of IS 456).

Grade of concrete	Direct tension σ_{td} (N/mm^2)	Bending compression σ_{cbc} (N/mm^2)	Direct compression σ_{cc} (N/mm^2)	Permissible bond stress in τ_{bd} for plain bars in tension (N/mm^2)
M 20	2.8	7.0	5.0	0.8
M 25	3.2	8.5	6.0	0.9
M 30	3.6	10.0	8.0	1.0
M 35	4.0	11.5	9.0	1.1
M 40	4.4	13.0	10.0	1.2

- From the values of the permissible stresses and the respective characteristic strengths for different grades of concrete, it may be seen that the factors of safety of concrete in direct tension, bending compression, direct compression and in bond for plain bars in tension are from 8.5 to 9.5, 3, 4 and from 25 to 35, respectively.

PERMISSIBLE STRESSES IN STEEL REINFORCEMENT

- Permissible stresses in steel reinforcement for different grades of steel, diameters of bars and the types of stress in steel reinforcement are given in table IS 456.
- Selective values of permissible stresses of steel of grade Fe 250 (mild steel) and Fe 415. (high yield strength deformed bars) in tension (σ_{st} and σ_{sh}) and compression in column (σ_{sc}) are furnshed in Table 1.6 as a ready reference.
- It may be noted from the values of Table 1.6 that the factore of safety in steel for these stresses is about 1.8, much lower than concrete due to high quality control during the production of steel in the industry in comparison to preparing of concrete.

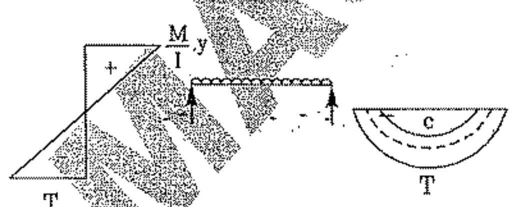
Table 1.6: Permissible stress in steel reinforcement.

Type of stress in steel reinforcement	*Mild steel bars, Fe 250, (N/mm²)*	*High yield strength deformed bars, Fe 415, (N/mm²)*
Tension σ_{st} or σ_{ss}		
(a) up to and including 20 mm diameter	140	230
(b) over 20 mm diameter	130	230
Compression in column bars σ_{sc}	130	190

***Note:** It can be observed that for a given grade of concrete $\sigma_{cc} < \sigma_{cbc}$ i.e. a gretar F.o.S. is adopted for direct stress than for a bending stress.*

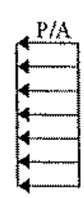
Explaination

When a c/s is subjected to bending stress, the stress induced on it is variable maximum at extreme fibre and zero at. N.A.



When the maximum stress exceeds the permissible value, the extreme fibre will not fail actually, but will transfer the additional force to the inner fibre which has a lower stress.

When the section is subjected to a direct stress, all points of the section have uniform stress having no scope for such a transfer of the force.



It is for this reason larger F.O.S is adopted for directe stress than Bending stress.

***Note :** In case of steel reinforcement of small diameter. The stress will be uniform for Direct stress as well or for bending stress. Therefore in steel bars, the permissible stresses in bending and direct stresses are the same for lower dia bars up to 20 mm ϕ.*

MODULAR RATIO (*m*)

In the elastic theory, structures having different materials are made equivalent to one common meterial. In the reinforced concrete structure, concrete and reinforcing steel are, therefore, converted into one material. This is done by transformation using the modular ratio m which is the ratio of modulus of elasticity of steel and concrete. Thus, $m = E_s/E_c$, where E_s is the modulus of elasticity of steel which is not a perfectly elastic material.

The short-term modulus of concrete $E_c = 5000\sqrt{f_{ck}}$ in N/mm^2, where f_{ck} is the characteristic strength of concrete. However, the short-term modulus does not take into account the effects of creep, shrinkage and other long-term effects Accordingly, the modular ratio 'm' is not computed as $m = E_s/E_c = 200000/(50000\sqrt{f_{ck}})$. i.e., $280/3\,\sigma_{cbc}$, partially takes into account long term effect. This is also mentioned in the note of cl. B-1.3 of IS 456.

TRANSFORMED SECTION

Consider a Reinforced concrete section and load applied on this section is P.

A = Total area

A_C = Area of concrete

A_S = Area of steel

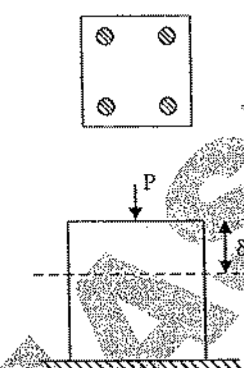


Fig. 1.33

When load P is applied then load taken by concrete = P_c and load taken by steel = P_s, then

$$\boxed{P = P_C + P_S}$$

$$\text{We know that stress} = \frac{P}{A}$$

$$\boxed{P = \text{Stress} \times \text{Area}}$$

p_s = stress in steel

p_c = stress in concrete

$$P = p_s.A_s + p_c.A_c \qquad \text{---(1)}$$

Deformation δ is same for steel and concrete from the compatibility equation

$$\delta_s = \delta_c$$

$$\frac{P_s L}{A_s E_s} = \frac{P_c L}{A_c E_c}$$

$$\frac{p_s}{E_s} = \frac{p_c}{E_c}$$

$$\frac{p_s}{p_c} = \frac{E_s}{E_c}$$

$$\because \quad m = \frac{E_s}{E_c}$$

$\therefore$ $\boxed{p_s = mp_c}$ Stress in steel = m × stress in concrete put this value in equation (1)

$$P = mp_c A_s + p_c A_c$$

$$\boxed{\begin{array}{l} P = p_c(mA_s + A_c) \\ P = p_c \times \text{equivalent area in terms of concrete} \end{array}}$$

For A_s area of steel equivalent area in terms of concrete Area = mA_s

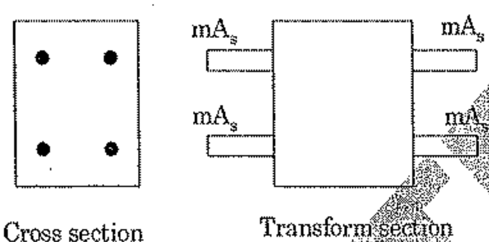


Fig. 1.34

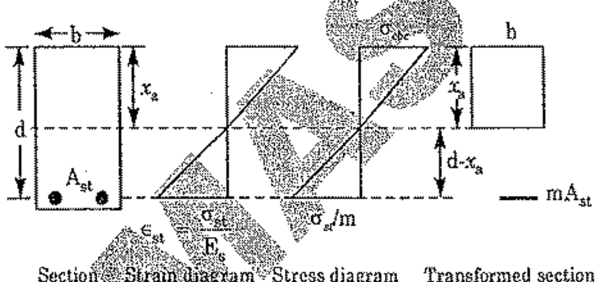


Fig. 1.35

where σ_{cbc} and σ_{st} is stresses in concrete and steel respectively.

Transformed Section or equivalent Homogeneous Section are:

- In which the steel area is replaced by an equivalent concrete area.
- Transform Section (TS) consists of a single material, therefore theory of simple bending can be applied (Assumption of simple bending).
- TS may be of steel when concrete is replaced by steel or it may be of concrete when area is replaced by concrete.
- It is usually replace steel area by concrete hence a transformed section would mean to a homogenous concrete section.

Note: *Line area developes same strain as steel area of original section develops*

Assumption (P 80 Annex-B)

The following assumptions are made for a section resisting bending moment in elastic theory.

1. **Adhesion of concrete to steel is perfect within tbe elastic limit of steel.**
It means that if a concrete member, with reinforcement bars embeded in it, is pulled, the strain induced in fibres of concrete as well as those of steel are same. This will happen only if concrete and steel are fully bonded together. If this bond is not there, the concrete will not be able to pull the steel bar with it. In such a case, the entire pull will be borne by concrete and the steel bar will remain unstressed. This may result in the breakdown of the member due to weakness of concrete in tension and concrete will slip over the steel bar. If the bond between concrete and steel is perfect, the entire pull will be borne by the steel bar after concrete cracks. Hence, for transferring the stress from concrete to steel or from steel to concrete, it is necessary that there exists a perfect bond between the two materials.

2. **A section, wbich is plane before bending, remains plane after bending.**
This assumption is an enunciation of the well-known straight line theory for homogeneous materials.

3. **Modulus of elasticity of concrete remains the same at all stresses and does not change with tbe duration of stress.**
This assumption is not quite correct, as the elastic modulus of concrete change with age, its quality, intensity of stress and its quation. But the formulae for design work derived on the basis of changeing modulus would be too complicated to be used in practice, as in that case the bending stress diagram for a section will not be linear; i.e. though the strain diagram will be triangular, the stress diagram will have some other shape. For ordinary design work, this assumption is justified and adopted, since it does not introduce large errors.

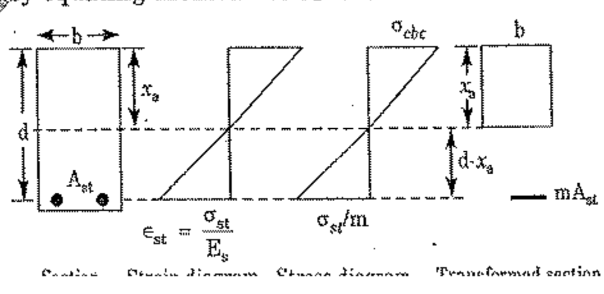
4. **Tension is borne entirely by steel.**
This assumption is not quite correct in as much as concrete is also capable of taking tension upto a certain limit (upto about 10% of the compressive strength) and actually shares the tensile stesses with steel in practice. But its share is so small that it can safely be neglected for convenience and simplicity in design work. In the fibres, below the neutral plane, where the tensile stress in concrete is more than its strength, it cracks and does nottake any tensile stress. So it is only a small portion of concrete near the neutral axis that bears the tensile stress, and is not much effective in resisting the moment. Besides the shrinkage of concrete induces tensile stress in concrete and reduces the capacity to take an appreciable share of tensile stress. The assumption is therefore, justified.

5. **There are no initial stresses in steel when it is embeded in concrete.**
This assumption is fairly well realised in practice unless of course, the workmanship is defective. The initial stresses, in steel will limit its capacity to bear further tensile stresses when used as a reinforcement.

Analysis

(1) Find Actual Depth of Nutral Axis

Depth of N.A. is calculated by equating moment of area of both side of transformed section in figure.



$$b.x_a.\frac{x_a}{2} = mA_{st}(d-x_a)$$

$$\boxed{\frac{b.x_a^2}{2} = m\,A_{st}(d-x_a)}$$

where b = Width of section
x_a = Actual depth of N.A.
d = Effective depth
D = Overall depth

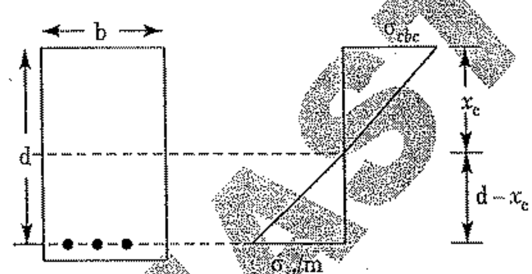
$\boxed{k_a = \frac{x_a}{d}}$ $k_a \rightarrow$ Neutral Axis depth factor

Note actual depth of N.A. depends on the area of steel ($x_a \propto A_{st}$)
More reinforcement is provided actual depth of N.A. is moves towards the critical depth of N.A.

(2) Find Critical Depth of Neutral Axis

It is such depth of actual N.A for which stress in concrete and steel increases in such a manner that maximum permissible stress in concrete and steel are attains simultaneously.

σ_{cbc} = Maximum permissible stress is concrete



Cross section Stress diagram

Fig. 1.37

t = maximum permissible stress in steel.

From similar triangle in stress diagram.

$$\frac{\sigma_{cbc}}{x_c} = \frac{\sigma_{st}/m}{d-x_c}$$

$$\frac{d-x_c}{x_c} = \frac{\sigma_{st}}{m\sigma_{cbc}}$$

$$\frac{d}{x_c}-1 = \frac{\sigma_{st}}{m\sigma_{cbc}}$$

$$\frac{d}{x_c} = 1+\frac{\sigma_{st}}{m\sigma_{cbc}}$$

$$\frac{d}{x_c} = \frac{m\sigma_{cbc}+\sigma_{st}}{m\sigma_{cbc}}$$

$$\boxed{\frac{x_c}{d} = \frac{m\sigma_{cbc}}{\sigma_{st}+m\sigma_{cbc}}} \quad \text{---(i)}$$

$$k_c = \frac{x_c}{d} = \text{critical neutral axis depth factor.}$$

we know that $$m = \frac{280}{3\sigma_{cbc}}$$

$m\sigma_{cbc} = 93.33$ put this value in equation (i), we will get

$$\boxed{k_c = \frac{93.33}{\sigma_{st} + 93.33}}$$

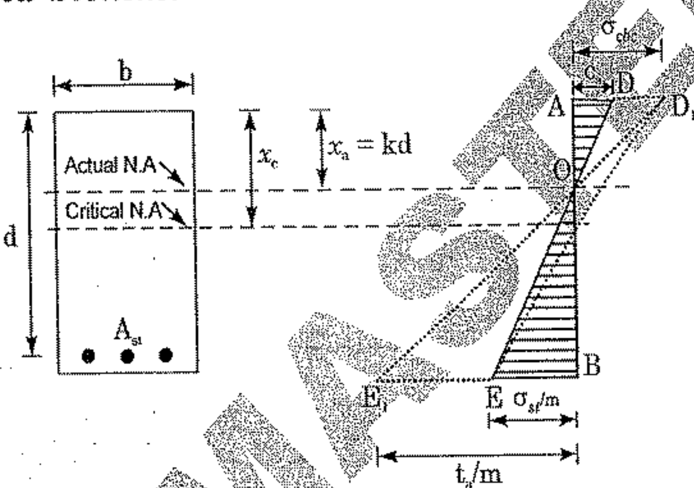
σ_{cbc} = Maximum stress in concrete

σ_{st} = maximum stress in steel

Note: *From above it is clear that k_c will depends only on stresses in steel (t).*

(3) Cases of Singly Reinforced Beam

(i) Under Reinforced Sections:



(a) (b)

Fig. 1.38

Following points may be noted for under reinforced section:

(*i*) When the value of $x_a < x_c$ it is a under reinforced section.

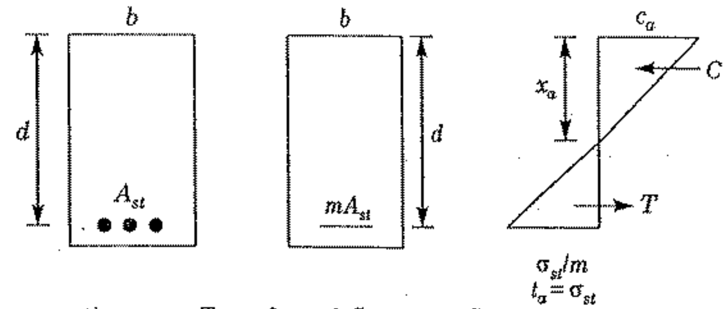
(*ii*) Area of steel is less.

(*iii*) Maximum permissible stresses in steel is attains first, at the same time actual stress in concrete is much lower than the maximum permissible value $c_a < \sigma_{cbc}$.

(*iv*) Failure of under reinforce section occurs due to failure of steel so it is a case of ductile failure.

(*v*) Due to duclile failure that is not sudden, sufficient time is available before failure.

Moment of Resistance (For under rainforced section)



Cross section Transformed Section Stress diagram

Fig. 1.39

For under reinforced section

$$x_a < x_c$$
$$c_a < \sigma_{cbc}$$
$$t_a = \sigma_{st}$$

where x_a = actual depth of N.A

x_c = critical depth of N.A

c_a = actual stress in concrete

t_a = actual stress in steel

σ_{st} = maximum stress in steel

σ_{cbc} = maximum stress in concrete

$$M.R_c = \text{Force} \times \text{Lever Arm (LA)}$$

Force on compressive side = Area of stress diagram × Width of the section.

$$= \frac{1}{2} c_a x_a b$$
$$= \frac{1}{2} b x_a c_a$$
$$\text{L.A} = d - \frac{x_a}{3}$$
$$\boxed{M.R_c = \frac{1}{2} b x_a c_a \left(d - \frac{x_a}{3}\right)}$$
$$\text{MR}_t = \frac{t_a}{m} \times m\text{A}_{st}\left(d - \frac{x_a}{3}\right)$$
$$\boxed{MR_t = \sigma_{st} A_{st}\left(d - \frac{x_a}{3}\right)}$$
$$(t_a = \sigma_{st})$$

where $M.R_c$ = Moment of resistance of compression side and $M.R_t$ is moment of resistance of tension side.

$\mathbf{c_a}$ can be obtained from similar triangle

$$\frac{c_a}{x_a} = \frac{\sigma_{st}/m}{d - x_a}$$
$$\boxed{c_a = \frac{\sigma_{st}}{m(d - x_a)} x_a}$$

(ii) Balanced Section

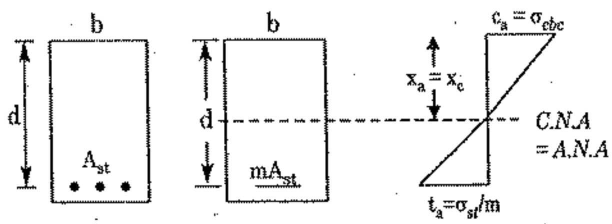


Fig. 1.40

C.N.A. Critical Neutral Axis Depth

A.N.A. Actual Neutral Axis Depth

Following points may be noted for balanced section:

(i) $x_a = x_c$

(ii) Steel is such that critical depth of N.A. is same as x_a.

(iii) The value of stress in steel and concrete are at its Maximum permissible values.

(iv) Both section is prone to failure.

(v) Due to brittle behaviour concrete is likely to fails first.

Moment of Resistance

For balanced section $x_a = x_c$

$$c_a = \sigma_{cbc}$$

$$t_a = \sigma_{st}$$

where x_a = actual depth of N.A

x_c = critical depth of N.A

c_a = actual stress in concrete

t_a = actual stress in steel

σ_{st} = maximum stress in steel

σ_{cbc} = maximum stress in concrete

$$M.R_c = \frac{1}{2}\sigma_{cbc} \cdot x_c b\left(d - \frac{x_c}{3}\right) \quad \text{---(i)}$$

Critical depth factor $k_c = \frac{x_c}{d}$ $\quad x_c = k_c.d$ put this value in equation (i)

$$M.R_c = \frac{1}{2}b\sigma_{cbc}.k_c.d\left(d - \frac{k_c.d}{3}\right)$$

$$= \frac{1}{2}\sigma_{cbc}bk_c.d^2\left(1 - \frac{k_c}{3}\right) \text{ put } 1 - \frac{k_c}{3} = j$$

$$= \frac{1}{2}bk_c.d^2\,j.\sigma_{cbc}$$

$$\boxed{M.R_c = Qbd^2}$$

where $Q = \frac{1}{2} j k_c . \sigma_{cbc}$

$$\boxed{M.R_t = \sigma_{st} A_{st}\left(d - \frac{x_c}{3}\right)}$$

where $M.R_c$ = Moment of resistance of compression side and $M.R_t$ is moment of resistance of tension side.

(iii) Over Reinforced Section

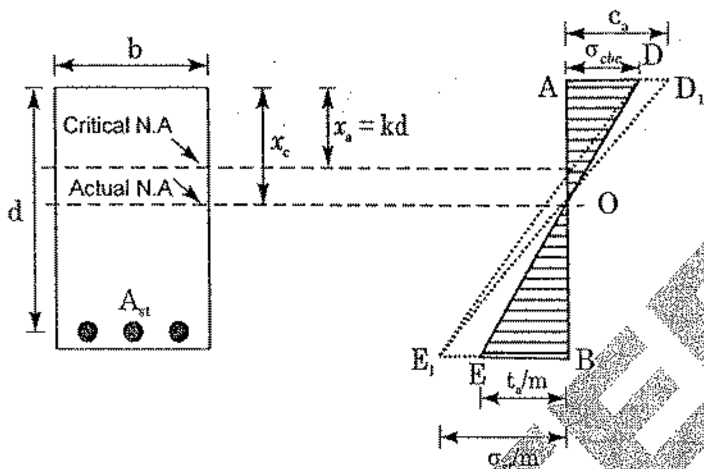


Fig. 1.41

Following points may be noted for over reinforced section:

(i) $x_a > x_c$

(ii) Area of steel is very high

(iii) In this case stress in concrete attains its maximum permissible value first and at this time actual stress is steel in much less than maximum permissible. Stress in steel $t_a < \sigma_{st}$

(iv) Concrete fail first

(v) Failure is brittle so it is not desirable and not prefered in design.

Moment of Resistance

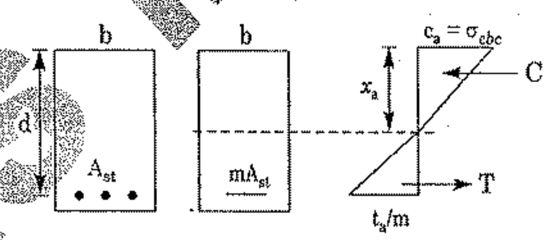


Fig. 1.42

where C = compressive force, T = Tensile force

For over reinforced section

$$x_a > x_c$$
$$c_a = \sigma_{cbc}$$
$$t_a < \sigma_{st}$$

where x_a = actual depth of N.A

x_c = critical depth of N.A

c_a = actual stress in concrete

t_a = actual stress in steel

σ_{st} = maximum stress in steel

σ_{cbc} = maximum stress in concrete

$$M.R_c = \frac{1}{2}c.bx_a\left(d-\frac{x_a}{3}\right)$$

$$M.R_t = \frac{t_a}{m}mA_{st}\left(d-\frac{x_a}{3}\right)$$

$$M.R_t = t_aA_{st}\left(d-\frac{x_a}{3}\right)$$

t_a can be calculated from similar triangle

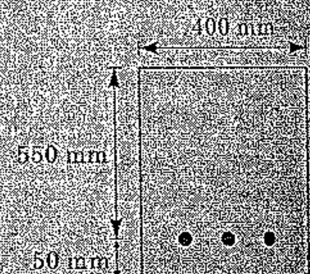
$$\frac{\sigma_{cbc}}{x_a} = \frac{t_a/m}{d-x_a}$$

$$\boxed{t_a = \frac{\sigma_{cbc}.m\,(d-x_a)}{x_a}}$$

Example 1

Find the moment of resistence of the section as shown in figure.

Use of grade of concrete M 25 and Fe 415 grade of steel.



For area of steel

Case 1: 3 nos. 16 mm ϕ

Case 2: 3 nos. 25 mm ϕ

s_{cbc} = *8.5 N/mm²*

σ_{st} = *230 N/mm²*

Also calculate Moment of Resistance of the balanced section and area of steel required for the same?

Sol: *Case (1):*

$$\text{Area of steel} = 3\times\frac{\pi}{4}\times16^2 = 603.18\text{ mm}^2$$

$$\text{Critical depth of N.A. } \frac{x_c}{d} = \frac{m\sigma_{cbc}}{\sigma_{st}+m\sigma_{cbc}}$$

$$\sigma_{st} = 230 \text{ N/mm}^2$$

$$m = 11$$

$$\frac{x_c}{d} = \frac{11 \times 8.5}{230 + 11 \times 8.5} = 0.289$$

$$x_c = 0.289 \times 550$$

$$x_c = 158.96 \text{ mm}$$

Actual depth of N.A.

$$\frac{bx_a^2}{2} = mA_{st}(d - x_a)$$

$$\frac{400}{2}x_a^2 = 11 \times 603.18\ (550 - x_a)$$

$$x_a = 119.50 \text{ mm}$$

$x_a < x_c$ section is under reinforced

For under rainforced section

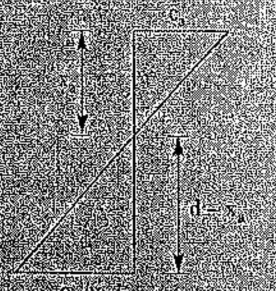
$$x_a < x_c$$

$$c_a < \sigma_{cbc}$$

$$t_a = \sigma_{st}$$

$M.R_c$ (Moment of resistance of compressive side) $= \frac{1}{2} bx_a \cdot c_a \left(d - \frac{x_a}{3}\right)$ ... (i)

From similar triangle $\quad \frac{c_a}{x_a} = \frac{\sigma_{st}/m}{d - x_a}$



$$\frac{c_a}{119.50} = \frac{\frac{230}{11}}{550 - 119.50}$$

$c_a = 5.80$ N/mm² put this value in equation (i)

$$M.R_c = \frac{1}{2} \times 400 \times 119.50 \times 5.80\left(550 - \frac{119.50}{3}\right)$$

$$M.R_c = 70.76 \text{ kN-m}$$

OR

$$M.R_t = \sigma_{st} A_{st}\left(d - \frac{x_a}{3}\right)$$

$$= 230 \times 603.18\left(550 - \frac{119.50}{3}\right)$$

$$M.R_t = 70.77 \text{ kN-m}$$

Case (2) : $A_{st} = 3 \times \frac{\pi}{4} \times 25^2 = 1472.6 \text{ mm}^2$

For critical depth of neutral axis

$$\frac{x_c}{d} = \frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}}$$

$$= \frac{11 \times 8.5}{230 + 11 \times 8.5}$$

$$x_c = 158.96 \text{ mm}$$

For actual depth of N.A.

$$\frac{bx_a^2}{2} = mA_{st}(d - x_a)$$

$$400\frac{x_a^2}{2} = 11 \times 1472.6\,(500 - x_a)$$

$$200\ x_a^2 + 16198.6\ x_a - 8909230$$

$$x_a = 174.41 \text{ mm}$$

$x_a > x_c$ section is over reinforced.

For over reinforced section

$$x_a > x_c$$

$$c_a = \sigma_{cbc}$$

$$t_a = \sigma_{st}$$

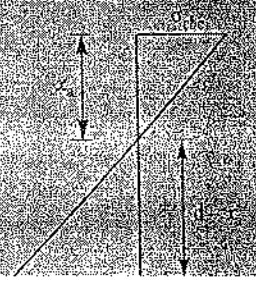
$$M.R_c = \frac{1}{2} b\sigma_{cbc} x_a \left(d - \frac{x_a}{3}\right) \quad \text{---(i)}$$

$$= \frac{1}{2} \times 400 \times 8.5 \times 174.41\left(550 - \frac{174.41}{3}\right)$$

$$= 145.83 \text{ kN-m}$$

or $\quad M.R_t = t_a A_{st}\left(d - \frac{x_a}{3}\right) \quad$ ---(ii)

For similar triangle $\quad \dfrac{\sigma_{cbc}}{x_a} = \dfrac{t_a/m}{d - x_a}$



$$\frac{t_a}{m} = \frac{\sigma_{cbc}(d - x_a)}{x_a}$$

$$= \frac{8.5(550 - 174.41)}{174.41}$$

$$\frac{t_a}{m} = 18.30$$

$t_a = 18.3 \times m$

$t_a = 18.3 \times 11$

$t_a = 201.35$ N/mm² put this value in equation (ii)

$$M.R. = 201.35 \times 1472.6 \times \left(550 - \frac{174.41}{3}\right)$$

$= 145.84$ kN-m

Case (3) : $M.R._{\text{balance}} = Qbd^2$

$$Q = \frac{1}{2}\sigma_{cbc} J k_c$$

$$k_c = \frac{x_c}{d} = \frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}}$$

$$= \frac{11 \times 8.5}{230 + 11 \times 8.5} = 0.289$$

$$j = 1 - \frac{k_c}{3} = 1 - \frac{0.289}{3}$$

$= 0.9036$

$$Q = \frac{1}{2} \times 8.5 \times 0.9036 \times 0.289$$

$= 1.109$

$M.R._{\text{balance}} = 1.109 \times 400 \times 550^2$

$= 134.30$ KN-m

Now $$M.R._{\text{balance}} = \sigma_{st} A_{st}\left(d - \frac{x_c}{3}\right)$$

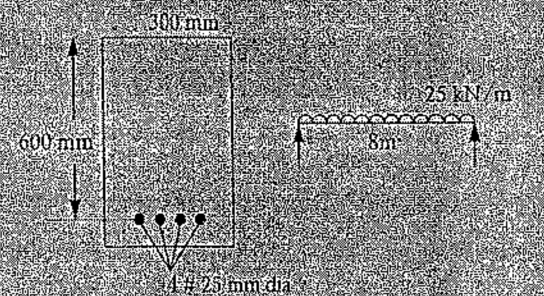
$$134.3 \times 10^6 = 230 A_{st}\left(550 - \frac{0.289 \times 550}{3}\right)$$

$= A_{st} \times 230 \times (550 - 52.98)$

$$A_{st} = \frac{134.3 \times 10^6}{114313.83} = 1174.83 \text{ mm}^2$$

Example 2

A rectangular beam 300 mm × 600 mm (effective depth) consists of 4 nos. 25 mm dia steel. if the beam is S.S over a span of 8 m and carries UDL 25 kN/m inclusive self weight. Calculate the stresses developed take m = 15



Sol.

$$B.M. = \frac{wl^2}{8} = \frac{25 \times 8^2}{8} = 200 \text{ kN-m}$$

Actual depth of N.A

$$\frac{bx_a^2}{2} = mA_{st}(d - x_a)$$

$$\frac{300}{2}x_a^2 = 15 \times 4 \times \frac{\pi}{4} \times 25^2(600 - x_a)$$

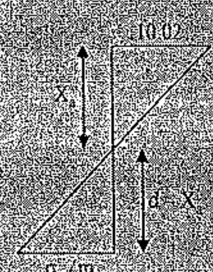
$$150x_a^2 = 17671458.68 - 29452.43\, x_a$$

$$x_a = 258.81 \text{ mm}$$

$$\text{B.M.} = \frac{1}{2} bx_a \sigma_{cbc}\left(d - \frac{x_a}{3}\right)$$

$$200 \times 10^6 = 300 \times 258.81 \frac{\sigma_{cbc}}{2}\left(600 - \frac{258.81}{3}\right)$$

$$\sigma_{cbc} = 10.02 \text{ N/mm}^2$$



$$\frac{10.02}{258.81} = \frac{\sigma_{st}}{15 \times (600 - 258.81)}$$

$$\sigma_{st} = 198.30 \text{ N/mm}^2$$

$$\text{or } 200 \times 10^6 = \sigma_{st} A_{st}\left(d - \frac{x_a}{3}\right)$$

$$200 \times 10^6 = \sigma_{st} \cdot \pi \times 25^2 \left(600 - \frac{258.81}{3}\right)$$

$$\sigma_{st} = 198.30 \text{ N/mm}^2$$

Example 3

Find out area of steel for b = 350 mm and D = 700 mm

(i) 100 kN-m

(ii) 160 kN-m

Take d′ = 50 mm (effective cover)

Given grade of concrete M20 and grade of steel - Fe 415 take $\sigma_{cbc} = 7$ *N/mm² and* $\sigma_{st} = 230$ *N/mm²*

Sol: ***Case (I):***

$$d = 700 - 50 = 650 \text{ mm}$$

$$M.R_{\text{balance}} = Qbd^2$$

$$Q = \frac{1}{2}\sigma_{cbc} jk_c$$

$$k_c = \frac{x_c}{d} = \frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}}$$

For M 20
$$\sigma_{cbc} = 7 \text{ N/mm}^2$$

$$\sigma_{st} = 230 \text{ N/mm}^2$$

$$m = \frac{280}{3\sigma_{cbc}}$$

$$m = \frac{280}{3 \times 7} = 13.33 \simeq 13$$

$$\frac{x_c}{d} = \frac{13 \times 7}{13 \times 7 + 230} = 0.2834$$

$$x_c = 0.2834 \times 650 = 184.26 \text{ mm}$$

$$j = 1 - \frac{k_c}{3}$$

$$= 1 - \frac{0.2834}{3} = 0.9056$$

$$Q = \frac{1}{2} \times 7 \times 0.9055 \times 0.2834$$

$$= 0.898$$

$$M.R_b = 0.898 \times 350 \times 650^2$$

$$= 132.82 \times 10^6 \text{ N-mm}$$

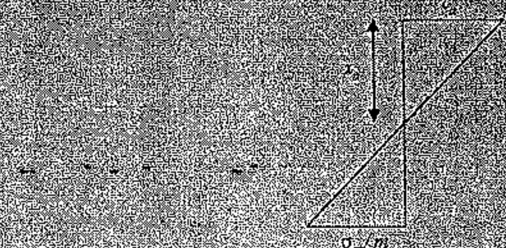
$$= 132.82 \text{ kN-m}$$

For under reinforced section

$$x_a < x_c$$
$$c_a < \sigma_{cbc}$$
$$t_a = \sigma_{st}$$

$$\text{B.M}_t = \sigma_{st} A_{st}\left(d - \frac{x_a}{3}\right) \quad \text{—(i)}$$

$$\text{B.M}_c = \frac{1}{2} b x_a c_a \left(d - \frac{x_a}{3}\right) \quad \text{—(ii)}$$



$$\frac{c_a}{x_a} = \frac{\sigma_{st}/m}{d - x_a}$$

$$c_a = \frac{\sigma_{st}}{m}\left(\frac{x_a}{d - x_a}\right)$$

$$\text{B.M}_c = \frac{1}{2} b x_a \frac{\sigma_{st}}{m} \frac{x_a}{(d - x_a)}\left(d - \frac{x_a}{3}\right) \quad \text{—(iii)}$$

From Eq. (i) and (iii) L.H.S. = R.H.S

$$\sigma_{st} A_{st}\left(d - \frac{x_a}{3}\right) = \frac{1}{2} b x_a \frac{\sigma_{st}}{m} \frac{x_a}{d - x_a}\left(d - \frac{x_a}{3}\right)$$

$$A_{st} = \frac{1}{2} \times 350 \frac{x_a^2}{13(650 - x_a)}$$

$$A_{st} = 13.46 \frac{x_a^2}{(650 - x_a)}$$

Put this in equation (i)

$$100 \times 10^6 = \frac{\sigma_{st} \times 13.46}{(650 - x_a)} x_a^2 \left(d - \frac{x_a}{3}\right)$$

$$100 \times 10^6 (650 - x_a) = 230 \times 13.46\, x_a^2 \left(650 - \frac{x_a}{3}\right)$$

$$6.5 \times 10^{10} - 100 \times 10^6 x_a = 2.01 \times 10^6 x_a^2 - 1031.93\, x_a^3$$

$$1031.93 x_a^3 - 2.01 \times 10^6 x_a^2 - 100 \times 10^6 x_a + 6.5 \times 10^{10} = 0$$

$x_a = 162.65$ mm again put this in eq. (i), we get

$$\left(\quad 162.65 \quad \right)$$

$$A_{st} = 729.76 \text{ mm}^2$$

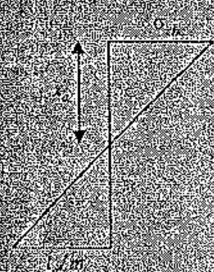
Case (2):

160 kN-m > 132.82 kN-m

B.M. > M.R. So, the section is over reinforced

For over reinforced section

$$x_a > x_c$$
$$c_a = \sigma_{cbc}$$
$$t_a < \sigma_{st}$$
$$BM_T = t_a A_{st}\left(d - \frac{x_a}{3}\right)$$
$$BM_C = \frac{1}{2} b x_a \sigma_{cbc}\left(d - \frac{x_a}{3}\right)$$



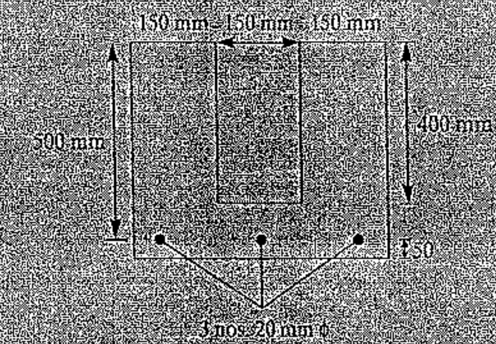
$$BM_C = \frac{1}{2} b x_a\left(d - \frac{x_a}{3}\right) \cdot \sigma_{cbc}$$
$$160 \times 10^6 = \frac{1}{2} \times 350\, x_a\left(650 - \frac{x_a}{3}\right) \times 7$$
$$= 796.25 \times 10^3\, x_a - 408.33\, x_a^2$$
$$x_a = 227.47 \text{ mm}$$
$$\frac{\sigma_{cbc}}{x_a} = \frac{t_a/m}{d - x_a}$$
$$t_a = m\sigma_{cbc}\left(\frac{d - x_a}{x_a}\right)$$
$$t_a = 13 \times 7\left(\frac{650 - 227.47}{227.47}\right)$$
$$t_a = 169.03 \text{ N/mm}^2$$

Now

$$160 \times 10^6 = 169.03 \times A_{st}\left(650 - \frac{227.47}{3}\right)$$
$$A_{st} = 1648.6 \text{ mm}^2$$

Example 4

Find the moment of resistance of the beam section as shown in figure below. Also state whether the beam is under-reinforcement or over-reinforcement. The materials are M 20 grade of concrete and HYSD of grade of steel used.



Sol. For M20

$$\sigma_{cbc} = 7 \text{ N/mm}^2$$
$$\sigma_{st} = 23 \text{ N/mm}^2$$
$$\frac{x_c}{d} = \frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}}$$
$$m = \frac{280}{3\times 7} = 13.33 \simeq 13$$
$$\frac{x_c}{d} = \frac{13\times 7}{13\times 7 + 230} = 0.283$$
$$x_c = 0.283 \times 500 = 141.74 \text{ mm}$$

Actual neutral axis depth

Assume neutral axis is less than 400 mm

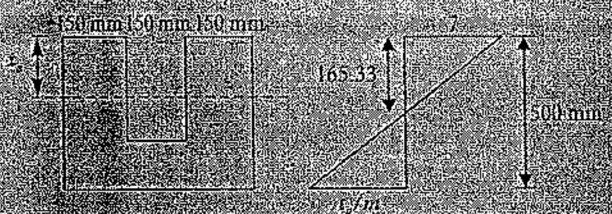
$$\frac{bx_a^2}{2} = mA_{st}(d - x_a)$$
$$\left(150x_a \cdot \frac{x_a}{2}\right)\times 2 = mA_{st}(d - x_a)$$
$$150x_a^2 = 13\times 3\times \frac{\pi}{4}\times 20^2(500 - x_a)$$
$$150x_a^2 = 6.126\times 10^6 - 12252.21x_a$$
$$x_a = 165.33 \text{ mm}$$
$$x_a > x_c \text{ section is over reinforced}$$

and x_a is less than 400 mm our assumption is correct

For over reinforce section

$$t_a < \sigma_{st}$$
$$x_a > x_c$$
$$c_a = \sigma_{cbc}$$
$$MR_c = \frac{1}{2}bx_a c\left(d - \frac{x_a}{3}\right) \times 2$$
$$= \left[\frac{1}{2}\times 150\times 165.33\times 7\left(500 - \frac{165.33}{3}\right)\right]\times 2$$
$$= 77.91 \text{ kN/m.}$$

OR



From similar triangle

$$\frac{7}{165.33} = \frac{t_a}{13\times(500-165.33)}$$

$$t_a = 184.20 \text{ N/mm}^2$$

$$M.R._T = t_a A_{st}\left(d - \frac{x_a}{3}\right)$$

$$= 184.20 \times 3 \times \frac{\pi}{4} \times 20^2 \left(500 - \frac{165.33}{3}\right)$$

$$M.R._T = 77.23 \text{ kN-m}$$

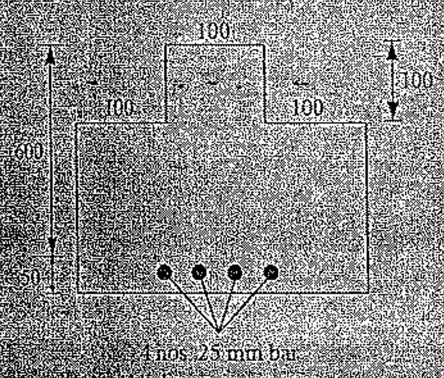
Example 5

Find M.R. of the section given below. Take

$$\sigma_{cbc} = 7\ N/mm^2$$

$$\sigma_{st} = 140\ N/mm^2$$

$$m = 13$$



Sol. Assume N.A. depth x_a is less then 100 mm

For actual depth of N.A.

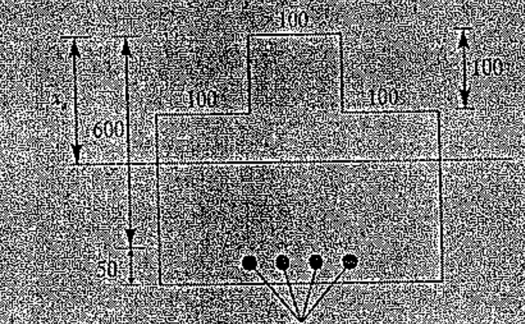
$$\frac{bx_a^2}{2} = mA_{st}(d - x_a)$$

$$\frac{100x_a^2}{2} = 13 \times \pi \times 25^2(600 - x_a)$$

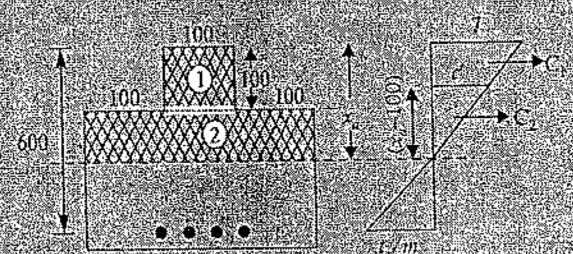
$$x_a = 353.97 \text{ mm}$$

$x_a > 100$ mm, so our assumption is wrong and N.A. is greater than 100 mm.

Again calculate actual depth of N.A. (assume x_a is greater than 100 mm)



4 nos 25 mm bar
All dimension in mm



For actual depth of N.A. equate moment of area about N.A. of tension and compression side

$$100 \times 100 \times (x_a - 50) + 300 \times (x_a - 100)\frac{(x_a - 100)}{2} = 13 \times \pi \times 25^2 \times (600 - x_a)$$

$$10^4 x_a - 50 \times 10^4 + 150\,(x_a^2 + 100^2 - 2 \times 100\, x_a) = 15.32 \times 10^6 - 25525.4\, x_a$$

$$150x_a^2 + 5525.4x_a - 14.32 \times 10^6 = 0$$

$$x_a = 291.10 \text{ mm}$$

$$\frac{x_c}{d} = \frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}} = \frac{13 \times 7}{13 \times 7 + 140} = 0.393$$

$$x_c = 0.393 \times 600 = 236.36 \text{ mm}$$

and $x_a > x_c$ so the section is over reinforced.

For over reinforce section

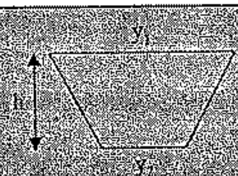
$$x_a > x_c$$
$$c_a = \sigma_{cbc}$$
$$t_a < \sigma_{st}$$

Fist find c' from similar triangle

$$\frac{7}{291.10} = \frac{c'}{x_a - 100}$$

$$c' = 4.595 \text{ N/mm}^2$$

$$M.R_c = M_1 + M_2$$

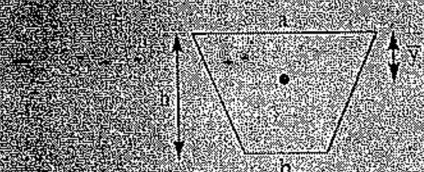


Area of trapezoidal section = $(y_1+y_2)\frac{h}{2}$

$$\text{Area} = (y_1+y_2)\cdot\frac{h}{2}$$

$$= (7+4.595)\times\frac{100}{2}$$

$$= 579.75\ \text{mm}^2$$



$$\bar{y}\ \text{(c.g. of trapezoidal section from top)} = \left(\frac{a+2b}{a+b}\right)\cdot\frac{h}{3}$$

$$\bar{y} = \left(\frac{7+2\times4.595}{7+4.595}\right)\frac{100}{3} = 46.54\ \text{mm}$$

$$L.A.\ \text{(lever arm)} = d-\bar{y} = 600-46.54 = 553.46\ \text{mm}$$

$$M_1 = 579.75\times100\times553.46$$

$$M_1 = 57975\times553.46 = 32.08\ \text{kN-m}$$

$$M_2 = \text{Area of strees diagram}\times\text{Width of section}\times L.A_2$$

$$= \frac{1}{2}\times4.595\,(x_a-100)\times300\cdot LA_2$$

$$LA_2 = \left[d-100-\frac{(x_a-100)}{3}\right]$$

$$= 600-100-\left(\frac{291.10-100}{3}\right)$$

$$L.A._2 = 436.3\ \text{mm}$$

$$M_2 = \frac{1}{2}\times4.595\times(291.10-100)\times300\times436.3$$

$$M_2 = 57.46\ \text{kN-m}$$

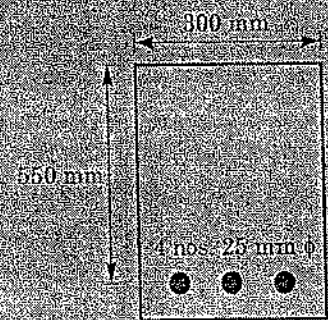
$$M = M_1+M_2$$

$$= 32.08+57.46$$

$$M = 89.54\ \text{kN-m}$$

Example 6

A rectangular RC beam of concrete grade M 20 is 300 mm wide and 550 mm deep (effective depth) as shown in figure. It is provided with 4 nos. of 20 mm φ mild steel bars as tension reinforcements. Determine the MR of the beam. Take $\sigma_{cbc} = 7\ N/mm^2$, $\sigma_{st} = 140\ N/mm^2$ and $m = 13$.



Sol:

$$A_{st} = 4 \times \frac{\pi}{4} \times 20^2 = 1256.64\ mm^2$$

$$b = 300\ mm$$

$$d = 550\ mm$$

$$\sigma_{cbc} = 7\ N/mm^2$$

$$\sigma_{st} = 140\ N/mm^2$$

$$m = 13$$

(i) Critical neutral axis

$$x_c = \left(\frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}}\right) d = \left[\frac{13 \times 7}{140 + (13 \times 7)}\right] 550 = 216.67\ mm$$

(ii) Actual neutral axis (x_a)

Equation MR of compression and tension side about NA

$$\frac{b}{2}(x_a)^2 = m.A_t \times (d - x_a)$$

$$\Rightarrow \frac{300}{2}(x_a)^2 = 13 \times 1256.64 \times (550 - x_a)$$

$$\Rightarrow 150x_a^2 + 108.91\ x_a^1 - 59899.84 = 0$$

$$\Rightarrow x_a^2 + 108.91\ x_a - 59899.84 = 0$$

$$\Rightarrow x_a = 196.27$$

$x_a < x_c$, hence section is under reinforced.

(iii) Finding MR

$$MR = \sigma_{st} A_t \left(d - \frac{x_a}{3}\right) \text{ [from tension side]}$$

$$= 140 \times 1256.64 \left(550 - \frac{196.27}{3}\right)$$

$$\frac{c_a}{x_a} = \frac{\sigma_{st}/m}{d - x_a} \text{ (from similar triangle)}$$

$$c_a = \frac{x_a \sigma_{st}}{m(d - x_a)} = \frac{196.27 \times 140}{13 \times (550 - 196.27)} = 5.975 \text{ N/mm}^2$$

$$\text{MR} = b\, x_a \frac{c_a}{2}\left(d - \frac{x_a}{3}\right) \quad \text{[from compressive side]}$$

$$= 300 \times 196.27 \times \frac{5.975}{2}\left(550 - \frac{196.27}{3}\right)$$

$$\text{MR} = 85.24 \text{ kNm}$$

Example 7

For the beam shown in previous question determine the area of tension reinforcement required to have it as a balanced section. Determine also the moment of resistance of the balanced section.

Sol:

$$\text{Critical neutral axis } x_c = \left(\frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}}\right) d$$

$$= \left[\frac{13 \times 7}{140 + 13 \times 7}\right] 550$$

$$x_c = 216.67 \text{ mm}$$

$$\text{MR} = b\, x_c \frac{\sigma_{cbc}}{2}\left(d - \frac{x_c}{3}\right)$$

$$= 300 \times 216.67 \times \frac{7}{2}\left(550 - \frac{216.67}{3}\right)$$

$$\text{MR} = 108.69 \text{ kNm}$$

$$\text{Again MR} = \sigma_{st} A_{st}\left(d - \frac{x_c}{3}\right)$$

$$\Rightarrow 108.69 \times 10^6 = 140 \times A_{st}\left(550 - \frac{216.67}{3}\right)$$

$$\Rightarrow A_{st} = 1624.94 \text{ mm}^2$$

Example 8

Design a RC beam with balanced sections for flexure by WSM for the data given below.

Effective span (SS) = 8 m

Live load = 12 kN/m

Breadth of the beam = 200 mm

Concrete grade = M 15

Reinforcement steel grade = Fe 415

Sol:

$$b = 300 \text{ mm}$$

$$l_{eff} = 8 \text{ m}$$

$$\text{Assuming overall depth } D = \frac{1}{10} \times l_{eff} = \frac{1}{10} \times 8000 = 800 \text{ mm}$$

(i) Self weight of the beam = 0.8 × 0.3 × 1 × 25 = 6 kN/m

$$\text{Live load} = 12 \text{ kN/m}$$

$$\text{Total load} = 6 + 12 = 18 \text{ kN/m}$$

$$\text{BM}_{max} = \frac{wl_{eff}^2}{8} = \frac{18 \times 8^2}{8} = 144 \text{ kNm}$$

(ii) Design constants

For M 15 and Fe 415

$$m = 19$$

$$\sigma_{cbc} = 5 \text{ N/mm}^2$$

$$\sigma_{st} = 230 \text{ N/mm}^2$$

$$k_c = \frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}} = \frac{19 \times 5}{230 + (19 \times 5)} = 0.2923$$

$$j = 1 - \frac{k_c}{3} = 1 - \frac{0.2923}{3} = 0.9025$$

$$Q = \frac{1}{2}\sigma_{cbc} j k_c = \frac{1}{2} \times 5 \times 0.9025 \times 0.2923 = 0.6595$$

(iii) Depth calculation

$$d = \sqrt{\frac{MR}{Qb}}$$

$$\Rightarrow \quad d = \sqrt{\frac{144 \times 10^6}{0.6595 \times 300}}$$

$$\Rightarrow \quad d = 853 \text{ mm}$$

Use clear cover for beam is 25 mm and maximum dia. of bar is 20 mm.

$$\text{Adding cover} = 25 + \frac{20}{2} = 35 \text{ mm}$$

$$D = 853 + 35 = 888 \text{ mm} > 800 \text{ mm}$$

Hence, redesign the depth, $D_{required} > D_{provided}$

2nd trial

$$\text{Assuming } D = 1000 \text{ mm}$$

$$\text{Self wt} = \frac{1000}{1000} \times \frac{300}{1000} \times 1 \times 25 = 7.5 \text{ kN/m}$$

$$\text{L.L.} = 12 \text{ kN/m}$$

$$\text{Total load} = 12 + 7.5 = 19.5 \text{ kN/m}$$

$$\text{BM}_{max} = \frac{19.5 \times 8^2}{8} = 156 \text{ kNm}$$

$$d = \sqrt{\frac{156 \times 10^6}{0.6595 \times 300}}$$

$$\Rightarrow \quad d = 887.96 \text{ mm} \approx 888 \text{ mm}$$

$$\text{Adding cover} = 25 + \frac{20}{2} = 35 \text{ mm}$$

$$D = 888 + 35 = 923 \text{ mm} < 1000 \text{ mm (hence OK)}$$

Keeping $D = 1000$ mm, $d = 1000 - 35 = 965$

$$\text{MR} = \sigma_{st} A_{st}\left(d - \frac{x_a}{3}\right)$$

for a balanced section $x_a = x_c = k\,d = 0.2923 \times 965 = 282.06$

$$A_{st} = \frac{MR}{\sigma_{st}\left(d - \frac{x_c}{3}\right)}$$

$$= \frac{156 \times 10^6}{230 \times \left(965 - \frac{277.68}{3}\right)}$$

$$A_{st} = 777.42 \text{ mm}^2$$

$$\frac{A_{st,min}}{bd} = \frac{0.85}{f_y}$$

$$A_{st,min} = \frac{0.85\,bd}{f_y}$$

$$= \frac{0.85 \times 965 \times 300}{415} = 592.95 \text{ mm}^2$$

$$A_{st,min} = 592.15 \text{ mm}^2$$

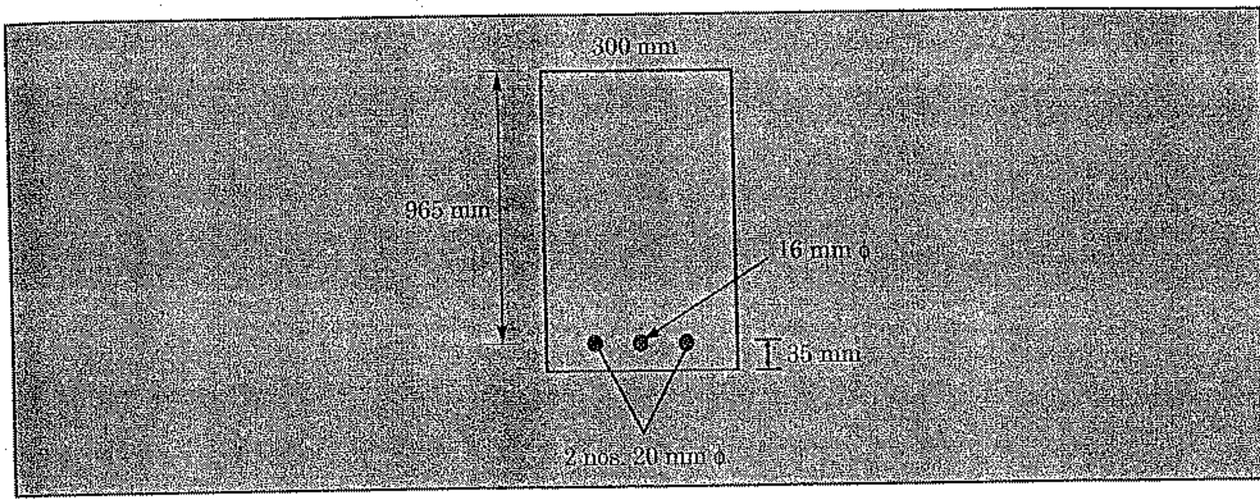
$$A_{st} > A_{st,min} \quad \text{OK}$$

Provide 2 nos. 20 mm dia and one 16 mm dia bar total area of steel provide $= 2 \times \frac{\pi}{4} \times 20^2 + \frac{\pi}{4} \times 16^2$

$$= 829.38 \text{ mm}^2$$

% of steel is provided

$$\frac{829.38}{300 \times 965} \times 100 = 0.29\% < 4\% \text{ Maximum \% of tension reinforcement}$$



Example 9

For the beam given in Q. 8 determine the main reinforcement required by LSM. If over all depth of the beam is 925 mm and partial safety factor of 1.5 for both DL and LL.

Sol: *Given*

$$b = 300 \text{ mm}$$

$$D = 925 \text{ mm}$$

$$l_{eff} = 8 \text{ m}$$

$$LL = 12 \text{ kN/m}$$

Using M15/Fe 415

(i) Load calculations

$$\text{Self weight of beam} = 0.3 \times 0.925 \times 1 \times 25 = 6.9375 \text{ kN/m}$$

$$\text{LL on the beam} = 12 \text{ kN/m}$$

$$\text{Total load} = 12 + 6.9375 = 18.9375 \text{ kN/m}$$

$$w_u = 1.5 \times 18.9375 = 28.41 \text{ kN/m}$$

$$M_{u,lim} = \frac{w_u l_{eff}^2}{8} = \frac{24.41 \times 8^2}{8} = 227.25 \text{ kN/m}$$

(ii) Designing balanced section

$$x_u = x_{u,lim} = 0.48\, d = 0.48 \times 925 = 444 \text{ mm}$$

(iii) MR of the section

$$M_u = 0.87 f_y A_{st} (d - 0.42\, x_u)$$

for the balance case

$$M_u = M_{u,lim} = BM_{max} = 0.87 f_y A_{st} (d - 0.42\, x_{u,lim})$$

$$\Rightarrow \quad 227.25 \times 10^6 = 0.87 \times 415 \times A_{st} \times (925 - 0.42 \times 444)$$

$$\Rightarrow \quad A_{st} = \frac{227.25 \times 10^6}{266642.646}$$

$$\Rightarrow \quad A_{st} = 852.26 \text{ mm}^2$$

Example 10

A rectangular RC section 25 cm wide and 50 cm overall deep is reinforced with 3-16 mm HYSD bars as effective cover of 4 cm from bottom face. If permissible stresses in concrete and steel in bending compression are 50 kg/cm² and 2300 kg/cm² respectively and m = 19 calculate MR of the section using WSM.

Sol:

$$b = 25 \text{ cm} = 250 \text{ mm}$$

$$D = 50 \text{ cm} = 500 \text{ mm}$$

$$A_{st} = 3\times\frac{\pi}{4}\times 16^2 = 603.18 \text{ mm}^2$$

$$\text{Effective cover} = 4 \text{ cm} = 40 \text{ mm}$$

$$\sigma_{cbc} = 50 \text{ kg/cm}^2 = 5 \text{ N/mm}^2$$

$$\sigma_{st} = 2300 \text{ kg/cm}^2 = 230 \text{ N/mm}^2$$

$$m = 19$$

$$d = D - 40 = 500 - 40 = 460 \text{ mm}$$

(i) Calculating critical depth of NA

$$x_c = \left(\frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}}\right)d$$

$$= \left[\frac{19\times 5}{230 + (19\times 5)}\right]460$$

$$x_c = 134.46 \text{ mm}$$

(ii) Calculating actual depth of NA by equating moment of area of compression sides and tension side about NA

$$\frac{bx_a^2}{2} = mA_{st}(d - x_a)$$

$$\Rightarrow \frac{250}{2}x_a^2 = 19\times 603.18\times(460 - x_a)$$

$$\Rightarrow 0.011x_a^2 + x_a - 460 = 0$$

$$\Rightarrow x_a = 164.03$$

$x_a > x_c$, Hence section is over reinforced

i.e. $c_a = \sigma_{cbc}$

$t_a < \sigma_{st}$

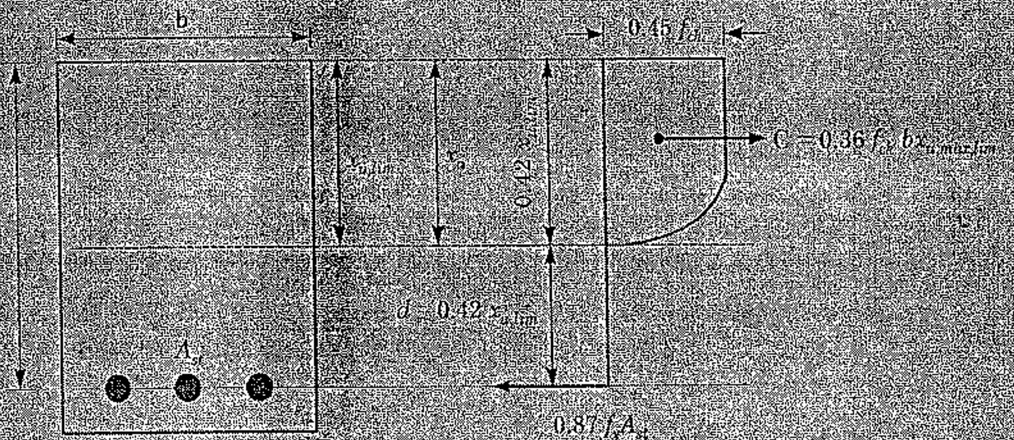
$$\text{MR} = b x_a \frac{c_a}{2}\left(d - \frac{x_a}{3}\right)$$

$$= 250\times 164.03\times\frac{5}{2}\times\left(460 - \frac{164.03}{3}\right)$$

MR = 41.55 kN

Example 11

Derive an expression for limiting percentage of tensile reinforcement in a flexural RC members.



Sol: Limiting percentage of tensile steel is calculated for a balanced section. Since in flexure total compressive force is equal to total tensile force. We get,

$$C = T$$

$$0.36 \times f_{ck}\,b\,x_{u,lim} = 0.67\,f_y\,A_{st}$$

$$\Rightarrow \quad x_{u,lim} = \frac{0.87\,f_y\,A_{st}}{0.36\,f_{ck}\,b}$$

$$\Rightarrow \quad \frac{x_{u,lim}}{d} = \frac{0.87 f_y}{0.36\,f_{ck}}\frac{A_{st}}{bd}$$

$$\Rightarrow \quad \frac{A_{st}}{bd} = \frac{0.36}{0.87}\frac{f_{ck}}{f_y}\frac{x_{u,lim}}{d}$$

$$\Rightarrow \quad \frac{A_{st}}{bd} \times 100 = \frac{0.36}{0.87} \times 100 \times \frac{f_{ck}}{f_y}\frac{x_{u,lim}}{d}$$

$$\Rightarrow \quad \boxed{P_t = 41.38\,\frac{f_{ck}}{f_y}\frac{x_{u,lim}}{d}}$$

If M 20/Fe 415 is used then

$$P_t = 41.38 \times \frac{20}{415} \times \frac{0.48\,d}{d}$$

$$\boxed{P_t = 0.957\%}$$

DOUBLY REINFORCED SECTION

When ever the size of beam is restricted and beam has to bear higher value of B.M. than the moment of Resistance of the ($M.R._{balance}$) balance section of given beam, then to resist this higher moment doubly reinforced section can be used.

$$\boxed{B.M. > M.R._{\text{balanced}}}$$

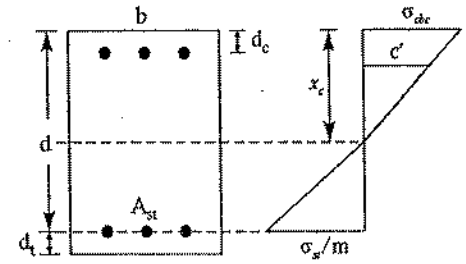


Fig. 1.43

***Note:** In tension steel, shrinkage reduces the tensile stress and creep produces additional tensile stress but in compressive steel both shrinkage and creep add additional stresses so we have to use different value of modular ratio (m).*

In case of Doubly Reinforced section stress in steel in tension side = m × stress in surrounding concrete (mc). But as per IS-456, for steel in compression zone.

Stress in steel = 1.5 m × stress in surrounding concrete = 1.5 m × c'

ANALYSIS OF DOUBLY REINFORCED BEAM

1. Find critical depth of N.A.

$$\boxed{\frac{x_c}{d} = \frac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}}}$$

2. Find actual depth of N.A

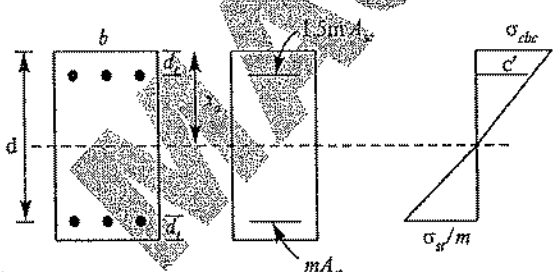


Fig. 1.44

where, A_{sc} = area of compression steel
A_{st} = area of tension steel
σ_{cbc} = maximum permissible value of concrete
c' = stress at the level of compression steel
σ_{st} = maximum permissible value of steel
d_c = effective cover of compression side
d_t = effective cover of tension side

For actual depth of N.A. equate moment of area of tension and compression side about N.A.

A_{sc} is deducted for the holes that concrete leaves for the compression reinforcement

$$\boxed{\frac{bx_a^2}{2}+A_{sc}(1.5m-1)(x_a-d_c)=mA_{st}(d-x_a)}$$

3. Comparision

(a) If $x_a < x_c$ section is under reinforce and for this type of section

$c_a < \sigma_{cbc}$ and

$t_a = \sigma_{st}$

(b) If $x_a = x_c$ section is balanced and for this type of section

$c_a = \sigma_{cbc}$ and

$t_a = \sigma_{st}$

(c) If $x_a > x_c$ section is over reinforce and for this type of section

$c_a = \sigma_{cbc}$

$t_a < \sigma_{st}$

4. Moment of resistance

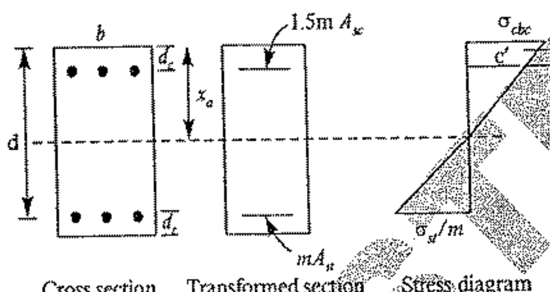


Fig. 1.45

$M.R. = C_1.LA_1 + C_2 \times L.A_2$

C_1 = Force in compression side

C_1 = Area of stress block × Width of section

$C_1 = \frac{1}{2}\sigma_{cbc}x_a.b$

$LA_1 = d - \frac{x_a}{3}$

C_2 = Force in compression steel

C_2 = Area of compression steel × stress at the level of compression steel

$C_2 = (1.5\ m - 1)\ A_{sc}.\ c'$

$LA_2 = d - d_c$

$$\boxed{M.R=\frac{1}{2}bx_a.\sigma_{cbc}.\left(d-\frac{x_a}{3}\right)+(1.5m-1)A_{sc}\ c'(d-d_c)}$$

(ii) M.R. for under reinforced section

$x_a < x_c$,

$c_a < \sigma_{cbc}$

and $t_a = \sigma_{st}$

$$M.R. = \frac{1}{2} b x_a . c_a \left(d - \frac{x_a}{3} \right) + (1.5\ m.1) A_{sc} c'(d - d_c)$$

(iii) M.R. for over reinforced section

$$x_a > x_c;$$
$$c_a = \sigma_{cbc};$$
$$t_a < \sigma_{st}$$

$$\boxed{M.R = \frac{1}{2} b x_a . \sigma_{cbc} \left(d - \frac{x_a}{3} \right) + (1.5m - 1)\ A_{sc}\ c'(d - d_c)}$$

5. **Design of Doubly Reinforced Section:** It is converted in rectangular section with tension reinforcement A_{st1} for giving balanced condition of moment of resistance M_1 + an auxiliary section reinforced with compression reinforcement A_{sc} and tensile reinforcement A_{st2} given moment of resistance M_2

$$M = M_1 + M_2$$

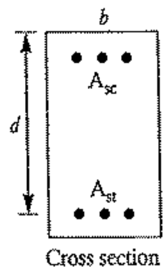


Fig. 1.46

Above cross section converted into following section that gives the moment M_1 and M_2

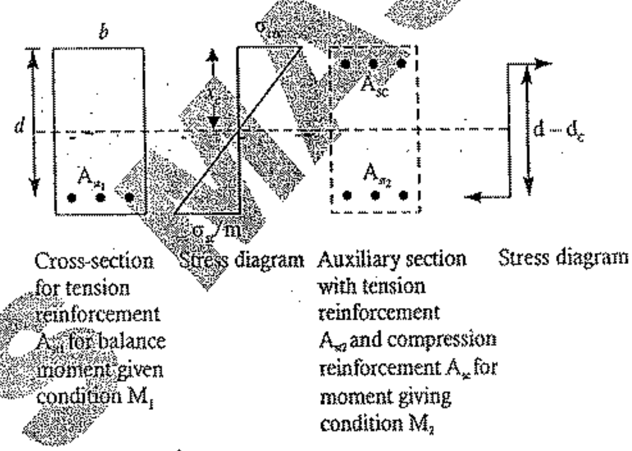


Fig. 1.47

$$M_1 = \frac{1}{2} b x_c . \sigma_{cbc} \left(d - \frac{x_c}{3} \right)$$

For A_{st1} equate

$$M_1 = \sigma_{st} A_{st1} \left(d - \frac{x_c}{3} \right)$$

$$A_{st1} = \frac{M_1}{\ldots}$$

$$M_2 = M - M_1$$

For A_{st2} equate

$$M_2 = \sigma_{st} A_{st2}(d - d_c)$$

$$A_{st2} = \frac{M_2(M - M_1)}{\sigma_{st}(d - d_c)}$$

For A_{sc},

$$C = T \text{ (of auxiliary section)}$$

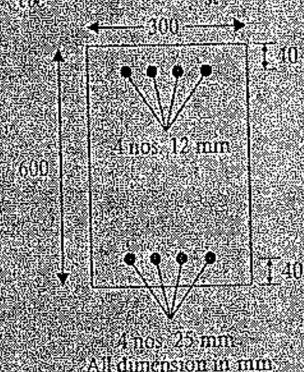
$(1.5m - 1)A_{sc}.c' = \sigma_{st}A_{st2}$

$$\boxed{A_{sc} = \frac{\sigma_{st} A_{st_2}}{(1.5m - 1)c'}}$$

$$A_{st} = A_{st1} + A_{st2}$$

Example 12

Find out the moment of resistance of the section given below? Grade of concrete is M 20 and Fe 415 grade of steel is used. Take, $m = 13$, $\sigma_{cbc} = 7\ N/m^2$, $\sigma_{st} = 230\ N/m^2$



Sol:

$$d = 600 - 40 = 560 \text{ mm}$$

Critical depth of N.A.

$$x_c = \left(\frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}}\right)d = \frac{13 \times 7}{230 + 13 \times 7} \times 560$$

$$x_c = 158.75 \text{ mm}$$

Actual depth of N.A.

$$\frac{bx_a^2}{2} + (1.5m - 1)A_{sc}(x_a - d_c) = mA_{st}(d - x_a)$$

$$\frac{300x_a^2}{2} + (1.5 \times 13 - 1) \times \pi \times 12^2(x_a - 40) = 13 \times \pi \times 25^2(560 - x_a)$$

$$150x_a^2 + 8369.20(x_a - 40) = 25525.4(560 - x_a)$$

$$150x_a^2 + 8369.20x_a - 334768 = 14.29 \times 10^6 - 25525.4x_a$$

$$150x_a^2 + 33894.6x_a - 14.62 \times 10^6 = 0$$

$$x_a > x_c \text{ section is over reinforced}$$

For over reinforced section

$$x_a > x_c$$
$$c_a = \sigma_{cbc}$$
$$t_a < \sigma_{st}$$

$$MR_c = \frac{1}{2} b x_a \sigma_{cbc}\left(d - \frac{x_a}{3}\right) + (1.5\,m - 1)A_{sc}\,c'(d - d_c)$$

c' can be find similar triangle

$$\frac{\sigma_{cbc}}{x_a} = \frac{c'}{x_a - d_c}$$

$$c' = \frac{7 \times (219.03 - 40)}{219.03} = 5.72 \text{ N/mm}^2$$

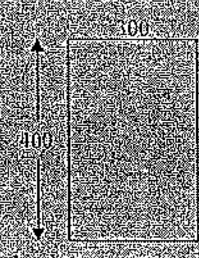
$$MR_c = \frac{1}{2} \times 300 \times 219.03 \times 7\left(560 - \frac{219.03}{3}\right) +$$
$$+ (1.5 \times 13 - 1) \times \pi \times 12^2 \times 5.72(560 - 40)$$
$$= 111.99 \times 10^6 + 24.89 \times 10^6$$
$$MR_c = 136.9 \text{ kN-m}$$

Example 13

A Reinforced beam S.S over a span of 5m carrying a UDL of 2000 kg/m (i/c self wt) $\sigma_{cbc} = 70$ kg/cm², $\sigma_{st} = 1900$ kg/cm² and $m = 13$. Design the beam for flexure only by working stress method. It is restricted to a size of depth 40 c.m. effective cover 40 mm both side compression and tension.

Sol: Assume

$$b = 300 \text{ mm}$$



$$d = 400 - 40 = 360 \text{ mm}$$
$$w = 2000 \text{ kg/m} = 20 \text{ kN/m}$$

$$\text{B.M.} = \frac{wl^2}{8} = \frac{20 \times 5^2}{8} = 62.5 \text{ kN-m}$$

$$MR = Qbd^2$$

$$Q = \frac{1}{2}\sigma_{cbc} j k_c$$

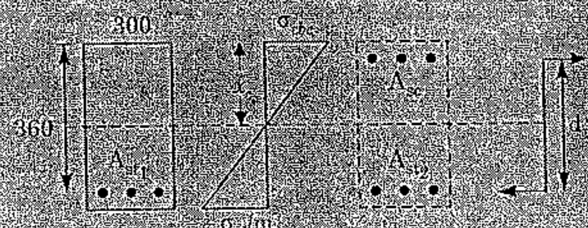
$$k_c = \frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}} = \frac{13 \times 7}{190 + 13 \times 7} = 0.3238$$

$$j = 1 - \frac{k_c}{3} = 1 - \frac{0.3238}{3} = 0.892$$

$$Q = \frac{1}{2} \times 7 \times 0.892 \times 0.3238 = 1.01$$

$$M.R. = 1.01 \times 300 \times 360^2 = 39.30 \text{ kN-m}$$

B.M > $M.R_{balance}$ so designed for doubly reinforce section.



Cross section for tension reinforcement A_{st} for balance moment given condition M_1 — Stress diagram — Auxiliary section with tension reinforcement A_{st} and compression reinforcement A_{sc} for moment giving condition M — Stress diagram

$$M = M_1 + M_2$$
$$M_1 = Qbd^2$$
$$M_1 = 39.30 \text{ kN-m}$$

For A_{st1} equate

$$M_1 = \sigma_{st} A_{st1}\left(d - \frac{x_a}{3}\right)$$

$$= 190\, Ast_1\left(360 - \frac{0.3238 \times 360}{3}\right)$$

$$39.30 \times 10^6 = 61017.36\, A_{st1}$$
$$A_{st1} = 644.07\, mm^2$$

For A_{st2} equate

$$M_2 = \sigma_{st} A_{st2}(d - d_c)$$
$$M_2 = B.M - M_1$$
$$M_2 = 62.5 - 39.30 = 23.2 \text{ kN-m}$$

$$A_{st2} = \frac{M_2(M - M_1)}{\sigma_{st}(d - d_c)} = \frac{23.2 \times 10^6}{190 \times (360 - 40)}$$

$$A_{st2} = 381.57 \text{ mm}^2$$

For A_{sc}

$$C = T \text{ (auxiliary section)}$$

$$(1.5\, m - 1)\, A_{sc}\, c' = \sigma_{st} A_{st2} \quad \text{...(i)}$$

c' from similar triangle

$$\frac{7}{x_c} = \frac{c'}{x_c - d_c}$$

$$c' = \frac{7 \times (x_c - d_c)}{x_c}$$

$$= \frac{7 \times (0.3238 \times 360 - 40)}{0.3238 \times 360}$$

$$c' = 4.59 \text{ N/mm}^2 \text{ put this value in eq. (i)}$$

$$(1.5 \times 13 - 1) \times A_{sc}\ 4.59 = 190 \times 381.57$$

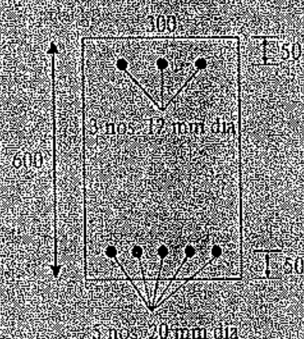
$$A_{sc} = 853.77 \text{ mm}^2$$

$$A_{st} = A_{st1} + A_{st2} = 644.07 + 381.57$$

$$= 1025.64 \text{ mm}^2$$

Example 14

What will be stress in concrete and steel for a beam of M 20 grade of concrete and Fe 415 grade of steel? For (I) 80 kN-m use m = 13.



Sol.

$$d = 600 - 50 = 550 \text{ mm}$$

$$d_c = 50 \text{ mm}$$

Actual Depth of N.A.

$$\frac{bx_a^2}{2} + (1.5\,m - 1)A_{sc}\,(x_a - d_c) = mA_s(d - x_a)$$

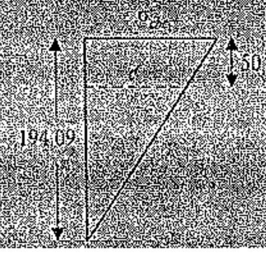
$$\frac{300x_a^2}{2} + (1.5 \times 13 - 1) + \frac{\pi}{4} \times 16^2 \times 3 \times (x_a - 50) = 13 \times 5 \times \frac{\pi}{4} \times 20^2\,(550 - x_a)$$

$$150\,x_a^2 + 11158.93(x_a - 50) = 20420.35(550 - x_a)$$

$$150x_a^2 + 11158.93\,x_a - 557946.5 = 11.23 \times 10^6 - 20420.35\,x_a$$

$$150x_a^2 + 31579.28x_a - 11.78 \times 10^6 = 0$$

$$x_a = 194.09 \text{ mm}$$



$$B.M._C = bx_a \frac{\sigma_{cbc}}{2}\left(d - \frac{x_a}{3}\right) + (1.5\,m - 1)\,A_{sc}\,c'(d - d_c)$$

$$\frac{\sigma_{cbc}}{194.09} = \frac{c'}{194.09 - 50}$$

$$c' = 0.742\sigma_{cbc}$$

$$80 \times 10^6 = 300 \times 194.09 \times \frac{\sigma_{cbc}}{2}\left(550 - \frac{194.09}{3}\right) + (1.5 \times 13 - 1) \times 3 \times \frac{\pi}{4} \times 16^2 \times 0.742 \times (550 - 50)$$

$$= 14128878.6\sigma_{cbc} + 4139965.67\sigma_{cbc}$$

$$\sigma_{cbc} = 4.379 \text{ N/mm}^2$$

Example 15

A doubly reinforce concrete beam having width = b, effective depth = d.

A_{st} = *1.5% of c/s area.*

A_{sc} = *1.0% of c/s area*

$\sigma_{cbc} = 5.5 \text{ N/mm}^2$, $\sigma_{st} = 195 \text{ N/mm}^2$, $m = 18$

Effective cover = 0.1 d

Calculate M.R.?

Sol:

$$A_{st} = \frac{1.5 \times bd}{100} = 0.015bd$$

(Note. It is given c/s area not gross area so c/s area is $b \times d$)

$$A_{sc} = \frac{bd}{100} = 0.01\,bd$$

Critical depth of N.A

$$x_c = \left(\frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}}\right)d = \left(\frac{18 \times 5.5}{195 + 18 \times 5.5}\right)d$$

$$= 0.337\,d$$

II. Actual depth of N.A

$$\frac{bx_a^2}{2} + (1.5\,m - 1)A_{sc}\,(x_a - d_c) = mA_{st}\,(d - x_a) \quad \text{(i)}$$

$$k_a = \frac{x_a}{d}$$

$x_a = k_a d$ put this value in eq. (i)

$$\frac{b.k_a^2 d^2}{2} + (1.5 \times 18 - 1) \times 0.01bd(k_a d - 0.1d) = 18 \times 0.015bd(d - k_a d) \text{ both side} \div bd^2$$

$$k_a^2 + 0.52\,(k_a - 0.1) = 0.54\,(1 - k_a)$$

$$k_a^2 + 1.06\,k_a - 0.592 = 0$$

$$\therefore\ k_a = 0.459$$

$$x_a = 0.459d$$

$x_a > x_c$ section is over reinforced

For over reinforce section

$$x_a > x_c$$

$$c_a = \sigma_{cbc}$$

$$t_a < \sigma_{st}$$

$$M.R. = b x_a \frac{\sigma_{cbc}}{2}\left(d - \frac{x_a}{3}\right) + (1.5\,m - 1)\,A_{sc}\,c'(d - d_c)$$

$$\frac{\sigma_{cbc}}{x_a} = \frac{c'}{x_a - d_c}$$

$$c' = (x_a - d_c)\frac{\sigma_{cbc}}{x_a}$$

$$M.R. = b \times 0.459\,d \times \frac{5.5}{2}\left(d - \frac{0.459d}{3}\right) + (1.5 \times 18 - 1) \times 0.01\,bd\,\frac{(0.459d - 0.1d) \times 5.5}{0.459\,d} \times (d - 0.1d)$$

$$\boxed{M.R. = 2.075bd^2}$$

Example 16

*A RC beam of 250 mm width has an effective depth of 500 mm, effective cover to centre of comp. Reinforcement = 50 mm. Maximum BM carried by the beam under working conditions = **120 kNm**. M 15 grade concrete and Fe 415 steel is used. Determine the reinforcement required.*

Sol. *Given:*

$$b = 250\text{ mm}$$

$$d = 500\text{ mm}$$

$$d_c = 50\text{ mm}$$

$$BM = 120\text{ kNm}$$

(i) Design constants

$$\sigma_{cbc} = 5\text{ N/mm}^2$$

$$\sigma_{st} = 230\text{ N/mm}^2\ [\text{for Fe 415}]$$

$$m = 19$$

$$k = \frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}} = \frac{19 \times 5}{230 + (19 \times 5)} = 0.2923$$

$$j = 1 - \frac{k_c}{3} = 1 - \frac{0.2923}{3} = 0.9025$$

$$Q = \frac{1}{2}\sigma_{cbc}jk_c = \frac{1}{2}\times 5\times 0.9025\times 0.2923 = 0.6595$$

(ii) Calculating MR of balanced section

$$MR_{\text{balanced}} = Qbd^2$$
$$= 0.6595 \times 250 \times 500^2$$
$$= 41.22 \text{ kNm}$$

(iii) BM is more than MR hence a doubly reinforced section is required.

(iv) Calculating area of steel requred for singhy reinforced balanced section

$$A_{st1} = \frac{M.R.}{\sigma_{st}(d - x_a/3)}$$

$$x_a = x_c = k_c d$$
$$= 0.2923 \times 500 = 146.15 \text{ mm}$$

$$= \frac{41.22\times 10^6}{230\left(500 - \frac{146.15}{3}\right)}$$

$$A_{st_1} = 397.13 \text{ mm}^2$$

(v) $$A_{st_2} = \frac{BM - M_2}{\sigma_{st}(d - d_c)}$$

$$= \frac{(120 - 41.22)\times 10^6}{230\times(500 - 50)}$$

$$A_{st_2} = 761.16 \text{ mm}^2$$

(vi) $$A_{st} = A_{st_1} + A_{st_2}$$
$$= 397.13 + 761.16$$
$$= 1158.29 \text{ mm}^2$$

(vii) $$A_x = \frac{m(d - x_a)}{(1.5m - 1)(x_a - d_c)} A_{st_2}$$

$$= \frac{19(500 - 146.15)}{(1.5\times 19 - 1)(146.15 - 50)}\times 761.16$$

$$= \frac{19\times 353.85\times 761.16}{2644.125}$$

$$A_s' = 1935.38 \text{ mm}^2$$

Example 17

A rectangular RC beam SS at ends over an effective span of 5.0 m carries a load of 2000 kg/m including its own weight. It $\sigma_{cbc} = 70$ kg/cm², $\sigma_{st} = 1900$ kg/cm², m = 13. Design the beam section for flexure only by WMS. The side of the beam is restricted to 40 cm × 40 cm (overall depth). Assume effective cover to be 4.0 cm. stress in compression reinforcement, if needed may be taken as 1.5 m times, the stress

Sol:

$$l_{eff} = 5 \text{ m}$$
$$w = 2000 \text{ kg/m} = 20 \text{ kN/m}$$
$$\sigma_{cbc} = 70 \text{ kg/cm}^2 = 7 \text{ N/mm}^2$$
$$\sigma_{st} = 1900 \text{ kg/cm}^2 = 190 \text{ N/mm}^2$$
$$m = 13$$
$$b = 40 \text{ cm} = 400 \text{ mm}$$
$$D = 40 \text{ cm} = 400 \text{ mm}$$
$$\text{Effective cover} = 4.0 \text{ cm} = 40 \text{ mm}$$
$$d = D - 40 = 400 - 40 = 360 \text{ mm}$$

(i) Max BM $= \dfrac{wl_{eff}^2}{8} = \dfrac{20\times5\times5}{8} = 62.5$ kNm

Design constants

$$k_c = \left(\frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}}\right) = \frac{13\times7}{190+(13\times7)} = 0.3238$$

$$j = 1 - \frac{k_c}{3} = 0.8920$$

$$Q = \frac{1}{2}\sigma_{cbc} jk_c = \frac{1}{2}\times7\times0.8920\times0.3238 = 1.011$$

(ii) MR of the balance section

$$\text{MR} = Qbd^2$$
$$= 1.011 \times 400 \times 360 \times 360$$
$$= 52.41 \text{ kNm}$$

(iii) BM > MR, hence a doubly reinforced section is required.

(iv) Calculating area of steel for singhy reinforced balanced section,

$$A_{st_1} = \frac{M_1}{\sigma_{st}(d - x_a/3)}$$

$$= \frac{52.41\times10^6}{190\left(360 - \dfrac{116.57}{3}\right)}$$

$$A_{st_1} = 858.93 \text{ mm}^2$$

$$x_a = x_c = k_c d$$
$$= 0.3238 \times 360 = 116.57 \text{ mm}$$

(v) A_{st_2} = area of remaining tensile steel in the section with compression reinforcement

$$A_{st_2} = \frac{M_2}{\sigma_{st}(d - d_c)}$$

$$= \frac{BM - M_1}{\sigma_{st}(d - d_c)}$$

$$= \frac{(62.5 - 52.41)\times 10^6}{190 \times (360 - 40)}$$

$$A_{st_2} = 165.95 \text{ mm}^2$$

(vi)
$$A_{st} = A_{st_1} + A_{st_2}$$
$$= 858.93 + 165.95$$
$$= 1024.88 \text{ mm}^2$$

(vii) A_{sc}, area of compression reinforcement

$$A_{sc} = \frac{m(d - x_a)}{(1.5m - 1)(x_a - d_c)} \qquad x_a = x_c = 0.3238\, d = 116.57 \text{ mm}$$

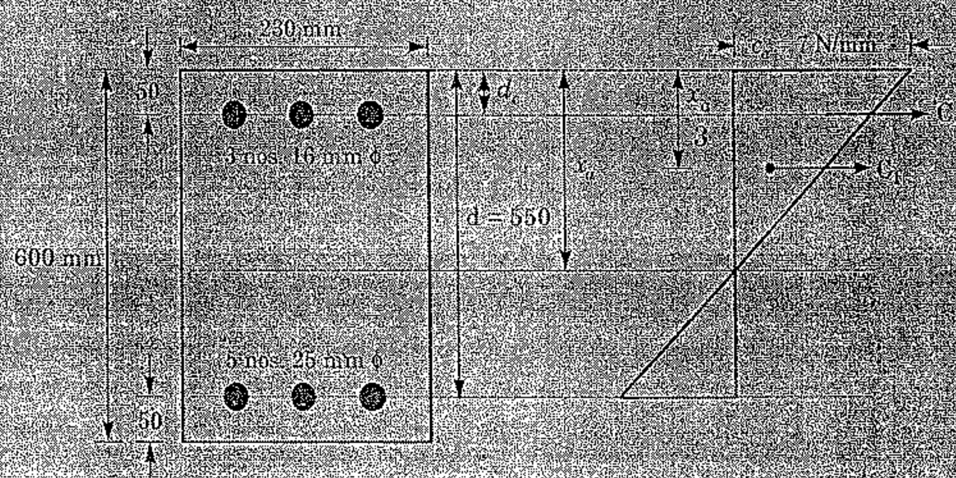
$$= \frac{13 \times (360 - 116.57)}{(1.5 \times 13 - 1)(116.57 - 40)} \times 165.95$$

$$A_{sc} = 523.03 \text{ mm}^2$$

Example 18

A rectangular beam of overall cross-sections dimensions 230 mm × 60 mm with 50 mm effective cover is reinforced with 5 nos. of 25 mm diameter bars in the tension side and 3 nos. of 16 mm φ bars on the compression side. It is carrying an imposed load of 50 kN/m over an effective span of 7.0 m. Check the adequacy of the design. The permissible stresses in concrete and steel are 7 N/mm² and 230 N/mm² respectively, m = 15. Compressive stress in steel bars = 1.5 m times the compressive stress in surrounding concrete.

Sol:



$$A_{st} = 5 \times \frac{\pi}{4} \times 25^2 = 2454.37 \text{ mm}^2$$

1. $BM_{max} = \frac{wl_{eff}^2}{8}$

Self weight of the beam = 0.230 × 0.600 × 1 × 25 = 3.45 kN/m

Imposed live load = 50 kN/m

∴ Total load w = 53.45 kN/m

$$BM_{max} = \frac{53.45 \times 7 \times 7}{8} = 327.38 \text{ kNm}$$

2. Critical neutral axis:

$$x_c = \left[\frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}}\right] d$$

$$= \left[\frac{15 \times 7}{230 + (15 \times 7)}\right] \times 550$$

$$= \left[\frac{105}{230 + 105}\right] \times 550$$

$$x_c = 172.39 \text{ mm}$$

3. Calculating actual depth of neutral axis.

Equating moment of area of compression side and tension side take about NA (actual) i.e.

$$\frac{bx_a^2}{2} + (1.5m - 1) A_{sc} (x_a - d_c) = mA_{st} (d - x_a)$$

$$\Rightarrow \frac{230}{2} x_a^2 + (1.5 \times 15 - 1) \times 603.18 \times (x_a - 50) = 15 \times 2454.37 \times (550 - x_a)$$

$$\Rightarrow 115x_a^2 + 12968.37 (x_a - 50) - 36815.55 (550 - x_a) = 0$$

$$\Rightarrow x_a^2 + 112.77 (x_a - 50) - 320.13 (550 - x_a) = 0$$

$$\Rightarrow x_a^2 + 432.90 x_a - 181710 = 0$$

$$x_a = 261.63 \text{ mm}$$

$x_a > x_c$. Hence the section is over reinforced

$c_a = \sigma_{cbc} = 7 \text{ N/mm}^2$

$t_a < \sigma_{st} = 230 \text{ N/mm}^2$

4. Let c' be the stress at the level of compression steel, then from similar triangles

$$\frac{c'}{x_a - d_c} = \frac{\sigma_{cbc}}{x_a}$$

$$\Rightarrow c' = \frac{7}{261.63} \times (261.63 - 50)$$

$$\Rightarrow c' = 5.66 \text{ N/mm}^2$$

5. Moment of resistance:

$$MR = x_a \frac{c_a}{2}\left(d - \frac{x_a}{3}\right) + (1.5m - 1)\, A_{sc}\, c'(d - d_c)$$

$$= 230 \times 261.63 \times \frac{7}{2} \times \left(550 - \frac{261.63}{3}\right) + (1.5 \times 15 - 1) \times (603.18 \times 5.66) \times (150 - 50)$$

$$= 134.17 \text{ kNm}$$

∴ Moment of resistance of the given section is 134.17 kNm which is less than max. bending moment applied (327.38 kNm) on the section. Hence it is unsafe for the given loading.

Example 19

The size of a RC beam is restricted to 250 mm × 500 mm. If carries a super imposed load of 25 kN/m over a span of 6 m. Determine the reinforcements for the beam by LSM of design. M 20 concrete and Fe 415 steel are used. Effective cover to steel = 40 mm.

Sol: (i) Load calculations

$$\text{Self weight of the beam} = 0.25 \times 0.5 \times 1 \times 25 = 3.125 \text{ kN/m}$$

$$\text{Super imposed} = 25 \text{ kN/m}$$

$$\text{Total load, } w = 28.125 \text{ kNm}$$

$$d = 500 - 40 = 460 \text{ mm}$$

(ii) Max. bending moment

$$BM_{max} = \frac{wl^2}{8} = \frac{28.125 \times 6^2}{8} = 126.56 \text{ kNm}$$

$$\text{Factored } BM_{max} = 1.5 \times 126.56 = 189.8 \text{ kNm}$$

(iii) Calculating moment of resistance for balanced section:

$$M_{u,lim} = 0.36 f_{ck}\, b x_{u,lim} (d - 0.42\, x_{u,lim})$$

$$= 0.36 \times 20 \times 250 \times 0.48 \times 460\, (460 - 0.42 \times 0.48 \times 460)$$

$$M_{u,lim} = 149.96 \text{ kNm}$$

B.M. > M.R. So this is designed as doubly reinforced section otherwise section will be come over reinforced.

(iii) Now taking $M_{u,lim} = M_1$ and providing area of steel A_{st1} as per the value of M_1

$$A_{st1} = \frac{M_1}{0.87 f_y (d - 0.42\, x_{u,lim})}$$

$$= \frac{149.96 \times 10^6}{0.87 \times 415 \times (460 - 0.42 \times 0.48 \times 460)}$$

$$= 1130.91 \text{ mm}^2$$

(iv) Now remaining moment which is to be resisted is

$$M_2 = BM_{max} - M_1$$

$$= 189.84 - 149.96$$

$$= 39.88 \text{ kNm}$$

(v) Providing tension steel as per the value of M_2

$$A_{st2} = \frac{M_2}{0.87f_y(d-d')}$$

$$A_{st2} = \frac{39.88\times10^6}{0.87\times415\times(460-40)}$$

$$= 262.99 \text{ mm}^2$$

(vi) But as we are designing a doubly reinforced section. Hence the compression steel is also to be provided as per the value of M_2

$$A_{sc} = \frac{M_2}{(f_{sc}-0.45f_{ck})(d-d')}$$

f_{sc} can be found with the help of strain in compression steel.

Designing as a balanced section

$$\varepsilon_{sc} = \left[\frac{x_{u,lim}-d_c'}{x_{u,lim}}\right]\times0.0035$$

$$= \left[\frac{0.48\times460-40}{0.48\times460}\right]\times0.0035$$

$$= 0.00287$$

$$f_{sc} \text{ for } 0.0087 = 352.08+\frac{(361.05-352.08)}{(0.00380-0.00276)}\times(0.00287-0.00276)$$

$$= 352.08+0.94875$$

$$= 353.03 \text{ N/mm}^2$$

$$A_{sc} = \frac{39.88\times10^6}{(353.03-0.45\times20)(460-40)}$$

$$= \frac{39.88\times10^6}{144492.6}$$

$$= 276.00 \text{ mm}^2$$

FLANGED BEAM

INTRODUCTION

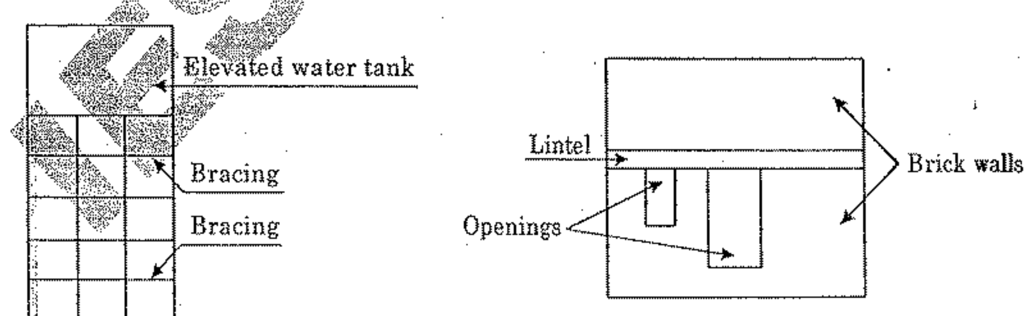
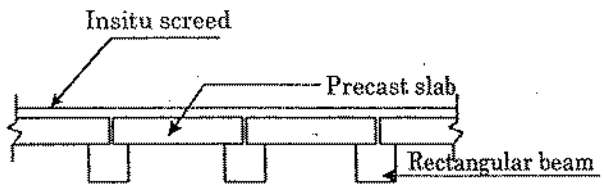


Fig. 1.48: Bracings of elevated water tank. (b) Lintels over openings (without effetive chajjas).



(c) Precast slab on rectangular beams.

- Reinforced concrete slabs used in floors, roofs and decks are mostly cast monolithic from the bottom of the beam to the top of the slab.
- Such rectangular beams having slab on top are different from others having either no slab (bracings of elevated tanks, lintels etc.) or having disconnected slabs as in some pre-cast systems (Figs. 1.48(a)-(c)).
- Due to monolithic casting, beams and a part of the slab act together. Under the action of positive bending moment, i.e. between the supports of a continuous beam, the slab, up to a certain width greater than the width of the beam, forms the top part of the beam.
- Such beams having slab on top of the rectangular rib are designated as the flanged beams-either T or L type depending on whether the slab is on both sides or on one side of the beam (Figs. 1.49(a)-(e)).
- Over the supports of a continuous beam, the bending moment is negative and the slab, therefore, is in tension while a part of the rectangular beam (rib) is in compression.
- The continuous beam at support is thus equivalent to a rectangular beam (Figs. 1.49(a), (c), (f) and (g)).

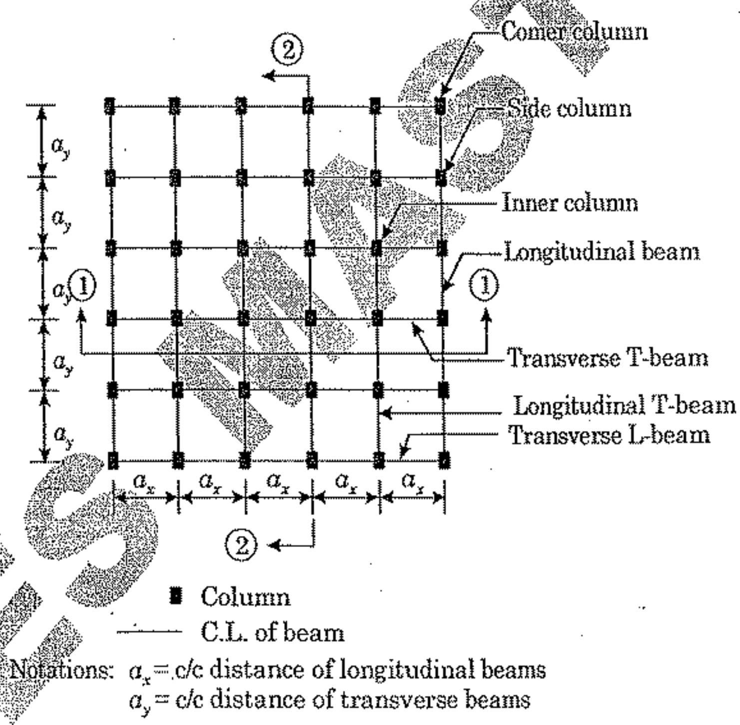


Fig. 1.49(a): Key plan.

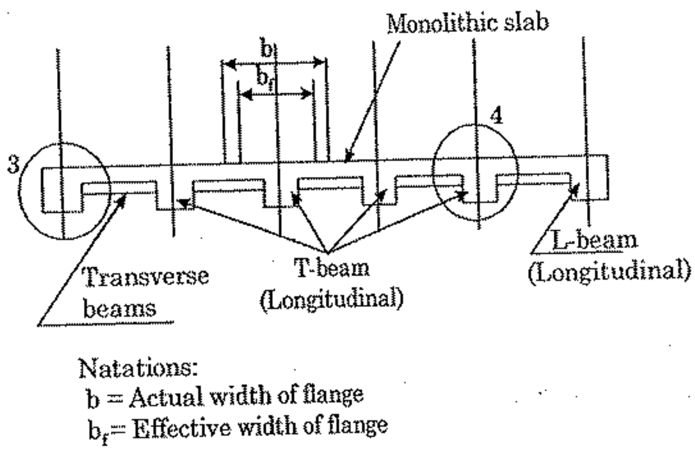


Fig. 1.49(b): Section 1-1

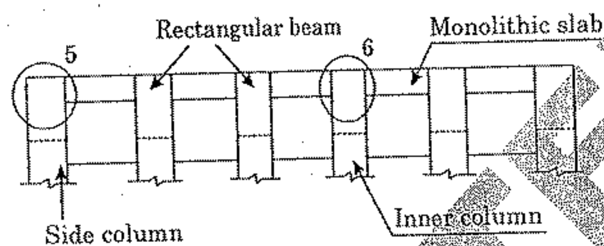


Fig. 1.49(c): Section 2-2.

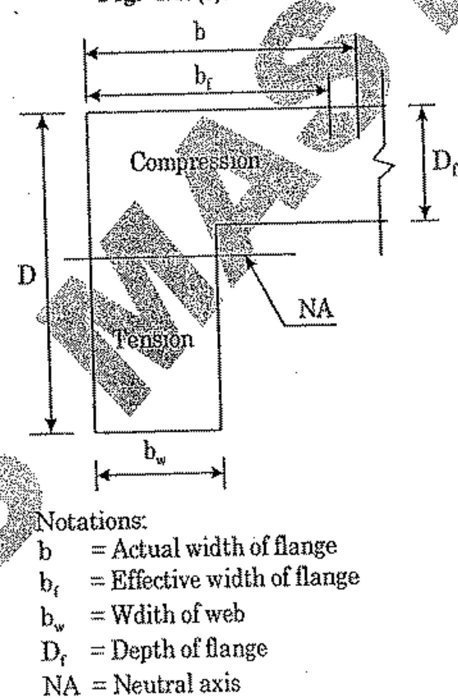
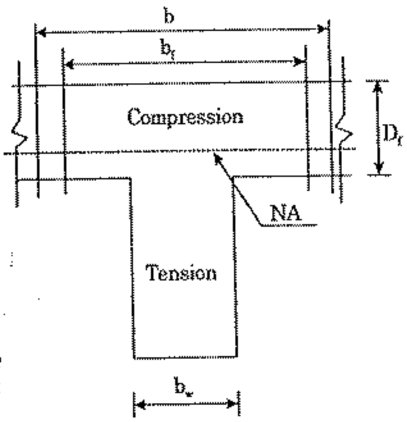
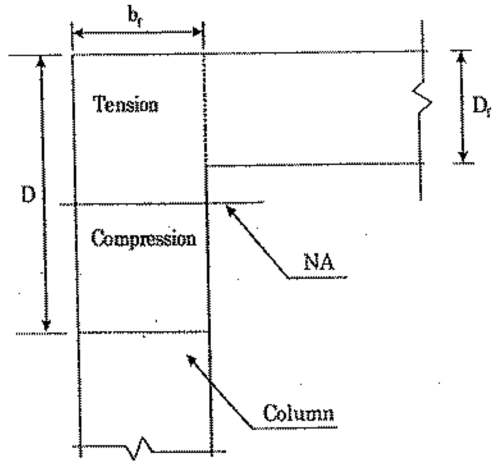


Fig. 1.49(d): Detail at 3 (L-beam).



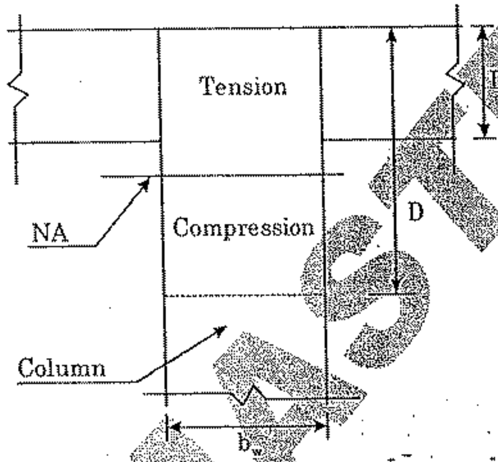
Notations:
b = Actual width of flange
b_f = Effective width of flange
b_w = Wdith of web
D_f = Depth of flange
NA = Neutral axis

Fig. 1.49(e): Detail at 4(T-beam).



Notations:
b_f = Effective width of flange
b_w = Wdith of web
D_f = Depth of flange
NA = Neutral axis

Fig. 1.49(f): Detail at 5 (rectangular beam).



Notations:
b_f = Effective width of flange
b_w = Wdith of web
D_f = Depth of flange
NA = Neutral axis

Fig.1.49(g): Detail at 6 (rectangular beam).

- The actual width of the flange is the spacing of the beam, which is the same as the distance between the middle points of the adjacent spans of the slab, as shown in Fig. 1.33(b).
- However, in a flanged beam, a part of the width less than the actual width, is effective to be considered as a part of the beam.
- This width of the slab is designated as the effective width of the flange.

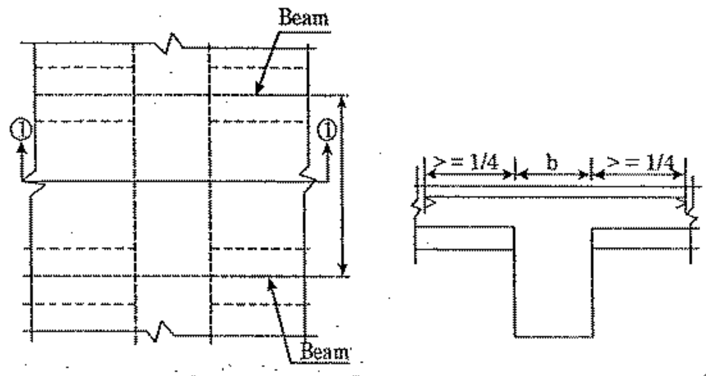
EFFECTIVE WIDTH

IS Code Requirements

The following requirements (cl. 23.1.1 of IS 456) are to be satisfied to ensure the combined action of the

(a) The slab and the rectangular beam shall be case integrally or they shall be effectively bonded in any other manner.

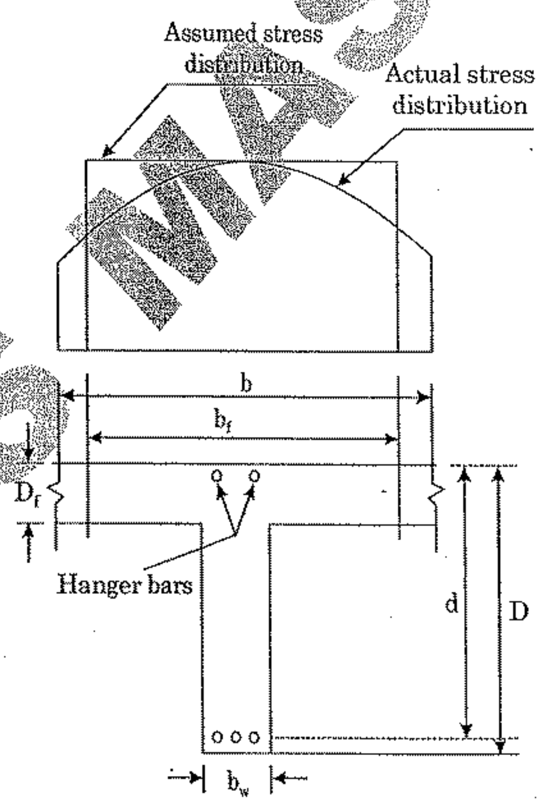
(b) Slabs must be provided with the traverse reinforcement of at least 60 per cent of the main reinforcement at the mid span of the slab if the main reinforcemtn of the slab is parallel to the traverse beam (Figs. 1.50(a) and (b)).



(a) Plan of slab-beam (b) Section 1-1

Fig. 1.50: Traverse reinforcement of flange of T-beam.

- The variation of compressive stress (Fig. 1.51) along the actual width of the flange shows that the compressive stress is more in the flange just above the rib than the same at some distance away from it.
- The nature of variation is complex and, therefore, the concept of effective width has been introduced.
- The effective width is a convenient hypothetical width of the flange over which the compressive stress is assumed to be uniform to give the same compressive force as it would have been in case of the actual width with the true variation of compressive stress.



IS Code Specifications

Clasue 23.1.2 of IS 456 specifies the following effective widths of T and L beams:

(a) For T-beams, the lesser of

(i) $b_f = l_o/6 + b_w + 6\ d_f$

(ii) b_f = Actual width of the flange

(b) For isolated T-beams, the lesser of

(i) $b_f = \dfrac{l_o}{(l_o/b)+4} + b_w$

(ii) b_f = Actual width of the flange

(c) For L-beams, the lesser of

(i) $b_f = l_o/12 + b_w + 3d_f$

(ii) b_f = Actual width of the flange

(d) For isolated L-beams, the lesser of

(i) $b_f = \dfrac{0.5\,l_o}{(l_o/b)+4} + b_w$

(ii) b_f = Actual width of the flange

where b_f = effective width of the flange,

l_o = distance between points of zero moments in the beam, which is the effective span for simply supported beams and 0.7 times the effective span for continuous beams and frames,

b_w = beadth of the web,

d_f = thickness of the flange,

and b = actual width of the flange.

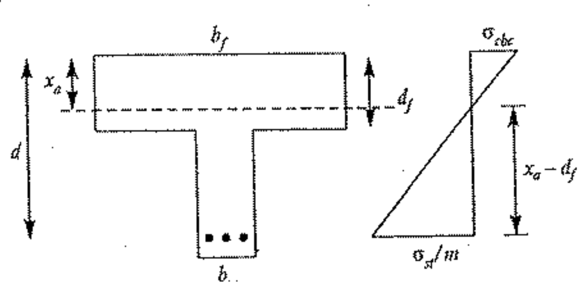
Analysis of T-beam

1. Calaculate critical depth of N.A.

$$x_c = \left(\frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}}\right)d$$

2. Actual depth of N.A.

Case (1): When N.A. lies in flange section



It is a simple case of rectangular section

For actual depth of N.A.

$$b_f . x_a . \frac{x_a}{2} = mA_{st}\,(d - x_a)$$

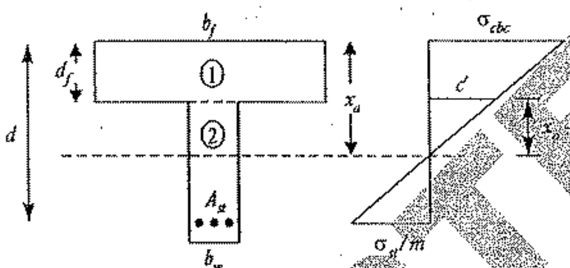
$$\boxed{b_f \frac{x_a^2}{2} = mA_{st}(d - x_a)}$$

Moment of resistance

$$MR_c = \frac{1}{2}\sigma_{cbc} x_a . b_f \left(d - \frac{x_a}{3}\right)$$

$$MR_t = \sigma_{st} A_{st}\left(d - \frac{x_a}{3}\right)$$

Case (ii): When N.A. lies in web area



Cross section of T-beam Stress diagram

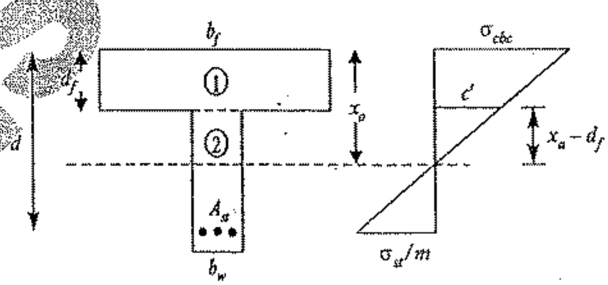
For actual detph of N.A.

Taking moment of area from both side compression and tension.

$$b_f . d_f\left(x_a - \frac{d_f}{2}\right) + \underbrace{b_w \times (x_a - d_f)\left(\frac{x_a - d_f}{2}\right)}_{\text{Can be neglected}} = mA_{st}(d - x_a)$$

$$b_f d_f \left(x_a - \frac{d_f}{2}\right) = mA_{st}\,(d - x_a)$$

Moment of resistance: When web area is neglected



Cross section of T-beam Stress diagram

M.R. = Area of stress diagram × Width of section × L.A.

Area of stress diagram = $(\sigma_{cbc} + c') . d_f$

$$\text{L.A.} = d - \bar{y}$$

$$\bar{y} = \left(\frac{\sigma_{cbc} + 2c'}{\sigma_{cbc} + c'}\right) \cdot \frac{d_f}{3}$$

$$\therefore \boxed{M.R_c = (\sigma_{cbc} + c')\frac{d_f}{2} \cdot b_f \left[d - \left(\frac{\sigma_{cbc} + 2c'}{\sigma_{cbc} + c'}\right) \cdot \frac{df}{3}\right]}$$

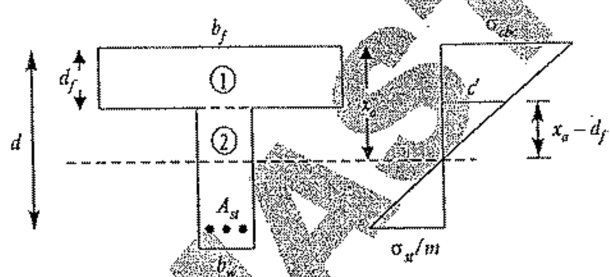
$$M.R_t = \sigma_{st} A_{st} \left[d - \left(\frac{\sigma_{cbc} + 2c'}{\sigma_{cbc} + c'}\right) \cdot \frac{d_f}{3}\right]$$

c' can be found from similar triangle

$$\frac{\sigma_{cbc}}{x_a} = \frac{c'}{x_a - d_f}$$

$$\boxed{c' = \frac{\sigma_{cbc}}{x_a}(x_a - d_f)}$$

2. Moment of resistance considering the web area



Cross section of T-beam Stress diagram

$$M.R_c = \text{Compressive force} \times \text{L.A}$$

Compressive force for trapezoidal section = Area of stress diagram × Width of section

$$= \left(\frac{\sigma_{cbc} + c'}{2}\right) \cdot d_f \times b_f$$

$$\bar{y} = \left(\frac{\sigma_{cbc} + 2c'}{\sigma_{cbc} + c'}\right) \cdot \frac{df}{3}$$

$$\therefore \quad \text{L.A.} = d - \bar{y}$$

Compressive force for tranglular section = Area of stress diagram × Width of section

$$= \frac{1}{2} c'(x_a - d_f) \times b_w$$

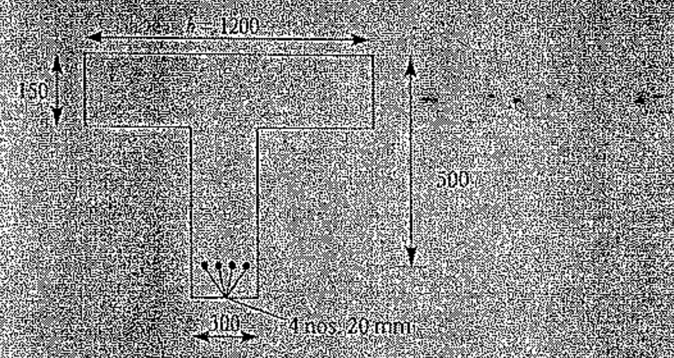
$$\bar{y} = \frac{x_a - d_f}{3}$$

$$\boxed{L.A = d - d_f - \left(\frac{x_a - d_f}{3}\right)}$$

$$MR_c = \left(\frac{\sigma_{cbc} + c'}{2}\right) d_f . b_f \left[d - \left(\frac{\sigma_{cbc} + 2c'}{\sigma_{cbc} + c'}\right)\right] \frac{d_f}{3} + \frac{1}{2} c'(x_a - d_f) b_w \left[d - d_f - \left(x_a - \frac{d_f}{3}\right)\right]$$

Example 20

Find the moment of the resistance of isolated T beam shown in figure below is simply supported over a span of 6 m use M 20 grade of concrete and Fe 415 grade of steel. Take σ_{cbc} = 7 N/mm², σ_{st} = 230 N/mm², m = 13.



All dimension in mm

Sol. Effective width of flange (b_f) $= \dfrac{l_0}{\dfrac{l_0}{b} + 4} + b_w$

$$= \frac{6000}{\dfrac{6000}{1200} + 4} + 300 = 966.67 \text{ mm}$$

Critical depth of N.A. b_f < 1200 mm O.K.

$$x_c = \left(\frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}}\right) d = \left(\frac{13 \times 7}{230 + 13 \times 7}\right) \times 500$$

$$x_c = 141.74 \text{ mm}$$

Actual depth of N.A.

Assume N.A. is lies in web portion (neglecting the web area)

$$b_f d_f \left(x_a - \frac{d_f}{2}\right) = mA_{st}(d - x_a)$$

$$966.67 \times 150 \times \left(x_a - \frac{150}{2}\right) = 13 \times \frac{\pi}{4} \times 20^2 (500 - x_a)$$

$$145000.5x_a - 10.87 \times 10^6 = 8.168 \times 10^6 - 16.33 \times 10^3 x_a$$

$$161333.678x_a = 19.03 \times 10^6$$

So our assumption is wronge.

Again assume N.A. is lies in flange portion

$$b_f\frac{x_a^2}{2} = mA_{st}(d - x_a)$$

$$966.67\frac{x_a^2}{2} = 13 \times 1256\,(500 - x_a)$$

$$483.3\,x_a^2 = 8.168 \times 10^6 - 16336.28x_a$$

$x_a = 114.19$ mm less than 150 mm so our assumption in correct and $x_a < x_c$ section is under reinforced

For under reinforced section

$$x_a < x_c$$

$$c_a < \sigma_{cbc}$$

$$t_a = \sigma_{st}$$

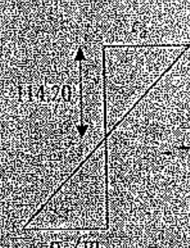
$$\text{MR} = \sigma_{st}A_{st}\left(d - \frac{x_a}{3}\right)$$

$$= 230 \times \pi \times 20^2\left(520 - \frac{114.20}{3}\right)$$

$$= 133.51 \text{ kN-m}$$

OR

$$\text{MR} = \frac{1}{2}b_f x_a c_a\left(d - \frac{x_a}{3}\right)$$



$$\frac{c_a}{114.20} = \frac{\frac{230}{13}}{500 - 114.20}$$

$$c_a = 5.24 \text{ N/mm}^2$$

$$M.R. = \frac{1}{2} \times 966.67 \times 114.20 \times 5.24\left(500 - \frac{114.20}{3}\right)$$

$$M.R. = 133.60 \text{ kN-m}$$

Example 21

Solve the Same problem while taking d_f = 80 mm.

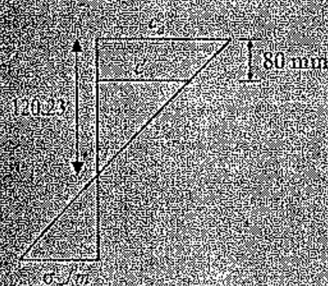
Sol: Assume N.A. lies in web (neglecting web portion)

$$b_f d_f\left(x_a - \frac{d_f}{2}\right) = mA_{st}(d - x_a)$$

$$x_a = 120.23 > d_f \text{ (80 mm)}$$

Our assumption is correct and $x_a < x_c$ section is under reinforced

M.R. (neglecting the web area)



$$\frac{c_a}{120.23} = \frac{\frac{230}{13}}{500 - 120.23}$$

$$c_a = 5.6 \text{ N/mm}^2$$

Again for c'

$$\frac{c_a}{120.23} = \frac{c'}{120.23 - 80}$$

$$c' = 1.87 \text{ N/mm}^2$$

$$\text{M.R.} = (c_a + c') \times \frac{80}{2} \times b_f \left[d - \left(\frac{c_a + 2c'}{c_a + c'}\right)\frac{80}{3}\right]$$

$$= (5.6 + 1.87) \times 40 \times 966.67 \left[500 - \left(\frac{5.6 + 1.87 \times 2}{5.6 + 1.87}\right)\frac{80}{3}\right]$$

$$= 134.80 \text{ kN-m}$$

Example 22

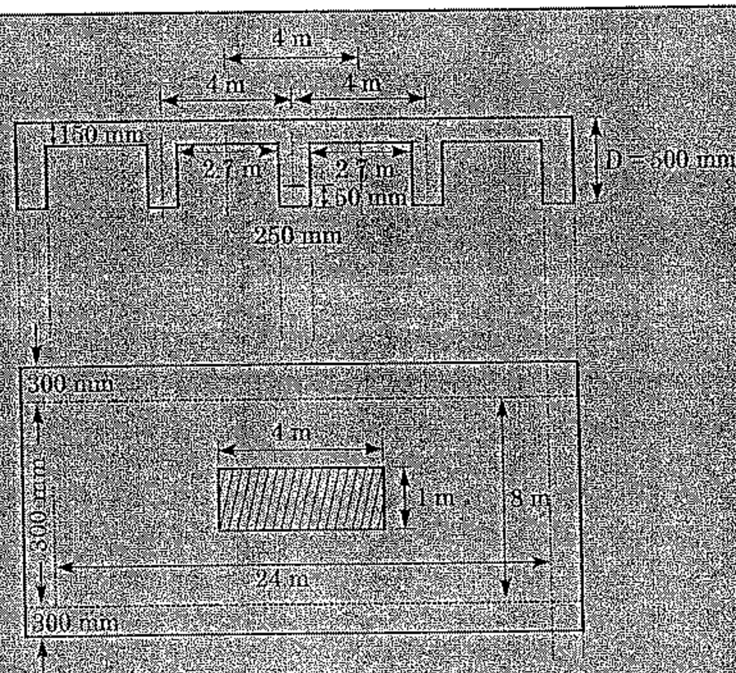
A hall of clear dimensions 8 × 24 mm is covered by a concrete slab 150 mm thick supported on T-beams spaces at 4 m intervals. The beam are resting on walls 300 mm wide. The rib width is to be resisted to 250 mm and overall depth of the T-beam is to be 500 mm. The beam is reinforced with 6 nos. of 22 mm ϕ high strength deformed bars. Live load on the roof including the weathering course is 3 kN/m². Grade of concrete used is M 15, $\sigma_{cbc} = 5\ N/mm^2$, $\sigma_{st} = 140\ N/mm^2$, m = 19. Check the safety of the beam.

Sol:

(i) Effective span

$$l_s < \frac{1}{12} l_{clear}$$

$$300 < \frac{1}{12} \times 8000 \Rightarrow 300 < 666.67, \text{ Hence O.K.}$$



$$A_{st} = 6\times\frac{\pi}{4}\times 22^2 = 2280.79\ \text{mm}^2$$

Assuming effective cover = 50 mm, d = 500 − 50 = 450 mm

Effective span will be $\begin{Bmatrix}8000+450\\8000+300\end{Bmatrix}$ whichever less

Hence $\qquad l_{\text{eff}} = 8000 + 300 = 8300\ \text{mm} = 8.3\ \text{m}$

(ii) Load calculations

Self wt. of slab = 4 × 1 × 0.150 × 25 = 15 kN/m

LL on slab including FF = 3 × 1 × 4 = 12 kN/m

Self of beams = (0.5 − 0.150) × 0.25 × 1 × 25 = 2.1875 kN/m

Total load on the beam = 15 + 12 + 2.1875

w = 29.19 kN/m

(iii) Calculating bending moment

$$\text{BM}_{max} = \frac{wl_{eff}^2}{8} = \frac{29.19\times 8.3^2}{8} = 251.36\ \text{kNm}$$

(iv) Calculation of effective width (b_f)

$$b_f = \frac{l_0}{6} + b_w + 6d_f$$

According to IS 456-2000

$$l_0 = 0.7\,l_{\text{eff}}$$
$$= 0.7 \times 8 \times 3$$
$$= 5.81\,m = 5810\ \text{mm}$$

5810

$$= 968.33 + 250 + 400$$

$$= 2118.33 \text{ mm} < b_w + \frac{l_1}{2} + \frac{l_2}{2} = 250 + \frac{3750}{2} + \frac{3750}{2}$$

$$= 4000 \text{ mm}$$

$$= 2118.33 < 4000. \text{ Hence OK}$$

$$\text{Adopt } b_f = 2118.33 \text{ mm}$$

(v) Critical neutral axis of the beam

$$x_c = \left(\frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}}\right) d$$

$$= \left[\frac{19 \times 5}{140 + (19 \times 5)}\right] \times 450$$

$$x_c = 181.91 \text{ mm}$$

(vi) Calculation of actual depth of neutral axis

Assuming NA in the flange i.e. $x_a < d_f$

$$b_f \frac{x_a^2}{2} = m A_{st} (d - x_a)$$

$$\Rightarrow \quad 2118.33 \frac{x_a^2}{2} = 19 \times 2280.79 (450 - x_a)$$

$$\Rightarrow \quad 0.024 x_a^2 + x_a - 450 = 0$$

$$\Rightarrow \quad x_a = 116.76 \text{ mm}$$

$\because$ $x_a < x_c$. Hence the section is under reinforced and as $x_a < d_f$, hence the actual neutral axis lies in the flange.

Hence

$$\text{MR} = \sigma_{st} A_t \left(d - \frac{x_a}{3}\right)$$

$$= 140 \times 2280.79 \left(450 - \frac{116.76}{3}\right)$$

$$\text{MR} = 131.26 \text{ kNm}$$

$\because$ Moment of resistance of the beam (131.26 kNm) is much less than BM_{max} (251.36 kNm). Hence the given beam is not safe and it is to be redesigned.

Some Important Points

- Concrete is superior to stone, timber and steel because:

  (i) Stone, timber and steel cannot be fitted to any mould, but concrete during its green stage can fit to anymould.

  (ii) Structures made of stone, timber and steel have servel joints, but different elements of concrete structures can be cast monolithically.

- **Integratged structure is one** where each part of the structure satisfies both the structureal

- **The objectives of the design of reinforced concrete structure are:**
  - (i) Have acceptable probability of performing satisfactorily during their intended life,
  - (ii) Sustain all loads with limited deformations during construction and use,
  - (iii) be durable,
  - (iv) adequately resist the effects of mis use and fire.
- **The four objectives of the design of reinforced concrete structures can be fulfilled by:**
  - (i) understanding the strength and deformation characteristics of concrete and steel,
  - (ii) following the clearly defined standards for materials, production workmanship and maintenance, and use of structures in service,
  - (iii) adopting measures needed for durability.
- **The three methods of design of reinforced concrete beam are:**
  - (i) limit state method,
  - (ii) working stress method,
  - (iii) method based on experimental approach
- **Out of above three methods** the Limit state method is the best method.
- The design loads in (i) limit state method, and (ii) working stress method are calculated by in limit state method,

  Design loads = Characteristic loads multiplied by the partial safety factor for loads

  (ii) In working stress method,

  Design loads = Characteristic loads
- Characteristic load is the value of load which has 95% probability of not being exceeded during the life of the structure.
- The main (i) loads, (ii) forces and (iii) effects to be considered while designing the structures are:
  1. **Loads:**
     - (a) Dead load
     - (b) Imposed loads or live loads
     - (c) Wind loads
     - (d) Show loads
     - (e) Erection loads
  2. **Force**
     - (a) Earthquake force
  3. **Effects**
     - (a) Shrinkage, creep and temperature effects
     - (b) Foundation movements
     - (c) Elastic axial shortening
     - (d) Soil and fluid pressures
     - (e) Vibration
     - (f) Fatigue
     - (g) Impact
     - (h) Stress concentration effects due to application of points

- **The common steps of design of structure by any metbod of design are:**
  - (i) To assess the dead loads and other external loads and forces likely to be applied on the structure,
  - (ii) To determine the design load from different combinations of loads,
  - (iii) To estimate structural responses (bending moment, shear force, axial thrust etc.) due to the design loads.
  - (iv) To determine the cross-sectional areas of concrete sections and amount of reinforcement needed.
- The characteristic strength of concrete is determined from the results of tests conducted on cube specimens of 150 mm dimension. The dimension of concrete structures or in members of structure are different and widely varying. This has an effect on the strength of concrete in the structure. This is known as size effect.

  Due to the size effect, the characteristic strength of concrete is reduced to 2/3 of its value and then further divided by the partial safety factor of the concrete (γm = 1.5) to get the design strength of concrete (f_d). Thus,

$$f_d = (f_{ck})\frac{(2/3)}{1.5} = \frac{0.67\ f_{ck}}{1.5} = 0.446\ f_{ck}$$

- Linear elastic theory should be employed for the analysis of structural system subjected to design loads in limit state theory.
- **The reason to justify the design of structures by limit state method are:**
  - (i) Concept of separate partial safety factors of loads of different combinations in the two limit statre methods.
  - (ii) Concept of separate partial safety factors of materials depending on their quality control during preparation. Thus, γ_m for concrete is 1.5 and the same for steel is 1.15. The is more logical than one arbitary value in the name of safety factor.
  - (iii) A structure designed by employing limit state method of collapse and checked for other limit states will ensure the strength and stability requirements at the collapse under the design loads and also deflection and cracking at the limit state of serviceability. This will help to achieve the structure with acceptable probabilities that the sturcture will not become unfit for the use for which it is intended.
  - (iv) The stress block represents in a more realistic manner when the structure is at the collapsing stage (limit state of collapse) subjected to designl loads.
- **The beams and slabs carry the transverse loads primarily by bending.**
- **The three different types of reinforced concrete beams and their specific applecations are:**
  - (i) Singly reinforced and doubly reinforced rectangular beams - used in resisting negative moments in intermediate spans of continuous beam over the supports or elsewhere in slab-beam monolithic construction, and positive moments in midspan of isolated or intermediate spans of beam with inverted slab (monolithic) constructions and lintels.
  - (ii) Singly reinforced and double reinforcved T-beams - used in resisting positive moments in isolated or intermediate spans (midspan) in slab-beam monolithic constructions and negative moments over the support for continuous spans with inverted slab (monolithic) constructions.
  - (iii) Singly reinforced and double reinforced L-beams - Same as (ii) above except that these are for end spans instead of intermediate spans.
- **The four parameters which determine the effective widths of T and L-beams are:**

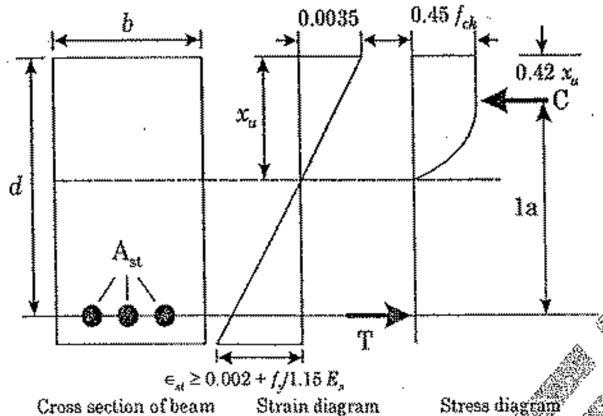
(i) Isolated or continuous beams,

(ii) the distance between points of zero moments in the beam,

(iii) the breadth of the web,

(iv) the thickness of the flange.

- **The cross-section of single reinforced rectangular beam and show the strain and stress diagrams is**



Strain and stress diagrams for a cross-section.

- **The three equations of equilibrium needed to design the reinforced concrete beams are:**

  (i) Equilibrium of horizontal forces : $\sum H = 0$ gives $C = T$

  (ii) Equilibrium of vertical shear forces : $\sum V = 0$

  This equation gives an identity 0 = 0 as there is no shear in the middle third zone of the beam.

  (iii) Equilibrium of moments $\sum M = 0$,

  This equation shows that the applied moment at the section is fully resisted by moment of the resisting couple T z = C z, where z is the operating lever arm between T and C.

- **The final expression of tbe total compressive force C and tansile force T for a rectangular reinforced concrete beam in terms of the designing parameters are:** $C = 0.36\, f_{ck}\, x_u\, b$ and $T = 0.87\, f_y\, A_{st}$.

- **The equation is needed to determine the depth of the neutral axis is**

$$0.87\, f_y\, A_{st} = 0.36\, x_u\, f_{ck}$$

$$x_u = \frac{0.87\, f_y\, A_{st}}{0.36\, b\, f_{ck}}$$

we can also write in non-dimmensional form $\frac{x_u}{d} = \frac{0.87 \quad f_y\, A_{st}}{0.36 \quad bd\, f_{ck}}$.

- **Tbe lever arm is the difference between centroid of the compressive force and the centroid of the tensile force, and denoted by z and can be find out by**

$$z = d - 0.42\, x_u$$

- The expression of M_u when (i) $x_u < x_{u,\,lim}$, and (ii) $x_u = x_{u,\,lim}$ are:

  (i) $M_u = 0.36\, f_{ck}\, x_u\, b(d - 0.42\, x_u)$

  OR

  $M_u = 0.87\, f_y\, A_{st}\, (d - 0.42\, x_u)$, we can also write this as

  $$M_u = 0.87\, f_y\, A_{st}\, d\left(1 - \frac{A_{st}\, f_y}{f_{ck}\, b\, d}\right)$$

  (ii) $M_{u,\,lim} = 0.36\, f_{ck}\, x_{u,\,lim}\, b\, (d - 0.42\, x_{u,\,lim})$

- **When $x_u > x_{u,\,lim}$, the section has to be redesigned as this does not ensure ductile failure of tbe beam.**

- We go for doubly reinforced beams when the depth of the beams may be restricted for architectural and/ or functional requirments. Doubly reinforced beam are designed if such beams of restricted depth are required to resist that its $M_{u,lim}$.

- **Tbe tbree situations otber than doubly reinforced beams where the compression reinforcement is provided when**

  (i) Some sections of a continuous beam with moving loads undergo change of sign of the bending moment which makes compression zone as tension zone,

  (ii) the ductility requirement has to be satisfied,

  (iii) the reduction of long term deflection is needed.

- **The following procedure may be followed to determine the value of f_{sc} and f_{cc} for tbe design type of problems (and not for analysing a given section). For the design problem the deptb of the neutral axis may be taken as $x_{u,lim}$. The strain at the level of compression steel reinforcement ε_{sc} may be written as**

$$\varepsilon_{sc} = 0.0035\left(1 - \frac{d'}{x_{u,lim}}\right)$$

The stress in compression steel f_{sc} is corresponding to the strain ε_{sc} of and is determined for (a) mild steel and (b) cold worked bars Fe 415 and 500 as given below:

(a) Mild steel Fe 250

The strain at the design yield stress of 217.39 N/mm² ($f_d = 0.87\, f_y$) is 0.0010869 ($= 217.39/E_s$). The f_{sc} is determined from the idealized stress-strain diagram of mild steel (Figure 23B of IS 456) after computing the value of ε_{sc}.

(i) If the computed value of $\varepsilon_{sc} \le 0.0010869$, $f_{sc} = \varepsilon_{sc}\, E_s = 2\,(10^5)\,\varepsilon_{sc}$

(ii) If the computed value of $\varepsilon_{sc} > 0.0010869$, $f_{sc} = 217.39$ N/mm².

(b) Cold worked bars Fe 415 and Fe 500)

The stress-strain diagram of these bars is (given in figure 1.3b) and in figure 23A of IS 456. It shows that stress is proportional to strain up to a stress of 0.8 f_y. The stress-strain curve for the design purpose is obtained by substituting f_{yd} for f_y in the figure up to 0.8 f_{yd}. Thereafter, from 0.8 f_{yd} to f_{yd}, Table A of SP-16 gives the values of total strains and design stresses for Fe 415 and Fe 500. Table 1.4 presents these values as a ready reference here.

- The two type of problems of doubly reinforced beams specifying the given data and the values to be determined are:

(i) Design type of problems and

(ii) Analysis type of problems

(i) ***Design type of problems and:*** The given data are $b, d, D\ f_{ck}, f_y$ and M_u. It is required to determine A_{sc} and A_{st}.

(ii) ***Analysis type of problems*** : The given data are b, d, D f_{ck}, f_y, A_{sc} and A_{st}. It is required to determine the M_u of the beam.

- The effective width is an imaginary width of the flange over which the compressive stress is assumed to be uniform to give the same compressive force as it sould have been in case of the actual width with the true variation of compressive stress.

- The expression of effective widths of T and L-beams and isolated beams according to Clasue 23.1.2 of IS 456 are:

(a) For *T*-beams, the lesser of

(i) $b_f = l_o/6 + b_w + 6\ d_f$

(ii) b_f = Actual width of the flange

(b) For isolated *T*-beams, the lesser of

(i) $b_f = \dfrac{l_o}{(l_o / b) + 4} + b_w$

(ii) b_f = Actual width of the flange

(c) For *L*-beams, the lesser of

(i) $b_f = l_o/12 + b_w + 3d_f$

(ii) b_f = Actual width of the flange

(d) For isolated *L*-beams, the lesser of

(i) $b_f = \dfrac{0.5\, l_o}{(l_o/b) + 4} + b_w$

(ii) b_f = Actual width of the flange

where b_f = effective width of the flange,

l_o = distance between points of zero moments in the beam, which is the effective span for simply supported beams and 0.7 times the effective span for continuous beams and frames,

b_w = beadth of the web,

d_f = thickness of the flange,

and b = actual width of the flange.

Objective Practice Questions

1. The limiting compressive strain inconcrete is

(a) 0.0035 (b) 0.0015

(c) 0.0025 (d) 0.015

(a) 2.0
(b) 1.5
(c) 1.15
(d) 1.00

3. Combination of partial safety factors for loads under limit state of collapse and limit state of serviceability will be

(a) 1.5 (DL + LL) or 1.5 (DL + WL) or 1.2 (DL + LL + WL) and DL + 0.8 (LL + WL)
(b) 1.5 (DL + LL) and DL + 0.8 (LL + WL)
(c) 1.5 (DL + LL) or 1.5 (DL + WL) or 1.2 (DL + LL + WL) and 1.0 (DL + LL) or 1.0 (DL + WL) or DL + 0.8 (LL + WL)
(d) 1.2 (DL + LL + WL) and 1.0 (DL + LL) or 1.0 (DL + WL) or DL + 0.8 (LL + WL)

4. The assumption that the plane sections normal before bending remain normal after bending is used

(a) only in the working stress method of design
(b) only in the limit-state method of design
(c) in both working stress and limit-state methods of design
(d) only in the ultimate load method of design

5. In the limit state design method of concrete structure, the recommended partial meterial safety factor (γ_m) for steel according to IS:456-2000 is

(a) 1.5
(b) 1.15
(c) 1.00
(d) 0.87

6. For avoiding the limit state of collapse, the safety of RC structures is checked for appropriate combinations of Dead Load (DL), Imposed Load or Live Load (IL), Wind Load (WL) and Earthquake Load (EL). Which of the following load combinations is NOT considered?

(a) 0.9 DL + 1.5 WL
(b) 1.5 DL + 1.5 WL
(c) 1.5 DL + 1.5 WL + 1.5 FL
(d) 1.2 DL + 1.2 IL + 1.2 WL

7. The partial factor of safety for concrete as per IS : 456-2000 is

(a) 1.50
(b) 1.15
(c) 0.87
(d) 0.446

8. If the characteristic strength of concrete f_{ck} is defined as the strength below which not more than 50% of the test results are expected to fall, the expression for f_{ck} in terms of mean streangh f_m and standard deviation S would be

(a) $f_m - 0.1645S$
(b) $f_m - 1.645S$
(c) f_m
(d) $f_m + 1.645S$

9. For limit state of collapse, the partial safety factors recommended by IS 456: 2000 for estimating the design strength of concrete and reinforcing steel are respectively

(a) 1.15 and 1.5
(b) 1.0 and 1.0
(c) 1.5 and 1.15
(d) 1.5 and 1.0

10. If f_{cu} and f_y are cube compressive strength of concrete and yield stres of steel respectively and E_s is the modulus of elasticity of steel for all grades of concrete, the ultimate flexural strain in concrete can be taken as

(a) 0.002
(b) $\frac{f_{cu}}{1000}$
(c) 0.0035
(d) $\frac{f_y}{1.15E_s} + 0.002$

11. A T-beam roof section has the following particulars:

Thickness of slab	:	100 mm
Width of rib	:	300 mm
Depth of beam	:	500
Centre to centre distance of beams	:	3.0 m
Effective span of beam	:	6.0 m
Distance between points of the beam is	:	3.60 m

The effective width of flange of the beam is

(a) 3000 mm
(b) 1900 mm
(c) 1600 mm
(d) 1500 mm

12. The maximum strain in concrete at the outermost compression fiber in the limit state desing offlexural member is (as per IS : 456 - 1978)

(a) 0.0020
(b) 0.0035
(c) 0.0065
(d) 0.0050

13. In limit state approach, spacing of main reinforcement controls primarily

(a) collaps
(b) cracking
(c) deflection
(d) durability

14. The effective width 'b_f' of flange of a continuous T-beam in a floor system is given by

$$b_f = \frac{L_0}{6} + b_w + 6D_f$$

where L_0 representrs the

(a) distance between points of contraflexure in a span
(b) effective span of beams
(c) clear span of beams
(d) spacing between beams

15. A doubly reinforced beam is considered less economical than a singly reinforced beam because

(a) tensile steel required is more than that for a balanced section
(b) shear reinforcement is more
(c) concrete is not stressed to its full value

16. As per IS:456, for a singly reinforced rectangular section,

(a) $\frac{x_{u,lim}}{d}$ for Fe 415 steel is 0.48

(b) the depth of the centroid of compression is 0.43 $x_{u,lim}$

(c) the depth of the rectangular position of the stress block is 0.38 $x_{u,lim}$

(d) the maximum value of lever arm is $d - x_{u,lim}$

17. In a reinforced concrete T-beam (in which the flange is in compression). The position of neutral axis will

(a) be within the flange

(b) be within web

(c) depend on the thickness of flange in relation to total depth and percentage of reinforcement

(d) at the junction of flange and web

18. A reinforced concrete beam is subjected to the following bending moments:

Dead load – 20 kN-m

Live load – 30 kN-m

Seismic load – 10 kN-m

The design bending moment for limit state of collapse is

(a) 60 kN-m (b) 75 kN-m

(c) 72 kN-m (d) 80 kN-m

19. Which one of the following sections performs better on the ductility criterion?

(a) Balanced section (b) Over-reinforced section

(c) Under-reinforced section (d) Non-prismatic section

20. A doubly reinforced concrete beam has efective cover d' tothe centre of compression reinforcement. 'x_u' is the depth of neutral axis, and 'd' is the effective depth to the centre of tension reinforcement. What is the maximum strain in concrete at the level of compression reinforcement?

(a) 0.0035 (1 – d'/d) (b) 0.0035 (1 – d'/x_u)

(c) 0.002 (1 – d'/x_u) (d) 0.002 (1 – d'/d)

21. Consider the following statements:

In an under-reinforced concrete beam,

1. actual depth of neutral axis is less than the critical depth of neutral axis.
2. concrete reaches ultimate stress prior to steel reaching the ultimate stress.
3. moment of resistance is less than that of balanced section.
4. lever arm of resisting couple is less than of balanced section.

Which of these statements is/are correct?

(a) 1 and 2 (b) 1 and 3

(c) 2, 3 and 4 (d) 1 and 4

22. The maximum strain in the tension reinforcement in the section at failure when designed for the limit state of collapse should be

(a) $>\left(\frac{f_y}{1.15E_s}+0.002\right)$ (b) $<\left(\frac{f_y}{1.15E_s}+0.002\right)$

(c) exactly equal to $\left(\frac{f_y}{1.15E_s}+0.002\right)$ (d) < 0.002

23. A t-beam roof section has the following partriculars:

Thickness of slab	:	100 mm
Width of rib	:	300 mm
Depth of beam	:	500 mm
Centre to centre distance of beam	:	3.0 mm
Effective span of beam	:	6.0 mm
Distance between points of contraflexure	:	3.6 mm

What is the effective flages width of the T-beam?

(a) 3000 mm (b) 1900 mm

(c) 1600 mm (d) 1500 mm

24. A T-beam behaves as a rectangular beam of width equal to its flange if its neutral axis

(a) coincides with centriod of reinforcement

(b) coincides with centroid of T-section

(c) remains within the flange

(d) remains in the web

25. In a singly reinforced beam, the tensile steel reaches its maximum allowable stress earlier than concrete. What is such a section known as?

(a) Under-reinforced section (b) Over-reinforced section

(c) Balanced section (d) Economic section

26. Why is the design of a RC section as over reinforced undesirable?

(a) It consumes more concrete (b) It undergoes high strains

(c) It fails suddenly (d) Its appearance is not good

27. What is the moment capacity of an under reinforced rectangular RCC beam?

(a) Rbd^2 (b) Rdb^2

(c) $A_{st}\,\sigma_{sb}jd$ (d) $A_{sb}jd$

28. In limit state design method, the moment of resistance for a balanced section using M20 grade concrete and HYSD steel of grade Fe 415 is given by $M_{u,lim} = Kbd^2$, what is the value of K?

(a) 2.98 (b) 2.76

Common data for Questions 29 and 30:

A single reinforced rectangular concrete beam has a width of 150 mm and an effective depth of 330 mm. The characteristic compressive strengthof concrete is 20 MPa and the characteristic tensile strength of steel is 415 MPa. Adopt the sress block for concrete as given in IS:456: 2000 and take limiting value of depth of neutral axis as 0.48 time the effective depth of the beam.

29. The limiting value of the moment of resistance of the beam in kN-m is

(a) 0.14 (b) 0.45

(c) 45.08 (d) 156.82

30. The limiting area of tension steel in mm^2 is

(a) 473.9 (b) 412.3

(c) 373.9 (d) 312.3

Conventional Type Questions

1. A T-beam having an effective flange width of 2500 mm is required to resist an ultimate moment of 1200 kN-m. Thickness of flange is 150 mm; width of beam is 300 mm and effective depth is 900 mm. Using M15 grade concrete and Fe250 grade steel, determine the area of steel reinforcement required. The following formula may be used:

(i) Rectangular sections without compression reinforcement

Depth of neutral axis = $(0.87\ f_y\ A_{st})/0.36\ f_{ck}b$

(ii) Moment of resistance when depth of neutral exis is less than the limiting value

$$M_u = 0.87\ f_y\ A_{st}\ d\left(1-\frac{A_{st}\ f_y}{bd\ f_{ck}}\right)$$

(iii) Moment of resistance for limiting value of depth of neutral axis

$M_{u,\ lim} = 0.148\ f_{ck}\ bd^2$

Flanged section-if $x_u < D_f$ the moment of resistence may be calculated using above equations. The limiting value of the moment of resisitacne of the section may be obtained by the following equation when the ratio D_f/d does not exceed 0.2.

$$M_u = 0.36\frac{x_{u,lim}}{d}\left(1-0.42\frac{x_{u,lim}}{d}\right) f_{ck}\ b_w\ d^2 + 0.45\ f_{ck}\ (b_f - b_w)D_f\left(d-\frac{D_f}{2}\right)$$

where,

M_u = limiting value of resisting moment without compression reinforcement

$x_{u,lim}$ = limiting value of NA

b_f = breadth of the compression face/flange

D_f = thickness of flange

d = effective depth.

2. A rectangular beem 350 mm × 750 mm has to support a working load of 80 kN/m in addition to its dead load over a simply supported span of 7.5 m. Determine the reinforcements required for collaps stage using a load factor of 1.5. Adopt M20 grade concrete and Fe 415 grade steel. Cover for the reinforcements is 40 mm.

3. What are the various assumptions on which the design for the limit state of collapse in flaxure is based?

4. What are the three assumptions made for design of reinforced concrete section for limit state of colapse in flaxure that lead to the limiting value of depth of neutral axis? Calculate the limiting values of neutral axis in terms of effective depth for two grades of steel having characteristic strength f_y = 250 and 415 N/mm^2.

5. Enumerate the situations in which doubly reinforced concrete beams become necessary. What is the rold of compression steel?

6. State the assumptions made in the design for the limit state of collapse in flaxure. Are these assumptions justified? Derive the stress block parameters for a rectangular cross-section.

7. Explain 'under-reinforced', 'balanced' and over-reinforced sections in the ultimate load theory.

8. Determine ultimate moment capacity of a doubly reinforced beam with b = 300 mm, D = 600 mm, A_{st} = 2060 mm^2, A_{sc} = 804 mm^2 and effective cover of 50 mm for both tension and compression steels. The materials used are M20 concrete and HYSD steel of grade Fe 415.

Answers

1.	(a)	9.	(c)	17.	(c)	25.	(a)
2.	(c)	10.	(d)	18.	(b)	26.	(c)
3.	(c)	11.	(d)	19.	(c)	27.	(c)
4.	(c)	12.	(b)	20.	(b)	28.	(b)
5.	(a)	13.	(b)	21.	(b)	29.	(c)
6.	(a)	14.	(a)	22.	(a)	30.	(a)
7.	(a)	15.	(d)	23.	(d)		
8.	(b)	16.	(a)	24.	(c)		

2

Limit State of Collapse in Shear

- A beam, when loaded, is subjected to a varying bending moment from section to section and has different stresses at any two adjacent sections in the same fibres.
- This inequality of stresses produces a tendency in each fibre to slide over the lower one in a horizontal plane i.e. horizontal shear stress is created and it is also accompanied by a complementary shear between two adjacent sections in the vertical direction.
- The horizontal and vertical shear stresses are to be accounted for in the designs of beams.

  Shear force is present in beam where there is a change in bending moment along the span.

$$\frac{dM}{dx} = V$$

- Exact analysis of shear in a reinforced concrete beam is quite complex.
- Several experimental studies have been conducted to understood the various modes of failure.

  Which could occure due to possible combination of shear and bending momentum acting at a given section.
- These are consider a small element ABCD along the length of the beam which is taken between the neutral axis and extreme fibres (at top or bottom). This is subjected to the shear stress (τ) parallel to four sides and the tensile bending stress (σ) along the length of the beam as shown in Fig. 2.1.

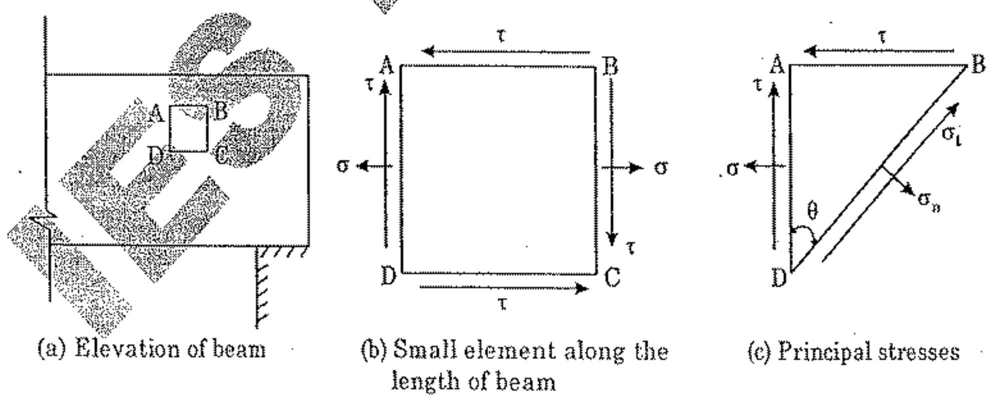


Fig. 2.1: Forces on small element along the length of beam.

The principle stresses on this element are given by

$$\sigma_n = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

This stress σ_n is tensile. There will be another principal stress (σ_t) at right angle to σ_n, which will be compressive.

$$\sigma_t = \frac{\sigma}{2} - \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

and the inclination of principal plane is given by

$$\tan 2\theta = \frac{2\tau}{\sigma}$$

Two important cases are discussed below:

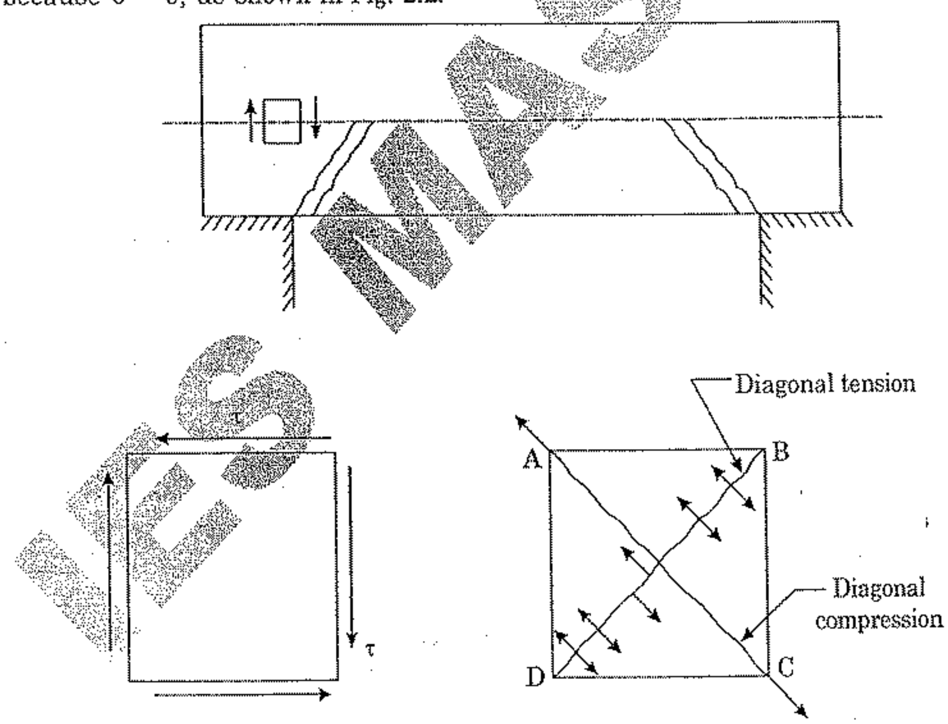
(i) If B.M. = 0 i.e. $\sigma = 0$

then $\sigma_n = \tau$ and $\sigma_t = -\tau$

$\tan 2\theta = \theta = \tan 90°$

i.e. $\theta = 45°$ or $135°$

This means that near the support for a simply supported beam, where bending moment is zero, the principal tension (σ_n) is equal to shear stress (τ) and is inclined at 45°. This is known as **diagonal tension** and the other principal stress (σ_t) will be at right angle to σ_n, which will be compressive. This is known as **diagonal compression** and is of the same value as the shear stress. The principal tensile stresses and compressive stresses act along the diagonal BD and AC respetively. This is the case of pure shear because $\sigma = 0$, as shown in Fig. 2.2.



(b) Pure shear (c) Cracking under pure shear

Fig. 2.2

As the concrete is weak in tension, the concrete near the support cracks at 45° (i.e. perpendicular to the diagonal BD) with horizontal. These are known as web-shear cracks or diagonal tension cracks. To avoid the shear cracks, the beam should be reinforced across the cracks.

(ii) When bending moment is maximum at mid-span of a simply supported i.e. consider an element at the bottom fibre at the mid-section of the beam. The bending stress (σ) is maximum, while shear stress is zero (i.e. $\tau = 0$), we get,

$$\sigma_n = \sigma, \quad \sigma_t = 0$$

and $\theta = 90°$ i.e. principal plane is perpendicular to the beam axis.

This means that principal tensile stress acts in horizontal direction and shear cracks will be vertical as shown in Fig. 2.3.

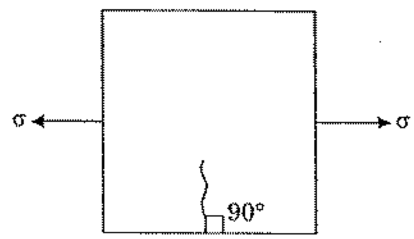


Fig. 2.3: Pure flexure.

Thus at mid-section, where bending stresses are predominant, the cracks will start developing vertically. These cracks are called **flexural cracks.**

From above discussion, it is clear that between the two limits, if concrete cracks due to diagonal tension, the cracks will change from a vertical direction at a point of zero shear to a direction inclined at an angle of 45° at a point where bending stress is zero. This is illustrated in Fig. 2.4, which shows diagonal tension failure of a beam.

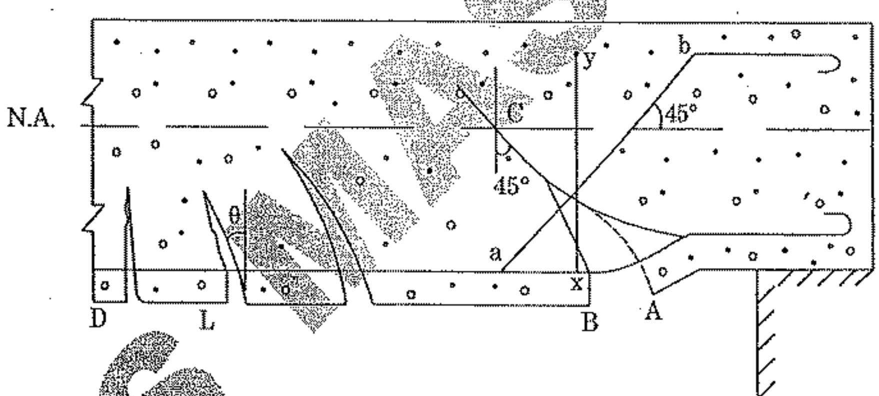
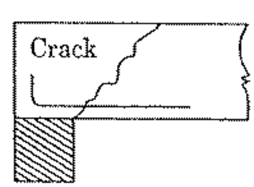


Fig. 2.4: Diagonal tension cracks in a beam.

From above the discussion different modes of failure are:

1. **Diagonal Tension failure:** Which occure under large shear force and less bending momentum. Such craks or normaly at 45° with Hz.



2. **Flexural shear failure:** Which occurs under large bending momentum and less shear force. Which occurse normaly at 90° with Hz.

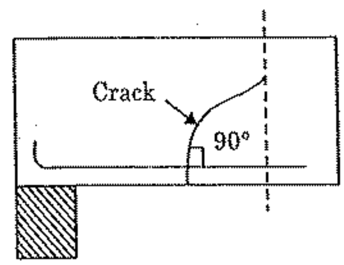


Fig. 2.6

3. **Diagonal Component failure:** Which occurs under large shear force. It is chatacterized by the crushing of concrete. Normally it occurs in beams which are reinforced against heavy shear.

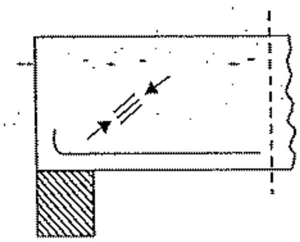
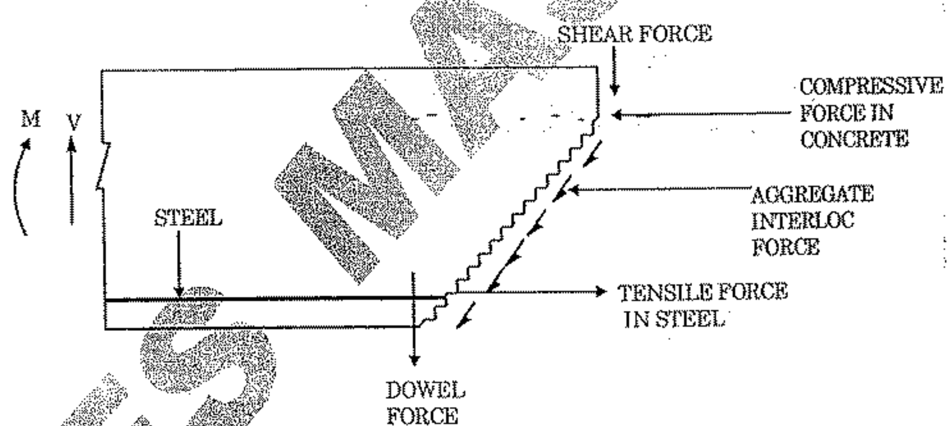


Fig. 2.7

- Studies have shown that shear force is resisted by the uncracked concrete in compression region, the aggregate interlocking and the shear acting across the longitudinal steel bars as shown in figure.
- The shear force across the steel bars is also known as dowel force. Shear reinforcment, if present, will also resist the shear force.

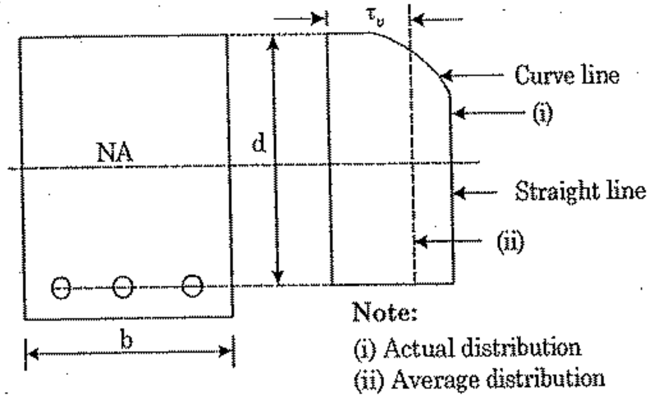


Forces acting on beam when resisting shear forces

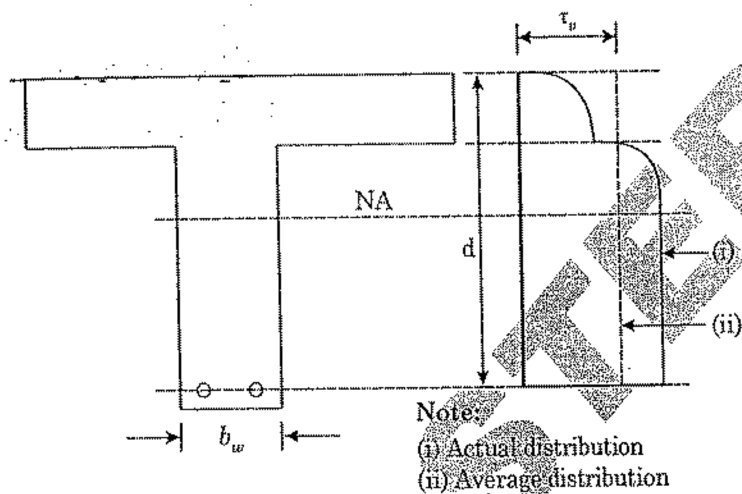
Fig. 2.8

SHEAR STRESS

The distribution of shear stress in reinforced concrete rectangular, T and L-beams of uniform and varying depths depends on the distribution of the normal stress. However, for the sake of simplicity the nominal shear stress τ_v is considered which is calculated as follows (IS 456, cls. 40.1 and 40.1.1):



(a) Rectangular beam



(b) T-beam

Fig 2.9: Distribution of shear stress and average shear stress.

(i) In beams of uniform depth:

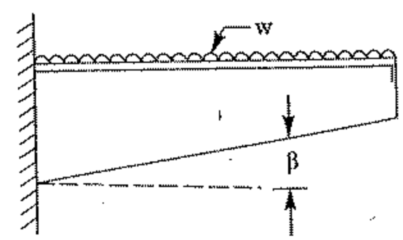
$$\tau_v = \frac{V_u}{bd}$$

where V_u = shear force due to design loads,

b = breadth of rectangular beams and breadth of the web b_w for flanged beams, and

d = effective depth

(ii) In beams of varying depth:



$$\tau_v = \frac{V_u \pm \dfrac{M_u}{d}\tan\beta}{bd}$$

where, τ_v, V_u, b or b_w and d are the same as in (i),

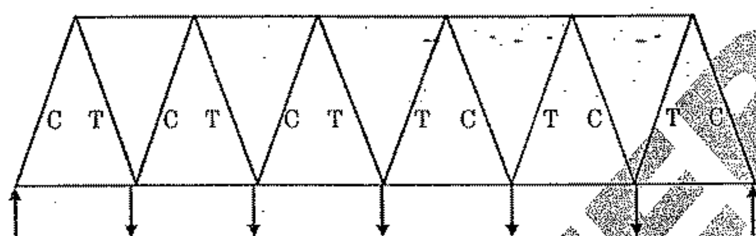
M_u = bending moment at the section, and

β = angle between the top and the bottom edge.

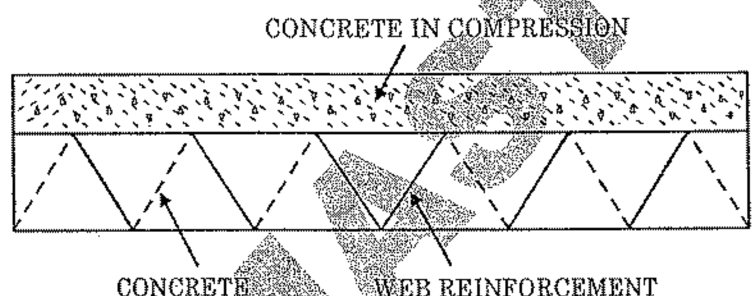
The positive sign is applicable when the bending moment M_u decreases numerically in the same direction as the effective depth increases, and the negative sign is applicable when the bending moment

SHEAR REINFORCEMENT

- The recommendations in IS : 456 Code for shear reinforcement are based on the truss analogy. In a steel truss as shown in figure (a).
- The upper and lower chords are in compression and tension respectively.
- The analogy is that tensile forces in the alternatively in compression and tension.
- The analogy is that tensile forces in the diagonal tie members are resisted by reinforcment and compression forces in the diagonal struts are resisted by the concrete as a shown in figure b.
- When shear stress exceeds the shear capacity of the concrete, shear reinforcement is provided.
- To prevent the possibility of crushing of concrete in the web of a member, maximum shear stress values are limited as shown in table 19. The shear strength of concrete τ_c based on the percentage of longitudinal tensile reinforcement is shown in table 20.



(a) A Steel Truss



(b) Truss Action in A renforced concrete beam

Fig. 2.10

Thus, when shear reinforcement is necessary, the shear strength of the beam is calculated on the following basis :

Total shear strength = sbear resistance of effective concrete area as a function of longitudinal main steel bars+ shear resistance of vertical sbear stirrups + shear resistance of inclined shear stirrups.

DESIGN SHEAR STRENGTH OF REINFORCED CONCRETE

- Recent laboratory experiments confirmed that reinforced concrete in beams has shear strength even without any shear reinforcement.
- This shear strength (τ_c) depends on the grade of concrete and the percentage of tension steel in beams.
- On the other hand, the shear strength of reinforced concrete with the reinforcement is restricted to some

- These minimum and maximum shear strengths of reinforced concrete (IS 456, cls. 40.2.1 and 40.2.3, respetively) are given below:

Design Shear Strength without Shear Reinforcement (IS 456, cl. 40.2.1)

Table 19 of IS 456 stipulates the design shear strength of concrete τ_c for different grades of concrete with a wide range of percentages of positive tensile steel reinforcement. It is worth mentioning that the reinforced concrete beams must be provided with the minimum shear reinforcement as per cl. 40.3 even when τ_v is less than τ_c given in Table 19.

Table 19: Design shear strength of concrete, τ_c in N/mm^2

Grade of concrete (100 A_s/bd)	M 20	M 25	M 30	M 35	M 40 and above
≤ 0.15	0.28	0.29	0.29	0.29	0.30
0.25	0.36	0.36	0.37	0.37	0.38
0.50	0.48	0.49	0.50	0.50	0.51
0.75	0.56	0.57	0.59	0.59	0.60
1.00	0.62	0.64	0.66	0.67	0.68
1.25	0.67	0.70	0.71	0.73	0.74
1.50	0.72	0.74	0.76	0.78	0.79
1.75	0.75	0.78	0.80	0.82	0.84
2.00	0.79	0.82	0.84	0.86	0.88
2.25	0.81	0.85	0.88	0.90	0.92
2.50	0.82	0.88	0.91	0.93	0.95
2.75	0.82	0.90	0.94	0.96	0.98
≥ 3.00	0.82	0.92	0.96	0.99	1.01

In Table, A_{st} is the area of longitudinal tension reinforcement which continues at least one effective depth beyond the section considered except at support where the full area of tension reinforcement may be used provided the detailing is as per IS 456, cls. 22.2.2 and 26.2.3.

Maximum Shear Stress $\tau_{c\,max}$ with Shear Reinforcement (cls. 40.2.3, 40.5.1 and 41.3.1)

Table 20 of IS 456 stipulates the maximum shear stress of reinforced concrete in beams τ_{cmax} as given below in Table. Under no circumstances, the nominal shear stress in beams τ_v shall exceed $\tau_{c\,max}$ given in Table for different grades of concrete.

Table 20: Maximum shear stress, $\tau_{c\,max}$ in N/mm^2

Grade of concrete	M 20	M 25	M 30	M 35	M 40 and above
τ_{cmax}, N/mm^2	2.8	3.1	3.5	3.7	4.0

Critical Section for Shear

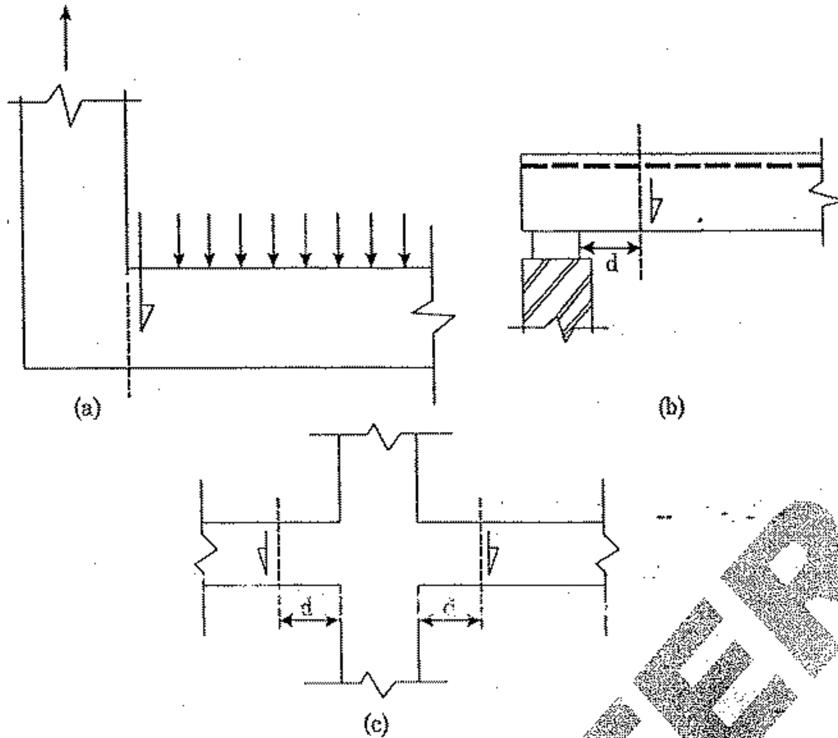


Fig. 2.11: Support conditions for locating factored shear force.

Clauses 22.6.2 and 22.6.2.1 stipulate the critical section for shear and are as follows:

For beams generally subjected to uniformly distributed loads or where the principal load is located further than $2d$ from the face of the support, where d is the effective depth of the beam, the critical sections depend on the conditions of supports as shown in figures (a), (b) and (c) and are mentioned below.

(i) When the rection in the direction of the applied shear introduces tension (figure (a)) into the end region of the member, the shear force is to be computed at the face of the support of the member at that section.

(ii) When the reaction in the direction of the applied shear introduces compression into the end region of the member (figures (b) and (c)), the shear force computed at a distance d from the face ofthe support is to be used for the design of sections located at a distance less than d from the face of the support.

Minimum Shear Reinforcement (cls. 40.3, 26.5.1.5 and 26.5.1.6 of IS 456)

Minimum shear reinforcement has to be provided even when τ_v is less than τ_c given in Table as recommended in cl. 400.3 of IS 456. The amount of minimum shear reinforcement, as given in cl. 26.5.1.6, is given below.

The minimum shear reinforcement in the form of stirrups shall be provided such that:

$$\frac{A_{sv}}{bs_v} \geq \frac{0.4}{0.87\, f_y} \quad \text{---(a)}$$

where A_{sv} = total cross-sectional area of stirrup legs effective in shear,

s_v = stirrup spacing along the length of the member,

b = breadth of the beam or breadth of the web of flanged beam b_w, and

f_y = characteristic strength of the stirrup reinforcement in N/mm² which shall not be taken

The above provision is not applicable for members of minor structural importance such as lintels where the maximum shear stress calculated is less than half the permissible value.

The minimum shear reinforcement is provided for the following:

(i) Any sudden failure of beams is prevented if concrete cover bursts and the bond to the tension steel is lost.

(ii) Brittle shear failure is arrested which would have occurred without shear reinforcement.

(iii) Tension failure is prevented which would have occurred due to shrinkage, thermal stresses and internal cracking in beams.

(iv) To hold the reinforcement in place when concrete is poured.

(v) Section becomes effective with the tie effect of the compression steel.

Further, cl. 26.5.1.5 of IS 456 stipulates that the maximum spacing of shear reinforcement measured along the axis of the member shall not be more than 0.75 d for vertical stirrups and d for inclined stirrups at 45°, where d is the effective depth of the section. However, the spacing shall not exceed 300 mm in any case.

PLACEMENT OF STIRRUPS

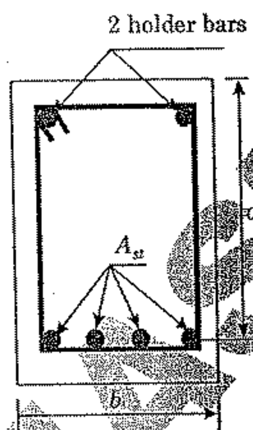


Fig. 2.12: Placement of stirrups.

- The stirrups in beams shall be taken around the outer-most tension and compression bars.
- In T and L-beams, the stirrups will pass around longitudinal bars located to the outer face of the flange.
- In the rectangular beams, two holder bars of diameter 10 to 12 mm are provided if there is no particular need for compression reinforcement (figure).

IMPORTANT POINTS

- Shear strength of concrete is depends on the compressive strength, Higher the compressive strength greater the shear strength because shear strength $\alpha\sqrt{f_{ck}}$.
- **Shear strength is also depends upon % of tensile steel: Larger the steel area, greater the shear strength.**

Note: 1. Mini-shear reinforcement is provided to resist the principal tension.

2. Upper limit to shear strength [τ_{cmax}] is specified to prevent the failure of beam by diagonal

3. In any case shear stress developed τ_v in the the section should not be more than τ_{cmax}
$\tau_v \ngtr \tau_{cmax}$
if $\tau_v > \tau_{c\,max}$ only option is to revised the section change b or D or both so that $\tau_v < \tau_{max}$
4. Shear span is defined as the zone where shear force is constant.

DESIGN STEPS

(i) Calculate $\tau_v = \dfrac{V_u}{bd}$

(ii) Calculate % of steel (tension reinforcement)

$$p_t = \frac{Ast}{bd} \times 100\%$$

Find out value of τ_c based on % of steel and grade of concrete.

(iii) Find shear strength of concrete

$V_c = \tau_c.bd$

(iv) Design shear reinforcement:

Case I if $V < V_c$ or $\tau_v < \tau_c$

Shear reinforcement is not required, but provide mini. Shear reinforcement

$$\boxed{\frac{A_{sv}}{bS_V} \geq \frac{0.4}{0.87f_y}} \text{ same to WSM and LSM}$$

As per 26.5.1.6 P 48

Note: 1. Mini. shear reinforcement in the form of strrups shall be provides as concrete alone cannot be relied upon to take up shear stresses safely.
2. Shear design for a prestressed concrete beam is based on elastic theory.

$$s_v \leq \frac{0.87f_y A_{sv}}{0.4b}$$

s_v = Spacing of shear reinforcement
b = Width of section
A_{sv} = c/s area of all shear reinf at one section.

Case II if $V > V_u$ or $\tau_v > \tau_c$

When τ_v is more than τ_c given in Table 20, shear reinforcement shall be provided in any of the three following forms: (cl. 40.4 of IS 456)

(a) Vertical stirrups,
(b) Bent-up bars along with vertical stirrups, and
(c) Inclined stirrups.

- In the case of bent-up bars, it is to be seen that the contribution towards shear resistance of bent-up bars should not be more than fifty per cent of that of the total shear reinforcement.
- The amount of shear reinforcement to be provided is determined to carry a shear force V_{us} equal to

$$V_{us} = V_u - \tau_c\, bd$$

where b is the breadth of rectangular beams or b_w in the case of flanged beams.

(a) Vertical stirrups:

$$V_{us} = \frac{0.87 f_y A_{sv} d}{s_v}$$

(b) For inclined stirrups or a series of bars bent-up at different cross-sections:

$$V_{us} = \frac{0.87\, f_y\, A_{sv} d}{s_v}(\sin\alpha + \cos\alpha)$$

(c) For single bar or single group of parallel bars, all bent-up at the same cross-section:

$$V_{us} = 0.87\, f_y\, A_{sv}\, s_v \sin\alpha$$

where A_{sv} = total cross-sectional area of stirrup legs or bent-up bars within a distance s_v,

s_v = spacing of stirrups or bent-up bars along the length of the member,

τ_v = nominal shear stress,

τ_c = design shear strength of concrete,

b = breadth of the member which for the flanged beams shall be taken as the breadth of the web b_w,

f_y = characteristic strength of the stirrup or bent-up reinforcement which shall not be taken greater than 415 N/mm^2,

α = angle between the inclined stirrup or bent-up and the axis of the member, not less than 45°, and

d = effective depth.

The following two points are to be noted:

(i) The total shear resistance shall be computed as the sum of the resistance for the various types separately where more than one type of shear reinforcement is used. (vertical + inclined stirrups)

(ii) The area of stirrups shall not be less than the minimum specified in cl. 26.5.1.6. (equation 'a' for minimum shear reinforcment)

Where Bent-up Bars along with Vertical Stirups or Inclined Stirrups at *Different Cross-sections are Provided*

The above mentioned types of provision may be made where the total shear force to be resisted by steel (V_{us}) is born either by bent-up bars and stirrups together or by inclined stirrups only.

The shear force V_{us_1} to be resisted by inclined reinforcement is determined as follows (figure).

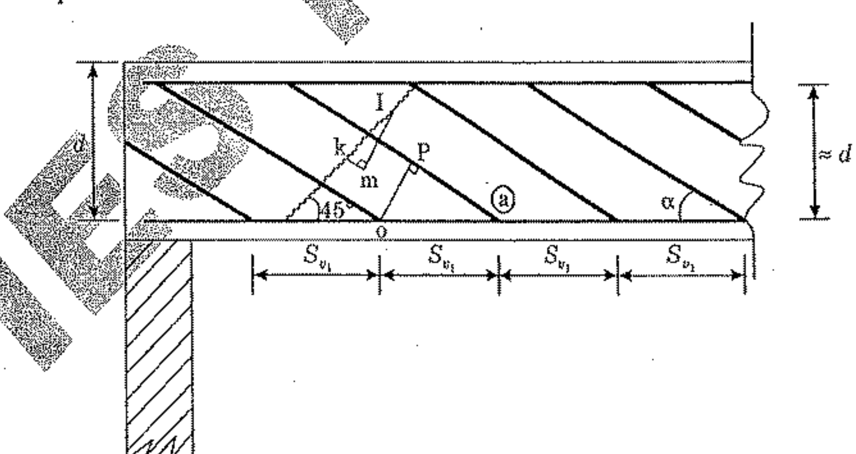


Fig. 2.13: Inclined reinforcement to resist a part of shear forces.

$$V_{us_1} = \frac{0.87 f_y A_{sv_1} d}{s_{v_1}} (\sin\alpha + \cos\alpha) \quad \text{---(i)}$$

(a) If inclined bars are provided in combination with vertical stirrups, total shear resistance of both types together = $V_{us} = V_{us_1} + V_{us_2}$

where V_{us_1} = shear resistance of inclined bars, and

V_{us_2} = shear resistance of vertical stirrups.

It may be noted here that:

(i) the area of stirrups shall not be less than the minimum specified in equation 'a' and

(ii) where bent-up bars are provided their contribution towards shear resistance shall not be more than *half* that of the *total reinforcement*.

Note: Inclined bars are ineffective in case of reversal of shear force and their exact behaviour in resisting shear is still controversial; that is why contribution of bent up bars towards shear resistance is limited to only half that of total reinforcement.

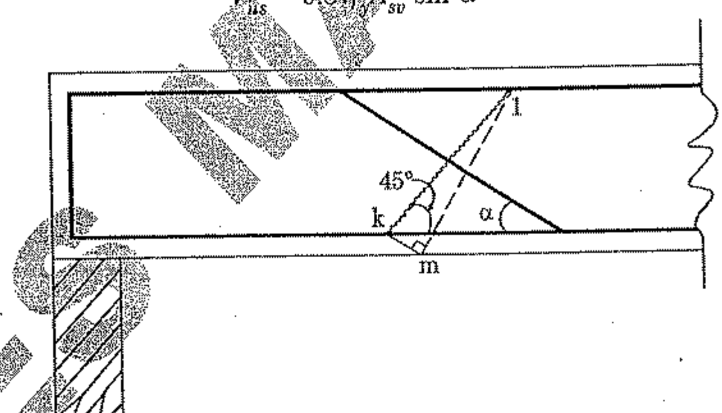
(b) But if *inclined stirrups* are only provided V_{us_1} and A_{sv_1} in eq. (i) are to be substituted as V_{us} and A_{sv} respectively to have eq. (ii) for such reinforcement i.e.

$$V_{us} = \frac{0.87 f_y A_{sv}}{s_v} (\sin\alpha + \cos\alpha) \quad \text{---(ii)}$$

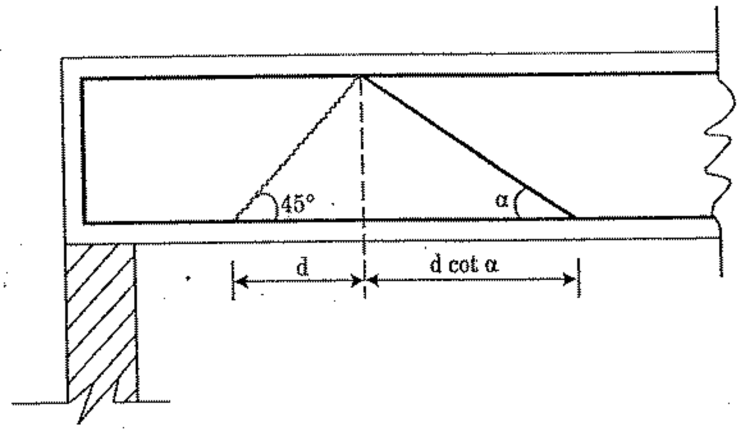
Where Single or Group of Parallel Bars, all bent-up at the same cross-section, are provided

If a single or group of parallel bars are bent-up at the *same* cross-section to resist a total shear of V_{us} (figure (a)) then

$$V_{us} = 0.87 f_y A_{sv} \sin\alpha$$



(a) Single or group of parallel inclined bars to resist sheat at the same cross-section.



(b) Maximum distance between bent-up or inclined stirrups.

Fig. 2.14

Such reinforcement is effective upto a distance (d + $d$cot a) along the span as shown in figure.

Example 1

Design a beam for shear reinforcement having a cross-section of $b \times D = 250 \times 500$ reinforced with 4 nos. 20 mm φ. The factored shear force = 130 kN. Use M 15 concrete and Fe 250 steel. Provide vertical stirrups only.

Sol: Assuming clear cover of 35 mm

$$d = 500 - 35 - \frac{20}{2} = 455 \text{ mm}$$

$$\text{Nominal shear stress} = \tau_u = \frac{V_u}{bd} = \frac{130 \times 1000}{250 \times 455} = 1.143 \text{ MPa}$$

$\tau_{c\,max}$ for M 15 concrete = 2.5 MPa

Percentage of tensile reinforcement

$$p\% = \frac{100 A_s}{bd} = \frac{100 \times 1256.64}{250 \times 455} = 1.1\%$$

Design shear strength for 1.1% tensile reinforcement and M 15 concrete

$$= 0.6 + \frac{(0.64 - 0.6)}{(1.25 - 1.0)} \times (1.1 - 1.0)\,2 = 0.616 \text{ MPa}$$

$\tau_{c\,max} > \tau_u > \tau_c$ therefore, shear reinforcement is required for $V_{us} = V_u - \tau_c d$

$$= 130 \times 1000 - 0.616 \times 250 \times 455$$

$$= 59930 \text{ N}$$

Providing 8 mm dia 2 legged vertical stirrups having

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.53 \text{ mm}^2$$

From

$$s_v = \frac{0.87 f_y A_{sv} d}{V_{us}} = \frac{0.87 \times 250 \times 100.53 \times 455}{59930}$$

$$= 166 < 300 < 0.75d\ (341.25)$$

The maximum spacing, $s_{v\,max}$, required for minimum shear reinforcement according to eq.

$$s_{v\,max} = \frac{0.87 f_y A_{sv}}{0.4b} = \frac{0.87 \times 250 \times 100.53}{0.4 \times 250} = 218.65 > s_v\,(166)$$

Hence, provided 8 mm dia 2 legged vertical stirrups at 160 c/c.

Example 2

Design the beam for shear at support for the arrangement of tensile reinforcement shown in figure below. Take b = 300; d = 550; d = 550; f_y = 250 MPa; f_{ck} = 20 MPa, V_u at support = 250 kN.

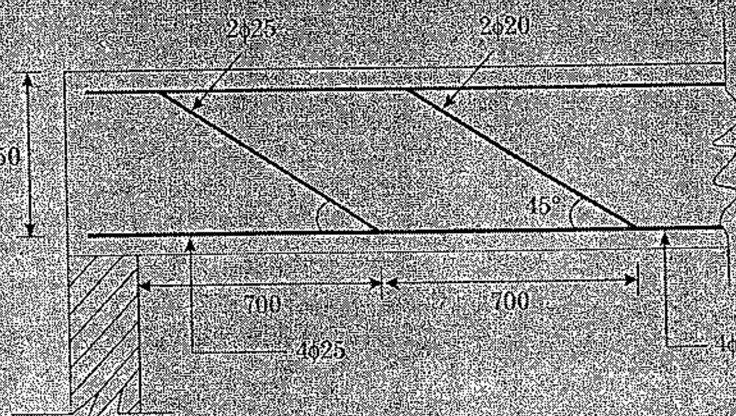


Fig. Showing bent-up bars at different cross section

Sol: Percentage of tensile reinforcement at support $= \dfrac{4 \times \frac{\pi}{4} \times 25^2}{300 \times 550} \times 100 = 1.19\%$

For M 20 concrete and 1.19% tensile reinforcement

$$\tau_c = 0.62 + \frac{(0.67 - 0.62)}{(1.25 - 1.0)} \times (1.19 - 1.0) = 0.658\ \text{N/mm}^2$$

For M 20 concrete, $\tau_{c\,max}$ = 2.8 MPa

$$\tau_v = \frac{V_u}{bd} = \frac{250 \times 10^3}{300 \times 550} = 1.52\ \text{MPa}$$

$\tau_{c\,max} > \tau_v > \tau_c$, therefore shear reinforcement is to be provided for

$$V_{us} = V_u - \tau_c bd = 250 \times 1000 - 0.658 \times 300 \times 550 = 141430\ \text{N}$$

Shear resistance for a series of bent-up bars at different cross-section

$$V_{us_1} = \frac{0.87 f_y A_{sv_1} d}{s_v} (\sin\alpha + \cos\alpha)$$

$$= \frac{0.87 \times 250 \times 2 \times \frac{\pi}{4} \times 20^2 \times 550}{700} (\sin 45^\circ + \cos 45^\circ)$$

As the shear resistance of bent-up bars cannot exceed $0.5 \times 141430 = 70715$ N, vertical stirrups are to be provided for $V_{us_2} = V_u - V_{us_1} = 141430 - 70715 = 70715$ N

Providing 6 mm dia 2 legged vertical stirrups

$$A_{sv_2} = 2 \times \frac{\pi}{4} \times 6^2 = 56.55 \text{ mm}^2$$

$$s_{v_2} = \frac{0.87 f_y A_{sv_2} d}{V_{us_2}}$$

$$= \frac{0.87 \times 250 \times 56.55 \times 550}{70715} = 95.66 < 300 < 0.75d\,(412.5)$$

From minimum reinforcement consideration the maximum spacing of vertical reinforcement

$$s_{v_{2\max}} = \frac{0.87 \times 250 \times 56.55}{0.4 \times 300} = 102.49 > 95.66$$

Hence, provided 6 mm dia 2 leggled vertical stirrups @ 95 c/c in addition to bent-up bars at *different* cross-sections to resist total shear force.

This type of problem specified as bent-up Bars at Different Cross-sections along with Vertical Stirrups as the Shear Reinforcements.

Example 3

Design a beam for shear reinforcements for the following data:

$b = 250$; $d' = 540$; $d = 500$; $f_y = 415$ MPa and $f_{ck} = 15$ MPa

At a cross-section shear force, V_u, equal to 93 kN is to be resisted by two bent up bars of f18 inclined at 45° in combination with vertical stirrups. The tensile reinforcement of 2f18 is available at the section.

Sol: Nominal shear stress $= \tau_v = \frac{93 \times 1000}{250 \times 500} = 0.744$ MPa

Percentage of tensile reinforcement

$$p_t = \frac{2 \times \frac{\pi}{4} \times 18^2}{250 \times 500} \times 100 = 0.407\%$$

According, $\tau_c = 0.35 + \frac{(0.46 - 0.35)}{(0.5 - 0.25)} \times (0.407 - 0.25)$

$$= 0.419 \text{ MPa}$$

For M 15 concrete, $\tau_{c,\max} = 2.5$ MPa

Since $\tau_{c\max} > \tau_v > \tau_c$, shear reinforcement is to be designed for

$$V_{us} = V_u - V_c = 93000 - 0.419 \times 250 \times 500 = 40628 \text{ N}$$

Shear resistance of 2 nos 18 mm at the same cross section

$$= 0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 18^2 \times \sin 45°$$

$$= 129932.33 \text{ N} > 40625 \ (V_{us})$$

The effective shear resistance of bent-up bars, $V_{us_1} = 0.5 \times 40625 = 20312.5$ N

Vertical stirrups are to be provided for $V_{us_2} = V_{us} - V_{us_1} = 40625 - 20312.5$

$$= 20312.5 \text{ N}$$

If 6 mm dia two-legged stirrups are adopted, spacing of vertical stirrups

$$s_{v_2} = \frac{0.87 f_y A_{su_2} d}{V_{us_2}}$$

$$= \frac{0.87 \times 415 \times \left(2 \times \frac{\pi}{4} \times 6^2\right) \times 500}{2031.5} = 502.58 > 375 \ (0.75d) > 300$$

The maximum spacing of vertical stirrups from equation.

$$s_{v_2} = \frac{0.87 f_y A_{su_2} d}{0.4b} = \frac{0.87 \times 415 \times 56.55}{0.4 \times 250} = 204.17 < 375$$

Hence, provided two bent-up bars inclined at 45° at the section along with 6 mm dia two-legged vertical stirrups @ 200c/c at the section.

This type of problem is specified as bent-up Bars at the Same Cross-section along with Vertical Stirrups as the Shear Reinforcements

Example 4

Design tensile and shear reinforcements at support for a cantilever beam (figure) of constant width 300. The depth of a beam is linearly varying from 800 at support to 350 at free end. The beam is loaded with a U.D.L. of 39 kN/m including its self weight. Use M 20 concrete and Fe 415 steel.

Sol: (i) Design of tensile reinforcement at support

Factored B.M. at support

Taking effective cover = 40, $d = 800 - 40 = 760$ mm

$$M_u = -1.5 \times \frac{W l_{eff}^2}{2} = -1.5 \times \frac{30 \times 3^2}{2} = -202.5 \text{ kN/m}$$

Tensile reinforcement for M_u is given by the equation

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{bd \, f_{ck}}\right)$$

$$202.5 \times 10^6 = 0.87 \times 415 \times A_{st} \times 760 \left(1 - \frac{A_{st} \times 415}{300 \times 760 \times 20}\right)$$

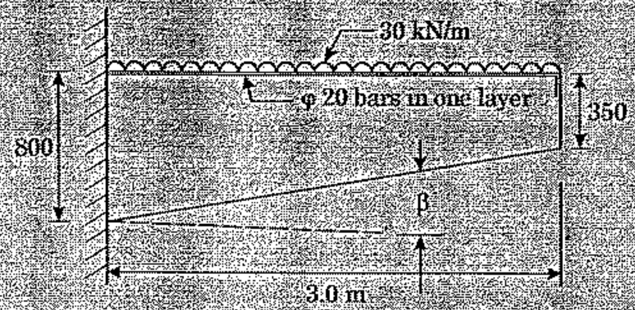


Fig. Details of a cantilever beam.

or, $24.97 A_{st}^2 - 274398 A_{st} + 202.5\times10^6 = 0$

or, $A_{st} = 795.58\ \text{mm}^2$

$$\text{Min. tensile reinforcement} = \frac{0.85}{f_y}\times100 = \frac{0.85\times100}{415} = 0.2\%$$

i.e. $A_{st} = 0.002 \times 300 \times 760 = 456\ \text{mm}^2$

Hence provided 3nos 20 mm dia (A_{st} = 942 mm²)

(ii) Design of shear reinforcement at support

$$V_u = 1.5 \times 30 \times 3 = 135\ \text{kN}$$

$$\tau_v = \frac{V_u - \frac{M_u}{d}\tan\beta}{bd}$$

$$= \frac{135\times10^3 - \frac{202.5\times10^6}{760}\times\frac{450}{3000}}{300\times760}$$

$$= 0.42\ \text{N/mm}^2$$

$$\%\text{ of tensile steel} = \frac{942\times100}{300\times760}\% = 0.413\%$$

$$\text{Accordingly,}\quad \tau_c = 0.36 + \frac{0.48-0.36}{(0.5-0.25)}\times(0.413-0.25)$$

$$= 0.438\ \text{N/mm}^2 > 0.42\ (t_c)$$

Hence only nominal shear reinforcement is to be provided.

Providing 8 mm dia two legged stirrups

$$A_{sv} = 2\times\frac{\pi}{4}\times8^2 = 100\ \text{m}^2$$

$$s_{v\max} = \frac{0.87 f_y A_{sv}}{0.4b} = \frac{0.87\times415\times100}{0.4\times300}$$

$$= 300.8 < 450 < 570\ (0.75d)$$

Hence provided 8 mm dia two-legged stirrups @ 300 c/c.

Example 5

A recgangular RC beam of concrete grade M 15 is 250 mm wide and 500 mm deep (effective depth). it is provided with 3 nos. of 16 mm ϕ HYSD bar (Fe 415) as tension reinforcement. The cross-section has to carry a shearing force of 60 kN under service load conditions. Calculate the spacing of 8 mm diameter 2-legged shear stirrups to be provided in the beam. The maximum shear stress $\tau_{c,max}$ for M 15 is 2.5 N/mm². Use partial factor of safety for load as 1.5.

Sol:

$$b = 250 \text{ mm}$$

$$d = 500 \text{ mm}$$

$$A_{st} = 3\times\frac{\pi}{4}\times16^2 = 603.18 \text{ mm}^2$$

$$A_{sv} = 2\times\frac{\pi}{4}\times8^2 = 100.53 \text{ mm}^2$$

$$v = 60 \text{ kN}$$

$$V_u = 1.5 \times 60 = 90 \text{ kN}$$

$$\tau_{c,max} = 2.5 \text{ N/mm}^2$$

(i) Nominal shear stress

$$\tau_v = \frac{V_u}{bd} = \frac{90\times10^3}{250\times500} = 0.72 \text{ N/mm}^2$$

(ii) Percentage area of steel $= \frac{A_{st}}{bd}\times100 = \frac{603.18}{250\times500}\times100 = 0.48\%$

For M 15 and % of steel τ_c is

%	τ_c
0.25	0.35
0.50	0.46

$$\tau_c \text{ for } 0.48\% = 0.35 + \frac{(0.46-0.35)}{(0.50-0.25)}(0.48-0.35)$$

$$= 0.35 + 0.0572$$

$$= 0.407 \text{ N/mm}^2$$

$\because \tau_v > \tau_c$. Hence shear reinforcement is to be provided.

$$V_c = \tau_c bd = 0.407 \times 250 \times 500 = 50.9 \text{ kN}$$

$$V_{us} = V_u - V_c = 90 - 50.9 = 39.1 \text{ kN}$$

$$s_v = \frac{A_{sv}\times0.87\times f_y\times d}{V_{us}}$$

$$= \frac{100.53\times0.87\times415\times500}{39.1\times10^3}$$

$$s_v = 464.15 \text{ mm}$$

But as per IS-456-2000 specifications

$$\frac{As_v}{Bs_v} \geq \frac{0.4}{0.87 f_y}$$

$$\Rightarrow \quad s_v < \frac{A_{sv} \times 0.87 f_y}{b \times 0.4}$$

$$\Rightarrow \quad s_v < \frac{100.53 \times 0.87 \times 415}{250 \times 0.4}$$

$$\Rightarrow \quad s_v \leq 362.96 \text{ mm}$$

Spacing $s_v = 0.75\, d = 0.75 \times 500 = 375$ mm

Hence s_v = 300 mm limited to maximum 300 mm should be adopted as the spacing.

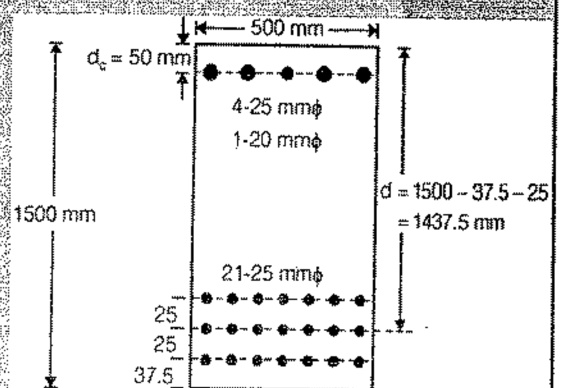
Example 6

A simply supported 18 m effective span RCC rectangular beam of 500 mm × 1500 mm (overall depth) section is reinforced throughout with 21 numbers of 25 mm diameter bars in three layers of 7 bars each at a clear cover of 37.5 mm on tensile face. The reinforcement in the compression face is 4 numbers 25 mm diameter and 1 number 20 mm diameter bars in one layer at an effective cover of 50 mm. The clear cover between the different layers on tension face is 25 mm. M 25 grade concrete and Fe 415 grade steel bars are used in the beam throughout. The beam is laterally restrained throughout the span.

(a) *Find out the maximum bending moment M_u at limit state of collapse for the beam section assuming the stress in compression reinforcement $\sigma_{sc} = 0.9566\, f_y$ corresponding to tensile stress f_y in tensile reinforcement at the limit state.*

(b) *What should be the superimposed uniformly distributed load 'w', the beam can carry at working conditions?*

(c) *Design the shear reinforcement at support if design shear strength of concrete τ_c is given as follows for different values of $p = \dfrac{100\, A_s}{bd}$*



p	*1.25*	*1.5*	*1.75*
τ_c, MPa	*0.70*	*0.74*	*0.78*

Sol:

(a)

$$f_y = 415 \text{ N/mm}^2$$

$$x_{u,lim} = 0.48d = 0.48 \times 1437.5 = 690 \text{ mm}$$

$$b = 500 \text{ mm}$$

$$f_{ck} = 25 \text{ N/mm}^2$$

$$d = 1437.5 \text{ mm}$$

$$d_c = 50 \text{ mm}$$

$$f_{sc} = \sigma_{sc} = 0.9566\, f_y = 0.9566 \times 415 = 396.989 = 397 \text{ N/mm}^2$$

$$A_{st} = 21 \times \frac{\pi}{4} \times 25^2 = 10308.35 \text{ mm}^2$$

$$A_{sc} = 4 \times \frac{\pi}{4} \times 25^2 + 1 \times \frac{\pi}{4} \times 20^2 = 2277.65 \text{ mm}^2$$

$$MR = 0.36 f_{ck} b x_{u,lim} (d - 0.42 x_{u,lim}) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d_c)$$
$$= 0.36 \times 25 \times 500 \times 690 (1437.5 - 0.42 \times 690) + (397 - 0.45 \times 25) \times 2277.65 \times (1437.5 - 50)$$
$$= (0.36 \times 25 \times 500 \times 690 \times 1147.70) + (385.75 \times 2277.65 \times 1387.5)$$
$$MR = 4782.67 \text{ kN-m}$$

(b) $$M_{u,lim} = \frac{w_u l^2}{8}$$

$$\Rightarrow \quad \frac{47.82.67 \times 8}{18 \times 18} = w_u$$
$$\Rightarrow \quad w_u = 118.09 \text{ kN/m}$$
$$\text{load} = \frac{w_u}{1.5} = \frac{118.09}{1.5} = 78.73 \text{ kN/m}$$

Working load is super imposed UDL along with the self weight of the beam. Hence, the self weight of beam should be deducted.

Self weight of beam = 0.5 × 1.5 × 1 × 25 = 18.75 kN/m

Hence, w = working load – 18.75 = 78.73 – 18.75 = 59.98 kN/m

(c) $\because$ $w_u = 118.09$ kN/m

$$V_u = \frac{w_u l}{2} = \frac{118.09 \times 15}{2} = 885.67 \text{ kN}$$

$\therefore$ Nominal shear stress $\tau_v = \dfrac{V_u}{bd} = \dfrac{885.67 \times 10^3}{500 \times 1437.5} = 1.23$ N/mm²

Now $p_t = 100 \times \dfrac{A_{st}}{bd} = \dfrac{100 \times 10308.35}{500 \times 1437.5} = 1.43\%$

For $p_t = 1.25\%$, we have $\tau_c = 0.70$ N/mm²

For $p_t = 1.50\%$, we have $\tau_c = 0.74$ N/mm²

$\therefore$ $V_c = \tau_c bd = 0.73 \times 500 \times 1437.5 = 523.82$ kN

$\therefore$ $\tau_v > \tau_c$, hence shear reinforcement is required.

$$V_{us} = V_u - V_c = 885.67 - 523.82 = 361.84 \text{ kN}$$

Now shear reinforcement at supports should be provided according to IS: 456–2000 i.e.

$$S_v = \frac{0.87 f_y A_{sv} d}{V_{us}}$$

Adopting 2-legged 10 mm ϕ bars for vertical stirrups.

$$A_{sv} = 2 \times \frac{\pi}{4} \times 10^2 = 157.08 \text{ mm}^2$$

$$\therefore \quad s_v = \frac{0.87 \times 415 \times 157.08 \times 1437.5}{361.84 \times 10^3}$$
$$s_v = 225.31 \text{ mm}$$

Hence, providing 220 mm spacing at supports.

Minimum shear reinforcement,

$$s_v \le \frac{2.5\,A_{sv} f_y}{b}$$

$$s_v \le \frac{2.5 \times 157.08 \times 415}{500} \qquad [\because b = B]$$

$$s_v \le 325.94 \text{ mm}$$

Hence, at mid span spacing between stirrups can be 325 mm c/c.

Maximum spacing = $0.75d = 0.75 \times 1437.5 = 1078.12$ mm

Hence provide reinforcement OK.

***Note :** The Hz distance between two sucussive craks is approximately equal to effective depth. The spacing of stirrups shall be such that it crosses the crack and also no crack shall remain unreinforced. To ensure this, the spacing of volume stirrups is limited to 0.75 d.*

Some Important Points

- The shear associated with change of bending moment along the span is known as flexural shear or simply shear.
- The shear associated with the possibility of punching a thin member by a concentrated load is called punching shear.
- When a member is subjected to torsion, it is subjected to torsion shear. In RCC structures, the torsion shear is usually accompanied with flexural shear.
- Usually the shear failures of shallow RCC beams may not lead to immediate failure, however, it considerably reduces its flexural strength and thus there is a state of impending failure. Hence, the shear design is considered as a limit state of collapse.
- The shear in concrete beam is resisted by:
  (a) Above neutral axis the shear resistance is provided by the uniform shear stress in uncracked concrete (20% to 40% of total resistance).
  (b) Along the crack, the shear resistance is provided by the vertical component of force due to the interlocking of aggregates (33% to 50% of total resistance).
  (c) At the tensile reinforcements, shear is resisted by dowel action of the longitudinal bars (15% to 25% of total resistance).
- The increase in tension steel improve shear capacity of a concrete beam by following action
  (a) When amount of tension steel increases, the depth of neutral axis increases and thus the depth of uncracked concrete increases. This increases the capacity of concrete in shear.
  (b) When amount of tension steel increases, the cracks formed are smaller which improves the aggregate inter-lock. Also because of larger steel area, the dowel action is improved. This further improves the capacity of section in shear.
- The nominal shear stress τ_v in beams of uniform depth shall be obtained from the equation

$$\tau_v = \frac{V}{bd}$$

where V = shear force due to design loads

b = width of member in case of rectangular beam and width of rib in case of flanged beam

d = effective depth.

- The upper limit (table 19) is specified to prevent the failure of beam by diagonal compression.
- The design for shear is empirical for the following reasons:
  (a) The reinforced concrete is a non-homogeneous material and does not resist tension without cracking. It is therefore difficult to form the equations to find out the maximum shear stress on a given plane.
  (b) Shear failures do not occur in the direction of shear force, but along the diagonal planes.
  (c) The tensile strength of concrete is highly variable. Thus, the shear resisted by dowel action and by aggregate inter-lock cannot be accurately determined.
- The primary functions of the stirrups are:
  (i) to resist a part of the shear.
  (ii) to resist the growth of the inclined cracks and improve aggregate interlock.
  (iii) to tie the longitudinal bars in place, thereby increase the dowel action.
- The minimum shear reinforcement required when $\tau_v < \tau_c$ because, the instant a shear crack is formed, the tension carried by concrete is transferred to the shear reinforcement. In other words, the shear reinforcement restrains the growth of shear crack and increases the ductility of the beam. In case of overloading, this provides the warning before sudden failure. To ensure that stirrups will have sufficient strength to absorb the diagonal tension in concrete, the minimum shear reinforcement is required.
- The characteristic strength of shear reinforcement limited to 415 N/mm^2 because, the width of shear crack is proportional to the strain in shear reinforcement. To limit the width of shear crack, strain in shear reinforcement shall be limited. To ensure this, code requires that the characteristic strength in shear reinforcement shall not exceed 415 N/mm^2.
- The bent bars alone is not satisfactory as shear reinforcement because, they should be designed to carry a maximum of 50% of design shear. This is because the exact behaviour of bent bars in resisting shear is not clearly understood. Also, the bent bars do not resist the reversal of shear force.
- The maximum spacing of vertical stirrups limited to 0.75d because, the horizontal distance between two successive cracks is approximately equal to effective depth es.3des0. The spacing of stirrups shall be such that it crosses the crack and also no crack shall remain unreinforced. To ensure this, the spacing of vertical stirrups is limited to 0.75d.

Practice Objective Questions

1. If the nominal shear stress (τ_v) at a section does not exceed the permissible shear stress (τ_c)
   (a) minimum shear reinforcement is still provided
   (b) shear reinforcement is provided to resist the nominal shear stress
   (c) no shear reinforcement is provided
   (d) shear reinforcement is provided for the difference of the two

2. Shear span is defined as the zone where
   (a) bending moment is zero
   (b) shear force is zero
   (c) shear force is constant
   (d) bending moment is constant

3. Which one of the following statements is correct?
   (a) Web shear cracks start due to high diagonal tension in case of beams with their webs and high prestressing force.

(b) Shear design for a prestressed concrete beam is based on elastic theory.
(c) In the zone where bending moment is dominant and shear is insignificant, cracks occur at 20° to 30°.
(d) After diagonal cracking, the mechanism of shear transfer in a prestressed concrete member is very much different from that in reinforced concrete members.

4. While checking shear resistance of reinforced concrete beams for limit state of collapse as per **IS:456**, which one of the following nominal shear stress recommendsations is to be adhered to? (V_u is shear force at vertical cross-section, 'b' and 'd' are overall breadth and effective depth of beam respectively)

(a) $\frac{0.5V_u}{bd}$
(b) $\frac{2V_u}{5bd}$
(c) $\frac{V_u}{0.5\,bd}$
(d) $\frac{V_u}{bd}$

5. **Assertion (A):** Shear capacity of a concrete beam increases with the increase in tension reinforcement.
Reason (R): Increase in tension reinforcement increases aggregate interlocking force.

6. The maximum permissible shear stress $\tau_{c,max}$ given in **BIS : 456-1978** is based on
(a) diagonal tension failure
(b) diagonal compression failure
(c) flexural tension failure
(d) flexural compression failure

7. In case of deep beam or in thin webbed RCC members, the first crack form is
(a) flexural crack
(b) diagonal crack due to compression
(c) diagonal crack due to tension
(d) shear crack

8. The chances of diagonal tension cracks in R.C.C. member reduce when
(a) axial compression and shear force act simultaneously
(b) axial tension and shear force act simultaneously
(c) only shear force act
(d) flexural and shear force act

9. The codal provisions recommend minimum shear reinforcement in the form of stirrups in the beams
1. to cater for any torsion in the beam section
2. to improve ductility of the cross-section
3. to improve dowel action of longitudinal tesnion bars

Select the correct answer using the codes given below:
(a) 1, 2 and 3
(b) 2 and 3
(c) Only 1
(d) Only 2

10. A reinforced concrete beam of 10 m effective span and 1 m effective depth is supported on 500 mm × 500 mm columns. If the total uniformly distributed load on the beam is 10 MN/m, the design shear force for the beam is
(a) 50 MN
(b) 47.5 MN
(c) 37.5 MN
(d) 43 MN

11. **Assertion (A):** Minimum shear reinforcement as stirrups must be provided in beams, even if the shear stress τ_v is less than the shear strength of concrete τ_c.
Reason (R): The bending of beams creates a tendnecy in the particles to slide upon each other with the beam. This tendnency is called shear.

12. Minimum shear reinforcement in beams is provided in the form of stirrups
(a) to resist extra shear force due to live load
(b) to resist the effect of shrinkage of concrete
(c) to resist principal tension
(d) to resist shear cracks at the bottom of beam

13. Diagonal tension reinforcement is provided in a beam as
(a) longitudinal bars
(b) bent up bars
(c) helical reinforcement
(d) 90° bend at the bends of main bars

14. Minimum shear reinforcement is provided to
(a) resist shear force at the support
(b) resist shear on account of accidental torsion
(c) arrest the longitudinal cracks on side faces due to shrinkage and temperature variation
(d) resist shear in concrete developing on account of non-homogeneity of concrete

15. **Assertion (A):** Minimum shear reinforcement in all shallow beams is provided when shear stress exceeds 0.5 τ_c (where τ_c is design shear stress).
Reason (R): Minimum shear reinforcement prevents formation of inclined cracks and avoids abrupt failures and introduces ductility in shear.

16. Shear strength of concrete in a reinforced concrete beam is a function fo which of the following:
1. Compressive strength of concrete
2. Percentage of shear reinforcement
3. Percentage of longitudinal reinforcement in tension in the section
4. Percentage total longitudinal reinforcement in the section

Select the correct answer using the codes given below:
(a) 1, 2 and 4 (b) 1, 2 and 3
(c) Only 1 and 3 (d) Only 1 and 4

17. What is the adoptable maximum spacing between vertical stirrups in an RCC beam of rectangular cross-section having an effective depth of 300 mm?
(a) 300 mm (b) 275 mm
(c) 250 mm (d) 225 mm

18. Consider the following statements dealing with flexural reinforcement to be terminated in the tension zone:
1. The shear at the cut-off point not to exceed two-third of the otherwise permitted value.

2. Shear reinforcement is provided along each terminated bar overlapping three fourth of the appropriate distance from the cut-off point.
3. For 36 mm and smaller bars, the continuiing bars sball provide double the area required for flexure at the cutoff and shear does not exceed three-fourth of the permitted value.

Which of these statements is/are correct?

(a) 1, 2 and 3
(b) 1 and 2 only
(c) 2 and 3 only
(d) 3 only

Common Data for Questions 19 and 20:

A reinforced concrete beam of rectangular cross section of breadth 230 mm and effective depth 400 mm is subjected to a maximum factored shear force of 120 kN. The grades of concrete, main steel and stirrup steel are M 20, Fe 415 and Fe 250 respectively. For the area of main steel provided, the design shear strength τ_c as per **IS : 456-2000** is 0.48 N/mm^2. The beam is designed for collapse limit state.

19. The spacing (mm) of 2-legged 8 mm stirrups to be provided is

(a) 40
(b) 115
(c) 250
(d) 400

20. In addition, the beam is subjected to a torque whose factored value is 10.90 kN-m. The stirrups have to be provided to carry a shear (kN) equal to

(a) 50.42
(b) 130.56
(c) 151.67
(d) 200.23

Answers

1. (a)	6. (b)	11. (b)	16. (c)
2. (c)	7. (c)	12. (c)	17. (d)
3. (b)	8. (a)	13. (b)	18. (a)
4. (d)	9. (b)	14. (b)	19. (b)
5. (b)	10. (c)	15. (a)	20. (c)

3

Bond and Anchorage

INTRODUCTION

- One of the most important assumptions in the behaviour of R.C. structures is that there is proper bond between concrete and reinforcing bars.
- When steel bars are embedded in concrete, the concrete (after setting) adheres to the surface of the bars and thus resist any force that tends to push or pull this rod.
- Thus the term '*bond*' describes the means by which the relative movement between concrete and steel is prevented and the intensity of adhesive force is called *bond stress*.
- **Bond stress is defined as longitudinal shear stress acting on the surface between steel and concrete.**
- Bond between steel and concrete is due to combined effect of adhesive resistance, frictional resistance and mechanical resistance (for deformed bars).
- The adhesive resistance is provided by 'chemical gum' produced by concrete during setting.
- The bond due to friction is provided by gripping of bars due to shrinkage.
- The friction gives considerable bond resistance. With increasing force in bar, the adhesion is loose first than the friction between concrete and steel.
- The mechanical resistance is provided by deformed bars only (not by the plain bars). The deformed bars have lugs, or corrugations and give higher bond resistance by providing an interlock between steel and concrete.
- In deformed bar, adhesion and friction become minor elements, and the bond strength is primarily dependent on bearing of concrete against the lugs or corrugations.

TYPES OF BOND STRESS

The bond stress in reinforced concrete members arises due to two distinct situations.

(a) The change in the bar force along its length due to variation in bending moment in this length. This type of bond force is called *flexural bond stress.*

(b) From the anchorage of bar in case of tension or compression. This type of bond force is known as *Anchorage bond stress.*

Flexural Bond Stress

As discussed above, the flexural bond arises along the length of the bar when change in bar force occurs due to variation in bending moment.

Consider the segment between two sections *ab* and *cd*, spaced *dx* apart of a R.C. beam, as shown in Fig. 3.1.

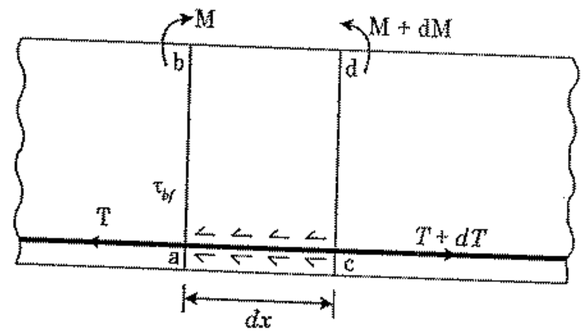


Fig. 3.1

Let M be the bending moment at section ab and $M + dM$ be the bending moment at section cd. Let T and $T + dT$ be the tensile forces developed in steel reinforcement at ab and cd respectively.

Now, $M = T.jd$

and $(M + dM) = (T + dT).jd$

Hence, $dM = dT.jd$

or, $dT = \dfrac{dM}{jd}$ = change in bar force

If τ_{bf} is the bond stress acting along the surface of bar, then for equilibrium,

Bond force acting along periphery of bar = Change in bar force dT.

or $\tau_{bf}[dx\,\Sigma O)] = dT$

where, ΣO = Sum of perimeters of all steel bars resisting tension,

or, $\tau_{bf}(dx\,\Sigma O) = \dfrac{dM}{jd}$

or, $\tau_{bf} = \dfrac{dM}{dx}\cdot\dfrac{1}{jd\,\Sigma\,O}$

But, $\dfrac{dM}{dx}$ = Shear force = V

Hence, $\tau_{bf} = \dfrac{V}{jd\,\Sigma\,O}$

The stress τ_{bd} at a particular section is called *local bond stress*. Thus, the flexural bond stress is directly proportional to the shear force and inversely proportional to the sum of perimeters of the bars at the section.

Anchorage Bond Stress and Development Length

Anchorage bond stress arises when a bar is carrying certain force. In anchorage bond, it is necessary to transfer this force in the bar to the surrounding concrete over a certain length. The length of bar 'L_d' is required to transfer the force in the bar.

Figure 3.2 shows a steel bar embedded in concrete and subjected to a tensile force T. Due to this force, there will be a tendency of the bar to slip out and this tendency is resisted by the bond stress developed over the perimeter of the bar, along its length of embedment. This required length 'L_d' is called *anchorage length* in case of axial tension (or compression) and *development length* in case of flexural tension.

The development length is an embedded length of the bar required to develop the design strength of reinforcement at the critical section.

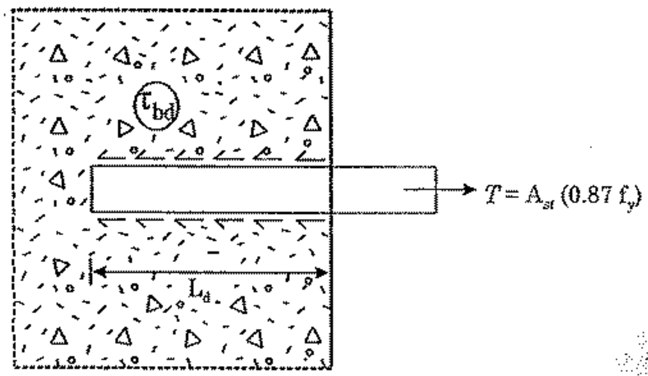


Fig. 3.2

If ϕ is the nominal diameter of a bar, then

$$\text{Tension, } T = (0.87\, f_y)\, A_{st}$$

$$T = (0.87\, f_y)\left(\frac{\pi}{4}\phi^2\right)$$

This force must be transferred from steel to concrete through bond acting over the perimeter of the bar along its length of embedment L_d.

If τ_{bd} is the average bond stress, then

$$\text{Force} = \tau_{bd} \times (\pi\phi)\, L_d$$

For equilibrium

$$0.87\, f_y\left(\frac{\pi}{4}\phi^2\right) = \tau_{bd}\,(\pi\phi)\, L_d$$

$$\therefore \quad L_d = \frac{0.87 f_y}{4\tau_{bd}}\phi$$

The value of design bond stress for plain bars in tension prescribed by IS code are reproduced in Table 3.1.

Table 3.1: Permissible bond stress in tension

Grade of concrete	M 20	M 25	M 30	M 35	M 40 and above
Design bond stress τ_{bd}, (N/mm^2)	1.2	1.4	1.5	1.7	1.9

Note : 1. τ_{bd} for deformed bar is increased by 60%. This is because for deformed bars, the actual contact area of a bar with concrete is taken into account which is much more than contanct area based on nominal diameter.

2. For deformed bars, these values shall be increased by 60 percent.

3. For bars in compression, the above values may be increased by 25 percent.

DEVELOPMENT LENGTH FOR FLEXURAL BOND

Following equation gives the flexural bond stress in the tension reinforcement at any section of the beam. In case, there is a group of bars of mixed size, the largest flexural bond stress occurs at the bar with the largest diameter. If there are N bars of equal size,

$$\tau_{bd} = \frac{V}{Jd(\Sigma O)} \quad \text{---(1)}$$

where $\Sigma O = \pi\phi N$ = total permeter of all bars in tension at the given section

Similarly, for a group of N bars with equal diameter, equation for anchorage bond can be written as:

$$0.87 f_y A_t = \tau_{bd}\, L_d\, \pi\phi N$$

$$= \tau_{bd}\, L_d\, (\Sigma O)$$

$$\text{or} \quad \tau_{bd} = \frac{0.87 f_y A_t}{L_d(\Sigma O)} \quad \text{---(2)}$$

Equating τ_{bd} from equation (1) and (2), yields

$$\frac{0.87 f_y A_t}{L_d(\Sigma O)} = \frac{V}{J(\Sigma O)}$$

$$\text{or} \quad \frac{0.87 f_y A_t J}{V} = L_d$$

$$\text{or} \quad L_d = \frac{M_1}{V} \quad \text{---(3)}$$

where J = lever arm

where M_1 = moment of resistance with respect to tension steel alone at the section under consideration (assuming all reinforcement at the section to be stressed $0.87\, f_y$)

V = factored shear force at the same section

if the design bond stress τ_{bd} is not to be exceeded, the ratio M_1/V must be equal to or greater than L_d. As an additional factor of safety, the anchorage length L_0 has been added to the right hand side of equation (3) in the code, that is

$$\boxed{L_d \le \frac{M_1}{V} + L_0} \quad \text{or}$$

The code IS: 456 (clause 26.2.3.3) stipulates that at simple supports and at point of inflexion, positive moment tension reinforcement shall be limited to a diameter such that

$$L_d \le \frac{M_1}{V} + L_0$$

where, L_0 = Sum of anchorage beyond the centre of the support and the equivalent anchorage value of any hook or mechanical anchorage at simple support, and at a point of inflexion L_0 is limited to the effective depth of the members or 12 ϕ, whichever is greater; and

ϕ = Diameter of bar

From equation (1), it is clear that flexural bond stress is maximum at the section where shear force is large or where the sum of the perimeters of bars is minimum. Therefore, the check for flexural bond becomes necessary at the section where shear force is maximum and where bending moment is zero.

The locations of such sections are:

(a) Simple supports.

(b) Points of contraflexure.

Note : At simple support, for the computation of L_0, the support width should be known. Figure shows a beam with end support, in which x' is the side cover and x_0 is the distance of the beginning of the hook from the centre line of the support.

Now, L_0 = Sum of the anchorage beyond the centre of the support and the equivalent anchorage value.

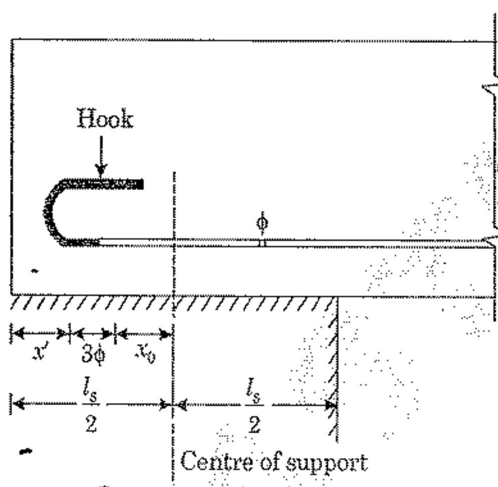


Fig. 3.3

The dark portion shows the hook which has an anchorage value of 16ϕ. The distance of the beginning of the hook from its apex of the semi-circle can be taken to be equal to 3ϕ.

Let, l_s be the length of support

$$\therefore \quad L_0 = x_0 + 16\,\phi \text{ where } x_0 = \frac{l_s}{2} - x' - 3\phi$$

$$\therefore \quad L_0 = \left(\frac{l_s}{2} - x' - 3\phi\right) + 16\phi$$

$$\therefore \quad L_0 = \frac{l_s}{2} - x' + 13\phi$$

If no hook is provided, as in case of deformed bars,

$$L_0 = \frac{l_s}{2} - x'$$

(2) The code further recommends that the value of $\frac{M_1}{V}$ may be increased by 30 percent when the ends of the reinforcement are confined by a compressive reaction. This condition of 'confinement' of reinforcing bars may

not be available at all the types of simple supports. In simple supports of beams resting on walls or columns, the reaction induces compressive stress due to which bar gets confined, as shown in Fig. 3.4.

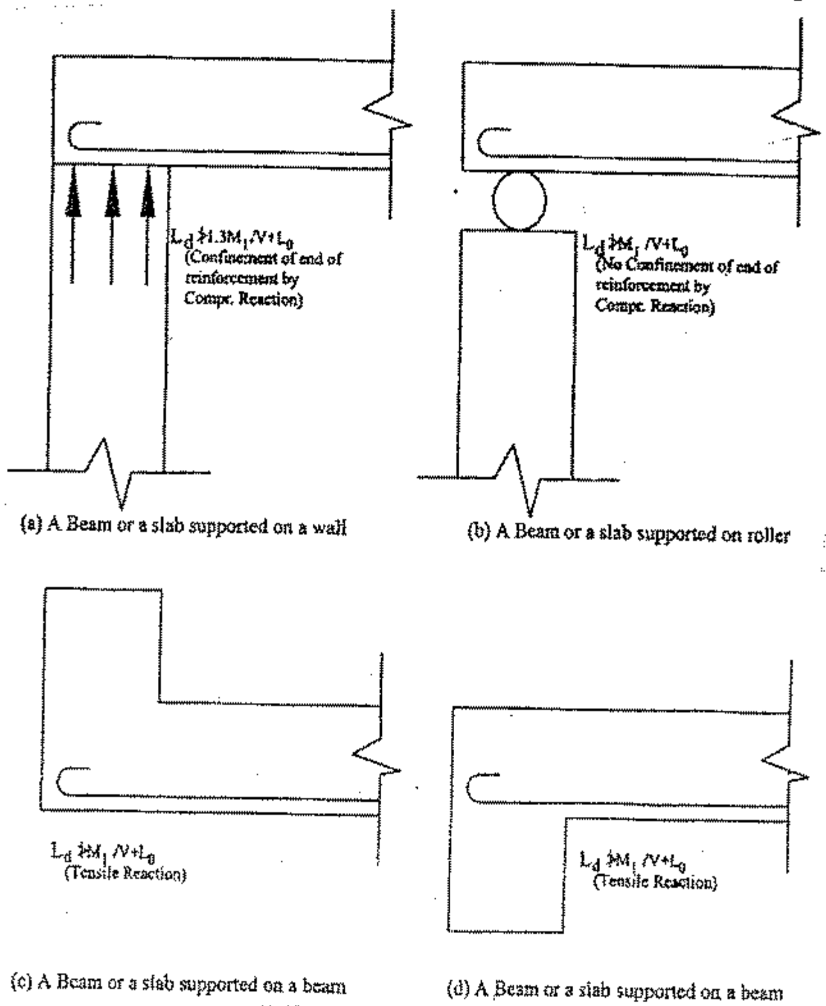


Fig. 3.4: Different support conditions for a beam of a slab.

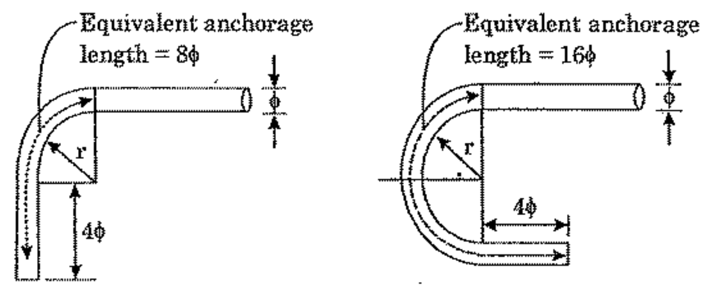
Thus, at simple supports, where the compressive reaction confines the ends of reinforcing bars, we have

$$L_d \leq 1.3\,\frac{M_1}{V} + L_0$$

BENDS AND HOOKS

- When the required straight development length (or anchorage length) cannot be provided due to limited space at the end of beam, in that case, standard hooks or bends are provided to find the full development length.
- The common types of anchorage provided are U-type hook and L-type bend. The bends and hooks shall conform to IS : 2502.
- The anchorage value of bend shall be taken as 4 times the diameter of the bar for each 45° bend subject to a maximum of 16 times the diameter of bars.
- The anchorage value of a standard U-type hook shall be equal to 16 times the diameter of the bar.

- Thus the anchorage value of the standard hook and standard bend shall be taken equal to 16ϕ and 8ϕ respectively. Following figure shows the dimensions of the hook and bend.



(a) Standard 90° bend (b) Standard U-type hook

Fig. 3.5: Anchorage lengths of standard bends and hooks.

The minimum (internal) turning radius 'r' specified for a hook is 2ϕ for plain mild steel and 4ϕ for cold worked deformed bars.

Anchoring Bars in Tension (Clause 26.2.2.1a)

Deformed bars may be used without end anchorages, provided development length required is satisfied. Normally hooks should be provided for plain bars in tension.

Anchoring Bars in Compression (Clause 26.2.2.2)

The anchorage length of straight bar in compression shall be equal to the development lengths of bars in compression. The projected length of hooks, bends and straight lengths beyond bends, if provided for a bar in compression, shall only be considered for development length.

Example 1

A simply supported beam is 25 cm by 50 cm and has 2-20 mm TOR bars going into the support. If the shear force at the centre of support is 110 kN at working loads, determine the anchorage length. Assume M20 mix and Fe 415 grade TOR steel.

Sol: For a load factor equal to 1.5, the factored SF = 1.5 × 110 = 165 kN

Assume 25 mm clear cover to the longitudinal bars

$$\text{Effective depth} = 500 - 25 - \frac{20}{2} = 465 \text{ mm}$$

Characteristic strength of TOR steel $f_{ck} = 415$ N/mm^2

$$\text{Moment of resistance } M_1 = 0.87\, f_y A_{st}\, (d - 0.42\, x_u)$$

$$x_u = \frac{0.87\, f_y\, A_{st}}{0.36\, f_{ck} b} = \frac{0.87 \times 415 \times 628}{0.36 \times 20 \times 250} = 126 \text{ mm} < x_{u,lim}\ (0.48 \times 465 = 223.2 \text{ mm})$$

$$\text{or} \quad M_1 = 0.87 \times 415 \times 2 \times \pi/4 \times 20^2\ (465 - 0.42 \times 126)$$

$$= 93.45 \times 10^6 \text{ Nmm}$$

Bond stress $\tau_{bd} = 1.2$ N/mm^2 for M 20 mix. It can be increased by 60% in case of TOR bars.

$$\text{Development length } L_d = \frac{0.87 f_y \phi}{4\tau_{bd}} = \frac{0.87 \times 415\phi}{4 \times (1.6 \times 1.2)} = 47\,\phi$$

If the bar is given a 90° bend at the centre of support, its anchorage value

$$L_0 = 8\,\phi = 8 \times 20 = 160 \text{ mm}$$

$$L_d \le 1.3\, M_1/V + L_0$$

$$47\,\phi \le \left[\frac{1.3 \times 93.45 \times 10^6}{165 \times 1000}\right] + 160$$

$$\text{or } \phi \le 19 \text{ mm}$$

Since actual bar diameter of 20 mm is greater than 19 mm, there is a need to increase the anchorage length. Let us increase the anchorage length L_0 to 240 mm. It gives

$$\phi \le 20.8 \text{ mm}$$

The arrangement of 90° bend is shown in Fig. 3.6(a).

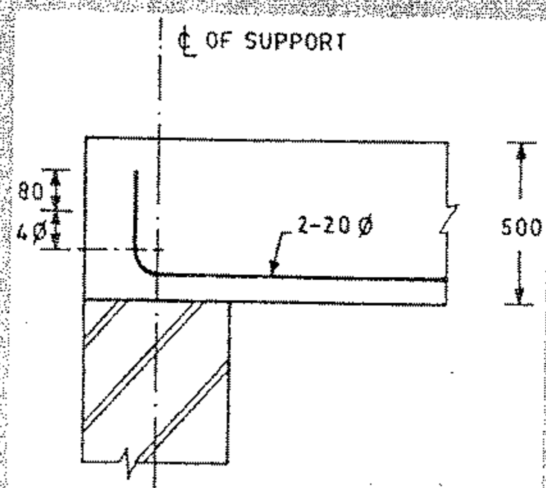


Fig. 3.6(a): Details of 90° hook.

Alternatively

Provide a U bend at the centre of support, its anchorage value,

$$L_d \le 1.3\, M_1/V + L_0$$

$$47\,\phi \le \left[\frac{1.3 \times 93.45 \times 10^6}{165 \times 1000}\right] + 320$$

$$\text{or} \quad \phi \le 22.47 \text{ mm}$$

Actual bar diameter provided is 20 mm < 22.47 mm.

The arragement of U bend is shown in Fig. 3.6(b).

In high strength reinforcement bars U-bend should be avoided as far as possible since they may be brittle and may fracture while bending.

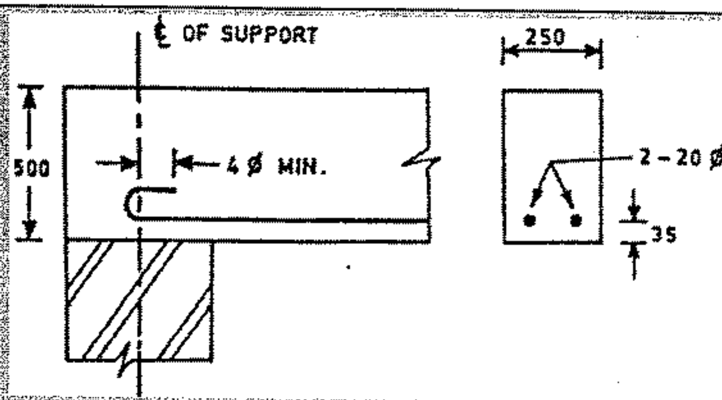


Fig. 3.6(b): Details of U hook reinforcement.

Example 2

A continuous beam 25 cm by 40 cm carries 3-16 mm longitudinal bars beyond the point of inflection in the sagging moment region as shown in Fig. 3.7. If the factored SF at the point of inflection is 150, f_{ck} = 20 N/mm² and f_y = 415 N/mm², check if the beam is safe in bond?

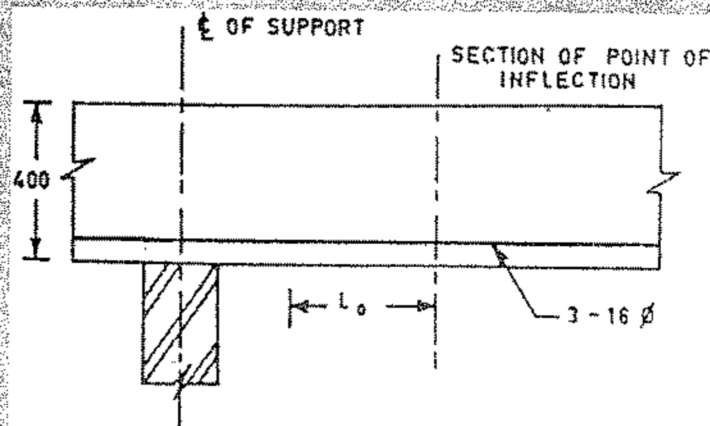


Fig. 3.7: Section of continuous beam.

Sol: Assuming 25 mm clear cover to the longitudinal bars.

$$\text{Effective depth } d = 400 - 25 - \frac{16}{2} = 367 \text{ mm}$$

$$\text{Depth of neutral axis } x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87 \times 415 \times 3 \times \pi/4 \times 16^2}{0.36 \times 20 \times 250}$$

$$= 120 \text{ mm} < x_{u,lim} = (0.48\text{ d}) \quad \text{OK}$$

$$\text{Moment of resistance } M_1 = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$= 0.87 \times 415 \times 603 (367 - 0.42 \times 120) = 68.90 \times 10^6 \text{ Nmm}$$

$$\text{Development length } L_d = \frac{0.87 f_y \phi}{4\tau_{bd}}$$

Bond stress $\tau_{bd} = 1.6 \times 1.2$ N/mm² for M 20 mix and HSD steel

$$\text{or} \quad L_d = \frac{0.87 \times 415\,\phi}{4 \times 1.6 \times 1.2} = 47\,\phi$$

$$\text{Anchorage length } L_0 = \text{greater of d or } 12\,\phi$$

$$= \text{greater of 367 mm, or } 12 \times 16 = 192 \text{ mm}$$

$$= 367 \text{ mm}$$

$$L_d \le \frac{M_1}{V} + L_0$$

$$\text{or} \quad 47\phi \le \frac{68.9 \times 10^6}{150 \times 1000} + 367 \quad \text{or,} \quad \phi \le 17.6 \text{ mm}$$

Thus, 16 mm bars are safe in bond at the point of inflection.

Some Important Points

- A length of reinforcement embedded in concrete so that it can develop the stress by bond is termed as developmeent length and is denoted by L_d.
- The development lenght for single bar is obtained from the formula

$$L_d = \frac{\phi\sigma_s}{4\,\tau_{bd}}$$

where, ϕ = nominal diameter of the bar

σ_s = stress in bar at the section considered at design load

τ_{bd} = design bond stress.

For bundled bars, the development length of each bar of bundled bars shall be that for the individual bar, increased by 10% for two bars in contact, 20% for three bars in contact and 33% for four bars in contact.

- The grip of the reinforcement and concrete due to adhesion or bearing is termed as bond. The magnitude of the bond stress at a point is called local bond stress which baries with the bending moment. Local bond is therefore, also termed as flexure bond.
- The factors influencing the stress transfer by bond are:
  - (i) adhesion of concrete and steel.
  - (ii) shear strength of concrete.
  - (iii) interlocking of surface ribs of HYSD (CTD or TMT) bars with concrete. The design bond strength can be determined considering these factors into account.
- Design bond strength in compression is increased by 25%. This is because the end bearing of the bar helps in resisting compression. Also the possibility of cracking the concrete is nil and hence, the allowable bond stress is increased.
- Design bond stress for deformed bars is increased by 60. This is because for deformed bars, the actual contact area of a bar with concrete is taken into account which is much more than conctact area based on nominal diameter.
- The reinforcement with surface deformation is used to increase the bond with the concrete.

Practice Objective Questions

1. In limit state design, permissible bond stress in the case of deformed bars is more than that in plain bars by

   (a) 60% (b) 50%

   (c) 40% (d) 25%

2. Consider the following statements:

   1. At a point or inflection, the embedment length need not exceed the development length L_d.
   2. The condition that $L_d \le \left(\frac{M_1}{V} + L_0\right)$ need not be checked for negative reinforcement.
   3. At least one-third of the total negative reinforcement provided must extend beyond the point of inflection for a distance not less than 'd' or 12ϕ or clear-span whichever is larger.

   Which of these statements are correct?

   (a) 1, 2 and 3 (b) 1 and 2

   (c) 1 and 3 (d) 2 and 3

3. Consider the following statements:

   Bars that extend into a simple support must be able to develop their full strength at a designated point L so that their moment capacity is more than the bending moment at that point. The clauses of the code require that ($\sigma_s = 0.85\ \sigma_{sy}$)

   1. $L_d \le \frac{1.3\ M_1}{V} + L_0$
   2. $\frac{\phi\sigma_s}{4\tau_{bd}} \le \frac{1.3\ M_1}{V} + L_0$
   3. $\phi \le \frac{4\tau_{bd}}{\sigma_s}\left(\frac{1.3\ M_1}{V} + L_0\right)$

   Which of these statements are correct?

   (a) 1 and 2 (b) 2 and 3

   (c) 1 and 3 (d) 1, 2 and 3

4. Consider the following statements:

   1. Reinforcement that is no longer required for flexure beyond a certain section, shall however be extended by d or 12 ϕ, whichever is greater, before being curtailed.
   2. At least half the bars should be bend up at the cut-off point.
   3. The shear capacity at cut-off point should at least be 1.5 times the shear force at that section.

   Which of these statements are correct?

   (a) 1 and 2 (b) 1 and 3

   (c) 2 and 3 (d) 1, 2 and 3

5. In a reinforced concrete member, the best way to ensure adequate bond is

(a) to provide minimum number of large diameter bars
(b) to provide large number of smaller diameter bars
(c) to increase the cover for reinforcement
(d) to provide additional stirrups

6. A beam is designed for uniformly distributed loads causing compression in the supporting columns. Where is the critical section for shear? (d is effective depth of beam and L_d is development length)

(a) A distance $L_d/3$ from the face of the support
(b) A distanced from the face of the support
(c) At the centre of the support
(d) At the mid span of the beam

7. Which one of the following is the correct expression to estimate the development length of deformed reinforcing bar as per IS code in limit state design?

(a) $\frac{\phi\sigma_s}{4.5\,\tau_{bd}}$ (b) $\frac{\phi\sigma_s}{5\tau_{bd}}$

(c) $\frac{\phi\sigma_s}{6.4\,\tau_{bd}}$ (d) $\frac{\phi\sigma_s}{8\tau_{bd}}$

8. When HYSD bars are used in place of mild steel bars in a beam, the bond strength

(a) does not change (b) increases
(c) decreases (d) becomes zero

9. Match List-I with List-II and select the correct answer using the codes given below the lists:

List-I	List-II
A. $\frac{V_u}{bd}$	1. Modulus of rupture
B. $0.7\sqrt{f_{ck}}$	2. Development length
C. $5000\sqrt{f_{ck}}$	3. Nominal shear stress
D. $\frac{\phi f_s}{4\tau_b}$	4. Hook anchorage value
	5. Modulus of concrete

Codes:

	A	B	C	D
(a)	3	1	5	2
(b)	2	1	4	3
(c)	3	5	1	4
(d)	2	4	1	3

10. What is the anchorage value of a standard hook of a reinforcement bar of diameter D?

(a) 4D
(b) 8D
(c) 12D
(d) 16D

Answers

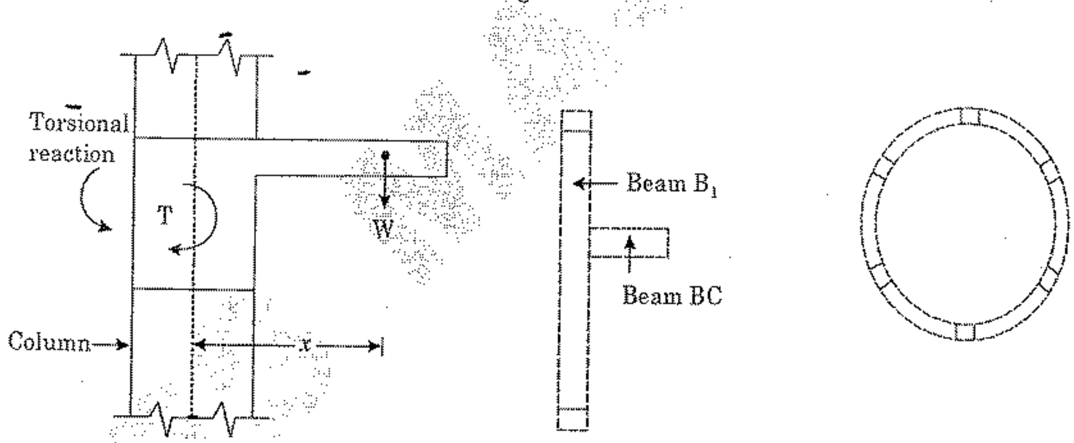
1. (a)	4. (a)	7. (c)	10. (d)
2. (b)	5. (b)	8. (b)	
3. (d)	6. (b)	9. (a)	

4

Torsion

INTRODUCTION

- In addition to axial force, shear force, and bending moment, the reinforced concrete elements are also sometimes subjected to torsion.
- However, members in structures are rarely subjected to pure torsion. In most of the cases, bending and shear are predominant, and torsion has only secondary effect.
- Torsion occurs either due to eccentric load lying in the plane contained by the cross-section or due to rotational compatibility between interconnected members.
- Many theories have been proposed to explain the phenomenon of torsion in reinforced concrete, such as: Skew bending theory and space truss analogy.
- Some examples of torsion are illustrated in Fig. 4.1.



(a) Cantilever slab and ell beam (b) Cantilever BC supported on B_1 (c) Circular beam

Fig. 4.1: Torsion in structural systems.

In Fig. 4.1(a) shows a slab cantilevered from a beam which is assumed to be fixed at supporting columns. Slab load W induces a torsional moment of Wx in beam.

If beam is supported throughout its length by a load bearing masonry wall, this is the case of *pure torsion*.

In Fig. 4.1(b) a small beam BC is cantilevered from beam B_1. Beam B_1 is considered fixed at columns. Here negative moment of beam BC will be the torsional moment of beam B_1.

In Fig. 4.1(c) shows a circular ring beam supported on columns. In this case, because of the shape of the beam, torsional moment is induced.

The maximum torsional shear stress occurs at the middle of the wider face and has a value given by $\tau_{t,max} = \dfrac{T}{\alpha b^2 D}$.

EFFECT OF TORSIONAL MOMENT

- Torsional moment induces shear stresses in the beam because of the torsion a beam fails in diagonal tension forming spiral cracks around the beam as shown in Fig. 4.2.

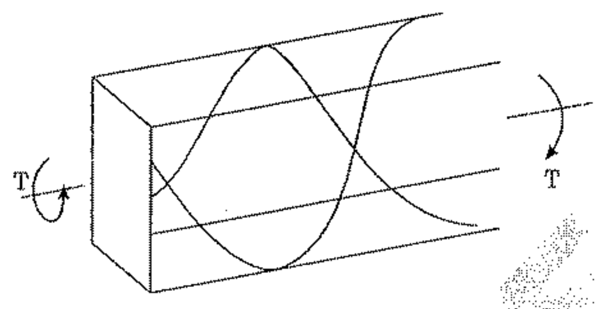


Fig. 4.2

- The behaviour of concrete structures subjected to torsion is complicated and a number of theories have been developed. Out of these, a space truss analogy for the behaviour of torsion have received great attention.
- In this analogy, the longitudinal reinforcements, the stirrups and strut of concrete in compression form a space truss to resist torsion.
- The longitudinal reinforcement helps in reducing the crack width through dowel action and stirrups crossing the cracks resist shear due to vertical loads and torsion.

  As a simplification, the effect of torsional moment is split up into

  (a) Equivalent shear, and

  (b) Equivalent bending moment

Code Provisions: (Clause 41)

- If in the analysis of structure, the torsional resistance or stiffness of members has not been taken into account, no specific calculation for torsion is necessary, adequate control of any torsional cracking being provided by the required nominal shear reinforcement.
- Where the torsional resistance or stiffness of members is taken into account in the analysis, the members shall be designed for design.
- For design of torsion, section located at a distance less than '*d*' from the face of the support may be designed for the same torsion as computed at a distance '*d*', where *d* is the effective depth.

The design rules for torsion are based on equivalent moment. They are explained as:

(a) **Shear and Torsion (Clause 41.3.1):** The combined effect of torsion and shear is considered and called equivalent shear.

Equivalent shear V_e is calculated from the formula

$$V_e = V_u + 1.6\frac{T_u}{b}$$

where, V_e = Equivalent shear
V_u = Ultimate vertical shear
T_u = Ultimate torsional moment
and b = Breadth of beam.

The equivalent nominal shear stress τ_{ve} is given by,

$$\tau_{ve} = \frac{V_e}{b.d}$$

The equivalent nominal shear stress τ_{ve} shall not exceed the values of $\tau_{c,max}$ as given in table 20 of IS 456.

If τ_{ve} does not exceed τ_c as given in table 19 of IS 456, only minimum shear reinforcement shall be provided (clause 41.3.2).

It τ_{ve} exceeds τ_c as given in table 19 of IS 456, both longitudinal and traverse reinforcement shall be provided (clause 41.3.3).

(b) Longitudinal Reinforcement (Clasuse 41.4.2): The longitudinal reinforcement shall be designed to resist an equivalent bending moment M_{e_1}, given by

$$M_{e_1} = M_u + M_t$$

where, M_u = Bending moment at the cross-section

$$M_t = T_u\left(\frac{1+D/b}{1.7}\right)$$

where, T_u = Torsional moment
D = Overall depth of beam
b = Breadth of the beam

When $M_t > M_u$, there is resultant torsion due to $(M_t - M_u)$. Therefore, the longitudinal reinforcement shall be provided on the flexural compression face, to resist the equivalent moment

$$M_{e_2} = M_t - M_u$$

When $M_t < M_u$, no additional steel is required on bending compression side.

(c) Transverse reinforcement (Clause 41.4.3): Two-legged closed hoops enclosing the corner longitudinal bars shall have an area of cross-section A_{sv}, given by

$$A_{sv} = \frac{T_u.S_v}{b_1 d_1(0.87\, f_y)} + \frac{V_u.S_v}{2.5 d_1(0.87 f_y)}$$

But the total transverse reinforcement shall not be less than

$$\frac{(\tau_{ve} - \tau_c)\, b.S_v}{0.87\, f_y}$$

where, T_u = Ultimate torsional moment

V_u = Ultimate shear force
S_v = Spacing of the stirrup reinforcement
b_1 = c/c distance between corner bars in the direction of width
d_1 = c/c distance between corner bars in the direction of depth
b = Breadth of the member
f_y = Characteristic strength of shear reinforcement ≤ 415 N/mm²

$\tau_{ve} = \frac{V_e}{bd}$ = Equivalent shear stress

τ_c = Shear strength of concrete, as in table 19 of IS 456.

1. **The transverse reinforcement** for torsion shall be rectangular closed stirrups placed perpendicular to the axis of the member. The spacing of the stirrups shall not exceed the least of x_1, $\frac{x_1 + y_1}{4}$ and 300 mm, where x_1 and y_1 are the short and long dimensions of the stirrups, as shown in Fig. 4.3 shown below.

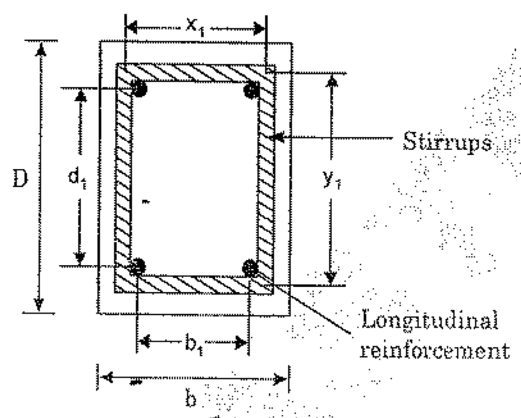


Fig. 4.3

where,

b_1 = c/c distance between corner bars in the direction of the width.

$= b - \text{clear cover} - 2\ \text{diameter of stirrups} - 2\left(\frac{\text{diameter of longitudinal bar}}{2}\right)$

d_1 = c/c distance between corner bars in the direction of depth

$d_1 = D - \text{clear cover} - 2\ \text{diameter of stirrups} - 2\left(\frac{\text{diameter of longitudinal bar}}{2}\right)$

x_1 and y_1 = short and long dimesions of the stirrups.

$$x_1 = b_1 + 2\left(\frac{\text{diameter of longitudinal bar}}{2}\right) + 2\left(\frac{\text{diameter of stirrups}}{2}\right)$$

and

$$y_1 = d_1 + 2\left(\frac{\text{diameter of longitudinal bar}}{2}\right) + 2\left(\frac{\text{diameter of stirrups}}{2}\right)$$

2. **Longitudinal reinforcement**

- Shall be placed as close as practicable to the corners of the cross section and in all cases, there shall be at least one longitudinal bar in each corner of the ties

- When the cross-sectional dimensions either b or D of the member exceeds 450 mm, additional longitudinal bars shall be provided along the two faces. The total area of such reinforcement shall not be less than 0.1% of the web area and shall be distributed equally on two faces at a spacing not exceeding 300 mm or web thickness, whichever is less.
- Although code does not specify the minimum diameter of this bar, it may not be less than 10 mm.

Example 1

Design a reinforced concrete beam of rectangular cross-section for the following data :
$b = 300$; $d = 800$; $D = 850$; $f_{ck} = 15$; $f_y = 250$; $M_u = 200$ kNm; $V_u = 100$ kN and $T_u = 50$ kNm.

Sol: Equivalent shear

$$V_e = V_u + 1.6\frac{T_u}{b}$$

$$= 100 + 1.6 \times \frac{50}{0.3} = 366.67 \text{ kN}$$

$$\tau_{ve} = \frac{366.67 \times 10^3}{300 \times 800} = 1.53 \text{ N/mm}^2$$

For M 15 concrete, $\tau_{c,max} = 2.5$ MPa

Since tensile reinforcement is not known at the outset, therefore, for the minimum % of tensile steel, i.e. for

$$100\frac{A_{st}}{bd} = 100 \times \frac{0.85}{f_y} = 100 \times \frac{0.85}{250} = 0.34\%$$

$$\tau_c = 0.35 + \frac{(0.45 - 0.35)}{(0.5 - 0.25)} \times (0.34 - 0.25) = 0.39 \text{ MPa} < \tau_{ve}$$

Hence, both longitudinal and traverse reinforcement shall be provided.

Equivalent Bending Moment

$$M_{e_1} = M_u + M_t = 200 + T_u \cdot \frac{(1 + D/b)}{1.7}$$

$$= 200 + 50 \times \frac{(1 + 850/300)}{1.7} = 200 + 112.75 = 312.75 \text{ kNm}$$

Since $M_u > M_t$, *no* longitudinal reinforcement will be required on compression flange.

Longitudinal Reinforcement

$$M_{e_1} = 0.87 f_y A_{st} d\left(1 - \frac{A_{st} f_y}{b d f_{ck}}\right)$$

$$\text{or,} \quad 312.75 \times 10^6 = 0.87 \times 250 \times A_{st} \times 800 - \frac{A_{st} \times 250}{300 \times 800 \times 15}$$

$$\text{or} \quad 13.08\, A_{st}^2 - 17400\, A_{st} + 31275 + 10^6 = 0$$

$$\text{or,} \quad A_{st} = 2105.06 \text{ mm}^2$$

Provided 4 nos. 28 mm dia bar

$$100\times\frac{A_{st}}{bd}\% = 100\times\frac{4\times\frac{\pi}{4}\times 28^2}{300\times 800}\% = 1.03\% > 0.34\%\ (A_{st,\min})$$

Now revised τ_c is given as

$$\tau_c = 0.6+\frac{(0.64-0.6)}{(1.25-1.0)}\times(1.03-1.0) = 0.605\text{ MPa}$$

Transverse Reinforcement

$$A_{sv} = \frac{T_u s_v}{b_1 d_1(0.87f_y)}+\frac{V_u s_u}{2.5d_1(0.8f_y)}$$

Providing side and top clear cover of 30 mm and 2 nos. 10 mm dia bars at the top

Assume 2 nos. 8 mm dia two legged stirrups

$$A_{sv} = 2\times\frac{\pi}{4}\times 8^2 = 100.53\text{ mm}^2$$

$$b_1 = 300-2\times 30-2\times 8-2\left(\frac{28}{2}\right) = 196\text{ mm}$$

$$d_1 = 850-2\times 30-2\times 8-\frac{28}{2}-\frac{10}{2} = 755\text{ mm}$$

Substituting these values in the above equation

$$100.53 = \frac{50\times 10^6 s_v}{196\times 755\times(0.87\times 250)}+\frac{100\times 1000 s_v}{2.5\times 755\times(0.87\times 250)}$$

$$s_v = 55.94\text{ mm}$$

Minimum shear reinforcement τ_{ve}

$$A_{sv} = \frac{(\tau_{ve}-\tau_c)\,bs_v}{0.87\,f_y}$$

or, $$100.53 = \frac{(1.53-0.605)\times 300\,s_v}{0.87\times 250}$$

or, $$s_v = 78.79\text{ mm} > 55.94\text{ mm}$$

(i) $s_{v,\max}$ is least of $x_1 = \left[196+2\times\left(\frac{28}{2}\right)+2\left(\frac{8}{2}\right)\right] = 232\text{ mm}$

(ii) $$s_{v,\max} = \frac{x_1+y_1}{4}$$

where $$y_1 = 755-\frac{28}{2}-\frac{10}{2}-2\times\left(\frac{8}{2}\right) = 728\text{ mm}$$

Substituting y_1 in the above equation

$$s_{v,\max} = \frac{232+728}{4} = 240\text{ mm}$$

(iii) $s_{v,max} = 300$ O.K. [s_v (50 mm) < $s_{v,max}$ (232 mm)]

As side reinforcement

$= 0.1 \times$ web area

$= 0.1 \times 300 \times 850$

$= 250$ mm^2

Provided 2ϕ 10 mm on each face.

The arrangement of reinforcements is shown in Fig. 4.2 below.

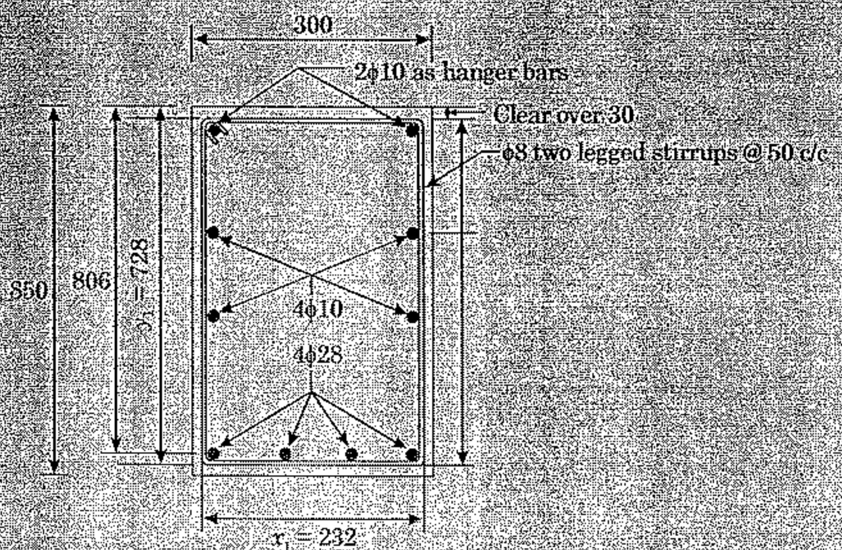


Fig. 4.2: Details of reinforcement for combined M_u, V_u and T_u having $M_t < M_u$

Design of a Beam in which $M_t > M_u$

Example 2

Design a rectangular R.C. beam for the following data:

$b = 300$; $d = 800$; $D = 850$; $M_u = 200$ kNm; $V_u = 100$ kN; $T_u = 95$ kNm; $f_{ck} = 20$ MPa; and $f_y = 415$ MPa.

Sol: Equivalent shear

$$V_e = V_u + 1.6\frac{T_u}{b}$$

$$= 100 + 1.6 \times \frac{95}{0.3} = 606 \text{ kN}$$

$$\therefore \quad \tau_{ve} = \frac{606.67 \times 10^3}{300 \times 800} = 2.53 \text{ MPa}$$

For M 20 concrete, $\tau_{c,max} = 2.8 > \tau_{ve}$

Since tensile reinforcement is not known at the outset; therefore, for minimum % of tensile reinforcement

$$\text{i.e. for } 100\,\frac{A_{st}}{bd}\% = 100 \times \frac{0.85}{415}\% = 0.205\%$$

$$\tau_c = 0.33 + \frac{(0.205-0.2)}{(0.25-0.2)} \times (0.36-0.33)$$

$$= 0.333 < \tau_{ve}\ (2.53\ \text{MPa})$$

Hence, both longitudinal as well as transverse reinforcement shall be provided.

Equivalent Bending Moment

$$M_{e1} = M_u + M_t = M_u + T_u\left(\frac{1+D/b}{1.7}\right)$$

$$= 200 + 95\frac{(1+850/300)}{1.7} = (200+214.22)\ \text{kNm}$$

$$= 414.22\ \text{kNm}$$

Here, $M_t > M_{e1}$, therefore, longitudinal reinforcement on compression face will *also* be provided for a bending moment of $(M_t - M_u)$

$$M_{e2} = M_t - M_u = 214.22 - 200 = 14.22\ \text{kNm}$$

$$m_{u,\text{lim}} = 0.36\frac{x_{u,\text{lim}}}{d}\left(1 - 0.42\frac{x_{u,\text{lim}}}{d}\right)bd^2 f_{ck}$$

Here for Fe 415 $\frac{x_{u,\text{lim}}}{d} = 0.48$

$$\therefore \quad m_{u,\text{lim}} = 0.36 \times 0.48\ (1 - 0.42 - 0.48) \times 300 \times 800^2 \times 20$$

$$= 529.92\ \text{kNm} > M_{e1}\ (414.22\ \text{kNm})$$

Hence, the beam will be under reinforced.

Longitudinal Reinforcement on Tension Side

$$M_{e1} = 0.87 f_y A_{st}\left(1 - \frac{A_{st} f_y}{bdf_{ck}}\right)$$

or, $$414.22 \times 10^6 = 0.87 \times 415 \times A_{st} \times 800\left(1 - \frac{A_{st} f_y \times 415}{300 \times 800 \times 20}\right)$$

or, $$24.97 A_{st}^2 - 288840 A_{st} + 414.22 \times 10^6 = 0$$

or, $$A_{st} = 1677.29\ \text{mm}^2$$

Provided 5ϕ22 (= 1900 mm^2)

Longitudinal Reinforcement on Compression Face

$$M_{e2} = 0.87 f_y A_{st}\ (d - d')$$

or, $$14.22 \times 10^6 = 0.87 \times 415\ A_{st}\ (800 - 25)$$

or, $$A_{st} = 50.82\ \text{mm}^2$$

Provided 2ϕ10 (= 157 mm^2)

Transverse Reinforcement

$$A_{st} = \frac{T_u s_v}{b_1 d_1 (0.87 f_y)} + \frac{V_u s_v}{2.5 d_1 (0.87 f_y)}$$

Providing 8ϕ two legged stirrups

$$A_{sv} = 2\times\frac{\pi}{4}\times 8^2 = 100.53\ \text{mm}^2$$

Again taking clear cover on top and on sides = 25 mm

$$b_1 = 300 - 25 - 25 - 2\times 8 - 2\times\left(\frac{22}{2}\right) = 212\ \text{mm}$$

$$d_1 = 850 - 2\times 25 - 2\times 8 - \frac{22}{2} - \frac{10}{2} = 768\ \text{mm}$$

Substituting these values in the above eq.

$$100.53 = \frac{95\times10^6 s_v}{212\times768\,(0.87\times415)} + \frac{100\times10^3 s_v}{2.5\times768\,(0.87\times415)}$$

or, $s_v = 57.10$

As $$P_t\% = \frac{5\times\frac{\pi}{4}\times22^2}{300\times800}\times100\% = 0.79\%$$

$$\tau_c = 0.56 + \frac{(0.62-0.56)}{(1-0.75)}\times(0.79-0.75) = 0.57\ \text{MPa}$$

Now, $$A_{sv} = \frac{(\tau_{ve}-\tau_c)\,bs_v}{0.87 f_y}$$

or, $$100.53 = \frac{(2.53-0.57)\times300\,s_v}{0.87\times415}$$

or, $s_v = 61.73 > 57.10$

Again $$x_1 = 212 + 2\times\frac{22}{2} + 2\times\frac{8}{2} = 242\ \text{mm}$$

$$y_1 = 768 + \frac{22}{2} + \frac{10}{2} + 2\times\frac{8}{2} = 792\ \text{mm}$$

Spacing of transverse reinforcement shall be the least of the following:

(i) 57.10

(ii) $x_1 = 242$

(iii) $\frac{x_1+y_1}{4} = \frac{242+792}{4} = 258.5$ mm

Therefore, provided φ8 two legged stirrups @ 50 mm c/c.

Since, the depth of the beam is greater than 450, side face reinforcement area

$$A_s = \frac{0.1}{100}\times bD = \frac{0.1}{100}\times300\times850 = 255\ \text{mm}^2$$

Hence provided 2φ10 on each side face.

The arrangement of reinforcement is shown in Fig. 4.3 below.

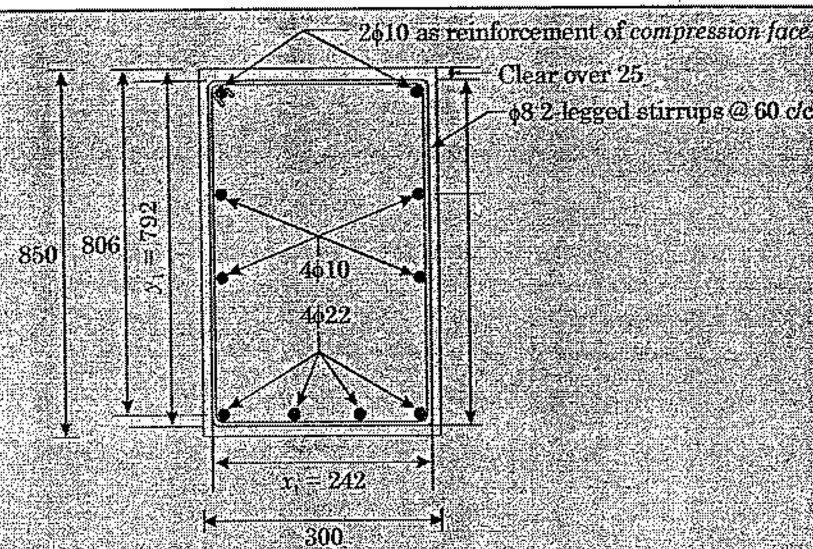


Fig. 4.3: Details of reinforcements for combined M_u, V_u and T_u having $M_t > M_u$

Example 3

Design a beam for following moment/S.F./Torsional moment acting at certain location.

$$B.M. = 200\ kN\text{-}m$$
$$S.F = 120\ kN$$
$$T.M. = 48\ kN\text{-}m$$

Use M 25/Fe 415, b = 400 mm, Cover is 50 mm (effective cover)

Sol:

$$M_u = 200 \times 1.5 = \text{kN-m}$$
$$V_u = 120 \times 1.5 = 180 \text{ kN-m}$$
$$T_u = 48 \times 1.5 = 72 \text{ kN-m}$$
$$M_{ue1} = M_u + \frac{T_u}{1.7}\left(+\frac{D}{b}\right)$$

Assume $D = 500$ mm

$$d = 500 - 50 = 450 \text{ mm}$$
$$M_{ue1} = 300 + \frac{72}{1.7}\left(1+\frac{500}{400}\right)$$
$$M_{ue1} = 395.29 \text{ kN-m}$$
$$M_{ue,lim} = 0.138 f_{ck} bd^2 \text{ for M 25}$$
$$\text{or } M_{u,lim} = 0.36 f_{ck} x_{u,lim}\, b\, (d - 0.42\, x_{u,lim})$$
$$395.29 \times 10^6 = 0.138 \times 25 \times 400 \times d^2$$
$$d = 535.61 \quad > 450 \text{ mm}$$

Take $D = 700$ mm (if you take $D = 550$ or 600 mm then again you have to revised the depth)

$$d = 700 - 50 = 650 \text{ mm}$$
$$M_{ue} = M_u + M_T$$

$$= 300 + \frac{72}{1.7}\left(1+\frac{700}{400}\right)$$

$$= 416.47 \text{ kN-m}$$

$M_T < M_u$ compressive reinforcement is not required

$$M_{ue} = 0.138 f_{ck} bd^2$$

$$d = \sqrt{\frac{416.47\times10^6}{0.138\times25\times400}} = 549.35 \text{ mm}$$

$d_{\text{Required}} < d_{\text{provided}}$ (650)

$$d' = 50$$

$$D = 549.35 + 50 = 599.35 \text{ mm} \simeq 600 \text{ mm}$$

$$d = 550 \text{ mm}$$

$$V_{ue} = V_u + 1.6\frac{T_u}{b} = 180 + 1.6 \times \frac{72}{0.40}$$

$$= 468 \text{ kN}$$

$$\tau_{ve} = \frac{V_u}{bd} = \frac{468\times10^3}{400\times550} = 2.13 \text{ N/mm}^2$$

Minimum % of tensile reinforcement

$$\frac{A_{st}}{bd}\times100 = \frac{0.85}{f_y}\times100$$

$$= \frac{0.85}{415}\times100$$

$$p_t = 0.205\%$$

$$\tau_c = 0.33 + \frac{(0.205-0.2)}{0.25-0.2}\times(0.36-0.33)$$

$$\tau_c = 0.333 < \tau_{ve}\ 2.13 \text{ N/mm}^2$$

Hence both longitudinal as well as transverse reinforcement shall be provided.

$$\tau_{c,\max} = 3.10 \text{ N/mm}^2 \text{ (M 25)}$$

$$\tau_{ve} < \tau_{c,\max} \text{ O.K.}$$

$$M_{ue} = 0.87 f_y A_{st} (d - 0 - 0.42\, x_u)$$

Take $x_u = x_{u,lim}$

$$A_{st} = \frac{416.47\times10^6}{0.87\times415}\ (550 - 0.42 \times 0.48 \times 550)$$

$$= 2626 \text{ mm}^2$$

Privide 5 nos. 28 mm ϕ reinforced (3078.76 mm^2) and 2ϕ 10 mm hanger bar at top.

$$d_1 = 600 - 50 - 50$$

$$= 500 \text{ mm}$$

$$b_1 = 400 - 50 - 50$$

$$= 300 \text{ mm}$$

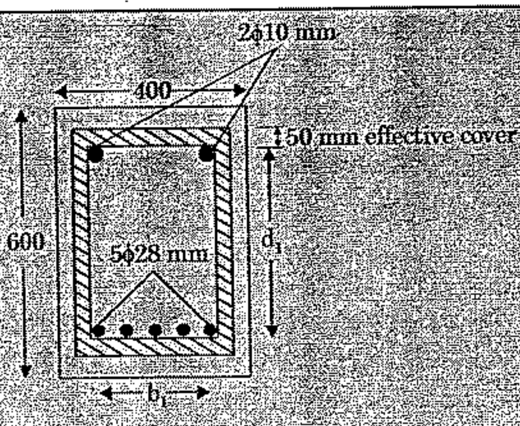


Fig. 4.4

Use 10 mm dia 2 legged stirrups

$$x_1 = 300 + \frac{28}{2} + \frac{28}{2} + \frac{10}{2} + \frac{10}{2} = 338 \text{ mm}$$

$$500 + \frac{28}{2} + \frac{10}{2} + \frac{10}{2} + \frac{10}{2} = 529 \text{ mm}$$

(*v*) Transverse Reinforcement

$$s_v = \frac{A_{sv}(0.87 f_y) d_1}{V_{net}}, \; V_{net} = \frac{T}{b_1} + \frac{V}{2.5}$$

$$V_{net} = \frac{72}{0.3} + \frac{180}{2.5} = 312 \text{ kN}$$

$$S_v = \frac{2 \times \frac{\pi}{4} \times 10^2 (0.87 \times 415) \times 500}{312 \times 10^3} = 90.88 \text{ mm}$$

This spacing $\not> x_1 = 338$ mm

$\frac{x_1 + y_1}{4}$ and 300 mm

$$\frac{338 + 529}{4} = 216.75 \text{ mm}$$

Transverse reinforcement should not less than $(A_{sv}) \frac{(\tau_{ve} - \tau_c) b - s_v}{0.87 f_y}$

$$p_t = \frac{A_{st}}{bd} \times 100 = \frac{5 \times \frac{\pi}{4} \times 28^2}{400 \times 550} \times 100$$
$$= 1.399\%$$

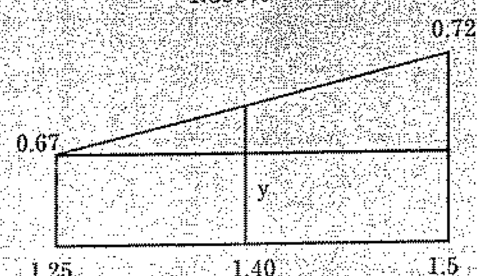


Fig. 4.5

$$\tau_c = \frac{0.72-0.67}{1.5-1.25} = \frac{y-0.67}{1.4-1.25}$$

$$y = \tau_c = 0.7\ \text{N/mm}^2$$

Now,

$$A_{sv} = \frac{(\tau_{ve}-\tau_c)\,bs_v}{0.87f_y} = \frac{(2.13-0.7)\times 400\times s_v}{0.87\times 415} = 2\times\frac{\pi}{4}\times 10^2$$

$$\therefore s_{v,max} \text{ or } s_v = 99.14\ \text{mm} > 90.88\ \text{O.K.}$$

$$\text{Side face reinforcement} = \frac{0.1\,bD}{100} = \frac{0.1\times 400\times 600}{100} = 240\ \text{mm}^2$$

Hence provide 2ϕ10 mm bar on each side of face.

Example 4

A rectangular concrete beam 300 mm wide is subjected to a sagging moment of 20 kNm and a torsional moment of 30 kNm. Design a suitable depth for the beam and the longitudinal reinforcement required. Allowable stresses are $\sigma_{cbc} = 5\ N/mm^2$, $\sigma_{st} = 140\ N/mm^2$, $m = 19$.

Sol: (i) Design constants

$$k_c = \frac{mc}{t+mc} = \frac{19\times 5}{140+(19\times 5)} = 0.4042$$

$$j = 1-\frac{k_c}{3} = 1-\frac{0.4042}{3} = 0.8652$$

$$Q = \frac{1}{2}cjk_c = \frac{1}{2}\times 5\times 0.8652\times 0.4042 = 0.8743$$

$$M = 20\ \text{kNm}$$

$$T = 30\ \text{kNm}$$

(ii) Equivalent moment

$$M_e = M_u + M_T$$

where

$$M_T = \frac{T}{1.7}\left[1+\frac{D}{b}\right]$$

Assume $D = 700$ mm

$$M_T = \frac{T}{1.7}\left[1+\frac{700}{300}\right]$$

$$= \frac{30}{1.7}\left[1+\frac{700}{300}\right]$$

$$M_T = 58.82\ \text{kN-m}$$

$Me_1 = 20 + 58.82 = 78.82$ kN-m and $M_T > M_u$ so compression reinforcement is required.

Check d

$$Me_1 = 0.8743 \times 300\ d^2$$

$$d = \sqrt{\frac{78.82\times10^6}{0.8743\times300}} = 548.18 \text{ kNm}$$

Use nominal cover 25 mm for beam and 20 mm dia bar

$$D = 548.18 + 25 + \frac{20}{2} = 583.18$$

Adopt $D = 600 \text{ mm} < D_{\text{provided}}\,(700)$

$$d = 600 - 35 = 565 \text{ mm}$$

(iv) Tension reinforcement

$$A_{st} = \frac{M_e}{t_s jd}$$

$$= \frac{78.82\times10^6}{140\times0.8652\times565}$$

$$A_{st} = 1151.71 \text{ mm}^2$$

Provide 20 mm ϕ bars, no. of bars $= \frac{1151.71}{\frac{\pi}{4}\times20^2} = 3.67 = 4$ bars

(v) Compression reinforcements

$$M_T = 58.82 \text{ Nmm}$$

$$M_u = 20\times10^6 \text{ Nmm}$$

$$Me_2 = M_T - M_u$$

$$= 58.82 - 20$$

$$Me_2 = 38.82 \text{ kN-m}$$

$$A_{sc} = \frac{Me_2}{t(d-dc)}$$

$$= \frac{38.82\times10^6}{140\times(565-35)}$$

$$A_{sc} = 523.18 \text{ mm}^2$$

No. of 20 mm ϕ bars $= \frac{523.18}{\frac{\pi}{4}\times20^2} = 1.66 = 2$ bar

Proving two bars in compression reinforcements.

Example 5

A reinforced concrete beam of rectangular section is 550 mm wide and has an overall depth of 750 mm. It is subjected to an ultimate BM of 150 kNm and an ultimate twisting moment of 50 kNm. M 15 grade concrete and Fe 415 grade steel are used. Determine the necessary longitudinal reinforcement.

Sol:

$$b = 550 \text{ mm}$$

$$D = 750 \text{ mm}$$

$M_u = 150$ kNm

$\tau_u = 50$ kNm

Assuming effective cover = 35 mm

M/15 grade concrete and Fe 415 are used.

$$M_e = M_u + M_t$$

where $$M_t = \frac{T_u}{1.7}\left[1+\frac{D}{b}\right]$$

$$= \frac{50}{1.7}\left[1+\frac{750}{550}\right]$$

$$= 69.52 \text{ kNm}$$

$$M_e = 150 + 69.52 = 219.52 \text{ kNm}$$

$M_u > M_T$ so compression reinforcement is not required.

$$M_{u,lim} = 0.36\, f_{ck}\, b x_{u,lim}\,(d - 0.42\, x_{u,lim})$$

$$= 0.36 \times 25 \times 550 \times 0.48 \times 715\,(715 - 0.42 \times 0.48 \times 715)$$

$$= 581.87 \text{ kN-m}$$

$M_e < M_{u,lim}$. Hence section is under reinforced.

Now force or under reinforced section

$$0.36\, f_{ck} b\, x_u = 0.87\, f_y\, A_{st}$$

$$\Rightarrow \quad x_u = \frac{0.87\, f_y A_{st}}{0.36\, f_{ck} b}$$

$$= 0.1215\, A_{st}$$

Also $$MR = 0.87\, f_y\, A_{st}\,(d - 0.42\, x_u)$$

$\Rightarrow$ $$219.52 \times 10^6 = 0.87 \times 415 \times A_{st}\,(700 - 0.42 \times 0.1215\, A_{st})$$

$\Rightarrow$ $$608004.43 = 700\, A_{st} - 0.051\, A_{st}^2$$

$\Rightarrow$ $$0.051\, A_{st}^2 - 700\, A_{st} + 608004.43 = 0$$

$\Rightarrow$ $$A_{st} = 931.84 \text{ mm}^2$$

Practice Objective Questions

1. Torsion resisting capacity of a given RC section
   (a) decreases with decrease in stirrup spacing
   (b) decreases with increase in longitudinal bars
   (c) does not depend upon stirrup and longitudinal steels
   (d) increases with the increase in stirrup and longitudinal steels

2. A beam of rectangular cross-section ($b \times d$) is subjected to torque T. What is the maximum torsional stress induced in the beam ($b < d$ and α is a constant)?

(a) $\frac{T}{\alpha b^2 d}$

(b) $\frac{T}{\alpha b d^2}$

(c) $\frac{T}{\alpha b d}$

(d) $\frac{T}{bd}$

3. An RC structural member rectangular in cross section of which b and depth D is subjected to a combined action of bending moment M and torsional moment T. The longitudinal reinforcement shall be designed for a moment M_e given by

(a) $M_e = M + \frac{T(1 + D/b)}{1.7}$

(b) $M_e = M + \frac{T(1 - D/b)}{1.7}$

(c) $M_e = \frac{T(1 + D/b)}{1.7}$

(d) $M_e = \frac{T(1 - b/D)}{1.7}$

Common Data For Questions 4 and 5:

At the limit state of collapse, an RC beam is subjected to flexural moment 200 kN-m, shear force 20 kN and torque 9 kN-m. The beam is 300 mm wide and has a gross depth of 425 mm, with an effective cover of 25 mm. The equivalent nominal shear stress (τ_{ve}) as calculated by using the design code turns out to be lesser than the design shear strength (τ_c) of the concrete.

4. The equivalent shear force (V_e) is

(a) 20 kN

(b) 54 kN

(c) 56 kN

(d) 68 kN

5. The equivalent flexural moment (M_{eq}) for designing the longitudinal tension steel is

(a) 187 kN-m

(b) 200 kN-m

(c) 209 kN-m

(d) 213 kN-m

Answers

1.	(d)	2.	(a)	3.	(a)	4.	(d)	5.	(d)

5

Design of Beam and Slab

A beam is a horizontal structural element that is capable of withstanding load primarily by resisting bending. The bending force induced into the material of the beam as a result of the external loads, own weight, span and external reactions to these loads is called a bending moment.

I.S 456 Provision

1. Effective Span (Clause 22.2)

(a) Simply supported Beam or Slab

The effective span of a member that is not built integrally with its supports shall be taken as clear span plus the effective depth of slab or beam or centre of supports, whichever is less.

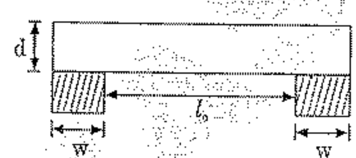


Fig. 5.1

$$l_{eff}\text{ (Effective span)} = l_0 + d$$

OR

$$l_{eff.} = l_0 + \frac{w}{2} + \frac{w}{2}$$

which ever is less

d = effective depth
w = width of support
l_0 = clear span
l_{eff} = effective span

(b) Continuous Beams or Slab

In case of continuous beam or slab, if the width of support is less than $\frac{1}{12}$ of the clear span, the effective span shall be as in (a) above. If the supports are wider than $\frac{1}{12}$ of the clear span or 600 mm, whichever is less, the effective span shall be taken as under:

Case (1): If width of support < $\frac{\text{span}}{12}\left(w<\frac{l_0}{12}\right)$

Then effective span is calculated same as for simply supported case.

$$l_{eff} = l_0 + d$$

or

$$l_{eff} = l_0 + \frac{w}{2} + \frac{w}{2}$$

which ever in less

Case (2): If width of support > $\frac{\text{span}}{12}\left(w>\frac{l_0}{12}\right)$

(a) (i) For one end fixed other continuous

(*ii*) Both end continuous (Intermediate span)

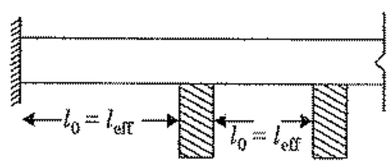


Fig. 5.2

$$l_{eff} = l_0 = \text{clear span}$$

(b) One end discontinuous other continuous (simply supported)

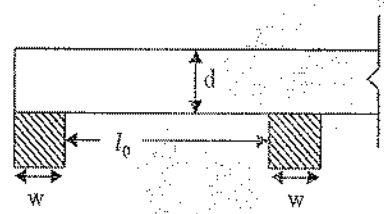


Fig. 5.3

$$\left.\begin{aligned} l_{eff} &= l_0 + d/2 \\ \text{or} \quad l_{eff} &= l_0 + \frac{w}{2} \end{aligned}\right\} \text{which ever is less}$$

Case 3: Cantilever

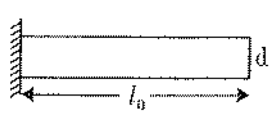
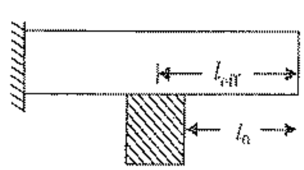


Fig. 5.4

$$l_{eff} = l_0 + d/2$$



$$l_{eff} = l_0 + \frac{w}{2}$$

Case (4): For frame it's centre to centre distence between member.

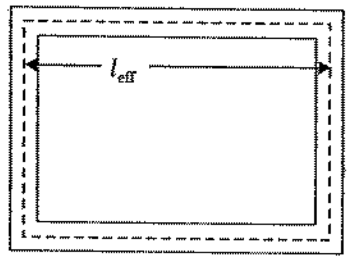


Fig. 5.6

l_{eff} = Centre to Centre distance between members.

Case 5: In the case of spans with roller or rocket bearings, tbe effective span sball always be the distance between tbe centres of bearings.

2. Check For Deflection (cl 23.2/P–37)

The deflection shall generally be limited to the following:

1. The final deflection due to all loads including the effect of temperature, creep and shrinkage and measured from at cast level of support of floor, roof and all Hz. Deflection should not normally exceed span/250.
2. The deflection including the effect of creep, temperature and shrinkage occuring after erection of portion and application of finishes.

   $\ngtr \frac{\text{span}}{350}$ or 20 mm which ever is less.

Note 1: Deflection can be reduced by providing more tensile steel than required for flexure, the service stress in steel is reduced and hence the deflection. However care must be taken so that section will not become over reinforced.

Note 2. Calculation of Deflection are required.

1. When designer wishes to exceed span depth ratio
2. Where specific deflection control is required.
3. Where the structure is abnormal due to loading or behaviour.

CONTROL OF DEFLECTION

The deflection of structure of part thereof shall not adversely affect the appearance or efficiency of the structure or finishes of partitions.

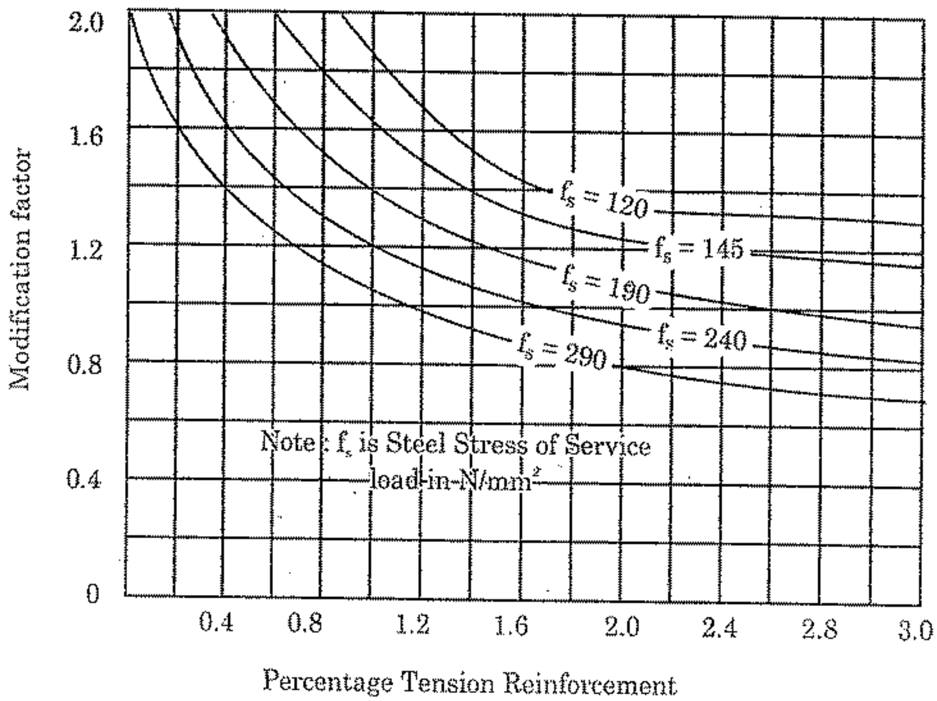
- For beam and slabs, the vertical deflection limits may generally be assumed to be satisfied, provided that the span to depth ration are not greater than the values obtained as below

  (a) Basic values of span toeffective depth ratios for spans upto 10m.

Cantilever	7
simply supported	20
Continuous	26

(b) For spans above 10m, the values in (a) may be multiplied by 10/span in meters, except for cantilever in which case deflection calculations should be made.

(c) Depending on the area and the stress of steel for tension reinforcement, the values in (a) or (b) shall be modified by multiplying with the modification factor obtained as per Fig. 5.7.



$$f_s = 0.58\, f_y \frac{\text{Area of cross - section of steel required}}{\text{Area of cross - section of steel provided}}$$

Fig. 5.7

(d) Depending on the area of compression reinforcement, the value of span to depth ratio be further modified by multiplying with the modification factor obtained as per Fig. 5.8.

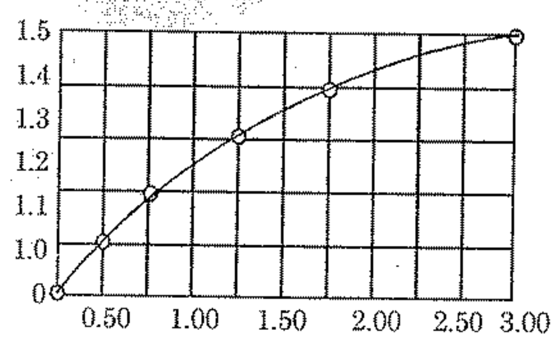


Fig. 5.8

So from (c) and (d), it is clear that

$$\left(\frac{l}{d}\right)_{max} = \left(\frac{l}{d}\right)_{basic} \times k_t \times k_c$$

where, $\left(\frac{l}{d}\right)_{basic} = 7$ for cantilever span

20 for simply supported spans

26 for continuous spans

And the modification factor k_t (which varies with p_t and f_s) and k_c (which varies with p_c) are as given in Fig. 5.8 of IS : 456.

- For flanged beams, the values of (a) or (b) be modified as per Fig. 5.9 and the reinforcement percentage for use Fig. 5.7 and 5.8 should be based on area of section equal to b_f d.

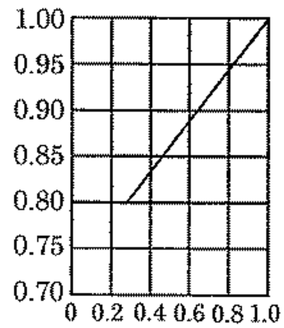


Fig. 5.9: Ratio of web width to flange width.

Note : (1) For slabs spanning in two directions, the shorter of the two spans should be used for calculating the span of effective depth ratios.

(2) For two-way slabs of shorter spans (upto 3.5m) with mild steel reinforcement, the span to overall detpth ratios given below may generally be assumed to satisfy vertical deflection limits for loading class upto 3 kN/m^2.

Simply supported slab	25
Continuous slab	40

For HYSD of grade Fe 415, the values given above should be multipled by 0.8.

- **Slenderness Limits for Beams to Ensure Lateral Stability (Clause 23.3)**

A simply supported or continuous beam shall be so proportioned that the clear distance between the lateral restraints does not exceed 60 b or $\frac{250\,b^2}{d}$, whichever is less, where d is the effective depth of the beam and b is the breadth of the compression face midway between the lateral restraints.

For a cantilever, the clear distance from the free and of the cantilever to the lateral restraint shall not exceed 25 b or $\frac{100 b^2}{d}$, whichever is less.

b → Width of the compression face midway between the lateral resitants.

Steel Reinforcement: For beams (cl. 26.51 Pg 46)

(a) Tension reinforcement

(1) Minimum tension reinforcement

$$\frac{A_{st}}{bd} = \frac{0.85}{f_y} \qquad (1)$$

Minimum tension reinforcement is provide to avoid the sudden fracture of steel due to transfer of full tension to steel on cracking of concrete when the beam is loaded.

(2) Max. Tension reinforcement = 4% of total C/s area = 0.04 BD

- The maximum tension reinforcment is provide to avoid jumbling up of reinforcement and consequent diffculting in compaction.

(b) Maximum compression reinforcement = 0.04 BD

Side Face Reinforcement

Where the depth of the web in a beam exceeds 750 mm, side face reinforcement shall be provided along the two faces.

The total area of such reinforcement shall be not less than 0.1 percent of the web area and shall be distributed equally on two faces at a spacing not exceeding 300 mm or web thickness, whichever is less.

Transverse Reinforcement in Beams for Shear and Torsion

The transverse reinforcement in beam shall be taken around the outermost tension and compression bars. In T-beams and I-beams, such reinforcement shall pass around longitudinal bars located close to the outer face of the flange.

Maximum Spacing of Shear Reinforcment

The maximum spacing of shear reinforcement measured along the axis of the member shall not exceed 0.75 d for vertical stirrups and 'd' for inclined stirrups at 45°, where d is the effective depth of the section under consideration. In no case, shall the spacing exceed 300mm.

Minimum Shear Reinforcement

Minimum shear reinforcement in the form of stirrups shall be provided such that :

$$\frac{A_{sv}}{b.s_v} \geq \frac{0.4}{0.87 f_y}$$

where, A_{sv} = Total cross-sectional area of stirrup legs effective in shear

s_v = Stirrup spacing along the length of the member

b = Breadth of the beam or breadth of the web of flanged beam, and

f_y = Characteristic strength of the stirrup reinforcement in N/mm², Which shall not be taken greater than 415 N/mm².

Where the maximum shear stress calculated is less than half the permissible value and in members of minor structural importance such as lintels, this provision need not be complied with.

Cover: Certain minimum thickness of concrete cover has to be provided over reinforcement bars in order to ensure protection of steel against corrosion which may be caused due to the ingress of environment element.

Nominal or Clear cover	
Slab	20 mm
Beam	25 mm
Column	40 mm
Foundation	50 mm

Minimum Distance Between Individual Bars

The following shall apply for spacing of bars.

(i) The horizontal distance between two parallel main reinforcing bars shall usually be not less than the greatest of the following :

(a) The diameter of the bar if the diameters are equal.

(b) The diameter of the larger bar if the diameters are unequal, and

(c) 5 mm more than the nominal maximum size of coarse aggregate.

(ii) Greater horizontal distance then the minimum specified in (i) should be provided wherever possible.

- However, when needle vibrators are used, the horizontal distance between bars of a group may be reduced to two-thirds the nominal maximum size of the coarse aggregate, provided that sufficient space is left between groups of bar to enable the vibrator to be immersed.

(iii) Where there are two or more rows of bars, the bars shall be vertically in line and the minimum vertical distance between the bars shall be 15 mm, two-thirds the nominal maximum size of aggregate or the maximum size of bars, whichever is greater.

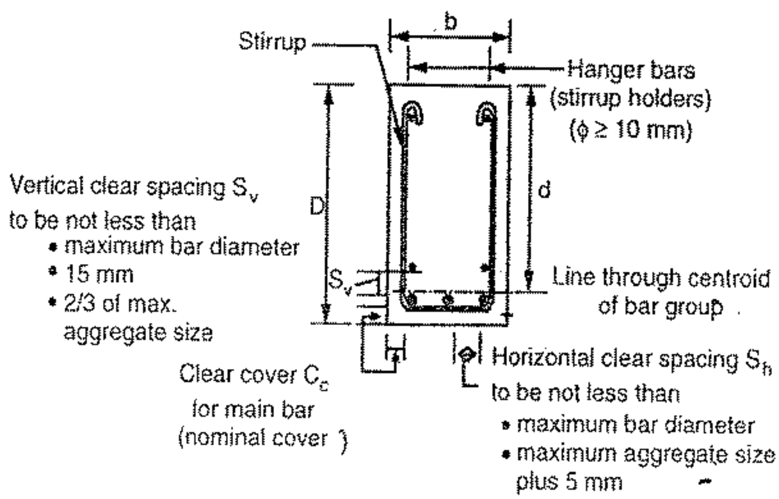


Fig. 5.10

CONTINUOUS BEAM

Moment and shear coefficients for Continuous Beams (Clause 22.5)

(a) Unless more exact estimates are made, for beams of uniform cross-section which support substantially uniformly distributed loads over three or more spans which do not differ by more than 15 percent of the longest the bending moments and shear forces used in the design may be obtained by using the coefficients given in Table 5.1 and 5.2 respectively. (It is also shown in Fig. 5.11).

For moment at supports where two unequal spans meet or in case where the spans are not equally loaded, the average of the two values for negative moment at the support may be taken for design.

(b) Where a member is built into a masonry wall which develops only partial restraint, the member shall be designed to resist a negative moment of the face of the support of $\frac{Wl}{24}$, where W is the total design load and l is the effective span, or such other restraining moment as may be shown to be applicable. For such a condition, shear coefficient given in table at the end support may be increased by 0.05.

Table 5.1

Type of load	Span moments		Support moments	
	Near middle of end span	at middle of interior span	At support next to the end support	At other interior supports
(1)	(2)	(3)	(4)	(5)
Dead load and imposed load (fixed)	$+\frac{1}{12}$	$+\frac{1}{16}$	$-\frac{1}{10}$	$-\frac{1}{12}$
Imposed load (not fixed)	$+\frac{1}{10}$	$+\frac{1}{12}$	$-\frac{1}{9}$	$-\frac{1}{9}$

Table 5.2

Type of load	At end support	At support next to the end support		At all other interior supports
		Outer side	Inner side	
(1)	(2)	(3)	(4)	(5)
Dead load and imposed load (fixed)	0.4	0.6	0.55	0.5
Imposed load (not fixed)	0.45	0.6	0.6	0.6

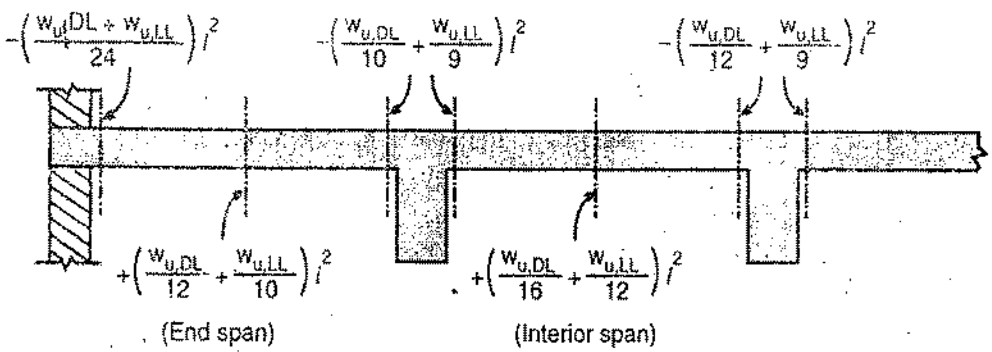


Fig. 5.11

For Interior Span

$$\text{Bending moment at support } (-\text{ve}) = \frac{w_d l^2}{12} + \frac{w_l l^2}{9}$$

$$\text{Bending moment at mid span } (+\text{ ve}) = \frac{w_d l^2}{16} + \frac{w_l l^2}{12}$$

$$\text{Shear force } V = 0.5\, w_d.l + 0.6\, w_l.l$$

For End Span

$$\text{Bending moment at support } (-\text{ve}) = \frac{w_d l^2}{10} + \frac{w_l l^2}{9}$$

$$\text{Bending moment at mid span } (+\text{ ve}) = \frac{w_d l^2}{12} + \frac{w_l l^2}{10}$$

$$\text{Shear force } V_1 = 0.4\, w_d.l + 0.45\, w_l.l$$

$$V_2 = 0.6\, w_d l + 0.6\, w_l.l$$

DESIGN TYPE OF PROBLEMS

- The designer has to make preliminary plan lay out including location of the beam, its span and spacing, estimate the imposed and other loads from the given functional requirement of the structure.
- The deed loads of the beam are estimated assuming the dimensions b and d initially. the bending moment, shear force and axial thrust are determined after estimating the different loads.
- In this illustrative problem, let us assume that the imposed and other loads are given.
- Therefore, the problem is such that the designer has to start with some initial dimensions and subsequently revise them, if needed.
- The following guidelines are helpful to assume the design parameters initially.

Selection of Breadth of the Beam b

Normally, the breadth of the beam b is governed by:

(i) proper housing of reinforcing bars and

(ii) architectural considerations.

- It is desirable that the width of the beam should be less than or equal to the width of its supporting structure like coloumn width, or width of the wall etc.
- Practical aspects should also be kept in mind. It has been found that most of the requirements are satisfied with b as 150, 200, 230, 250 and 300 mm.
- Again, width to overall depth ratio is normally kept between 0.5 and 0.67.

Selection of Depths of the Beam d and D

The effective depth has the major role to play in satisfying:

(i) the strength requirements of bending moment and shear force, and

(ii) deflection of the beam.

- The initial effective depth of the beam, however, is assumed to satisfy the deflection requirement depending on the span and type of the reinforcement. IS 456 stipulates the basic ratios of span to effective depth of beams for span up to 10m as (Clause 23.2.1)

Cantilever	7
Simply supported	20
Continuous	26

- Force spans above 10 m, the above values may be multiplied with 10/span in meters, except for cantilevers where the deflection calculations should be made.

- Further, these ratios are to be multiplied with the modification factor depending on reinforcement percentage and type.
- The total depth D can be determined by adding nominal cover and half of the diameter of the bar used to the effective depth.

Selection of Diameters of Bar of Tension Reinforcement

- Reinforcement bars are available in different diameters such as 6, 8, 10, 12, 14, 16, 18, 20, 22, 25, 28, 30, 32, 36 and 40 mm.
- Some of these bars are less available.
- The selection of the diameter of bars depends on its availability, minimum stiffness to resist while persons walk over them during construction, bond requirement etc.
- Normally, the diameters of main tensile bars are chosen from 12, 16, 20, 22, 25 and 32 mm.

Selection of Grade of Concerate

Besides strength and deflection, durability is a major factor to decide on the grade of concrete. Table of IS 456 recommends M 20 as the minimum grade under mild environmental exposure and other grades of concrete under different environmental exposures also.

Selection of Grade of Steel

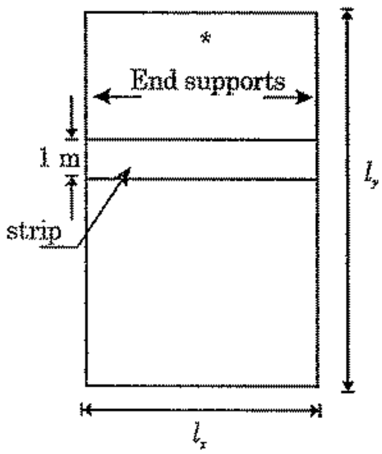
Normally, Fe 250, 415 and 500 are in used in reinforced concrete work. Mild steel (Fe 250) is more ductile and is preferred for structures in earthquake zones or where there are possiblities of vibration, impact, blast etc.

ONEWAY SLAB

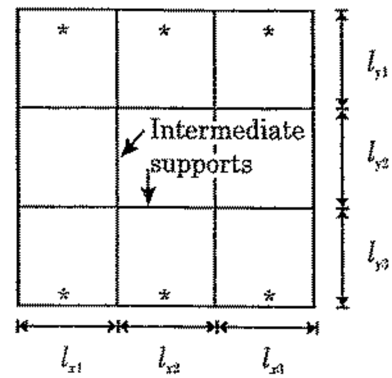
Slabs are plate elements having depth much smaller than its other two dimensions. So slab is a two dimensiional element. Slabs form roof or floor of the building. Slabs are designed same as beams with unit width.

INTRODUCTION

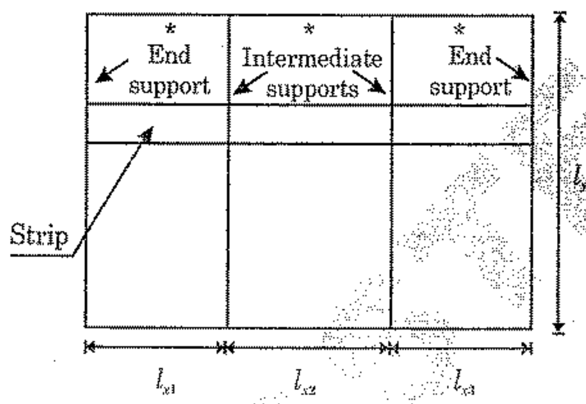
- Slabs, used in floors and roofs of buioldings mostly integrated with the supporting beams, carry the distributed loads primarily by bending.
- It has been mentioned that a part of the integrated slab is considered as flange of T - or L - beams because of monolithic construction.
- However, the remaining part of the slab needs design considerations.
- Thewse slabs are either single span or continuous having different support conditions like fixed, hinged or free along the edges. though normally these slabs are horizontal, inclined slabs are also used in ramps, stair cases and inclined roofs.
- While square or rectangular plan forms are normally used, triangular, circular and other plan forms are also needed for different functional requirements.
- This lesson takes up horizontal and rectangular/square slabs of buildings supported by beams in one or both directions and subjected to uniformly distributed vertical loadings.



(a) One Span



(b) Continuous in both directions



(c) Continuous in one direction

Fig. 5.12

Note:

1. One - way slab if $l_y > 2l_x$
2. *indicates that no support is needed if $l_y > 2l_x$ and is needed if $l_y <= 2l_x$
3. End supports may be simply supported or clamped.

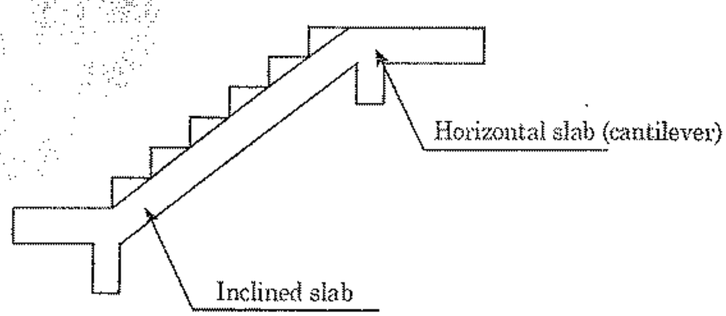
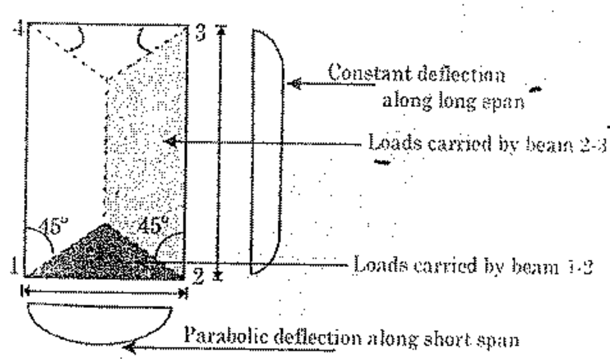


Fig. 5.13

ONE-WAY AND TWO-WAY SLABS

- Figures 5.13 (a) and (b) explain the share of loads on beams supporting solvid slabs along four edges when vertical loads are uniformly distributed.
- It is evident from the figures that the share of loads on beams in two perpendicular directions depends upon the aspect ratio l_y/l_x of the slab, l_x being the shorter span.
- For large values of l_y, the triangular area is much less than the trapezoidal area. Hence, the share of loads on beams along shorter span will gradually reduce with increasing ratio of l_y/l_x.
- In such case, it may be said that the loads are primarily taken by beams long longer span. The deflection profiles of the slab along both directions are also shown in the figure.
- The deflection profile is found to be constant along the longer span except near the edges for the slab panel of Fig. 5.13 (a).
- These slabs are designated as one-way slabs as they span one direction (shorter one) only for a large part of the slab when $l_y/l_x > 2$.
- On the other hand, for square slabs of $l_y/l_x = 1$ and rectangular slabs of l_y/l_x up to 2, the deflection profile in the two directions are parabolic figure b.
- Thus, they are spanning in two directions and these slabs with l_y/l_x up to w are designated as two-way slabls, when supported on all edges.
- It would be noted that an entirely one-way slab would need lack of support on short edges. Also, even for $l_y/l_x < 2$, absence of supports in two parallel edges will rander the slab one-way.
- In Fig. 5.13 (b), the separating line at 45° is tentative serving purpose of design. Actually, this angle is a function of l_y/l_x.



Gta

Fig. 5.13 (a): One-way slab ($l_y/l_x > 2$).

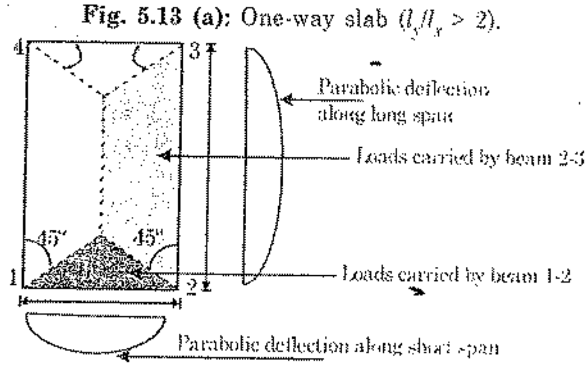


Fig. 5.13 (b): Two-way slab ($l_y/l_x <= 2$).

STRUCTURE ANALYSIS

- As explained earlier one-way slabs subjected to mostly uniformly distributed vertical loads carry them primarily by bending in the shorter direction.
- Therefore, for the design, it is important to analyse the slab to find out the bending moment (both positive and negative) depending upon the supports.
- Morever, the shear forces are also to be computed for such slabs.
- These internal bending moments and shear forces can be determined using elastic method of analysis considering the slab as beam of unit width i.e. one meter .

Design Considerations

- The primary design considerations of both one and two-way slabs are strength and deflection. The depth of the slab and areas of steel reinforcement are to be determined from these two aspects However, the following aspects are to be decided first.

Effective span (Clause 22.2 of IS 456)

The effective span of a slab depends on the boundary condition. Table gives the guidelines stipulated in Cl.22.2 of IS 456 to determine the effective span of a slab.

Table Effective span of slab (cl.22.2 of IS 456)

S.No.	Support condition	Effective span
1	Simply supported not built integrally with its supports	Lesser of (i) clear span+ effective depth of slab, and (ii) centre to centre of supports
2.	Continuous when the width of the support is < 1/12th of clear span	Do
3.	Continuous when the width of the support is > lesser of 1/12th of clear span or 600 mm (i) for end span with one end fixed and the other end continuous or for intermediate spans, (ii) for end span with one end free and the other end continuous, (iii) spans with roller or rocker bearing.	(i) Clear span between the supports (ii) Lesser of (a) clear span+ half the effective depth of slab, and (b) clear span + half the width of the discontinuous support (iii) The distance between the centres of bearings
4.	Cantilever slab at the end of a continuous slab	Length up to the centre of support
5.	Centilever span	Length up to the face of the support + half the effective depth
6.	Frames	Centre to centre distance

Effect Span to Effective Depth Ratio (cls.23.2.1a-e of IS 456)

The deflection of the slab can be kept under control if the ratios of effective span to effective depth of one-way slabs are taken up from the provisions in cl.23.2.1 a-e of IS 456. these stipulations are for the beams and are also applicable for one-way slabs as they are designed considering them as beam of unit width.

Nominal Cover (cl.26.4 of IS 456)

The nominal cover to be provided depends upon durability and fire resistance requirements. Table 16A of IS 456 provide the respective values. Appropriate value of the nominal cover is to be provided from these tables for the particular requirement of the structure. In general condition it is taken as 20 mm.

Minimum Reinforcement (cl.26.5.2.1 of IS 456)

Both for one and two-way slabs, the amount of mininum reinforcement in either direction shall not be less than 0.15 and 0.12 per cents of the total cross-sectional area for mild steel (Fe 250) and high strength deformed bars (Fe 415 and Fe 500)/welded wire bavric, respectively.

Maximum Diameter of Reinforcing Bars

The maximum diameter of reinforcing bars of one and two-way slabs shall not exceed one-eighth of the total depth of the slab.

Maximum Distance between Bars

The maximum horizontal distance between parallel main reinforcing bars shall be the lesser of (i) three times the effective depth, or (ii) 300 mm. However, the same for secondary/ distribution bars for temperature, shrinkage etc. shall be the lesser of (i) five times the effective depth, or (ii) 450 mm.

Design of One-way Slabs

The procedure of the design of one-way slab is the same as that of beams. However, the amounts of reinforcing bars are for one meter width of the slab as to be determined from either the governing design moments (positive or negative) or from the requirement of minimum reinforcement. The different steps of the design are explained below.

Step 1: Determination/checking of the effective and total depths of slabs

- The effective depth of the slab shall be determined by

  $m_{u,\lim} = 0.36\, f_{ck} x_{u,lim} b\ (d - 0.42\ x_{u,lim}) = R_{lim} bd^2$

  where the values of $R_{,\lim}$ for three different grades of concrete and three different graades of steel.
- The total depth of the slab shall then be determined adding appropriate nominal cover and half of the diameter of the larger bar if the bars are of different sizes.
- Nomally, the computed depth of the slab comes out to be much less than the assumed depth in step. However, final seslection of the depth shall be done after checking the depth for shear force.

Step 2: Depth of the Slab for Shear Force

- Theoreticlly, the depth of the slab can be checked for shear force if the design shear strength of concrete is known.
- Since this depends upon the percentage of tensile reinforcement, the design shear strength shall be assumed considering the lowest percentage of steel k from the depth tentatively selected for the slab in step.
- If necessary, the depth of the slab shall be modified.
- The above equation is applicable as the slab in most of the cases is under reinforced due to the selection of depth larger than the computed value in step 1.
- The area of steel so determined should be checked whether it is at least the minimum area of steel as mentioned in cl.26.5.2.1 of IS 456.

Step 3: Determination of Areas of Steel

Area of steel reinforcement along the direction of one-way slab should be determined by

$$M_u = 0.87 f_y A_{st} d \left\{1 - \frac{A_{st} f_y}{f_{ck} bd}\right\}$$

The above equation is applicable as the slab in most of the cases is under-reinforced due to the selection of depth larger than the computed value in Step 1. The area of steel so determined should be checked whether it is at least the minimum area of steel as mentioned in cl. 26.5.2.1 of IS 456.

Step 4: Selection of Diameters and Spacings of Reinforcing Bars

The diameter and spacing of bars are to be determined as per cls.26.3.3 of IS 456. As mentioned in this step may be avoided when using the table and charts of SP - 16.

Example 1

Design a rectangular beam simply supported over a clear span of 6 m if superimposed load is 30 kN/m and support width is 500 mm each. Use M 15 mix and HYSD steel.

Sol: For simply supported beam take $d = \frac{\text{Span}}{10}$

$$d = \frac{6000}{10} = 600 \text{ mm}$$

Take clear cover for beam is 25 mm and 20 mm dia bar:

$$D = 600 + 25 + \frac{20}{2} = 635 \text{ mm}$$

Adopt $D = 650$ mm

$$d = 650 - 25 - \frac{20}{2} = 615 \text{ mm}$$

and $b = 0.5D = 0.5 \times 650 = 325$ mm

(ii) Effective span is minimum of

(i) $6 + \frac{0.5}{2} + \frac{0.5}{2} = 6.5$ m

(ii) $6 + 0.615 = 6.615$ m

$\therefore$ Effective span = 6.5 m

(iii) **Load calculation**

$$\text{Dead load of beam} = b \times D\gamma = 0.325 \times 0.650 \times 25 = 5.28 \text{ kN/m}$$

Super imposed load = 30 kN/m

Total load = 30 + 5.28 = 35.28 kN/m

$$\text{B.M.} = \frac{wl_{eff}^2}{8} = \frac{35.28 \times 6.5^2}{8} = 186.32 \text{ kNm}$$

Factored B.M. = 1.5 × 186.32 = 279.48 kN-m

(iv) **Check for d**

Take $(\text{B.M.})_u = M_{u,lim}$

$$M_{u,lim} = 0.36\, f_{ck}\, x_{u,lim}.b\,(d - 0.42\, x_{u,lim})$$
$$279.48 \times 10^6 = 0.36 \times 15 \times 0.48\, d \times 325\,(d - 0.42 \times 0.48\, d)$$
$$279.48 \times 10^6 = 672.57\, d^2$$
$$d = \sqrt{\frac{279.48 \times 10^6}{672.57}}$$
$$d = 644.62 \text{ mm} > d_{provided}\ (615 \text{ mm})$$

Let us revised the section, take $D = 700$ mm

$$d = 700 - 25 - \frac{20}{2} = 665 \text{ mm}$$
$$b = 0.05\, D = 0.5 \times 700 = 350 \text{ mm}$$

(v)
$$\text{Dead load} = b \times D \cdot \gamma$$
$$= 0.350 \times 0.7 \times 25$$
$$= 6.125 \text{ kN/m}$$
$$\text{Super imposed load} = 30 \text{ kN/m}$$
$$\text{Total load} = 30 + 6.125 = 36.125 \text{ kN/m}$$

(vi) Effective span is minimum of

(i) $6 + \frac{0.5}{2} + \frac{0.5}{2} = 6.5$ m

(ii) $6 + 0.665 = 6.665$ m

$$l_{eff} = 6.5 \text{ m}$$
$$\text{B.M.} = \frac{wl_{eff}^2}{8} = \frac{36.125 \times 6.5^2}{8} = 190.78 \text{ kN-m}$$
$$\text{Factored B.M.} = 1.5 \times 190.78 = 286.18 \text{ kN-m}$$

(vii) **Check for d**

$$M_{u,lim} = 0.36\, f_{ck}\, x_{u,lim}.b\,(d - 0.42\, x_{u,lim})$$
$$286.18 \times 10^6 = 0.36 \times 15 \times 0.48\, d.350\,(d - 0.42 \times 0.48\, d)$$
$$286.18 \times 10^6 = 724.30\, d^2$$
$$d = \sqrt{\frac{286.18 \times 10^6}{724.30}} = 628.57 \text{ mm}$$
$$D = 628.57 + 25 + \frac{20}{2} = 663.57 \text{ mm} < \text{assumed value of } D\ (700 \text{ mm})$$

Adopt $D = 665$ mm

$$d = 665 - 25 - \frac{20}{2} = 630 \text{ mm}$$

(viii) **Area of steel**

$$M_{u,lim} = 0.87\, f_y\, A_{st}\,(d - 0.42\, x_{u,lim})$$
$$286.18 \times 10^6 = 0.87 \times 415\, A_{st}\,(630 - 0.42 \times 0.48 \times 630)$$
$$A_{st} = \frac{286.18 \times 10^6}{181605.26} = 1575.8 \text{ mm}^2$$

OR

Equate C = T

$$0.36\, f_{ck}\, x_{u,lim}\, b = 0.87\, f_y\, A_{st}$$

$$A_{st} = \frac{0.36 f_{ck} x_{u\,lim} b}{0.87 f_y}$$

$$A_{st} = \frac{0.36 \times 15 \times 0.48 \times 630 \times 350}{0.87 \times 415}$$

$$A_{st} = 1582.98 \text{ mm}^2$$

Check

$$A_{st,min} = \frac{0.85}{f_y} bd$$

$$= \frac{0.85}{415} \times 350 \times 630$$

$$= 451.62 \text{ mm}^2$$

A_{st} (1582.98 mm^2) > $A_{st,min}$ (451.62 mm^2) O.K.

Provide 4 nos. 20 mm and 2 nos. 16 mm bars giving steel area = 1659 mm^2.

$$\text{Maximum tension steel} = 0.04\, bD$$

$$= 0.04 \times 350 \times 665$$

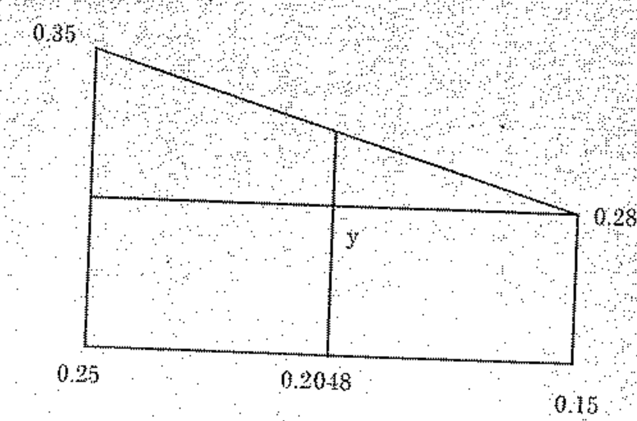
$$= 9310 \text{ mm}^2 > 1659 \text{ mm}^2$$

Check for Shear

$$\tau_v = \frac{V_u}{bd} = \frac{162.56 \times 10^3}{350 \times 630} = 0.737 = 0.74 \text{ N/mm}^2$$

$$\text{Minimum \% of steel} = \frac{0.85}{f_y} \times 100 = \frac{0.85}{415} \times 100 = 0.2048\%$$

τ_c corresponding to 0.2048% of steel is



$$\tau_c = ?$$

$$\frac{0.35 - 0.28}{0.25 - 0.15} = \frac{y - 0.28}{0.2048 - 0.15}$$

$$y = \tau_c = 0.318 \text{ N/mm}^2$$

$\tau_v > \tau_c$ so shear reinforcement is required

$$V_{us} = V_u - V_c$$
$$= 162.56 \times 10^3 - 0.318 \times 350 \times 630$$
$$= 92.44 \text{ kN}$$

Use 8 mm bar 2 legged.

$$s_v = \frac{0.87 f_y \times d \times A_{sv}}{V_{us}} = \frac{0.87 \times 415 \times 630 \times 2 \times \frac{\pi}{4} \times 8^2}{92.44 \times 10^3}$$

$$s_v = 247.37 \text{ mm} < 300 \text{ m} < 0.75\, d\ (472.5)$$

From minimum shear reinforcement criteria

$$\frac{A_{sv}}{b.s_v} \geq \frac{0.4}{f_y}$$

$$s_{v,max} = \frac{A_{sv} f_y}{b \times 0.4} = \frac{2 \times \frac{\pi}{4} \times 8^2 \times 415}{350 \times 0.4} = 298 \text{ mm}$$

247.37 mm < $s_{v,max}$ (298 mm) O.K.

Provide 2 legged 8 mm dia stirrups @ 240 mm c/c.

Check for Development Length

$$l_d \leq 1.3 \frac{M_1}{V} + l_0$$

$$l_d = \frac{0.87 f_y \phi}{4 \tau_{bd}}$$

$$= \frac{0.87 \times 415 \times 20}{4 \times 1.6 \times 1}$$

$$l_d = 1128.3 \text{ mm}$$

$$1.3 \frac{M_1}{V} + l_0$$

l_0 = Maximum of 12 ϕ or d

l_0 = Max. of 12 × 20 = 240 mm or 630 mm

l_0 = 630 mm

$$M_1 = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$\text{But} \quad x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$M_1 = 0.87 \times 415 \times 1659 \left(630 - \frac{0.87 \times 415 \times 1659}{0.36 \times 15 \times 350}\right)$$

$$M_1 = 187.52 \text{ kN-m}$$

$$V = 1.5 \times \frac{w.l.}{2} = \frac{1.5 \times 36.125 \times 6}{2} = 162.56 \text{ kN}$$

$$\therefore 1.3\frac{M_1}{V}+l_0$$

$$1.3\times\frac{187.52\times10^6}{162.56\times10^3}+630$$

$$= 1499.60 + 630$$

$$= 2129.6 \text{ mm}$$

L_d (1128.3 mm) < 2129.6 mm O.K.

Example 2

Design a simply supported roof slab for a room 8 m × 3.5 m clear in size if the superimposed load is 5 kN/m². Use M15 mix and Fe 415 grade steel.

Sol:

$$\text{for slab } d = \frac{\text{Span}}{20} = \frac{3500}{20} = 175 \text{ mm}$$

Assume clear cover for slab is 20 mm and 10 mm dia bar

$$D = 175 + 20 + \frac{10}{2}$$

$$D = 200 \text{ mm}$$

1. Effective span in x direction:

l_x is minimum of (i) $3.5 + d$ and (ii) 3.5

$$l_x = 3.5 \text{ m}$$

and l_y is 8 m

$\frac{l_y}{l_x} = \frac{8}{3.5} = 2.28 > 2$ so designed as a one way slab.

2. Load calculation (per meter run)

$$\text{Dead load} = b \times D \times \gamma$$

$$= 1 \times 0.2 \times 25 = 5 \text{ kN/m}$$

$$\text{Superimposed load} = 1 \times 5 = 5 \text{ kN/m}$$

$$\text{Total load} = 10 \text{ kN/m}$$

$$\text{B.M.} = \frac{wl_{eff}^2}{8} = \frac{10\times3.5^2}{8} = 15.31 \text{ kN-m}$$

$$\text{Factored B.M.} = 1.5 \times 15.31 = 22.96 \text{ kNm}$$

$$\text{Factored S.F.} = 1.5\frac{wl}{2} = 1.5\times\frac{10\times3.5}{2} = 26.25 \text{ kN}$$

3. Check depth of slab

$$M_{u,lim} = 0.36 f_{ck}\, x_{u,lim}\, b\,(d - 0.42\, x_{u,lim})$$

$$22.96 \times 10^6 = 0.36 \times 15 \times 0.48\, d\,(d - 0.42 \times 0.48\, d) \times 1000$$

$$d = \sqrt{\frac{22.96\times10^6}{2069.45}} = 105.33 \text{ mm}$$

$$d = 105.33 < \text{assumed value of } d = 175 \text{ mm}$$

$$D = 105.33 + 20 + \frac{10}{2} = 130.33 \text{ mm}$$

Adopt $D = 150$ mm

$$d = 150 - 20 - \frac{10}{2} = 125 \text{ mm}$$

4. Area of steel

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

from C = T

$$0.36 f_{ck} x_u b = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$M_u = 0.87 f_y A_{st} \left(d - \frac{0.42 \times 0.87 f_y A_{st}}{0.36 f_{ck} b}\right)$$

$$22.96 \times 10^6 = 0.87 \times 415 A_{st} \left(125 - \frac{0.42 \times 0.87 \times 415 A_{st}}{0.36 \times 15 \times 1000}\right)$$

$$22.96 \times 10^6 = 45131.25 A_{st} - 10.13 A_{st}^2$$

$$A_{st} = 585.75 \text{ mm}^2$$

$$A_{st,min} = \frac{0.12bd}{1000} = 0.12 \times \frac{1000 \times 150}{100} = 180 \text{ mm}^2$$

$$A_{st} > A_{stmin} \text{ O.K.}$$

$$\text{Spacing of 10 mm } \phi \text{ bar} = \frac{\frac{\pi}{4} \times 10^2}{585 \times 75} \times 1000 = 134.08 \text{ mm}$$

Spacing $\ngtr$ (i) $3d = 3 \times 125 = 375$ mm

(ii) 300 mm

∴ Provide 10 mm ϕ @ 130 mm c/c as main reinforcement.

5. Distribution steel

Is is always $A_{st,min}$ and

$$A_{st,min} = 0.12 \frac{bD}{100} = \frac{0.12 \times 1000 \times 150}{100} = 180 \text{ mm}^2$$

$$\text{Use 6 mm } \phi \text{ bar spacing} = \frac{\frac{\pi}{4} \times 6^2}{180} \times 1000 = 157.07 \text{ mm}$$

Spacing $\ngtr$ (i) 5d

(ii) $5 \times 125 = 625$ mm

(ii) 450 mm

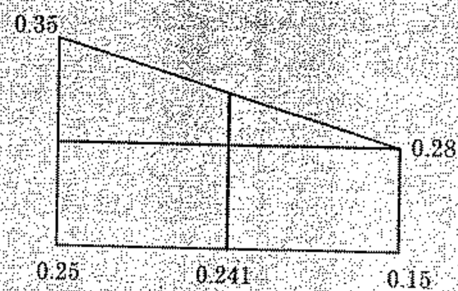
∴ Provide 6 mm dia @ 150 mm centre to centre as a distribution steel.

6. Check for shear

$$p' = \frac{A_{st}}{bd} \times 100$$

$$= \frac{100 \times \left(\frac{1000}{260} \times \frac{\pi}{4} \times 10^2\right)}{1000 \times 125} = 0.241$$

$\tau_c = ?$ for M15



$$\frac{0.35 - 0.28}{0.25 - 0.15} = \frac{y - .28}{.241 - .15}$$

$$y = \tau_c = 0.34 \text{ N/mm}^2$$

$$\tau_v = \frac{V_u}{bd} = \frac{26.25 \times 10^3}{1000 \times 125} = 0.21 \text{ N/mm}^2$$

$\tau_v < \tau_c$ O.K.

7. Check for Development length

M_1 = Moment of resistance offered by 10 mm bars @ 260 mm c/c (M_1 is calculated where bar is bent)

$M_1 = 0.87\, f_y\, A_{st}\, (d - 0.42\, x_u)$

$$M_1 = 0.87 \times 415 \times \frac{\pi}{4} \times 10^2 \times \left(\frac{1000}{260}\right)\left[125 - \frac{0.42 \times 0.87 \times 415 \times \frac{\pi}{4} \times 10^2 \times \frac{1000}{260}}{0.36 \times 15 \times 1000}\right]$$

$$M_1 = 12.707 \text{ kN-m}$$

$$V = 26.25 \text{ kN}$$

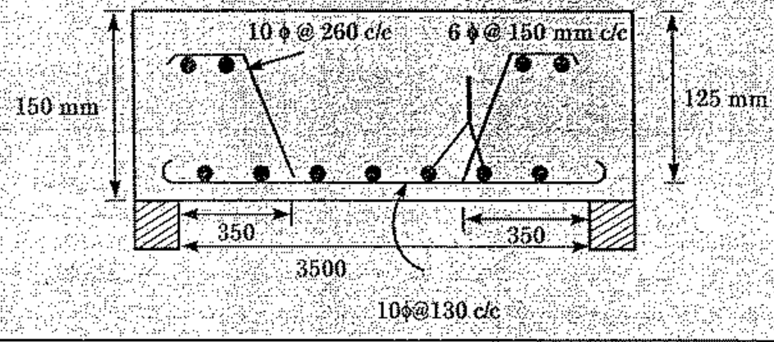
Let us assume anchorage length = 0

$$L_d \le 1.3\, \frac{M_1}{V}$$

$$\frac{0.87\, f_y\, \phi}{4 \tau_{bd}} \le 1.3 \times \frac{12.707 \times 10^6}{26.25 \times 10^3}$$

$$\frac{0.87 \times \phi \times 415}{4 \times 1.6 \times 1} \le 629.29$$

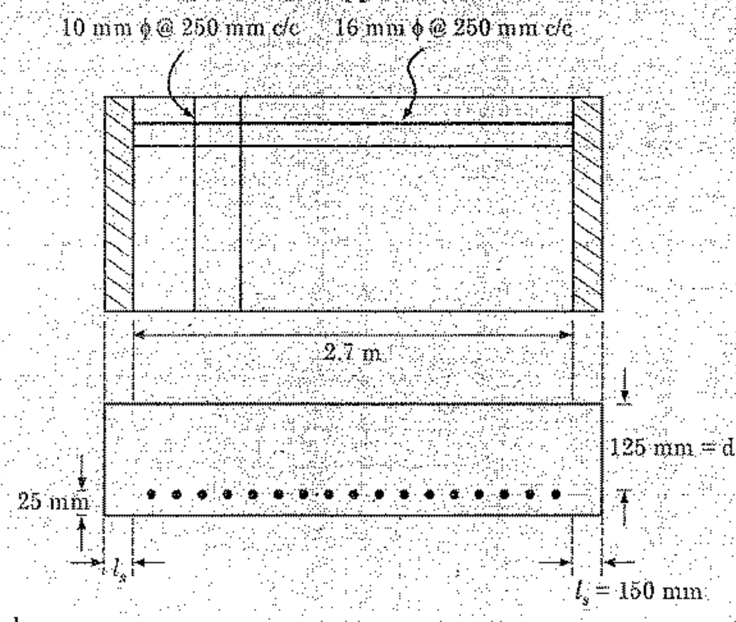
$$\phi < 11.15 \text{ mm} \quad \text{O.K.}$$



Example 3

A rectangular RC slab 2 m × 3 m is simply supported along shorter edges such that clear distance between the supporting wall is 2.7 m. The slab is 15 cm thick and reinforced with 16 mm diameter mild steel bars spaced at 25 cm c/c. concrete used is M 15 grade for which permissible stresses in bending shear (nominal) and bond 50, 3 and 6 kg/cm² respectively. Permissible tensile stresses in mild steel = 1400 kg/cm², m = 19. Calculate maximum safe intensity of load that the slab can carry in addition to its self weight.

Sol: Assuming clear cover= 25 mm, width of support = 150 mm



Effective span for slab

(i) Clear span + effective depth = $l_0 + d$

$= 2.7 + 0.125$

$= 2.825$ m

(ii) Centre to centre distance between support = $l_0 + l_s$

$= 2.7 + 0.15$

$= 2.850$ m

Lesser of (i) and (ii) is adopted.

∴ Effective span, $l_{eff} = 2.825$ m

Let the total load including self weight of slab that can be carried by slab = w kN/m².
(i) Bending

$$\text{Moment bending moment} = \frac{wl_{eff}^2}{8}$$

$$= \frac{w \times (2.825)^2}{8}$$

$$= 0.997\, w \text{ kN-m}$$

$$= 0.997 \times 10^6\, w \text{ N/mm}$$

Now, $m = 19$, $c = 50$ kg/cm² = 5 MPa, $t = 140$ kg/cm² = 140 MPa

$$x_c = \left(\frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}}\right)d = \left[\frac{19 \times 5}{140 + (19 \times 5)}\right] \times 125 = 50.53 \text{ mm}$$

$$A_{st} = \frac{1000}{250} \times \frac{\pi}{4} \times 16^2 = 804 \text{ mm}^2$$

Actual neutral axis

$$\frac{bx_a^2}{2} = mA_{st}(d - x_a)$$

$$\Rightarrow \quad \frac{1000 \times x_a^2}{2} = 19 \times 804\,(125 - x_a)$$

$$\Rightarrow \quad 500\, x_a^2 + 15276 x_a - 1909500 = 0$$

$$\Rightarrow \quad x_a = 48.38 \text{ mm}$$

$\because x_a < x_c$, section is under reinforced

Thus, $\sigma_{st} = 140$ and $\sigma_{cbc} > c_a$

$$\frac{c_a}{x_a} = \frac{\sigma_{st}/m}{d - x_a}$$

$$\Rightarrow \quad c_a = \frac{140}{9} \times \frac{48.38}{(125 - 48.38)}$$

$$\Rightarrow \quad c_a = 4.65 \text{ N/mm}^2$$

Thus equating with MR formula, we have

$$0.997 \times 10^6\, w = b.x_a \frac{c_a}{2}\left(d - \frac{x_a}{3}\right).$$

$$\Rightarrow \quad 0.997 \times 10^6 \times w = 1000 \times 48.38 \times \frac{4.65}{2}\left(125 - \frac{48.38}{3}\right)$$

$$\Rightarrow \quad w = 12.28 \text{ kN/m}^2$$

Checking for shear

$$\text{Maximum shear force} = \frac{wl}{2} = \frac{w \times 2.7}{2} = 1.35w \text{ kN}$$

$$\tau_v = \frac{V}{bd} = \frac{1.35\,w \times 10^3}{1000 \times 125} = 0.0108\,w$$

Permissible stress in shear = 3 kg/cm^2 = 0.3 MPa

$$\therefore \quad 0.0108\,w = 0.3$$

$$\Rightarrow \quad w = 27.78 \text{ kN/m}^2$$

Check for bond

$$k_c = \frac{mc}{t + mc} = \frac{19 \times 5}{140 + (19 \times 5)} = 0.404$$

$$j = 1 - \frac{k_c}{3} = 1 - \frac{0.404}{3} = 0.865$$

$$\text{Number of bars} = \frac{1000}{250} = 4 \text{ nos.}$$

Shear force, V = 1.35w kN

Σo = sum of perimeters of steel bars

$= 4 \times \pi \times 16$

$= 64\,\pi$

Now,

$$\tau_{bd} = \frac{V}{\Sigma o . jd} = \frac{1.35\,w \times 10^3}{64\pi \times 0.865 \times 125}$$

Permissible value of bond stress = 6 kg/cm^2 = 0.6 MPa

$$\therefore \quad \frac{1.35\,w \times 10^3}{64\pi \times 0.865 \times 125} = 0.6$$

$$\Rightarrow \quad w = 9.66 \text{ kN/m}^2$$

The value of total load will be the lesser of the loads calculated in bending, shear and bond.

$$\therefore \quad w = 9.66 \text{ kN/m}^2$$

Maximum safe intensity of load excluding self weight

$= 9.66 - (0.150 \times 1 \times 1 \times 25)$

$= 5.91$ kN/m^2

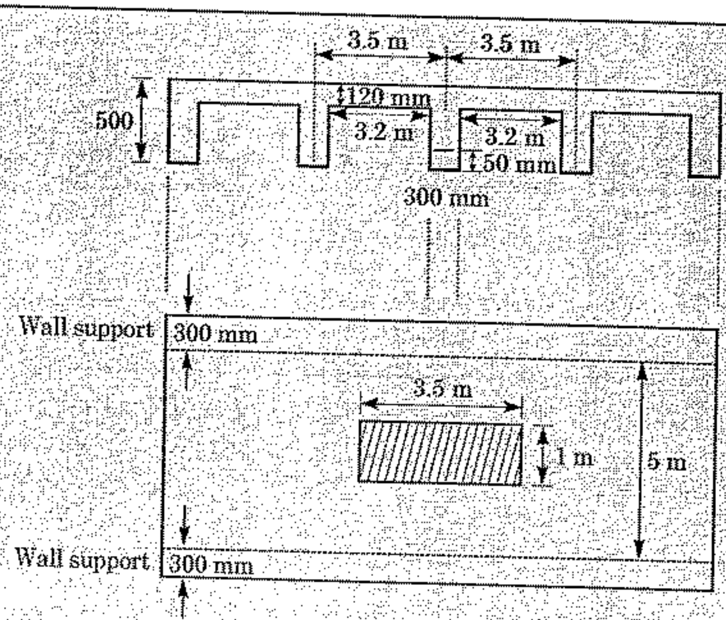
or 5.91 kN/m (per meter run)

Example 4

A RC beam of overall dimensions 300 × 500 mm rests on a brick wall 300 mm thick clear span is 5 m. The beams are spaced at 3.5 m intervals. Thickness of slab supported by the beam is 120 mm. Live load of the (on the) slab is 200 kg/m². Floor finish weighs 60 kg/m². Concrete adopted is M 20 and mild steel is used for reinforcements. Effective cover to the centre of reinforcement is 50 mm. Design the reinforcements required for flexure and shear. Draw a net longitudinal section of the beam and indicate the placement of reinforcements.

Sol:

(i) Using LSM



Live load = 200 kg/m^2 = 2 kN/m^2

Floor finishing = 60 kg/m^2 = 0.6 kN/m^2

Clear spear l_0 = 5 m

l_s = width of support = 300 mm

d_f = 120 mm

M 20/Fe 250 are used

b_w = 300 m

Effective cover = 50 cm

D = 500 mm

$d = D - 50 = 500 - 50 = 450$ mm

Step (i) Effective span

If $l_s < \frac{1}{12}$ clear span (l_0), then effective span will be

$$\left.\begin{matrix} l_0 + d \\ l_0 + l_s \end{matrix}\right\} \text{ lesser of the two}$$

Now $300 < \frac{1}{12} \times 5000$

$$\Rightarrow \quad 300 < 416.66$$

$l_{\text{eff.}}$ is mini. of (i) $5000 + 450 = 5450$

(ii) $5000 + 450 = 5300$

Hence $l_{\text{eff.}}$ = 5300 mm = 5.3 m

Step (2) Load calculations

Load for slab (load per meter run of beam) = load on slab per unit area × centre to centre distance between beams

Self wt. of slab = 3.5 × 1 × 0.12 × 25 = 10.5 kN

Live load on slab = 3.5 × 1 × 2 = 7 kN

Load for beam

Self weight of beam = $(0.5 - 0.12) \times 0.3 \times 1 \times 25 = 2.85$ kN

Floor finishing load = $0.6 \times 3.5 \times 1 = 2.1$ kN

Total load on the beam 1 m length = $10.5 + 7 + 2.85 + 2.1 = 22.45$ kN

Step (3)

$$\text{Max BM} = \frac{wl_{eff}^2}{8} = \frac{22.45 \times 5.3 \times 5.3}{8} = 78.83 \text{ kNm}$$

$$\text{Max shear force } V = \frac{wl}{2} = \frac{22.45 \times 5}{2} = 56.125 \text{ kN}$$

Step (4)

Effective width of flange (b_f)

$$b_f = \frac{l_0}{6} + b_w + 6d_f$$

$$l_0 = 0.7\, l_{\text{eff}} = 0.7 \times 5300 = 3710 \text{ mm}$$

$$b_f = \frac{3710}{6} + 300 + (6 \times 120)$$

$$b_f = 1638.33 \text{ mm}$$

Also, $b_f \not> b_w + \frac{l_1}{2} + \frac{l_2}{2} \Rightarrow 1638.33 \not> 300 + \frac{3200}{3} + \frac{3200}{3} = 3500$

$$b_f = 1638.33 \not> 3500 \text{ Hence OK.}$$

Assuming the neutral axis lies in flange

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$\text{and} \quad x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f}$$

$$x_u = 0.01587 A_{st}$$

$$M_u = 0.87 f_y A_{st} (d - 0.42 \times 0.01587 A_{st})$$

$$78.83 \times 1.5 \times 10^6 = 0.87 \times 250 A_{st} (450 - 0.42 \times 0.1587 A_{st})$$

$$= 97875 A_{st} - 1.45 A_{st}^2$$

$$A_{st} = 1230.55 \text{ mm}^2$$

$$\text{and} \quad x_u = 0.01587 \times 1230.55$$

$$= 19.52 \text{ mm} < 120 \text{ mm} \quad \text{N.A. lies in flange}$$

$$x_{u,lim} = 0.53 \times 450 = 238.5 \text{ mm}$$

$x_u < x_{u,lim}$ section is under reinforced.

∴ NA lies in the flange

$$A_{st,min} = \frac{0.85\, bd}{f_y} = \frac{0.85\, b_w d}{f_y}$$

$$= \frac{0.85 \times 300 \times 561}{250} = 572.22 \text{ mm}^2$$

A_{st} (1230.55) > $A_{st,min}$ (572.22)

Provide 4 nos. 20 mm dia bar (1256.63 mm²) > 1230.55 mm²
Maximum area of tension steel = $0.04\, b_w . D$
$= 0.04 \times 300 \times 600 = 7200$ mm²
> 1256.63 mm²

Step (5): Shear reinforcement

$$V_u = 56.125 \times 1.5 = 84.187 \text{ kN}$$

$$\tau_v = \frac{V_u}{b_w d} = \frac{84.187 \times 10^3}{300 \times 450} = 0.623 \text{ N/mm}^2$$

for τ_c, we have $\frac{A_{st} \times 100}{b_w d} = \frac{1256.63 \times 100}{300 \times 450} = 0.93\%$

$$\tau_c = 0.35 + \left(\frac{0.39 - 0.35}{1.00 - 0.75}\right)(0.93 - 0.75)$$

$$\tau_c = 0.3788 \text{ N/mm}^2$$

$$\therefore \quad V_c = \tau_c b_w d = 0.3788 \times 300 \times 450 = 51.138 \text{ kN}$$

$\because \; V_u > V_c$ hence shear reinforcement is provided

$$V_{us} = V_u - V_c = 84.187 - 51.138 = 33.049 \text{ kN}$$

Proving 2-legged 8 mm ϕ stirrups

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.53 \text{ mm}^2$$

$$\text{Spacing } s_c = \frac{A_{sv} \times d \times 0.87 \times 250}{33.049 \times 10^3}$$

$$= 297.72 \simeq 300 \text{ mm}$$

Check

$$s_v \le \frac{A_{sv} \times 0.87 f_y}{0.4 \times bw}$$

$$s_v \le \frac{100.53 \times 0.87 \times 250}{0.4 \times 300}$$

$$s_v \le 182 \text{ mm } (s_{v,\max})$$

$$\text{Max. spacing} = 0.75d = 0.75 \times 450 = 337.7 \text{ mm}$$

Hence provide 180 mm c/c.

Example 5

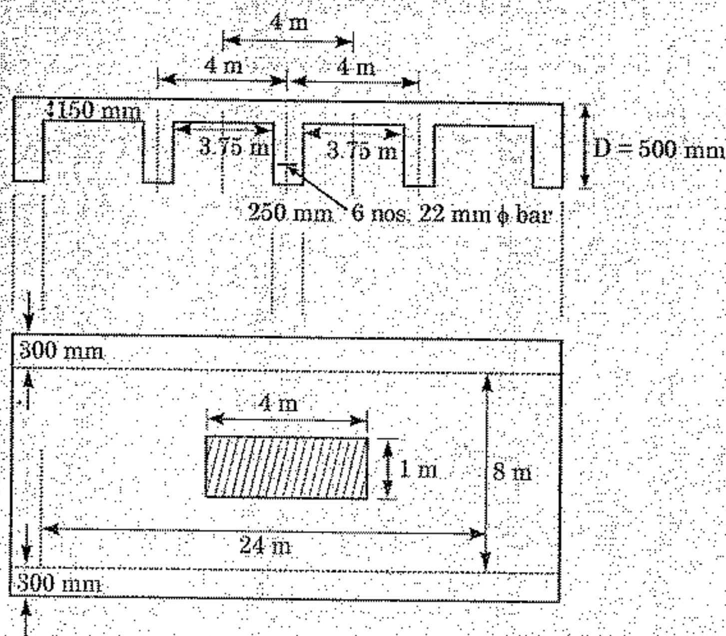
A hall of clear dimensions 8 × 24 mm is covered by a concrete slab 150 mm thick supported on T-beams spaces at 4 m intervals. The beam are resting on walls 300 mm wide. The rib width is to be resisted to 250 mm and overall depth of the T-beam is to be 500 mm. The beam is reinforced with 6 nos. of 22 mm ϕ high strength deformed bars. Live load on the roof including the weathering course is 3 kN/m². Grade of concrete used is M 15. σ_{cbc} = 5 N/mm², σ_{st} = 140 N/mm², m = 19. Check the safety of the beam.

Sol:

(i) Effective span

$$l_s < \frac{1}{12} l_{clear}$$

$$300 < \frac{1}{12} \times 8000 \Rightarrow 300 < 666.67 \text{ , Hence O.K.}$$



Take clear cover = 25 mm and 20 mm dia bar

$$A_{st} = 6 \times \frac{\pi}{4} \times 22^2 = 2280.79 \text{ mm}^2$$

Effective cover = 35 mm, $d = 500 - 35 = 465$ mm

Effective span will be $\begin{Bmatrix} 8000 + 465 \\ 8000 + 300 \end{Bmatrix}$ whichever less

Hence $l_{eff} = 8000 + 300 = 8300$ mm = 8.3 m

(ii) Load calculations

Self wt. of slab = 4 × 1 × 0.150 × 25 = 15 kN/m

LL on slab including FF = 3 × 1 × 4 = 12 kN/m

Self of beams = (0.5 – 0.150) × 0.25 × 1 × 25 = 2.1875 kN/m

Total load on the beam = 15 + 12 + 2.1875

w = 29.19 kN/m

(iii) Calculating bending moment

$$BM_{max} = \frac{w l_{eff}^2}{8} = \frac{29.19 \times 8.3^2}{8} = 251.36 \text{ kNm}$$

(iv) Calculation of effective width (b_f)

$$b_f = \frac{l_0}{6} + b_w + 6d_f$$

According to IS 456-2000

$$l_0 = 0.7\, l_{eff}$$

$= 5.81\text{ M} = 5810\text{ mm}$

Hence
$$b_f = \frac{5810}{6} + 250 + 6\times150$$
$$= 968.33 + 250 + 400$$
$$= 2118.33\text{ mm} < b_w + \frac{l_1}{2} + \frac{l_2}{2} = 250 + \frac{3750}{2} + \frac{3750}{2}$$
$$= 4000\text{ mm}$$
$$= 2118.33 < 4000.\text{ Hence OK}$$

Adopt $b_f = 2118.33$ mm

(v) Critical neutral axis of the beam

$$x_c = \left(\frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}}\right)d$$
$$= \left[\frac{19\times5}{140+(19\times5)}\right]\times465$$
$$x_c = 187.97\text{ mm}$$

(vi) Calculation of actual depth of neutral axis.

Assuming NA in the flange i.e. $x_a < d_f$

$$b_f.\frac{x_a^2}{2} = m.A_{st}\,(d - x_a)$$
$$\Rightarrow\ 2118.33\,\frac{x_a^2}{2} = 19\times2280.79\,(465 - x_a)$$
$$\Rightarrow\ 0.024\,x_a^2 + x_a - 465 = 0$$
$$\Rightarrow\ x_a = 119.91\text{ mm}$$

$\because x_a < x_c$ Hence the section is under reinforced and as $x_a < d_f$ hence the actual neutral axis lies in the flange.

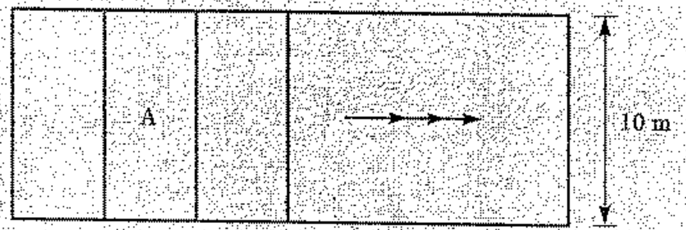
Hence

$$\text{MR} = \sigma_{st}A_{st}\left(d - \frac{x_a}{3}\right)$$
$$= 140\times2280.79\left(465 - \frac{119.91}{3}\right)$$
$$\text{MR} = 135.71\text{ kNm}$$

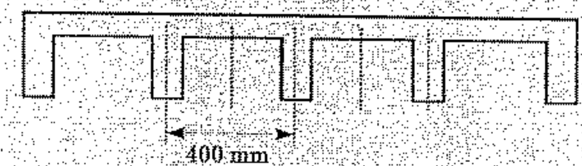
$\because$ Moment of resistance of the beam (131.26 kNm) is much less than BM_{max} (251.36 kNm). hence the given beam is not safe and it is to be redesigned.

Example 6

A hall of 10 × 40 mm is covered by a RC slab supported over T-beams spaced at 4 m intervals. Floor finish weight 50 kg/cm²; live load is 200 kg/cm²; concrete used is M 15; reinforcement used is mild steel. Calculate the maximum positive and negative bending moments in the slab for design. Design the depth of slab and the reinforcement requirement for max. negative bending moment. Sketch typical arrangement of reinforcements in the panels.



Choosing a panel from the above diagram and design all the panels accordingly.



Assuming the width of the T-beams = 300 mm

Clear span of slab = $l_{x0} = 4 - 0.300 = 3.70$ m

$l_{y0} = 10$ m

Assuming clear span as effective span = 3.7 m

$l_{y0} = 10$ m

$$\frac{l_{y0}}{l_{x0}} = \frac{10}{3.7} = 2.702 > 2.$$ Hence design a one way slabs.

(i) Effective depth $= \dfrac{\text{Span}}{26 \times MF}$

Assuming MF = 1.0

$$= \frac{3700}{26 \times 1}$$

$d = 142.30$ mm

Take clear cover = 20 mm and 10 mm ϕ bar

Effective cover $= 20 + \dfrac{10}{2} = 25$ mm

Hence total depth = 142.30 + 25

$D = 167.3$ mm

Adopt $D = 175$ mm

Effective cover = 25 mm

Effective depth $d = 175 - 25 = 150$ mm

(ii) Loads

Self wt of slab = $0.175 \times 1 \times 1 \times 25 = 4.375$ kN/m

Floor finish = $0.5 \times 1 \times 1$

Total dead load $w_d = 4.875$ kN/m

$w_{ud} = 1.5 \times 4.875 = 7.3125$ kN/m

Live load = $w_L = 2 \times 1 \times 1 = 2$ kN/m

$w_{uL} = 1.5 \times 2 = 3$ kN/m

Let us design end panel

$$BM \begin{cases} 0 \quad +\frac{1}{12} \quad -\frac{1}{10} \quad -\frac{1}{16} \quad -\frac{1}{12} \\ 0 \quad +\frac{1}{10} \quad -\frac{1}{9} \quad +\frac{1}{12} \quad -\frac{1}{9} \end{cases}$$

$$SF \quad \begin{array}{cc|cc|} 0.4 & 0.6 & 0.55 & 0.5 \\ 0.45 & 0.6 & 0.6 & 0.6 \end{array}$$

$$\text{Max. negative BM} = -\frac{1}{10}\times w_{uD}l_x^2 - \frac{1}{9}\times w_{uL}\times l_x^2$$

for end panel

$$= -\frac{1}{10}\times 7.3125\times 3.7^2 - \frac{1}{9}\times 3\times 3.7^2$$

$$= -14.57 \text{ kNm}$$

Maximum positive bending moment will be in the middle of 1st panel

$$\text{Maximum positive BM} = \frac{1}{12}\times w_{uD}l_x^2 + \frac{1}{10}w_{uL}\times l_x^2$$

at mid for end panel

$$= \frac{1}{12}\times 7.3125\times 3.7^2 + \frac{1}{10}\times 3\times 3.7^2$$

$$= 12.45 \text{ kNm}$$

$$\text{Maximum positive shear force} = 0.6\times w_{uD}l_x + 0.6\times w_{uL}\times l_x$$

$$= 0.6\times 7.3125\times 3.7 + 0.6\times 3\times 3.7$$

$$V_1 = 22.89 \text{ kN}$$

$$V_2 = 0.4\times 7.3125\times 3.7 + 0.45\times 3\times 3.7 = 15.8 \text{ kN}$$

(iv) Depth calculation

$$Q = 0.148 f_{ck}$$

$$= 0.148\times 15$$

$$= 2.22$$

$$d = \sqrt{\frac{M}{Qb}}$$

$$d = \sqrt{\frac{14.57\times 10^6}{2.22\times 1000}}$$

$$d = 81.01 \text{ mm} \le 150 \text{ mm} \quad \text{O.K.}$$

So we have to use the effective depth as calculated from deflection point of view. This thickness of slab is very less so use

$$d = 150 \text{ mm}$$

(v) Area of steel for maximum –ve B.M.

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

from $C = T$

$$0.36 f_{ck} x_u b = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} . b}$$

$$\therefore \quad M_u = 0.87 f_y A_{st}\left(d - \frac{0.42 - 0.87 f_y A_{st}}{0.36 f_{ck} . b}\right)$$

$$M_u = 0.87 \times 250 A_{st}\left(150 - \frac{0.42 \times 0.87 \times 250\, A_{st}}{0.36 \times 15 \times 1000}\right)$$

$$M_u = 32625\, A_{st} - 3.679\, A_{st}^2$$

$$14.57 \times 10^6 = 32625\, A_{st} - 3.679\, A_{st}^2$$

$$A_{st} = 471.67 \text{ mm}^2$$

$$A_{st,min} = \frac{0.15 bD}{100} = \frac{0.15 \times 1000 \times 175}{100} = 262.5 \text{ mm}^2$$

$$A_{st} > A_{st,min} \quad \text{O.K.}$$

$$\text{Spacing of 10 mm dia bar} = \frac{\frac{\pi}{4} \times 10^2 \times 1000}{471.67} = 166.51 \text{ mm}$$

This spacing $\ngtr$ (i) 3d

$$3 \times 150 = 450 \text{ mm}$$

(ii) 300 mm

So provide 10 mm bar @ 160 mm c/c

Area of steel for maximum +ve B.M.

From eq. (i)

$$M_u = 32625\, A_{st} - 3.679\, A_{st}^2$$

$$M_u = 12.45 \text{ kN-m}$$

$$12.45 \times 10^6 = 32625\, A_{st} - 3.679\, A_{st}^2$$

$$A_{st} = 399.61 \text{ mm}^2 > A_{st,min}\ 262.5 \text{ mm}^2$$

$$\text{Spacing} = \frac{\frac{\pi}{4} \times 10^2}{399.61} \times 100 = 196.54 \text{ mm}$$

This spacing $\ngtr$ (i) 450 mm

(ii) 300 mm

Provide 10 dia @ 190 mm c/c

Area of distribution steel

$$\text{Area of distribution steel} = \frac{0.15\, bD}{100}$$

$$= \frac{0.15 \times 1000 \times 175}{100}$$

$$= 262.5 \text{ mm}^2$$

$$\text{Spacing of 8 mm bar} = \frac{\frac{\pi}{4}\times 8^2}{262.5}\times 1000 = 191.48$$

Provide 8 mm ϕ @ 190 mm c/c.

Example 7

Design a interior slab of continuous slab of span 4.2 M for a residential building use M 20 grade of concrete, HYSD (FE 415) grade of steel. Live load = 2 kN/m², modification factor 1.2 flor finish = 1 kN/m².

Sol:

$$d = \frac{\text{Span}}{26\times MF} \text{ for continuous slab}$$

$$d = \frac{4200}{26\times 1.2} = 134.6 \text{ mm}$$

Use clear cover is 20 mm for slab and 10 mm dia bar.

$$D = 134.6 + 20 + \frac{10}{2} = 159.6 \text{ mm}$$

$$\text{Adopt } D = 165 \text{ mm}$$

$$d = 165 - 20 - \frac{10}{2} = 140 \text{ mm}$$

Load calculation

Dead load

(i) Self weight of slab = 0.165 × 1 × 25 = 4.125 kN/m

(ii) Floor finish = 1 × 1 = 1 kN/m

$$\text{Total dead load } w_d = 5.125 \text{ kN/m}$$

Live load

$$\text{Live load } 1\times 2 = 2 \text{ kN/m}$$

(iii) Bending moment calculation

For interior slab

$$(-\text{ve moment of support})\ M_{support} = \frac{w_d\, l_{eff}^2}{12} + \frac{w_l\, l_{eff}^2}{9}$$

$$= \frac{5.125\times 4.2^2}{12} + \frac{2\times 4.2^2}{9}$$

$$= 7.53 + 3.92$$

$$= 11.45 \text{ kN-m}$$

$$(+\text{ ve moment near mid span})\ M_{centre} = \frac{w_d\, l_{eff}^2}{16} + \frac{w_l l_{eff}^2}{12}$$

$$= \frac{5.125\times 4.2^2}{16} + \frac{2\times 4.2^2}{12}$$

$$= 5.65 + 2.94$$
$$= 8.59 \text{ kN.m}$$

$$\text{Shear force V} = 0.5\, w_d L + 0.6\, w_L l$$
$$= 0.5 \times 5.125 \times 4.2 + 0.6 \times 2 \times 4.2$$
$$= 15.80 \text{ kN}$$

Depth requirement

$$M_u = 0.36\, f_{ck}\, x_{u,lim}\, b\, (d - 0.42\, x_{u,lim})$$
$$11.45 \times 1.5 \times 10^6 = 0.36 \times 20 \times 0.48\, d \times 1000\, (d - 0.42 \times 0.48\, d)$$
$$d = \sqrt{\frac{11.45 \times 1.5 \times 10^6}{2759.27}}$$
$$= 78.89 \text{ mm} < \text{assumed depth (140 mm) O.K.}$$

Thickness of slab is very less so take $d = 140$ mm only.

Area of steel required at support

$$M_u = 0.87\, f_y\, A_{st}\, (d - 0.42\, x_u)$$
$$0.36\, f_{ck}\, x_u\, b = 0.87\, f_y\, A_{st}$$
$$x_u = \frac{0.87\, f_y\, A_{st}}{0.36\, f_{ck}\, b}$$
$$11.45 \times 1.5 \times 10^6 = 0.87\, f_y\, A_{st} \left(\frac{d - 0.42 \times 0.87 f_y A_{st}}{0.36 f_{ck}\, b} \right)$$
$$= 0.87 \times 415\, A_{st} \left(140 - \frac{0.42 \times 0.87 \times 415\, A_{st}}{0.36 \times 20 \times 1000} \right)$$
$$= 361.05\, A_{st}\, (140 - 0.02106\, A_{st})$$
$$17.175 \times 10^6 = 50547\, A_{st} - 7.60\, A_{st}^2$$
$$A_{st} = 359.18 \text{ mm}^2$$
$$A_{st,min} = \frac{0.12 bD}{100} = \frac{0.12 \times 1000 \times 165}{100} = 198 \text{ mm}^2$$

$A_{st} > A_{st,min}$ O.K.

$$\text{Use 10 mm dia bar spacing} = \frac{\frac{\pi}{4} \times 10^2}{359.18} \times 100 = 218.66 \text{ mm}$$

Spacing $\not>$ (i) 3d (3 × 140 = 420 mm)

(ii) 300 mm O.K.

Hence provide 10 mm dia bar @ 210 mm c/c

$$\text{Distribution steel} = \frac{0.12\, bD}{100} = 198 \text{ mm}^2$$

$$\text{Use 6 mm dia bar, spacing} = \frac{\frac{\pi}{4} \times 6^2}{198} \times 1000 = 142.79 \text{ m}$$

Spacing $\not>$ (i) 5d (5 × 140 = 700 mm)

(ii) 450 mm O.K.

Hence provide 6 mm dia bar @ 140 mm c/c.

Area of steel required at mid span

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$8.5 \times 1.5 \times 10^6 = 50547 A_{st} - 7.60 A_{st}^2$$

$$A_{st} = 265.51 \text{ mm}^2 > A_{st,min} (198 \text{ mm}^2) \quad \text{O.K.}$$

Use 10 mm dia bar, spacing $= \dfrac{\frac{\pi}{4} \times 10^2}{265.51} \times 1000 = 295.80$ mm

Spacing $\not>$ (i) 3d (3 × 140 = 420 mm)

(ii) 300 mm O.K.

Hence provide 10 mm dia bar @ 290 mm c/c

$$\text{Distribution steel} = \frac{0.12\, bD}{100} = \frac{0.12 \times 1000 \times 165}{100}$$

$$= 198 \text{ mm}^2$$

$$\text{Spacing} = \frac{\frac{\pi}{4} \times 6^2}{198} \times 1000 = 142.79 \text{ mm}$$

Spacing $\not>$ (i) 5d (700 mm)

(ii) 450 mm O.K.

Hence provide 6 mm dia bar @ 140 mm c/c

Check for Shear

$$V_u = 1.5 \times 15.80 \text{ kN}$$

$$V_u = 23.7 \text{ kN}$$

$$\tau_V = \frac{V_u}{bd} = \frac{23.7 \times 10^3}{1000 \times 140} = 0.1692 \text{ N/mm}^2$$

$$P_t' = \frac{A_{st}}{b.d} \times 100$$

$$= \frac{\frac{1000}{290} \times \frac{\pi}{4} \times 10^2}{1000 \times 140} \times 100 = 0.193\%$$

$$\tau_c = 0.3144 \text{ N/mm}^2$$

$$\tau_v < \tau_c \quad \text{Hence O.K.}$$

Check for Development length

$$l_d \le 1.3 \frac{M_1}{V} + l_0$$

$$l_d = \frac{0.87 f_y \phi}{4\, \tau_{bd}} = \frac{0.87 \times 415 \times 10}{4 \times 1.6 \times 1.2} = 470.116 \text{ mm}$$

$$M_1 = 0.87 f_y A_{st} (d - 0.42 x_u)$$

Let us certail 50% bar at a distance of $0.1l$ from the centre of support.

$$M_1 = 0.87 f_y A_{st}\left[d-\left(\frac{0.42\times 0.87 f_y A_{st}}{0.36 f_{cb} b}\right)\right]$$

$$= 0.87\times 415\times\left(\frac{1000}{580}\right)\times\frac{\pi}{4}\times 10^2\left[140-\left(\frac{0.42\times 0.87\times 415\times\frac{1000}{580}\times\frac{\pi}{4}\times 10^2}{0.36\times 20\times 1000}\right)\right]$$

$$M_1 = 6.705 \text{ kN-m}$$

$$V = 23.7 \text{ kN}$$

$$1.3\ \frac{M_1}{V}+l_0$$

l_0 is greater of d or $12\ \phi$

$$l_0\ (140 \text{ or } 12\times 10 = 120 \text{ mm})$$

$$l_0 = 140 \text{ mm}$$

$$\frac{1.3\times 6.705\times 10^6}{23.7\times 10^3}+140$$

$$= 507.78 \text{ mm}$$

l_d (470.116 mm) < 507.78 so safe in development length.

Example 8

Design the RC floor slab for a room of internal dimensions of 4.0 m × 9.5 m. Assume the slab to be simply supported on 230 mm thick masonry walls. the slab is to support live loads of 4.0 kN/m² and surface finish 1 kN/m². Use M 20 grade concrete. HYSD steel of Fe 415 grade. Draw reinforcement details.

Sol:

1. As per the vertical deflection criterion, the span to effective depth ratio for spans upto 10 m for a ss slab is given by

$$\frac{l}{d} = 20$$

$$\Rightarrow\ d = \frac{l}{20} = \frac{4000}{20} = 200 \text{ mm}$$

2. Effective span in x direction

l_{x_0}

(i) clear span + effective depth = 4.00 + 0.2 = 4.2 m

(ii) centre to centre distance between supports = 4.0 + 0.23 = 4.23 m

Hence lesser of the above two will be adopted i.e. $l_e = 4.2 \text{ m} = l_{x_0}$.

l_{y_0} (i) 9.5 + 0.2

(ii) 9.5 + 0.23

$$\frac{l_{y_0}}{l_{x_0}} = \frac{9.7}{4.2} = 2.30 > 2 \text{ Design as a one way slab.}$$

Assuming a nominal cover of 20 mm and 10 mm bar

$$D = 200 + 20 + \frac{10}{2} = 225 \text{ m}$$

3. Bending moment and shear force

(a) Load calculation

$$\text{Load due to self weight of slab} = 0.225 \times 1 \times 25 = 5.625 \text{ kN/m}$$

$$\text{Superimposed load} = 4 \times 1 = 4 \text{ kN/m}$$

$$\text{Surface finishes} = 1 \times 1 = 1 \text{ kN/m}$$

$$\text{Total load} = 10.625 \text{ kN/m}$$

(b) $\text{B.M.} = \dfrac{wl_{eff}^2}{8} = 10.625 \times \dfrac{4.2^2}{8} = 23.428 \text{ kN-m}$

(c) Shear force $V = \dfrac{wl}{2} = \dfrac{10.625 \times 4}{2} = 21.25 \text{ kN}$

$$M_u = 0.36 f_{ck}\, x_{u,lim}\, b(d - 0.42\, x_{u,lim})$$

$$23.428 \times 1.5 \times 10^6 = 0.36 \times 20 \times 0.48\, d \times 1000\, (d - 0.42 \times 0.48 d)$$

$$= 3456d\, (d - 0.42 \times 0.48d)$$

$$M_u = 3456d \times 0.7884d$$

$$23.428 \times 1.5 \times 10^6 = 2759.27d^2$$

$$d = \sqrt{12735.97}$$

$$d = 112.85 \text{ mm} < \text{assumed depth (200 mm)}$$

$$D = 112.85 + 20 + \frac{10}{2} = 137.85 \text{ mm}$$

Adopt $D = 150$ mm and

$$d = 150 - 20 - \frac{10}{2} = 125 \text{ mm}$$

5. Area of reinforcement

$$M_u = 0.87 f_y A_{st} (d - 0.42\, x_u)$$

$$0.36 f_{ck} x_u b = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} \text{ put this in above equation}$$

$$M_u = 0.87 f_y A_{st}\left(d - \frac{0.42 \times 0.87 f_y A_{st}}{0.36 f_{ck} b}\right)$$

$$= 0.87 \times 415\, A_{st}\left(125 - \frac{0.42 \times 0.87 \times f_y A_{st}}{0.36 \times 20 \times 1000}\right)$$

$$= 361.05\, A_{st}\, (125 - 0.021\, A_{st})$$

$$23.428 \times 1.5 \times 10^6 = 45131.25\, A_{st} - 7.60\, A_{st}^2$$

$$A_{st} = 921.73 \text{ mm}^2$$

$$A_{st,min} = 0.12\frac{bd}{100} = 0.12\times\frac{1000\times150}{100} = 180 \text{ mm}^2$$

$$A_{st} > A_{st,min} \quad \text{O.K.}$$

Use 10 mm dia bar with a spacing $= \dfrac{\frac{\pi}{4}\times10^2}{921.73}\times1000 = 85.20$ mm

This spacing $\ngtr$ (i) $3d = 3 \times 125 = 375$ mm

(ii) 300 mm O.K.

Hence provide 10 mm dia @ 80 mm c/c.

$$\text{Distribution steel} = 0.12\frac{bD}{100} = 0.12\times\frac{1000\times150}{100} = 180 \text{ mm}^2$$

Use 6 mm dia bar as a distribution steel

$$\text{Spacing} = \frac{\frac{\pi}{4}\times6^2}{180}\times1000 = 157 \text{ mm}$$

Spacing $\ngtr$ (i) $5\,d = 625$ mm

(ii) 450

Provide 6 mm dia bar @ 150 mm c/c

Check for shear

$$\tau_v = \frac{V_u}{bd}$$

$$V_u = \frac{wl}{2} = \frac{10.625\times4}{2} = 21.25 \text{ kN (for shear force take clear span)}$$

$$V_u = 1.5 \times 21.25 \times 10^3 = 31.87 \text{ kN}$$

$$\tau_v = \frac{V_u}{bd} = \frac{31.87\times10^3}{1000\times125} = 0.254 \text{ N/mm}^2$$

$$P_t = \frac{A_{st}}{bd}\times100 = \frac{\left(\frac{1000}{160}\right)\times\frac{\pi}{4}\times10^2}{1000\times125}\times100 = 0.392\%$$

(50% bar bent at 0.1 l distance from support)

$\tau_c = ?$

$$\frac{0.56-0.48}{0.50-0.25} = \frac{y-0.48}{0.392-0.25}$$

$$\tau_c = y = 0.525 \text{ N/mm}^2$$

τ_v (0.254 N/mm²) < τ_c (0.525 N/mm²)

So safe in shear.

Check for development length

$$l_t \le 1.3\frac{M_1}{V} + l_0$$

$$l_d = \frac{0.87 f_y \phi}{4\tau_{bd}} = \frac{0.87\times415\times10}{4\times1.6\times1.2} = 470.117 \text{ mm}$$

$$M_1 = 0.87\, f_y A_{st}\,(d - 0.42\, x_u)$$

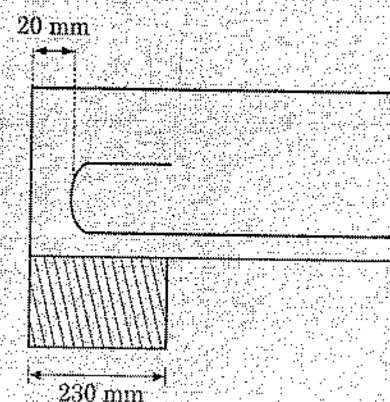
$$M_1 = 0.87\, f_y A_{st}\left(d - \frac{0.42\times0.87 f_y A_{st}}{0.36 f_{ck}.b}\right)$$

$$= 0.87\times415\times\frac{(1000)}{160}\times\frac{\pi\times10^2}{4}\left(125 - \frac{0.42\times0.87\times415\times\left(\frac{1000}{160}\right)\times\frac{\pi}{4}\times10^2}{0.36\times20\times1000}\right)$$

$$M_1 = 177230.00 \times 114.66$$

$M_1 = 20.32$ kN-m [50% bar bent at 0.1 l distance from the support]

$V = 21.25 \times 1.5 = 31.87$ kN



Assuming a clear cover of 20 mm is provided at the side (end) and providing a U-hook given width of support = 230 mm = l_s

$$\therefore\ l_0 = \frac{l_s}{2} - x' + 13\,\phi$$

$$= \frac{230}{2} - 20 + 13\times10$$

$$l_0 = 225 \text{ mm}$$

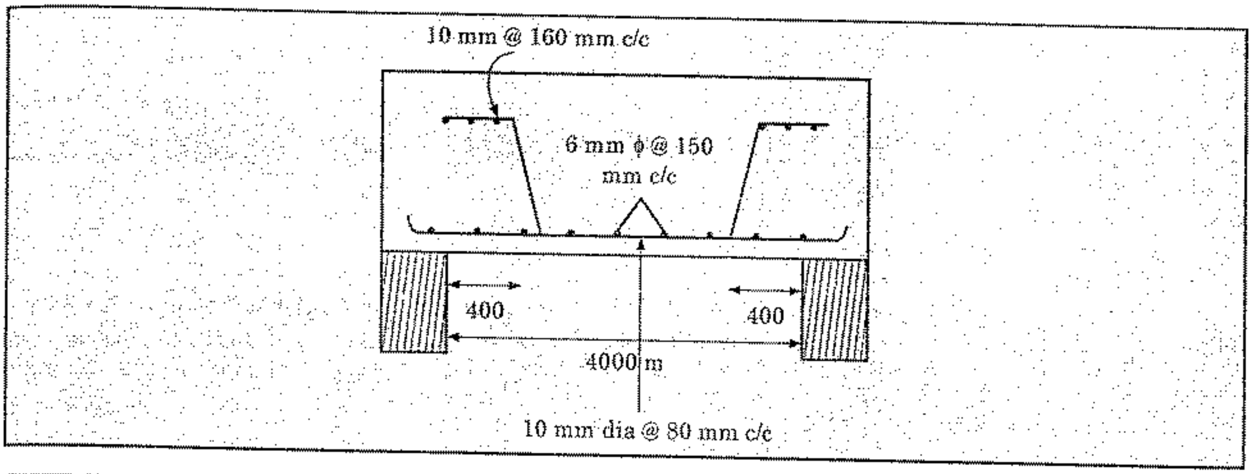
$$1.3\frac{M_1}{V} + l_0$$

$$= 1.3\times\frac{20.32\times10^6}{31.87\times10^3} + 225$$

$$l_0 = 1053.86 \text{ mm}$$

l_d (470.11 mm) < 1053.86 mm

Safe in development length.



TWO WAY SLAB

- When the slab is supported on all the four edges, and when the ratio of long span to short span is small (say less than 2), bending takes place along both the spans.
- Such a slab is known as a two-way slab or a slab spanning in two directions. The maximum bending moment and deflection for such a slab is much smaller than that of a one-way slab and hencve a thinner slab a slab is loaded, the corners get lifted up.
- If the corners get lifted up the corners are held down, by fixidity at the wall support etc., the bending moment and deflection are further reduced, thus requiring still thinner slab. In that case, special torsional reinforcement at the corner has to be provided to check the cracking of corners. We will divide the two-way slabs in the following three heads :

1. Slabs simply supported on the four edges, with corners not held down, and carrying uniformly distributed load.
2. Slabs simply supported on the four edges, with corners held down and carrying uniformly distributed load.
3. Slabs with edges fixed or continuous and carrying uniformly distributed load.

Slab Thickness Based on Deflection Control Criterion

The initial proportioning of the slab thickness may be done by adopting the same guidelines regardingspan/effective depth ratios, as applicable in the case of one-way slabs. The effective span in the short span direction should be considered for this purpose. However, the percentage tension reinforcement requirement in the short span direction for a two-way slab is likely to be less than that required for a one-way slab with slabs. A value of $k_t \approx 1.5$ may be considered for preliminary design.

For the special case of two-way slabs with spans up to 3.5 m and live loads not exceeding 3.0 kN/m^2, the code(Cl. 24.1, Note 2) permits the slab thickness (overall depth D) to be calculated directly as follows, without the need for subsequent checks on deflection control:

(i) using mild steel (Fe 250 grade),

$$D \geq \begin{cases} l_x / 35 & \text{for simply supported slabs} \\ l_x / 40 & \text{for continuous slabs} \end{cases}$$

(ii) using Fe 415 grade steel.

$$D \geq \begin{cases} l_x/28 & \text{for simply supported slabs} \\ l_x/32 & \text{for continuous slabs} \end{cases}$$

Slab Simply Supported on the four edges, with corners not held down and carrying U.D.L.

Such slabs are commonly used in single storey buildings, where the supporting walls do not cause any fixidity. Grashoff Rankine

The moment coefficients prescribed in the Code (Cl. D–2) to estimate the maximum moments (per unit width) in the short span and long span directions are based on the Rankine-Grashoff theory. According to this theory, the slab can be divided into a series of orthogonal crossing unit (beam) strips, and the load can be apportioned to the short span and long span strips such that the deflections δ of the two strips along the two centre-lines is the same at their intersection [Fig. 11.4.]

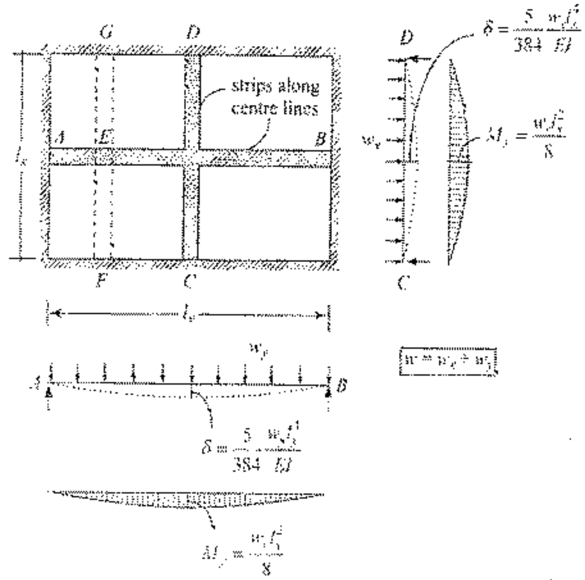


Fig. 5.14: Concept underlying rankine-grashoff theory.

If the effect of secondary (compatibility) torsion arising due to the interaction between the interconnecting strips and the influence of adjoining strips of either side are ignored, each of the two strips along the centrelines can be isolated and considered to act as simply supported beams, subjected to uniformly distrubuted loads w_x (on the short span strip) and w_y (on the long span strip). Hence, the mid-point deflection d is easily obtained as

$$\delta = \frac{5}{384}\frac{w_x l_x^4}{EI} = \frac{5}{384}\frac{w_y l_y^4}{EI}$$

where it is assumed that the second moment of area, I, is the same for both strips. A simple relation between w_x and w_y is obtainable.

$$w_x = w_y \left(\frac{l_y}{l_x} \right)^4$$

Also, in order to satisfy force equilibrikum,

$$w_x = w_y = w$$

$$\left.\begin{aligned} w_x &= w\left[\frac{r^4}{1+r^4}\right] \\ w_y &= w\left[\frac{1}{1+r^4}\right] \end{aligned}\right\} \Rightarrow$$

where $r = l_y / l_x$

The maximum short span moment M_x (per unit width) and maximum long span moment M_y (per unit width) are easily obtained as

$$\left.\begin{aligned} M_x &= w_x l_x^2 / 8 \\ M_y &= w_y l_y^2 / 8 \end{aligned}\right\}$$

The following expressions (in the formet presented in the Code) are obtained :

$$\left.\begin{aligned} M_x &= \alpha_x \; w l_x^2 \\ M_y &= \alpha_y \; w l_x^2 \end{aligned}\right\}$$

where the 'moment coefficient' α_x and α_y are given by

$$\left.\begin{aligned} \alpha_x &= \frac{1}{8}\left[\frac{r^4}{1+r^4}\right] \\ \alpha_y &= \frac{1}{8}\left[\frac{r^2}{1+r^4}\right] \end{aligned}\right\}$$

It is important to note that both M_x and M_y are given in terms of the short span l_x.

Slab Simply Supported on the Four Edges with Corners Held Down and Carrying U.D.L.

Such a case arises when the slab is built into brick walls or when they are case monolithically into thin beam, provided the slab is not continuous over its edges. The exact analysis of such a slab, based on theory of elasticity, is difficult and complicated. We shall discuss in brief the following three methods of analysis of such slabs:

1. Pigeaud's method 2. Marcus's method. 3. IS Code method

1. Pigeaud's Method

When corners are held down, the maximum bending moment and maximum deflection are further reduced. However, holding down of corners super-imposes a twisting moment in the slab, for which special reinforcement is needed at top and bottom faces of the slab at each corner. The behaviour of such a slab is similar to that of a uniformly loaded thin elastic plate, and has been analysed by pigeaud. According to this method, the bending moments M_x and M_y along the short and long spans, per unit width are given by the following expressions.

$$M_x = r_x' \frac{wL_x^2}{8} = \frac{w_x L_x^2}{8} \qquad \text{where } w_x = r_x' w$$

$$M_y = r_y' \frac{wL_x^2}{8} = \frac{w_y L_x^2}{8} \qquad \text{where } w_y = r_y' w$$

The coefficients r_x' and r_y' depend on l_y/l_x.

2. Marcus's Method

Dr. Marcus evolved a simplified but approximate method by which maximum bending moment in a slab with corners held down can be determined. According to this method, the bending moments M_x and M_y calculated by Rankine-Grashof method are multiplied by a reduction factor C, the value of which is always less than unity since the positive bending moments in the slab are reduced due to corner restraint. The value of C depends upon L_y/L_x ratio, and is given by

$$C = 1-\frac{5}{6}\frac{r^2}{1+r^4}$$

The value of C for various values of l_y/l_x ratio are tabulated in Table 5.3.

Table 5.3: Value of factor C.

$r = L_y/L_x$	C	$r = L_y/L_x$	C
1	0.861	1.4	0.888
1.05	0.862	1.5	0.898
1.1	0.864	1.6	0.907
1.15	0.866	1.745	0.919
1.2	0.871	2.0	0.935
1.25	0.874	2.5	0.957
1.3	0.879	3.0	0.970

Thus, the midspan bending moment per unit width are given by

$$M_x = \left(1-\frac{5}{6}\frac{r^2}{1+r^4}\right)\frac{r^4}{1+r^4}\frac{wl_x^2}{8}$$

or $$M_x = C\frac{r^4}{1+r^4}\frac{wl_x^2}{8} = C\,r_x\frac{wl_x^2}{8}$$

$$M_y = \left(1-\frac{5}{6}\frac{r^2}{1+r^4}\right)\frac{1}{1+r^4}\frac{w.l_y^2}{8}$$

or $$M_y = C\frac{1}{1+r^4}\frac{wl_x^2}{8} = C.r_y.\frac{wl_y^2}{8}$$

The resulting bending moments calculated by Marcus's method and by the exact theory, with Poisson's ratio equal to zero, are almost identical. If, however Poisson's ratio is taken equal to 0.15, the resulting bending moments are given by the following expressions:

$$M_x = Cr_x\left(1+\frac{0.15}{r^2}\right)\frac{wl_x^2}{8}$$

and $$M_y = Cr_y\left(0.15+\frac{1}{r^2}\right)\frac{wl_y^2}{8}$$

Shear Force: The shear force, per unit width, along the long and short span is calculated by

$$F_x = \frac{1}{3}wL_x$$

$$F_y = wL_x\frac{r}{2+r} \quad \text{(subject to maximum of } 0.5\,wL_x)$$

Uniformly Loaded 'Restrained' Rectangular Slabs

The code (Cl. D–1) uses the term restrained slabs to refer to slabs whose concreters are prevented from lifting and contain suitable reinforcement to resist torsion. All the four edges of the rectangular 'restrained' slab are assumed to be suported rigidly against vertical translation, and the edges may be either continuous/fixed or discontinuous (simply supported). Accordingly, nine different configurations of restrained rectangular slab panels are possible on the number of discontinuous edges also depending on whether the discontinuous edge is short or long.

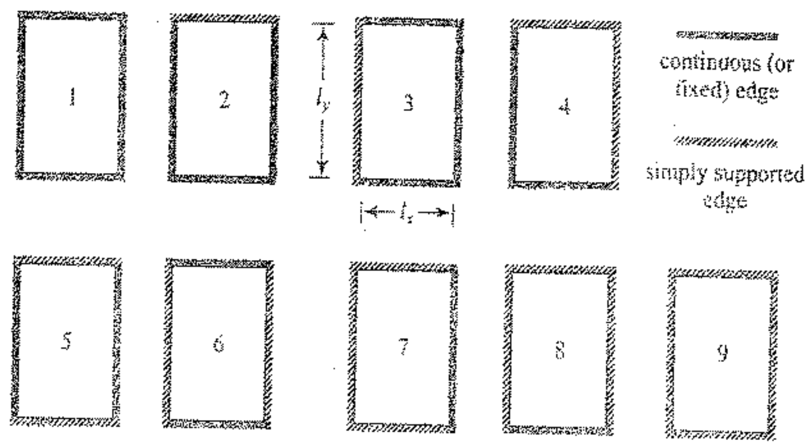


Fig. 5.15: Nine different types of 'restrained' rectangular slab panels.

Panel type '1' corresponds to the slab with all four edges continuous/fixed, and panel type '9' corresponds to the slab with all four edges simply supported incidentally, there willbe serveral more cases if combinations involving free (unsupported) edges are also considered.

The torsional restraint at the corner calls for the provision of special corner reinforcement. Approximate solution based on the Rankine-Grashoff theory are also available. Modifications to these solutions were proposed by Marcus ('Marcus correction'), whereby the moment coefficients are reduced to account for the effects of torsional restraint at the corners, as will as torsional resistance of the transverse and longitudinal unit strips

However, the moment coefficients recommended in the Code (Cl. D–1) are based on inelastic analysis (yield line analysis, rather than elastic theory.

This analysis is based on the following assumptions:

- the bottom steel in either direction is uniformly distributed over the 'middle strip' which spreads over 75 per cent of the span;
- the 'edge strip' lies on either side of the middle strip, and has a width equal to $l_x/8$ or $l_y/8$ [Fig. 5.16];
- top steel is provided in the edge strip adjoining a continuous edge (and at right angles to the edge) such that the corresponding flexural strength ultimate 'negative' moment capacity) is 4/3 times the coresponding ultimate 'positive' moment capacity due to the bottom steel provided in the middle strip in the direction under consideration;

The resulting moment coefficients α_x^+, α_y^+ for 'positive' moments at midspans in the short span and long span directions respectively, and the coefficients α_x^-, α_y^- for 'negative' moments at the continuous edge(s) in the two directions, for the nine different sets of boundary conditions [Fig. 5.15] are listed in Table 26 of the Code. The design factored moment is obtained as

$$M_u = \alpha w_u l_x^2$$

where w_u is the uniformly distributed factored load and l_x the effective short span and α is the appropriate moment coefficient.

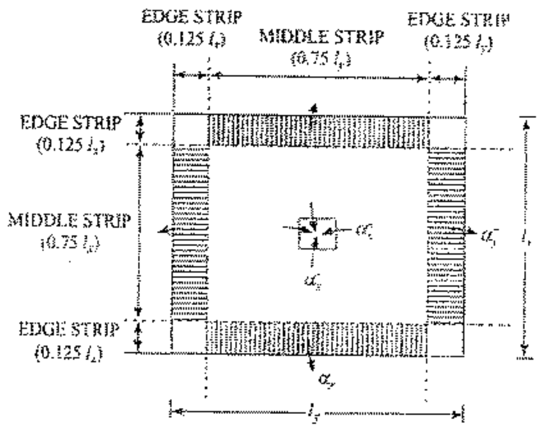


Fig. 5.16: Basis for code moment coefficients for 'restrained' two-way slabs.

Detailing of Flexural Reinforcement

The bottom steel for the design moments (per unit width) $M_{ux}^{+} = \alpha_x^{+}\ w_u l_x^2$ and $M_{uy}^{+} = \alpha_y^{+}\ w_u l_x^2$ should be uniformly distrubuted across the 'middle strips' in the short span and long span directions respectively. the Code (Cl. D–1.4) recommends that these bars should extend to within $0.25l$ of a continuous edge or $0.15l$ of a discontinuous edge. It is recommended that alternate bars should extend fully into the support, as shown in Fig. 5.17.

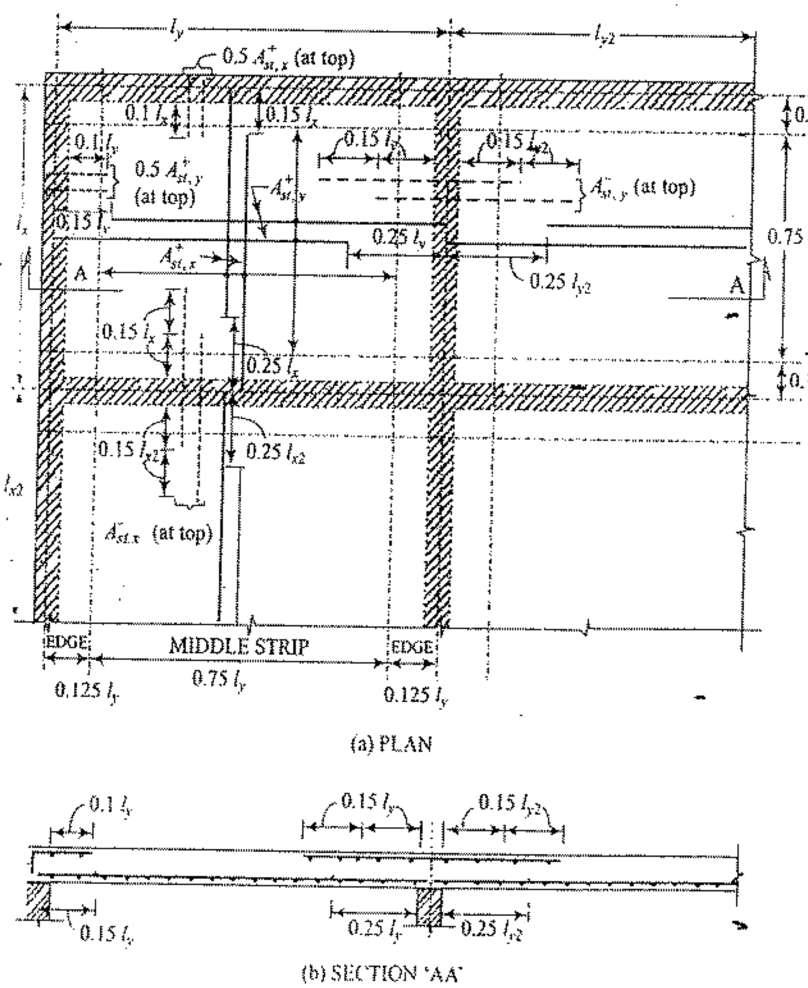


Fig. 5.17: Detailing of flexural reinforcement in two-way 'restrained' rectangular slabs (excluding corner reinforcement).

- The top steel calculated for the design moments $M_{ux}^{-} = \alpha_x^{-} dw_u l_x^2$ and $M_{uy}^{-} = \alpha_y^{-} w_u l_x^2$ at continuous supports should be uniformly across the edge strips in the long span and short span of these bars should extend to a distance of $0.3l$ from the face of the continuous support, on either side. The remaining bars may be curtailed at a distance of $0.15l$ from the face of the continuous support as shown in Fig. 5.17.
- To safeguard against possible 'negative' moments at a discontinuous edge due to partial fixity, the code (Cl. D –1.6) recommends that top steel with area equal to 50 percent of that of the bottom steel at mid-span (in the same direction) should be provided, extending over a lingth of $0.1l$, as shown in Fig. 5.17.
- In the edge strip, distribution bars parallel to that edge should be provided — at top and bottom — to tie up with the main bars as shown in Fig. 5.17.

Detailing of Torsional Reinforcement at Corners

Torsional reinforcement is required at the corners of rectangular slab panels whose edges are discontinuous. This can conveniently be provided in the form of a mesh (or grid pattern) at top and bottom. The bars can be made U-shaped (wherever convenient) and provided in the two orthogonal directions as shown in Fig. 5.18. The Code (Cl. D–1.8) recommends that the mesh should extend beyond the edge Over a distance not less than one-fifth of the shorter span (l_x). The total area of steel to be provided in *each* of the four layers should be not less than

- $0.75\, A_{st,x}^{+}$ if both edges meeting at the corner are discontinous; or
- $0.375\, A_{st,x}^{+}$ if one edge is continuous and the other discontinuous.

Here, $A_{st,x}^{+}$ is the area of steel required for the maximum midspan moment in the slab.

It may be noted that if both edges meeting at a corner are continuous, torsional reinforcement is not called for at the corner [refer Cl. D–1.10]. This is indicated in Fig. 5.18. [However, this area will have some reinforcement provided anyway, because of the 'negative' moment reinforcements over supports in the middle strips and the distributor reinforcements in the edge strips.]

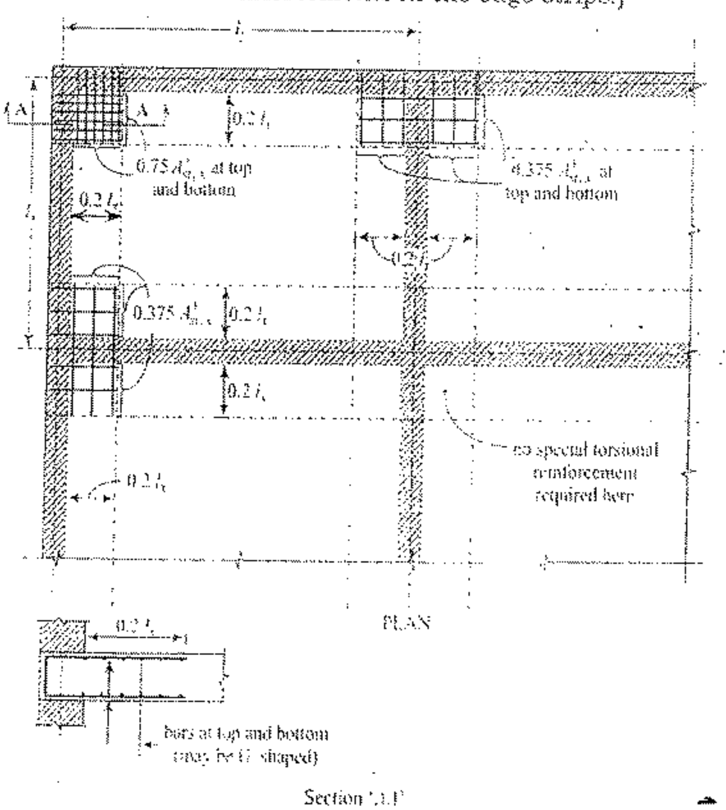


Fig. 5.18: Detailing of torsional reinforcement at corners.

Objective Practice Questions

1. If the nominal shear stress (τ_v) at a section does not exceed the permissible shear stress (τ_c)
   (a) minimum shear reinforcement is still provided
   (b) shear reinforcement is provided to resist the nominal shear stress
   (c) no shear reinforcement is provided
   (d) shear reinforcement is provided for the difference of the two

2. In limit state design, permissible bond stress in the case of deformed bars is more than that in plain bars by
   (a) 60% (b) 50%
   (c) 40% (d) 25%

3. Limit state of serviceability for deflection including the effects due to creep, shrinkage and temperature occurring after erection of partitions and application of finishes as applicable to floors and roofs is restricted to
   (a) $\frac{\text{Span}}{150}$ (b) $\frac{\text{Span}}{200}$
   (c) $\frac{\text{Span}}{250}$ (d) $\frac{\text{Span}}{350}$

4. Shrinkage deflection in case of rectangular beams and slabs can be eliminated by putting
   (a) compression steel equal to tensile steel
   (b) compression steel more than tensile steel
   (c) compression steel less than tensile steel
   (d) compression steel 25% greater than tensile steel

5. From limiting deflection point of view, use of high strength steel in RC beam results in
   (a) reduction in depth (b) no change in depth
   (c) increase in depth (d) increase in width

6. In case of a composite construction, the effect of creep and shrinkage
   (a) can be ignored even at the limit state of serviceability
   (b) can be ignored at the ultimate limit state due to large inelastic strains induced
   (c) can be completely eliminated if the props are removed after 28 days
   (d) in the in-situ concrete has no interaction on the stresses in the pre-cast component at any stage

7. For maximum sagging bending moment at support in a continuous RC beam, live load should be placed on
   (a) spans adjacent to the support plus alternate spans
   (b) all the spans except the spans adjacent to the support
   (c) spans next to the adjacent spans of the support plus alternate spans
   (d) spans adjacent to supports only

8. In Pigeaud's coefficient method for the analysis of an interior panel of a T-beam bridge
   (a) notation for coefficients as αx^4 and αx^4 includes suffix 4 since the panel is continuous on all the four edges
   (b) Poisson's ratio of concrete has no contribution
   (c) the applicability is restricted to the case when the wheel load is centrally placed
   (d) dispersion of load as considered through the wearing coat only.

9. Shear span is defined as the zone where
   (a) bending moment is zero (b) shear force is zero
   (c) shear force is constant (d) bending moment is constant

10. Which one of the following statements is correct?
   (a) Web shear cracks start due to high diagonal tension in case of beams with their ewbs and high prestressing force.
   (b) Shear design for a prestressed concrete beam is based on elastic theory.
   (c) In the zone where bending moment is dominant and shear is insignificant, cracks occur at 20° to 30°.
   (d) After diagonal cracking, the mechanism of shear transfer in a prestressed concrete member is very much different from that in reinforced concrete members.

11. Unequal top and bottom reinforcement in a reinforced concrete section leads to
   (a) creep deflection (b) shrinkage deflection
   (c) long-term deflection (d) large deflection

12. The final deflection due to all loads including the effects of temperature, creep and shrinkage and measured from as cast level of supports of floors, roofs and all other horizontal members should NOT exceed
   (a) span/350 (b) span/300
   (c) span/250 (d) span/200

13. Side face reinforcement is provided in a beam when the depth of web exceeds
   (a) 300 mm (b) 450 mm
   (c) 500 mm (d) 750 mm

14. Flexural collapse in over reinforced beams is due to
   (a) primary compression failure
   (b) secondary compression failure
   (c) primary tension failure
   (d) bond failure

15. A reinforced cantilever beam of span 4 m, has a cross-section of 150 mm × 500 mm. If checked for lateral stability and deflection, the beam will
   (a) fail in delfection only
   (b) fail in lateral stability only
   (c) fail in both deflection and lateral stability
   (d) satisfy the requirements of deflection and lateral stability

16. In an RCC beam, side face reinforcement is provided if its depth exceeds

(a) 300 mm (b) 500 mm

(c) 700 mm (d) 750 mm

17. In the limit state method of design, the failure criterion for reinforced concrete beams and columns is

(a) maximum principal stress theory

(b) maximum principal strain theory

(c) maximum shear stress theory

(d) maximum strain energy theory

18. In an RCC beam of breadth 'b' and overall depth D exceeding 750 mm, side face reinforcement required and the allowable area of maximum tension reinforcement shall be respectively

(a) 0.2% and 0.02bD (b) 0.3% and 0.03bD

(c) 0.1% and 0.04bD (d) 0.4% and 0.01bD

19. Consider the following statements:

1. At a point or inflection, the embedment length need not exceed the development length L_d.

2. The condition that $L_d \le \left(\frac{M_1}{V} + L_0\right)$ need not be checked for negative reinforcement.

3. At least one-third of the total negative reinforcement provided must extend beyond the point of inflection for a distance not less than 'd' or 12ϕ or clear-span whichever is larger.

Which of these statements are correct?

(a) 1, 2 and 3 (b) 1 and 2

(c) 1 and 3 (d) 2 and 3

20. As per IS:456-1978, the ratio of stress in concrete to its characteristic strength at collapse in flexure for design purposes taken

(a) 0.67 (b) 0.576

(c) 0.447 (d) 0.138

21. Consider the following statements:

Bars that extend into a simple support must be able to develop their full strength at a designated point L so that their moment capacity is more than the bending moment at that point. The clauses of the code require that ($\sigma_s = 0.85\ \sigma_{sy}$)

1. $L_d \le \frac{1.3M_1}{V} + L_0$

2. $\frac{\phi\sigma_s}{4\tau_{bd}} \le \frac{1.3M_1}{V} + L_0$

3. $\phi \le \frac{4\tau_{bd}}{\sigma_s}\left(\frac{1.3\ M_1}{V} + L_0\right)$

Which of these statements are correct?

(a) 1 and 2 (b) 2 and 3

(c) 1 and 3 (d) 1, 2 and 3

22. Match **List-I** with **List-II** and select the correct answer using the codes given below the list:

List-I

A. Minimum percentage of tension reinforcement of RC bam

B. Minimum percentage of shear reinforcement of RC beam

C. Maximum allowable percentage of tension reinforcement of RC beam

D. Maximum allowable percentage of compression reinforcement of RC beam

List-II

1. 4%
2. $\frac{0.85}{f_y}$
3. $\frac{0.40\, S_V}{f_y d}$

Codes:

	A	B	C	D
(a)	2	1	3	1
(b)	2	3	1	1
(c)	1	3	1	2
(d)	3	2	1	1

23. According to Whitney's theory, the maximum depth of concrete stress block in a balanced RCC beam section of depth 'd' is

(a) 0.3d (b) 0.43d

(c) 0.5d (d) 0.53d

24. While checking shear resistance of reinforced concrete beams for limit state of collapse as per IS:456, which one of the following nominal shear stress recommendations is to be adhered to? (V_u is shear force at vertical cross-section, 'b' and 'd' are overall breadth and effective depth of beam respectively)

(a) $\frac{0.5V_u}{bd}$ (b) $\frac{2V_u}{5bd}$

(c) $\frac{V_u}{0.5bd}$ (d) $\frac{V_u}{bd}$

25. As per **IS:456**, side face reinforcement, not less than 0.05% of web area, is provided on each side when the depth of web is not less than

(a) 300 mm (b) 400 mm

(c) 500 mm (d) 750 mm

26. Consider the following statements:

The reinforcement in reinforced concrete shall have concrete cover, the thickness of such cover shall be not less than

1. 25 mm
2. the diameter of bar
3. the spacing between bars
4. 5 mm

Which of these statements are correct?

(a) 3 and 4 (b) 1 and 4

(c) 2 and 3 (d) 1 and 2

27. The maximum permissible shear strain $\tau_{c,max}$ given in **BIS : 456-1978** is based on

(a) diagonal tension failure (b) diagonal compression failure

(c) flexural tension failure (d) flexural compression failure

28. The design for the limit state of collapse in flexure is based on the following assumptions:

1. Plane sections normal to the axis remain plane after bending
2. The maximum strain in concrete at the outermost tension fibre is 0.0035.
3. The relationship between the compressive stress distribution in concrete and the strain in concrete may be assumed to be rectangular, trapezoida, parabolic or any other shape which reeults in prediction of strength in substantialaggreeement with the results of tests.

Select the correct answer using the codes given below:

(a) 1 and 3 (b) 1, 2 and 3

(c) 2 and 3 (d) 1 and 2

29. The chances of diagonal tension cracks in R.C.C. memeber reduce when

(a) axial compression and shear force act simultaneously

(b) axial tension and shear force act simultaneously

(c) only shear force act

(d) flexural and shear force act

30. The maximum depth of neutral axis for a beam with 'd' as the effective depth, in limit state method of design for Fe 415 steel is

(a) 0.46d (b) 0.48d

(c) 0.50d (d) 0.53d

31. A simply supported rectangular beam of span 20.0 m is subjected to UDL. The minimum effective depth required to check deflection of this beam, when modification factor for tension and compression are 0.9 and 1.1 respectively, will be

(a) 2.0 m (b) 1.8 m

(c) 1.3 m (d) 1.0 m

32. A continuous RC beam spans six span segments, each supporting a monolithic reinforced concrete slab. The beam will best be designed

(a) as a rectangular one throughout its span

(b) as a tee-beam throughout its span

(c) as a rectangular beam for span moments and tee-beam for support moments

(d) as a tee-beam for span moments and as a rectangular beam for support moments

33. The reinforcement for tension, when required in members, shall consist of

(a) only longitudinal reinforcement in the tension face

(b) only longitudinal reinforcement in the compressionf ace

(c) only two legged closed loops enclosing the corner reinforcement

(d) both longitudinal and transverse reinforcement

34. The codal provisions recommend minimum shear reinforcement in the form of stirrups in the beams
    1. to cater for any torsion in the beam section
    2. to improve ductility of the cross section
    3. to improve dowel action of longitudinal tension bars

    Select the correct anser using the codes given below:

    (a) 1, 2 and 3 (b) 2 and 3
    (c) Only 1 (d) Only 2

35. A reinforced concrete beam of 10 m effective span and 1 m effective depth is supported on 500 mm × 500 mm columns. If the total uniformly distributed load on the beam is 10 MN/m, the design shear force for the beam is

    (a) 50 MN (b) 47.5 MN
    (c) 37.5 MN (d) 43 MN

36. Consider the following statements:

    Under-reinforced concrete flexural members
    1. are deeper
    2. are stiffer
    3. can undergo larger deflection

    Which of these statements is/are correct?

    (a) 1, 2 and 3 (b) 1 and 2
    (c) 2 only (d) 1 and 3

37. Minimum tension steel in RC beam needs to be provided to

    (a) prevent sudden failure (b) arrest crack width
    (c) control excessive deflection (d) prevent surface hair cracks

38. Mininum shear reinforcement in beams is provided in the form of stirrups

    (a) to resist extra shear force due to live load
    (b) to resist the effect of shrinkage of concrete
    (c) to resist principal tension
    (d) to resist shear cracks at the bottom of beam

39. Diagonal tension reinforcement is provided in a beam as

    (a) longitudinal bars (b) bent up bars
    (c) helical reinforcement (d) 90° bend at the bends of main bars

40. Doubly reinforced beams are recommended when:

    (a) the depth of the beam is restricted
    (b) the breadth of the beam is restricted
    (c) both depth and breadth are restricted
    (d) the shear is high

41. In a reinforced concrete member, the best way to ensure adequate bond is

    (a) to provide minimum number of large diameter bars
    (b) to provide large number of smaller diameter bars
    (c) to increase the cover for reinforcement
    (d) to provide additional stirrups

42. Minimum shear reinforcement is provided to
(a) resist shear force at the support
(b) resist shear on account of accidental
(c) arrest the longitudinal cracks on side faces due to shrinkage and temperature variation
(d) resist shear in concrete developing on account of non-homogeneity of concrete

43. Match **List-I** (Beam variable) with **List-II** (Design provision0 and select the correct answer using the codes given below the lists:

	List-I		**List-II**
A.	Flexure	**1.**	Minimum depth of section
B.	Shear	**2.**	Longitudinal steel reinforcement
C.	Bond	**3.**	Stirrups
D.	Deflection	**4.**	Anchorage in support

Codes:

	A	B	C	D
(a)	3	2	1	4
(b)	2	3	1	4
(c)	3	2	4	1
(d)	2	3	4	1

44. A beam of rectangular cross-section (b × d) is subjected to a torque T. What is the maximum torsional stress induced in the beam (b < d and α is a constant)?
(a) $\frac{T}{\alpha b^2 d}$ (b) $\frac{T}{\alpha b d^2}$
(c) $\frac{T}{\alpha bd}$ (d) $\frac{T}{bd}$

45. Match **List-I** (Codal Parameter) with **List-II** (Structural Member) and select the correct answer using the codes given below the lists:

	List-I		**List-II**
A.	0.04 bD	**1.**	Column
B.	$250\ b^2/d$	**2.**	Cantilever
C.	$100\ b^2/d$	**3.**	Continuous beam
D.	$(k_x l_x)/D_x$	**4.**	Beam

Codes:

	A	B	C	D
(a)	4	1	2	3
(b)	2	3	4	1
(c)	4	3	2	1
(d)	2	1	4	3

46. A square column section of size 350 mm × 350 mm is reinforced with four bars of 25 mm diameter and four bars of 16 mm diameter. Then the transverse steel should be
(a) 5 mm dia @240 mm c/c (b) 6 mm dia @250 mm c/c
(c) 8 mm dia @250 mm c/c (d) 8 mm dia @350 mm c/c

47. Shear strength of concrete in a reinforced concrete beam is a function of which of the following:
   1. Compressive strength of concrete
   2. Percentage of shear reinforcement
   3. Percentage of longitudinal reinforcement in tension in the section
   4. Percentage total longitudinal reinforcement in the section

   Select the correct answer using the codes given below:
   (a) 1, 2 and 4 (b) 1, 2 and 3
   (c) Only 1 and 3 (d) Only 1 and 4

48. A beam is designed for uniformly distributed loads causing compression in the supporting columns. Where is the critical section for shear? (d is effective depth of beam and L_d is development length)
   (a) A distance $L_d/3$ from the face of the support
   (b) A distance d from the face of the support
   (c) At the centre of the support
   (d) At the mid span of the beam

49. An RC structural member rectangular in cross section of width b and depth D is subjected to a combined action of bending moment M and torsional moment T. The longitudinal reinforcement shall be designed for a moment M_e given by
   (a) $M_e = M + \frac{T(1+D/b)}{1.7}$ (b) $M_e = M + \frac{T(1-D/b)}{1.7}$
   (c) $M_e = \frac{T(1+D/b)}{1.7}$ (d) $M_e = \frac{T(1-b/D)}{1.7}$

50. Which one of the following is the correct expression to estimate the development length of deformed reinforcing bar as per IS code in limit state design?
   (a) $\frac{\phi\sigma_s}{4.5\tau_{bd}}$ (b) $\frac{\phi\sigma_s}{5\tau_{bd}}$
   (c) $\frac{\phi\sigma_s}{6.4\tau_{bd}}$ (d) $\frac{\phi\sigma_s}{8\tau_{bd}}$

51. The cover of longitudinal reinforcing bar in a beam subjected to sea spray should not be less than which one of the following?
   (a) 30 mm (b) 70 mm
   (c) 75 mm (d) 80 mm

52. How can shear strength be ensured in a beam?
   (a) By providing binding wire on main bars
   (b) By providing HYSD bars instead of mild steel bars
   (c) By providing rounded aggregate
   (d) By providing stirrups

53. Usually stiffness of a simply supported beam is satisfied if the ratio of its span to depth does not exceed which one of the following?
   (a) 7 (b) 10
   (c) 20 (d) 26

54. When HYSD bars are used in place of mild steel bars in a beam, the bond strength
    (a) does not change (b) increases
    (c) decreases (d) becomes zero

55. What is the bond stress acting parallel to the reinforcement on the interface between bar and concrete?
    (a) Shear stress (b) Local stress
    (c) Flexural stress (d) Bearing stress

56. Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

List-I	**List-II**
A. $\frac{V_u}{bd}$	**1.** Modulus of rupture
B. $0.7\sqrt{f_{ck}}$	**2.** Development length
C. $5000\sqrt{f_{ck}}$	**3.** Nominal shear stress
D. $\frac{\phi f_s}{4\tau_b}$	**4.** Hook anchorage value
	5. Modulus of concrete

Codes:

	A	B	C	D
(a)	3	1	5	2
(b)	2	1	4	3
(c)	3	5	1	4
(d)	2	4	1	3

57. What shall be the maximum area of reinforcement (i) in compression and (ii) in tension to be provided in an RC beam, respectively, as per **IS:456**?
    (a) 0.08% and 2% (b) 2% and 4%
    (c) 4% and 2% (d) 4% and 4%

58. What is the adoptable maximum spacing between vertical stirrups in an RCC beam of rectangular cross-section having an effective depth of 300 mm?
    (a) 300 mm (b) 275 mm
    (c) 250 mm (d) 225 mm

59. Consider the following statements dealing with flexural reinforcement to be terminated in the tension zone:
    1. The shear at the cut-off point not to exceed two-third of the otherwise permitted value.
    2. Shear reinforcement is provided along each terminated bar overlapping three-fourth of the appropriate distance from the cut-off point.
    3. For 36 mm and smaller bars, the continuing bars shall provide double the area required for flexure at the cutoff and shear does not exceed three-fourth of the permitted value.

    Which of these statements is/are correct?
    (a) 1, 2 and 3 (b) 1 and 2 only
    (c) 2 and 3 only (d) 3 only

60. What is the anchorage value of a standard hook of a reinforcement bar of diameter D?
(a) 4D
(b) 8D
(c) 12D
(d) 16D

61. The maximum per cent of moment redistritution allowed in RCC beams is
(a) 10%
(b) 20%
(c) 30%
(d) 40%

Common Data for Questions 62 and 63:

At the limit state of collapse. an RC beam is subjected to flexural moment 200 kN-m, shear force 20 kN and torque 9 kN-m. The beam is 300 mm wide and has a gross depth of 425 mm, with an effective cover of 25 mm. The equivalent nominal shear stress (τ_{ve}) as calculated by using the design code turns out to be lesser than the design shear strength (τ_c) of the concrete.

62. The equivalent shear force (V_e) is
(a) 20 kN
(b) 54 kN
(c) 56 kN
(d) 68 kN

63. The equivalent flexural moment (M_{eq}) for designing the longitudinal tension steel is
(a) 187 kN-m
(b) 200 kN-m
(c) 209 kN-m
9d) 213 kN-m

Statement for Linked Anser Questions 64 and 65:

In the design of beams for the limit state of collapse in flexure as per **IS:456-2000**, let the maximum strain in concrete be limited to 0.0025 (in place of 0.0035). For this situation, consider a rectangular beam section with breadth as 250 mm, effective depth as 350 mm, area of tension steel as 1500 mm^2, and characteristic strengths of concrete and steel as 30 MPa and 250 MPa respectively.

64. The depth of neutral axis for the balanced failure is
(a) 140 mm
(b) 156 mm
(c) 168 mm
(d) 185 mm

65. At the limiting state of collapse in flexure, the force acting on the compression zone of the section is
(a) 326 kN
(b) 389 kN
(c) 424 kN
(d) 542 kN

Common Data for Questions 66 and 67:

A reinforced concrete beam of rectangular cross section of breadth 230 mm and effective depth 400 mm is subjected to a maximum factored shear force of 120 kN. The grades of concrete, main steel and stirrup steel are M20, Fe415 and Fe250 respectively. For the area of main steel provided, the design shear strength τ_c as per **IS:456-2000** is 0.48 N/mm^2. The beam is designed for collapse limit state.

66. The spacing (mm) of 2-legged 8 mm stirrups to be provided is
(a) 40
(b) 115
(c) 250
(d) 400

67. In addition, the beam is subjected to a torque whose factored value is 10.90 kN-m. The stirrups have to be provided to carry a shear (kN) equal to
(a) 50.42
(b) 130.56
(c) 151.67
(d) 200.23

Answers

1. (a)	18. (c)	35. (c)	52. (d)
2. (a)	19. (b)	36. (a)	53. (c)
3. (d)	20. (c)	37. (a)	54. (b)
4. (a)	21. (d)	38. (c)	55. (c)
5. (c)	22. (b)	39. (b)	56. (a)
6. (b)	23. (d)	40. (a)	57. (d)
7. (c)	24. (d)	41. (b)	58. (c)
8. (c)	25. (d)	42. (b)	59. (a)
9. (c)	26. (d)	43. (d)	60. (d)
10. (b)	27. (b)	44. (a)	61. (c)
11. (b)	28. (a)	45. (c)	62. (d)
12. (c)	29. (a)	46. (c)	63. (d)
13. (d)	30. (b)	47. (c)	64. (b)
14. (b)	31. (a)	48. (b)	65. (b)
15. (c)	32. (c)	49. (a)	66. (b)
16. (d)	33. (d)	50. (c)	67. (c)
17. (b)	34. (b)	51. (b)	

5

6 Column

INTRODUCTION

- A 'compression member' is a structural element which is subjected predominatily to axial compressive forces. Compression members are most commonly encountered in reinforced concrete buildings as columns (and sometimes as reinforced concrete walls).
- The coloumn is representative of all types of compression members, and hence, sometimes, the terms column and compression member are used interchangeably.
- The code (Cl. 25.1.1) defines the column as a compression member, the effective length of which exceed three times the least lateral diamension.
- The term pedestal is used to describe a vertical compression member whose effective length is less than three times its least lateral dimension (Cl. 26.5.3.1) of the code.

ASSUMPTIONS

The following assumptions are made for the limit state of collapse in compression:

1. Plane sections normal to the axis remain plane after bending.
2. The relationship between stress-strain distribution in concrete is assumed to be parabolic. The maximum compressive stress is equal to $0.67\, f_{ck}/1.5$ or $0.446\, f_{ck}$.
3. The tensile strength of concrete is ignored.
4. The stresses in reinforcement are derived from the representative stress-strain curve for the type of steel used.
5. The maximum compressive strain in concrete in axial compression is taken as 0.002.
6. The maximum compression strain at the highly compressed extreme fibre in concrete subjected to axial compression and bending, but when there is no tension on the section, is taken as 0.0035 minus 0.75 times the strain at the least compressed extreme fibre.
7. The maximum compressive strain at the highly compressed extreme fibre in concrete subjected to axial compression and bending, when part of the section is in tension, is taken as 0.0035.

CLASSIFICATION OF COLUMNS BASED ON TYPE OF REINFORCEMENT

Reinforced concrete columns may be classified into the following three types based on the type of reinforcement provided:

1. Tied columns: where the main longitudinal bars are enclosed within closely spaced *lateral ties* [Fig. 6.1(a)]
2. Spiral columsn (spirally reinforced columns): where the main longitudinal bars are enclosed within spiral reinforcement [Fig. 6.1(b)]
3. Composite columns: where the reinforcement is in the form of structural steel sections or pipes, with or without longutidinal bars [Fig. 6.1(c)].

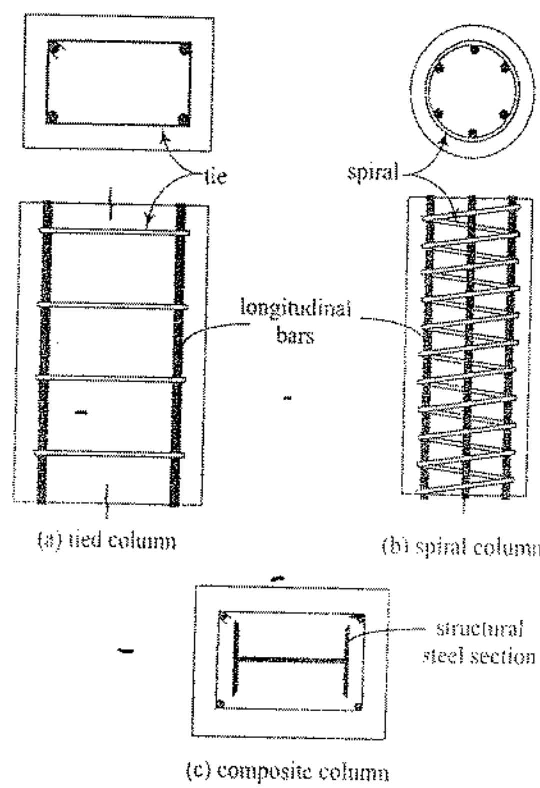


Fig. 6.1: Types of columns—tied, spiral and composite.

Classification of Columns Based on Type of Loading

Columns may be classified into the following three-types. based on the natural of loading.

1. Columns with axial loading (applied concentrically)
2. Columns with uniaxial eccentric loading.
3. Columns with biaxial eccentric loading

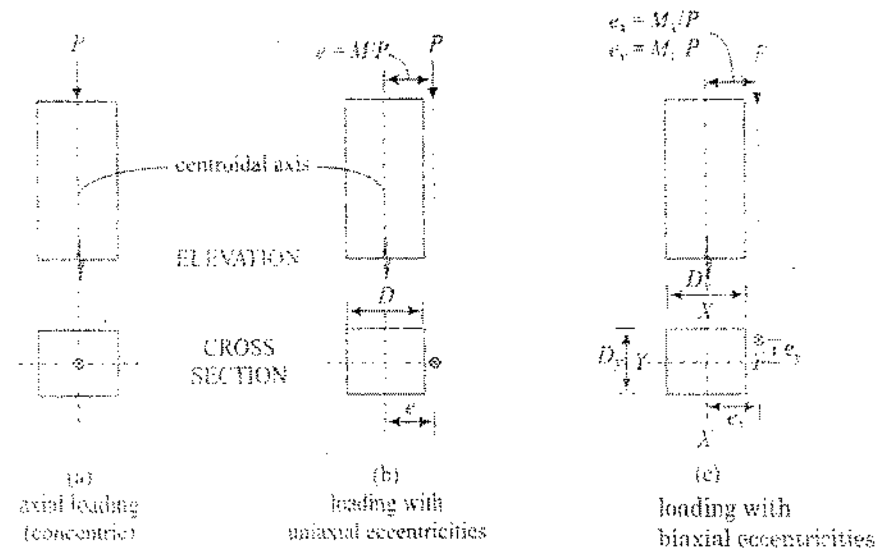


Fig. 6.2: Different loading situations in columns.

Classification of Columns Based on Slenderness Ratios

Columns (i.e.. compression memberers) may be classified into the following two types, depending on whether slenderness effects are considered insignificant or significant:

1. Short columns; and
2. Slender (or long) columns.

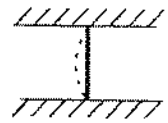
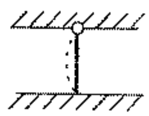
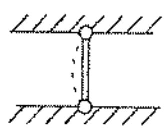
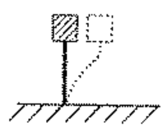
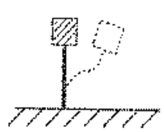
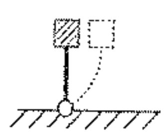
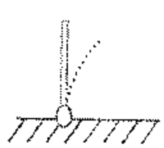
- Slenderness is a geometrical property of a compression member which is related to the ratio of its effective length to its lateral dimension.
- This ratio, called slenderness ratio, also provides a measure of the vulnerability to failure of the column by elastic instability (buckling) - in the plane in which the slenderness ratio is computed.
- Columns with low slenderness ratios, ie., relatively short and stockly columns, invariably fail under ultimate loads with the material reaching its ultimate strength, and not by buckling.
- On the other hand, columns with very high slenderness ratio are in large lateral deflection) under relatively low compressive loads, and thereby failing suddenly.
- Design codes attempt to preclude such failure by specifying slenderness limits' to columns.

Effective Length

Effective length is length between the points of contraflexures of a buckled column. It depends upon the conditions.

The effective length (l_{eff}) of al column length (l) may be determined with the help of Table 6.1.

Table 6.1: Effective length of column.

Degree of end restraint of compression member	Symbol	Theoretical value of effective length	Recommended value of effective length
(1)	(2)	(3)	(4)
Effectively held in position and restrained		0.50 l	0.65 l
Effectively held in position at both ends, rotation at one end		0.70 l	0.80 l
Effectively held in position at both ends, but not restrained against rotation.		1.00 l	1.00 l
Effectively held in position and restrained against rotation at one end, and at the other restrained against rotation but not held in position.		—	1.50 l
Effectively held in position and restrained against rotation in one end, and at the other partially restrained against rotation but not held in position		2.00 l	2.00 l
Effectively held in position at one end but not restrained against rotation, and at the other end restrained against rotation but not held in position.		2.00 l	2.00 l
Effectively held in position and restrained against rotation at one end but not held in position nor restrained against rotation at the other end.		2.00 l	2.00 l

Note : l is the unsupportede length of compression member. It is the clear distance between the floor.

Minimum Eccentricities

Eccentricities not explicitly arising out of structural analysis calculations act on the column but eccentricity may concrete due to various reasons, such as

- lateral loads not considered in design;
- live load placements not considered in design;
- accidental lateral/eccentric loads;
- errors in costruction (such as misalignments):
- slenderness effects underestimated in design.

- For this reason, the Code (Cl. 25.4) requires every column to be designed for a minimum eccentricity e_{min} (in any plane) equal to the unsupported length/500 plus lateral dimension/30, subject to a minimum of 20 mm. For a column with a rectangular section this implies.

$$e_{x.min}\begin{cases}l/500+D_x/30\\20mm\end{cases}\quad\text{(which ever is greater)}$$

$$e_{y.min}\begin{cases}l/500+D_y/30\\20mm\end{cases}\quad\text{(which ever is greater)}$$

- For non-rectangular and non-circular cross-sectional shapes, it is recommended that, for any given plane,

$$e_{min}\begin{cases}l_e/300\\20mm\end{cases}\quad\text{(which ever is greater)}$$

where l_e is the effective length of the column in the plane considered.

Code Requirements on Reinforcement and Detailing

Longitudinal Reinforcement

Minimum Reinforcement

The longitudinal bars must, in general, have a cross-sectional area not less than 0.8% of the gross area of the column section. Such a minimum limit is specified by the Code :

- to ensure nominal flexural resistance under unforeseen eccentricities in loading; and
- to prevent the yielding of the bars due to creep and shrinkage effects, which result in a transfer of load from the concrete to the steel.

- However, in the case of pedestals which are designed as plain concrete columns, the minimum requirement of longitudinal bars may be taken as 0.15 per cent of the gross area of cross-section.
- In the case of reinforcement concretre walls, the Code (Cl.32.5) has introduced detailed provisions regarding minimum reinforcement requirements for vertical (and horizontal) steel.
- The vertical reinforcement shold not be less than 0.15% of the gross area in general.
- This may be reduced to 0.12% if welded wire fabric or deformed bars (Fe 415/Fe 500 grade steel) is used, provided the bar diameter does not exceed 16 mm.
- This reinforcement should be placed in two layers if the wall is more than 200 mm thick.
- In all cases, the bar spacing should not exceed three times the wall thickness 450 mm, whichever is less.

Maximum Reinforcement:

- The maximum cross-sectional area of longutidinal bars should not exceed 6 per cent of the gross area of the column section.
- However, a reduced maximum limit of 4 per cent is recommended in general in the interest of better placement and compaction of concrete—and, in particular, at lapped splice locations.
- In tall buildings, columns located in the lowermost storeys generally carry heavy reinforcement (~ 4 per cent). The bars are progressively curtailed in stages at higher levels.

Minimum diameter/number of bars and their location

- Longitudinal bars in column (and pedestals) should not be less than 12 mm in diamter and should not be spaced more than 300 mm apart (centre-to-centre) along the periphery of the column [Fig. 6.3(a)].
- At least 4 bars (one at each corner) should be provided in a column with rectangular cross-section, and at least 6 bars (eqully spaced near the periphery) in a circular column.
- In 'spiral columns' (including noncircular shapes), the longitudinal bars should be placed in contact with the spiral reinforcement, and equidistant around its inner circumference [Fig. 6.3(b)]. In columns with T-, L-, other cross sectional shapes, at least one bar should be located at each corner or apex [Fig. 6.3(c)].
- Longitudinal bars are usually located close to the periphery (for better flexural resistance), but may be placed in the interior of the column when eccentricities in loading are minimal. When a large number of bars need to be accommodated, they may be *bundled*, or, alternatively, *grouped*, as shown in [Fig. 6.3(d)].

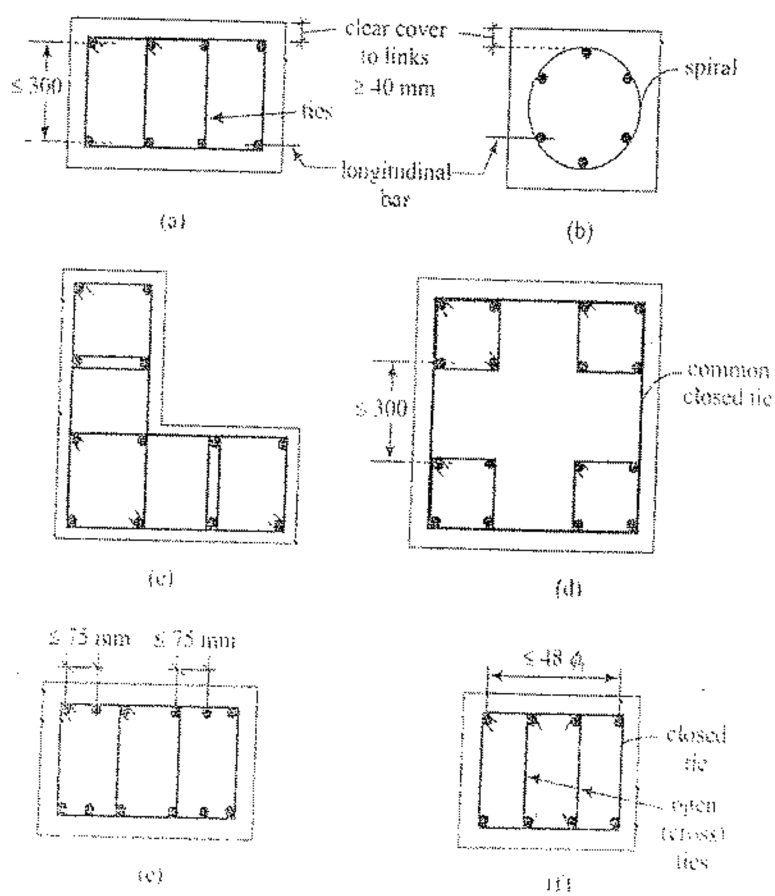


Fig. 6.3: Some code recommendations for detailing in columns.

Cover to Reinforcement: A minimum clear cover of 40 mm or bar diameter (whichever is greater), to the column ties is recommended by the Code (Cl. 26.4.2.1) for columns in general; a reduced clear cover of 25 mm is permitted in small-sized columns ($D \le 200$ mm and whose reinforcing bars do not exceed 12 mm) and a minimum clear cover of 15 mm (or bar diameter, whichever is greater) is specified for walls. however, in aggressive environments, it is desirable, in the interest of durability, to provide increased cover but preferably not greater than 75 mm.

Transverse Reinforcement (Cl. 26.5.3.2 of the Code)

- General All longitudinal reinforcement in a compression member must be enclosed within transverse reinforcement, comprising either lateral ties (with internal angles not exceeding 135°) or sprials. This is required:
  - to prevent the premature buckling of individual bars;
  - to confine the concrete in the 'core', thus improving ductility and strength;
  - to hold the longitudinal bars in position during construction; and
  - to provide resistance against shear and torsion, if required.

Lateral Ties

- The arrangement of lateral ties should be effective in fulfilling the above requirements.
- They should provide adequate lateral support to each longitudinal bar, thereby preventing the outward movement of the bar.
- The diameter of the tie ϕ_t is governed by requirments of stiffness, rather than strength, and so is independent of the grade of steel.
- The pitch σ_t (centre-to centre spacing along the longitudinal axis of the column) of the ties should be small enough to reduce adequately the unsupported length (and hence, slenderness ratio) of each longitudinal bar. The code recommendations are as follows:

$$\text{tie diameter } \phi_t \ge \begin{cases} \phi_{long,max}/4 \\ 6\text{mm} \end{cases}$$

$$\text{tie spacing } \quad S_t \le \begin{cases} D \\ 16\phi_{long,min} \\ 300\text{mm} \end{cases}$$

(i) Minimum tie diameter was specified as 5 mm, instead of 6 mm in earlier version of code.
(ii) Maximum tie spacing was specified as 48 ϕ_t instead of 300 mm in earlier version of code.

where ϕ_{long} denotes the diameter of longitudinal bar to be tied and D denotes the least lateral dimension of the column.

- When the spacing of longitudinal bars is less than 75 mm, lateral support need only be provided for the corner and alternate bars.
- If the longitudinal bars spaced at a distance not exceeding 48ϕ are effectively tied in two directions, then the additional longitudinal bars in between these bars need be tied only in one direction by open ties.

Spirals

Helical reinforcement provides very good confinement to the concrete in the 'core' and enhances significantly the ductility of the column at ultimate loads. The diameter and pitch of the spiral may be computed as in the case of ties—except when the column is desiged to carry a 5 per cent overaload (as permitted by the Code), in which case

$$\text{pitch} \quad S_t < \begin{cases} 75 \text{ mm} \\ \text{core diameter}/6 \end{cases} \qquad \text{and} \qquad S_t > \begin{cases} 25\text{mm} \\ 3\phi_t \end{cases}$$

The ends of the spiral should be anchored properly by providing one and a half extra turns.

Design of Short Columns under Axial Compression

It load P is applied on a column of width B and depth D

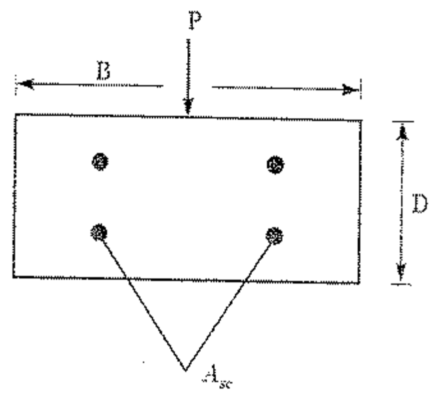


Fig. 6.4

Then
$$P_u = P_c + P_{sc} \qquad \text{---(i)}$$

P_c = load taken by concrete
P_{sc} = load taken by steel
P_u = total load taken up by the column (ultimate load)

$$\text{We know that stress} = \frac{\text{Load}}{\text{Area}}$$

$$\sigma_{cc} = \frac{P_c}{A_c}$$

and
$$\sigma_{sc} = \frac{P_{sc}}{A_{sc}}$$

where σ_{cc} = stress in concrete
σ_{sc} = stress in steel
A_c = Area of concrete
A_{sc} = Area of steel
A = Gross area

$$P_u = \sigma_{cc}A_c + \sigma_{sc}A_{sc} \qquad \text{---(ii)}$$

The design stress in concrete is $\dfrac{0.67\, f_{ck}}{1.5} = 0.447\, f_{ck}$

and design stress in steel is $0.87\, f_y$ in case of Fe 250 under 'pure' axial loading conditions, the design strength of a short column is

$$P_u = 0.447\, f_{ck}A_c + 0.87\, f_y A_{sc} \qquad \text{---(iii)}$$

However the code requires all columns to be designed for minimum eccentricities in loading hence equation (iii) can not be applied directly. Nevertheless, where the calculated minimum eccentricity (in any plane) does not exceeds 0.05 time the least lateral dimension (in the plane considered). the code (Cl. 39.3) permits the use of the following simplified formula, obtained by reducing the P_u by 10%.

Now, $$\boxed{P_u = 0.4\, f_{ck} A_c + 0.67\, f_y A_{sc}}$$

Example 1

Design A R-C-column of size 450 mm × 600 mm subject to an axial total of 2000 kN under service load condition. Unsupported length of column is 3m, use M 20 concrete and Fe 415 steel.

Sol:

(1) Check whether long or short column $= \frac{3000}{450} = 6.67 < 12$ so column is short

(2) Check e_{min} along 450 mm

e_{min} is greater of

(i) $\frac{3000}{5000} + \frac{450}{30} = 21$ mm

(ii) 20 mm

$$e_{min} = 21$$

e_{min} must be less than $0.05 \times 450 = 22.5$ mm

$$e_{min}\ (21) < 0.05B\ (22.5)\ \text{O.K}$$

e_{min} along 600 mm

e_{min} is greater of (1) $\frac{3000}{500} + \frac{600}{30} = 26$

(1) 20 mm

$$e_{min} = 26 < 30\ (0.05\ D)$$

So we can use

$$P_u = 0.4\, f_{ck} \times A_c + 0.67\, f_y\, A_{sc}$$

$$= 0.4 \times 20 \times (A - A_{sc}) + 0.67 \times 415 \times A_{sc}$$

$$2000 \times 10^3 \times 1.5 = 0.4 \times 200 \times (450 \times 600 - A_{sc}) + 0.67 \times 415\, A_{sc}$$

$$= 216 \times 10^4 - 8A_{sc} + 278.05\, A_{sc}$$

$$A_{sc} = 3110.5\ \text{mm}^2$$

Provide 4 nos. 25 mm ϕ at corner

$$4 \times 491 = 1964\ \text{mm}^2$$

and additional 4 nos. 20 ϕ = $4 \times 314 = 1256$ mm^2

$$A_{sc} = 3220\ \text{mm}^2$$

$$> 3110.5\ \text{mm}^2$$

$$p' = \frac{A_{sc}}{bd} \times 100 = \frac{3220}{450 \times 600}$$

$$= 1.192 > 0.8 \text{ minimum reinforcement}$$

Lateral ties is greater of

(i) 25/4 = 6.25

(ii) 6 mm use 8 mm ϕ

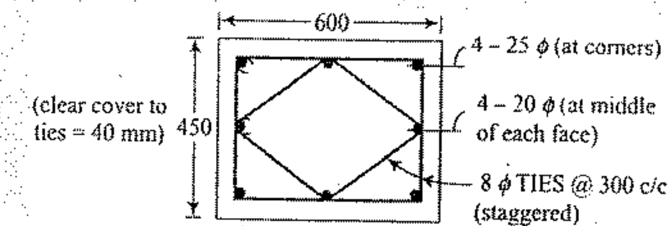
Spacing is mini. of

(i) 450 mm

(ii) 16 × 20 = 320 mm

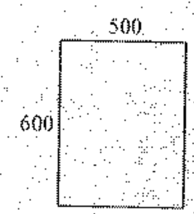
(iii) 300 mm

Hence use 8 mm ϕ @ 300 mm c/c



Example 2

Design a short axially loaded column section of size 500 × 600 m for a service load of 2200 kN use M20/ Fe 415 steel.



Sol.

Factored load = 2200 × 1.5 = 3300 kN.

Short Column is given so

$$P_u = 0.4\, f_{ck}\, A_c + 0.67\, f_y\, A_{sc}$$

$$3300 \times 10^3 = 0.4 \times 20 \times (A - A_{sc}) + 0.67 \times 415 \times A_{sc}$$

$$A_{sc} = 3333 \text{ mm}^2$$

$$\text{No. of bar of 25 mm } \phi = \frac{3333}{\frac{\pi}{4} \times 25^2} = 6.78 = 8 \text{ nos.}$$

$$p' = \frac{A_{st}}{bd} \times 100 = \frac{8 \times \frac{\pi}{4} \times 25^2}{500 \times 600} \times 100$$

$$= 1.30 > 0.8 \text{ minimum reinforcement}$$

Lateral ties is greater is

(i) $\frac{\phi}{4}\left(\frac{25}{4} = 6.25\right)$

(ii) 6 mm

Use 8 mm ϕ bar as a lateral ties.

Spacing

Spacing is minimum of

(i) least lateral dimension (500 mm)

(ii) $16 \times \phi_{min}$ (16 × 25 = 400 mm)

(iii) 300 mm

Hence provide 8 mm ϕ @ 300 mm c/c.

Example 3

An RCC column 3.5 m length is required to resist an axial ultimate load of 1750 kN. Design column use M 20 and Fe 415 grade of steel?

Sol: Assume column is short and design for square column

$$\frac{l_{eff}}{b} \le 12$$

$$b \ge \frac{l_{eff}}{12}$$

$$b \ge \frac{3500}{12}$$

$$b \ge 291.6 \text{ mm}$$

Take $b \times b = 450$ mm

(Note if we take $b = 300$ mm and 400 mm it will fail in eccentricity check.

Calculating e_{min}

e_{min} is greater of

(i) $\frac{L}{500} + \frac{b}{30}$

$\frac{3500}{500} + \frac{450}{30}$

$= 22$ mm

(ii) 20 mm

$$e_{min} = 22 \text{ mm}$$

This $e_{min} < 0.05\,b$

$$0.05 \times 450 = 22.5 \text{ mm}$$

$$e_{min} (22 \text{ mm}) < 0.05b \; (22.5 \text{ mm}) \text{ O.K.}$$

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$1750 \times 10^3 = 0.4 \times 20 \times (450 \times 450 - A_{sc}) + 0.67 \times 415 A_{sc}$$

$$1750 \times 10^3 = 1.62 \times 10^6 - 8A_{sc} + 278.05 A_{sc}$$

$$A_{sc} = 481.39 \text{ mm}^2$$

Use 12 mm ϕ then no. of bar $= \frac{481.39}{\frac{\pi}{4} \times 12^2} = 4.25 \simeq 6$

$$p' = \frac{6 \times \frac{\pi}{4} \times 12^2}{450 \times 450} \times 100 = 0.33 < 0.8 \text{ minimum reinforcement}$$

So take 0.8% of gross area

$$A_{st} = 0.8 \times 450 \times 450 = 1620 \text{ mm}^2$$

Use 20 mm ϕ bar

$$\text{Number of bar} = \frac{1620}{\frac{\pi}{4} \times 20^2} = 5.15 \simeq 6 \text{ nos.}$$

Lateral ties is greater of

(i) $\frac{1}{4} \times 20 = 15$ mm

(ii) 6 mm

Use 6 mm ϕ bar lateral ties

Spacing

Spacing is minimum of

(i) least lateral dimension (450 m)

(ii) $16 \times 20 = 320$ mm

(iii) 300 mm

Hence use 6 mm ϕ bar @ 300 mm c/c

COLUMN SUBJECTED TO AXIAL COMPRESSION AND UNIAXIAL BENDING (CLAUSE 39.5)

INTRODUCTION

- The load on the column is rarely axial. There is always some minimum inherent eccentricity on account of inaccuracies in loading, bad workmanship etc.
- In such cases, the column is subjected to axial compression P_u and bending moment M_u. This loading system can be reduced to a single resultant load P_u acting at an eccentricity $e = \frac{M_u}{P_u}$ as shown in Fig. 6.5.
- But the design of member subjected to combined axial load and uniaxial bending involves lengthy calculations by trial and error.
- In order to overcome these difficulties, interaction diagrams may be used. These have been prepared and published by BIS in "SP : 16 design aids for reinforced concrete to IS 456".

Interaction Curve

- The 'interaction curve' is a complete graphical representation of the design strength of a uniaxially eccentrically loaded column of given proportions.
- Each point on the curve corresponds to the design strength values of P_u and M_u assciated with a specific eccentricity (e) of loading.
- That is to say, if load 'P' is applied on a short column with an eccentricity 'e' and if this load is gradually increased till the ultimate limit state is reached and that ultimate load at failure is given by $P = P_u$ and the correspoinding moment by $M = M_u = P_u.e$, then the co-ordinates (M_u, P_u) form a unique point on the interaction diagram.
- The interaction curves define the different (M_u, P_u) combinations for all possible eccentricities of loading $0 \le e < \infty$.
- For design purposes, the calculations of M_u and P_u are based on the design stress-strain curves (including partial safety factors) and the resulting interaction curve is referred to as the 'design interaction curve' (which is different from the characteristic interaction curve).

- Using the design interaction curve for a given column section, it is possible to make a quick judgement as to whether or not the section is safe under a specified factored load effect combination (P_u, M_u).
- If the point given by the co-ordinates (M_u, P_u) falls within the design interaction curve, the column is 'safe', otherwise it is not.

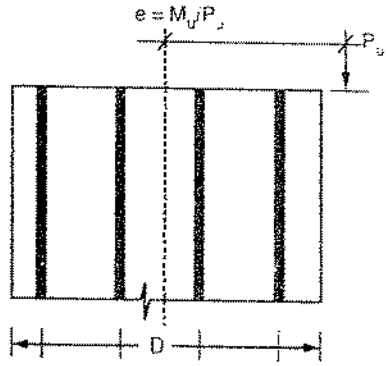


Fig. 6.5

Salient Point on the Interaction Curve

The salient points, marked 1 to 5 on the interaction curve corresponds to the failure strain profiles, marked 1 to 5 in Fig. 6.6.

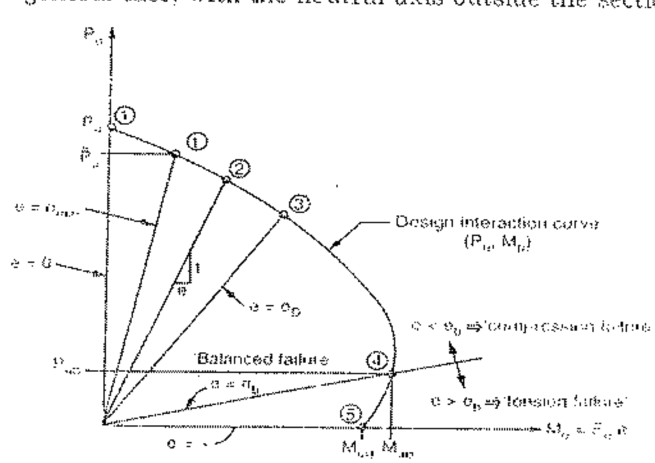
(a) The point 1 corresponds to the condition of axial loading with e = 0
For this case of 'pure' axial compression, $M_u = 0$
and $P_u = 0.45\ f_{ck}\ A_c + 0.75\ f_y\ A_{sc}$

(b) The point (1) corresponds to the condition of axial loading with the minimum eccentricity.
$e_{min} = 0.05 \times D$[clause 25.4 and 39.3]
The corresponding ultimate resistance is written as
$P_u = 0.4\ f_{ck}\ A_c + 0.67\ f_y\ A_{sc}$

(c) The point (3) corresponds to the condition $e = e_D$(i.e. $x_u = D$). For $e < e_D$, the entire section is in under compression and neutral axis is located outside the section ($x_u > D$) with $0.002 < \varepsilon_u < 0.0035$.

For $e > e_D$, the neutral axis is located within the section ($x_u < D$) and $\varepsilon_u = 0.0035$ at the "highly compressed edge".

Point (2) represents a general case, with the neutral axis outside the section ($e < e_D$).



Typical $P_u - M_u$ interaction diagram

Fig. 6.6

(d) The point (4) corresponds to the balanced failure condition, with $e = e_b$ and $x_u = x_{u'lim}$. The design strength values for this 'balanced failure' condition are denoted as p_{ub} and M_{ub}.

For $P_u < P_{ub}$ (i.e. $e > e_b$), the mode of failure is called tension failure.

(e) The point (5) corresponds to a 'pure' bending condition ($e = \infty$, $P_u = 0$). The resultig ultimate moment of resistance is denoted by M_{u0} and the corresponding neutral axis depth takes on a minimum value $x_{u'min}$. This case is the same as the doubly reinforced section of beam.

Example 4

Design a short reinforced concrete column of rectangular section to carry an ultimate load of 600 kN and ultimate moment of 100 kN-m, acting about an axis bisecting the depth of the column. Assume the effective length of column is 4.5 m. Width of the supported beam is 300 mm.

Use M-20 concrete and Fe 415 steel, provide equal steel on both tension and compression sides.

Sol: Given that $P_u = 600$ kN, $M_u = 100$ kN-m, $f_{ck} = 20$ N/mm², $l_{eff} = 4.5$ m and column is short.

Now, for column to be short,

$$\frac{l_{eff}}{D} \leq 12$$

$$\text{or} \quad D \geq \frac{l_{eff}}{12} \geq \frac{4500}{12} \geq 375 \text{ mm}$$

Since the width of support of beam = 300 mm, therefore, provide width of column = 300 mm.

Assume size of column = (300 × 400) mm and diameter of lateral ties = 6 mm.

Using 20 mm diameter bars,

$$\text{Effective cover} = 40 + 6 + \frac{\phi}{2}$$

$$= 40 + 6 + \frac{20}{2} = 56 \text{ mm}$$

Now,

$$\frac{d'}{D} = \frac{56}{400} = 0.14$$

$$f_{ck}.b.D = 20 \times 300 \times 400 = 2400 \text{ kN}$$

$$\frac{P_u}{f_{ck}.b.D} = \frac{600}{2400} = 0.25$$

$$\frac{M_u}{f_{ck}.b.D^2} = \frac{100 \times 10^6}{2400 \times 10^3 \times 400} = 0.104$$

Using chart 2 and chart 3, (Appendix)

$$\frac{d'}{D} = 0.10 \text{ and } 0.15, \text{ we get,}$$

for $\frac{d'}{D}$	$\frac{P}{f_{ck}}$
0.10	0.042
0.15	0.045

By interpolation, we get,

$$\frac{d'}{D} = 0.14, \quad \frac{P}{f_{ck}} = 0.044$$

$$\therefore \quad P = 0.044 \times f_{ck} = 0.044 \times 20 = 0.89\% > A_{st,min} = 0.8\% \text{ (OK)}$$

$$\therefore \quad A_s = \frac{PbD}{100} = 0.89 \times \frac{300 \times 400}{100} = 1066 \text{ mm}^2$$

Provide 4-20 mm ϕ bars = 4 × 314 = 1256 mm^2.

Lateral ties:

Using $\phi_t = 6$ mm

$$\text{Spacing} \quad S_t < \begin{cases} 300 \text{ mm} \\ 16 \times 20 = 320 \text{ mm} \\ 300 \text{ mm} \end{cases} \quad \therefore \text{ Provide 300 mm}$$

Provide 6 mm ϕ lateral ties @ 300 mm c/c.

Example 5

A reinforced concrete column 400 mm square is constructed of concrete M 20 and steel with characteristic strength of 250 N/mm². Design the reinforcement in the section if it has to support a design ultimate load of 800 kN at an eccentricity of 200 mm. Effective cover to steel = 40 mm.

Sol:

Given:

$$b = D = 400 \text{ mm}, \ P_u = 800 \text{ kN}$$
$$e = 200 \text{ mm}, \ d' = 40 \text{ mm}$$
$$f_{ck} = 20 \text{ N/mm}^2, \ f_y = 250 \text{ N/mm}^2$$

Required: Area of steel.

Now, referring to chart in SP : 16, for $f_y = 250$ N/mm² and $\frac{d'}{D} = \frac{40}{400} = 0.10$, the non-dimensional parameters are

$$\frac{P_u}{f_{ck}.b.D} = \frac{800 \times 10^3}{20 \times 400 \times 400} = 0.25$$

$$\frac{M_u}{f_{ck}.b.D^2} = \frac{800 \times 10^3 \times 200}{20 \times 400 \times (400)^2} = 0.125$$

The value of $\frac{P}{f_{ck}} = 0.09$

$$\therefore \quad P = 0.09 \times 20 = 1.8\% > 0.8\%$$

Therefore,
$$A_s = \frac{PbD}{100} = \frac{1.8 \times 400 \times 400}{100} = 2880 \text{ mm}^2$$

Provide 6-25 ϕ mm diameter bar = 6 × 491 = 2946 mm

(Distributing 3 bars on either face).

Lateral ties: Using 6 mm ϕ for lateral ties.

$$\text{Spacing} \quad S_t < \begin{cases} 400 \text{ mm} \\ 16 \times 25 = 400 \\ 300 \text{ mm} \end{cases} \quad \therefore \text{ Provide 300 mm}$$

Therefore, provide 6 mm ϕ @ 300 mm c/c.

COLUMN SUBJECTED TO AXIAL COMPRESSION AND BIAXIAL BENDING (CLAUSE 39.6)

INTRODUCTION

The condition of biaxial bending along with axial compression is of common occurrence for design of R.C. columns due to its monolithic construction with beams in two different directions. This is specially true for corner columns in frames.

The strength of the section under axial compression and biaxial bending is a function of P_u, M_{ux} and M_{uy}. The ultimate moment M_{ux} about the major axis of bending (i.e. X-axis) and M_{uy} about the minor axis of bending (i.e. Y-axis) can be expressed in terms of axial compression P_u acting at eccentricities $e_x = \frac{M_{ux}}{P_u}$ and $e_y = \frac{M_{uy}}{P_u}$.

CODE PROCEDURE FOR DESIGN OF BIAXIALLY LOADED COLUMNS

The column subjected to axial compression and biaxial bending may be designed by the fofllowing equation:

$$\left[\frac{M_{ux}}{M_{ux1}}\right]^{\alpha_n} + \left[\frac{M_{uy}}{M_{uy1}}\right]^{\alpha_n} \le 1.0$$

where M_{ux} and M_{uy} = Moment about X and Y axes due to design loads

M_{ux1} and M_{uy1} = Maximum uniaxial moment capacity for an axial load of P_u, bending about X and Y axis respectively

α_n is related to $\frac{P_u}{P_{uz}}$

where $P_{uz} = 0.45\, f_{ck} A_c + 0.75\, f_y A_{sc}$

$= 0.45\, f_{ck} A_g + (0.75\, f_y - 0.45\, f_{ck}) A_{sc}$

$= 0.45\, f_{ck} A_g + (0.75\, f_y - 0.45\, f_{ck})\, A_{sc}$

For values of $\frac{P_u}{P_{uz}}$ = 0.2 to 0.8, the values of α_n vary lineraly from 1.0 to 2.0. For values less then 0.2, α_n is 1.0; for values greater than 0.8, α_n is 2.0 as shown in Fig. 6.7.

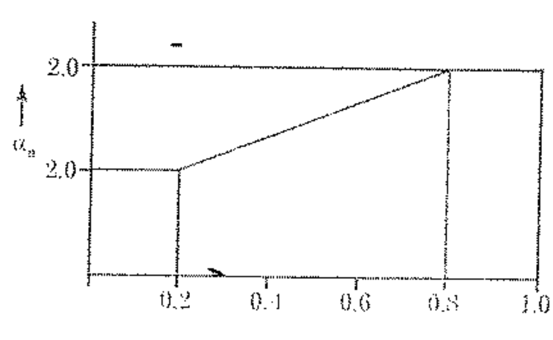


Fig. 6.7

$$\alpha_n = \begin{cases} 1.0 & \text{for } \dfrac{P_u}{P_{uz}} < 0.2 \\ 2.0 & \text{for } \dfrac{P_u}{P_{uz}} > 0.8 \\ 0.667 + 1.667\dfrac{P_u}{P_{uz}} & \text{otherwise} \end{cases}$$

Slender Columns

An essential step in the design of a column is to determine whether the proposed dimensions will make it a short column or a slender column. A short compression member, whose l/D ratio is less than 12, is not in danger of buckling prior to achieving its ultimate strength based on the properties of the cross-section. Moreover, the lateral deflection short compression members subjected to bending moments are small, thus, contributing little secondary bending moment P-Δ as shown in Fig. 6.8. These buckling and additional deflection effects are more pronounced in slender compression members and reduce their ultimate strength as compared to that of a short column having the same cross-section and amount of steel.

Fig. 6.8

P-Δ effect in a column

The CEB-FIP recommendations for buckling in compression and bending advise a check for columns with effective slenderness ratio more than 35. The effective slenderness ratio should not exceed 140 for normal aggregate concrete and 80 for light weight aggregate. However, clause 25.3.1 of the Code restricts maximum slenderness ratio of a column to 60. If in any given plane, one end of a column is unrestraine, its effective length l should not exceed.

$$100\frac{b^2}{D}$$

where, b = width of cross-section

D = depth of cross-section measured in the plane under consideration

In the code, the effect of slenderness is approximated by using a moment magnifier approach, whereby the sum of the primary and secondary moment is used as total design moment. The additional or secondary moments are given by:

$$M_{ay} = \frac{PD}{2000}\left(\frac{l_{ey}}{D}\right)^2$$

and $$M_{ax} = \frac{Pb}{2000}\left(\frac{l_{ex}}{b}\right)^2$$

where,

l_{ey} = effective length in respect of major axis, that is, bending occurs about the major axis
l_{ex} = effective length in respect of minor axis, that is, bending occurs about the minor axis
D = depth of cross-section at right angles to the major axis
b = breadth of cross-section

These expressions are applicable to a balance design of a slender column subjected to uniaxial bending as well as biaxial bending. As the axial load increases from zero, the tensile stress in the steel decreases to zero and changes to a compressive stress. As this occurs, the curvature and deflection decreases. Clause 39.7.1.1 of the code permits a reduction in the additional moments by a factor k given by:

$$k = \frac{P_z - P}{P_z - P_b} \le 1$$

P_z = capacity of cross section under pure axial load
$P_z = 0.45\, f_{ck}\, A_c + 0.75\, f_y\, A_{sc}$
P_b = balance axial load corresponding to the condition of maximum compressive strain of 0.0035 in concrete and tensile strain of 0.002 in the outermost layer of tension steel.

The value of P_b will depened on arrangement of reinforcement and the cover ratio d'/D, in addition to the grades of concrete and steel. The values of P_b can be determined using the following equations:

For rectangular sections, $$P_b = \left(q_1 + \frac{q_2 p}{\sigma_{ck}}\right) f_{ck} b D$$

For circular sections $$P_b = \left(q_1 + \frac{q_2 p}{\sigma_{ck}}\right) f_{ck} D^2$$

The coefficients q_1 and q_2 are given in tables.

DESIGN OF COLUMN BY WSM

SHORT AND LONG (OR SLENDER) COLUMNS

Columns are divided into two types: (i) short column and (ii) long or slender column. When the ratio of effective length of the column to its least lateral dimension is less than 12, it is termed as a short column. A long or slender column is the one whose ratio of effective length to its least lateral dimension is not less than 12. In long columns, the permissible values of stresses in concrete and steel, used for short columns, should be multiplied by a coefficient C_r given by the following formula:

$$C_r = 1.25 - \frac{l_{eff.}}{48b}$$

C_r = reduction coefficient
$l_{eff.}$ = effective length of column
b = least lateral dimension of column.

When in a column, having helical reinforcement, the permissible load is based on core area, the least lateral dimension should be taken as the diameter of the core.

For more exact calculations, the maximum permissible stresses in a reinforced concrete column or part there of having a ratio of effective column length to least lateral radius of gyratio above 50 should not exceed those which result from the multiplication of the appropriate maximum permissible stresses by the coefficient C_r given by the following formula:

$$C_r = 1.25 - \frac{l_{ef}}{160\, i_{min}}$$

where, i_{min} = least radius of gyration

LOAD CARRYING CAPACITY OF SHORT COLUMNS

(a) Short columns with lateral ties: According to the classical elastic theory or the compatible strain theory, when a reinforced concrete column is loaded, both concrete and steel have equal strains during initial stage of loading, because they are well bonded. Hence if P is the load on the column, the loads carried by steel bars (P_s) and that carried by surrounding concrete (P_c) bears the relation

$$P = P_c + P_s \quad \text{---(i)}$$

From compatibility requirements, $\varepsilon_s = \varepsilon_c$

$$\therefore \quad \frac{P_s}{E_s . A_{sc}} = \frac{P_c}{E_c . A_c}$$

or $$\frac{\sigma_{sc}}{E_s} = \frac{\sigma_{cc}}{E_c}$$

or $$\sigma_{sc} = \frac{E_s}{E_c} . \sigma_{cc} = m . \sigma_{cc}$$ where m is the modular ratio

Hence from (i) $P = \sigma_{cc} . A_c + \sigma_{sc} . A_{sc}$

or $$P = \sigma_{cc} . A_c + m \sigma_{cc} . A_{sc} \quad \text{---(ii)}$$

where A_{sc} = Area of steel reinforcement: A_c = Area of concrete

σ_{sc} = Compressive stress in steel; σ_{cc} = Compressive stress in concrete.

It has been found that the steel reinforcement is restrained against lateral expansion, due to concrete surrounding it, reducing the problem to be a plain strain problem rather than a plain stress problem. It is therefore essential to moderate the compatible stress in steel by restrained Poisson's effect, using modified Possion's ration m_c = 1.5 m. Thus stress in steel is given by $\sigma_{sc} = m_c . \sigma_{cc} = 1.5m\, \sigma_{cc}$ and equation (ii) reduces to the form

$$P = \sigma_{cc} . A_c + 1.5m\, \sigma_{cc} . A_{sc} \quad \text{---(iii)}$$

The above theoretical approach, based on strain compatibility, has been proved to behighly unrealistic due to creep and shrinkage behaviour of concrete. The above formula therefore gives very conservative results. It is therefore recommended to use equilibrium approach based on allowable stresses in the two materials. Such an equilibrium approach, however, violates the compatibility of strains. According to this approach, the load carrying capacity of a short column, axially loaded and provided with lateral ties, is given by

$$P = \sigma_{cc} . A_c + \sigma_{sc} . A_{sc} \quad \text{---(iv)}$$

where P = permissible axial load

σ_{sc} = permissible compressive stress in column bars

σ_{cc} = permissible compressive stress in concrete

IS : 456-2000 recommends eq. (iv) for the determination of permissible axial load P. The minimum eccenttricity mentioned in 13.5.7 may be deemed to be incoporated in the above equation.

(b) Short column with helical reinforcement: For a column having longitudinal reinforcement tied with spirals (i.e. helical reinforcement), the load carrying capacity is taken as 1.05 times the strength of similar member with lateral ties, provided the requirements laid down in para 8b of 13.5 are satisfied.

(c) Composite columns: The allowable load P on a composite column, consisting of a structural steel or cast-iron column thoroughly encased in concrete reinforced with both longitudinal and spiral reinforcement is given by the following expression:

$$P = \sigma_{cc}.A_c + \sigma_{sc}.A_{sc} + \sigma_{mc}.A_m$$

where A_c = net area of concrete section

A_{sc} = cross-sectional area of longitudinal bar reinforcement

σ_{mc} = allowable unit stress in metal core, not to exceed 125 N/mm² for a steel core, or 70 N/mm² for a cast iron core.

A_m = Cross-sectional area of steel or cast iron core.

Example 6

Design a square section column using M 15 concrete and mild steel bars to cary an axial load (P) of 30,000 Kg. Effective length of column (l_{eff}) = 4 m. Assume permissible stresses in direct compression in M 15 concrete (σ_{cc}) and in mild steel bars (σ_{sc}) as 40 and 1300 kg/cm² respectively. As per IS code

$$P = C_R\,(\sigma_{cc}A_c + \sigma_{sc}A_s)$$

where A_c and A_s are areas of cross-section of concrete and steel respectively.

$$C_R = \text{reduction coefficient} = 1.25 - \frac{l_{eff}}{48b} \ngtr 1.0$$

b = lateral dimension of column. Sketch the arragement for longitudinal and lateral reinforcement

Sol: Designing the column as a short column.

Assuming area of steel = 1% of gross area of column

Let us design a square section of size = $b \times b$

$$A = B^2$$

$$A_{sc} = \frac{1}{100}b^2 = 0.01b^2$$

$$A_{cc} = A - A_{sc} = b^2 - 0.01b^2 = 0.99b^2$$

Now, for a short column

$P = \sigma_{cc}A_{cc} + \sigma_{sc}A_c$ $\quad [\because \sigma_{cc} = 40 \text{ kg/cm}^2 = 4 \text{ MPa}, \; \sigma_{sc} = 1300 \text{ kg/cm}^2 = 130 \text{ MPa}]$

$$\Rightarrow \quad 30{,}000 \times 10 = 4 \times 0.99b^2 + 130 \times 0.01b^2$$

$$\Rightarrow \quad b = 238.81 \approx 240 \text{ mm}$$

Ckeck

$\frac{l_{eff}}{b} = \frac{4000}{240} = 16.67 > 12$. Hence column is a long column.

Now,

$$C_R = 1.25 - \frac{l_{eff}}{48b}$$

$$= 1.25 - \frac{4000}{48 \times 240}$$

$$C_R = 0.903$$

$$\therefore \quad P = C_R(\sigma_{cc}A_c + \sigma_{sc}A_{sc})$$

$$\Rightarrow \quad 30{,}000 \times 10 = 0.903\left[4(240 \times 240 - A_{sc}) + 130A_{sc}\right]$$

$$\Rightarrow \quad \frac{30{,}000 \times 10}{0.903} = 230400 + 126\,A_{sc}$$

$$\Rightarrow \quad A_{sc} = 808.14 \text{ mm}^2$$

Adopting 16 mm ϕ bar, we get A $\phi = \frac{\pi}{4} \times 16^2 = 201.06$ mm²

$\therefore$ Number of 16 mm ϕ bars $= \frac{808.14}{201.06} = 4.0$ bars

Diameter of stirrups or lateral ties

(i) $\frac{1}{4} \times 16 = 4$ mm

(ii) 6 mm

$\therefore$ Provide 6 mm ϕ bars as laterla ties

Spacing of lateral ties.

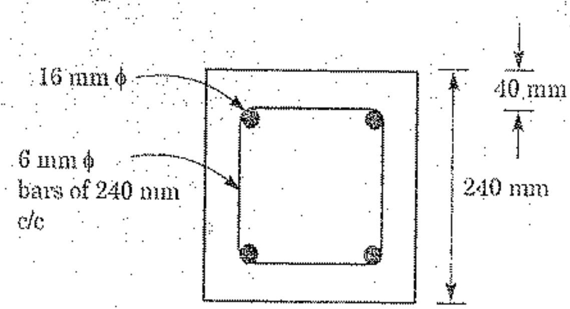
(i) least lateral dimension $= b = 240$ mm

(ii) $16 \times 16 = 256$ mm

(iii) 300 mm

Hence providing 6 mm bars at 240 mm c/c

The reinforcement details are as follows:



Example 7

A RC column 4 m long and 400 mm in dia. is reinforced is 8 bar of 20 mm dia. Find safe load of column concrete carry $\sigma_{cc} = 4$ N/mm², $\sigma_{sc} = 130$ N/mm².

Sol: $A_{sc} = 8 \times \frac{\pi}{4} \times 20^2 = 2513.27$ mm², $A_c = \frac{\pi}{4} \times 400^2 - 2531.27 = 123068.73$ mm²

$P = \sigma_{cc}A_c + \sigma_{sc}A_{sc} = 4 \times A_c + 130\,A_{sc}$

$P = 4 \times 123068.73 + 130 \times 2513.27$

$P = 818.72$ kN

Example 8

Design a short squre column to carry on axial load 1200 kN. Use M25. $\sigma_{cc} = 6\ N/mm^2$, $\sigma_{sc} = 130\ N/mm^2$.

Sol: For short column $P = \sigma_{cc}A_c + \sigma_{sc}A_{sc}$

Assume $A_{sc} = 0.8A \times \frac{1}{100} = 0.008A$

$\Rightarrow$ $120 = 6 \times (A - 0.008A) + 130 \times 0.008\ A \Rightarrow A = 171624.7\ mm^2$

$\therefore$ $B \approx \sqrt{A} = 420$ mm

$\therefore$ $A_{sc} = 0.8\%$ of $A = 1411.2\ mm^2$

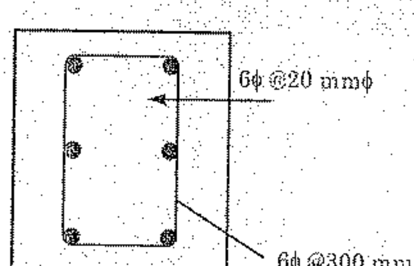
Use 20 mm ϕ bar no. of bar = 4.49 $\simeq$ 6 nos.

% of steel = $A_{sc}/BD \times 100 = 1.068\% > 0.8$ so O.K.

Transverse reinforced max. of (i) $\frac{1}{4}$ of 20 = 5 (ii) 6 mm provide 6 mm.

Spacing min. of (i) 420 (ii) 16 × 20 = 320 (iii) 300 m.

Provide 6 mm ϕ bar 300 mm, c/c as ties.



Example 9

If the column is 12 m long and effective held in position at both end and restrain against rotation at one end. And carries load of 1200 kN. Design a column using M25 and $\sigma_{sc} = 130\ N/mm^2$ and $\sigma_{cc} = 6\ N/mm^2$.

Sol: $L_{eff} = 0.8 \times 12 = 9.6$ m. Let us design a long and square column $c_r = 1.25 - \frac{9[illegible]}{B}$

$$P = c_r(\sigma_{cc}A_c + \sigma_{sc}A_{sc})$$

Assume $A_{sc} = 0.008A$ then, $1200 \times 10^3 = \left(125 - \frac{9600}{B}\right)\left[6(A - 0.008A) + 130 \times 0.008A\right]$

$\Rightarrow$ $1200 \times 10^3\ B = (1.25B - 9600)(7.0352)A$

$\Rightarrow$ $1200 \times 10^3 B = 8.8.B.B^2 - 67537.9.B \Rightarrow 8.8B^2 - 67537.9B - 1200 \times 10^3 = 0$

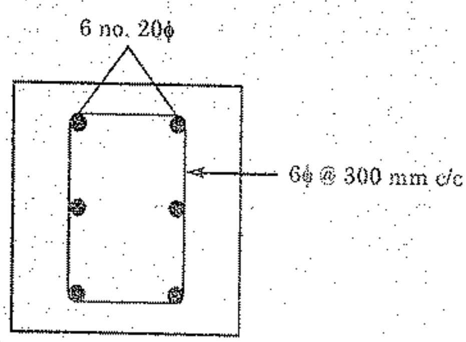
$B = 470$ mm, $S.R. = \frac{9600}{470} = 20.45 > 12$ O.K.

$\therefore A_{sc} = 0.008 \times B^2 = 1767.2$

Use 20 ϕ bar then no. of bar = 5.6 = 6 nos.

Transverse reinforced max of (i) $\frac{1}{4}$ of 20 = 5 (ii) 6 mm provide 6 mm spacing min. of (i) $B = 470$ (ii) $16 \times 20 = 320$ (iii) 300 mm

Provide 6 mm transverse reinforced 300 mm c/c as ties.



Design of Spacing of Helical Reinforcement for a Circular Column

Following criteria requried to be satisfied for design of helical reinforcement

$$0.36 \frac{f_{ck}}{f_y}\left(\frac{A_g}{A_c} - 1\right) \le \frac{V_h}{V_c}$$

where f_{ck} = characteristic st. of concrete

f_y = characteristic st. of steel.

A_g = gross c/s area.

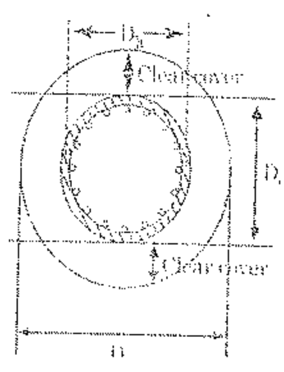
A_c = Core area.

V_h = Vol. of helical R/F in unit length (1m)

V_c = Volume of core for unit length of column.

(1) Gross dia (D_g) = Total dia. of column

$$\text{Gross Area} = A_g = \frac{\pi}{4} \times (D_g)^2$$



Core diameter $D_c = D_g - 2 \times$ clear cover to tie

$$D_c = D_g - 2\ d_c$$

$$A_c = \frac{\pi}{4} D_c^2$$

Core volume V_c for unit length (for 1 m) of column

$$\boxed{V_c = 1000 \times A_c}$$

Now,

$$D_h = D_c - \phi_h$$

V_h = (No. of turns) × length is one turn × c/s area of helical bar

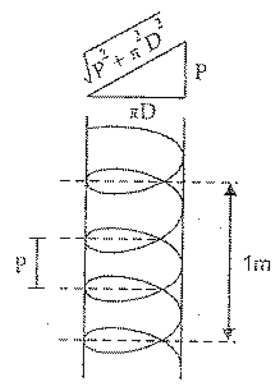
$$= \frac{1000}{p} \times \pi D_h \times \frac{\pi}{4} \phi_h^2$$

$$\boxed{\frac{1000}{p} \times \pi D_h \times \frac{\pi}{4} \phi_h^2}$$

where P = pitch of helix

D_h = diameter of helix

ϕ_h = diameter of helical reinforcement



Example 10

Design a short circular col. of dia. 700 mmfor a load 5000 kN use helical reinf. Using M25/mild steel. Calculate area of steel require and spacing of helical reinf. Use clear cover to ties = 40 mm.

Sol.

$$P = [A_c \sigma_{cc} + A_{sc} \sigma_{sc}] \times 1.05$$

(Short Column)

$$A = \frac{\pi}{4} \times 700^2 = 384845.0 \text{ mm}^2$$

For M25 → σ_{cc} = 6 N/mm² (Direct stress)

Fe250 → σ_{sc} = 130 N/mm²

$$A_c = A - A_{sc}$$

$$5000 \times 10^3 = [(A - A_{sc}) \times 6 + A_{sc} \times 130] \times 1.05$$

$$= [(384845 - A_{sc}) \times 6 + A_{sc} \times 130] \times 1.05$$

$$A_{sc} = 19780.9 \text{ mm}^2$$

$$\text{Use 40 mm } \phi \text{ bar} = \frac{19780.9}{\frac{\pi}{4} \times 40^2}$$

$$= 15.74 = 16 \# 40$$

% of steel]

$$\frac{A_{st}}{A} \times 100 = \frac{19780.9}{384845} \times 100 = 5.14\%$$

[can be designed if bars are not lapped]

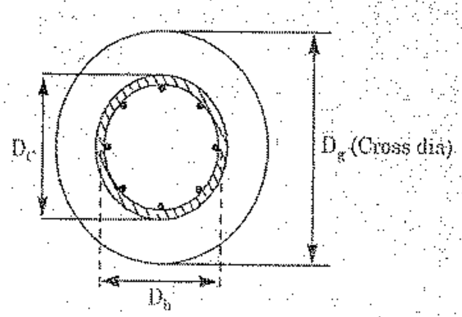
Helical Reinforcement

(1) (i) $\frac{1}{4}d = \frac{1}{4} \times 40 = 10$ mm
(ii) 6 mm] Use 10 mm dia.

(2) 6 mm

For helical Reinforce

$$0.36\frac{f_k}{f_y}\left[\frac{A_g}{A_c}-1\right] \le \frac{V_h}{V_c}$$



$$A_g = \frac{\pi}{4} \times 700^2 = 384845 \text{ mm}^2$$

$$D_c = 700 - 40 - 40 = 620 \text{ mm}$$

$$V_c = A_c \times 1m = \frac{\pi}{4} \times 620^2 \times 1000$$

$$= 301907054 \text{ mm}^3$$

Assuming spacing = S

$$V_h = \text{No. of turn} \times \text{length in on turn} \times \text{c/s area.}$$

$$= \frac{1000}{S} \times \pi D_h \times \frac{\pi}{4}\phi_h^2$$

$$D_h = 700 - 2 \times 40 - \frac{10}{2} - \frac{10}{2}$$

$$= 610 \text{ mm}$$

$$\therefore \quad V_h = \frac{1000}{S} \times \pi \times 610 \times \frac{\pi}{4} \times 10^2$$

$$= \frac{150511467.1}{S}$$

$$0.36\left[\frac{Ag}{Ac}-1\right]\frac{f_k}{f_y} \le \frac{V_h}{V_c}$$

$$0.36\left[\frac{384845}{301907.05}-1\right]\times\frac{25}{250} \le \frac{150511467.1}{S\times 301907050}$$

$$S = 50.4 \text{ mm}$$

Spacing (i) $\not> 75$ mm

(ii) $\not> \frac{1}{6}D_c = \frac{1}{6}\times 610 = 103$ mm

(iii) $\not< 3\phi = 3\times 10 = 30$ mm

Use 10 mm ϕ helical reinf. @ 50 mm c/c

COMBINED DIRECT & BENDING STRESS

INTRODUCTION

- In many cases, a reinforced concrete member is subjected to both direct load and bending moment, or to a direct load acting eccentrically.
- Exterior columns of a framed structure, arches, chimneys, silos, bunkers etc. are some of the common examples of such combination of loading. In some cases, the forces may be compression and bending, while in other cases, the forces may be tension and bending.
- The effect of the centrally applied load P and a moment (M) to the direct load (P) is generally termed as eccentricity.
- The analysis and design of such members evidently depend upon the value of eccentricity.
- If the eccentricity is small so that section is subjected to the same kind of stress (i.e. either compressive or tensile), the analysis is slightly simplied.
- If the eccentricity is large, stress reversal may take place in the section, necessitating the determination of neutral axis.
- Based on the value of eccentricity, and also upon the nature of direct force (i.e. compressive or tensile), six cases may arise.

(a) Combined Compression and Bending

Case 1: *Eccentricity small :* When eccentricity is smaller than one-fourth the depth of the section: In such a case, no reversal of stresses takes place, or even if tension is induced, the resultant tension in concrete is not greater than 35 percent and 25 percent of the resultant compression for biaxial and uniaxial bending respectively, or exceeds three-fourths the 7-day modulus of rupture of concrete. The design of such a section is based on uncracked section.

Case 2: *Eccentricity large:* When the eccentricity is larger than 1.5 times the depth of the section.

Case 3: *Intermediate eccentricity:* When the eccentricity is between 0.25 and 1.5 times the depth of the section.

Case 4: *Eccentricity small:* The section is subjected to tensile stresses throughout.

Case 5: *Eccentricity large.*

Case 6: *Intermediate eccentricity.*

CASE 1: COMPRESSIVE LOAD AT ECCENTRICITY SMALLER THAN D/4

Figure (6.9) shows a R.C. section of size b × D, subjected to a compressive load P at a distance e from the middle of the section. This eccentric load is equivalent to a direct load P acting through the C.G. of the section, and a bending moment equal to P × e′ where e′ is the eccentricity of the load measured from the centre of gravity of the section. It is assumed that eccentricity is small so that no stress reversal takes place, or even if tension is developed the whole area of concrete is effective.

Let A_{sc1}, A_{sc2}, A_{sc3} be the steel areas at various levels y_1, y_2, y_3, etc. from the top fibre (AB) as marked in Fig. 6.9.

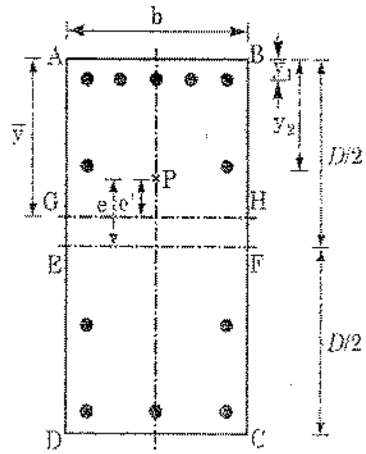


Fig. 6.9

The equivalent area of section is given by

$$A_e = bD + \Sigma(1.5m - 1)A_{sc}$$

The distance $\bar{y}$ of the centre of gravity, measured from AB, is

$$\bar{y} = \frac{(1.5\,m-1)(A_{sc1}y_1 + A_{sc2}y_2 + ...) + bD^2/2}{(1.5m-1)(A_{sc1} + A_{sc2} + ...) + bD}$$

$$= \frac{bD^2/2 + \Sigma(1.5\,m-1)A_{sc}y}{bD + \Sigma(1.5m-1)A_{sc}}$$

where $y = \Sigma y_1 + y_2 + y_3 + ...$

$\Sigma A_{sc} = A_{sc1} + A_{sc2} + ...$

The equivalent moment of inertia of the section is given by

$$I_e = (1.5\,m-1)(A_{sc1}a_1^2 + A_{sc2}a_2^2 + ...) + \frac{bD^3}{12} + BD\left(\bar{y} - \frac{D}{2}\right)^2$$

$$I_e = \Sigma(1.5\,m-1)A_{sc}a^2 + \frac{bD^3}{12} + BD\left(\bar{y} - \frac{D}{2}\right)^2$$

where a_1, a_2, a_3, ... etc. are the distances of the steel areas from the centre of gravity of the section.

Now M = load × eccentricity.

$$\therefore \quad M = P \times e' = P\left\{e - \left(\frac{D}{2} - \bar{y}\right)\right\}$$

The stress distribution in the column may be determined from the expression:

$$\sigma = \frac{P}{A_e} \pm \frac{M.y}{I_e}.$$

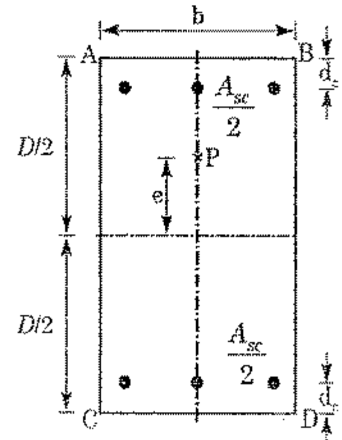


Fig. 6.10

Hence the maximum and minimum compressive stresses are given by:

$$\sigma_1 = \frac{P}{A_e} + \frac{P\left\{e - \left(\frac{D}{2} - \bar{y}\right)\right\}}{I_e}.\bar{y}$$

$$\sigma_2 = \frac{P}{A_e} - \frac{P\left\{e - \left(\frac{D}{2} - \bar{y}\right)\right\}}{I_e}.(D - \bar{y})$$

Figure 6.10 shows a symmetrically reinforced section, where $\bar{y} = D/2$

$$A_e = bD + (1.5\,m - 1)A_{sc}$$

$$I_e = \frac{1}{12}bD^3 + (1.5\,m - 1)A_{sc}\left(\frac{D}{2} - d_c\right)^2$$

Hence maximum compressive stress at AB is given by

$$\sigma_1 = \frac{P}{A_e} + \frac{P.e}{I_e}.\frac{D}{2}$$

Minimum stress at DC is

$$\sigma_2 = \frac{P}{A_e} - \frac{P.e}{I_e}.\frac{D}{2}$$

If no tension is to develop in the section, σ_2 must be equal to zero.

Hence $\frac{P}{A_e} - \frac{P.e}{I_e}.\frac{D}{2} = 0.$

which gives

$$e = \frac{2I_e}{A_e.D}$$

A member subjected to axial load and bending (due to eccentricity of load, monolithic construction, lateral forces, etc.) should be considered safe provided the following condition is satisfied:

$$\frac{\sigma_{cc'}}{\sigma_{cc}}+\frac{\sigma_{cbc'}}{\sigma_{cbc}}\le 1$$

where, σ_{cc} = permissible direction compressive stress in concrete

$\sigma_{cc'}$ = calculated direct compressive stress in concrete

σ_{cbc} = permissible bending compressive stress in concrete

$\sigma_{cbc'}$ = calculated bending compressive stress in concrete.

$$\text{Stress in steel} = \sigma_{s'} = 1.5m\left[\frac{P}{A_e}+\frac{P.e}{I_e}\left(\frac{D}{2}-d_c\right)\right]$$

6.2 BENDING ABOUT TWO AXES

Consider a symmetrically reinforced column subjected to an axial load P, a bending moment M_x about x–axis and a moment M_y about y-axis. Let,

I_{ex} = equivalent moment of inertia of the section about x–x axis.

I_{ey} = equivalent moment of inertia of the section about y–y axis.

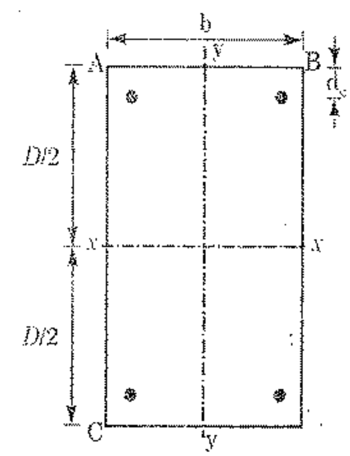


Fig. 6.11

The stress f at any point (x, y) can be determined from the expression:

$$f = \frac{P}{A_e}\pm\frac{M_x.y}{I_{ex}}\pm\frac{M_y.x}{I_{ey}}$$

The maximum direct stress $\sigma_{cc'} = \frac{P}{A_e}$

Maximum bending stress $\sigma_{cbc'} = \frac{M_x}{I_{ex}}.\frac{D}{2}+\frac{M_y}{I_{ey}}.\frac{B}{2}$

The section will be safe if $\frac{\sigma_{cc'}}{\sigma_{cc}}+\frac{\sigma_{cbc'}}{\sigma_{cbc}}\le 1$

6.3 DESIGN OF COLUMNS SUBJECTED TO COMBINED BENDING AND DIRECT STRESSES [IS: 456-2000]

The design of a column subjected to combined bending and direct stresses is that of trial and check. A suitable size of the column and steel reinforcement are first chosen. The preliminary section so selected is then checked for stresses in concrete and steel. The following notes refer to the design of such sections.

1. Design Based on Uncracked Section

A member subjected to axial load and bending (due to eccentricity of load, monolithic construction, lateral forces etc.) shall be considered safe provided the following conditions are satisfied:

(a) $\frac{\sigma_{cc'}}{\sigma_{cc}} + \frac{\sigma_{cbc'}}{\sigma_{cbc}} \le 1$

where

$\sigma_{cc'}$ = calculated direct compressive stress in concrete

σ_{cc} = permissible axial compressive stress in concrete

$\sigma_{cbc'}$ = calculated bending compressive stress in concrete

σ_{cbc} = permissible bending compressive stress in concrete.

(b) The resultant tension in concrete is not greater than 35 percent and 25 percent of the resultant compression for biaxial and uniaxial bending respectively, or does not exceed three-fourths the 7-day modulus of rupture of concrete.

Note: (i) $\sigma_{cc'} = \dfrac{P}{A_c + 1.5mA_{sc}}$ for columns with ties.

(ii) $\sigma_{cc'}$ = M/Z, where M equals the moment and Z equals modulus of section. In the case of sections subject to moments in two directions, the stress shall be calculated separately and added algebraically.

2. Design Based on Cracked Section

If the requirements specified in clause 1 above are not satisfied, the stresses in concrete and steel shall be calculated by theory of cracked section in which tensile resistance of concrete is ignored. If the calculated stresses are within the permissible stresses, the section may be assumed to be safe.

Note: The maximum stress in concrete and steel may be found from tables and charts based on the uncracked section theory or directly by determining the no-stress line which should satisfy the following requirements:

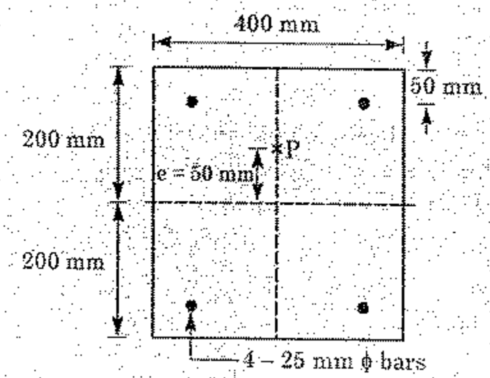
(a) The direct load should be equal to the algebraic sum of the forces on concrete and steel.

(b) The moment of the external loads about any reference line should be equal to the algebraic sum of the moment of the forces in concrete (ignoring the tensile force in concrete) and steel about the same line, and

(c) The moment of the external loads about any other reference lines should be equal to the algebraic sum of the moment of the forces in concrete (ignoring the tensile force in concrete) and steel about the same line.

Example 11

A R.C. column 400 mm × 400 mm is reinforced with 4 bars of 25 mm diameter, placed at a cover of 50 mm to the centre of steel bars. Determine the maximum and minimum stresses in concrete if the column is subjected to a load of 400 kN at an eccentricity of 50 mm about on of the axes. Also, check whether the section is safe or not. Use M 15 concrete, taking m = 19.



Sol. For M 15 concrete, $\sigma_{cc} = 4\text{N/mm}^2$ and $\sigma_{cbc} = 5\text{N/mm}^2$. This is the case of small eccentricity $\left(e < \frac{D}{4}\right)$.

However while calculating the equivalent area and moment of inertia, modular ratio 1.5 m should be used with the steel area in compression.

$$A_{sc} = 4\frac{\pi}{4}(25)^2 = 1960 \text{ mm}^2$$

$$A_e = A_c + 1.5m\ A_{sc} = (A - A_{sc}) + 1.5\ m\ A_{sc}$$

$$= A + (1.5m - 1)A_{sc}$$

$$= (400 \times 400) + (1.5 \times 19 - 1)1960 = 213900 \text{ mm}^2$$

$$I_e = \frac{1}{12}bD^3 + (1.5m - 1)A_{sc}\left(\frac{D}{2} - d_c\right)^2$$

$$= \frac{1}{12} \times 400(400)^3 + (1.5 \times 19 - 1) \times 1960(200 - 50)^2$$

$$= 33.46 \times 10^8 \text{ mm}^4$$

$$\sigma_{cc'} = \frac{P}{A_e} = \frac{400000}{213900} = 1.87 \text{ N/mm}^2$$

$$\sigma_{cbc'} = \frac{P.e}{I_e} \times \frac{D}{2} = \frac{400000 \times 50}{33.46 \times 10^8} \times 200 \approx 1.20 \text{ N/mm}^2$$

$\therefore$ Max. stress $= \sigma_{cc'} + \sigma_{cbc'} = 1.87 + 1.20 = 3.07 \text{ N/mm}^2$

Min. stress $= \sigma_{cc'} - \sigma_{cbc'} = 1.87 - 1.20 = 0.67 \text{ N/mm}^2$

For the section to be safe, we have $\frac{\sigma_{cc'}}{\sigma_{cc}} + \frac{\sigma_{cbc'}}{\sigma_{cbc}} \le 1$ or $\frac{1.87}{4} + \frac{1.20}{5} \le 1$

or $0.71 \le 1$. Hence safe.

Example 12

Determine the maximum eccentricity of load in Example 11, if the column is to be safe as uncracked section.

Sol. For safety, we have $\frac{\sigma_{cc'}}{\sigma_{cc}}+\frac{\sigma_{cbc'}}{\sigma_{cbc}}\le 1$

Hence $$\frac{\sigma_{cbc'}}{\sigma_{cbc}} = 1-\frac{\sigma_{cc'}}{\sigma_{cc}} = 1-\frac{1.87}{4} = 1-0.4675 = 0.5325$$

$$\therefore \quad \sigma_{cbc'} = 0.5325\sigma_{cbc} = 0.5325\times 5 = 2.66\ \text{N/mm}^2$$

$$\therefore \quad \sigma_{cbc'} = \frac{P.e}{I_e}\times\frac{D}{2} = 2.66$$

$$e = 2.66\times\frac{2I_e}{P.D} = \frac{2.66\times 2\times 33.46\times 10^8}{400000\times 400} = 111\ \text{mm}^2$$

Example 13

If the column of Example 11 is subjected to an axial load of 400 kN along with a bending moment at 25 kN-m, determine the maximum and minimum stresses in concrete. Will the section be safe as an uncracked section?

Sol. A_e = 213900 mm^2 and I_e = 33.46 × 10^8 mm^4, as found in example 14.1.

$$\therefore \quad \sigma_{cc'} = \frac{P}{A_e} = \frac{400000}{213900} = 1.87\ \text{N/mm}^2$$

$$\text{B.M.} = M = 25\ \text{kN-m} = 25\times 10^6\ \text{N-m}$$

$$\therefore \quad \sigma_{cbc'} = \frac{M}{I_e}\cdot\frac{D}{2} = \frac{25\times 10^6}{33.46\times 10^8}\times\frac{400}{2} = 1.49\ \text{N/mm}^2$$

$$\text{Max. Stress} = \sigma_{cc'}+\sigma_{cbc'} = 1.87+1.49 = 3.36\ \textbf{N/mm}^2$$

$$\text{Min. Stress} = \sigma_{cc'}-\sigma_{cbc'} = 1.87-1.49 = \mathbf{0.38\ N/mm^2}$$

For the section to be safe as an uncracked section,

$$\frac{\sigma_{cc'}}{\sigma_{cc}}+\frac{\sigma_{cbc'}}{\sigma_{cbc}}\le 1$$

$$\text{or} \quad \frac{1.87}{4}+\frac{1.49}{5}\le 1$$

$$\text{or} \quad 0.766\le 1$$

Hence safe

Example 14

A rectangular section 300 mm × 400 mm is reinforced with 8 bars of 20 mm ϕ as shown in figure. Taking σ_{cc} = 4N/mm², σ_{cbc} = 5 N/mm² and m = 19, determine

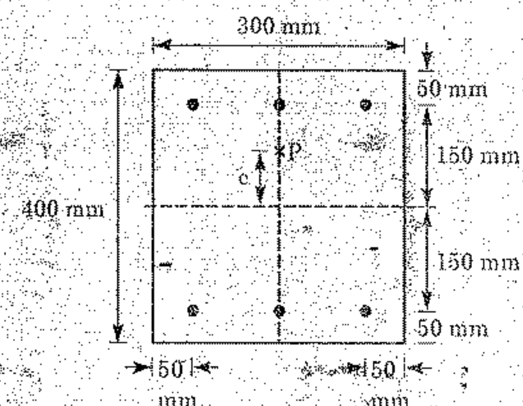
(i) maximum eccentricity about x-axis at which load can be applied without developing tension in the section,

(ii) the magnitude of the load

Sol. $A_{sc} = 8\times\frac{\pi}{4}(20)^2 = 2513 \text{ mm}^2$

$A_e = bD + (1.5m - 1)A_{sc} = (300 \times 400) + (1.5 \times 19 - 1) \times 2513 = 189108 \text{ mm}^2$

$I_e = \frac{1}{12}\times 300(400)^3 + (1.5\times 19 - 1)\times 6\times\frac{\pi}{4}(20)^2(150)^2 = 27.663 \times 10^8 \text{ mm}^4$



For no tension to develop, we have

$$\frac{P}{A_e} - \frac{P.e}{I_e}\cdot\frac{D}{2} = 0$$

$$\text{or} \quad e = \frac{2I_e}{A_e D} = \frac{2\times 27.663\times 10^8}{189108\times 400} = 73.14 \text{ mm}$$

Again, if P is the maximum load, we have

$$\sigma'_{cc} = \frac{P.e}{I_e}\cdot\frac{D}{2} = \frac{P}{I_e}\left[\frac{2I_e}{A_e.D}\right]\times\frac{D}{2} = \frac{P}{A_e} = \frac{P}{189108}$$

(**Note:** Both $\sigma_{cc'}$ and $\sigma_{cbc'}$ will be equal at the critical value of e). Hence,

Now, $\frac{\sigma_{cc'}}{\sigma_{cc}} + \frac{\sigma_{cbc}'}{\sigma_{cbc}} \le 1$

or $\quad \frac{P}{189108}\left[\frac{1}{4}+\frac{1}{5}\right] = 1$

or $\quad P = \frac{189108\times 4\times 5}{9} = 420240 \text{ N} = 420.24 \text{ kN}$

Example 15

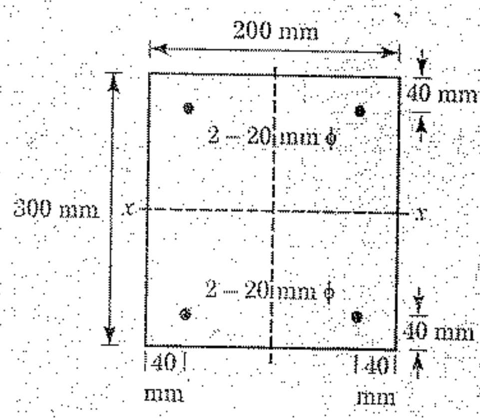
A R.C. section 200 mm × 300 mm is reinforced with 4 bars of 20 mm Φ arranged as shown in figure. The section is subjected to (i) axial load of 200 kN (ii) bending moment of 3 kN-m about x-axis, and (iii) bending moment of 2 kN-m about y-axis. Taking $\sigma_{cc} = 4\ N/mm^2$ and $\sigma_{cbc} = 5\ N/mm^2$, find whether the section is safe or not. Take m = 19.

Sol:

$$A_{sc} = 4 \times \frac{\pi}{4}\,(20)^2 = 1256\text{ mm}$$

$$A_e = (200 \times 300) + (1.5 \times 19 - 1) \times 1256\,(150 - 40)^2$$
$$= 8.679 \times 10^8\text{ mm}^4$$

$$I_{ey} = \frac{1}{12}\,300\,(200)^3 + (1.5 \times 19 - 1)\,1256\,(100 - 40)^2$$
$$= 3.243 \times 10^8\text{ mm}^4$$



$$\sigma_{cc}' = \frac{P}{A_e} = \frac{200000}{94540} = 2.12\text{ N/mm}^2$$

$$(\sigma_{cbc})_x' = \frac{M_x}{I_{ex}}\,\frac{D}{2} = \frac{3000 \times 1000}{8.679 \times 10^8} \times 150 = 0.52\text{ N/mm}^2$$

$$(\sigma_{cbc})_y' = \frac{M_y}{I_{ey}}\,\frac{b}{2} = \frac{2000 \times 1000}{3.243 \times 10^8} \times 100 = 0.62\text{ N/mm}^2$$

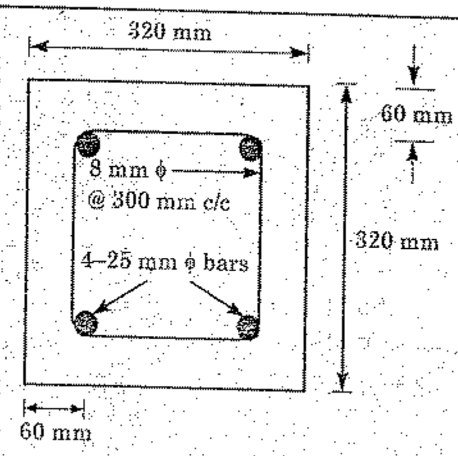
∴ Total bending stress $\sigma_{cbc}' = (\sigma_{cbc})_x' + (\sigma_{cbc})_y' = 0.52 + 0.62 = 1.14\text{ N/mm}^2$

Now $\frac{\sigma_{cc}'}{\sigma_{cc}} + \frac{\sigma_{cbc}'}{\sigma_{cbc}} \le 1$ or $\frac{2.12}{4} + \frac{1.14}{5} < 1$ or $0.757 < 1$. Hence safe.

Example 16

Design a suitable column section to carry an axial load of 430 kN and bending moment of 8 kN-m. The allowable stresses may be taken as follows: $\sigma_{cc} = 4\ N/mm^2$; $\sigma_{cbc} = 5\ N/mm^2$ and m = 19.

Sol: Let us assume a section of 320 mm × 320 mm, reinforced symmetrically with 4 bars of 25 mm Φ as shown in Fig. 14.10. provide a cover of 60 mm upto the centre of steel.



$$A_s = 4\ \frac{\pi}{4}\ (25)^2 = 1963.5\ \text{mm}^2$$

$$A_e = (320 \times 320) + (1.5 \times 19 - 1)\ 1963.5 = 156382\ \text{mm}^2$$

$$I_e = \frac{1}{12} \times 320\ (320)^3 + (1.5 \times 19 - 1)\ 1963.5 \left(\frac{320}{2} - 60\right)^2$$

$$= 14.138 \times 10^8\ \text{mm}^4. \text{ Hence,}$$

$$\sigma_{cc}' = \frac{P}{A_e} = \frac{430000}{156382} = 2.75\ \text{N/mm}^2$$

$$\sigma_{cbc}' = \frac{M}{I_e} \cdot \frac{D}{2} = \frac{8000 \times 1000}{14.138 \times 10^8} \times 160 = 0.905\ \text{N/mm}^2$$

Now $\frac{\sigma_{cc}'}{\sigma_{cc}} + \frac{\sigma_{cbc}'}{\sigma_{cbc}} = \frac{2.75}{4} + \frac{0.905}{5} = 0.869 < 1$

Hence safe. Provide 8 mm Φ ties @ 300 mm c/c.

Practice Objective Questions

1. Which one of the following statements is correct?
   (a) Maximum longitudinal reinforcement in an axially loaded short column is 6% of gross sectional area
   (b) Columns with circular section are provided with transverse reinforcement of helical type only
   (c) Spacing of lateral ties cannot be more than 16 times the diameter of the tie bar
   (d) Longitudinal reinforcement bar need not be in contact with lateral ties

2. The limits of percentage 'p' of the longitudinal reinforcement in a column is
   (a) 0.15% to 2%
   (b) 0.8% to 4%
   (c) 0.8% to 6%
   (d) 0.8% to 8%

3. Match **List-I** with **List-II** regarding the minimum concrete cover to reinforcing steel and select the correct answer using the codes given below the lists:

List-I

A. For longitudinal reinforcement in columns of size 200 mm and less, with 12 mm diameter bars as longitudinal steel.

B. For longitudinal reinforcement in beams.

C. For longitudinal bars in slabs.

D. For longitudinal bars in columns of size more than 200 mm.

List-II

1. 40 mm or diameter of bar whichever is more
2. 15 mm or diameter of bar whichever is more
3. 25 mm or diameter of bar whichever is more
4. 25 mm

Codes:

	A	B	C	D
(a)	4	3	2	1
(b)	1	2	3	4
(c)	1	3	2	4
(d)	4	2	3	1

4. The load carrying capacity of a column designed by working stress method is 500 kN. The collapse load of the column is

(a) 500.0 kN
(b) 662.5 kN
(c) 750.0 kN
(d) 1100.0 kN

5. The reduction coefficient of a reinforced concrete column with an effective length of 4.8 m and size 250 mm × 300 mm is

(a) 0.80
(b) 0.85
(c) 0.90
(d) 0.95

6. Given that d = effective depth, b = width and D = overall depth, the maximum area of compression reinforcement in a beam is

(a) 0.04bd
(b) 0.04bD
(c) 0.12bd
(d) 0.[illegible]bD

7. Match **List-I** (Stress) with **List-II** (Nature) and select the correct answer using the codes given below the lists:

List-I

A. Bend stress

B. Thermal stress

C. Hoop stress

D. Torsional stress

List-II

1. Zero at centre of cross-section
2. Circumferential stress
3. Linear stress
4. Longitudinal shear stress
5. Zero on the surface

Codes:

	A	B	C	D
(a)	5	2	4	1
(b)	4	3	2	1
(c)	4	5	2	3
(d)	1	3	4	5

8. Lateral ties in RC columns are provided to resist

(a) bending moment
(b) shear
(c) buckling of longitudinal steel bars
(d) both bending moment and shear

9. In an axially loaded spirally reinforced short column, the concrete inside the core is subjected to

(a) bending and compression
(b) biaxial compression
(c) triaxial compression
(d) uniaxial compression

10. In a Pedestal, the factor by which the effective length should not exceed the least lateral dimension is

(a) 2
(b) 3
(c) 4
(d) 5

11. Which of the following are the additional moments considered for design of slender compression member in lieu of deflection in x and y directions?

(a) $\frac{P_u l_{ex}^2}{2000\,D}$ and $\frac{P_u l_{ey}^2}{2000\,D}$
(b) $\frac{P_u l_{ex}}{2000\,D}$ and $\frac{P_u l_{ey}}{2000\,D}$
(c) $\frac{P_u l_{ex}^2}{2000\,D}$ and $\frac{P_u l_{ey}^2}{2000\,b}$
(d) $\frac{P_u l_{ex}^2}{200\,D}$ and $\frac{P_u l_{ey}^2}{200\,D}$

where P_u is axial load; l_{ex} and l_{ey} are effective lengths in respective directions; D is depth of section perpendicular to major axis; b is width of the member.

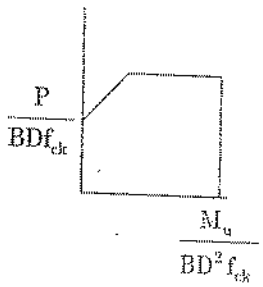
12. What is the minimum number of longitudinal bars provided in a reinforced concrete column of circular cross-section?

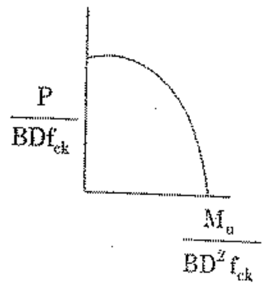
(a) 4
(b) 5
(c) 6
(d) 8

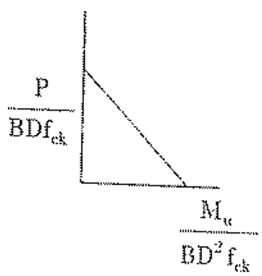
13. An axially loaded column is of 300 mm × 300 mm size. Effective length of column is 3 m. What is the minimum eccentricity of the axial load for the column?

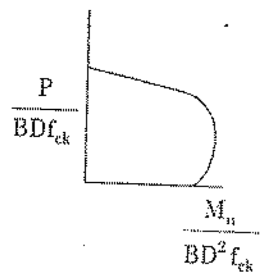
(a) 0
(b) 10 mm
(c) 16 mm
(d) 20 mm

14. A rectangular reinforced column (B × D) has been subjected to uniaxial bending moment M and axial load P. Characteristic strength of concrete = f_{ck}. Which one among the following column design curves shows the relation between M and P qualitatively?

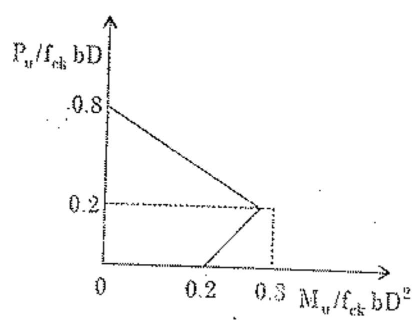
(a)


(b)


(c)


(d)


15. A RC column of square cross-section (400×400 mm^2) has its column load-moment interaction diagram as shown in figure below.



What is the maximum uniaxial eccentricity at which a factored load $P_u = 640$ kN can be applied safely? (Take $f_{ck} = 20$ MPa)

(a) 300 mm
(b) 400 mm
(c) 600 mm
(d) 800 mm

16. The load carrying capacity of a column designed by working stress method is 500 kN. The ultimate collapse load of the column is

(a) 500 kN
(b) 662.5 kN
(c) 750 kN
(d) 1100 kN

17. If the load acting on a commonly conventional sized RC column increases continuously from zero to higher magnitudes, the magnitude of the uniaxial ultimate moment that can allowed on the column

(a) increases
(b) decreases
(c) increases and then decreases
(d) remains constant

Conventional Questions

1. A circular RC column of M15 grade concrete and 300 mm diameter has 8 numbers of 12 mm diameter, Fe 415 grade steel bars as longitudinal reinforcement. The length of the column is 7 m. If its ends are effectively held in position and restrained against rotation, determine the strength of the column.

2. A column of dimensions 200 mm × 300 mm has an effective length of 3 m. It is reinforced with 6 numbers of 20 mm diameter high strength deformed bars. Determine the safe load the column can carry using WSM of design. Grade of concrete used is M20.

3. A rectangular column 240 mm × 400 mm overall is reinforced with 6 numbers of 16 mm diameter bars. The bars are equally distributed on the short faces with an effective cover of 40 mm. Determine the eccentricity at which a load of 400 kN should act so that the maximum stress in concrete is limited to 7 N/mm². Assume m = 13.

4. Check the safety of a rectangular RC columns 250 mm × 350 mm reinforced with 4 bars of 16 mm diameter on each of its shortest faces and subjected to an axial load of 450 kN and a bending moment of 15 kN-m acting about its major axis. Can the column carry any more axial load? If so how much more? Concrete is M20 grade for which $\sigma_{cbc} = 7$ N/mm², $\sigma_{cc} = 5$ N/mm², m = 19. Cover to centres of reinforcing bars = 40 mm.
   As per code section is safe if

$$\frac{\sigma'_{cc}}{\sigma_{cc}} + \frac{\sigma'_{cbc}}{\sigma_{cbc}} \le 1.0$$

   where σ'_{cc} and σ'_{cbc} are the calculated direct and bending compressive stresses in concrete respectively and σ_{cc} and σ_{cbc} are permissible direct and bending compressive stresses in concrete respectively. Equivalent area of column = $A_c + 1.5\, m\, A_{sc}$ where A_{sc} is the area of concrete and A_{sc} is the area of steel in the column.

5. Design a square section column using M15 concrete and mild steel bars to carry an axial load (P) of 30,000 kg. Effective length of column (left) = 4 m. Assume permissible stresses in direct compression in M15 concrete (σ_{cc}) and in mild steel bars (σ_{sc}) as 40 and 1300 kg/cm², respectively. As per IS code

$$P = C_R(\sigma_{cc}A_c + \sigma_{sc}A_s)$$

   where A_c and A_s are areas of cross-section of concrete and steel respectively.

$$C_R = \text{Reduction Coefficient} = 1.25 - \frac{L_{eff}}{48b} > 1.0$$

   b = lateral dimension of column.
   Sketch the arrangement for longitudinal and lateral reinforcement.

6. A RC column of effective length of 3.0 m is to be designed to support a factored load of 2400 kN. Using limit state method, determine the cross-sectional dimensions of the column and reinforcement when one side of the column is restricted to 350 mm. The concrete used is of grade M20 and reinforcement is of HYSD steel of grade Fe 415.

7. A RC column of size 460 mm × 600 mm having effective length of 3.6 m is to be designed using LSM to support an axial service load of 2500 kN. Use M20 grade concrete and HYSD steel of Fe 415 grade.

Answers

1. (a)
2. (c)
3. (a)
4. (c)
5. (b)
6. (b)
7. (b)
8. (c)
9. (c)
10. (b)
11. (c)
12. (c)
13. (d)
14. (d)
15. (c)
16. (c)
17. (c)

7

Footing

- Till now we discussed the different structural elements viz. beams, slabs, staircases and columns, which are placed above the ground level and are known as superstructure.
- The superstructure is placed on the top of the foundation structure, designated as substructure as they are placed below the ground level.
- The elements of the superstructure transfer the loads and moments to its adjacent element below it and finally all loads and moments come to the foundation structure, which in turn, transfers them to the underlying soil or rock.
- Thus, the foundation structure effectively supports the superstructure. However, all types of soil get compressed significantly and cause the structure to settle.
- Accordingly, the major requirements of the design of foundation structures are the two as given below (see cl.34.1 of IS 456):

  (i) Foundation structures should be able to sustain the applied loads, moments, forces and induced reactions without exceeding the safe bearing capacity of the soil.

  (ii) The settlement of the structure should be as uniform as possible and it should be within the tolerable limits. It is well known from the structural analysis that differential settlement of supports causes additional moments in statically indeterminate structures. Therefore, avoiding the differential settlement is considered as more important than maintaining uniform overall settlement of the structure.

- In addition to the two major requirements mentioned above, the foundation structure should provide adequate safety for maintaining the stability of structure due to either overturning and/or sliding (see cl.20 of IS 456).
- It is to be noted that this part of the structure is constructed at the first stage before other components (columns/beams etc.) are taken up.
- So, in a project, foundation design and details are completed before designs of other components are undertaken.
- However, it is worth mentioning that the design of foundation structures is different from the design of other elements of superstructure due to the reasons given below. Therefore, foundation structures need special attention of the designers:

  (i) Accurate estimations of all types of loads, moments and forces are needed for the present as well as for future expansion, if applicable. It is very important as the foundation structure, once completed, is difficult to strengthen in future.

(ii) Foundation structures, though remain underground involving very little architectural aesthetics, have to be housed within the property line which may cause additional forces and moments due to the eccentricity of foundation.

(iii) Foundation structures are in direct contact with the soil and may be affected due to harmful chemicals and minerals present in the soil and fluctuations of water table when it is very near to the foundation. Moreover, periodic inspection and maintenance are practically impossible for the foundation structures.

(iv) Foundation structures, while constructing, may affect the adjoining structure forming cracks to total collapse, particularly during the driving of piles etc.

However, wide ranges of types of foundation structures are available. It is very important to select the appropriate type depending on the type of structure, condition of the soil at the location of construction, other surrounding structures and several other practical aspects as mentioned above.

TYPES OF FOUNDATION STRUCTURES

Foundations are mainly of two types: (i) shallow and (ii) deep foundations. The two different types are explained below:

(A) Shallow Foundations

- Shallow foundations are used when the soil has sufficient strength within a short depth below the ground level.
- They need sufficient plan area to transfer the heavy loads to the base soil.
- These heavy loads are sustained by the reinforced concrete columns or walls (either of bricks or reinforced concrete) of much less areas of cross-section due to high strength of bricks or reinforced concrete when compared to that of soil.
- The strength of the soil, expressed as the safe bearing capacity of the soil is normally supplied by the geotechnical experts to the structural engineer.
- Shallow foundations are also designated as footings.
- The different types of shallow foundations or footings are discussed below.

1. Plain Concrete Pedestal Footings

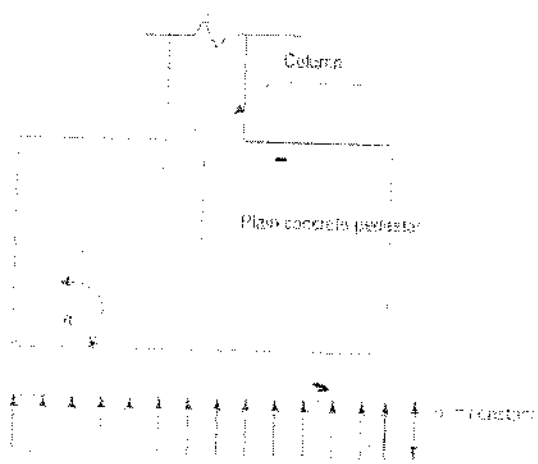


Fig. 7.1: Plain concrete pedestal.

Plain concrete pedestal footings (Fig. 7.1) are very economical for columns of small loads or pedestals without any longitudinal tension steel (see cls.34.1.2 and 34.1.3 of IS 456). In Fig. 7.1, the angle α between the plane passing through the bottom edge of the pedestal and the corresponding junction edge of the column with pedestal and the horizontal plane shall be determined from

$$\tan \alpha \leq 0.9 \sqrt{\frac{100\, q_a}{f_{ck}} + 1}$$

where q_a = calculated maximum bearing pressure at the base of pedestal in N/mm²

f_{ck} = characteristic strength of concrete at 28 days in N/mm²

2. Isolated Footings

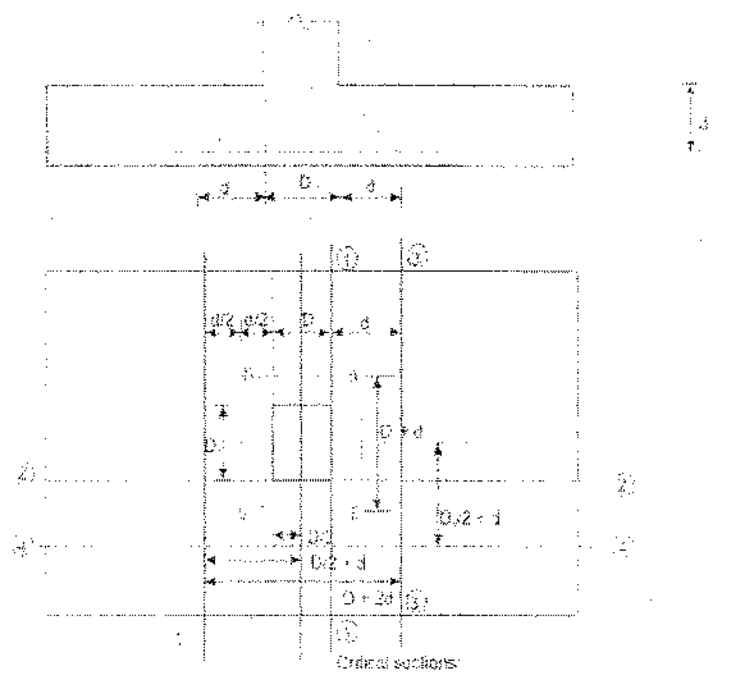


Fig. 7.2: Uniform and rectangular footing.

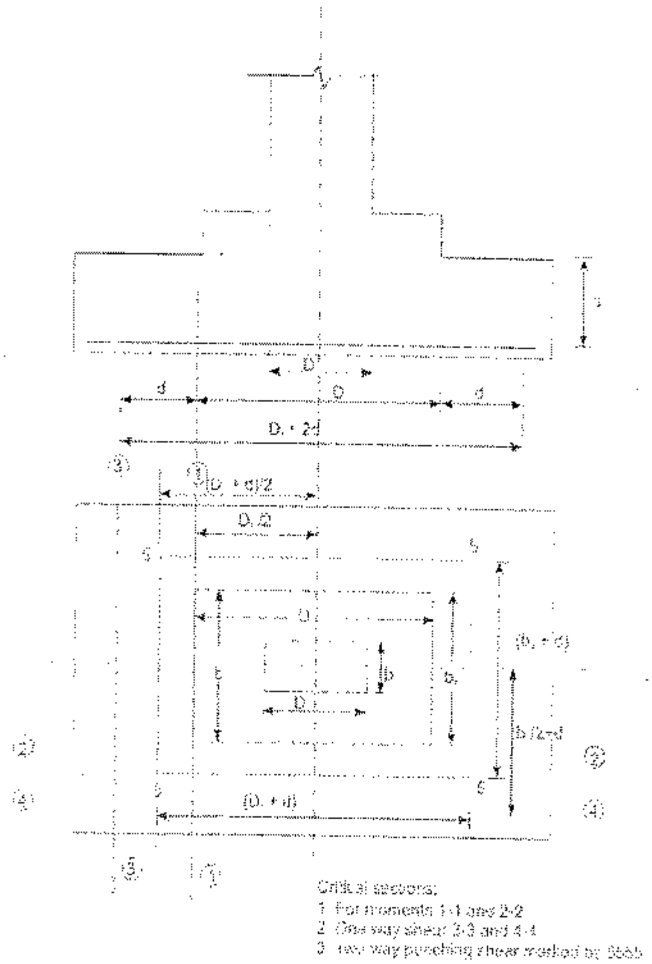


Fig. 7.3: Stepped and rectangular footing.

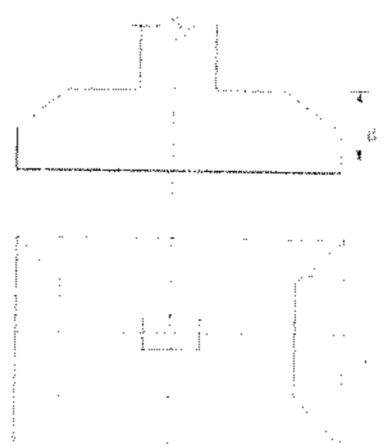


Fig. 7.4: Sloped and rectangular footing.

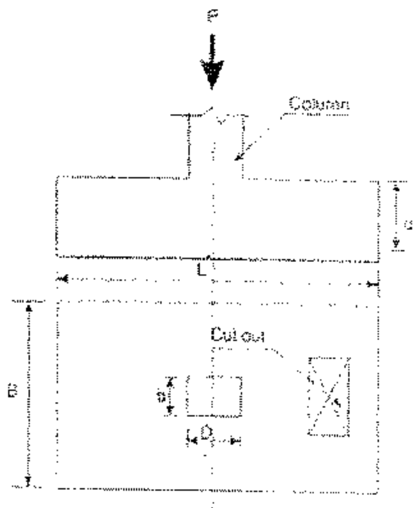


Fig. 7.5: Unsymmetrical footing about x-axis.

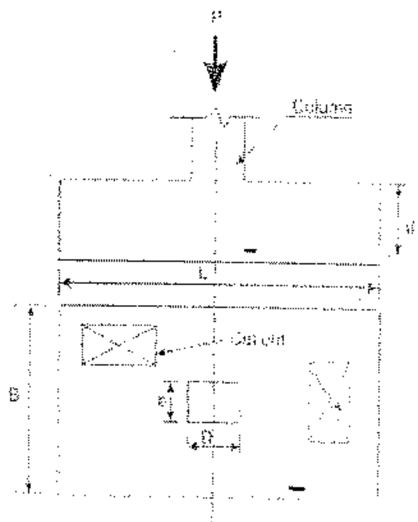


Fig. 7.6: Unsymmetrical footing about both axes.

- These footings are for individual columns having the same plan forms of square, rectangular or circular as that of the column, preferably maintaining the proportions and symmetry so that the resultants of the applied forces and reactions coincide.
- These footings, shown in Figs. 7.2 to 7.4, consist of a slab of uniform thickness, stepped or sloped.
- Though sloped footings are economical in respect of the material, the additional cost of formwork does not offset the cost of the saved material.
- Therefore, stepped footings are more economical than the sloped ones.
- The adjoining soil below footings generates upward pressure which bends the slab due to cantilever action.
- Hence, adequate tensile reinforcement should be provided at the bottom of the slab (tension face). Clause 34.1.1 of IS 456 stipulates that the sloped or stepped footings, designed as a unit, should be constructed to ensure the integrated action.
- Moreover, the effective cross-section in compression of sloped and stepped footings shall be limited by the area above the neutral plane.

- Though symmetrical footings are desirable, sometimes situation compels for unsymmetrical isolated footings (Eccentric footings or footings with cut outs) either about one or both the axes (Figs. 7.5 and 7.6).

3. Combined Footings

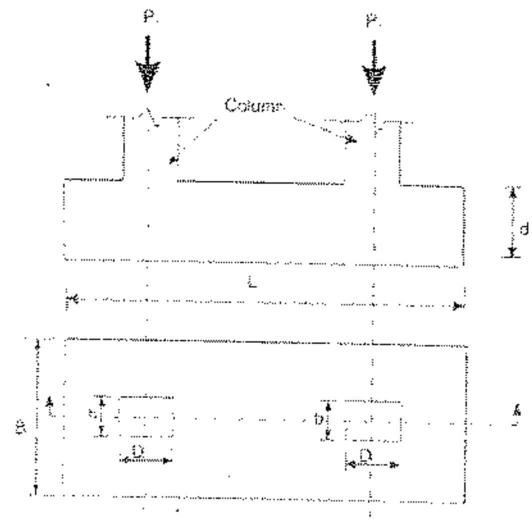


Fig. 7.7: Combined footing without a central beam.

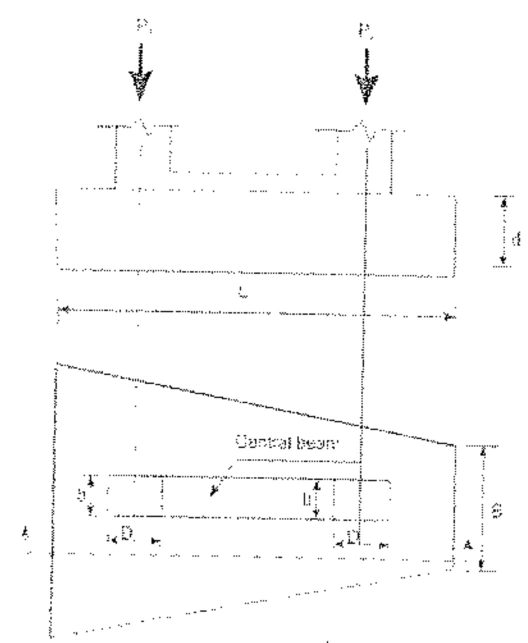


Fig. 7.8: Combined footing with a central beam.

- When the spacing of the adjacent columns is so close that separate isolated footings are not possible due to the overlapping areas of the footings or inadequate clear space between the two areas of the footings. combined footings are the solution combining two or more columns.
- Combined footing normally means a footing combining two columns.
- Such footings are either rectangular or trapezoidal in plan forms with or without a beam joining the two columns, as shown in Figs. 7.7 and 7.8.

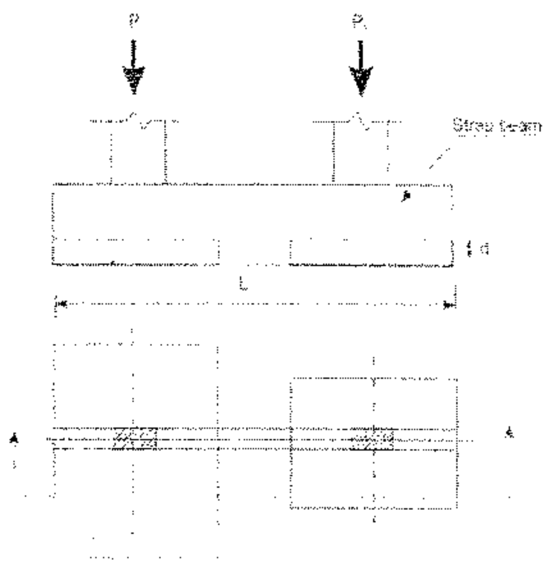


Fig. 7.9: Strap footings.

- When two isolated footings are combined by a beam with a view to sharing the loads of both the columns by the footings, the footing is known as strap footing (Fig. 7.9).
- The connecting beam is designated as strap beam.
- These footings are required if the loads are heavy on columns and the areas of foundation are not overlapping with each other.

4. Strip Foundation or Wall Footings

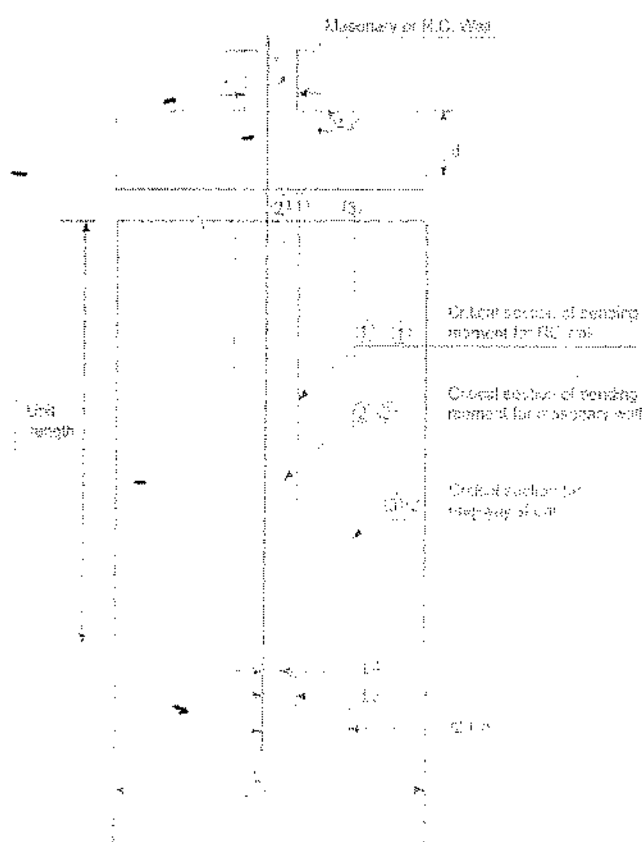


Fig. 7.10: Wall footing.

- These are in long strips especially for load bearing masonry walls or reinforced concrete walls (Figs. 7.10).
- However, for load bearing masonry walls, it is common to have stepped masonry foundations.
- The strip footings distribute the loads from the wall to a wider area and usually bend in transverse direction.
- Accordingly, they are reinforced in the transverse direction mainly, while nominal distribution steel is provided along the longitudinal direction.

5. Raft or Mat Foundation

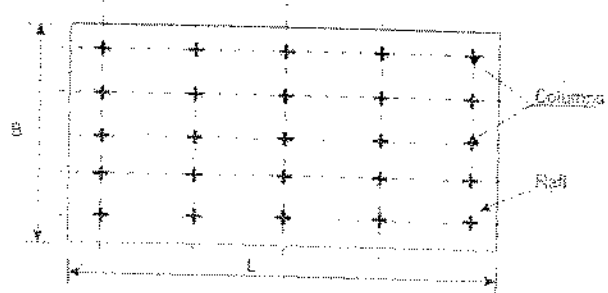


Fig. 7.11: Raft footing.

These are special cases of combined footing where all the columns of the building are having a common foundation (Fig. 7.11). Normally, for buildings with heavy loads or when the soil condition is poor, raft foundations are very much useful to control differential settlement and transfer the loads not exceeding the bearing capacity of the soil due to integral action of the raft foundation. This is a threshold situation for shallow footing beyond which deep foundations have to be adopted.

(B) Deep Foundations

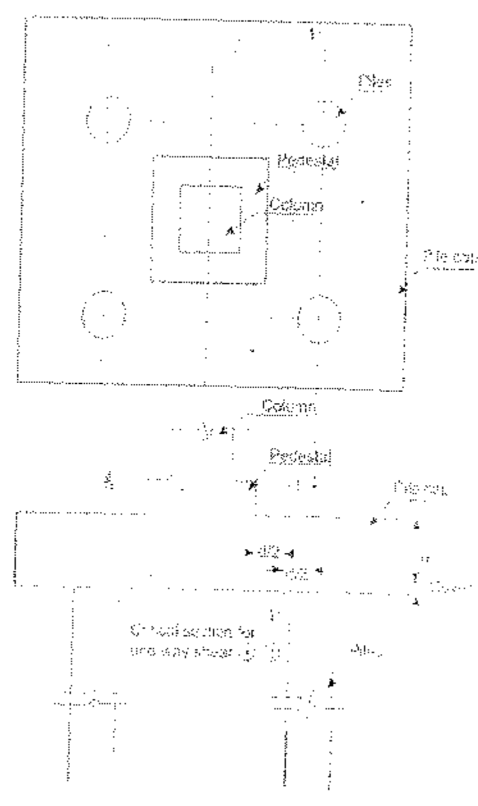


Fig. 7.12: Pile foundation.

- As mentioned earlier, the shallow foundations need more plan areas due to the low strength of soil compared to that of masonry or reinforced concrete.
- However, shallow foundations are selected when the soil has moderately good strength, except the raft foundation which is good in poor condition of soil also.
- Raft foundations are under the category of shallow foundation as they have comparatively shallow depth than that of deep foundation.
- It is worth mentioning that the depth of raft foundation is much larger than those of other types of shallow foundations.
- However, for poor condition of soil near to the surface, the bearing capacity is very less and foundation needed in such situation is the pile foundation (Figs.7.12).

DEPTH OF FOUNDATION

- All types of foundation should have a minimum depth of 50 cm as per IS 1080-1962. This minimum depth is required to ensure the availability of soil having the safe bearing capacity assumed in the design.
- Moreover, the foundation should be placed well below the level which will not be affected by seasonal change of weather to cause swelling and shrinking of the soil. Further, frost also may endanger the foundation if placed at a very shallow depth. Rankine formula gives a preliminary estimate of the minimum depth of foundation and is expressed as

$$d = \frac{q_c}{\gamma}\left(\frac{1-\sin\phi}{1+\sin\phi}\right)^2$$

where d = minimum depth of foundation

q_c = gross bearing capacity of soil

γ = density of soil

ϕ = angle of repose of soil

- Though Rankine formula considers three major soil properties q_c, γ and ϕ, it does not consider the load applied to the foundation. However, this may be a guideline for an initial estimate of the minimum depth which shall be checked subsequently for other requirements of the design.

11.28.5 DESIGN CONSIDERATIONS

(a) Minimum Nominal Cover (cl. 26.4.2.2 of IS 456)

The minimum nominal cover for the footings should be more than that of other structural elements of the superstructure as the footings are in direct contact with the soil. Clause 26.4.2.2 of IS 456 prescribes a minimum cover of 50 mm for footings. However, the actual cover may be even more depending on the presence of harmful chemicals or minerals, water table etc.

(b) Thickness at the Edge of Footings (cls. 34.1.2 and 34.1.3 of IS 456)

The minimum thickness at the edge of reinforced and plain concrete footings shall be at least 150 mm for footings on soils and at least 300 mm above the top of piles for footings on piles, as per the stipulation in cl.34.1.2 of IS 456.

For plain concrete pedestals, the angle α (see Fig. 7.1) between the plane passing through the bottom edge of the pedestal and the corresponding junction edge of the column with pedestal and the horizontal plane shall be determined from the following expression (cl.34.1.3 of IS 456)

$$\tan \alpha \leq 0.9\sqrt{\frac{100q_a}{f_{ck}} + 1} \qquad (11.3)$$

where q_a = calculated maximum bearing pressure at the base of pedestal in N/mm², and
f_{ck} = characteristic strength of concrete at 28 days in N/mm².

(c) Bending Moments (cl. 34.2 of IS 456)

1. It may be necessary to compute the bending moment at several sections of the footing depending on the type of footing, nature of loads and the distribution of pressure at the base of the footing. However, bending moment at any section shall be determined taking all forces acting over the entire area on one side of the section of the footing, which is obtained by passing a vertical plane at that section extending across the footing (cl.34.2.3.1 of IS 456).
2. The critical section of maximum bending moment for the purpose of designing an isolated concrete footing which supports a column, pedestal or wall shall be:
   (i) at the face of the column, pedestal or wall for footing supporting a concrete column, pedestal or reinforced concrete wall, (Figs. 7.2, 3 and 10), and
   (ii) halfway between the centre-line and the edge of the wall, for footing under masonry wall (Fig. 7.10). This is stipulated in cl.34.2.3.2 of IS 456.

   The maximum moment at the critical section shall be determined as mentioned in 1 above.

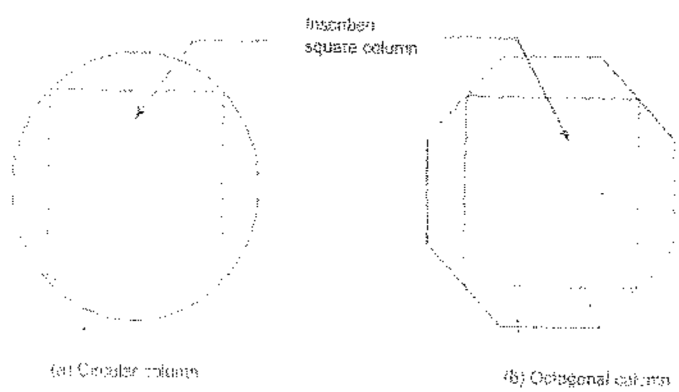


Fig. 7.13: Equivalent square columns (cl 34.2.2 of IS 456:2000).

For round or octagonal concrete column or pedestal, the face of the column or pedestal shall be taken as the side of a square inscribed within the perimeter of the round or octagonal column or pedestal (see cl.34.2.2 of IS 456 and Figs. 7.13 (a) and (b).

(d) Shear Force (cl. 31.6 and 34.2.4 of IS 456)

Footing slabs shall be checked in one-way or two-way shears depending on the nature of bending. If the slab bends primarily in one-way, the footing slab shall be checked in one-way vertical shear. On the other hand, when the bending is primarily two-way, the footing slab shall be checked in two-way shear or punching shear. The respective critical sections and design shear strengths are given below:

1. One-way Shear (cl. 34.2.4 of IS 456)

One-way shear has to be checked across the full width of the base slab on a vertical section located from the face of the column, pedestal or wall at a distance equal to (Figs. 7.2, 3 and 10):

(i) effective depth of the footing slab in case of footing slab on soil, and

(ii) half the effective depth of the footing slab if the footing slab is on piles (Fig. 7.12).

The design shear strength of concrete without shear reinforcement is given in Table 19 of cl.40.2 of IS 456.

2. Two-way or Punching Shear (cls.31.6 and 34.2.4)

Two-way or punching shear shall be checked around the column on a perimeter half the effective depth of the footing slab away from the face of the column or pedestal (Figs. 7.2 and 3).

The permissible shear stress, when shear reinforcement is not provided, shall not exceed $k_s \tau_c$, where $k_s = (0.5 + \beta_c)$, but not greater than one, β_c being the ratio of short side to long side of the column, and $\tau_c = 0.25(f_{ck})^{1/2}$ in limit state method of design, as stipulated in cl.31.6.3 of IS 456.

Normally, the thickness of the base slab is governed by shear. Hence, the necessary thickness of the slab has to be provided to avoid shear reinforcement.

(e) Bond (cl.34.2.4.3 of IS 456)

The critical section for checking the development length in a footing slab shall be the same planes as those of bending moments in part (c) of this section. Moreover, development length shall be checked at all other sections where they change abruptly. The critical sections for checking the development length are given in cl.34.2.4.3 of IS 456, which further recommends to check the anchorage requirements if the reinforcement is curtailed, which shall be done in accordance with cl.26.2.3 of IS 456.

(f) Tensile Reinforcement (cl.34.3 of IS 456)

The distribution of the total tensile reinforcement, calculated in accordance with the moment at critical sections, as specified in part (c) of this section, shall be done as given below for one-way and two-way footing slabs separately.

(i) In one-way reinforced footing slabs like wall footings, the reinforcement shall be distributed uniformly across the full width of the footing i.e., perpendicular to the direction of wall. Nominal distribution reinforcement shall be provided as per cl. 34.5 of IS 456 along the length of the wall to take care of the secondary moment, differential settlement, shrinkage and temperature effects.

(ii) In two-way reinforced square footing slabs, the reinforcement extending in each direction shall be distributed uniformly across the full width/length of the footing.

(iii) In two-way reinforced rectangular footing slabs, the reinforcement in the long direction shall be distributed uniformly across the full width of the footing slab. In the short direction, a central band equal to the width of the footing shall be marked along the length of the footing, where the portion of the reinforcement shall be determined as given in the equation below. This portion of the reinforcement shall be distributed across the central band:

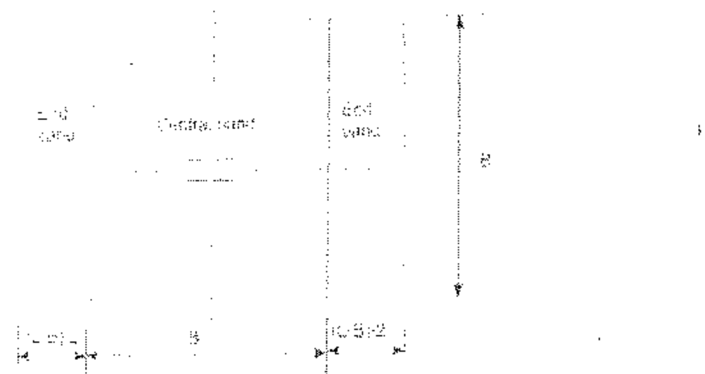


Fig. 7.14: Bands for reinforcement in a rectangular footing.

Reinforcement in the central band = {2/(β + 1)} (Total reinforcement in the short direction) (7.14)
where β is the ratio of longer dimension to shorter dimension of the footing slab (Fig. 7.14).
Each of the two end bands shall be provided with half of the remaining reinforcement, distributed uniformly across the respective end band.

(g) Transfer of Load at the base of Column (cl.34.4 of IS 456)

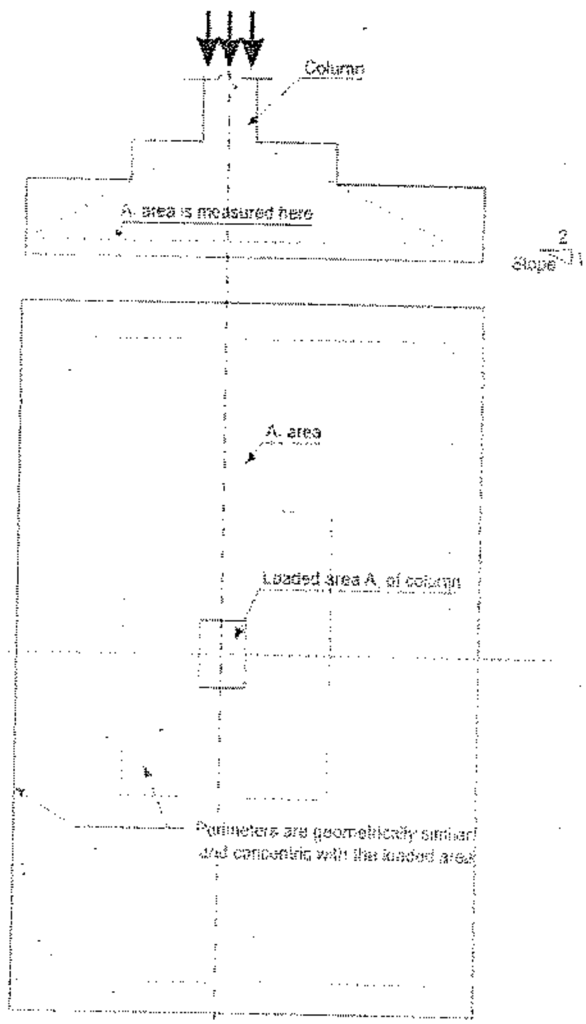


Fig. 7.15: Bearing area in sloped or stepped footing.

All forces and moments acting at the base of the column must be transferred to the pedestal, if any, and then from the base of the pedestal to the footing, (or directly from the base of the column to the footing if there is no pedestal) by compression in concrete and tension in steel. Compression forces are transferred through direct bearing while tension forces are transferred through developed reinforcement. The permissible bearing stresses on full area of concrete shall be taken as given below from cl.34.4 of IS 456:

$$\sigma_{br} = 0.25 f_{ck}, \text{ in working stress method, and} \quad \text{(i)}$$

$$\sigma_{br} = 0.45 f_{ck}, \text{ in limit state method} \quad \text{(ii)}$$

The permissible bearing stress of concrete in footing is given by (cl.34.4 of IS 456):

$$\sigma_{br} = 0.45 f_{ck} (A_1/A_2)^{1/2} \quad \text{(iii)}$$

with a condition that

$$(A_1/A_2)^{1/2} \leq 2.0 \quad \text{(iv)}$$

where A_1 = maximum supporting area of footing for bearing which is geometrically similar to and concentric with the loaded area A_2, as shown in Fig. 7.15

A_2 = loaded area at the base of the column.

The above clause further stipulates that in sloped or stepped footings, A_1 may be taken as the area of the lower base of the largest frustum of a pyramid or cone contained wholly within the footing and having for its upper base, the area actually loaded and having side slope of one vertical to two horizontal, as shown in Fig. 7.15.

If the permissible bearing stress on concrete in column or in footing is exceeded, reinforcement shall be provided for developing the excess force (cl.34.4.1 of IS 456), either by extending the longitudinal bars of columns into the footing (cl.34.4.2 of IS 456) or by providing dowels as stipulated in cl.34.4.3 of IS 456 and given below:

(i) Sufficient development length of the reinforcement shall be provided to transfer the compression or tension to the supporting member in accordance with cl.26.2 of IS 456, when transfer of force is accomplished by reinforcement of column (cl.34.4.2 of IS 456).

(ii) Minimum area of extended longitudinal bars or dowels shall be 0.5 per cent of the cross-sectional area of the supported column or pedestal (cl.34.4.3 of IS 456).

(iii) A minimum of four bars shall be provided (cl.34.4.3 of IS 456).

(iv) The diameter of dowels shall not exceed the diameter of column bars by more than 3 mm.

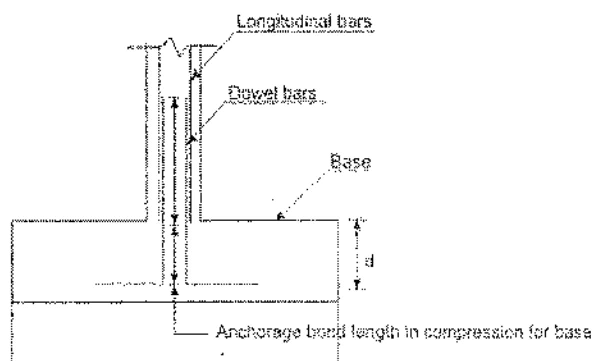


Fig. 7.16: Anchorage length of dowels.

(v) Column bars of diameter larger than 36 mm, in compression only can be doweled at the footings with bars of smaller size of the necessary area. The dowel shall extend into the column, a distance equal to the development length of the column bar and into the footing, a distance equal to the development length of the dowel, as stipulated in cl.34.4.4 of IS 456 and as shown in Fig. 7.16.

Example 1

Design a R.C. footing for a concrete wall of 400 mm width for a super-imposed load of 800 kN/m. The safe bearing capacity, p, of soil is 200 kN/m². Use M 15 concrete and Fe 415 steel.

Sol: *Design constants*

$$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3\times 5} = 1867 \approx 19$$

$$k_c = \frac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}} = \frac{19\times 5}{19\times 5 + 140} = 0.404$$

$$j = 1 - \frac{k_c}{3} = 1 - \frac{0.404}{3} = 0.865$$

$$Q = \frac{1}{2}\sigma_{cbc} j k_c = \frac{1}{2} \times 5 \times 0.865 \times 0.404 = 0.87$$

Fixing Base Size

Considering 1 m length of footing

Super-imposed load, $P = 800$ kN/m

Self-weight, P' assuming it to be 10% of $P = 80$ kN/m

Total load $(P + P') = 880$ kN/m

$$\text{Area of footing/m length; } A = \frac{(P+P')}{p} = \frac{880}{200} = 4.4 \text{ m}^2 \text{ per m width of footing}$$

$$\text{Net upward pressure, } p_0 = \frac{P}{A} = \frac{800}{4.4 \times 1} = 181.81 \text{ kN/m}^2 = 181.81 \times 10^{-3} \text{ N/mm}^2$$

∴ Provide 4.4 m width strip footing.

Determination of Depth

(i) From bending Moment Consideration

Design B.M.

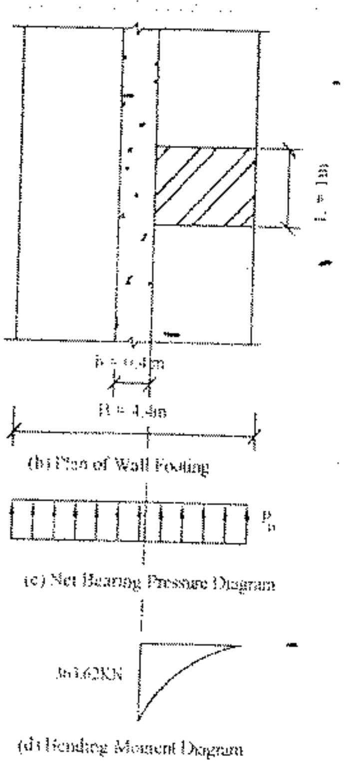


Fig. 1.1: Design B.M. for wall footing.

$$M = \frac{P_0}{8}(B-b)^2$$

$$= \frac{181.81}{8}(4.4-0.4)^2$$

$$= 363.62 \text{ kNm}$$

$$d = \sqrt{\frac{M}{Q \times L}} = \sqrt{\frac{363.62 \times 10^6}{0.87 \times 1000}} = 646.49 \approx 650 \text{ mm}$$

(ii) From Shear Force Consideration

Distance of critical section x-x at the edge of the footing (figure)

$$= \left(\frac{B-b}{2} - d\right)$$

$$= \left(\frac{4.4-0.4}{2} - d\right) = (2-d)\ m$$

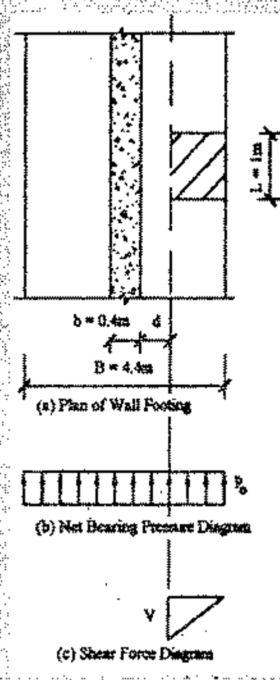


Fig. 1.2: Design S.F. for wall footing.

$$\therefore \quad V = p_0\left(\frac{B-b}{2} - d\right) = 181.81 \times (2-d) \text{ kN}$$

$$\tau_v = \frac{V}{L \times d} = \frac{181.81 \times (2-d)}{1 \times d}$$

From M 15 concrete $p_t = 0.72\%$ for which $\tau_c = 0.334$ N/mm² and for $D \geq 300$ (assumed) k = 1, permissible shear stress $k\tau_c = 1 \times 0.334$ N/mm².

Equating τ_v and τ_c, we get

Note: P_t is taken for balanced section general equation for P_t for balanced section is

$$\boxed{P_t = 41.38 \frac{f_{ck}}{f_y} \frac{x_{u,\lim}}{d}}$$

for M 15 and Fe 415 $P_t = 41.38 \times \frac{15}{415} \times 0.48 = 0.72$

$$\frac{181.81\times10^{-3}(2-d)}{1000\times d} = 0.334 \text{ (in mm units)}$$

or $d = 704.94 + \frac{16}{2} + 50 = 762.94$ mm

Use clear cover 50 mm and 16 mm dia bar.

Hence provided D = 765 mm total depth

$$d = 765 - \frac{16}{2} - 50 = 707 \text{ mm}$$

Main Reinforcement

$$A_{st} = \frac{M}{\sigma_{st} jd} = \frac{363.62\times10^6}{140\times0.865\times707} = 4247.01 \text{ mm}^2$$

Use 16 ϕ bars @ 45 c/c

Distribution reinforcement

$$A_s = \frac{0.15}{100} bD = \frac{0.15}{100}\times1000\times765 = 1147.50 \text{ mm}^2$$

$$\text{Spacing for } \phi \text{ 12 bars} = \frac{\frac{\pi}{4}\times12^2\times1000}{1147.50} = 98.56 \text{ mm}$$

Hence provided ϕ 12 @ 100 c/c

Check for Development Length

$$L_d = \frac{\phi\sigma_s}{4\tau_{bd}} = \frac{16\times140}{4\times0.6} = 933.33 \text{ mm}$$

Providing 60 mm of end cover length of bar beyond the face of wall (i.e. critical section fo bending) = $\frac{1}{2}(B-b) - 60 = \frac{1}{2}(4400 - 400) - 60 = 1940 > 933.33$ mm **Hence O.K.**

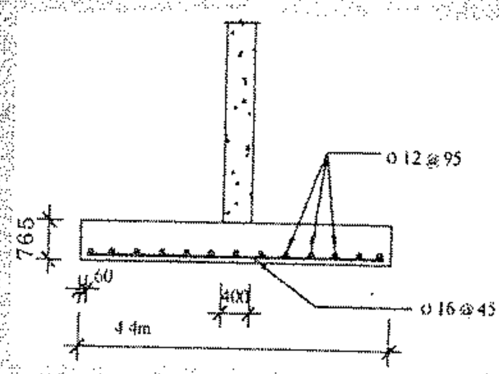


Fig. 1.3: Details of wall footing.

Example 2

Design a square footing for a column of cross-section 400 × 400 mm which transfers load of 800 kN inclusive of self-weight of footing. The safe bearing capacity of soil is 245 kN/m². Use M 20 concrete and Fe 250 steel.

Sol: *Design constants*

$$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3\times7} = 13.33 \approx 13$$

$$k_c = \frac{m\sigma_{cbc}}{m\sigma_{cbc}+\sigma_{st}} = \frac{13\times7}{13\times7+140} = 0.394$$

or

$$j = 1-\frac{k_c}{3} = 1-\frac{0.394}{3} = 0.869$$

$$Q = \frac{1}{2}cjk_c = \frac{1}{2}\times7\times0.394\times0.869 = 1.198$$

Fixing Base Size

$$\text{Total Load, } (P+P') = 800 \text{ kN}$$

$$\text{Area of footing, } A = \frac{P+P'}{p} = \frac{800}{245} = 3.265 \text{ m}^2$$

$$\text{Side of square footing, } B = \sqrt{3.265} = 1.81 \text{ m}$$

Hence provided 1.85 × 1.85 m footing.

$$\text{Self-weight of footing} = \frac{800}{11} = 72.7 \text{ kN}$$

(assuming self-weight = 10% of superimposed load)

$$\text{Net upward pressure, } p_0 = \frac{P}{A} = \frac{(800-72.7)}{1.85\times1.85} = 212.51 \text{ kN/m}^2$$

Determination of Depth

(i) From Bending Moment Consideration

The details of footing is shown in Fig. 2.1.

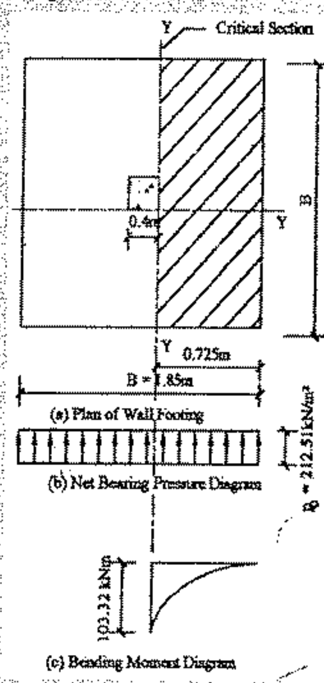


Fig. 2.1: B.M. diagram for column footing.

$$M = \frac{p_0 B(B-b)^2}{8} = \frac{212.51 \times 1.85 \times (1.85 - 0.4)^2}{8} = 103.32 \text{ kNm}$$

$$\therefore\ d = \sqrt{\frac{M}{Q \times B}} = \sqrt{\frac{103.32 \times 10^6}{1.198 \times 1850}} = 215.91 \text{ mm}$$

(ii) From Shear Force Consideration

(a) Distance of critical section from face of column for *one way shear* = d (Figure 2.2).

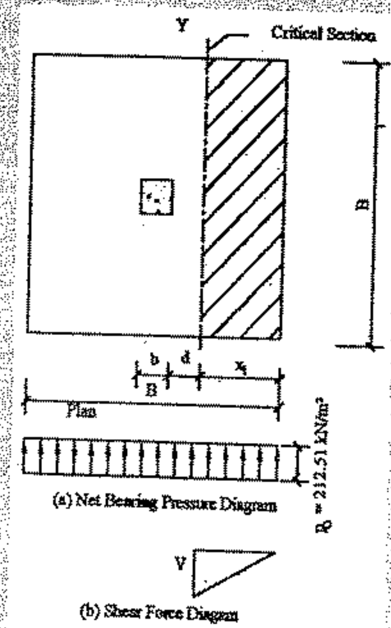


Fig. 2.2: S.F. diagram for column footing.

Breadth of loaded area from edge,

$$x_1 = \left(\frac{B-b}{2} - d\right) = \left(\frac{1850 - 400}{2} - d\right) = (725 - d) \text{ mm}$$

$$V = p_0 B x_1 = 212.51 \times 1.850 \times (0.725 - d) = 393.1435\ (0.725 - d)$$

$$\tau_v = \frac{V}{Bd} = \frac{393.1435 \times (0.725 - d)}{1.850 \times d} \text{ kN/m}^2 \qquad \text{(i)}$$

For M 20 concrete $p_t = 0.15\%$ minimum % of steel $\tau_c = 0.28 \text{ N/mm}^2$

Assuming $d > 300$, $k = 1$

$\therefore$ Permissible shear stress = $k\tau_c = 1 \times 0.28 \text{ N/mm}^2 = 0.28 \times 10^3 \text{ kN/m}^2$ (ii)

Equating τ_v and τ_c

$$\frac{393.1435 \times (0.725 - d)}{1.850 \times d} = 0.28 \times 1000, \text{ giving } d = 0.3128 \text{ m} = 312.8 \text{ mm}$$

or, $d = 312.8 > 215.91$ mm

(b) Critical section for *two way shear* is at *d*/2 from the face of the column (Fig. 2.3) perimeter of critical section

$b_0 = 4\,(0.4 + d)$, b_0 = Peripheri of critical section

$V = p_0\,(B^2 - b_0^2)$ or $V = P_0\,(L \times B - (a + d) \times (b + d)$

$= 212.51 \times 10^{-3}\,[1.850^2 - (0.4 + d)^2]$, here $L = B$ and $a = b$

$= [-212.51 \times d^2 - 170.008d + 693313.875]$

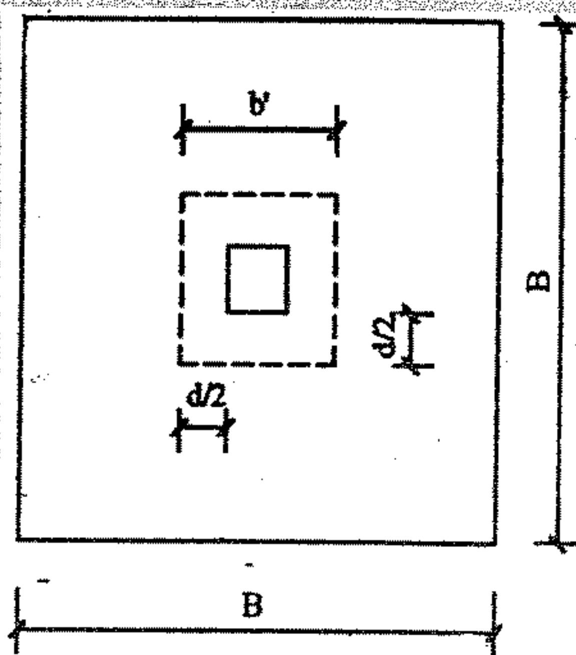


Fig. 2.3: Critical section for two-way shear.

$$\tau_v = \frac{V}{b_0 d} = \frac{-212.51 \times d^2 - 170.008d + 693.313}{4(0.4 + d)\,d}\ \text{kN/m}^2$$

Permissible shear stress = $k_s \tau_c$

where, $k_s = 0.5 + \beta_c = 0.5 + \frac{400}{400} = 1.5 > 1$

Hence $k_s = 1$

$$\tau_c = 0.16\sqrt{f_{ck}} = 0.16\sqrt{20} = 715.5\ \text{kN/m}^2$$

Equating τ_v and $k_s\,\tau_c$

$$\frac{(-212.51 \times d^2 - 170.008d + 693.313)}{4(0.4 + d) \times d} = 1 \times 0.715 \times 1000$$

or $d^2 + 0.4276d - 0.2255 = 0$

or $d = 0.30698\ m = 306.98$ mm

Providing ϕ 16 bars and 50 mm clear cover to bottom layer of reinforcement,

$$D = 312.8 + \frac{16}{2} + 16 + 50 = 386.8\ \text{mm}$$

Hence provided D = 390 mm

Effective depth for upper layer of reinforcement

$$d_u = 390 - \frac{16}{2} - 16 - 50 = 316 \text{ mm}$$

Effective depth for bottom layer of reinforcement

$$d_b = 316 + 16 = 332 \text{ mm}$$

Reinforcement of upper layer

$$A_{stu} = \frac{M}{\sigma_{st} d_u} = \frac{103.32 \times 10^6}{140 \times 0.869 \times 316} = 2687.50 \text{ mm}^2$$

$$\text{Spacing for } \phi \text{ 16 bars} = \frac{\frac{\pi}{4} \times 16^2 \times 1850}{2687.50} = 138.40 \text{ mm}$$

Hence provided φ 16 @ 135 c/c

Reinforcement of bottom layer

$$A_{stb} = \frac{M}{\sigma_{st} j d_b} = \frac{103.32 \times 10^6}{140 \times 0.869 \times 332} = 2557.98 \text{ mm}^2$$

$$\text{Spacing} = \frac{\frac{\pi}{4} \times 16^2 \times 1850}{2557.98} = 145.41$$

Hence provided φ 16 @ 140 c/c

Check for Development length

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd}} = \frac{16 \times 140}{4 \times 0.8} = 700 \text{ mm}$$

Providing 60 mm edge cover, length available beyond the critical section for bending (i.e. beyond the face of column)

$$= \frac{1}{2}(B - b) - 60 = \frac{1}{2}(1850 - 400) - 60 = 665 < 700 \text{ mm}$$

Providing 90° bend at ends, additional length available = 8φ = 8 × 16 = 128

∴ Total available length including 90° bend at ends of bars = 665 + 128 = 793 > 700 mm

Hence O.K.

Check for Bearing Stress

$$A_2 = 400 \times 400 = 16 \times 10^4 \text{ mm}^2$$
$$A_1 = (400 + 2 \times 2D)^2 = (400 + 4 \times 390)^2 = 384 \times 10^4 \text{ mm}^2$$

∴ Multiplication factor to bearing stress in direction compression (σ_{cbr})

$$= \sqrt{\frac{A_1}{A_2}} = \sqrt{\frac{384 \times 10^4}{16 \times 10^4}} = 4.89 > 2. \text{ Hence, adopt factor} = 2$$

$$\text{Hence permissible bearing stress} = \sqrt{\frac{A_1}{A_2}} \cdot \sigma_{cbr} = 2 \times (0.25 f_{ck}) = 2 \times (0.25 \times 20) = 10 \text{ N/mm}^2$$

$$\text{Actual bearing stress} = \frac{\text{Super-imposed load}}{\text{Loaded area of column bar}}$$

$$= \frac{800 \times 10^3}{400 \times 400} = 5 \text{ N/mm}^2 < 10 \text{ N/mm}^2. \quad \textbf{Hence O.K.}$$

The detailing of the footing has been shown in next Fig. 2.4.

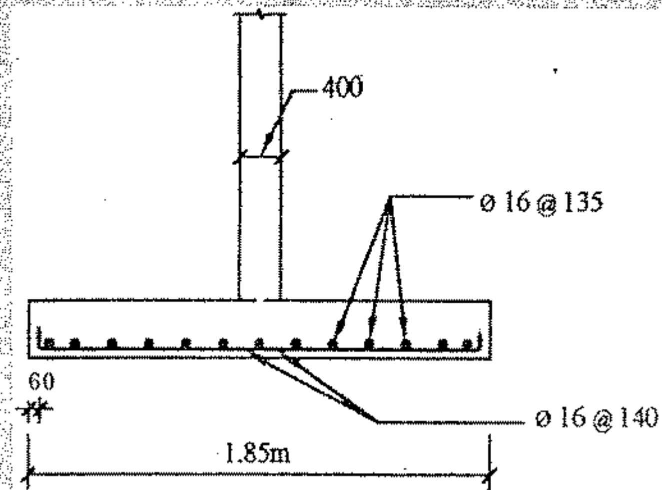


Fig. 2.4: Reinforcement detailing of the footing.

Example 3

Design a rectangular footing for a rectangular column of 300 × 400 mm carrying a load of 500 kN. The safe bearing capacity of soil is 150 kN/m². Use M 20 concrete and Fe 250 steel.

Sol: *Design constants*

$$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 7} = 13.33 = 13$$

$$k_c = \frac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}} = \frac{13 \times 7}{13 \times 7 + 140} = 0.394$$

$$j = 1 - \frac{k_c}{3}$$

$$j = 1 - \frac{k_c}{3} = 1 - \frac{0.394}{3} = 0.869$$

$$Q = \frac{1}{2} ckj = \frac{1}{2} \times 7 \times 0.869 \times 0.394 = 1.198$$

Fixing Base Size

Super-imposed load, $P = 500$ kN

Self-weight of footing, P'

(Assuming 10% of superimposed load = 50 kN

Total load $(P + P')$

$$\text{Area of footing} = \frac{P+P'}{p} = \frac{550}{150} = 3.67 \text{ m}^2$$

Assuming $\frac{L}{B}$ ratio of footing same as that for column

$$L = \frac{4}{3} B$$

$$A = L \times B = \frac{4}{3} B \times B = 3.67$$

$$\text{or} \quad B = 1.66 \text{ m and } L = \frac{4}{3} \times 1.66 = 2.2 \text{ m}$$

Hence provided $A = L \times B = 2.25$ m × 1.75 m size rectangular footing

Net upward soil pressure on footing,

$$p_0 = \frac{P}{A} = \frac{500}{2.25 \times 1.75} = 126.98 \text{ kN/m}^2$$

Determination of Depth

(i) From Bending Moment Consideration

Design

(a) Design B.M. at y-y

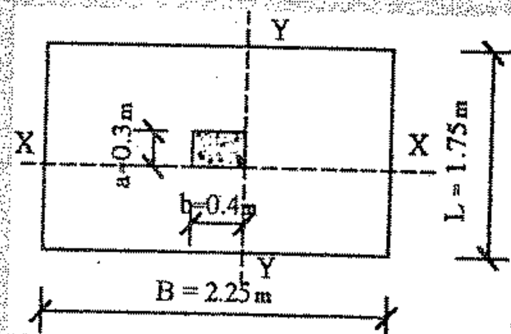


Fig. 3.1: Critical section for B.M.

$$M_1 = \frac{p_0 L}{8}(B-b)^2$$

$$= \frac{126.98 \times 1.75\,(2.25-0.4)^2}{8} = 95.066 \text{ kNm}$$

$$\therefore \quad d_1 = \sqrt{\frac{M}{L}} = \sqrt{\frac{95.066 \times 10^6}{1.198 \times 1750}} = 212.94 \text{ mm}$$

Similarly, design B.M. at x-x

$$M_2 = \frac{p_0 B}{8}(L-a)^2$$

$$= \frac{126.98 \times 2.25 (1.75-0.3)^2}{8} = 75.087 \text{ kNm}$$

$$d_2 = \sqrt{\frac{M}{B}} = \sqrt{\frac{75.087 \times 10^6}{1.198 \times 2250}} = 166.90 \text{ mm}$$

(ii) *From Shear Force Consideration*

(a) Distance of critical section for *one way shear* = d (Fig. 3.2)

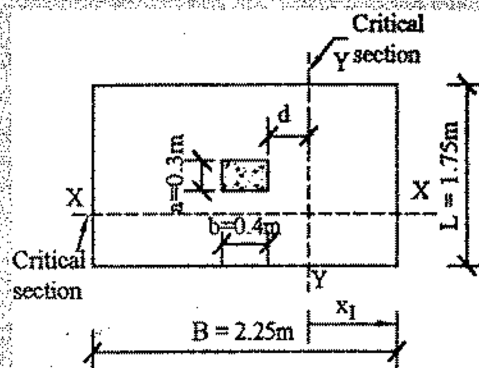


Fig. 3.2: Critical section for One-way S.F.

Breadth of loaded area from edge parallel to *B*,

$$x_1 = \left(\frac{B-b}{2} - d\right)$$

$$= \left(\frac{2.25-0.4}{2} - d\right)$$

Now

$$V_1 = p_0 L\left(\frac{B-d}{2} - d\right) = 126.98 \times 1.75\,(0.925-d) = 222.215\,(0.925\,d) \text{ kN}$$

$$\tau_{v1} = \frac{V_1}{Ld} = \frac{222.215\,(0.925-d)}{1750 \times d} = \frac{0.12698\,(0.925-d)}{d} \text{ N/mm}^2$$

From M 20 concrete and Fe 250 reinforcement minimum p_t = 0.15% and correspondingly τ_c = 0.28 N/mm². Assuming depth of footing to be greater than 300, k = 1

Permissible shear stress = $k\tau_c$ = 1 × 0.28 N/mm² = 0.28 N/mm²

Equating τ_{v1} and $k\tau_c$

$$\frac{0.12698\,(0.925-d)}{d} = 0.28$$

or d_1 = 288.46 mm

Similalry,

Breadth of loaded area parallel to *L*

$$y_1 = \left(\frac{L-a}{2} - d\right) = \left(\frac{1.75-0.3}{2} - d\right) = (0.725 - d)$$

Then from equation 16.7

$$V_2 = p_0 B\left(\frac{L-a}{2} - d\right) = 126.98 \times 2.25\ (0.725 - d)$$

$$= 285.705\ (0.725 - d)\ \text{kN}$$

$$\tau_{v2} = \frac{V}{bd} = \frac{285.705\,(0.725-d)\times 10^3}{2250 \times d \times 10^3}$$

$$= \frac{0.12698\,(0.725-d)}{d}\ \text{N/mm}^2$$

Equating τ_{v2} and $k\tau_c$

$$= \frac{0.12698\,(0.725-d)}{d} = 0.28$$

or $d = 226.20\ \text{mm} = d_2$

(iii) Critical section for *two way shear* is at $d/2$ from the face of the column (Fig. 3.3)

Perimeter of critical section,

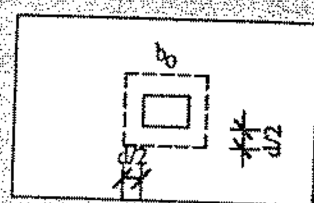


Fig. 3.3: Critical section for two-way shear

$$b_0 = 2[(a + d) + (b + d)]$$
$$= (1.4 + 4d)$$
$$V = p\{L \times B - (a + d)(b + d)\}$$
$$= 126.98\ [2.25 \times 1.75 - (d^2 + 0.7\ d + 0.12)]\ \text{kN}$$
$$= (-126.98\ d^2 - 88.886d + 484.746)\ \text{kN}$$

$$\tau_{v2} = \frac{V}{b_0 d} = \frac{(-126.98d^2 - 88.88d + 484.746)}{(1.4+4d)d}\ \text{kN/m}^2$$

Permissible shear stress $k_s \tau_c$

$$k_s = 0.5 + \frac{\text{Short side of column}}{\text{Long side of column}} = 0.5 + \frac{1.75}{2.25} = 1.278 >$$

Hence $k_s = 1$

$$\tau_c = 0.16\sqrt{f_{ck}} = 0.16\sqrt{20} = 0.7155\ \text{N/mm}^2$$

$$\therefore\ k_s\tau_c = 0.7155\ \text{N/mm}^2 = 0.7155 \times 103\ \text{kN/m}^2$$

Equating τ_v and $k_s\tau_c$

$$\frac{(-126.98d^2-88.886d+484.746)}{(1.4+4d)d}=0.7155\times10^3$$

or $\quad d^2+0.3649d-0.1622=0$; giving $d=0.25969$ m

or $\quad d=259.69$ mm

Hence the maximum value of $d = 259.69$ mm

Providing $\phi 12$ as main reinforcement and clear cover 50 mm to bottom layer of reinforcement

$$D=288.60+\frac{12}{2}+50=344.6\text{ mm}$$

Hence provided $D = 350$ mm

$$\therefore d_u \text{ for upper layer} = 350-50-\frac{12}{2}-12=282$$

$$\text{and } d_b \text{ for bottom layer} = 350-50-\frac{12}{2}=294\text{ mm}$$

Tensile reinforcement

(a) For bottom layer $A_{st}=\dfrac{M_1}{\sigma_{st}j_b d_b}=\dfrac{95.066\times10^6}{140\times0.869\times294}=2657.84\text{ mm}^2$

$$\therefore \text{Spacing of } \phi\,12 \text{ bars} = \frac{\frac{\pi}{4}\times12^2\times1750}{2657.84}=74.46\text{ mm}$$

Hence provided ϕ 12 @ 70 c/c

(b) For upper layer $A_{st}=\dfrac{M_2}{\sigma_{st}j_b d_u}=\dfrac{75.086\times10^6}{140\times0.869\times282}=2188.5\text{ mm}^2$

$$\text{No. of } \phi\,12 \text{ bars to be provided} = \frac{2188.5}{\frac{\pi}{4}\times12^2}=19.35=20$$

$$\text{No. of bars in the central band} = \frac{2}{\beta+1}\times20$$

$$=\frac{2}{\frac{2.25}{1.75}+1}\times20=17.5\approx18$$

$$\text{Spacing of bars in central band} = \frac{1750}{18}=97.22$$

Hence provided ϕ12 @ 90 c/c in the central band and one bar each on the two outer portions.

Check for Development Length

$$L_d=\frac{\phi\sigma_s}{4\tau_{bd}}=\frac{12\times140}{4\times0.8}=525$$

Providing 60 mm clear edge cover, length available beyond critical section for bending (i.e. beyond face of column) $= \frac{1}{2}\ (L - a) - 60$

$$\left(\frac{1750 - 300}{2}\right) - 60 = 665 > 525 \text{ mm. Hence O.K.}$$

Check for bearing stress

$$A_2 = 300 \times 400 = 12 \times 10^4 \text{ mm}^4$$

$$A_1 = (300 + 4D) \times (400 + 4D)$$

$$= (300 + 4 \times 350)\ (400 + 4 \times 350) = 306 \times 10^4 \text{ mm}^4$$

$$\sqrt{\frac{A_1}{A_2}} = \sqrt{\frac{306 \times 10^4}{12 \times 10^4}} = 5.049 > 2$$

Hence taking $= \sqrt{\frac{A_1}{A_2}} = 2$, permissible bearing stress $= 2 \times (0.25\ f_{ck}) = 2 \times (0.25 \times 20) = 10$ N/mm^2

$$\text{Actual bearing stress} = \frac{\text{Super-imposed load}}{\text{Loaded area of column bar}}$$

$$= \frac{500 \times 10^3}{300 \times 400} = 4.167 \text{ N/mm}^2 < 10 \text{ N/mm}^2. \quad \textbf{Hence O.K.}$$

The detailing of the footing has been shown in Fig. 3.4.

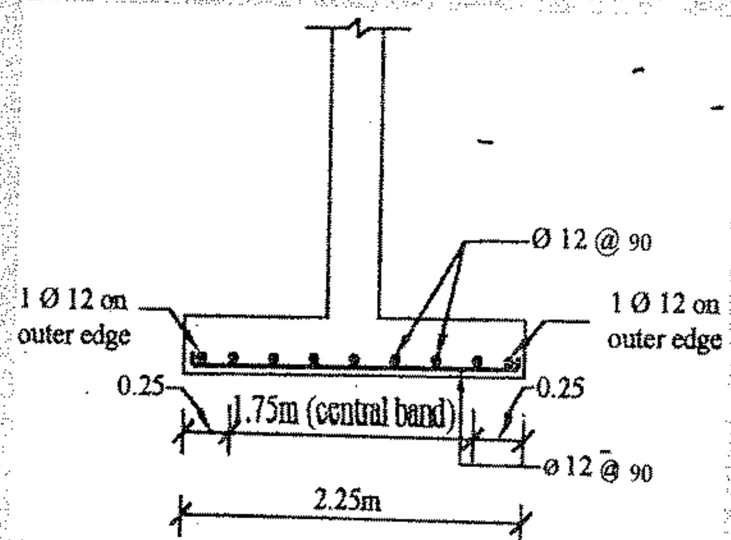


Fig. 3.4: Detailing of the footing.

Practice Objective Questions

1. Minimum clear cover (in mm) to the main steel bars in slab, beam, column and footing respectively are
   (a) 10, 15, 20, 25
   (b) 15, 25, 40, 40
   (c) 20, 25, 40, 50
   (d) 20, 35, 40, 75

2. Design of foundation for a large generator is guided, primarily, by
   (a) frequency
   (b) deformation
   (c) strength
   (d) stiffness

3. A trapezoidal combined footing for two axially loaded columns is provided when
   1. Width of the footing near the heavier column is restricted.
   2. Length of the footing is restricted.
   3. Projections of the footing beyond the heavier column are restricted.

   Select the correct answer using the codes given below:
   (a) 1 and 2
   (b) 1 and 3
   (c) 2 and 3
   (d) 1, 2 and 3

4. Match **List-I** (Reinforcement type) with **List-II** (Anchorage requirement) and select the correct answer using the codes given below the lists:

   List-I

   A. Footing slab, tensile reinforcement
   B. Cantilever beam, tensile reinforcement
   C. Simply supported beam, tensile reinforcement
   D. Beam, shear stirrup

   List-II

   1. $\frac{L_d}{3}$ into the support
   2. 6 ϕ for 135° bend
   3. L_d into the support
   4. L_d from the column face

   Codes:

	A	B	C	D
(a)	1	3	4	2
(b)	1	2	4	3
(c)	4	3	1	2
(d)	4	2	1	3

5. In the case of siolated square concrete footing, match the locations at which the stress resultants are to be checked, where d is effective depth of footing and select the correct answer using the codes given below the lists:

Stress	Resultant Location
A. Bending moment	1. At face of column
B. One way shear	2. At d/2 from face of column
C. Punching shear	3. At d from face of column

Codes:

	A	B	C
(a)	1	2	3
(b)	3	1	2
(c)	1	1	3
(d)	1	3	2

6. How is the depth of footing for an isolated column governed?
   1. By maximum bending moment
   2. By shear force
   3. By punching shear

   Select the correct answer using the codes given below:

   (a) 2 and 3 only
   (b) 1 and 2 only
   (c) 1 and 3 only
   (d) 1, 2 and 3

7. While designing combined footing, the resultant of the column loads passes through the centre of gravity of the footing slab such that the net soil pressure obtained is

   (a) parabolic
   (b) trapezoidal
   (c) uniform
   (d) non-uniform

8. The effective length of a column in a reinforced concrete building frame, as per **IS:456-2000**, is independent of the

   (a) frame type i.e., braced (no sway) or unbraced (with sway)
   (b) span of the beam
   (c) height of the column
   (d) loads acting on the frame

9. An RC square footing of side length 2 m and uniform effective depth 200 mm is provided for a 300 mm × 300 mm column. The line of action of the vertical compressive load passes through the centroid of the footing as well as of the column. If the magnitude of the load is 320 kN, the nominal transverse (one way) shear stress in the footing is

   (a) 0.26 N/mm^2
   (b) 0.30 N/mm^2
   (c) 0.34 N/mm^2
   (d) 0.75 N/mm^2

Conventional Questions

1. Design a square RC footing of uniform thickness for a RC column of 500 mm × 500 mm size carrying a total load of 2000 kN using M20 grade of concrete and mild steel reinforcement. The safe bearing capacity of the soil is 150 kN/m^2. Give a neat sketch of the footing with all the reinforcement details.

2. A rectangular RC column 60 cm × 40 cm carries a load of 80 tonnes. Design a rectangular footing to support the column using M15 concrete and mild steel reinforcement. Safe bearing capacity of soil = 20 t/m^2.

3. Design a square RC footing of uniform thickness for a RC column 500 mm × 500 mm size carrying a total load of 3000 kN, using M20 grade concrete and mild steel reinforcement. The safe bearing capacity of the soil is 225 kN/m^2. Give a neat sketch of the footing with all the reinforcement details.

Answers

1. (c)	3. (c)	5. (d)	7. (c)
2. (a)	4. (c)	6. (d)	8. (d)
			9. (a)

8

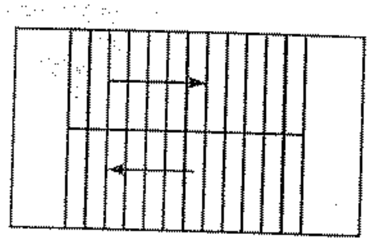
Staircase

INTRODUCTION

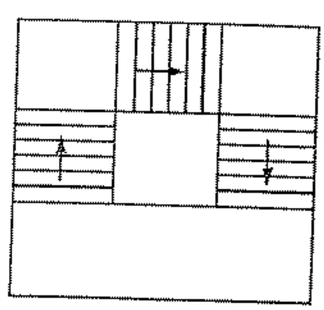
- Staircase is an important component of a building providing access to different floors and roof of the building. It consists of a flight of steps (stairs) and one or more intermediate landing slabs between the floor levels.
- Different types of staircases can be made by arranging stairs and landing slabs. Staircase, thus, is a structure enclosing a stair.
- The design of the main components of a staircase-stair, landing slabs and supporting beams or wall. The design of staircase, therefore, is the application of the designs of the different elements of the staircase.

TYPES OF STAIRCASES

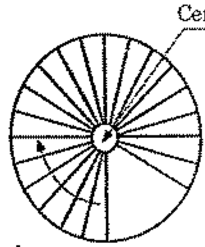
(a) Single flight staircase



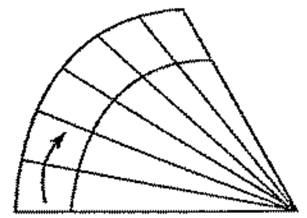
(b) Two flight staircase



(c) Open-well staircase



(d) Spiral staircase



(e) Helicoidal staircase

Fig. 8.1: Types of staircases.

Figures 8.1 (a) to (e) present some of the common types of staircases based on geometrical configurations:

(a) Single flight staircase (Fig. 8.1(a))

(b) Two flight staircase (Fig. 8.1(b))

(c) Open-well staircase (Fig. 8.1(c))

(d) Spiral staircase (Fig. 8.1(d))

(e) Helicoidal staircase (Fig. 8.1(e))

Architectural considerations involving aesthetics, structural feasibility and functional requirements are the major aspects to select a particular type of the staircase. Other influencing parameters of the selection are lighting, ventilation, comfort, accessibility, space etc.

CLASSIFICATION OF STAIRS

Different types of staircases are as follows:

(i) Straight stair.

(ii) Dog-legged stair.

(iii) Open-newel stair.

(iv) Geometrical stair.

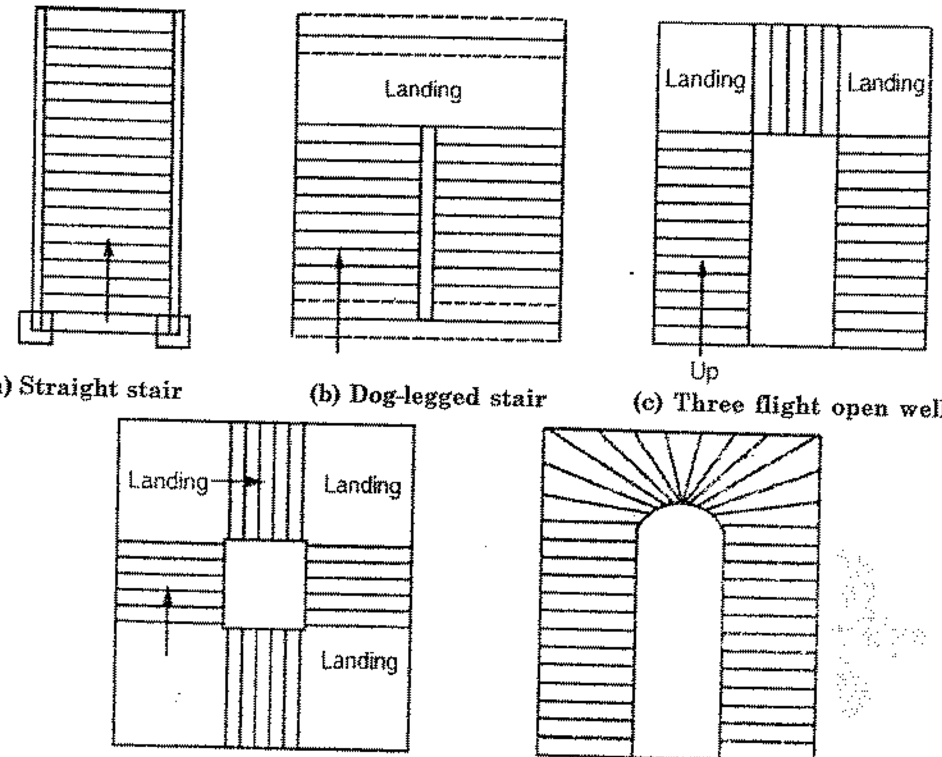
(v) Geometrical stair.

(i) Straight stair: When staircase room available is narrow, then straight stairs are preferable. In this type the steps are provided in the same direction. This type of stair consists of one flight without landing [Refer Fig. 8.2(a)].

(ii) Dog-legged stair: It consists of two flights in the opposite direction. The distance between two flights is from 0 to 150 mm. Landing is provided at the level from which the direction of flight changes. Generally dog-legged stairs are provided in residential building. [Refer Fig. 8.2(b)].

(iii) Open-newel stair: A rectangular space is provided between two flights and these two flights are connected by landing. If floor to floor height is large, then a flight with less steps can also be provided between two flights. The ventilation purpose can be served by open space. Generally, in public or commercial buildings, these types of staircase are provided. This space sometimes may be used as light well. [Refer Fig. 8.2 (c) and (d).

(iv) Geometrical stair: In this type of stair the well between the flights is curved Steps known as winders are provided in this curved portion with small width inner side for walking over the stair from inner edge. [Refer Fig. 8.2 (e)].



(a) Straight stair (b) Dog-legged stair (c) Three flight open well stair

(d) Four flight open well stair (e) Geometrical stair

Fig. 8.2: Different types of stairs.

A Typical Flight

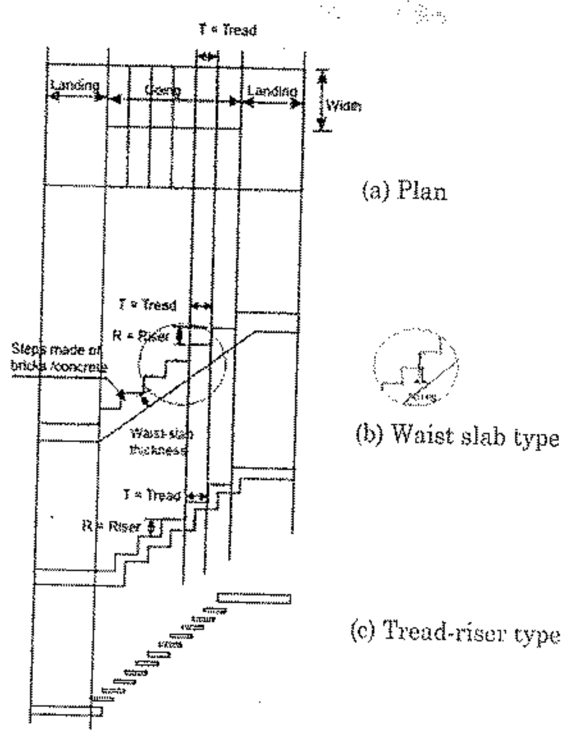


Fig. 8.3: A typical flight.

Figures 8.3(a) to (d) present plans and sections of a typical flight of different possibilities. The different terminologies used in the staircase are given below:

(a) **Tread:** The horizontal top portion of a step where foot rests (Fig. 8.3(b)) is known as tread. The dimension ranges from 270 mm for residential buildings and factories to 300 mm for public buildings where large number of persons use the staircase.

(b) **Nosing:** In some cases the tread is projected outward to increase the space. This projection is designated as nosing (Fig. 8.3(b)).

(c) **Riser:** The vertical distance between two successive steps is termed as riser (Fig. 8.3(b)). The dimension of the riser ranges from 150 mm for public buildings to 190 mm for residential buildings and factories.

(d) **Waist:** The thickness of the waist-slab on which steps are made is known as waist (Fig. 8.3(b)). The depth (thickness) of the waist is the minimum thickness perpendicular to the soffit of the staircase (cl. 33.3 of IS 456). The steps of the staircase resting on waist-slab can be made of bricks or concrete.

(e) **Going:** Going is the horizontal projection between the first and the last riser of an inclined flight (Fig. 8.3(b)).

The flight shown in Fig. 8.3(a) has two landings and one going. Figure 8.3(b) to (d) present the three ways of arranging the flight as mentioned below:

(i) waist-slab type (Fig. 8.3(b)).

(ii) tread-riser type (Fig. 8.3(c)), or free-standing staircase, and

(iii) isolated tread type (Fig. 8.3(d)).

General Guidelines

The following are some of the general guidelines to be considered while planning a staircase:

- The respective dimensions of tread and riser for all the parallel steps should be the same in consecutive floor of a building.
- The minimum vertical headroom above any step should be 2 m.
- Generally, the number of risers in a flight should be restricted to twelve.
- The minimum width of stair (Fig. 8.3(a)) should be 850 mm, though it is desirable to have the width between 1.1 to 1.6 m. In public building, cinema halls etc., large widths of the stair should be provided.

LOADS

DEAD LOAD

In consists of three loads.

(a) **Self weight of waist slab:** In case of flight, this is inclined slab so it is multiplied by $(\sqrt{R^2+T^2})/T$ to the dead load of landing.

(b) **Self weight of step:** Step is considered as a beam with depth equal to half the rise of step.

(c) **Floor finish:** Floor finish is considered on horizontal span. It is taken as 1 kN/m^2.

Live Load

Live load is taken according to I.S. 875-1987 (part II). The live load is considered as 3000 N/m^2 on the buildings where there is no possibility of overcrowding, such as residential buildings, hotels, hostels, club, etc. The live load on the buildings which are liable for overcrowding is taken as 4000 to 5000 N/m^2 such as public buildings, commercial complex etc.

Distribution of Loading on Stairs

Distribution of loading is according to IS 456-2000, clause no. 33.2, page 63.

In the case of stairs with open wells, where spans partly crossing at right angles occur, the load on areas common to any two such spans may be taken as one-half in each direction as shown in Fig. 8.4. Whwere flights or landing are embedded into walls for a length of not less than 110 mm and are designed to span in the direction of the flight, a 150 mm strip may be deducted from the loaded area and the effective breadth of the selection increased by 75 mm for purposes of design.

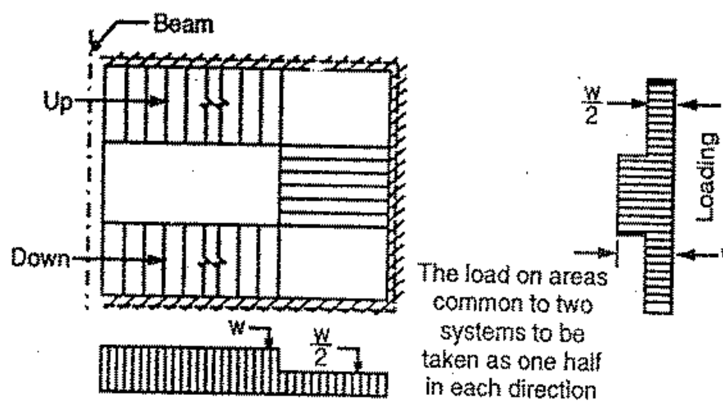


Fig. 8.4

EFFECTIVE SPAN OF STAIRS

(IS 456-2000, clause no. 33.1, page 63)

Stair slab may be divided structurally into two parts depending upon the direction in which the stair slab spans.

(i) Stair slab spanning horizontally.

(ii) Stair slab spanning longitudinally.

Stair Slab Spanning Horizontally

In this type of stair, slab is supported along the flight on each side. It may be supported by walls or by wall on one side and beam on the other side. In this type of stair, each step is designed as spanning horizontally considering equivalent to a rectangular beam with width b and effective depth equal to $\frac{D}{2}$.

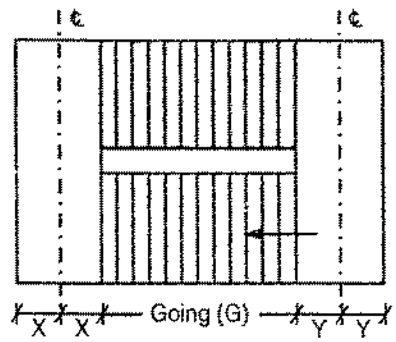
Width b is measured parallel to the slope of the stair. As slab is considered spanning parallel to step, so main reinforcement is provided along the step and distribution reinforcement is provided along the flight.

Stair Slab Spanning Longitudinally

In this type of stair, slab is supported at top and bottom of flight or landing and unsupported on sides. The effective span of slab in such cases is taken as follows:

(a) Where spanning at top and bottom risers by beams spanning parallel with the risers, the distance centre-to-centre of beams.

(b) Where spanning on to the edge of landing slab, which spans parallel with risers, a distance equal to the going of the stairs plus at each and either half the width of landing or one meter, whichever is smaller.



X	Y	SPAN IN METERS
< 1m	< 1m	G + X + Y
< 1m	≥ 1m	G + X + 1
≥ 1m	< 1m	G + 1 + Y
≥ 1m	≥ 1m	G + 1 + 1

Fig. 8.5: Effective span.

(c) Where the landing slab spans in the same direction as the stairs, they shall be considered as acting together to form a single slab and the span determined as the distance centre-to-centre of the supporting beams or walls, the going is measured horizontally.

STEPS TO BE FOLLOWED IN STAIRS

Step I: Design constants according to materials used.

Step II: Decide dimensions of risers and treads, number of risers and number of treads.

Let T be tread and R be the riser, then it should satisfy following condition:

$T \times R = 40000$ to 41000

and $2R + T = 600$

Generally tread is taken as 250 mm and riser as 150 mm for residential buildings, for public buildings tread as 270 mm to 300 mm.

Step III: From IS 456-2000 clause no. 33.1, calculate the effective span and assume trial depth as 40 mm to 50 mm per meter length.

Step IV: Calculation for loads:

(a) Dead loads : (per meter) and for B = 1 m

(i) Self weight of waist slab for flight $= \dfrac{25D \times \sqrt{R^2 + T^2}}{T}$

for loading = 25 D

(ii) Weight of one step $= 25 \times \dfrac{R}{2}$

(iii) Floor finish is generally taken as 0.75 kN/m^2 to 1.0 kN/m^2.

Total ultimate dead load = 1.5 × Total working load

= 1.5 × (i + ii + iii)

(b) Live loads : Live load is taken according to IS 875-1987 part II.

Total ultimate live load = 1.5 × Working live load

Step V: Bending moment calculation.

Step VI: Check for depth from B.M. consideration.

$\therefore$ B.M. $= R_u . b . d^2$ (b = 1000 mm)

Step VII: Steel reinforcement:

(a) Main reinforcement: $M_u = 0.87\, f_y\, A_{st}\, (d - 0.42\, x_u)$

$$\text{Spacing} = \frac{1000\,\phi}{A_{st}}$$

(b) Distribution reinforcement:

A_{st} = 0.12% bD → For Fe 415

A_{st} = 0.15% bD → For Fe 250.

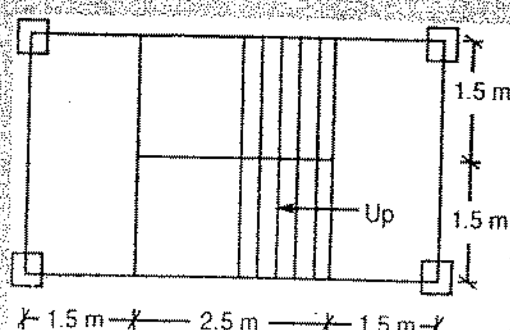
Consider diameter of bar ϕ

$$\text{Spacing} = \frac{1000\,\phi}{A_{st}}$$

Step VIII: Reinforcement detailing.

Example 1

Figure shows the general arragement of staircase room of a building. The columns are 230 mm × 230 mm. The floor to floor height is 3.3 m. The live load on staircase is 3.2 kN/m² and floor finish is 0.75 kN/m². Use M 20 concrete and Fe 415. Design two consecutive flights of the staircase. Prepare working drawings showing details of reinforcement and dowel bars.



Sol: ***Step I: Design constants:*** M 20, Fe 415

$$\frac{x_{u,\text{lim}}}{d} = 0.48$$

$$M_{u,lim} = 0.36\, f_{ck} x_{u,lim}\, (1 - 0.42\, x_{u,lim})$$

$$= 0.36 \times 20 \times 0.48\, (1 - 0.42 \times 0.48) = 2.76 \text{ N/mm}^2$$

Step II: Risers and treads:

Let riser = 150 mm, tread = 250 mm

$$\therefore \quad \text{No. of risers} = \frac{\text{Floor to floor height}}{\text{Rise}}$$

$$= \frac{3300}{150} = 22$$

Provide 11 risers in each flight.

∴ No. of treads = No. of risers − 1 = 11 − 1 = 10

Step III: Span and trial depth:

Assume landing slab spanning in the same direction as the stair slab.

∴ Effective span = 2500 + 1500 + 1500 = 5500 mm

Consider 45 mm thickness per meter.

∴ Slab thickness = 45 × 5.5 = 247.5 mm

∴ Provide overall depth of slab as 250 mm

∴ Effective depth = 250 – 20 = 230 mm

Step IV: Loads:

Load on going portion:

$$\text{Self weight of slab} = \text{Load on slope} \times \frac{\sqrt{R^2+T^2}}{T}$$

$$= 25D\times\frac{\sqrt{R^2+T^2}}{T}$$

$$= 25\times 0.25\,\frac{\sqrt{(0.15)^2+(0.25)^2}}{0.25}$$

$$= 7.29 \text{ kN/m}$$

$$\text{Weight of one step} = \frac{1}{2}\times 25\times R = \frac{25}{2}\times 0.15$$

$$= 1.875 \text{ kN/m}$$

Floor finish = 0.75 kN/m

Live load (considering 1 m) = 3.20 kN/m

Total working load = 13.115 kN/m

Ultimate load = 1.5 × 13.115

= 19.67 kN/m

Loads on landing:

Self weight of slab = 25D = 25 × 0.25 = 6.25 kN/m

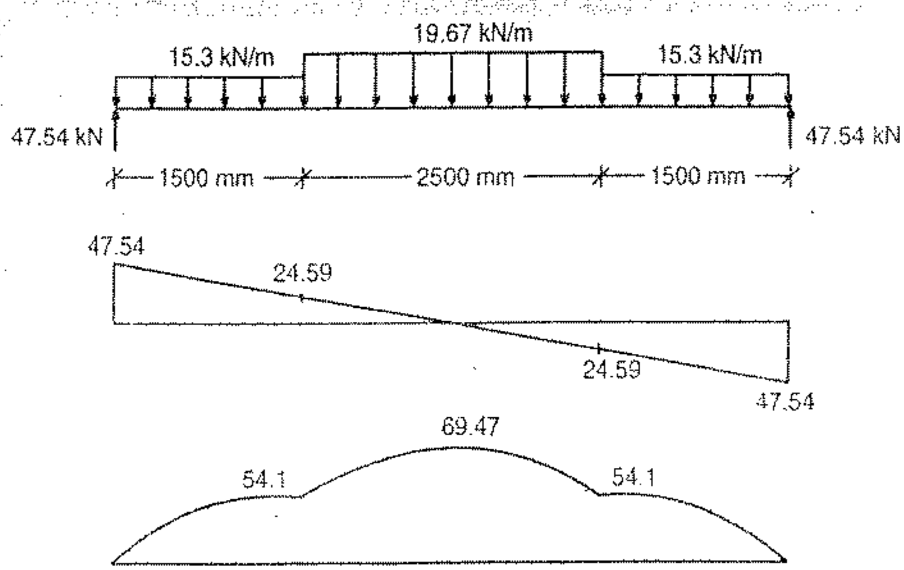
Floor finish = 0.75 kN/m

Live load = 3.20 kN/m

Total working load = 10.2 kN/m

∴ Ultimate load = 1.5 × 10.2 = 15.3 kN/m

Step V: Bending moment:



(a) Loading arrangement

(b) S.F.D.

(c) B.M.D.

$\therefore$ Maximum B.M. $= 47.54 \times 2.75 - 15.3 \times 1.5 \left(1.25 + \frac{1.5}{2}\right) - 19.67 \times \left[\frac{(1.25)^2}{2}\right]$

$= 69.47$ kN/m

Step VI: Check Depth from B.M. Consideration

$$\text{B.M.} = 0.36 f_{ck}\, x_{u,lim}\, b\,(d - 0.42\, x_{u,lim})$$

$$69.47 \times 10^6 = 0.36 \times 20 \times 0.48 \times d \times 1000\,(d - 0.42 \times 0.48\, d)$$

$$69.47 \times 10^6 = 2759.27\, d^2$$

$$d = \sqrt{\frac{69.47 \times 10^6}{2759.27}} = 158.67 \text{ mm}$$

$d_{\text{required}} < d_{\text{provided}}$ O.K.

Step VII: Steel Reinforcement

(a) Main steel:

$$M_u = 0.87 f_y A_{st}\,(d - 0.42 x_u) \quad \text{---(i)}$$

from $0.36 f_{ck} x_u\, b = 0.87 f_y A_{st}$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck}\, b} \text{ put in eq. (i)}$$

$$M_u = 0.87 f_y A_{st}\left(d - \frac{0.42 \times 0.87 f_y A_{st}}{0.36 f_{ck}\, b}\right)$$

$$= 0.87 \times 415\, A_{st}\left(230 - \frac{0.42 \times 0.87 \times 415\, A_{st}}{0.36 \times 20 \times 1000}\right)$$

$$69.47 \times 10^6 = 83041.5\, A_{st} - 7.60\, A_{st}^2$$

$$A_{st} = 912.829 \text{ mm}^2$$

$$\text{Spacing} = \frac{\phi}{A_{st}} \times 1000 = \frac{\frac{\pi}{4} \times 10^2}{912.829} \times 1000 = 86.04$$

Provide 10 mm ϕ @ 80 mm c/c

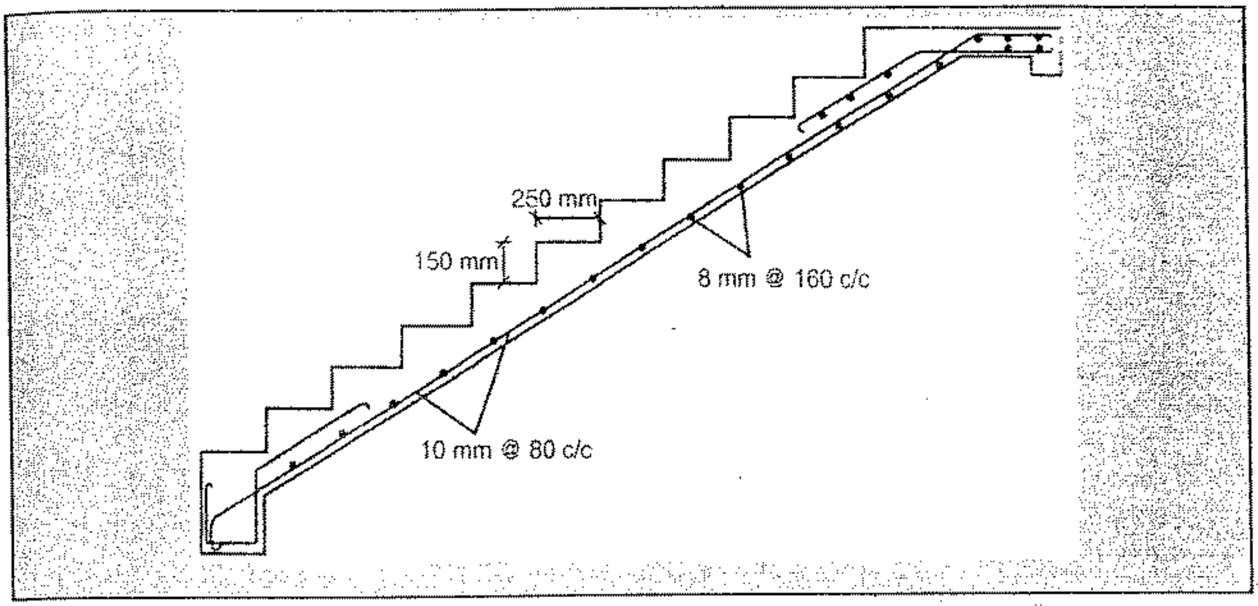
(b) Distribution steel:

$$A_{st,min} = 0.12\%\; bD$$

$$= \frac{0.12}{100} \times 1000 \times 250 = 300 \text{ mm}^2$$

$$\text{Spacing} = 1000 \times \frac{\frac{\pi}{4} \times 8^2}{300} = 166.67 \text{ mm}$$

$\therefore$ Provide 8 mm ϕ @ 160 mm c/c.


250 mm
150 mm
8 mm @ 160 c/c
10 mm @ 80 c/c

9

Prestress

PRESTRESS

Prestressed concrete is basically a concrete in which Internal stresses of suitable magnitude and distribution are introduced so that stresses resulting from external load are counteracted to a desired degree. In prestressed concrete member, the prestress is commonly introduced by tensioning steel reinforcement.

BASIC CONCEPT

- A prestressed concrete structure is different from a conventional reinforced concrete structure due to the application of an initial load on the structure prior to its use. The initial load or 'prestress' is applied to enable the structure to contract the streses arising during its service period.
- The proestruessing a structure is not the only instance of prestressing. The concept of prestressing existed before the application in concrete. Two examples of prestressing before the development of prestressed concrete are provided.

Force-Fitting of Metal Bands on Wooden Barrels

The metal bands induce a state of initial hoop compresion, to counteract the hoop tension caused by filling in the barrels.

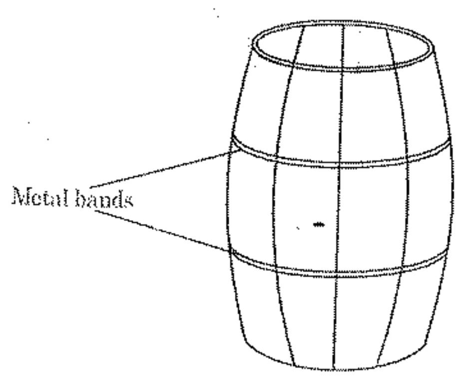
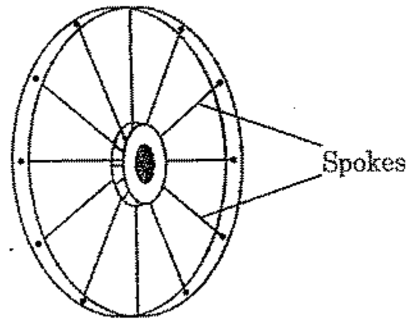


Fig. 9.1

Pre-Tensioning the Spokes in a Bicycle Wheel

The pre-tension of a spoke in a bicycle wheel is applied to such an extent that there will always be a residual tension in the spoke.



Pre-tensioning the spokes in a bicycle wheel

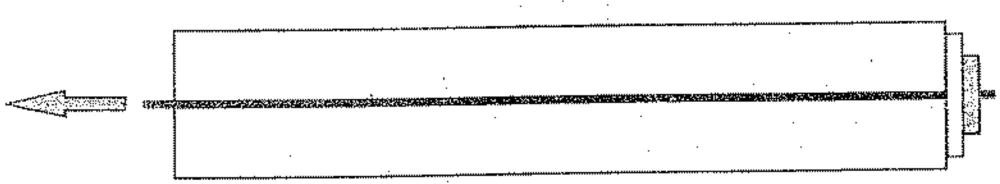
Fig. 9.2

For concrete, internal stress are induced (usually, by means of tensioned steel) for the following reasons.

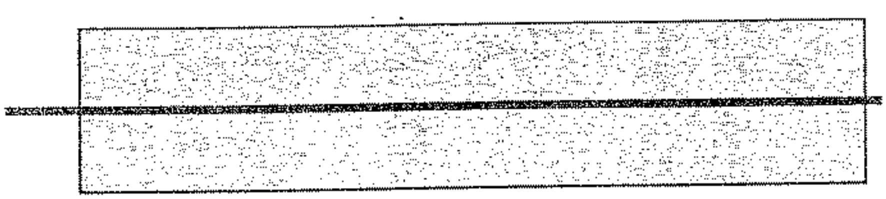
- The tensile strength of concrete is only about 8% to 14% of its compressive strength.
- Cracks tend to develop at early stages of loading in flexural meambers such as beem and slabs.
- To prevent such cracks, compressive force can be suitable applied in the perpendicular directions.
- Prestressing enhances the bending, shear and torsional capacities of the flexural members.
- In pipes and liquied storage tanks, the hoop tensile stresses can be effectively conteracted by circular prestressing.

Early Attempts of Prestressing

Prestressing of structures was introduced in late nineteenth century. the following sketch explains the application of prestres.



Place and stretch mild steel road, prior to concreting



Release the tension and cut the rods after concreting

Fig. 9.3

- Mild steel rods are stretched and concrete is poured around them. After hardening of concrete, the tension in the roads is released. The rods will try to regain their original length, but this is prevented by the surrounding concrete to which the steel is bonded. Thus, the concrete is now effectively in a state of pre-compression. It is capable of counteracting tensile stress, such as arising from the load shown in the following sketch.

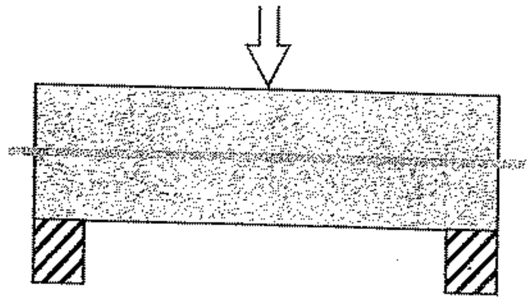


Fig. 9.4: A prestressed beam under an external load.

- But the early attempts of prestressing were not completely successful. It was observed that the effect of prestress reduced with time. The load resisting capacities of the members were limited. Under sustained loads, the members were found to fail. This was due to the following reason.
- Concrete shrinks with time. Morever under sustained load, the strain in concrete increases with increase in time. This is known as creep strain. The reduction in length due to creep and shrinkage is also applicable to the embedded steel, resulting in significant loss in the tensile strain.
- In the early applications, the strength of the mild steel and the strain during prestressing were less. The residual strain and hence, the residual prestress was only about 10% of the initial value. The following sketrches explain the phenomena.

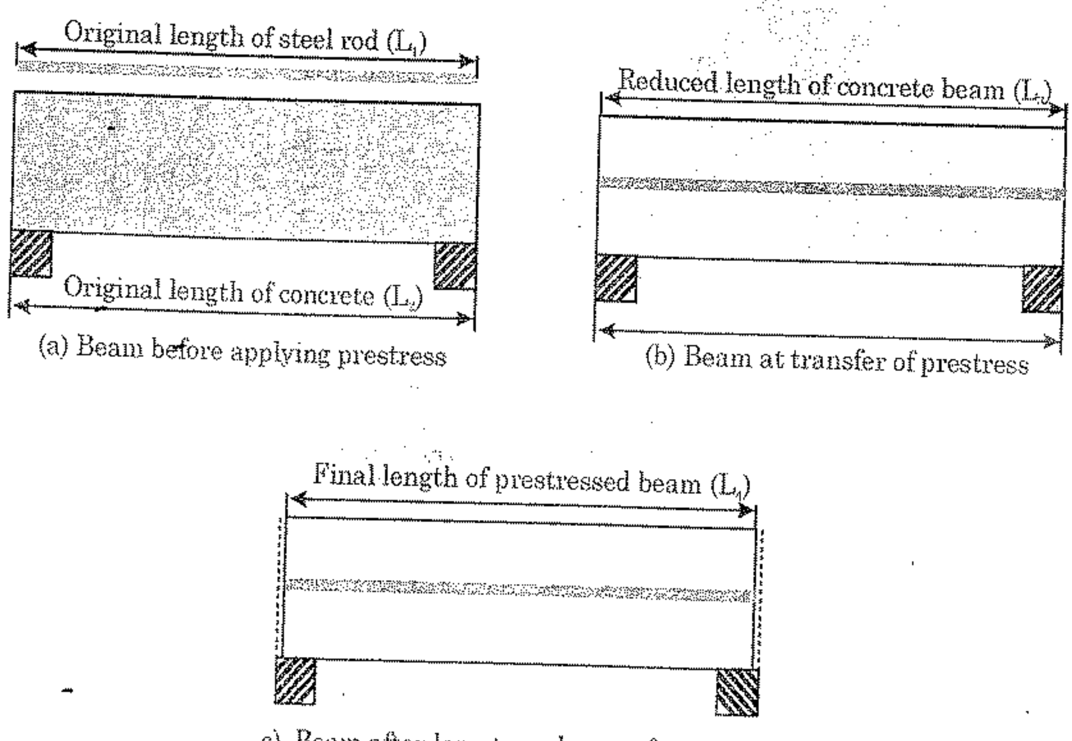


Fig. 9.5

The residual strain in steel = Original tensile strain in steel – Compressive strains corresponding to short-term and long term and long term losses.

Original tensile strain in steel = $(L_2 - L_1)/L_1$

Compressive strain due to elastic shortening of beam (short temm loss in prestress)

$= (L_2 - L_3)/L_1$

Compressive strain due to creep and shrinkage (long term losses in prestress

$= (L_3 - L_4)/L_1$

Therefore, residual strain in steel $= (L_4 - L_1)/L_1$

The maximum original tensile strain in mild steel

= Allowable stress/elastic modulus

$= 140 \text{ MPa}/2 \times 10^5 \text{ MPa}$

$= 0.0007$

The total loss in strain due to elasetic shortening, creep and shrinkage was also close to 0.0007. Thus, the residual strain was negligible.

The solution to increase the residual strain and the effective prestress are as follows.

- Adopt high strength steel with much higher original strain. This leads to the scope of high prestressing force.
- Adopt high strength concrete to withstand the high prestressing force.

Need for High Strength Steel

Most common method of introducing prestresses in concrete is by tensioning a steel reinforcement. In order to induce required magnitude of pre-stress the steel needs to be Tensioned to a very high degree i.e. toorder of 1200 N/mm^2 to 2000 N/mm^2, some amount of pre-stress generally of an amount 120 N/mm^2 to 200 N/mm^2 is lost during process of prestressing.

MS & HYSD bars have a working stress of 140 N/mm^2 and 230 N/mm^2 respectivity which are lost almost during proces of pre stressing itself.

Hence steel to be used for inducing pre-stress must be high tension steel.

Need for High Strength Concrete in Pre-stress Concrete

High strength concrete offers high resistance to tension, shear, bond and bearing.

In case of pre-tensioned members, tensile stress in steel of very high magnitude should be transferred to concrete as prestress through bonding between steel and surrounding concrete.

In post-tensioned members transfer of stress is through bearing at end sector. Hence concrete of appreciable bond and bearing strength is quite essential for pre-stressed concrete.

In addition to above use of high strength concrete have following advantages:

(1) High strength concrete is less liable to shrinkage cracks and has higher. Modulus of elasticity and smaller ultimate creep strain. As a result loss of prestress in steel is reduced.

(2) Use of high strength concrete results in reduction of cross sectional dimensions of prestressed concrete structural elements, with reduced dead weight longer spans becomes economically and practically viable.

Brief History

Before the development of prestressed concrete, two significant developments of reinforced concrete are the invention of Portland cement and introduction of steel in concrete. These are also mentioned as the part of the history. The key developments are mentioned next to the corresponding year.

Aspdin, J., (Dngland)

Obtained a patent for the manufacture of Portland cement.

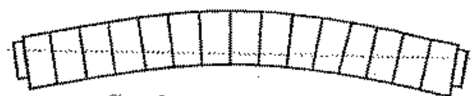
Monier, J., (France)

Introduced steel wires in concrete to make flower pots, pipes, arches and slabs.

The following events were significant in the development of prestressed concrete.

Jackson, P.H., (USA)

Introduced the concept of tightening steel tie rods in artificial stone and concrete arches.



Steel tie rods in arches

Fig. 9.6

Doehring, C.E.W., (Germany)

Manufactured concrete slabs and small beams with embedded tensionaed steel.

Stainer, C.R. (USA)

Recognised losses due to shrinkage and creep, and suggested retightening the rods to recover prestress.

Emperger, F., (Austria)

Developed a method of winding and pre-tensioning high tensile steel wires around concrete pipes.

Hewett, W.H., (USA)

Introduced hoop-stressed horizontal reinforcement around walls of concrete tanks through the use of turnbuckles.

Thousands of liquid storage tanks and concrete pipes were built in the two decades to follow.

Dill, R.H., (USA)

Used high strength unbonded steel rods. The rods were tensioned and anchored after hardening of the concrete.

Eugene Freyssinet (France)

Used high tensile steel wires, with ultimate strength as high as 1725 MPa and yield stress over 1240 MPa. In 1939, he developed conical wedges for end anchorages for post-tensioning and developed double-acting jacks. He is often referred to as the Father of Prestressed concrete.

Hoyer, E., (Germany)

Developed 'long line' pre-tensioning method.

Magnel, G., (Belgium)

Developed an anchoring system for post-tensioning, using flat wedges.

- During the Second world war, application of prestressed and precast concrete increased rapidly. The names of a few persons involved in developing prestressed concrete are mentioned. Guyon, Y., (France) built numerous prestressed concrete bridges in western and central Europe. abeles, P.W., (England) introduced the concept of pratial prestressing. Leonhardt, F., (Germany), Mikhailor, V., (Russia) and

Lin, T.Y., (USA) in 1954. The Precast/Prestressed Concrete Institute (PCI) was established in USA in 1954.

- The International Federation for Prestressing (FIP), a professional organisation in Europe was established in 1952. The Precast/ Prestressed Concrete Instutute (PCI) was established in USA in 1954.
- Prestressed concrete was started to be used in building frames, parking structures, stadium, railway sleepers, transmission line poles and other types of structres and elements.
- In India, the applications of prestressed concrete diversified over the years. The first prestressed concrete bridge was built in 1948 under the Assam Rail Link Project. among bridges, the Pamban Road Bridge at Rameshwaram, Tamilnadu, remains a classec example of theuse of prestressed concrete girders.

ADVANTAGES AND TYPES OF PRESTRESSING

Definitions

The terms commonly used in prestressed concrete are explained. The terms are placed in groups as per usage.

Prestressing Steel

The section covers the following topics.
Forms of Prestressing Steel
Types of Prestressing Steel
Properties of Prestressing Steel
Codal Provisions of Steel

Form of Prestressing Steel

The development of prestressed concrete was influenced by the invention of high strength steel. It is an alloy of iron, carbon, manganese and obtional materials. The following material describes the types and properties of prestressing steel.

In addition to prestressing steel, conventional non-prestressed reinforcement is used for flexural capacity (optional), shear capacity, temperature and shrinkage requirements.

Wires

A prestressing wire is a single unit made of steel. The nominal diameters of the wires are 2.5, 3.0, 4.0, 5,0, 7,0 and 8.0 mm. The different types of wires are as follows.

1. Plain wire: No indentations on the surface.
2. Indented wire: Thre are cirular or elliptical indentations on the surface.

Strands

A few wire are spun together in a helical form to form a prestressing strand. The different types of strands are as follows.

1. Two-wire strand: Two wires are spun together to form the strand.
2. Three-wire stand: Three wires are spun together to form the strand.
3. Seven-wire stands: In this type of strand, six wires are spun around a central wire. The central wire is larger than the other wires.

Tendons

A group of strands or wires are wound to form a prestressing tendon.

Cable

A group of tendons form a prestressing cable.

Bars

A tendon can be made up of a single steel bar. The diameter of a bar is much larger than that of a wire. Bars are available in the following sizes: 10, 12, 16, 20, 22, 25, 28, and 32 mm.

The following figure shows the different forms of prestressing steel.

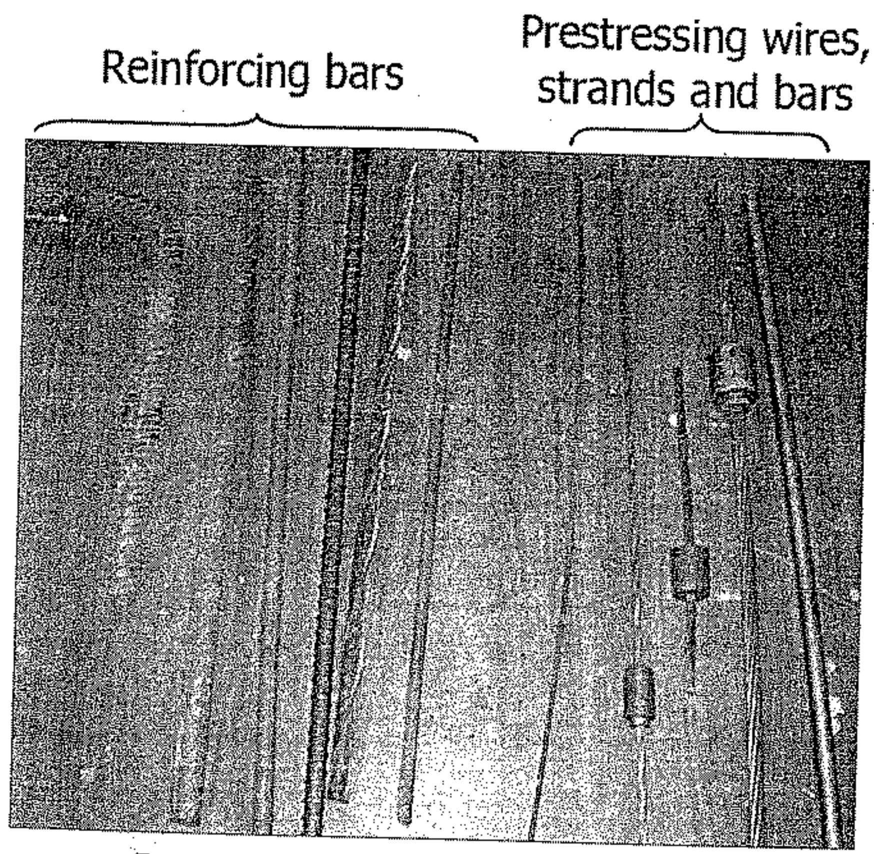


Fig. 9.7: Forms of reinforcing and prestressing steel.

Bonded Tendon

When there is adequate bond between the proestressing tendon and concrete, it is called a bonded tendon. Pre-tensioned and grouted post-tensioned tendons are bonded tendons.

Unbonded Tendon

When theire is no bond between the prestressing tendon and sconcrete, it is called unbonded tendon. When grout is not applied after post-tensioning, the tendon is an unbonded tenson.

Stages of Loading

The analysis of prestressed members can be different for the different stages of loading The stages of loading are as follows.

1. Initial: It can be subdivided into two stages
   (a) During tensioning of steel
   (b) At transfer of prestress to concrete.
2. Intermediate: This includes the loads during transportation of the prestressed members
3. Final : It can be subdivided into two stages.
   (a) At service, during operation.
   (b) At ultimate, during extreme events.

Advantages of Prestressing

The following text broadly mentions the advantages of a prestressed concrete member with an equivalent RC member. For each effect, the benefits are listed.

1. Section remains uncracked under service loads

(a) Reduction of steel corrosion
- Increase in durability.

(b) Full section is utilised
- Higher moment of inertia (higher stiffness)
- Less deformations (improved serviceability)

(c) Increase in shear capacity.

(d) Suitable for use in pressure vessels, liquid retaining structures.

(e) Improved performance (resilience) under dynamic and fatigue loading.

2. High span-to-depth ratios

Larger spans possible with prestressing (bridges, buildings with large colum-free spaces)

Typical values of span-to-depth ratios in slabs are given below:

Non-prestressed slab	28.1
Prestressed slab	45.1

For the same span, less depth compred to RC member.

- Reduction in self weight
- More aesthetic appeal due to slender sections
- More economical sections.

3. Suitable for Precast Construction

The advantages of precast construction are as follows.

- Rapid construction
- Better quality control
- Reduced maintenance
- Suitable for repetitive construction

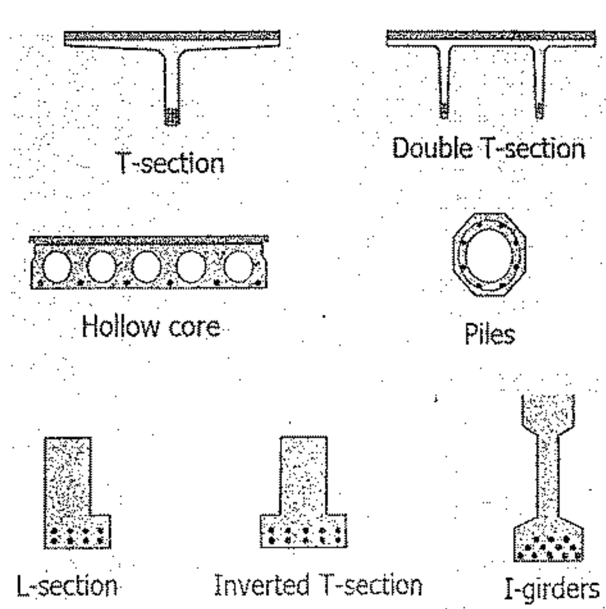


Fig. 9.8: Typical precast members.

- Reduction of formwork
- Availability of standard shapes.

The following figure shows the common types of precast sections.

Limitations of Prestressing

Although prestressing has advantages, some aspects need to be carefully addressed.

- Prestressing needs skilled technology. Hence, it is not as common as reinforced concrete.
- The use of high strength materials is costly.
- There is additional cost in auxiliary equipments.
- There is need for quality control and inspection.

Types of Prestressing

Prestressing of concrete can be classified in several ways. The following classifications are discussed.

Pre-tensioning or Post-Tensioning

This is the most important classification and is based on the sequence of casting the concrete and applying tension to the tendons.

Linear or Circular Prestressing

This classification is based on the shape of the member prestressed.

Full, Limited or Partial Prestressing

Based on the amount of prestressing force, three types of prestressing are defined.

PRE-TENSIONING OR POST-TENSIONING

Pre-Tensioning

The tension is applied to the tendons before casting of the concrete. The pre-compression is transmitted from steel to concrete through bond over the transmission length near the ends. The following figure shows manufactured pre-tensioned electric poles.

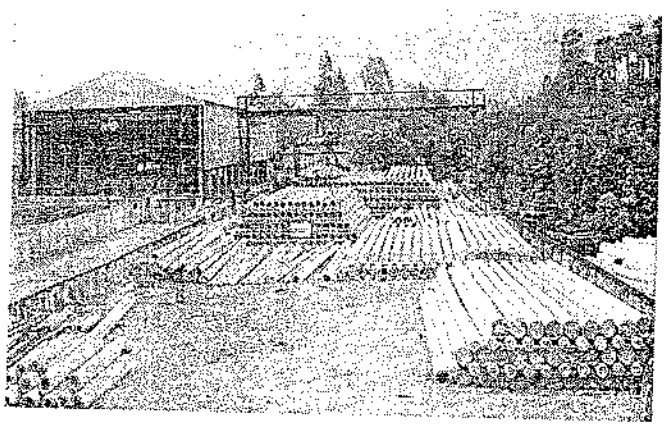


Fig. 9.9: Pre-tensioned electric poles.

Post-Tensioning

The tension is applied to the tendons (located in a duct) after hardening of the concrete. The pre-compression is transmitted from steel to concrete by the anchorage device (at the end blocks). The following figure shows a post-nsioned box girder of a bridge.

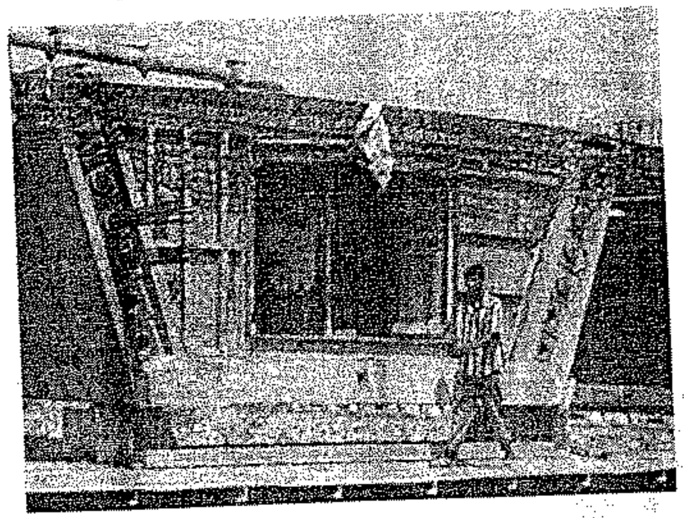


Fig. 9.10: Post-tensioning of a box girder.

Linear or Circular Prestressing

Linear Prestressing

When the prestressed members are straight or flat, in the direction of prestressing, the prestressing is called linear prestressing. For example, prestressing of beams, piles,poles and slabs. The profile of the prestressing tendon may be curved. The following figure shows linearly prestressed railway sleepers.

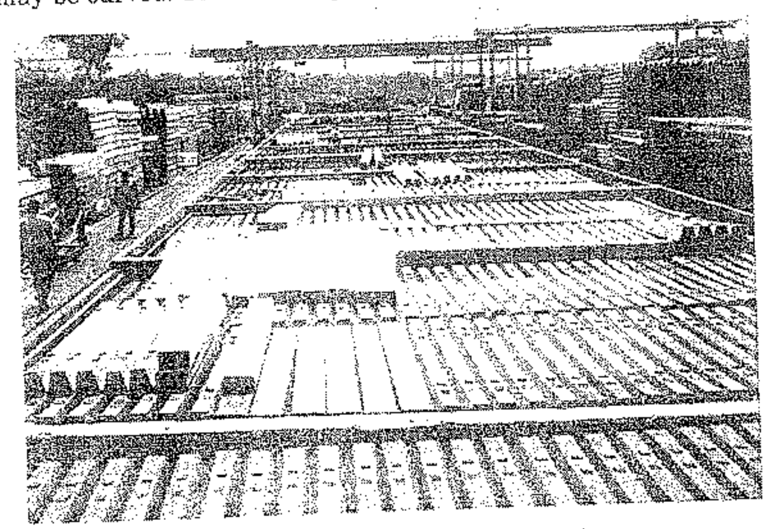


Fig. 9.11: Linearly prestressed railway sleepers.

Circular Prestressing

When the prestressed members are curved, in the direction of prestressing, the prestressing is called circular prestressing. For example, circumferential prestressing of tanks, silos, pipes and similar structures. The following figure shows the containment structure for a nuclear reactor which is circularly prestressed.

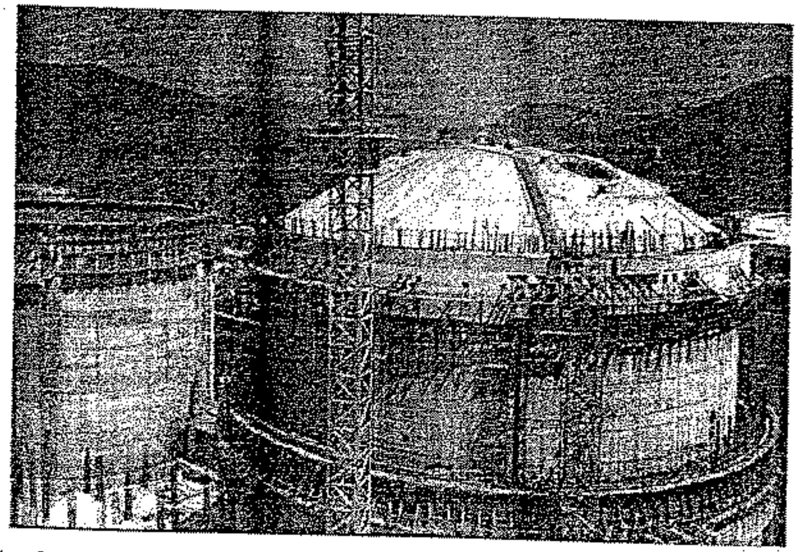


Fig. 9.12: Circularly prestressed containment structure, Kaiga Atomic Power Station, Karnataka.

FULL, LIMITED OR PARTIAL PRESTRESSING

Full Prestressing

When the level of prestressing is such that no tensile stress is allowed in concrete under service loads, it is called Full Prestressing (Type 1, as per **IS:1343 - 1980**).

Limited Prestressing

When the level of prestressing is such that the tensile stress under service loads is within the cracking stress of concrete, it is called Limited Prestressing (Type 2).

Partial Prestressing

When the level of prestressing is such that under tensile stresses due to service loads, the crack width is within the allowable limit, it is called Partial Prestressing (Type 3).

PRE-TENSIONING SYSTEMS AND DEVICES

Stages of Pre-tensioning

In pre-tensioning system, the high-strength steel tendons are pulled between two end abutments (also called bulkheads) prior to the casting of concrete. The abutments are fixed at the ends of a prestressing bed. Once the concrete attains the desired strength for prestressing, the tendons are cut loose from the abutments.

The prestress is transferred to the concrete from the tendons, due to the bond between them. During the transfer of prestress, the member undergoes elastic shortening. If the tendons are located eccentrically, the member is likely to bend and deflect (camber).

The various stages of the pre-tensioning operation are summarised as follows.

1. Anchoring of tendons against the end abutments
2. Placing of jacks
3. Applying tension to the tendons

4. Casting of concrete
5. Cutting of the tendons.

During the cutting of the tendons, the prestress is transferred to the concrete with elastic shortening and camber of the member.

The stages are shown schematically in the following figures.

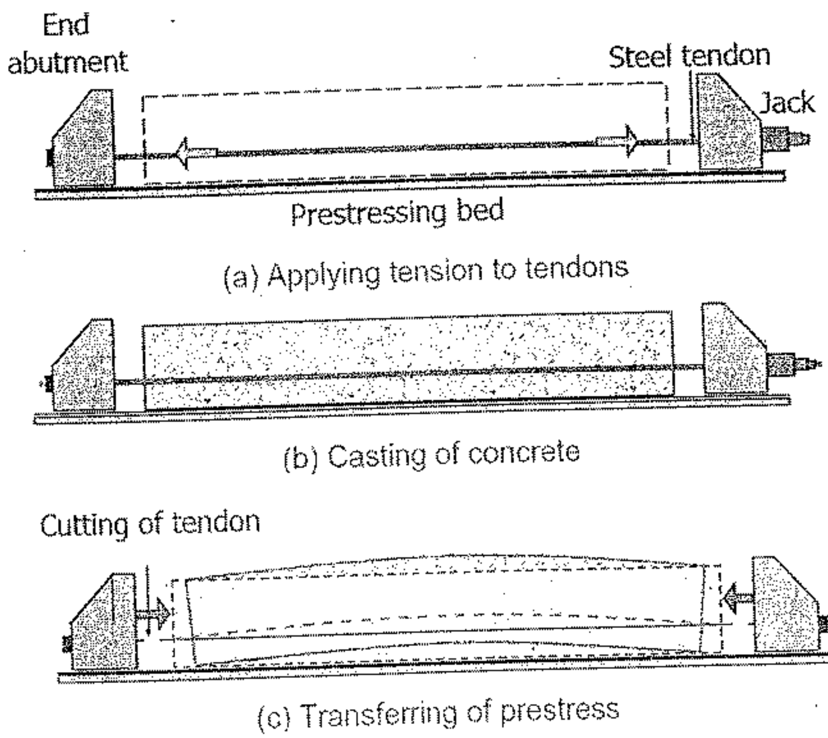


Fig. 9.13: Stages of pre-tensioning.

Advantages of Pre-Tensioning

The relative advantages of pre-tensioning as compared to post-tensioning are as follows.

- Pre-tensioning is suitable for precast members produced in bulk.
- In pre-tensioning large anchorage device is not present.

Disadvantages of Pre-Tensioning

The relative disadvantages are as follows.

- A prestressing bed is required for the pre-tensioning operation.
- There is a waiting period in the prestressing bed, before the concrete attains sufficient strength.
- There should be good bond between concrete and steel over the transmission length.

DEVICES

The essential devices for pre-tensioning are as follows.

- Prestressing bed
- End abutments
- Shuttering / mould

- Jack
- Anchoring device
- Harping device (optional)

Prestressing Bed, End Abutments and Mould

The following figure shows the devices.

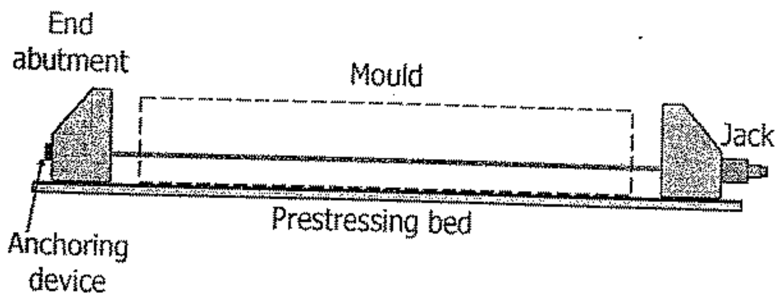


Fig. 9.14: Prestressing bed, end abutment and mould.

An extension of the previous system is the **Hoyer system**. This system is generally used for mass production. The end abutments are kept sufficient distance apart, and several members are cast in a single line. The shuttering is provided at the sides and between the members. This system is also called the **Long Line Method**. The following figure is a schematic representation of the Hoyer system

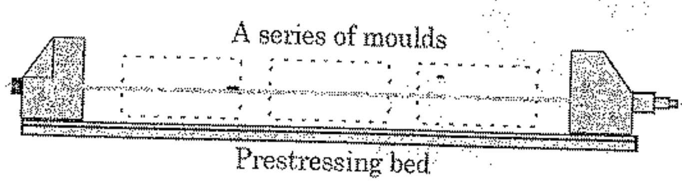


Fig. 9.15: Schematic representation of Hoyer system.

- The end abutments have to be sufficiently stiff and have good foundations.
- This is usually an expensive proposition, particularly when large prestressing forces are required.
- The necessity of stiff and strong foundation can be bypassed by a simpler solution which can also be a cheaper option.
- It is possible to avoid transmitting the heavy loads to foundations, by adopting self-equilibrating systems.
- This is a common solution in load-testing. Typically, this is done by means of a 'tension frame'.
- The following figure shows the basic components of a tension frame. The jack and the specimen tend to push the end members.
- But the end members are kept in place by members under tension such as high strength steel rods.
- The frame that is generally adopted in a pre-tensioning system is called a **stress bench**. The concrete mould is placed within the frame and the tendons are stretched and anchored on the booms of the frame. The following figures show the components of a stress bench.

Jacks

- The jacks are used to apply tension to the tendons. Hydraulic jacks are commonly used. These jacks work on oil pressure generated by a pump.

- The principle behind the design of jacks is Pascal's law. The load applied by a jack is measured by the pressure reading from a gauge attached to the oil inflow or by a separate load cell. The following figure shows a double acting hydraulic jack with a load cell.

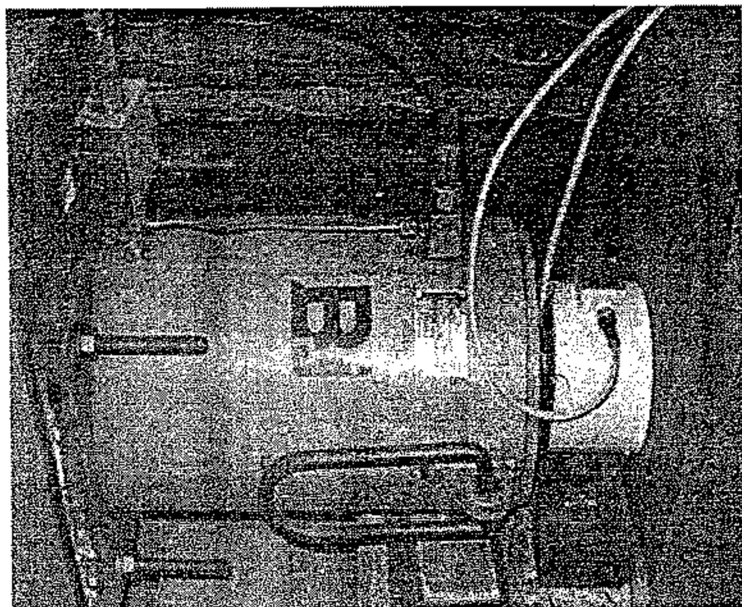


Fig. 9.16: A double acting hydraulic jack with a load cell.

Anchoring Devices

Anchoring devices are often made on the wedge and friction principle. In pre-tensioned members, the tendons are to be held in tension during the casting and hardening of concrete. Here simple and cheap quick-release grips are generally adopted. The following figure provides some examples of anchoring devices.

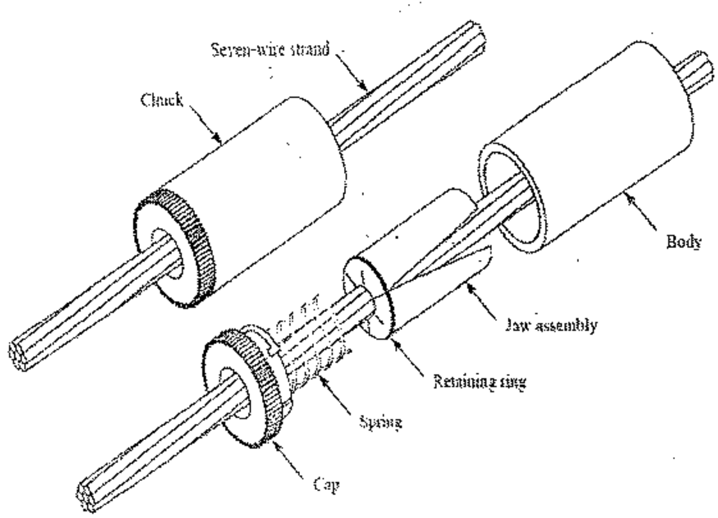


Fig. 9.17: Chuck assembly for anchoring tendons.

Harping Devices

The tendons are frequently bent, except in cases of slabs-on-grade, poles, piles etc. The tendons are bent (harped) in between the supports with a shallow sag as shown below.

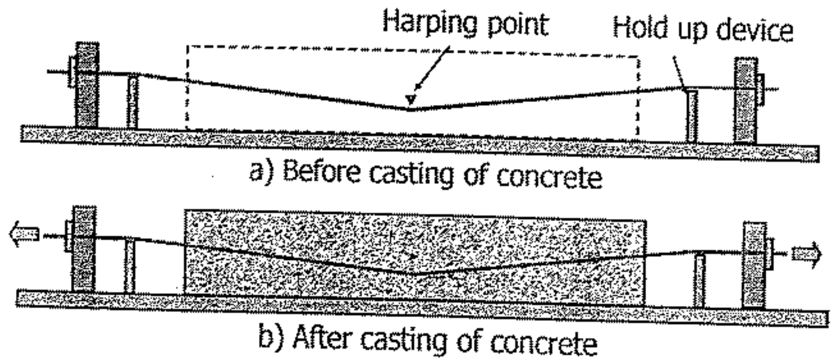
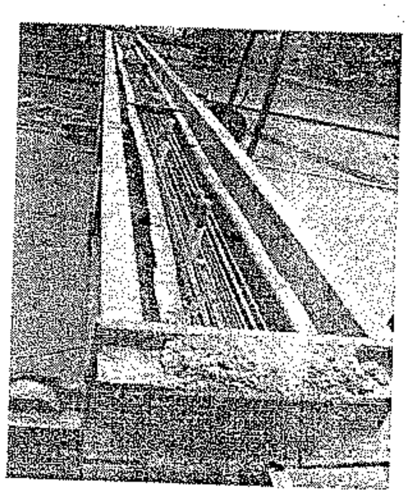


Fig. 9.18: Harping of tendons.

MANUFACTURING OF PRE-TENSIONED RAILWAY SLEEPERS

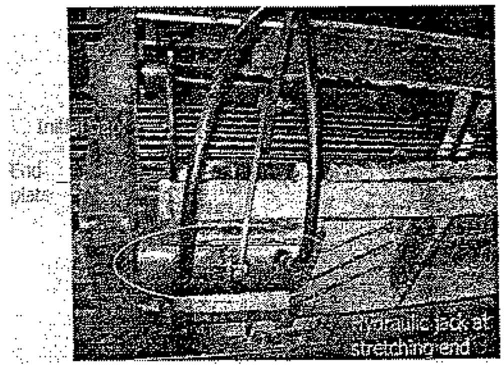
- The steel strands are stretched in a stress bench that can be moved on rollers. The stress bench can hold four moulds in a line.
- The anchoring device holds the strands at one end of the stress bench. In the other end, two hydraulic jacks push a plate where the strands are anchored.
- The movement of the rams of the jacks and the oil pressure are monitored by a scale and gauges, respectively.
- Note that after the extension of the rams, the gap between the end plate and the adjacent mould has increased. This shows the stretching of the strands.
- Meanwhile the coarse and fine aggregates are batched, mixed with cement, water and additives in a concrete mixer.
- The stress bench is moved beneath the concrete mixer. The concrete is poured through a hopper and the moulds are vibrated.
- After the finishing of the surface, the stress bench is placed in a steam curing chamber for a few hours till the concrete attains a minimum strength.



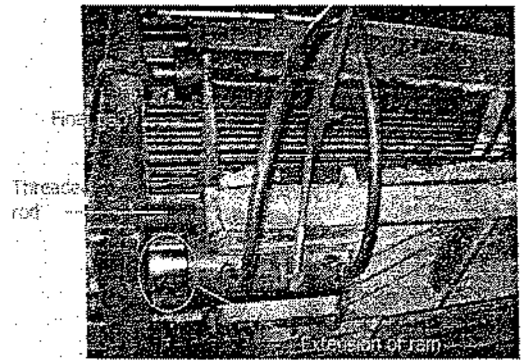
(a) Travelling pre-tensioning stress bench

Wedge and cylinder assembly at the dead end

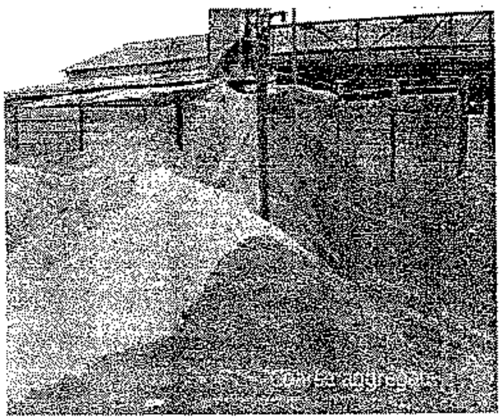
(b) Anchoring of strands



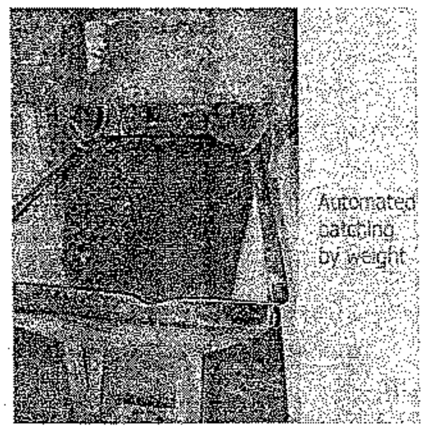
(c) Stretching of strands



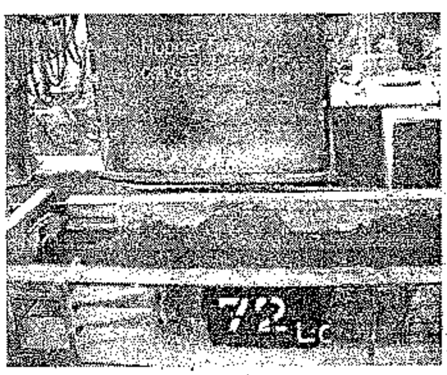
(d) Stretching of strands



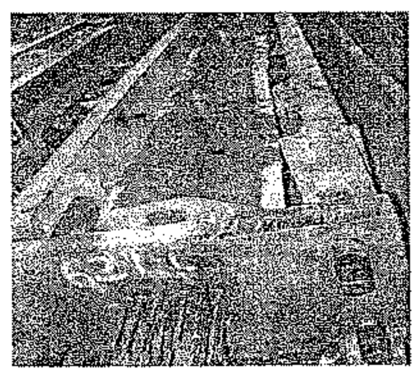
(e) Material storage



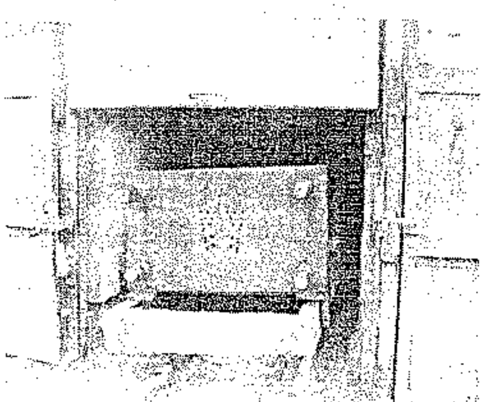
(f) Batching of materials



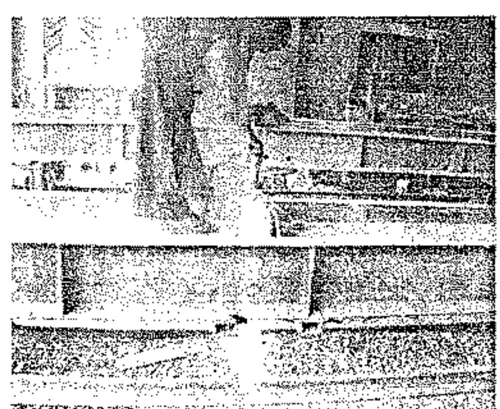
(g) Pouring of concrete



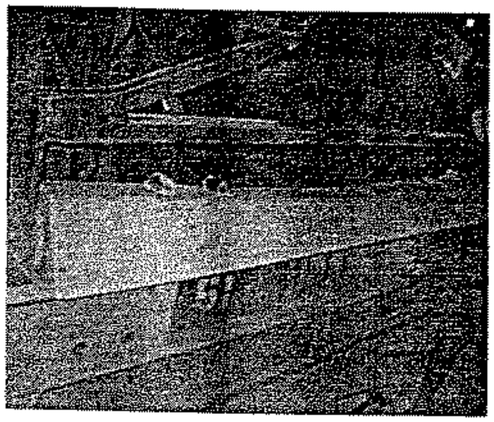
(h) Concrete after vibration of mould



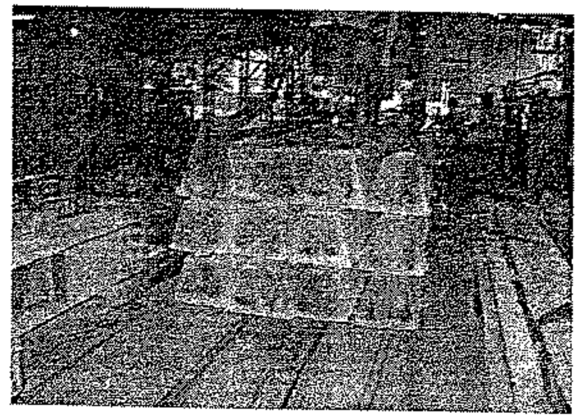
(i) Steam curing chamber



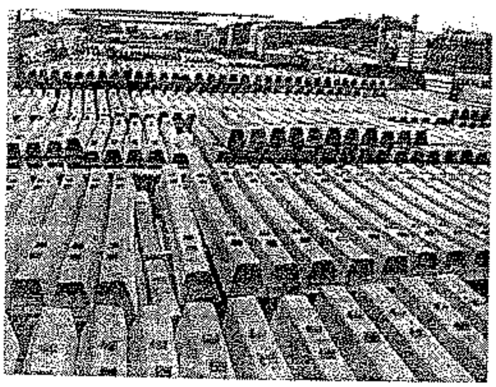
(j) Cutting of strands



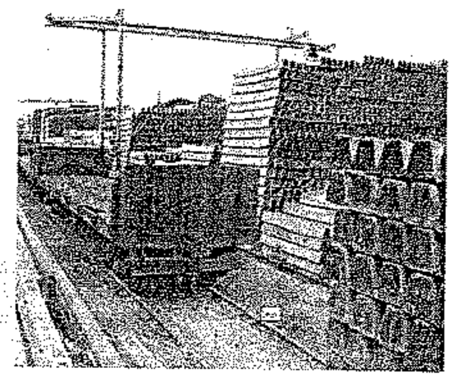
(k) Demoulding of sleeper



(l) Stacking of sleeper



(m) Water curing



(n) Storage and dispatching of sleepers

Fig. 9.19: Manufacturing of pre-tensioned railway sleepers

POST-TENSIONING SYSTEMS AND DEVICES

Stages of Post-Tensioning

- In post-tensioning systems, the ducts for the tendons (or strands) are placed along with the reinforcement before the casting of concrete.
- The tendons are placed in the ducts after the casting of concrete. The duct prevents contact between concrete and the tendons during the tensioning operation.
- Unlike pre-tensioning, the tendons are pulled with the reaction acting against the hardened concrete. If the ducts are filled with **grout**, then it is known as bonded post-tensioning.
- The grout is a neat cement paste or a sand-cement mortar containing suitable admixture. The grouting operation is discussed later in the section.
- In unbonded post-tensioning, as the name suggests, the ducts are never grouted and the tendon is held in tension solely by the end anchorages.
- The following sketch shows a schematic representation of a grouted post-tensioned member. The profile of the duct depends on the support conditions.
- For a simply supported member, the duct has a sagging profile between the ends. For a continuous member, the duct sags in the span and hogs over the support.

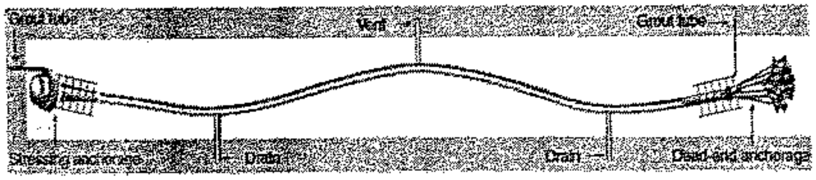


Fig. 9.20: Post-tensioning (Reference: VSL International Ltd.)

Among the following figures, the first photograph Fig. 9.21(a) shows the placement of ducts in a box girder of a simply supported bridge. The second photograph Fig. 9.21(b) shows the end of the box girder after the post-tensioning of some tendons.

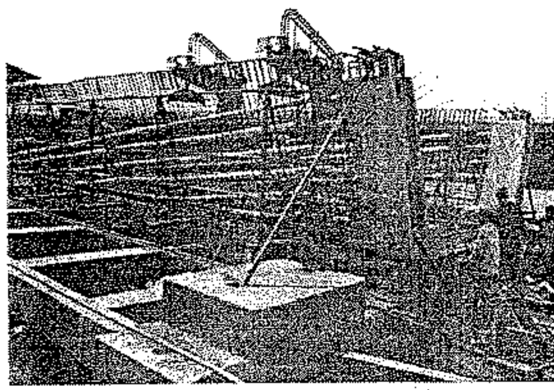


Fig. 9.21(a): Post-tensioning ducts in a box girder.

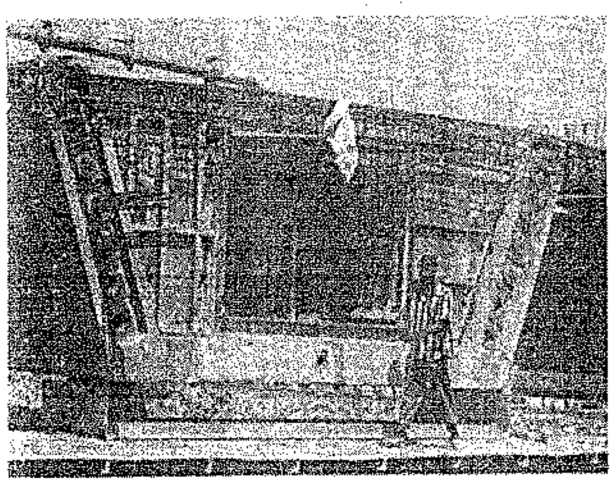
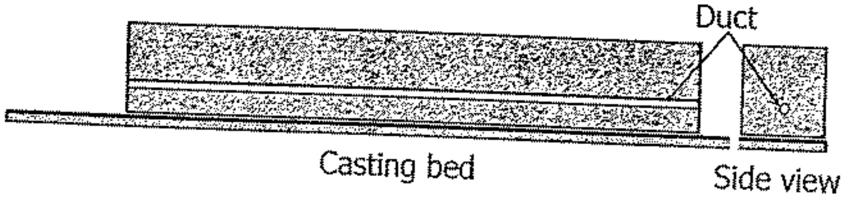


Fig. 9.21(b): Post- tensioning of a box girder
(Courtesy : Cochin Port Trust, Kerala)

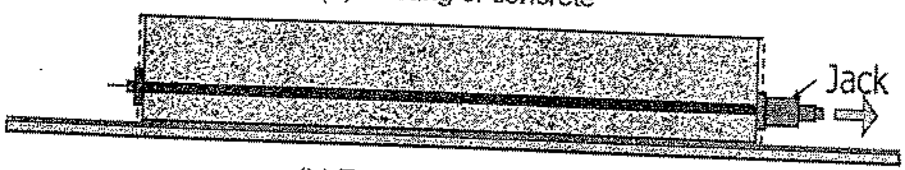
The various stages of the post-tensioning operation are summarised as follows.

1. Casting of concrete.
2. Placement of the tendons.
3. Placement of the anchorage block and jack.
4. Applying tension to the tendons.
5. Seating of the wedges.
6. Cutting of the tendons.

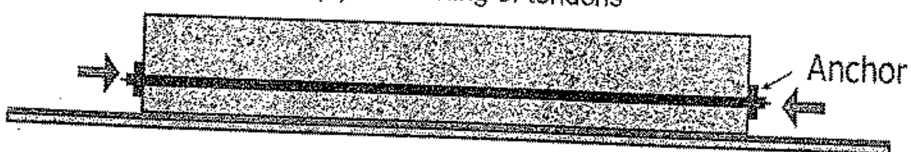
The stages are shown schematically in the following Figures 9.22(a to c). After anchoring a tendon at one end, the tension is applied at the other end by a jack. The tensioning of tendons and pre-compression of concrete occur simultaneously. A system of self-equilibrating forces develops after the stretching of the tendons.



(a) Casting of concrete



(b) Tensioning of tendons



(c) Anchoring the tendon at the stretching end

Fig. 9.22: Stages of post-tensioning (shown in elevation).

ADVANTAGES OF POST-TENSIONING

The relative advantages of post-tensioning as compared to pre-tensioning are as follows.

- Post-tensioning is suitable for heavy cast-in-place members.
- The waiting period in the casting bed is less.
- The transfer of prestress is independent of transmission length.

DISADVANTAGE OF POST-TENSIONING

The relative disadvantage of post-tensioning as compared to pre-tensioning is the requirement of anchorage device and grouting equipment.

DEVICES

The essential devices for post-tensioning are as follows.

1. Casting bed
2. Mould/Shuttering
3. Ducts
4. Anchoring devices
5. Jacks
6. Couplers (optional)
7. Grouting equipment (optional).

CASTING BED, MOULD AND DUCTS

The following figure shows the devices.

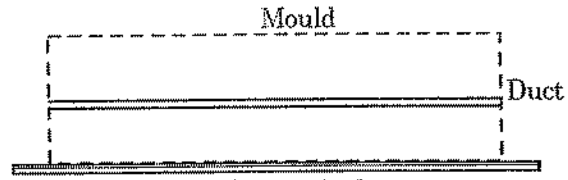


Fig. 9.23: Casting bed, mould and duct.

ANCHORING DEVICES

In post-tensioned members the anchoring devices transfer the prestress to the concrete. The devices are based on the following principles of anchoring the tendons.

WEDGE ACTION

The anchoring device based on wedge action consists of an anchorage block and wedges. The strands are held by frictional grip of the wedges in the anchorage block. Some examples of systems based on the wedge-action are Freyssinet, Gifford-Udall, Anderson and Magnel-Blaton anchorages. The following figures show some patented anchoring devices.

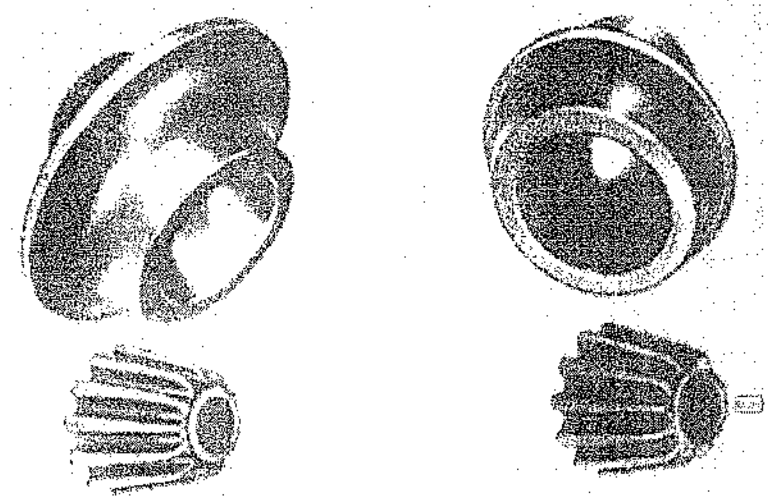


Fig. 9.24: Freyssinet 'T' system anchorage cones.
(Reference: Lin, T.Y. and Burns, N.H., *Design of Prestressed Concrete Structures*)

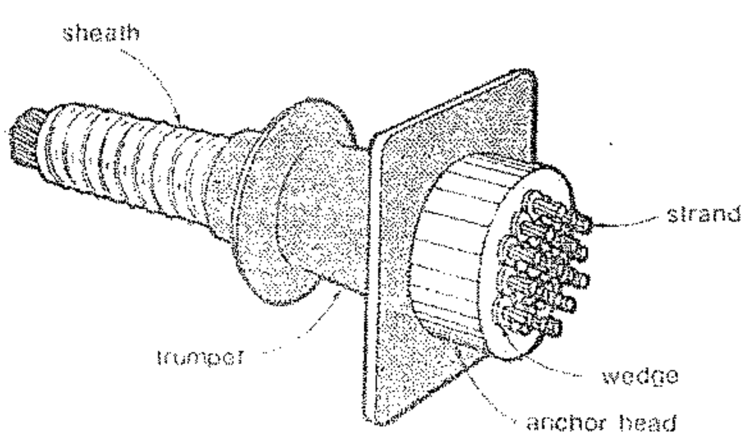


Fig. 9.25: Anchoring devices (Reference: Collins, M. P. and Mitchell, D., *Prestressed Concrete Structures*).

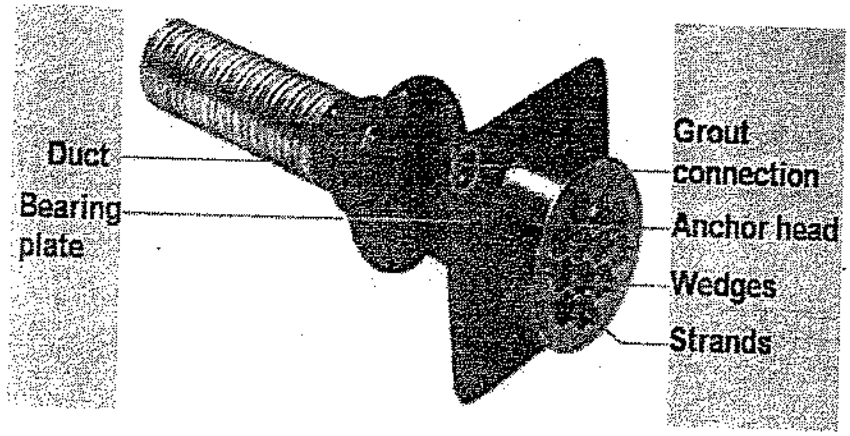


Fig. 9.26: Anchoring devices (Reference: VSL International Ltd).

Jacks

The following Fig. 9.27 shows an extruded sketch of the anchoring devices.

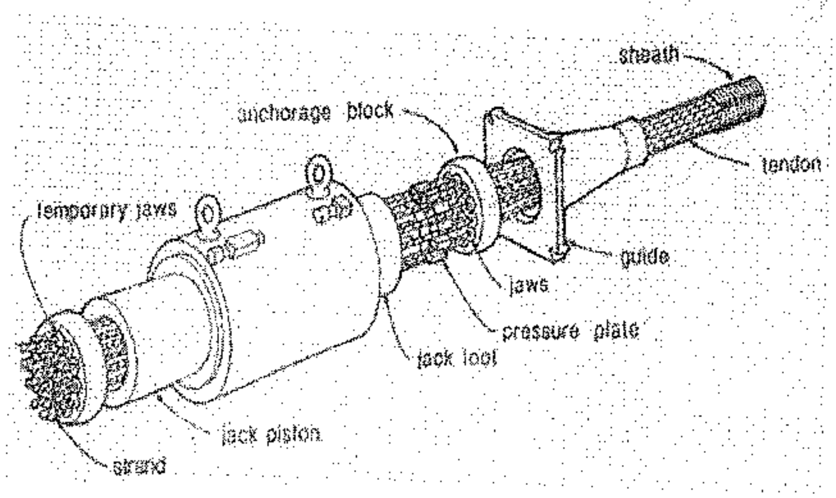


Fig. 9.27: Jacking and anchoring with wedges.
(Reference: Collins, M.P. and Mitchell, D., *Prestressed Concrete Structures*)

Couplers

The couplers are used to connect strands or bars. They are located at the junction of the members, for example at or near columns in post-tensioned slabs, on piers in post-tensioned bridge decks.

The couplers are tested to transmit the full capacity of the strands or bars. A few types of couplers are shown.

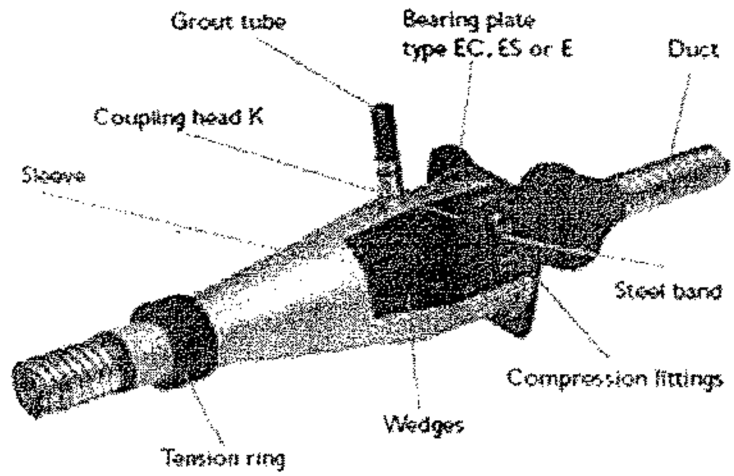


Fig. 9.28: Coupler for strands.(Reference: VSL International Ltd)

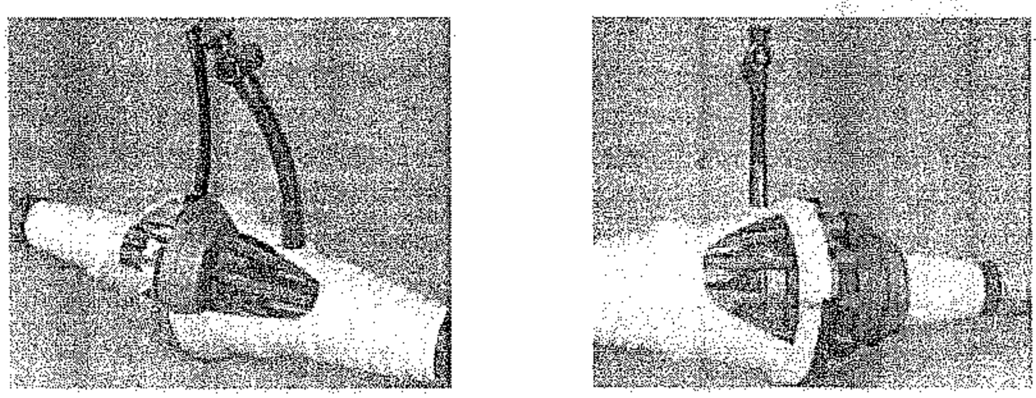


Fig. 9.29: Couplers for strands.
(Reference: Dywidag – Systems International)

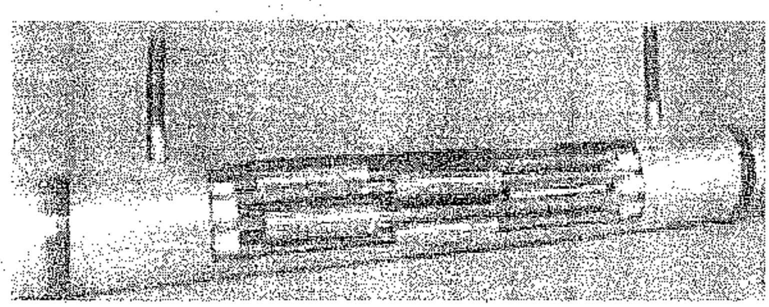


Fig. 9.30: Couplers for strands
(Reference: Dywidag – Systems International)

Grouting

Grouting can be defined as the filling of duct, with a material that provides an anti-corrosive alkaline environment to the prestressing steel and also a strong bond between the tendon and the surrounding grout.

Table 9.1: Post-tensioning system.						
Post tensioning system (country of origin)	*Type of tendon*	*Range of force*	*Cable duct*	*Arrangement of tendons in duct*	*Method of tensioning*	*Type of anchorage*
(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*
Freyssinet (France)	Wires and strands	Medium and large	Circular, formed by pneumatic tube or metal or plastic sheath around cable	Annular, spaced by helical wire core	Hydraulic jack tensioning all wires	Conical serrated concrete wedge driven by jack into female cone embedded at the end of the beam
Gifford-Udall-CCL (Great Britain)	Wires	Small and medium	Circular, formed by pneumatic tube or steel rod	Evenly spaced by perforated spacers at	Hydraulic tensioning wires singly	Split conical wedge and bush to each wire, bearing on anchor thrust plate and ring cast into end of beam
Lee-McCall (Great Britian)	Bars threaded at ends	Small, medium and large	Circular, formed by pneumatic tube or flexible corrugated	Single bars	Hydraulic jack screwed to threaded end of bar	High strength nut and spacing washers bearing on steel plate on end of beam
Magnel-Blaton (Belgium)	Wires	Small, medium and large	Rectangular, formed by solid rubber core or by metal sheath around cable	Horizontal rows of four wires spaced by metal grilles at intervals	Hydraulic jack tensioning two wires at a time	Pairs of wires held by flat steel wedges in sandwich plates bearing on distribution plates

Manufacturing of Post-tensioned Bridge Girders

The following photographs show some steps in the manufacturing of a post-tensioned I-girder for a bridge (Courtesy: Larsen & Toubro). The first photo 9.31(a) shows the fabricated steel reinforcement with the ducts for the tendons placed inside. Note the parabolic profiles of the duct for the simply supported girder. After the concrete is cast and cured to gain sufficient strength, the tendons are passed through the ducts, as shown in the second photo 9.31(b). The tendons are anchored at one end and stretched at the other end by a hydraulic jack. This can be observed from the third photo 9.31(c).

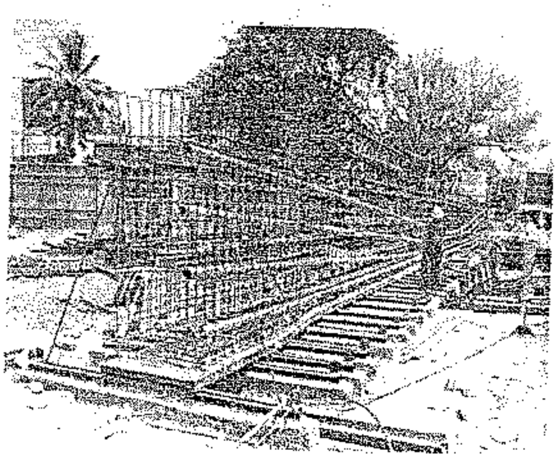


Fig. 9.31(a): Fabrication of reinforcement

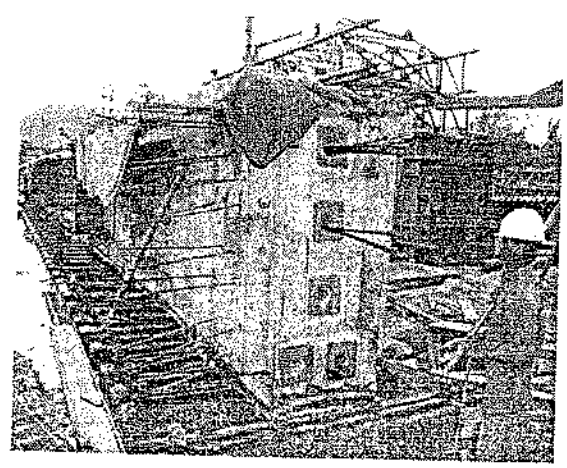


Fig. 9.31(b): Placement of tendons

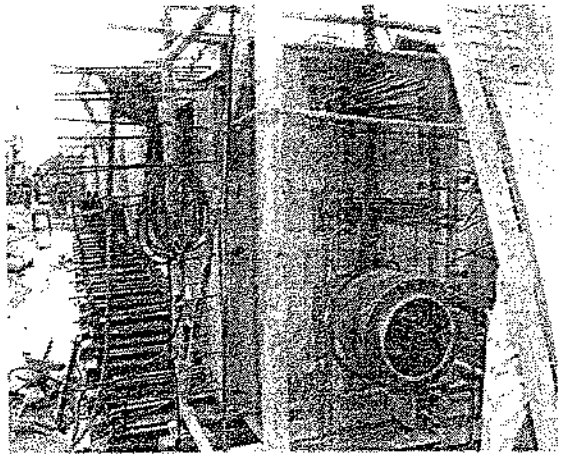
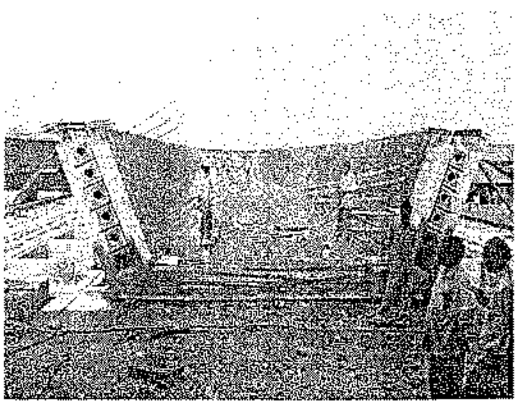
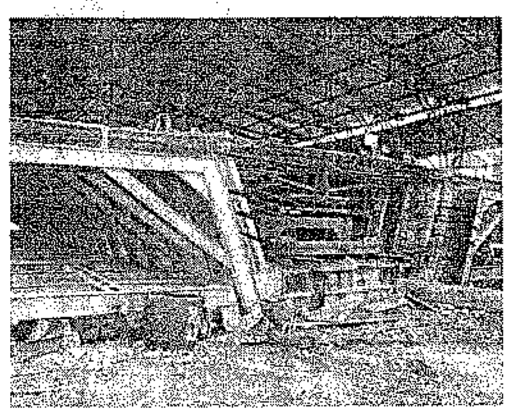


Fig. 9.31(c): Stretching and anchoring of tendons.

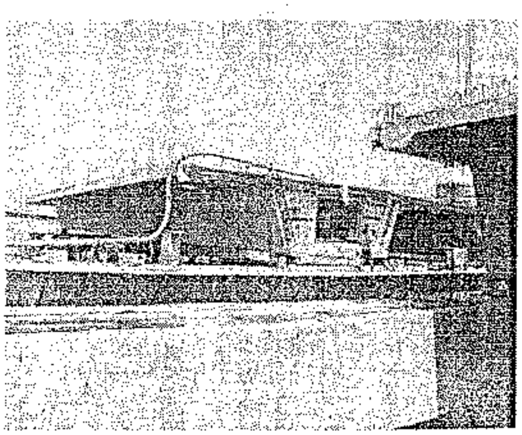
The following photos show the construction of post-tensioned box girders for a bridge (Courtesy: Cochin Port Trust). The first photo 9.32(a) shows the fabricated steel reinforcement with the ducts for the tendons placed inside. The top flange will be constructed later. The second photo 9.32(b) shows the formwork in the pre-casting yard. The formwork for the inner sides of the webs and the flanges is yet to be placed. In the third photo 9.32(c) a girder is being post-tensioned after adequate curing. The next photo 9.32(d) shows a crane on a barge that transports a girder to the bridge site. The completed bridge can be seen in the last photo 9.32(e).



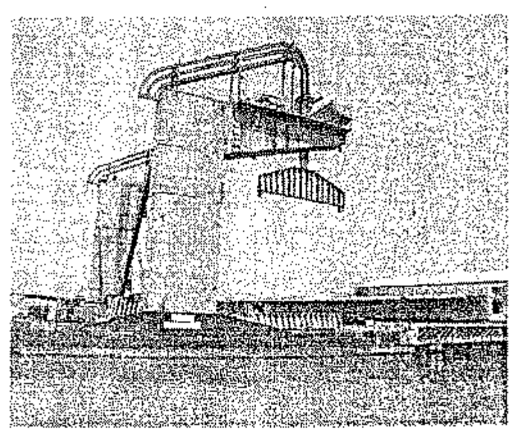
(a) Reinforcement cage for box girder.



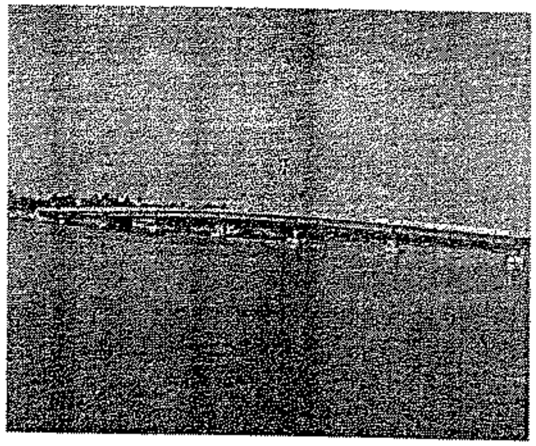
(b) Formwork for box girder.



(c) Post-tensioning of box girder.



(d) Transporting of box girder.



(e) Completed bridge.

Fig. 9.32: Manufacturing of post-tensioned bridge box girders.

IS Code Recomondation

- Concrete use for prestressed work should have a minimumof 28 days cube compressive strength of
  For Pre tensioning = 40 N/mm^2
  For Post-tensioning = 30 N/mm^2
- Ratio of standard cylinder Strength to cube Strength may be assumed to be 0.8 in the absence of data.

$$\frac{\text{Cylinder Strength}}{\text{Cube Strength}} = 0.8$$

$$\boxed{\text{Cylinder st} = 0.8 \times \text{cube st}}$$

- Mini. Cement content in concrete. of 300-360 kg/m^3 for the requirement of durability.
  Maxi. cement content in concrete of 530 Kg/m^3 far safe gaurd against excessive shrinkage.
- Pre stressing strand has greater tensile St. than prestressing wires.

PERMISSIBLE STRESSES IN HIGH TENSION STEEL

- At the time of initial tensioning, the permissible tensile stress is 80% of ultimate tensile St.
  At the times of initial tensioning.

  Permissible tensile stress = 0.80 of ultimate tensil strength
- Immediate after the tranfer, the permissible tensile stres is 70% of the ultimate tensile St.
  Immediates after the transfer

  Permissible tensile stress = 0.70 of ultimate tensil strength
- After allowing for all losses, tensile stress should not be less than 45% of ultimate tensile St.
  After allowing for all losses

  Tensile stress $\nless$ 0.45 of ultimate tensile st.

COVER

Minimum cover of

Pre tensioning member = 20 mm

Post tensioning member = 30 mm

Note: Is the members are exposed to atmosphear these values should be increased by 10 mm.

ANALYSIS OF PRESTRESS AND BENDING STRESSES

Assumption

The analysis of stresses developed in a prestressed concrete structural element is based on the following assumption:

1. Concrete is a homogeneous elastic material.
2. Within the range of working stresses, both concrete and steel behave elastically.
3. Plane Section before bending is assumed to remain plane even after bending (which implies a linear Strain distribution across the depth of the number.

ANALYSIS OF PRESTRESS

The stresses prestress in concrete evaluated by using the well know relationship for combined stresses used in the case of Columns three basic concept are used:

1. Stress concept
2. Strain concept
3. Load balancing concept

1. Stress Concept

(a) Concentric Tendon

Figure 9.33 shows a simply supported prestressed concrete beam of rectangular section prestressed by a tendon provided through its centroidal longitudinal axis. Let the beam be subjected to an external load system.

Fig. 9.33

Let P be the prestressing forcess supplied by the tendon. Due to this prestressing force, the compressiver stressed induced in concrete = $\sigma_c = \frac{P}{A}$ where A is the sectional area of the member.

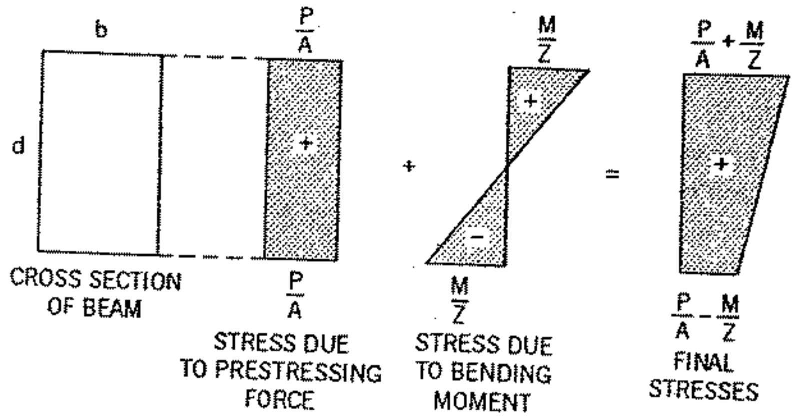


Fig. 9.34

Note: Stress developed due eccentricity (e) = 0 because in this case e = 0.

If due to the dead load and external loads, the bending moment at the section is M, then the extreme stresses at the section due to bending moment alone

$$\sigma_b = \pm\frac{M}{Z} \qquad (\sigma_b = \text{stress due to bending})$$

where, Z is the section modulus of the beam section.

$$Z = \frac{I}{Y}$$

Hence, the final extreme stresses on the beam section are given by

$$\sigma = \sigma_c \pm \sigma_b = \frac{P}{A} + \frac{M}{Z}$$

Stress at the extreme top edge $= \frac{P}{A} + \frac{M}{Z}$

and stress at the extreme bottom edge

$$= \frac{P}{A} - \frac{M}{Z}$$

The stresses due to direct load, bending and the final stresses are shown in Fig. 9.34.

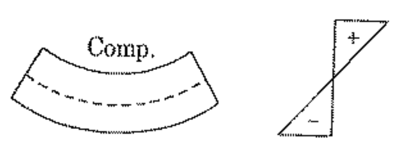


Fig. 9.34

- Used sign convention in whole prestress.

Example 1

A prestressed concrete beam 400mm × 600mm in section has a span of 6m and is subjected to uniformly distributed load of 16 kN/metre including the self-weight of the beam. The prestressing tendons which are located along the longitudinal centroidal axis provide an effective prestressing force of 960 kN. Determine the extreme fibre stresses in concrete at the mid span section.

Sol: Area of the beam section

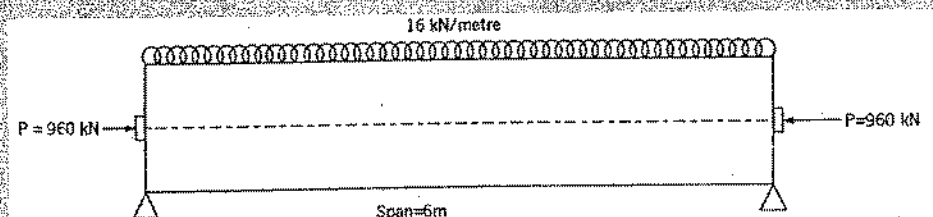
$$= 400 \times 600 = 2.4 \times 10^5 \text{ mm}^2$$

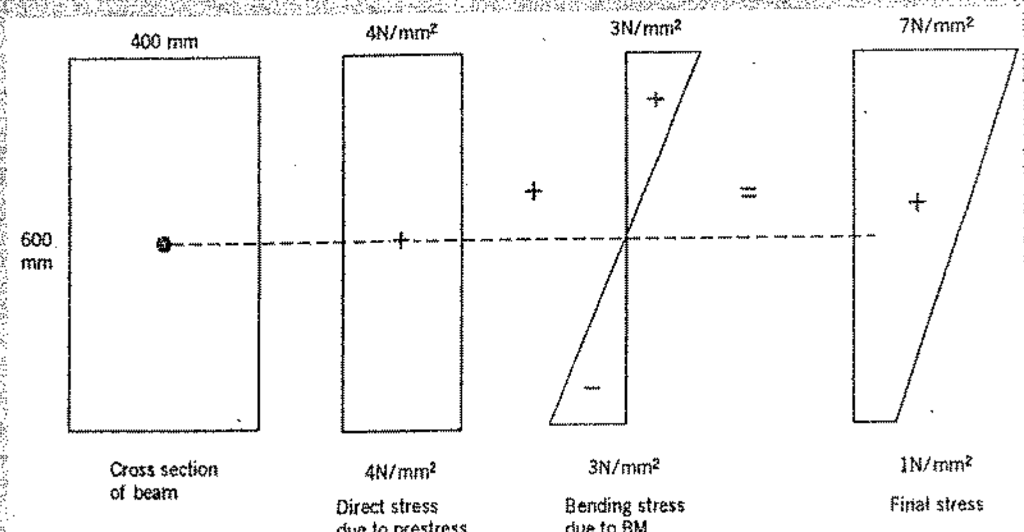
$$\text{Section modulus} = Z = \frac{bd^2}{6} = \frac{400 \times 600^2}{6} = 2.4 \times 10^7 \text{ mm}^3$$

$$\text{Maximum bending moment} = M = \frac{16 \times 6^2}{8} = 72 \text{ kNm}$$

$$\text{Direct stress due to prestressing force} = +\frac{P}{A} = +\frac{960 \times 10^3}{2.4 \times 10^5} \text{ N/mm}^2 = +4 \text{N/mm}^2$$

$$\text{Extreme stress due to bending moment} = \pm\frac{M}{Z} = \pm\frac{72 \times 10^6}{2.4 \times 10^7} = \pm 3 \text{ N/mm}^2$$





∴ Final stresses in the extreme fibres will be as follows:

Final stress in the extreme top fibres = + 4 + 3 = + 7 N/mm²

Final stress in the extreme bottom fibres = + 4 − 3 = + 1 N/mm²

(b) Analysis of Stress using Eccentric Tendon

It will be advantageous to place the tendon at an eccentricity. Figure 9.35 shows a beam of rectangular section prestressed by a tendon placed longitudinaly, at an eccentricity e from the centroidal longitudinal axis. Let the beam be subjected to an external load system. Let P be the prestressing force supplied by the tendon. Let due to the dead load and external loads the bending moment at a section be M.

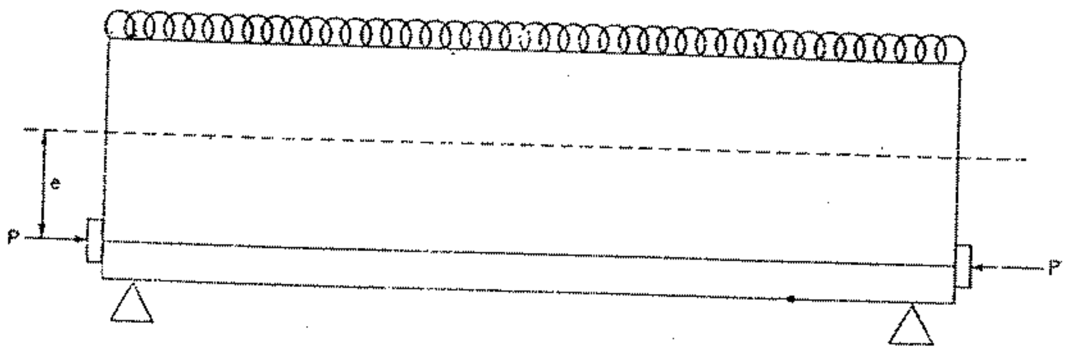


Fig. 9.35

The stresses on the section consist of the following

(i) Direct stress due to prestressing force

$$= +\frac{P}{A}$$

(ii) Extreme stresses due to eccentricity of the prestressing force

$$= \mp\frac{Pe}{Z}$$

(iii) Extreme stresses due to bending moment

$$= \pm\frac{M}{Z}$$

Final Stresses

$$\text{Stress at the extreme top edge} = \frac{P}{A} - \frac{Pe}{Z} + \frac{M}{Z}$$

$$\text{Stress at the extreme bottom edge} = \frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z}$$

These are shown in Fig. 9.36 below.

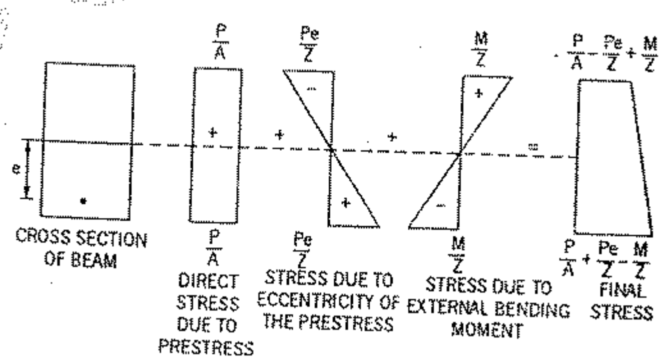


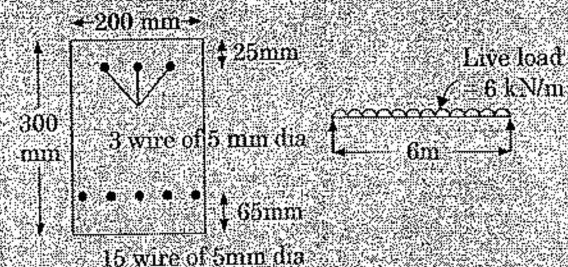
Fig. 9.36

Note: The stresses caused by external bending moment is counteract by Hogging moment developed due to eccentricity of the tendon.

Example 2

A rectangular concrete beam of cross-section 30 cm deep and 20 cm wide is prestressed by means of 15 wires of 5 mm diameter located 6.5 cm from the bottom of the beam and 3 wires of dia. of 5 mm, 2.5 cm from the top. Assuming the prestress in the steel as 840 N/mm², Calculate the Stresses at the extreme fibre at the mid span section when the beam is supporting its own weight over a span of 6 m. If a uniformly distributed live load of 6 kN/m is imposed, evaluate the maxi. working stress in concrete. The density of concrete is 24 kN/m³.

Sol:



Total load = Live load + Dead load

Dead looad = 0.2 × 0.3 × 24

= 1.44 kN/m

$w = 6 + 1.44 = 7.44$ kN/m

$$\text{B.M.} = \frac{wl^2}{8} = \frac{7.44\times6^2}{8}$$

= 33.48 kN-m

Total Prestressing force

$$P = \left(15\times\frac{\pi}{4}+5^2\times3\times\frac{\pi}{4}\times5^2\right)\times840$$

$P = 296.88$ KN.

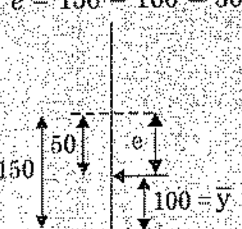
For eccentricity e

Distance of the centroid of the prestressing force from the base.

$$\bar{y} = \frac{15\times\frac{\pi}{4}\times5^2\times65+3\times\frac{\pi}{4}\times5^2\times275}{15\times\frac{\pi}{4}\times5^2+3\times\frac{\pi}{4}\times5^2}$$

$\bar{y} = 100$ mm.

$\therefore\ e = 150 - 100 = 50$ mm



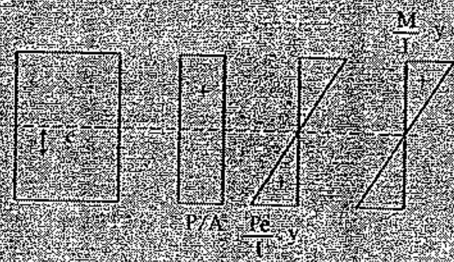
All dimensions in mm

$$I = \frac{bD^3}{12} = \frac{200 \times 300^3}{12}$$

$$= 450 \times 10^6 \text{ mm}^4$$

$$A = 200 \times 300$$

$$A = 60 \times 10^3 \text{ mm}^2$$



$$\frac{P}{A} = \frac{296.88 \times 10^3}{60 \times 10^3}$$

$$= 4.95 \text{ N/mm}^2$$

$$\frac{P.e}{I} y = \frac{296.88 \times 10^3 \times 50}{450 \times 10^6} 150$$

$$= 4.95 \text{ N/mm}^2$$

$$\frac{M}{I} y = \frac{33.48 \times 10^6}{450 \times 10^6} .150$$

$$= 11.16 \text{ N/mm}^2$$

$$\text{Total stress} = \frac{P}{A} \pm \frac{P.e}{I} y \pm \frac{M}{I} y$$

$$\text{Stress at top} = \frac{P}{A} - \frac{P.e}{I} .y + \frac{M}{I} .y$$

$$= 4.95 - 4.95 + 11.16$$

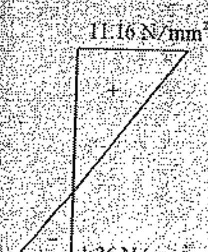
$$= 11.16 \text{ N/mm}^2$$

$$\text{Stress at bottom} = \frac{P}{A} + \frac{P.e}{I} .y + \frac{M}{I} .y$$

$$= 4.95 + 4.95 - 11.16$$

$$= -1.26 \text{ N/mm}^2$$

Resulstant stress



Maximum working stress in concrete is 11.16 N/mm² (compression).

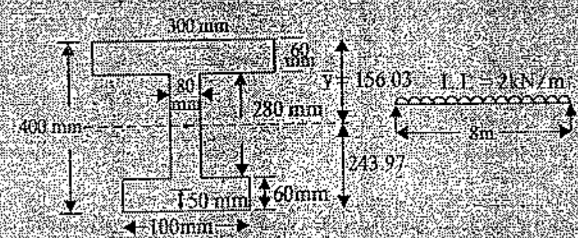
Example 3

An unsymmetrical I-section beam is used to support an imposed load of 2 kN/m over a span of 8 m. The sectional details are top flange, 300 mm wide and 60 mm thick, bottom flange 100 mm wide and 60 mm thick, thickness of the web 80 mm overall depth of beam 400 mm. At the centre of the span the effective prestressing force of 100 kN is located at 50 mm from the soffit of the beam. Estimate the stresses at the centre of span section of the beam for the following load conditions:

(a) Prestress + Self weight

(b) Prestress + Self wt + live load

Sol.



$\bar{y}$ = Distance of centroid of the prestressing force from top

$$\bar{y} = \frac{300\times60\times30+280\times80\times(60+140)+60\times100\times(400-30)}{300\times60+280\times80+100\times60}$$

$$\bar{y} = 156.03 \text{ mm}$$

from bottom $\quad \bar{y} = 243.97 \text{ mm } (400 - 156.03)$

$$e = 243.97 - 50 = 193.97 \text{ mm}$$

$$A = 0.3 \times 0.60 + 0.280 \times 0.80 + 0.10 \times 0.60 = 0.0464 \text{ m}^2$$

$$\text{Dead } wt = \begin{bmatrix} 0.300\times0.60+0.280 \\ \times0.80+0.100\times0.60 \end{bmatrix}\times24$$

$$= 1.114 \text{ KN/m}$$

$$(\text{B.M.})_{\text{LL}} = \frac{2\times8^2}{8} = 16 \text{ kN-m}$$

$$(\text{B.M.})_{\text{DL}} = \frac{1.114\times8^2}{8}$$

$$(\text{B.M.})_{\text{DL}} = 8.912 \text{ kN-m}$$

$$I = \frac{300\times60^3}{12}+300\times60\times(156.03-30)^2$$

$$+\frac{80\times280^3}{12}+280\times80\times(243.97-140-60)^2$$

$$+\frac{100\times60^3}{12}+100\times60\times(243.97-30)^2$$

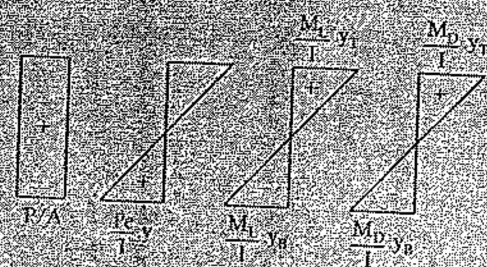
$$I = 757.45 \times 10^6 \text{ mm}^4$$

$$Z_T = \frac{I}{Y_T} = \frac{757.45\times10^6}{156.03}$$

$$= 4.85\times10^6\ mm^3$$

$$Z_B = \frac{I}{Y_B} = \frac{757.45\times10^6}{243.97} = 3.105\times10^6\ mm^3$$

Stress



$$\frac{P}{A} = \frac{100\times10^3}{46400} = 2.15\ N/mm^2$$

$$\frac{P.e}{I}.y_T = \frac{100\times10^3\times193.97}{757.45\times10^6}\times156.03 = 4.00\ N/mm^2$$

$$\frac{P.e}{I}.y_B = \frac{100\times10^3\times193.97\times243.97}{757.45\times10^6} = 6.25\ N/mm^2$$

$$\frac{M_L}{I}.y_T = \frac{16\times10^6\times156.03}{757.45\times10^6} = 3.29\ N/mm^2$$

$$\frac{M_L}{I}.y_B = \frac{16\times10^6}{757.45\times10^6}\times243.97 = 5.15\ N/mm^2$$

$$\frac{M_D.y_T}{I} = \frac{8.912\times10^6}{757.45\times10^6}\times156.03 = 1.835\ N/mm^2$$

$$\frac{M_D}{I}.Y_B = \frac{8.912\times10^6}{757.45\times10^6}\times243.97 = 2.87\ N/mm^2$$

Stress due to Prestress & Self *weight*

$$\text{at top} = \frac{P}{A} - \frac{Pe}{I}y_T + \frac{M_D.y_T}{I}$$

$$= 2.15 - 4 + 1.835 = -0.015\ N/mm^2$$

$$\text{at bottom} = \frac{P}{A} + \frac{Pe}{I}y_B - \frac{M_D.y_B}{I}$$

$$= 2.15 + 6.25 - 2.87$$

$$= 5.53\ N/mm^2$$

Stress due to Prestress + Self wt + Live load

$$\text{at top} = \frac{P}{A} - \frac{P.e}{I}\cdot y_T + \frac{M_L \cdot y_T}{I} + \frac{M_D \cdot y_T}{I}$$

$$= 2.15 - 4 + 3.29 + 1.835$$

$$= 3.275 \text{ N/mm}^2$$

$$\text{at bottom} = \frac{P}{A} + \frac{P.e}{I} y_B - \frac{M_L}{I} \cdot y_B - \frac{M_D}{I} y_B$$

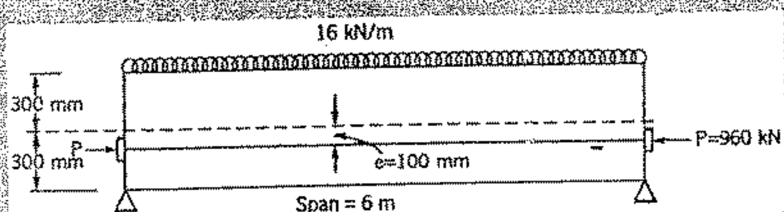
$$= 2.15 + 6.25 - 5.15 - 2.87$$

$$= 0.38 \text{ N/mm}^2$$

Example 4

A prestressed concrete beam 400 mm × 600 mm in section has a span of 6 m and is subjected to a uniformly distributed load of 16 kN/metre including the self weight of the beam. The prestressing tendons are located at the lower third point and provide an effective prestressing force of 960 kN. Determine the extreme fibre stresses in concrete at the mid span section.

Sol: Area of the beam section $= A = 400 \times 600 = 2.4 \times 10^5 \text{ mm}^2$



Section modulus
$$Z = \frac{bd^2}{6} = \frac{400 \times 600^2}{6} = 2.4 \times 10^7 \text{ mm}^3$$

Maximum bending moment due to external loading

$$= M = \frac{wl^2}{8} = \frac{16 \times 6^2}{8} = 72 \text{ kNm}$$

Direct stress due to prestressing force

$$= \frac{P}{A} = + \frac{960 \times 10^3}{2.4 \times 10^5} = +4 \text{ N/mm}^2$$

Extreme stress due to eccentricity of the prestressing force

$$= \mp \frac{M}{Z} = \mp \frac{960 \times 10^3 \times 100}{2.4 \times 10^7} = \mp 4 \text{ N/mm}^2$$

Extreme stress due to external bending moment

$$= \pm \frac{M}{Z} = \pm \frac{72 \times 10^6}{2.4 \times 10^7} = \pm 3 \text{ N/mm}^2$$

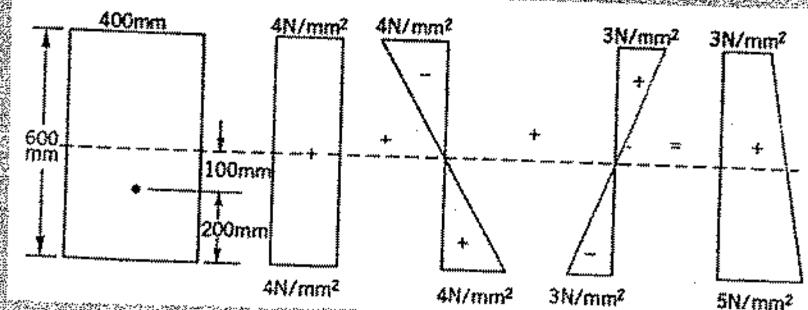
Final stresses for the extreme fibres will be as follows:

Final stress in the top extreme fibres

$$= +4 - 4 + 3 = +3 \text{ N/mm}^2$$

Final stress in the bottom extreme fibres

$$= + 4 + 4 - 3 = 5 \text{ N/mm}^2$$



Example 5

A prestressec concrete beam 300 mm wide and 600 mm deep is prestressed using 5 high tension bars of 12 mm φ provide at 220 mm from the soffit of the beam. The effective stress in the steel is 800 N/mm². Find the bending that must be applied to the section to just avoid tension at the soffit of the beam.

Sol: $A = 300 \times 600 = 18 \times 10^4$ mm², $Z = \dfrac{300 \times 600^2}{6} = 18 \times 10^6$ mm³

$P = 800 \times 5 \times \dfrac{\pi}{4} \times 12^2 = 452389$ N $\quad e = 300 - 220 = 80$ mm

Let M be the bending moment required so that the tension at the soffit in just avoided. For this condition.

$$\frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z} = 0$$

$$\frac{452389}{18 \times 10^4} + \frac{452389 \times 80}{18 \times 10^6} - \frac{M}{18 \times 10^6} = 0$$

$$2.51 + 2.01 - \frac{M}{18 \times 10^6} = 0$$

$\therefore M = 18 \times 10^6 \times 4.52 = 81.36 \times 10^6$ N/mm $= 81.36$ kNm

Example 6

A prestressed concrete beam 250 mm wide and 375 mm deep is prestressed by concentrically placed tendon. The span of the beam is 8 m and the beam has to support an imposed load of 4.25 kN/m. Find the prestressing force necessary so that the tension is just avoided at the soffit of the mid-section. If however, the tendon is provided at an eccentricity of 65 mm final the prestressing force necessary so that tension is just avoided at the soffit of the mid-section.
Concrte weights 24 kN/m³.

Sol: $A = 250 \times 375 = 93750$ mm²,

$$Z = \frac{250 \times 375^2}{6} = 5859375 \text{ mm}^3$$

$\therefore$ Dead load of the beam = 0.25 × 0.375 × 24 = 2.25 kN/m

Superimposed load of the beam = 4.25 kN/m

$\therefore$ Total load on the beam = 2.25 + 4.25 = 6.50 kN/m

Total bending moment due to dead load and superimposed load

$$= M = 650 \times \frac{8^2}{8} = 52 \text{ kNm}$$

Let P be the prestressing force so that the tensile stress at the soffit of the mid-section is just avoided.

Case (i): When the tendon is concentrically provided.

$$\text{For this case } \frac{P}{A} - \frac{M}{Z} = 0$$

$$\frac{P}{93750} = \frac{52\times10^6}{5859375} \quad P = \frac{52\times10^6\times93750}{5859375} = 832000 \text{ N} = 832 \text{ kN}$$

Case (ii): When the tendon is provided at an eccentricity of 65 mm

$$\text{For this case } \frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z} = 0$$

$$\frac{P}{93750} + \frac{P\times65}{5859375} - \frac{52\times10^6}{5859375} = 0 \quad 2.176\times10^{-5}\,P = 8.874667$$

$$P = 407873 \; N = \mathbf{407.843 \text{ kN}}$$

Example 7

A prestressed concrete beam of rectangular section 300 mm wide and 600 mm deep has a span of 10 m. The effective prestressing force is 980 kN at an eccentricity of 120 mm. The dead load of the beam is 4.5 kN/m and the beam has to carry a live load of 7.5 kN/m. Determine the extreme stresses:

(i) at the end section

(ii) at the mid section without the action of live load.

(iii) at the mid section with the action of live load.

Sol:

Area of the beam section = $A = 300 \times 600 = 1.8 \times 10^5 \text{ mm}^2$

Section modulus of the section = $Z = 300 \times \frac{600^2}{6} = 1.8 \times 10^7 \text{ mm}^3$

$$\text{D.L. moment at mid span} = \frac{4.5\times12^2}{8} = 81 \text{ kNm}$$

$$\text{L.L. moment at mid span} = \frac{7.5\times12^2}{8} = 135 \text{ kNm}$$

(i) Analysis of the end section

Direct stress due to prestressing force

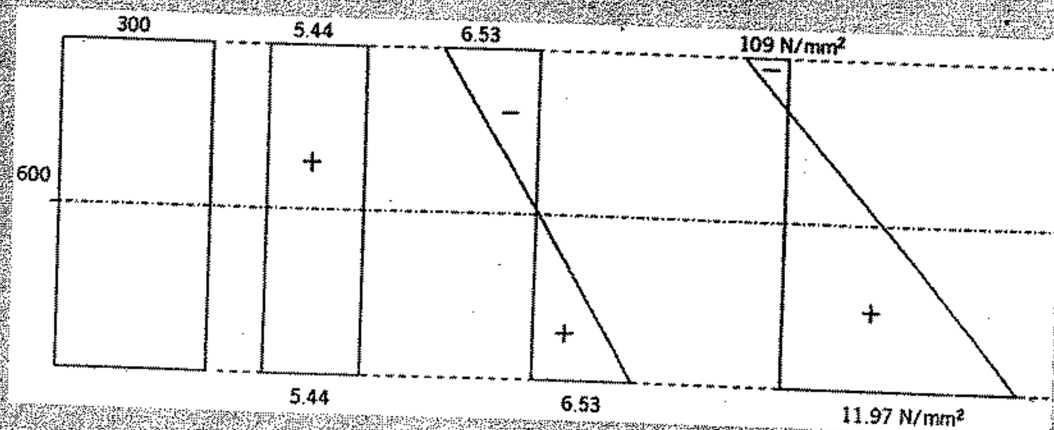
$$= +\frac{P}{A} = +\frac{980+10^3}{1.8\times10^5} = +5.44 \text{ N/mm}^2$$

Extreme stress due to eccentricity of the prestressing force

$$= \mp \frac{Pe}{Z} = \mp \frac{980 \times 10^3 \times 120}{1.8 \times 10^7} = \mp 6.53 \text{ N/mm}^2$$

Resultant stress at top edge = 5.44 − 6.53 = −1.109 N/mm² (tensile)

Resultant stress at bottom edge = 5.44 + 6.53 = + 11.97 N/mm² (compressive)



(ii) Analysis of the mid section subjected to prestressing force and dead load

Direct stress = + 5.44 N/mm²

Extreme stress due to eccentricity of the prestressing force

$$= \mp 6.53 \text{ N/mm}^2$$

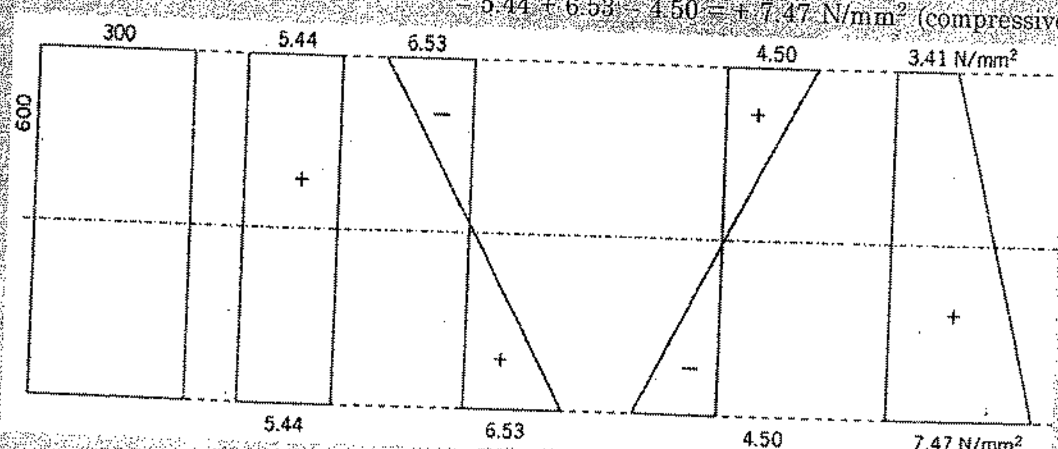
Extreme stress due to dead load moment

$$= \pm \frac{M_d}{Z} \pm \frac{81 \times 10^6}{1.8 \times 10^7} = \pm 4.50 \text{ N/mm}^2$$

Resultant stress at the top edge = 5.44 − 6.53 + 4.50 = 3.41 N/mm² (compressive)

Resultant stress at the bottom edge

= 5.44 + 6.53 − 4.50 = + 7.47 N/mm² (compressive)



(iii) Analysis of the mid section subjected to prestressing force, dead load and live load

Direct stress = + 5.44 N/mm²

Extreme stress due to eccentricity of the prestressing force

$$= \mp 6.53 \text{ N/mm}^2$$

Extreme stress due to dead load moment

$$= \pm 4.50 \text{ N/mm}^2$$

Extreme stress due to live load moment

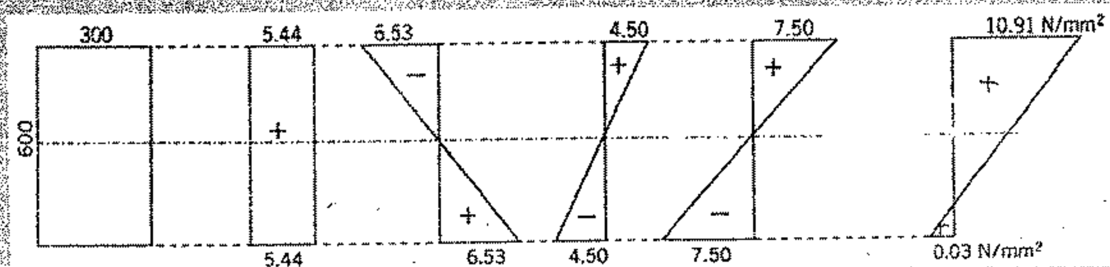
$$= \pm \frac{M_L}{Z} = \pm \frac{135\times10^6}{1.8\times10^7} = \pm 7.50 \text{ N/mm}^2$$

Resultant stress at the top edge

$$= 5.44 - 6.53 + 4.50 + 7.50 = + 10.91 \text{ N/mm}^2 \text{ (compressive)}$$

Resultant stress at the bottom edge

$$= 5.44 + 6.53 - 4.50 - 7.50 = -0.03 \text{ N/mm}^2 \text{ (tensile)}$$



STRAIN CONCEPT

Effect of loading on stress in tendon

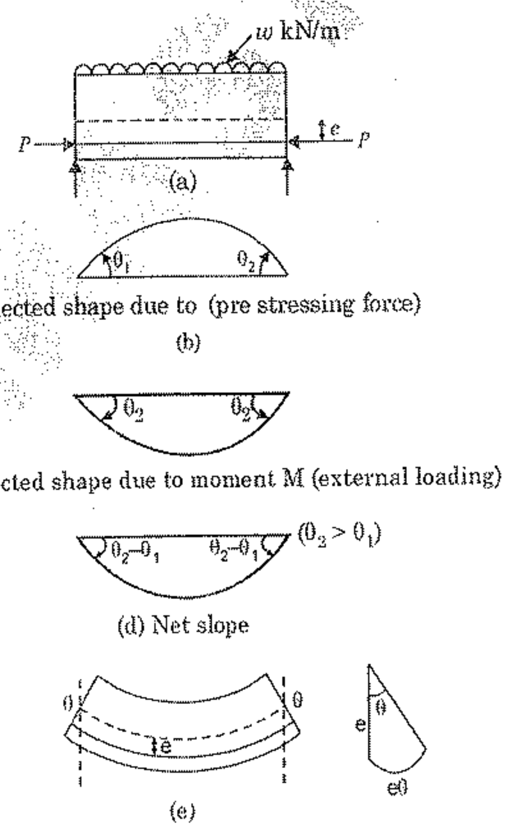
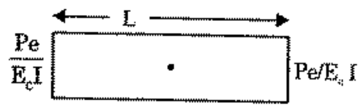


Fig. 9.37

- Now slope of beam due to prestressing moment (Pe)

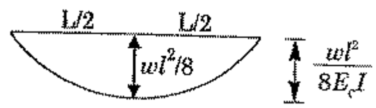


$$\text{Slope} = \text{area of } \frac{Pe}{EI} \text{ diagram}$$

$$\theta_1 = \frac{P.e}{E_c I} \cdot \frac{L}{2} = \frac{P.e.L}{2E_c I}$$

- Slope of beam due to moment M (external loading w)

$\theta_2 =$



$$\theta_2 = \frac{2}{3}\frac{wL^2}{8E_c I} \cdot \frac{L}{2} = \frac{wL^3}{24E_c I}$$

Net slope
$$\theta = \theta_2 - \theta_1 = \frac{wL^3}{24E_c I} - \frac{PeL}{2E_c I}$$

$$\theta_2 > \theta_1$$

- Net elongation of steel reinforcement due to θ slopes $= e\theta + e\theta = 2e\theta$

$$= 2e\left(\frac{wL^3}{24E_c I} - \frac{P.e.L}{2E_c I}\right)$$

- Stress in steel = Strain × E

$$= \frac{2e\theta}{L} \times E_s \left[\frac{\delta l}{l} = \frac{2e\theta}{l}\right]$$

Example 8

A prestresed concrete beam of size 300 × 400 mm is prestressed by steel tendons of area 100 mm².

$$P_o = 1200 \text{ N/mm}^2$$

Findout % increase of stress due to LL of 8 KN/m over a spanof 6 m

$$E_S = 210 \text{ kN/mm}^2$$
$$E_C = 30 \text{ kN/mm}^2$$
$$e = 60 \text{ mm}$$

Sol.

$$\text{Dead load} = 0.3 \times 0.4 \times 25 = 3 \text{ kN/m}$$
$$\text{LL} = 8 \text{ KN/m}$$
$$\text{Total load} = 11 \text{ KN/m}$$

Slope due to prestressing moment

$$\theta_1 = \frac{Pel}{2E_cI} = \frac{120\times10^3\times60\times6000}{2\times30\times\left(300\times\frac{400^3}{12}\right)\times10^3}$$

$$= 4.5\times10^{-4} \text{ radian}$$

$$\theta_2 = \frac{wl^3}{24E_cI} = \frac{11\times(6000)^3}{24\times30\times10^3\times300\times\frac{400^3}{12}}$$

$$= 2.062\times10^{-3} \text{ radian}$$

$$\theta = \theta_2 - \theta_1$$

$$= 2.062\times10^{-3} - 4.5\times10^{-4}$$

$$= 1.6125\times10^{-3} \text{ radian}$$

$$\text{elongation} = 2e\theta$$

$$= 2\times60\times1.6125\times10^{-3}$$

$$= 0.1935 \text{ mm}$$

Gain of stress is steel due to elongation

$$= \frac{2e\theta}{l}\times E_s = \frac{0.1935}{6000}\times210\times10^3 = 6.77 \text{ N/mm}^2$$

$$\% \text{ increase} = \frac{6.77}{1200}\times100 = 0.56\%$$

Example 9

A prestressed concrete beam of span 6 m has a width of 150 mm and a depth of 300 m. the initial stress in the tendons located at a constant eccentricity of 50 mm is 1000 N/mm². The sectional area of the tendons is 100 mm². Find the percentage increase in stress in the wires when the beam supports a live load of 5 kN/m. Unit weight of concrete is 24 kN/m³. Modulus of elasticity of steel is 210 kN/mm² and modulus of elasticity for concrete is 30 kN/mm².

Sol: D.L. of the beam = 0.15 × 0.30 × 24 = 1.08 kN/m

L.L. = 5 kN/m

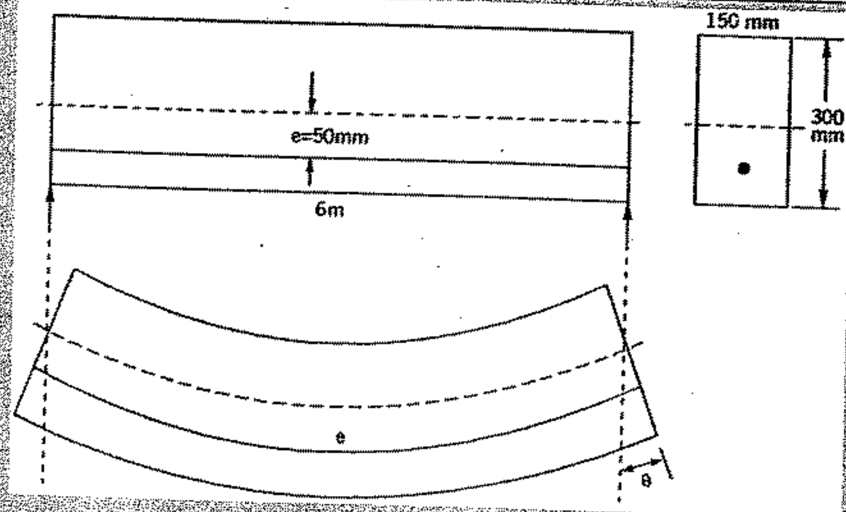
Total load = 5 + 1.08 = 6.08 kN/m

$$I = \frac{150\times300^3}{12} = 3.375\times10^8 \text{ mm}^4;\ e = 50 \text{ mm}$$

$$p = 1000 \text{ N/mm}^2;\ A_s = 100 \text{ mm}^2$$

$$P = 1000\times100 = 100000\ N = 100 \text{ kN}$$

$$\text{Rotation due to prestress } \theta = \frac{PeL}{2E_cI} = \frac{100\times50\times6000}{2\times36\times3.375\times10^8} = 0.00123 \text{ radians}$$



Rotation due to loads $\theta_2 = \frac{WL^3}{24 E_c I} = \frac{6.08\times6^3\times1000^2}{24\times36\times3.375\times10^8} = 0.00450$ radian

Net rotation = 0.00450 − 0.00123 = 0.00327 radian

Elongation of the cable = 2 × 50 × 0.00327 = 0.327 mm

$$\text{Increase in stress} = \frac{0.327}{6000} \times 2.1 \times 10^5 = 11.445\% \text{ N/mm}^2$$

$$\text{Percentage increase in stress} = \frac{11.445}{1000}\times100 = 1.1445\%$$

Example 10

A rectangular concrete beam 150 mm wide and 300 mm deep is prestressed by a straight cable carrying an effective prestressing force of 225 kN, at an eccentricity of 50 mm. The beam supports a uniformly distributed load of 7.20 kN/m inclusive of the self-weight of the beam. The span of the beam is 5m. If the modulus of rupture of concrete is 5 N/mm², calculate the load factor against cracking.

Sol:

$$A = 150 \times 300 = 45000 \text{ mm}^2$$

$$Z = \frac{150\times300^2}{6} = 2.25 \times 10^6 \text{ mm}^3$$

Let the ultimate load on the beam be w_u kN/m.

At cracking condition the tensile stress in concrete reaches 5 N/mm²

$$\therefore \frac{P}{A}+\frac{Pe}{Z}-\frac{M_u}{Z} = -5 \quad \therefore \frac{225\times10^3}{45\times10^3}+\frac{225\times10^3\times50}{2.25\times10^6}-\frac{M_u}{2.25\times10^6} = -5$$

$$\frac{M_u}{2.25\times10^6} = 15$$

$$M_u = 33.75 \times 10^6 \text{ Nmm} = 33.75 \text{ kNm}$$

Ultimate moment $\frac{wl^2}{8} = \frac{w_u \times 5^2}{8} = 33.75$, $\therefore w_u = 10.8$ kN/m

But the working load is only 7.20 kN/m

$\therefore$ Load factor against cracking $= \frac{10.8}{7.2} = 1.5$.

CASE 3: LOAD BALANCING CONCEPT

Prestressed Beam with Bent Tendon

- Sometimes it is convenient to provide bend tendons. By providing bent tendons, the tendons will exert an upward pressure on the concrete beam and will therefore counteract a part of the external downward loading.
- Figure 9.38(a) shows a prestressed beam with a bent tendon. The effect or providing a bent tendon can be best considered by studying the concrete as a free body subjected to forces as shown in Fig. 9.38(b) for the sake of discussion let us assume that the tendon forms a sharp bend and that there is no frictional loss along the tendon.
- Considering the concrete as a free body we find an upward force 2 P $\sin\theta$ is also available at mid span.
- This upward force can to some extent counteract a part of the external load. For this beam, the net downward load at the centre will be (W – 2P $\sin\theta$).

The axial longitudinal force provided by the tendon

$= P\cos\theta = P$ approximately since θ is a small angle

$\therefore$ Direct stress on the section $= \frac{P\cos\theta}{A} = \frac{P}{A}$ approximately.

$$\text{Net B.M.} = M = \frac{(W - 2P\sin\theta)}{4} + \frac{wl^2}{8}$$

where w = Dead load per unit length of the beam

$$\text{Total fibre stress} = \frac{P}{A} \pm \frac{M}{Z}$$

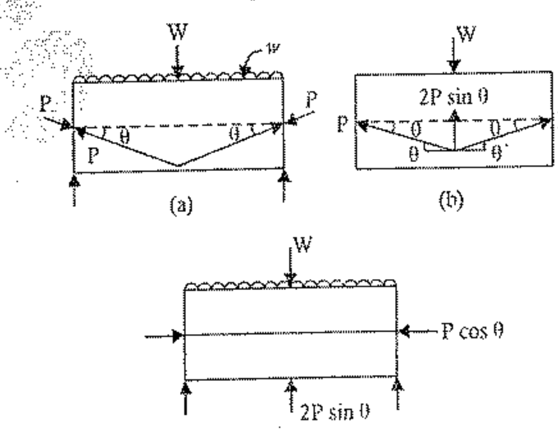
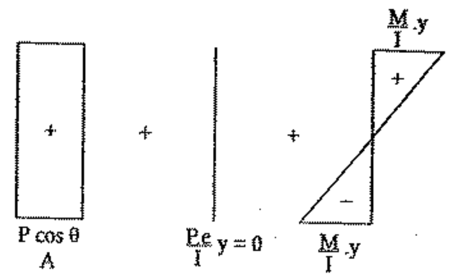


Fig. 9.38

Stress

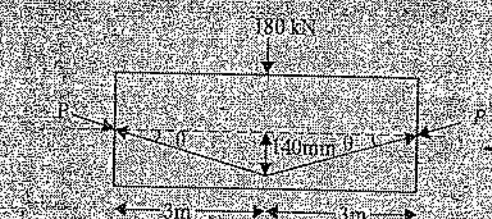


Stress at top fibre = $\frac{P}{A}+\frac{M}{I}.y$

Stress at bottom fibre = $\frac{P}{A}-\frac{M}{I}.y$

Example 11

A Prestress concrete beam 400 × 600 mm cross section has a span of 6 m and P = 1200 kN. Calculate the streses in a beam and mid span considering self weight as shown in figure below. Take unite wt of concrete 25 kN/m³.



Sol.

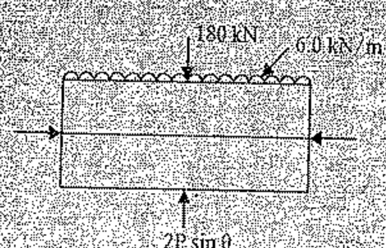
$$\tan\theta = \frac{140}{3000}$$

$$\theta = 2.67 \text{ radian}$$

$$\text{Self } wt = 0.4 \times 0.6 \times 1.0 \times 25$$

$$= 60 \text{ kN/m}$$

$$\text{Point load} = 180 \text{ kN}$$



$$\text{Net load} = 180 - 2\,P \sin\theta$$

$$= 180 - 2 \times 1200 \times \sin 2.67$$

$$W = 68.12 \text{ kN downward direction.}$$

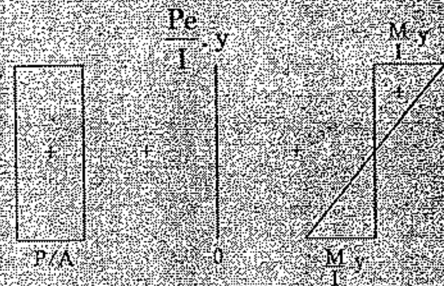
$$w = 6 \text{ kN/m}$$

$$M = \left(\frac{wl^2}{8} + \frac{Wl}{4}\right)$$

$$M = \frac{6\times6^2}{8} + \frac{68.12\times6}{4}$$

$$= 129.18 \text{ kN-m}$$

Stress



$$\frac{P}{A} = \frac{1200\times10^3}{400\times600} = 5 \text{ N/mm}^2$$

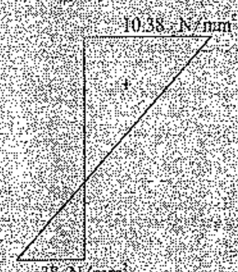
$$\frac{M}{I}y = \frac{129.18\times10^6}{\frac{400\times600^3}{12}}\times300$$

$$= 5.38 \text{ N/mm}^2$$

$$\text{Stress at top} = \frac{P}{A} + \frac{M}{I}y$$

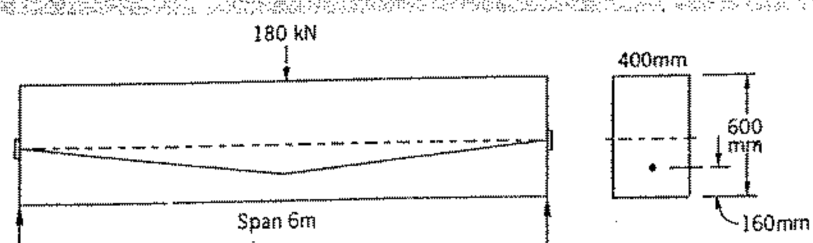
$$= 5 + 5.38 = 10.38 \text{ N/mm}^2$$

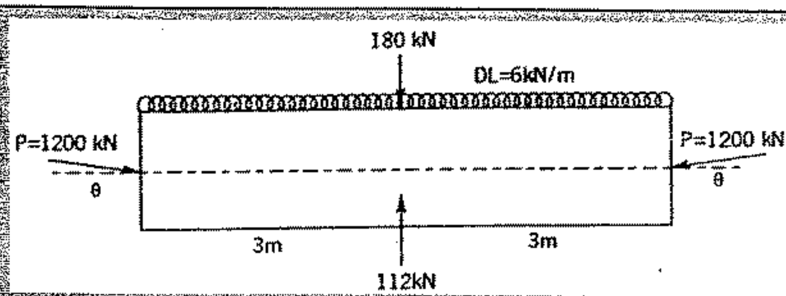
$$\text{Stress at bottom} = 5 - 5.38 = -0.38 \text{ N/mm}^2$$



Example 12

A prestressed concrete beam 400 mm wide and 600 mm deep has a span of 6 metres. The beam is prestressed with a tendon bent as shown in fig. The external laod on the beam consists of a concentrated load of 180 kN at mid span. If the effective prestressing force is 1200 kN, calculate the extreme stresses for the midspan section.





Sol. Let the tendon be at θ with the horizontal

$$\tan\theta = \frac{140}{3000} = \frac{7}{150}$$

The inclined tendon will provide an upward force at the midspan

This upward force = 2P sin θ = 2P tan θ (since θ is small)

$$= 2\times1200\times\frac{7}{150} = 112 \text{ kN}$$

Net vertical load at mid span = 180 − 112 = 68 kN (downward)

$$\text{B.M. due to net vertical load} = \frac{68\times6}{4} = 102 \text{ kNm}$$

Dead load of the beam = 0.4 × 0.6 × 25 = 6 kN/m

$$\text{B.M. due to dead load} = \frac{6\times6^2}{8} = 27 \text{ kNm}$$

Total B.M. = 102 + 27 = 129 kNm

Area of the beam section $A = 400 \times 600 = 2.4 \times 10^5 \text{ mm}^2$

$$\text{Section modulus of the beam section} = Z = \frac{bd^2}{6} = \frac{400\times600^2}{6} = 2.4\times10^7 \text{ mm}^3$$

Direct stress due to prestressing force

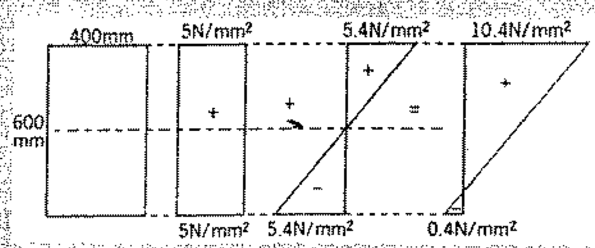
$$= +\frac{P}{A} = +\frac{1200\times10^3}{2.4\times10^5} = +5 \text{ N/mm}^2$$

$$\text{Extreme stress due to B.M} = \pm\frac{M}{Z} = \pm\frac{129\times10^6}{2.4\times10^7} = \pm5.4 \text{ N/mm}^2$$

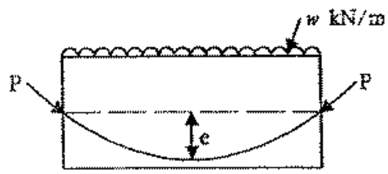
Resultant extreme stresses are

$5 + 5.4 = 10.4$ N/mm² (compressive) in the top most fibre

$5 - 5.4 = -0.4$ N/mm² (tensile) in the bottom-most fibre.

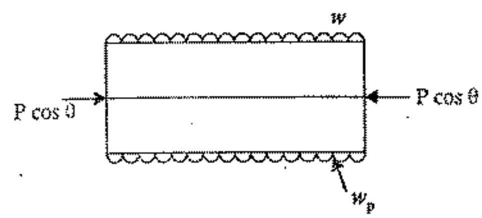


TENDONE WITH PARABOLIC PROFILE



Hz. reaction for each end of Cable is

$$p_x = \frac{wl^2}{8h}$$



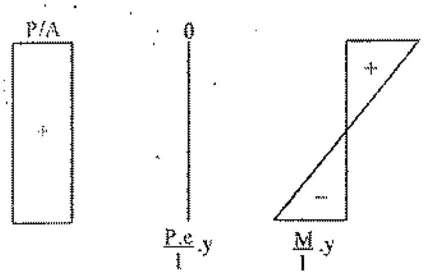
Here it is assumed that
$h = e$ and $w = w_p$

$$Pe = \frac{w_p l^2}{8}$$

$$P = \frac{w_p l^2}{8e}$$

$$w_p = \frac{8Pe}{l^2}$$

w = External loading kN/m
w_p = loading due to prestressed tendons.
Net load $w - w_p$ (assuming $w > w_p$)
Stresses



$$\text{Final stress} = \frac{P}{A} \pm 0 \pm \frac{M}{I}.y$$

$$\text{Equation of parabola } y = \frac{4h}{l^2}[x(l-x)]$$

When origin is at end.

$$\text{Slope} \quad \frac{dy}{dx} = \frac{4h}{l^2}(l-2x)$$

$$\text{at} \quad x = \frac{l}{2}, \ \tan\theta = 0$$

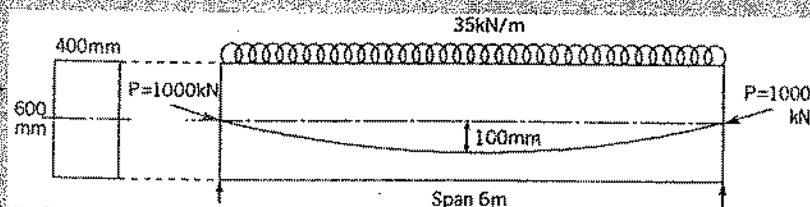
$$\text{at} \quad x = L, \tan\theta = \frac{dy}{dx} = \frac{4h}{l^2}(l-2l)$$

$$\tan\theta = \frac{dy}{dx} = -\frac{4h}{l}$$

$$\theta = \tan^{-1}\left(-\frac{4h}{l}\right)$$

Example 13

Figure shows a prestressed concrete beam provided with a tendon having a parabolic profile. If the total external load on the beam is 35 kN/m on the whole span, calculate the extreme stresses for the mid span section. The tendon carries a prestressing force of 1000 kN.



Sol. Area of the beam section $A = 400 \times 600 = 2.4 \times 10^5 \text{ mm}^2$

$$\text{Secction modulus of the secion} = Z = \frac{400 \times 600^2}{6}$$

$$= 2.4 \times 10^7 \text{ mm}^3$$

Span of tendon $= l = 6$m;

Dip of the tendon $e = 0.10$ m

Up ward uniformly distributed pressure provided by the cable

$$w_p = \frac{8Pe}{l^2} = \frac{8 \times 1000 \times 0.10}{6^2} = 22.22 \text{ kN/m}$$

Net downward load on the beam $= 35 - 22.22 = 12.78$ kN/m

Maximum B.M.

$$M = \frac{12.78 \times 6^2}{8} = 57.51 \text{ kNm}$$

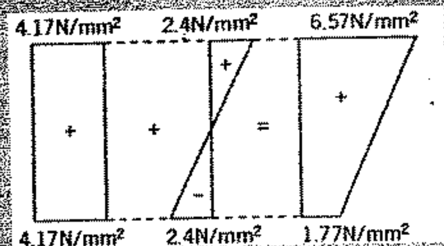
Extreme stresses for the mid section

$$= \frac{P}{A} \pm \frac{M}{Z} = \frac{1000 \times 10^3}{2.4 \times 10^5} \pm \frac{57.51 \times 10^6}{2.4 \times 10^7} \text{ N/mm}^2$$

$= 4.17 \pm 2.40 \text{ N/mm}^2$

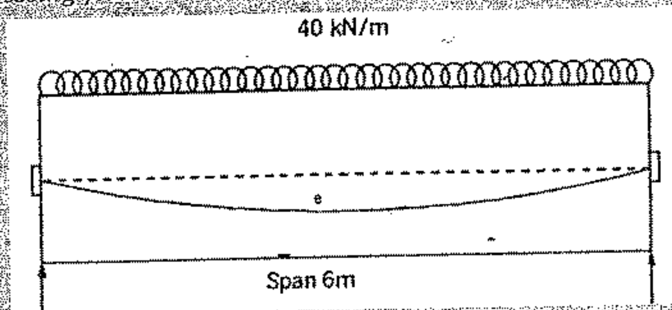
Extreme stress at top $= +4.17 + 2.40 = +6.57 \text{ N/mm}^2$

and Extreme stress at bottom $= +4.17 - 2.40 = +1.77 \text{ N/mm}^2$



Example 14

Determine the profile of a load balancing cable for a beam of span 6 metre carrying an all inclusive load of 40 kN/m. the prestressing force in the tendon is 1200 kN. The beam section is 400 mm × 600 mm.



Sol. We know that

$$Pe = \frac{wl^2}{8}$$

$$1200\, e = \frac{40 \times 6^2}{8}$$

$$e = 0.15 \text{ m}$$

Hence the cable should be provided with a central dip of 150 mm. The uniform stress for the beam section

$$= \frac{P}{A} = \frac{40 \times 3}{800} = 5 \text{ N/mm}^2 \text{ (compressive)}$$

Example 15

If the prestressing force in the tendon is P for the beam shown in figure find the central dip h required to fully balance a concentrated load W applied at mid span.

Sol. Let θ be the inclination of the tendon with the horizontal.

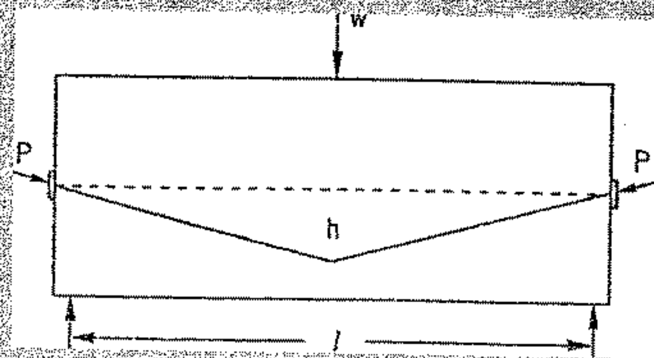
$\therefore$ Upward force provided by the tendon at the centre

$$= 2P\sin\theta = 2P\tan\theta = 2P\frac{h}{\left(\frac{l}{2}\right)} = \frac{4Ph}{l}$$

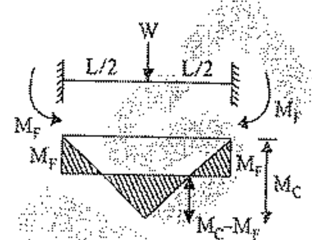
If this force should balance the applied downward load

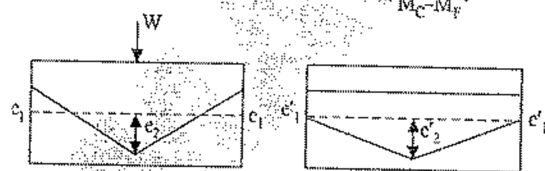
$$\frac{4Ph}{l} = W$$

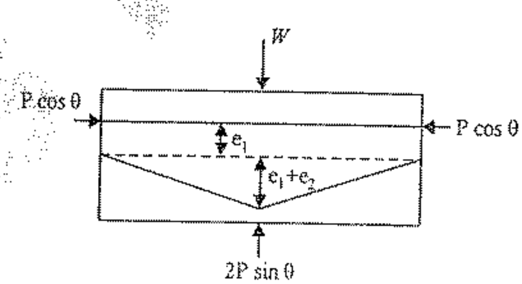
$$h = \frac{Wl}{4P}$$



SOME SPECIAL CASE







$$M_F = Pe_1 \quad \text{---(1)}$$

$$M_C - M_F = Pe_2 \quad \text{---(2)}$$

$$M_F = Pe_1' \quad \text{---(3)}$$

$$M_C = Pe_2' \quad \text{---(4)}$$

Now $e_1 = e_1'$

From equation (2)

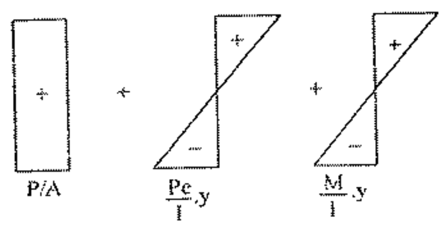
$$M_C - M_F = Pe_2$$

Put the value of M_c & M_F from equation (4) and (1)

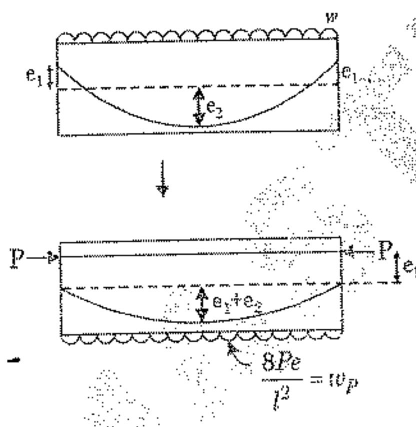
$$Pe_2' - Pe_1 = Pe_2$$

$$\boxed{e_2' = e_1 + e_2}$$

Stress



$$\text{Total Stress} = \frac{P}{A} \pm \frac{P.e_1}{I}.y \pm \frac{M}{I}.y$$



$$\tan\theta = \frac{4h}{l}$$

$$P = \frac{wl^2}{8e}$$

$$w_p = \frac{8pe}{l^2}$$

$$\text{Total stress} = \frac{P}{A} \pm \frac{Pe_1}{I}.y \pm \frac{M}{I}.y$$

STRESSES AT TRANSFER & FINAL STAGE

At Transfer Stage (Just after transfer of prestressing)

1. Losses are not considered.
2. Bending stress are considered due to Dead load only. (Live load not taken in to amount)

$$\text{Streses} = \frac{P}{A} \pm \frac{P.e.}{I} \cdot y \pm \frac{M_d}{I} \cdot y$$

At Final Stage of loading

(i) Losses are also considered.

(ii) Bending stress are considered due to dead load and live load.

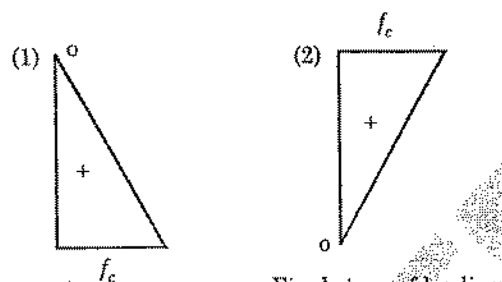
$$\text{Stresses} = \frac{\eta P}{A} \pm \frac{\eta Pe}{I} . y \pm \frac{M_d}{I} . y \pm \frac{M_L . y}{I}$$

$$\eta = \text{loss factor}$$

$$\therefore \quad \text{If loss} = 15\%$$

$$\text{then} \quad \eta = 1 - 0.15 = 0.85$$

Ideal stress diagram at



Transfer   Final stage of loading

Example 16

A Post tensioned prestressed concrete beam span 20 m and carries a uniformly distributed live load of 12 kN/m. Covering entire span besides its own-weight. The detail is as follows:

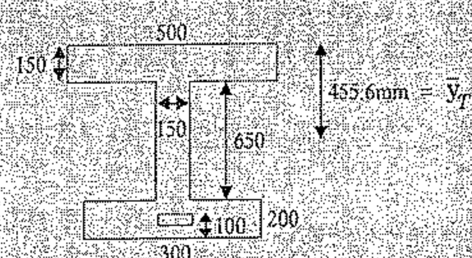
Top flange = 500 × 150 mm

Web = 150 × 650 mm

Bottom flange = 300 mm × 200 mm

Distance of C.G. of the section from the top edge = 455.6 mm, I_{xx} = 265.334 × 10^8 mm^4, Z_t = 5.8238 × 10^7 mm^3, Zb = 4.8739 × 10^7 mm^3. The prestressing force is applied by Cables of a total area = 1385.44 mm^2 streched initially to 1100 N/mm^2 and located at 100 mm from the bottom of edge of beam. Determine the stress at transfer and final stages of loading. Assume 15% loss of prestress in the final stage. Unit weight of concrete = 25 kN/m^3.

Sol.



$$\bar{Y}_B = 1000 - 455.60 = 544.40 \text{ mm}$$

$$A = 500 \times 150 + 150 \times 650 + 300 \times 200 = 232500 \text{ mm}^2$$

$$\text{Dead load} = A \times 25$$

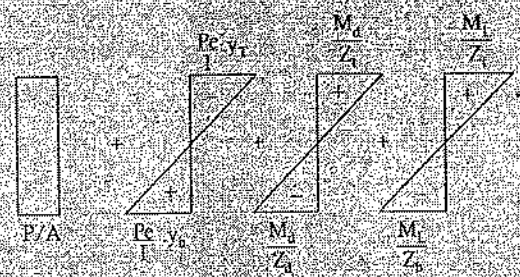
$$= \frac{232500}{10^6} \times 25$$

$$= 5.8125 \text{ kN/m}$$

$$M_d = \frac{5.8125 \times 20^2}{8}$$

$$= 290.625 \text{ kN-m}$$

$$M_L = \frac{12 \times 20^2}{8} = 600 \text{ kN-m}$$



$$e = 544.4 - 100 = 444.4 \text{ mm}$$

$$P = 1100 \times 232500 = 1523984 \text{ N}$$

$$\frac{P}{A} = \frac{1523984}{232500} = 6.55 \text{ N/mm}^2$$

$$\frac{P.e.}{Z_t} = \frac{1523984 \times 444.4}{5.8238 \times 10^7}$$

$$= 11.63 \text{ N/mm}^2$$

$$\frac{P.e.}{Z_b} = \frac{1523984 \times 444.4}{4.873 \times 10^7}$$

$$= 13.897 \text{ N/mm}^2$$

$$\frac{M_d}{Z_t} = \frac{290.625 \times 10^6}{5.8238 \times 10^7}$$

$$= 4.990 \text{ N/mm}^2$$

$$\frac{M_d}{Z_b} = \frac{290.625 \times 10^6}{4.873 \times 10^7}$$

$$= 5.964 \text{ N/mm}^2$$

$$\frac{M_L}{Z_t} = \frac{600 \times 10^6}{5.8238 \times 10^7} = 10.30 \text{ N/mm}^2$$

$$\frac{M_L}{Z_b} = \frac{600 \times 10^6}{4.873 \times 10^7} = 12.31 \text{ N/mm}^2$$

Transfer

Stress at top $= \frac{P}{A} - \frac{P}{Z_t} + \frac{M_d}{Z_t}$

$= 6.55 - 11.633 + 4.993$

$= -0.009 \text{ N/mm}^2$

Stress at bottom $= \frac{P}{A} + \frac{Pe}{Z_b} - \frac{M_d}{Z_b}$

$= 6.55 + 13.897 - 5.964$

$= \mathbf{14.98 \text{ N/mm}^2}$

At final stage

Stress at top $= \eta\frac{P}{A} - \frac{\eta P}{Z_t} + \frac{(M_d + M_L)}{Z_t}$

$= 0.85 \times 6.55 - 0.85 \times 11.633 + (4.993 + 10.30)$

$= 10.97 \text{ N/mm}^2$

Stress at bottom $= 0.85 \times 6.55 + 0.85 \times 13.897 - (5.694 + 12.31)$

$= -0.89 \text{ N/mm}^2$

THE CONCEPT OF C AND P LINE OR STRENGTH CONCEPT OR INTERNAL RESISTING COUPLE METHOD

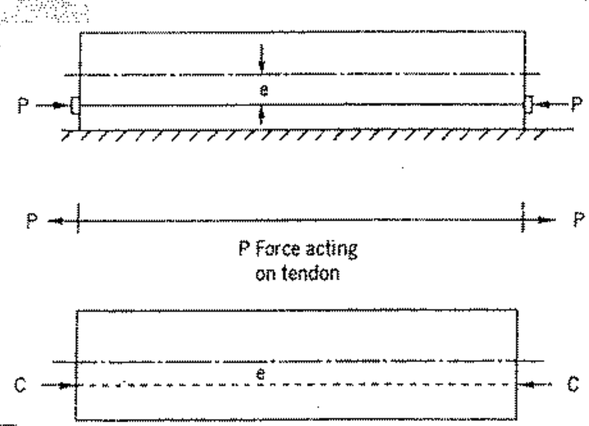
Consider a beam of length l provided with a tendon at an eccentricity e. Suppose the beam is lying on the ground *i.e.* the beam is not subjected to any external load, hence there is no external bending moment on the beam.

We should recognize the existence of the following forces, which are equal.

(i) The P-force which is the tension in the tendon.

(ii) The C-force which is the compressive force acting on concrete. Stresses in concrete are produced entirely due to the C-force.

In the absence of any external bending moment the C-force and the P-force act at the same level. The line of action of the P-force is called the P-line. The P-line is nothing but the tendon line itself. The line of action of the C-force is called the C-line or *Pressure line*. Hence, in the absence of any external bending moment the P-line and the C-line coincide.



P Force acting on tendon

C – Force acting on concrete

Suppose, the beam is subjected to a moment M, then the C-line will be shifted from the P-line by a distance a called the *lever arm*.

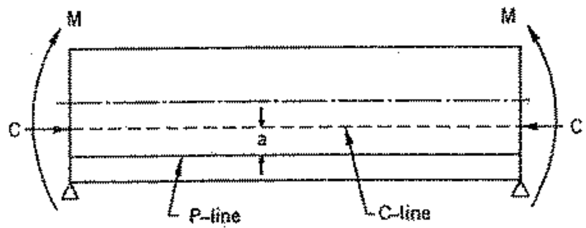
$$a = \text{Shift of the C-line from the } P\text{-line}$$

$$= \frac{\text{External moment}}{P} = \left(\frac{M}{P}\right)$$

In other words the effect of the moment may be considered by shifting the C-line by the distance M/P. Now corresponding to the new position of the C-line and its eccentricity the stress distribution for concrete can be determined.

$$\text{Extreme stresses in concrete} = \frac{C}{A} \pm \frac{C \times \text{eccentricity of C}}{Z}$$

This concept in the analysis is called *strength concept*.

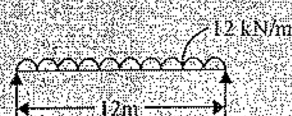


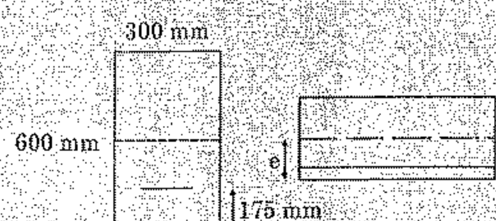
Example 17

In PSC beam with c/s 300 × 600 mm² is 12 m long. It carries a load of LL = 12 kN/m in addition to its self weight. It is prestresed with 2000 mm² high tensile steel located at 175 mm from soffit. The cable profile is straight for full length of the beam and stressed to a level of 800 N/mm². It is bonded to concrete. Determine the location of thrust line in the beam and plot its position at end, and at mid section.

Sol.
$$\frac{E_s}{E_c} = m = 6$$

$$P = 800 \times 2000 = 16000 \text{ kN}$$





$$e = 300 - 175$$
$$= 125 \text{ mm}$$

Shift of C line from P line

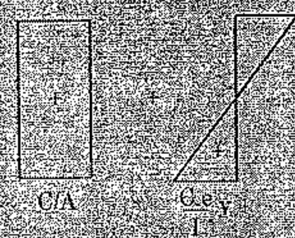
$$a = \frac{M}{P} \text{ at end}$$

$$= \frac{0}{P} = 0$$

$$e = 0 - 125$$

$$= -125 \text{ below N.A. (–ve sign means new eccentricity shifted to below the neutral axis)}$$

Stress



$$\text{Stress} = \frac{C}{A} + \frac{Ce}{I}.y$$

$$\text{at top} = \frac{C}{A} - \frac{Ce}{I}y$$

$$= \frac{1600\times10^3}{300\times600} - \frac{1600\times10^3\times125}{300\times\frac{600^3}{12}}\times300$$

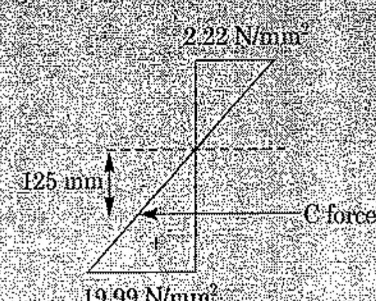
$$= 8.88 - 11.1$$

$$= -2.22 \text{ N/mm}^2$$

$$\text{at bottom stress} = \frac{C}{A} + \frac{C.e}{I}.y$$

$$= 8.88 + 11.11$$

$$= 19.99 \text{ N/mm}^2$$



At mid span

$$\text{Self } wt = 0.3 \times 0.6 \times 25$$

$$= 4.5 \text{ kN/m}$$

$$LL = 12 \text{ kN/m}$$

Total = 16.5 kN/m

$$M = \frac{wl^2}{8} = \frac{16.5 \times 12^2}{8}$$

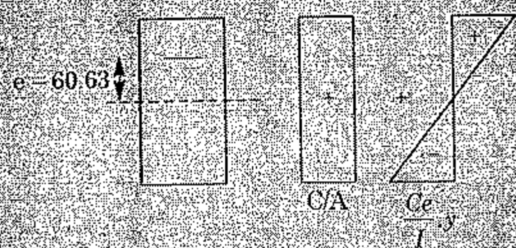
$$= 297 \text{ kN-m}$$

Shift of C line from P line

$$a = \frac{M}{P} = \frac{297 \times 10^6}{1600 \times 10^3} = 185.625 \text{ mm}$$

$$e = a - e' = 185.63 - 125$$

$$= 60.63 \text{ mm}$$ (+ve sign means new eccentricity shifted to above the neutral axis)



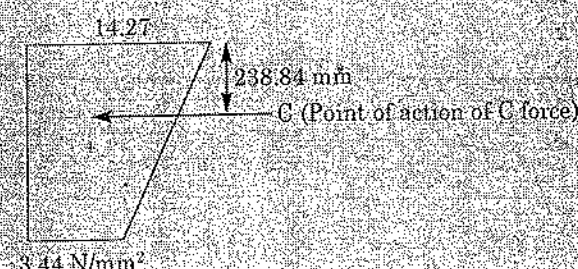
$$\text{Total stress} = \frac{C}{A} \pm \frac{Ce}{I} \cdot y$$

$$= \frac{1600 \times 10^3}{300 \times 600} \pm \frac{1600 \times 10^3 \times 60.63}{\frac{300 \times 600^3}{12}} \times 300$$

$$= 8.88 \pm 5.389$$

$$\text{Stress at top} = 8.88 + 5.389 = 14.27 \text{ N/mm}^2$$

$$\text{Stress at bottom} = 8.83 - 5.389 = 3.44 \text{ N/mm}^2$$



$$\bar{y} = \left(\frac{14.27 + 3.4492}{14.27 + 3.44}\right) \times \frac{600}{3}$$

$$= 238.84 \text{ mm} \quad \text{from Top.}$$

Example 18

A prestressed concrete beam 400 mm × 600 mm in section has a span of 6m and is subjected to a uniformly distributed load of 16 kN/m including the self-weight of the beam. The prestressing tendons which are located along the longitudinal centroidal axis provide an effective prestressing force of 960 kN. Determine the extreme stresses in concrete for mid span section. Using strength concept or internal resisting couple method.

Sol:
$$M = \frac{wl^2}{8} = \frac{16\times6^2}{8} = 72 \text{ kN-m}$$

Shift of C-line from the P line

$$a = \frac{M}{P} = \frac{72\times10^6}{960\times10^3} = 75 \text{ mm}$$

New eccentricity $e = a - e'$

$= 75 - 0$

$= 75$ mm (New eccentricity in shifted to above N-A)

Total stress $= \frac{C}{A} + \frac{C.e}{I}.y$

Stress at top $= \frac{C}{A} + \frac{C.e}{I}.y$

$$= \frac{960\times10^3}{400\times600} + \frac{960\times10^3}{400\times\frac{600^3}{12}}\times75\times300$$

$= 4 + 3$

$= 7 \text{ N/mm}^2$

Stress at bottom $= \frac{C}{A} - \frac{C.e}{I}.y$

$= 4 - 3$

$= 1 \text{ N/mm}^2$

Example 19

A prestressed concrete beam 400 mm × 600 mm in section has a span of 6m and is subject to a uniformly distributed load of 16 kN/m including the self-weight of the beam. The prestressing tendons are located at the lower third point and provide an effective prestressing force of 960 kN. Determine the extreme stresses at the mid span section. Using strength concept internal resisting couple method.

Sol:
$$M = \frac{wl^2}{8} = \frac{16\times6^2}{8} = 72 \text{ kN-m}$$

Shift of the C-line from the P-line

$$a = \frac{M}{P} = \frac{72\times10^6}{960\times10^3} = 75 \text{ mm}$$

New eccentricity $e = a - e'$

$= 75 - 100 = -25$ mm (New eccentricity is below N.A.)

Total stress $= \frac{C}{A} + \frac{C.e}{I}.y$

$$\text{Stress at top} = \frac{960\times10^3}{400\times600} - \frac{960\times10^3\times25}{400\times\frac{600^3}{12}}\times300$$

$$= 4 - 1$$

$$= 3 \text{ N/mm}^2$$

$$\text{Stress at bottom} = \frac{C}{A} + \frac{C.e}{I}.y$$

$$= 4 + 1$$

$$= 5 \text{ N/mm}^2$$

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Example 20

A prestressed concrete beam is prestressed with a tendon bent as shown in Fig. below. The external load on the beam consists of a concentrated load of 180 kN at mid span. It the effective prestressing force is 1200 kN, calculate the extreme stresses in concrete for the mid span section. Using strength concept or internal resisting couple method.

Sol:

$$\text{D.L. of the beam} = 0.4\times0.60\times25 = 6 \text{ kN/m}$$

$$\text{B.M. at centre} = \frac{WL}{4} + \frac{wl^2}{8}$$

$$= \frac{180\times6}{4} + \frac{6\times6^2}{8}$$

$$= 297 \text{ kN/m}$$

$$\text{Shift of } C \text{ line from } P \text{ line} = \frac{297\times10^6}{1200\times10^3} = 247.5 \text{ mm}$$

$$\text{New eccentricity} = a - e'$$

$$= 247.5 - 140$$

$$= 107.5 \text{ mm (New eccentricity is above N.A.)}$$

$$\text{Total stress} = \frac{C}{A} \pm \frac{C.e}{I}.y$$

$$= \frac{1200\times10^3}{400\times600} \pm \frac{1200\times10^3\times107.5}{400\times\frac{600^3}{12}}\times300$$

$$= 5 \pm 5.37 \text{ N/mm}^2$$

$$\text{Stress at top} = 5 + 5.37 = 10.37 \text{ N/mm}^2$$

$$\text{Stress at bottom} = 5 - 5.37 = -0.37 \text{ N/mm}^2$$

Example 21

A prestressed concrete beam with a rectangular section 120 mm wide by 300 mm deep supports a uniformly distributed load of 4 kN/m, which includes the self-weight of the beam. The effective span *of the beam is 6m. The beam is concentrically prestressed by a cable carrying a force of 180 kN. Locate the position of the pressure line in the beam.*

Sol:

$$B.M. = \frac{wl^2}{8} = \frac{4\times 6^2}{8} = 18\text{ kN-m}$$

Shift of C line from P line

$$a = \frac{M}{P} = \frac{18\times 10^6}{180\times 10^3} = 100\text{ mm}$$

$$\text{New eccentricity } e = a - e'$$

$$= 100 - 0 = 100\text{ mm (new eccentricity is above the N.A.)}$$

Total stress at centre of the span section:

$$= \frac{C}{A} \pm \frac{C.e}{I}.y$$

$$= \frac{180\times 10^3}{120\times 300} \pm \frac{180\times 10^3\times 100}{120\times \frac{300^3}{12}}\times 150$$

$$= 5 \pm 10\text{ N/mm}^2$$

$$\text{Stress at top} = 5 + 10 = 15\text{ N/mm}^2$$

$$\text{Stress at bottom} = 5 - 10 = -5\text{ N/mm}^2$$

Example 22

A prestressed concrete beam of section 120 mm wide by 300 mm deep is used over an effective span of 6 m to support a uniformly distributed load of 4 kN/m, which includes the self-weight of the beam. The beam is prestressed by a straight cable carrying a force of 180 kN and located at an eccentricity of 50 mm. Determine the location of the thrust-line in the beam and plot its position at quarter and central span sections.

Sol:

$$\text{B.M. at centre} = \frac{wl^2}{8} = \frac{4\times 6^2}{8} = 18\text{ kN-m}$$

$$\text{Shift of } C \text{ line from } P \text{ line } a = \frac{M}{P} = \frac{18\times 10^6}{180\times 10^3} = 100\text{ mm}$$

$$e = a - e'$$

$$= 100 - 50 = 50\text{ mm (eccentricity shifted above the N.A.)}$$

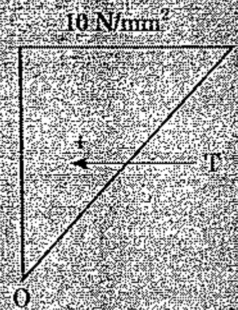
$$\text{Total stress} = \frac{C}{A} \pm \frac{C.e}{I}.y$$

$$= \frac{180\times 10^3}{120\times 300} \pm \frac{180\times 10^3\times 50}{120\times \frac{300^3}{12}}\times 150$$

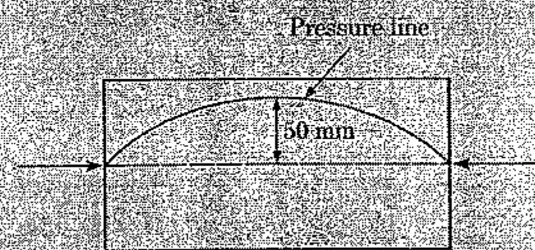
$$= 5 \pm 5\text{ N/mm}^2$$

Stress at top = 5 + 5 = 10 N/mm²

Stress at bottom = 5 − 5 = 0 N/mm²

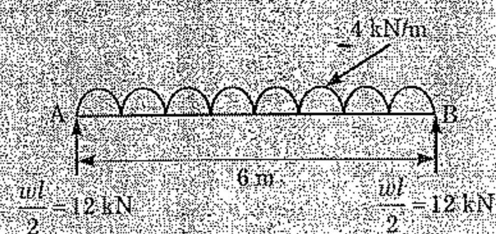


Action at a distance $\frac{300}{3} = 100$ mm from Top.



Location of pressure line and distribution of stresses at mid span.

At Quater span M



$$\text{Reaction } \frac{wL}{2} = \frac{4\times 6}{2} = 12 \text{ kN}$$

$$\text{Moment at 1.25 m from } A = 12 \times 1.5 - 4 \times \frac{1.5^2}{2}$$

$$= 13.5 \text{ kN-m}$$

$$\text{Shift of } C \text{ line from } P \text{ line } a = \frac{13.5\times 10^6}{180\times 10^3} = 75 \text{ mm}$$

$$e = a - e'$$

$$= 75 - 50 = 25 \text{ mm (eccentricity above the N.A.)}$$

$$\text{Total stress} = \frac{C}{A} \pm \frac{C.e}{I}.y$$

$$= \frac{180\times 10^3}{120\times 300} \pm \frac{180\times 10^3 \times 25}{120\times \frac{300^3}{12}} \times 150$$

$= 5 \pm 2.5 \text{ N/mm}^2$

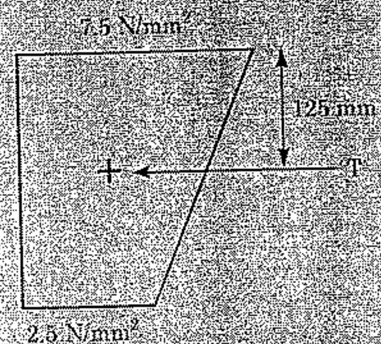
At top $= 5 + 2.5 = 7.5 \text{ N/mm}^2$

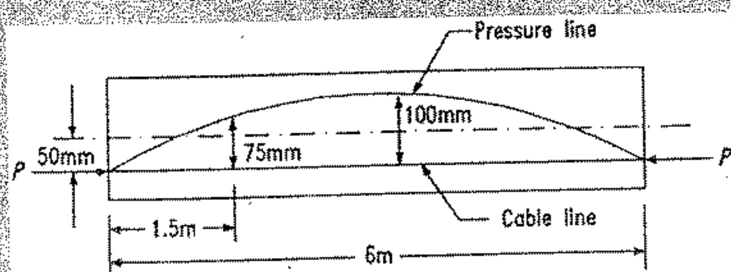
At bottom $= 5 - 2.5 = 2.5 \text{ N/mm}^2$

Thrust line acting at a distance

$$\bar{y} \text{ from top} = \left(\frac{7.5 + 2 \times 2.5}{7.5 + 2.5}\right) \times \frac{300}{3}$$

$$= 125 \text{ mm}$$





Example 23

A rectangular concrete beam 250 mm wide by 300 mm deep is prestressed by a force of 540 kN at a constant eccentricity of 60 mm. The beam supports a concentrated load of 68 kN at the centre of a span of 3 m. Determine the location of the pressure line at the centre, quarter span and support sections of the beam. Neglect the self-weight of the beam.

Sol: At support M = 0

$$a = \frac{M}{P} = 0$$

New eccentricity $= a - e'$

$= 0 - 60 = -60$ mm (eccentricity below the N.A.)

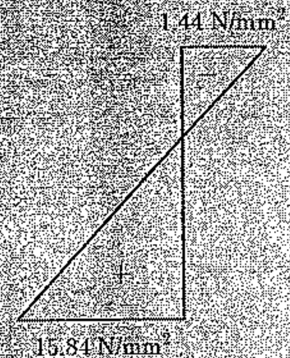
$$\text{Total stress} = \frac{C}{A} + \frac{C.e}{I}.y$$

$$= \frac{540\times10^3}{250\times300} \pm \frac{540\times10^3\times60}{250\times\frac{300^3}{12}}\times150$$

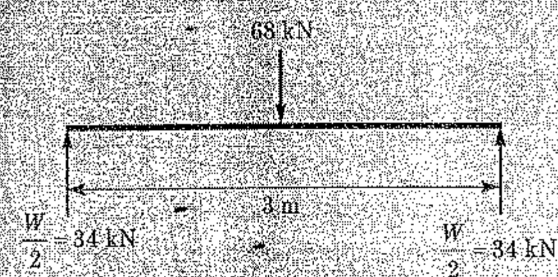
$$= 7.2 \pm 8.64\ \text{N/mm}^2$$

Stress at top $= 7.2 - 8.64 = -1.44\ \text{N/mm}^2$

Stress at bottom $= 7.2 + 8.64 = 15.84\ \text{N/mm}^2$



At Quater span



Moment at Quater span $= 34\times0.75 = 25.5$ kN-m

$$\text{Shift } a = \frac{M}{P} = \frac{25.5\times10^6}{540\times10^3} = 47.22 \text{ mm}$$

New eccentricity $e = a - e'$

$= 47.22 - 60$

$= -12.78$ mm (Eccentricity below the N.A.)

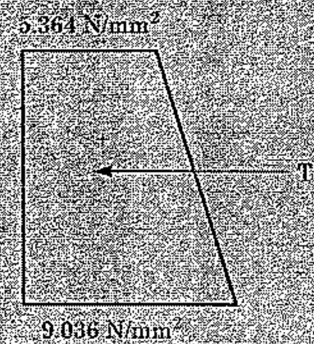
$$\text{Total stress} = \frac{C}{A} \pm \frac{C\,e}{I}\,y$$

$$= \frac{640\times10^3}{250\times300} \mp \frac{540\times10^3}{250\times\frac{300^3}{12}}\times12.78\times150$$

$= 7.2 \mp 1.836 \text{ N/mm}^2$

Stress at top $= 7.2 - 1.836 = 5.364 \text{ N/mm}^2$

Stress of bottom $= 7.2 + 1.836 = 9.036 \text{ N/mm}^2$



Pressure line act at distant $\bar{y}$ from the top

$$\bar{y} = \left(\frac{5.364 + 2\times 9.036}{5.364 + 9.036}\right)\times\frac{300}{3}$$

$$= 162.75 \text{ mm}$$

Moment at centre span

$$M = \frac{Wl}{4} = \frac{68\times 3}{4} = 51 \text{ kN-m}$$

$$\text{Shift } a = \frac{M}{P} = \frac{51\times 10^6}{540\times 10^3} = 94.44 \text{ mm}$$

New eccentricity $e = a - e'$

$= 94.44 - 60 = 34.44$ mm (new eccentricity is above the N.A.)

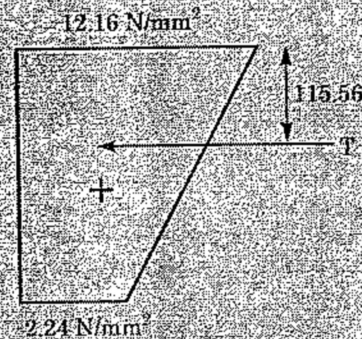
$$\text{Total stress} = \frac{C}{A} \pm \frac{C.e}{I}.y$$

$$= \frac{540\times 10^3}{250\times 300} \pm \frac{540\times 10^3\times 34.44}{250\times\frac{300^3}{12}}\times 150$$

$= 7.2 \pm 4.96 \text{ N/mm}^2$

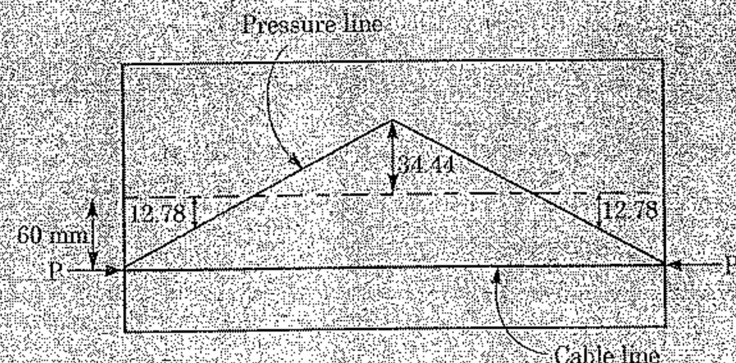
Stress at top $= 7.2 + 4.96 = 12.16 \text{ N/mm}^2$

Stress at bottom $= 7.2 - 4.96 = 2.24 \text{ N/mm}^2$



Pressure line act at a distance $\bar{y}$ from the top

$$\bar{y} = \left(\frac{12.16 + 2\times 2.24}{12.16 + 2.24}\right)\times\frac{300}{3} = 115.56 \text{ mm}$$



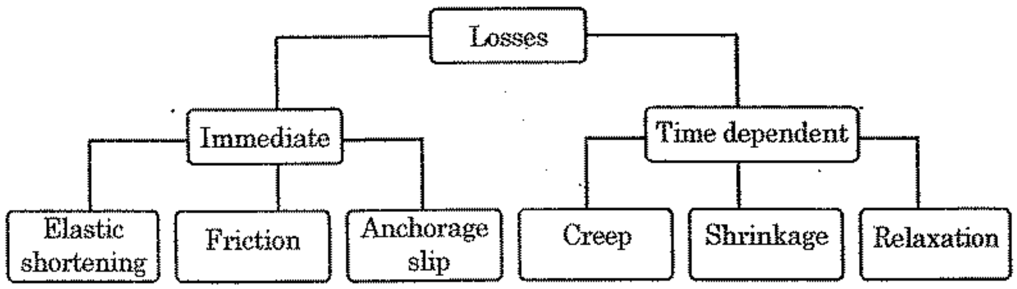
Location of pressure line in the prestressed beam

LOSSES IN PRESTRESS

INTRODUCTION

- In prestressed concrete applications, the most important variable is the prestressing force. In the early days, it was observed that the prestressing force does not stay constant, but reduces with time.
- Even during prestressing of the tendons and the transfer of prestress to the concrete member, there is a drop of the prestressing force from the recorded value in the jack gauge.
- The various reductions of the prestressing force are termed as the losses in prestress.
- The losses are broadly classified into two groups, immediate and time-dependent.
- The immediate losses occur during prestressing of the tendons and the transfer of prestress to the concrete member.
- The time-dependent losses occur during the service life of the prestressed member.
- The losses due to elastic shortening of the member, friction at the tendon-concrete interface and slip of the anchorage are the immediate losses.

- The losses due to the shrinkage and creep of the concrete and relaxation of the steel are the time-dependent losses.
- The causes of the various losses in prestress are shown in the following chart.



Causes of the various losses in prestress

Notations

Geometric Properties

The commonly used geometric properties of a prestressed member are defined as follows.

A_c = Area of concrete section
= Net cross-sectional area of concrete excluding the area of prestressing steel.

A_p = Area of prestressing steel
= Total cross-sectional area of the tendons.

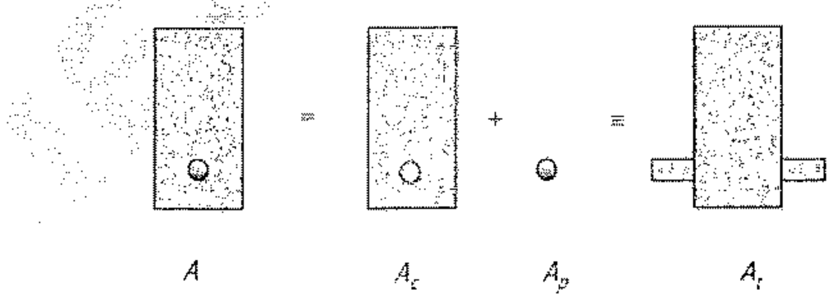
A = Area of prestressed member
= Gross cross-sectional area of prestressed member.
$= A_c + A_p$

A_t = Transformed area of prestressed member
= Area of the member when steel is substituted by an equivalent area of concrete.
$= A_c + mA_p$
$= A + (m - 1)A_p$

Here,

m = the modular ratio = E_p/E_c
E_c = short-term elastic modulus of concrete
E_p = elastic modulus of steel.

The following figure shows the commonly used areas of the prestressed members.



CGC = Centroid of concrete
= Centroid of the gross section. The CGC may lie outside the concrete

CGS = Centroid of prestressing steel

= Centroid of the tendons. The CGS may lie outside the tendons or the concrete

I = Moment of inertia of prestressed member

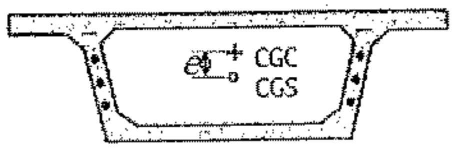
= Second moment of area of the gross section about the CGC.

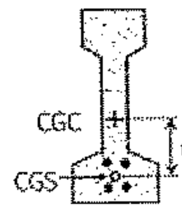
I_t = Moment of inertia of transformed section

= Second moment of area of the transformed section about the centroid of the transformed section.

e = Eccentricity of CGS with respect to CGC

= Vertical distance between CGC and CGS. If CGS lies below CGC, e will be considered positive and vice versa.





Load Variables

P_i = Initial prestressing force

= The force which is applied to the tendons by the jack.

P_0 = Prestressing force after immediate losses

= The reduced value of prestressing force after elastic shortening, anchorage slip and loss due to friction.

P_e = Effective prestressing force after time-dependent losses

= The final value of prestressing force after the occurrence of creep, shrinkage and relaxation.

Elastic Shortening

Pre-tensioned Members

When the tendons are cut and the prestressing force is transferred to the member, the concrete undergoes immediate shortening due to the prestress. The tendon also shortens by the same amount, which leads to the loss of prestress.

Post-tensioned Members

If there is only one tendon, there is no loss because the applied prestress is recorded after the elastic shortening of the member. For more than one tendon, if the tendons are stretched sequentially, there is loss in a tendon during subsequent stretching of the other tendons.

The elastic shortening loss is quantified by the drop in prestress (Δf_p) in a tendon due to the change in strain in the tendon ($\Delta\varepsilon_p$). It is assumed that the change in strain in the tendon is equal to the strain in concrete (ε_c) at the level of the tendon due to the prestressing force. This assumption is called **strain compatibility** between concrete and steel. The strain in concrete at the level of the tendon is calculated from the stress in concrete (f_c) at the same level due to the prestressing force. A linear elastic relationship is used to calculate the strain from the stress.

The quantification of the losses is explained below.

$$\Delta f_p = E_p \Delta\varepsilon_p$$

$$= E_p \varepsilon_c$$

$$= E_p\left(\frac{f_c}{E_c}\right)$$

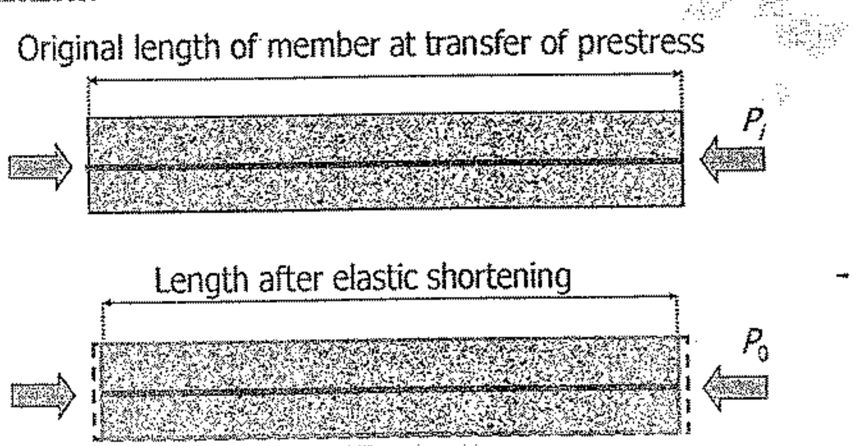
$$\Delta f_p = mf_c$$

For simplicity, the loss in all the tendons can be calculated based on the stress in concrete at the level of CGS controid of prestressing steel. This simplification cannot be used when tendons are stretched sequentially in a post-tensioned member. The calculation is illustrated for the following types of members separately.

- Pre-tensioned Axial Members
- Pre-tensioned Bending Members
- Post-tensioned Axial Members
- Post-tensioned Bending Members

Pre-tensioned Axial Members

The following figure shows the changes in length and the prestressing force due to elastic shortening of a pre-tensioned axial member.



Elastic shortening of a pre-tensioned axial member

The loss can be calculated as per Eqn. by expressing the stress in concrete in terms of the prestressing force and area of the section as follows.

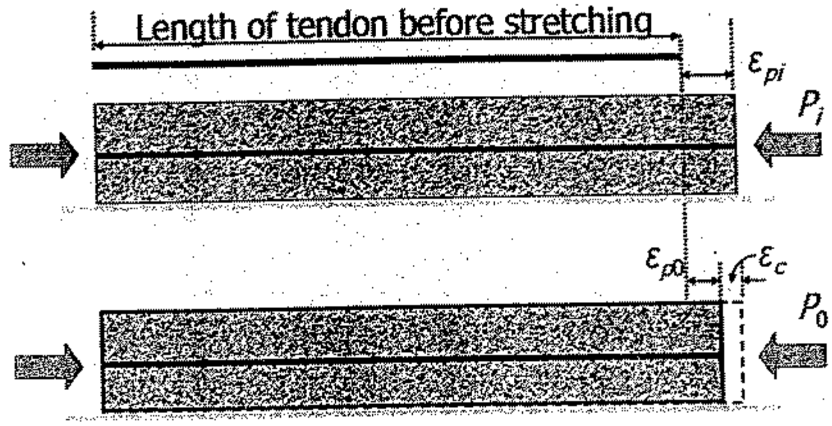
$$\Delta f_p = mf_c$$

$$= m\left(\frac{P_0}{A_c}\right)$$

$$\Delta f_p = m\left(\frac{P_i}{A_t}\right) \approx \left(\frac{P_i}{A}\right)$$

Note that the stress in concrete due to the prestressing force after immediate losses (P_0/A_c) can be equated to the stress in the transformed section due to the initial prestress (P_i/A_t). This is derived below. Further, the transformed area A_t of the prestressed member can be approximated to the gross area A.

The following figure shows that the strain in concrete due to elastic shortening (ε_c) is the difference between the initial strain in steel (ε_{pi}) and the residual strain in steel (ε_{p0}).



Strain variables in elastic shortening

The following equation relates the strain variables.

$$\varepsilon_c = \varepsilon_{pi} - \varepsilon_{P_o} \qquad \text{---(i)}$$

$$\varepsilon_c = \frac{P_0}{A_c E_c}$$

$$\varepsilon_{pi} = \frac{P_i}{A_p E_p}$$

$$\varepsilon_{po} = \frac{P_o}{A_p E_p}$$

$$\varepsilon_{po} = \frac{P_o}{A_p E_p}$$

Sbustituting the expressions of the strains in eq. (i)

$$\frac{P_o}{A_c E_c} = \frac{P_i}{A_p E_p} - \frac{P_o}{A_p E_p}$$

or, $$P_o\left(\frac{1}{A_c E_c} + \frac{1}{A_p E_p}\right) = \frac{P_i}{A_p E_p}$$

or, $$P_o\left(\frac{m}{A_c} + \frac{1}{A_p}\right) = \frac{P_i}{A_p}$$

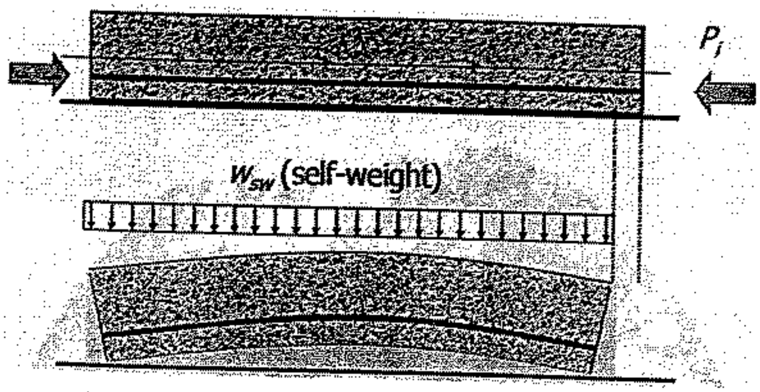
or, $$\frac{P_o}{A_c} = \frac{P_i}{mA_p + A_c}$$

or $$\frac{P_o}{A_c} = \frac{P_i}{A_t}$$

Thus, the stress in concrete due to the prestressing force after immediate losses (P_0/A_c) can be equated to the stress in the transformed section due to the initial prestress (P_i/A_t).

Pre-tensioned Bending Members

The following figure shows the changes in length and the prestressing force due to elastic shortening of a pre-tensioned bending member.



Elastic shortening of a pre-tensioned bending member

Due to the effect of self-weight, the stress in concrete varies along length (Figure). The loss can be calculated by Eqn. with a suitable evaluation of the stress in concrete. To have a conservative estimate of the loss, the maximum stress at the level of CGS at the mid-span is considered.

$$f_c = +\frac{P_i}{A} + \frac{P_i e.e}{I} - \frac{M_d e}{I}$$

Here, M_d is the moment at mid-span due to self-weight. Precise result using A_t and I_t in place of A and I, respectively, is not computationally warranted. In the above expression, the eccentricity of the CGS (e) was assumed to be constant.

For a large member, the calculation of the loss can be refined by evaluating the strain in concrete at the level of the CGS accurately from the definition of strain. This is demonstrated later for post-tensioned bending members.

Post-tensioned Axial Members

For more than one tendon, if the tendons are stretched sequentially, there is loss in a tendon during subsequent stretching of the other tendons. The loss in each tendon can be calculated in progressive sequence. Else, an approximation can be used to calculate the losses.

The loss in the first tendon is evaluated precisely and half of that value is used as an average loss for all the tendons.

$$\Delta f_p = \frac{1}{2}\Delta f_{p1}$$

$$= \frac{1}{2} m f_{c1}$$

$$= \frac{1}{2} m \sum_{j=2}^{n} \frac{P_{i,j}}{A}$$

Here,

$P_{i,j}$ = initial prestressing force in tendon j

n = number of tendons

The eccentricity of individual tendon is neglected.

Post-tensioned Bending Members

The calculation of loss for tendons stretched sequentially, is similar to post-tensioned axial members. For curved profiles, the eccentricity of the CGS and hence, the stress in concrete at the level of CGS vary along the length. An average stress in concrete can be considered.

For a parabolic tendon, the average stress ($f_{c,avg}$) is given by the following equation.

$$f_{c,avg} = f_{c1} + \frac{2}{3}(f_{c2} - f_{c1})$$

Here, f_{c1} = stress in concrete at the end of the member

f_{c2} = stress in concrete at the mid-span of the member.

Example 24

A pre-tensioned concrete beam, 100 mm wide and 300 mm deep, is prestressed by straight wires carrying an initial force of 150 kN at an eccentricity of 50 mm. the modulus of elesticity of steel and concrete are 210 and 35 kN/mm² respectively. Estimate the precentage loss of stress in steel due to elastic deformation of concretre if the area of steel wire is 188 mm².

Sol:

$$P = 150 \text{ kN}$$

$$e = \frac{d}{6} = \frac{300}{6} = 50\text{mm} \text{ (all ready given)}$$

$$A = (100 \times 300) = 3 \times 10^4 \text{ mm}^2$$

$$I = 225 \times 10^6 \text{ mm}^4$$

$$m = \left(\frac{E_s}{E_c}\right) = 6$$

$$\text{Initial stress insteel} = \left(\frac{150 \times 10^3}{188}\right) = 800 \text{N/mm}^2$$

$$\text{Stress in concrete, } f_c = \left(\frac{150 \times 10^3}{3 \times 10^4}\right) + \left(\frac{150 \times 10^3 \times 50 \times 50}{225 \times 10^6}\right)$$

$$= 6.66 \text{ N/mm}^2$$

$$\text{Loss of stress due to elastic deformation of concrere} = mf_c$$

$$= (6 \times 6.66) = 40 \text{ N/mm}^2$$

$$\text{Percentage loss of stress insteel} = \left(\frac{40 \times 100}{800}\right) = 5\%$$

Example 25

A rectangular concrete beam, 360 mm deep and 200 mm wide, is prestressed by mean of fifteen 5 mm diameter wires located 65 mm from the bottom of the beam and three 5 mm wires, located 25 mm from the top of the beam. If the wries are initially tensioned to a stress fo 840 N/mm², calculate the percentage loss of stress in steel immediately after transfer, allowing for the loss of stress due to elastic deformation of concrete only. Take E_s = 210 kN/mm², E_c = 31.5 kN/mm².

Sol:

$$E_s = 210 \text{ kN/mm}^2$$
$$E_c = 31.5 \text{ kN/mm}^2$$

Position of the centroid of the wires from the soffit of the beam,

$$y = \left[\frac{(15\times 65)+(3\times 275)}{(15+3)}\right] = 100\text{mm}$$

$$\text{Eccentricity } e = (150 - 100) = 50 \text{ mm}$$

$$\text{Area of concrete } A = (200 \times 300) = 6 \times 10^4 \text{ mm}^2$$

$$\text{Second moment of area } I = \frac{(200\times 300^3)}{12} = 45\times 10^7 \text{mm}^4$$

$$\text{Prestressing force } P = (840)(18 \times 19.7) = 3 \times 10^5 \text{ N} = 300 \text{ kN}$$

Stress in concrete

$$\text{At the level of top wires} = \left(\frac{300\times 10^3}{6\times 10^4}\right) - \left(\frac{300\times 10^3\times 50\times 125}{45\times 10^7}\right)$$

$$= 0.83 \text{ N/mm}^2$$

$$\text{At the level of bottom wires} = \left(\frac{300\times 10^3}{6\times 10^4}\right) + \left(\frac{300\times 10^3\times 50\times 85}{45\times 10^7}\right)$$

$$= 7.85 \text{ N/mm}^2$$

$$\text{Modular ratio } (m) = \left(\frac{210}{31.5}\right) = 6.68$$

Loss of stress in wires at

$$\text{top} = (6.68 \times 0.83) = 5.55 \text{ N/mm}^2$$

Loss of stress in wires at

$$\text{bottom} = (6.68 \times 7.85) = 52.5 \text{ N/mm}^2$$

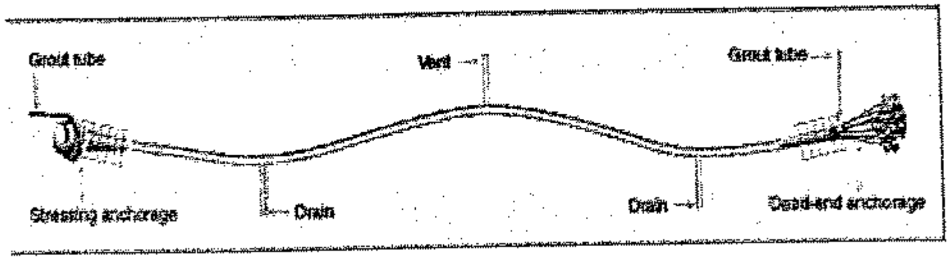
Percentage loss of stress:

$$\text{For wire at top} = \frac{5.55}{840}\times 100 = 0.66\%$$

$$\text{For wires at bottom} = \frac{52.5}{840}\times 100 = 6.25\%$$

FRICTION

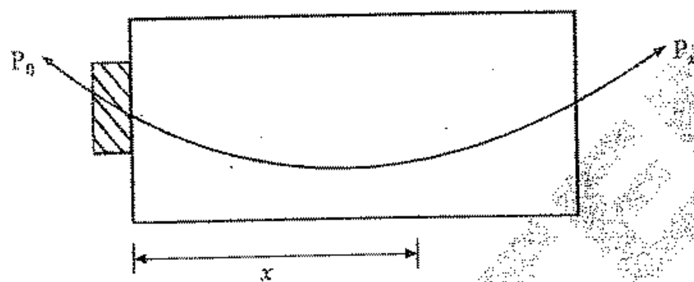
- The friction generated at the interface of concrete and steel during the stretching of a curved tendon in a post-tensioned member, leads to a drop in the prestress along the member from the stretching end.
- The loss due to friction does not occur in pre-tensioned members because there is no concrete during the stretching of the tendons.
- The friction is generated due to the curvature of the tendon and the vertical component of the prestressing force.
- The following figure shows a typical profile (laying pattern) of the tendon in a continuous beam.



A typical continuous post-tensioned member

(Reference: VSL International Ltd.)

In addition to friction, the stretching has to overcome the **wobble** of the tendon. The wobble refers to the change in position of the tendon along the duct. The losses due to friction and wobble are grouped together under friction.



Combined loss due to length and curvature $P_x = P_0 e^{-(\mu\alpha + kx)}$

where, P_x = Prestressing force at a distance x from jacking end.

P_0 = Prestressing force at jacking end.

k = Coefficient called wobble correction factor.

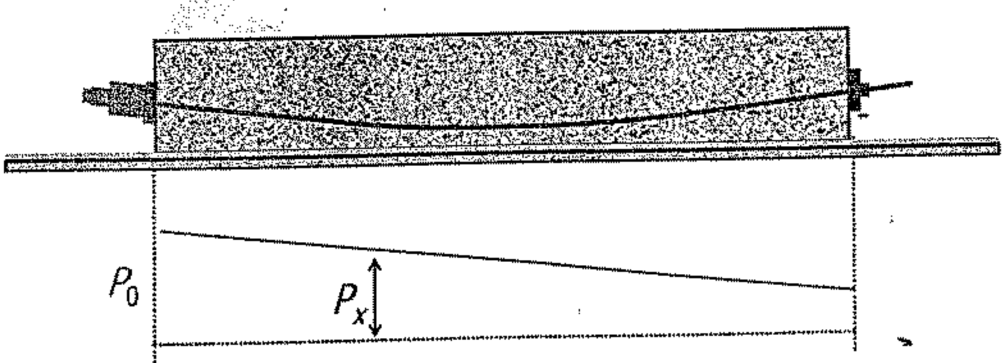
μ = Coefficient of friction in curve

α = Cumulative angle in radian through which the tangent to the cable profile has turned between any two point under consideration.

For small values of $\mu\alpha + kx$, the above expression can be simplified by the Taylor series expansion.

$$P_x = P_0 (1 - \mu\alpha - kx)$$

Thus, for a tendon with single curvature, the variation of the prestressing force is linear with the distance from the stretching end. The following figure shows the variation of



Variation of prestressing force after stretching

In the absence of test data, **IS:1343 - 1980** provides guidelines for the values of μ and k.

Table: Values of coefficient of friction

Type of interface	μ
For steel moving on smooth concrete	0.55.
For steel moving on steel fixed to duct	0.30
For steel moving on lead	0.25.

μ can be reduced by using lubricating like

(i) Greese (ii) Graphite (iii) Paraffin oil

Paraffin oil gives low value of μ and it is harmles to concrete.

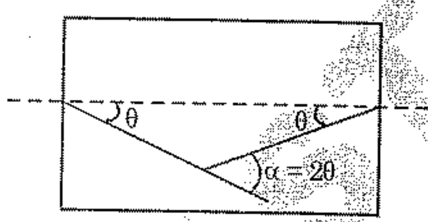
The value of k varies from 0.0015 to 0.0050 per meter length of the tendon depending on the type of tendon.

Value x

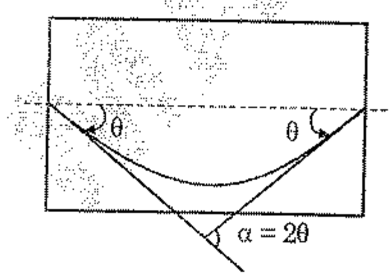
(a) When jacking from one end $x = L$

(b) When jacking from both the end $x = \frac{L}{2}$

Value of α



$$\alpha = \theta - (-\theta) = 2\theta$$



Jacking at one end

$$y = \frac{4h}{L}x(L-x)$$

$$\frac{dy}{dx} = \frac{4h}{L}(L-2x)$$

$$x = L$$

$$\frac{dy}{dx} = -\frac{4h}{L}$$

$$\boxed{\theta = -\frac{4h}{L}}$$

$$\boxed{\alpha = 2\theta = \frac{8h}{L}}$$

Jacking from both ends

$$x = \frac{L}{2}$$

$$\frac{dy}{dx} = \frac{4h}{L}\left(L - 2 \times \frac{L}{2}\right) = 0$$

$$\boxed{\theta = 0}$$

$$\alpha = \theta - (-0)$$

$$\alpha = \theta$$

Combined effect of length and curvature:

$$P_x = P_0 e^{-(\mu\alpha + kx)}$$

after expansion

$$P_x = P_0(1 - \mu\alpha + kx)$$
$$P_x = P_0 - P_0(\mu\alpha + kx)$$
$$P_0(\mu\alpha + kx) = P_0 - P_x$$
$$\mu\alpha + kx = \frac{P_0 - P_x}{P_0}$$

$$\frac{\text{loss of prestress}}{\text{Initial stress}} = \mu\alpha + kx$$

$$\boxed{\text{loss of prestress} = (\mu\alpha + kx)\ \text{Initial stress}}$$

Example 26

A concretre beam of 10 m span, 100 mm wide and 300 mm deep is prestressed by a cable with cross sectional area of 200mm². The cable profile is parabolic with an eccentricity of 50 mm above the centroid of the section at the supports and 50 mm below at mid span. If the cable is tensioned from one end only, estimate the percentage loss of prestress in the cable due to the effects of friction. Assume m = 0.35 and k = 0.0015 per m. the equation of the parabolic cable may be taken as $y = \frac{4h}{L^2}x(l-x)$.

Sol: The equation to the cable is

$$y = \frac{4h}{L^2}x(l-x)$$

Slope of the cable

$$\frac{dy}{dx} = \frac{4h}{L^2}(l-2x)$$

$e = h = e_1 + e_2 = 50 + 50 = 100$ mm

Slope of the cable at the end $= \tan\theta = \theta = \frac{4h}{l} = \frac{4\times100}{10\times1000} = \mathbf{0.04}$ **radian**

$\therefore \quad \alpha = 2\theta = 2\times0.04 = 0.08$ radian

Loss of stress due to friction $= f_o(\mu\alpha + kx) = f_o(0.35\times0.08 + 0.0015\times10) = 0.043f_o$

$\therefore$ Percentage loss of stress $= \frac{0.043f_o}{f_o}\times100 = 4.30\%$

Example 27

A simply supported post-tensioned concrete beam of span 15 m has a rectangular cross-section 300 mm × 800 mm. the prestres at ends is 1300 kN with zero eccentricity at the supports and an eccentricity of 250 mm at the centre, the cable profile being parabolic. Assuming k = 0.15 per 100 meters, and μ = 0.35, determine the loss of stress due to friction at the centre of the beam.

Sol: Equation to the cable with one end as origin is,

$$y = \frac{4h}{l^2}x(l - x)$$

Slope at any point
$$\frac{dy}{dx} = \frac{4h}{l^2}(l - 2x)$$

Slope at end
$$= \frac{4h}{l} = \frac{4\times0.25}{15} = \frac{1}{15}$$

$$\alpha = \theta - (0) = 0 \text{ at centre}$$

$$\text{Change of slope} = \frac{1}{15}$$

Angle of devation of cable from the end to the centre

$$\alpha = \frac{1}{15} \text{ radian (since } \alpha \text{ is small)}$$

$$k = \frac{0.15}{100} = 0.0015 \text{ per m}$$

$$\mu = 0.35$$

$$P_x = P_o(1 - \mu\alpha - kx)$$

At
$$x = \frac{l}{2} = 7.5\text{m (at centre)}$$

$$P_x = 1300\times10_3\left(1 - 0.35\times\frac{1}{15} - 0.0015\times7.5\right) = 1255046\text{N}$$

$\therefore$ Loss of prestressing force $= 1300000 - 1255046 = 44954$ N

$\therefore$ Percentage loss of prestress $= \frac{44954}{1300\times1000}\times100 = 3.46\%$

ANCHORAGE SLIP

In a post-tensioned member, when the prestress is transferred to the concrete, the wedges slip through a little distance before they get properly seated in the conical space. The anchorage block also moves before it settles on the concrete. There is loss of prestress due to the consequent reduction in the length of the tendon.

The total anchorage slip depends on the type of anchorage system. In absence of manufacturer's data, the following typical values for some systems can be used.

$$\text{This loss due to once slip} = \frac{E_s \Delta}{L}$$

E_s = Young modulus of steel in N/mm2
Δ = Anchorage slip in mm
L = Length of cable.

Table: Typical calues of anchorage slip

Anchorage system	Anchorage slip (Δ)
Freyssinet	
12·5 mm ϕ strands	4 mm
12·8 mm ϕ strands	6 mm
Magnel	8 mm
Dywidag system	1 mm

Example 28

In a post tensioned beam the cable is subjected to a stress of 1200 N/mm². If the slip at the jacking end is found to be 4mm, find the percentage loss of stress due to this case if the beam is 25 m long. Take E_s = 210 kN/mm².

Sol: Loss of stress due to anchorage slip

$$= \frac{\Delta}{L} E_s = \frac{4}{25 \times 1000} \times 210 \times 10^3 = 33.6 \text{N/mm}^2$$

$$\% \text{ loss} = \frac{33.6}{1200} \times 100 = 2.8\%$$

Creep of Concrete

Creep is the property of conrete by which it continues to deform with time under sustained loading.

$$\text{Creep coefficient is defined as } \phi = \frac{\text{creep strain}}{\text{elastic strain}} = \frac{\varepsilon_{cp}}{\varepsilon_c}$$

$$\varepsilon_{cp} = \phi \varepsilon_c \quad \text{---(i)}$$

$$\varepsilon_c = \frac{f_c}{\varepsilon_c}$$

$$\varepsilon_{cp} = \phi . \frac{f_c}{E_c}$$

$$\text{Stress} = \varepsilon_{cp} \times E_s$$

$$= \phi \cdot \frac{f_c}{E_c} \times E_s \quad m = \frac{E_s}{E_c}$$

$$\boxed{\text{stress} = m\phi f_c}$$

Example 29

A prestressed concrete beam of rectangular section 120 mm wide and 300 mm deep is prestressed by 6 wires of 6 mm diameter, provided at an eccentricity of 55 mm. The initial stress in the wires is 1150 N/mm². Find the loss of stress in steel due to creep of concrete. Take $E_s = 2 \times 10^5$ N/mm², $E_c = 3 \times 10^4$ N/mm², $\phi = 1.50$.

Sol: Modular ratio m

$$= \frac{E_s}{E_c} = \frac{2\times10^5}{3\times10^4} = \frac{20}{3}$$

Area of the beam section = A = 120 × 300 = 36000 mm²

Moment of Inertia of the beam section

$$I = \frac{120\times300^3}{12} = 2.70\times10^8\,\text{mm}^4$$

$$\text{Prestressing force} = P = 6\times\frac{\pi}{4}\times6^2\times1150 = 195093\text{N}$$

Stress in concrete at the level of steel

$$f_c = \frac{P}{A} + \frac{Pe^2}{I}$$

$$= \frac{195093}{36000} + \frac{195093}{2.70\times10^8} = 7.60\,\text{N/mm}^2$$

Loss of stress insteel due to creep of concrete

$$= \phi\, m\, f_c$$

$$= 1.50\times\frac{20}{3}\times7.60\,\text{N/mm}^2 = 76\,\text{N/mm}^2$$

Shrinkage of Concrete

Loss due to shrinkage

Shrinkage means a contraction of concrete due to chemical changes and drying. This depends on the interval of times and the moisture conditions.

$$\text{Loss of stress due to shrinkage} = \varepsilon_{cs} \times E_s.$$

ε_{cs} = residual shrinkage strain

for pre tension member = 3×10^{-4}

$$\text{for post tension member} = \frac{2\times10^{-4}}{\log_{10}(t+2)}$$

t = times in days.

Example 30

Calculate the loss of stress in the tendons due to shrinkage of concrete in a post tensioned beam if the age of concret at transfer is 15 days. Take $E_s = 2.10\times 10^5$ N/mm².

Sol: Shrinkage strain in concrete

$$= \frac{2\times10^{-4}}{\log_{10}(T+2)} = \frac{2\times10^{-4}}{\log_{10}(125+2)} = 1.6255\times10^{-4}$$

$\therefore$ Loss of stress in steel $E_s \times$ shrinkage strain in concrete

$$= 2.1\times10^5\times1.6255\times10^{-4} = 34.14 \text{ N/mm}^2.$$

Relaxation of Steel

Relaxation of steel is defined as the decrease in stress with time under constant strain. Due to the relaxation of steel, the prestress in the tendon is reduced with time. The relaxation depends on the type of steel, initial prestress (f_{pi}) and the temperature. To calculate the drop (or loss) in prestress (Δf_p), the recommendations of **IS:1343 - 1980** can be followed in absence of test data.

Final conclusion of above discussion.

S.No.	Type of loss	Equation
1	wobble & curvature effect	$(\mu\alpha + kx)P_0$
2.	Anchorage slip	$E_s\Delta/L$
3.	Shrinkage loss	$\varepsilon_{sc}\cdot E_s$
4.	Creep of concrete	$m\phi f_c$
5.	Elastic shortening of concrete	mf_c
6.	Relaxation in steel	4 to 5% for initial stress in steel.

General % of lossess that may be considered

Type of loss	Pretensioned (%)	Post tensioned (%)
electifc shorting of conc.	4	1
Shrinkage	7	6
Creep	6	5
Relaxation	8	8
Total loss	25%	20%

Losses	Prestensioning (only subsequent losses are taken)	Post tensioning
length effect	No	Yes
Curvature effect	No	Yes
Anchorage slip	No	Yes
Shrinkage of concrete	Yes	Yes
Creep of concrete	Yes	Yes
Elastic deformation or shortening of concrete	Yes	No (If all wires are simultaneously tensioned) Yes (If wires are successively tensioned)

Example 31

A prestressed concrete pile is 300 mm × 300 mm in section and is provided with 40 wires of 3 mm diametrer distributed uniformly over the section. Initially the wires are tensioned in the prestressing beds with a total pull of 450 kN. Determine the final stress in concrete and the percentage loss of stress in the wires.

Take $E_s = 2.08 \times 10^5$ N/mm², $E_c = 3.20 \times 10^4$ N/mm²
Creep shortening = 32×10^{-6} mm/mm per N/mm² of stress
Total shrinkage strain = 200×10^{-6}
Relaxation loss of stress in steel = 4.50% of the initial stress.

Sol: Initial prestressing force

$$= P_o = 450 \text{ kN}$$

$$\text{Area of 40 wires of 3mm } \phi = A_s = 40 \times \frac{\pi}{4} \times 3^2 = 282.743 \text{mm}^2$$

$$\text{Initial stress in concrete} = \frac{450 \times 10^3}{300^2} = 5 \text{N/mm}^2$$

$$\text{Initial stress in the wires} = \frac{450 \times 10^3}{282.743} = 1591.55 \text{N/mm}^2$$

$$\text{Modular ratio m} = \frac{2.08 \times 10^5}{3.20 \times 10^4} = 6.50$$

The vearous losses of stress that occur are the following

(i) Loss of stress due to elastic shortening = $6.50 \times 5 = 32.50$ N/mm²
(ii) Loss of stress due to creep of concrete = $(32 \times 10^{-6})\ 5 \times 2.08 \times 10^5 = 33.28$ N/mm²
(iii) Loss of stress due to shrinkage of concrete = $(200 \times 10^{-6})\ 2.08 \times 10^5 = 41.60$ N/mm²

(iv) Relaxation loss = $\frac{4.5}{100} \times 1591.55 = 71.62$ N/mm²

Total loss of stress in the wire = 179 N/mm²
Effective stress in the wire = 1591.55 − 179 = 1412.55 N/mm²
Percentage loss of stress in the wires

$$= \frac{179}{1591.55} \times 100 = 11.25\%$$

$$\text{Final stress in concrete} = \frac{1412.55 \times 282.743}{300^2} = 4.44 \text{N/mm}^2$$

Example 32

A pretensioned beam 250 mm wide and 360 mm deep is prestressed by 10 wires of 8 mm diameter initially stressed to 1000 N/mm². The centroid of the steel wires is located at 105 mm from the soffit. Determine the maximum stress in concrete immediately after transfer allowing elastic shortening of concrete only at th level of the centroid of steel.

If, however, the concrete is subjected to additional shortening due to creep and shrinkage and the steel is subjected to a relaxation of stress of 5 percent find the final percentage loss of stress in the steel wires.

Take $E_s = 210\ kN/mm^2$, $E_c = 36.85\ kN/mm^2$, creep coefficient $\phi = 1.60$. Total residual shrinkage strain $= 3 \times 10^{-4}$.

Sol: Area of the beam section

$$A = 250 \times 360 = 90000\ mm^2 = 9 \times 10^8\ mm^4$$

Moment of inertia of the beam section

$$I = \frac{250 \times 360^3}{12} = 9.72 \times 10^8\ mm^2$$

$$\text{Ecentriciricity } e = 180 - 105 = 75\ mm$$

$$\text{Modular ratio } m = \frac{210}{36.85} = 5.70$$

$$\text{Area of the steel wire} = A_s = 10 \times \frac{\pi}{4} \times 8^2 = 502.65\ mm^2$$

$$P_0 = 1000 \times 502.65 = 50265\ N$$

Stress in concrete at the level of steel

$$= \frac{P_o}{A} + \frac{P_o e^2}{I}$$

$$= \frac{502655}{9 \times 10^4} + \frac{502655}{9.72 \times 10^8} = 5.59 + 2.91 = 8.50\ N/mm^2$$

Loss of stress due to elastic shortening of concrete

$$= 5.70 \times 8.50 = 48.45\ N/mm^2$$

$$\text{Stress in the wire} = 1000 - 48.45 = 951.55\ N/mm^2$$

$$\text{Prestressing force } P = 951.55 \times 502.65 = 478296.6\ N/mm^2$$

$\therefore$ Actural stress in concrete at the level of steel

$$= \frac{P}{A} + \frac{Pe^2}{I}$$

$$= \frac{478296.6}{9 \times 10^4} + \frac{478296.6 \times 75^2}{9.72 \times 10^8} = 5.31 + 2.77 = 8.08\ N/mm^2$$

The various losses of stress which occur are the following.

(i) Loss of stress due to elastic shortening = 48.45 N/mm^2

(ii) Loss of stress due to creep of concrete $= \phi\, m f_c = 1.60 \times 5.70 \times 8.08$

$$= 73.69\ N/mm^2$$

(iii) Loss of stress due to shrinkage of concrete $= \varepsilon_{sc} E_s = 3 \times 10^{-4} \times 210 \times 10^3$

$$= 63.00\ N/mm^2$$

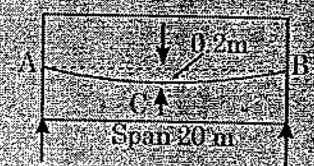
(iv) Relaxation loss $= \frac{5}{100} \times 1000 = 50.00\ N/mm^2$

Total loss of stress = 235.14 N/mm^2

$$\text{Percentage loss of stress} = \frac{235.14}{1000} \times 100 = 23.51\%$$

Example 33

A prestressed concrete beam is provided with a paracbolic tendon as shown in figure. Which is tensioned from both the ends. If the stress in the tendons at the ends is 1050 N/mm^2, calculate the loss of prestress from the ends to the centre. Take μ = 0.35 and k = 0.0015 per metre.



Sol: Equation to the tendon profile is

$$y = \frac{4h}{l^2}x(l-x)$$

Slope of cable is given by
$$\frac{dy}{dx} = \frac{4h}{l^2}(l-2x) = \frac{4\times 0.2}{20^2}(20-2x)$$

$$\frac{dy}{dx} = 2\times 10^{-3}(20-2x)$$

Putting $x = 0$,

Slope at A = $2\times 10^{-3}\times 20 = 0.04$

Slope at C = 0

Change of slope from A and C = 0.04

Angle of deviation of the tendon from A to C

= 0.04 radian

i.e., α = 0.04 radian

Loss of stress from A to C = ($\mu\alpha$ + kx) initial stress at end

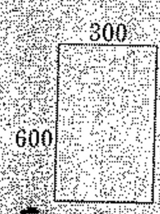
= [(0.35 × 0.04) + (0.0015 × 10)] × 1050 N/mm^2

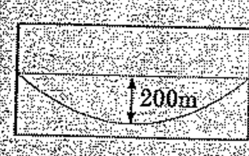
= 30.45 N/mm^2

Example 34

A simply supported post tensioned concrete beam, span = 15 m, 300 × 600 mm, force at ends = 1150 kN with zero ecentricity, cable is parabolic & ecentricity at mid span = 200 mm, K = 0.15 per 100 m μ = 0.35. Determine the loss due to friction of the centre of beam.

Sol: Post tensioned beam





oned
ss of

Loss due to friction = $(\mu\alpha + kx)$ initial stress

$$\mu = 0.35$$

$$\alpha = \theta$$

$$= \frac{4e}{L}$$

$$= \frac{4 \times 200}{15000} = 0.0533$$

$$k = \frac{0.15}{100} = 0.0015$$

$$x = \frac{L}{2}$$

(Case is similar to that – Jaking from both ends & losses are maximum at centre)

Loss due to friction = $(0.35 \times 0.0533 + 0.0015 \times 7.5) \times 1150 = 34.39$ kN

$$\% \text{ loss} = \frac{55.88}{1150} \times 100 = 4.85\%$$

$$\frac{34.39}{1150} = 2.99\% \approx 3\%$$

Example 35

A PSC pretensioned beam size 225 × 300 mm deep is prestressed by 12 wires of 5 mm ϕ, initially stressed at 1100 N/mm². Estimate the loss of prestress due to elastic deformation, creep shrinkage & relaxation of steel Grade of concrete = M40, Relaxation of steel = 5%

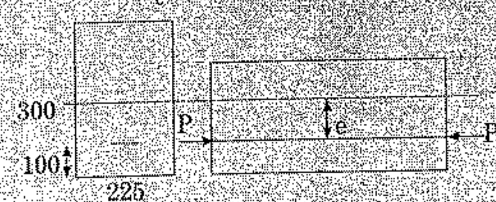
$E_S = 2 \times 10^5$ mpa, creep coefficient = 1.6, Residual shrinkage strain = 3×10^{-4}.

Sol. Prestressing force $= 1100 \times \frac{\pi}{4} \times 5^2 \times 12 = 259.18$ kN

(i) Loss due to shrinkage $= \varepsilon_{sc} \times E_s = 3 \times 10^{-4} \times 2 \times 10^5 = 60$ N/mm²

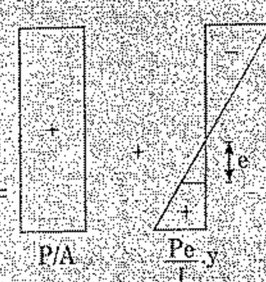
(ii) Loss due to creep $= m\phi f_c$

f_c = stress in concrete at the level of steel



e = 150 – 100 = 50 mm

Total stress =



1150
100 m

$$\text{Total stress} = P/A + \frac{Pe}{I}e$$

$$\text{Stress at bottom} = \frac{P}{A} + \frac{Pe}{I}e$$

$$= \frac{259.18\times10^3}{300\times225} + \frac{259.18\times10^3\times50}{\frac{300^3}{12}\times225}\times50$$

$$= 3.839 + 1.28 = 5.12 \text{ N/mm}^2$$

$$\text{Creep loss} = m\phi f_c$$

$$m = \frac{E_s}{E_c} = \frac{2\times10^5}{5000\sqrt{40}}$$

$$= 6.32$$

$$\text{Creep stress} = 6.32 \times 5.12 \times 1.6 = 51.81 \text{ N/mm}^2$$

$$\text{Loss due to Relaxation of steel} = 5\% \text{ of initial stress}$$

$$= \frac{5}{100}\times1100 = 55 \text{ N/mm}^2$$

$$\text{Loss due to elastic shorting} = mf_c = 6.32 \times 5.12 = 32.35 \text{ N/mm}^2$$

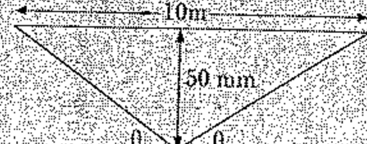
$$\text{Total loss} = 32.38 + 51.81 + 55 + 60 = 199.19 \text{ N/mm}^2$$

$$\% \text{ loss} = \frac{199.19}{1100}\times100 = 18.1\ \%$$

Example 36

A post tensioned beam of span 10m is provided with 3 cables each having area of 2000 mm² and each stress to a prestress of 1200 N/mm². Cable I is having '0' eccentricity as support & linearly varies to 50 mm below centroid as the centre, cable 2 is prabolic with zero ecentricity as support & 50 mm below centroid as the mid span, cable 3 is straight with a constant eccentricity of 50 mm below the axis. All the 3 cables are tensioned symultaneously. Compute the total % of loss of stress in each cable $E_s = 2 \times 10^5$ N/mm², $E_c = 30$ kN/mm², $\mu = 0.35$. Relaxation of steel = 6%, shrinkage strain = 2×10^{-4}, ϕ creep coefficient $\phi = 1.5$, wave effect = 0.0015 /m, $f_c = 6$ N/mm²

Sol: Cable (1)



Cable (1)

$$\alpha = 2\theta$$

$$\theta = \frac{50}{5\times10^3} = 0.01$$

$\alpha = 2\theta = 2 \times 0.01 = 0.02$ radian

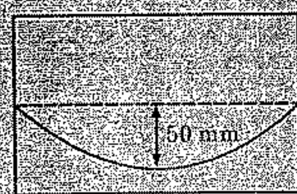
Loss due to friction $= (\mu\alpha + kx)f_0$ (Assuming Jaking from one end only)

$= (0.35 \times 0.02 + 0.0015 \times 10) \times 1200$

$= 26.4$ N/mm²

$$\% \text{ loss} = \frac{26.4}{1200} \times 100 = 2.27\%$$

Cable (2)



Cable (2)

$$\theta = \frac{4e}{L} = \frac{4 \times 50}{10000} = 0.02 \text{ radian}$$

$\alpha = 2\theta = 2 \times 0.02 = 0.04$

Loss due to friction

$= (\mu\alpha + kx)f_0 = (0.35 \times 0.04 + 0.0015 \times 10) \times 1200$

$= 34.8$ N/mm²

$$\% \text{ of loss} = \frac{34.8}{1200} \times 100 = 2.9\%$$

Cable 3

Cable (3)

$\alpha = 0$

Loss $= (0 + kx) \times$ initial stress

$= (0.0015 \times 10) \times 1200$

$= 18$ N/mm²

$$= \frac{18}{1200} \times 100 = 1.5\%$$

Subsequent loss

(i) Shrinkage loss

Loss in all 3 cables $= \varepsilon_{cs} \times E_c$

$= 2 \times 10^{-4} \times 210 \times 10^3 = 42$ N/mm²

$$\%\text{ of loss} = \frac{42\times100}{1200} = 3.5\%$$

(ii) Elastic starting = 0 cable is symultaneously tension

(iii) Loss due to creep = $m\phi f_c$

$$= \left(\frac{210}{30}\right)\times1.5\times6 = 63\text{ N/mm}^2$$

$$\%\text{ of loss} = \frac{63}{1200}\times100 = 5.25\%$$

(iv) Loss due to relaxation all three cable = $\frac{6}{100}\times1200 = 72\text{ N/mm}^2$

$$\%\text{ loss} = \frac{72}{1200}\times100 = 6\%$$

(v) Anchorage slip = $\frac{E_s\Delta}{l} = \frac{210\times1\times10^3}{10000} = 21\text{ N/mm}^2 = \%\text{ loss} = \frac{21}{1200}\times100 = 1.75\%$

Total losses

Cable (1) = 2.2 + 3.5 + 5.25 + 6 + 1.75 = 18.7%

Cable (2) = 2.0 + 3.5 + 5.25 + 6 + 1.75 = 18.5%

Cable (3) = 1.5 + 3.5 + 5.25 + 6 + 1.75 = 18%

DESIGN OF PRESTRESSED CONCRETE BEAMS

Let the effective total prestress be P at an eccentricity of e. Let M_d be the maximum bending moment due to dead load alone. The stresses due to direct load, eccentricity of the prestress and due to the dead load moment are shown in Fig. 9.39.

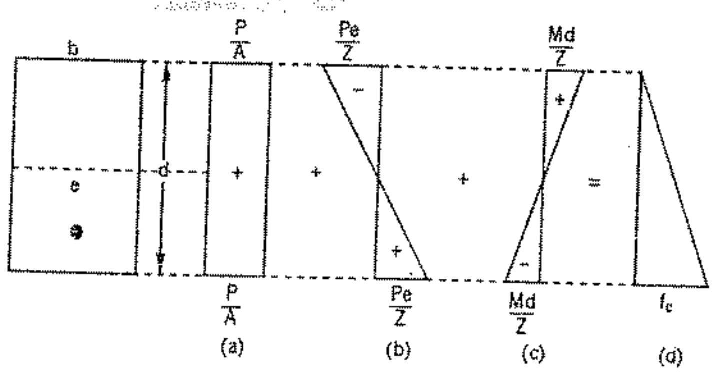


Fig. 9.39

The beam should be so designed that when the live load has not been applied, the resultant stress on the section due to prestress, eccentricity of the prestress and due to the dead load moment should be as shown in Fig. 9.39(a-c) so that the maximum compressive stress in the bottom fibre should be f_c the permissible stress in concrete. The stress in the top fibre wil be zero.

To suit this condition, we have

$$\boxed{\frac{P}{A}+\frac{M_d}{Z}=\frac{P_e}{Z}} \qquad \text{---(i)}$$

$$\boxed{\frac{P}{A}+\frac{Pe}{Z}-\frac{M_d}{Z}=f_c} \qquad \text{---(ii)}$$

Suppose now the live load is applied.
Let the bending moment due to live load be M_r.

The beam should be so designed that after the live load is applied, the bending stresses $\frac{M_l}{Z}$ caused by the live load should be such that the resultant stress should be as in Fig. 9.40(c) so that the compressive stress in the top fibre reaches f_c while the stress in the bottom fibre is zero.

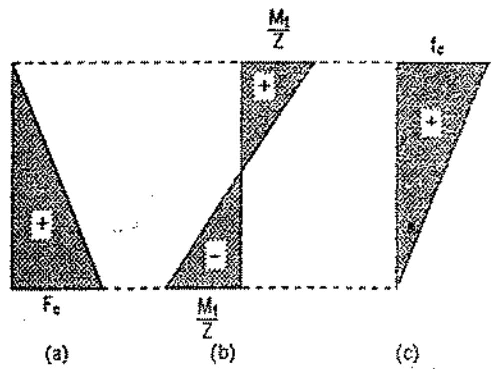


Fig. 9.40

(a) Resultant stress diagram before applying live load.
(b) Stresses due to live load.
(c) Resultant stress diagram after applying live load.

To fulfil this condition, we have

$$\boxed{\frac{M_l}{Z}=f_c} \qquad \text{---(iii)}$$

But from (2)

$$f_c=\frac{P}{A}+\frac{P_e}{Z}-\frac{M_d}{Z}$$

$$\frac{P}{A}+\frac{P_e}{Z}-\frac{M_d}{Z}=\frac{M_l}{Z}$$

$$\therefore \boxed{\frac{Pe}{Z}=\frac{M_d+M_l}{Z}-\frac{P}{A}} \qquad \text{---(iv)}$$

Rewritting equations (i) and (iv)

$$\frac{P}{A}+\frac{M_d}{Z}=\frac{Pe}{Z}$$

$$-\frac{P}{A}+\frac{M_d+M_l}{Z}=\frac{Pe}{Z}$$

Adding,

$$\frac{2M_d+M_l}{Z}=\frac{2Pe}{Z}$$

$$\therefore \quad \boxed{e = \frac{2M_d + M_l}{2P}} \qquad \text{---(v)}$$

In designing the beam, it is worth remembering the following formulae

$$Z = \frac{M_l}{f_c}$$

$$P = \frac{f_c A}{2}$$

(since the stress varies from 0 at one extreme fibre to f_c at the other extreme fibre) and

$$e = \frac{2M_d + M_l}{2P}$$

Hence the following steps may be adopted in designing a prestressed concrete beam.

Step 1: Calculate the live load moment M_l.

Step 2: Determine the section modulus required from the condition

$$Z = \frac{M_l}{f_c}$$

Choose a convenient depth $\frac{1}{20}$ or $\frac{1}{25}$ of the span. After choosing the depth we can determine the width b from the relation

$$\frac{bd^2}{6} = Z$$

Thus the width and the depth are both known.

Step 3: Now find the prestressing force P from the section $P = \frac{f_c}{2} A$.

Step 4: Find the amount of steel required from the relation $A_s = \frac{P}{\text{Safe stress in steel}}$.

Step 5: Determine the dead load moment M_d.

Step 6: Place the reinforcement at the eccentricity e given by

$$e = \frac{2M_d + M_l}{2P}$$

The following example shows the application of the above steps in the design of the beams.

Example 37

A prestressed concrete beam of uniform rectangular cross-section and span 15 metres supports a total distributed load of 272 kN excluding the weight of the beam. Determine the suitable dimensions of the beam and calculate the area of the tendons and their position. The permissible stresses are 14 N/mm² for concrete and 1050 N/mm² for the tendons.

Sol: B.M. due to live load $= M_l = \frac{Wl}{8} = \frac{272 \times 15}{8}$ kNm = 510 kNm

$\therefore$ Section modulus required $= Z = \frac{M_l}{f_c} = \frac{510 \times 10^6}{14} = 3.643 \times 10^7 \text{ mm}^3$

The depth of the beam may be taken from $\frac{1}{20}$ to $\frac{1}{25}$ of the span. In our case $\frac{1}{20}$ of the span $= \frac{15\times1000}{20}$ = 750 mm. Let the width of the beam be b mm. But, the section modulus of the section

$$Z = \frac{bd^2}{6}$$

$$\therefore \quad \frac{b\times750^2}{6} = 3.643\times10^7$$

$$\therefore \quad b = \frac{3.643\times10^7\times6}{750^2}\text{ mm} = 388\text{ mm say } 390\text{ mm}$$

hence the beam section will be 390 mm × 750 mm

$$\text{Prestressing force} = P = \frac{f_c}{2}.A = \frac{14}{2}\times390\times750N = 2047500\ N$$

$$\therefore \quad \textit{Area of the tendons} = A_s = \frac{P}{\text{Safe stress in tendons}}$$

$$= \frac{2047500}{1050}\text{ mm}^2 = 1950\text{ mm}^2$$

If 6 mm dia. bars be used.

$$\text{Area of one bar} = 28\text{ mm}^2$$

$$\therefore \quad \text{Number of tendons required} = \frac{1950}{28} = 70$$

Hence provide 70 wires of 6 mm diameter.

$$\text{D.L. of the beam} = 0.39\times0.75\times25000 = 7312.5\text{ Nm}$$

$$\text{D.L. moment} = \frac{7312.5\times15^2}{8} = 205664\text{ Nm} = 205.664\text{ kNm}$$

$$\therefore \quad \text{Eccentricity } e = \frac{2M_d+M_l}{2P} = \frac{2(205.664)+510}{2\times2047500}\times10^6\text{ mm} = 225\text{ mm}$$

Practice Objective Questions

1. If a simply supported concrete beam, prestressed with a force of 2500 kN is designed by load balancing concept for an effective span of 10 m and to carry a total load of 40 kN/m, the central dip of the cable profile should be

   (a) 100 mm (b) 200 mm
   (c) 300 mm (d) 400 mm

2. In case of pre-tensioned RC beams

   (a) shrinkage of concrete is of the order of 3×10^{-4}
   (b) relaxation of steel can be stretched at a time
   (c) only one wire can be stretched at a time
   (d) even mild steel can be used for restressing

3. For prestressed structural elements, high strength concrete is used primarily because

(a) both shrinkage and creep are more
(b) shrinkage is less but creep is more
(c) modulus of elasticity and creep values are higher
(d) high modulus of elasticity and low creep

4. In a load-balanced prestressed concrete beam under self load the cross-section is subjected to

(a) axial stress
(b) bending stress
(c) axial and shear stress
(d) axial and bending stress

5. A prestressed concrete beam 150 mm × 300 mm supports a live load of 5 kN/m over a simple span of 8 m. It has a parabolic cable having an eccentricity of 75 mm at mid-span and zero at the ends. The prestressing force required to maintain the net resultant stress at the bottom fibre at mid-span as zero under the action of DL + LL + prestress is

(a) 239 kN
(b) 293 kN
(c) 302 kN
(d) 392 kN

6. A partially prestressed member is one in which

(a) tensile stresses and cracking are permitted under service loads
(b) no tensile stresses are permitted under service loads
(c) mild steel is used in addition to prestressing steel
(d) tensile stresses are permitted but not cracking at service loads

7. For a pretensioned rectangular plank the uplift at centre on release of wires from anchors due to pretensioning only (force P, eccentricity e) will be

(a) $\frac{PeL^2}{6EI}$
(b) $\frac{Pe^2L}{6EI}$
(c) $\frac{PeL^2}{8EI}$
(d) $\frac{Pe^2L}{8EI}$

8. At the time of initial tensioning, the maximum tensile stress in tendon immediately behind the anchorage shall NOT exceed

(a) 50% of the ultimate tensile strength of the wire or bar or strand
(b) 80% of the ultimate tensile strength of the wire or bar or strand
(c) 40% of the ultimate tensile strength of the wire or bar or strand
(d) 60% of the ultimate tensile strength of the wire or bar or strand

9. The cable for a prestressed concrete simply supported beam subjected to uniformly distributed load over the entire span should ideally be

(a) placed at the centre of cross-section over the entire span
(b) placed at some eccentricity over the entire span
(c) varying linearly from the centre of cross-section at the ends to maximum eccentricity at the middle section
(d) parabolic with zero eccentricity at the ends and maximum eccentricity at the centre of the span

10. For a prestressed concrete bridge beam, a minimum clear spacing of the cable or group of cables should be

(a) 25 mm
(b) 25 mm or 6 mm plus the largest size of the aggregate
(c) 40 mm
(d) 50 mm

11. In the case of a P.S.C. beam, to satisfy the Limit State of Serviceability in cracking, match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

List-I

A. Class 1 structure
B. Class 2 structure
C. Class 3 structure

List-II

1. No visible cracking $\sigma_1 < 3$ N/mm^2
2. Concrete section uncracked; crack width to be calculated to check
3. No cracking under service loads

Codes:

	A	B	C
(a)	1	2	3
(b)	2	1	3
(c)	3	1	2
(d)	3	2	1

12. In the design of prestressed concrete structure, which of the following limit states will come under the limit states of serviceability?

1. Flexure
2. Shear
3. Deflection
4. Cracking

Select the correct answer using the codes given below:

(a) 1 and 4
(b) 3 and 4
(c) 2, 3 and 4
(d) 2 and 3

13. In pre-tensioning scheme, pre-stress load is transferred in

(a) a single stage process
(b) multi stage process
(c) either single stage or multi-stage process depending upon the magnitude of load transfer
(d) the same manner as in post-tensioning scheme

14. In the conventional prestressing, the diagonal tension in concrete

(a) increases
(b) decreases
(c) does not change
(d) may increase or decrease

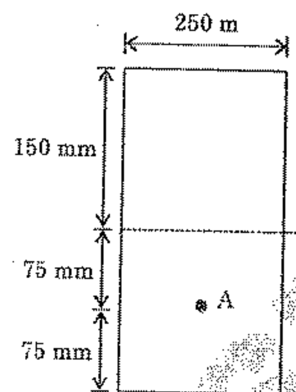
15. The ultimate strength of the steel used for prestressing is nearly

(a) 250 N/mm^2
(b) 415 N/mm^2
(c) 500 N/mm^2
(d) 1500 N/mm^2

16. If the loading on a simply supported prestressed concrete beam is uniformly distributed, the centroid of tendons should be preferably

(a) a straight profile along the centroidal axis
(b) a straight profile along with the lower kern
(c) a parabolic profile with convexity downard
(d) a circular profile with convexity upward

17. In the prestressed concrete beam section shown in the given figure (all dimensions in mm in the figure), if the net losses are 15% and final prestressing force applied at 'A' is 500 kN, the initial extreme fibre stresses at top and bottom will be respectively



(a) – 3.40 N/mm^2 and 16.70 N/mm^2
(b) – 3.40 N/mm^2 and 19.60 N/mm^2
(c) – 4.0 N/mm^2 and 16.70 N/mm^2
(d) – 4.0 N/mm^2 and 19.60 N/mm^2

18. The stress block in concrete for an estimate of ultimate strength in flexure of a prestressed beam

(a) should be parabolic
(b) should be parabolic-rectangular
(c) should be rectangular
(d) may be of any shape which provies agreement with the test data

19. When the tendon of a tectangular prestressed beam of cross-sectional area A is subjected to a load W through the centroidal longitudinal axis of beam, (where M = maximum bending moment and Z = section modulus) then the maximum stress in the beam section will be

(a) $\frac{W}{A} - \frac{M}{Z}$
(b) $\frac{W}{A} + \frac{M}{Z}$
(c) $\frac{A}{W} - \frac{Z}{M}$
(d) $\frac{A}{W} + \frac{Z}{M}$

20. The losses in prestress in pre-tensioning system are due to

1. elastic deformation of concrete when wires are tensioned successively
2. friction
3. shrinkage and creep of concrete

Select the correct answer using the codes given below:

(a) 1, 2 and 3 (b) 2 and 3
(c) 1 alone (d) 3 alone

21. Prestressing of indeterminate structures should take care of the following:

1. High strength concrete
2. High tensile steel
3. Load balancing
4. Partial safety factors

Select the correct answer using the codes given below:

(a) 1 and 3 (b) 2, 3 and 4
(c) 1, 2 and 4 (d) 1, 2, 3 and 4

22. Match **List-I** with **List-II** and select the correct answer using the codes gien below the lists:

List-I

A. Loss of prestress
B. End block
C. Transmission length
D. Partially pre-stressed

List-II

1. Class 3
2. Pretensioned members
3. Bursting tension
4. Elastic structures shortening

Codes:

	A	B	C	D
(a)	3	4	1	2
(b)	4	3	1	2
(c)	3	4	2	1
(d)	4	3	2	1

23. In pre-tensioning process of prestressing, the tendons are

(a) bonded to the concrete
(b) partially bonded to the concrete
(c) not bonded to the concrete
(d) generally bonded but sometimes remain unbonded to the concrete

24. In prestressed concrete, high grade concrete is used for

(a) controlling the press-stress loss
(b) having concrete of low ductility

(c) having concrete of high brittleness
(d) having low creep

25. In the Limit state design of prestressed concrete structures, the strain distribution is assumed to be

(a) Linear
(b) Non-linear
(c) Parabolic
(d) Parabolic and rectangular

26. A simply supported post-tensioned prestressed concrete beam of span L is prestressed by a straight tendon at a uniform eccentricity 'e' below the centroidal axis. If the magnitude of prestressing force is P and flexural rigidity of beam is EI, the maximum central deflection of the beam is

(a) $\frac{PeL^2}{8\,EI}$ (downwards)
(b) $\frac{PeL^2}{48\,EI}$ (upwards)
(c) $\frac{PeL^3}{8\,EI}$ (upwards)
(d) $\frac{PeL^2}{8\,EI}$ (upwards)

27. M 40 concrete is preferred to M 20 concrete for prestressed concrete to

(a) overcome bursting stresses at the ends
(b) avoid brittle failure of concrete
(c) eliminate the effect of shrinkage
(d) economize the use of cement

28. Match List-I (Post-tensioning system) with List-II (Type of anchorage) and select the correct answer using the codes given below the lists:

List-I

A. Freyssinet
B. Gifford-Udall
C. Lee-McCall
D. Magnel Blaton

List-II

1. Flat steel wedges in sandwich plates
2. High strength nuts
3. Split conical wedges
4. Conical serrated concrete wedges

Codes:

	A	B	C	D
(a)	2	1	4	3
(b)	4	3	2	1
(c)	2	3	4	1
(d)	4	1	2	3

29. Which one of the following systems of prestressing is suitable for pretensioned members?

(a) Freyssinet system
(b) Magnel-Blaton system
(c) Hoyer system
(d) GIfford-Udall system

30. Which one of the following method is employed to manufacture prestressed concrete sleepers for the railways?

(a) Post-tensioning
(b) Pre-tensioning
(c) Pre-tensioning followed by post tensioning
(d) Partial prestressing

31. Match **List-I** (Post-tensioning system) with **List-II** (Arrangement of tendons in the duct) and select the correct answer using the codes given below the lists:

List-I
- **A.** Freyssinet
- **B.** Gifford-Udall
- **C.** Lee-McCall
- **D.** Magnel-Blaton

List-II
1. Single bars
2. Wires evenly spaced by perforated spacers
3. Horizontal rows of four wires spaced by metal grills
4. Wires spaced by helical wire core in annular spacer

Codes:

	A	B	C	D
(a)	4	1	2	3
(b)	3	2	1	4
(c)	4	2	1	3
(d)	3	1	2	4

32. Concordant cable profile is

(a) a cable profile that produces no support reactions due to prestressing
(b) a cable profile which is parabolic in nature
(c) a cable profile which produces no bending moment at the supports of a beam
(d) a cable profile laid corresponding to axial stress diagram

33. The profile of the centroid of the tendon is parabolic with a central dip h. Effective prestressing force is P and the span L. What is the equivalent upward acting uniform load?

(a) $\frac{8hL}{P}$ (b) $\frac{8hP}{L^2}$

(c) $\frac{8h^2L}{P}$ (d) $\frac{8h^2P}{L}$

34. What is the uplift at centre on release of wires from anchors due to pretensioning only for force P and eccentricity e for a pre-tensioned rectangular plank?

(a) $\frac{PeL^2}{6EI}$
(b) $\frac{Pe^2L}{6EI}$
(c) $\frac{PeL^2}{8EI}$
(d) $\frac{Pe^2L}{8EI}$

35. An ordinary mild steel bar has been prestressed to a working stress of 200 MPa. Young's modulus of steel is 200 GPa. Permanent negative strain due to shrinkage and creep is 0.0008. How much is the effective stress left in steel?

(a) 184 MPa
(b) 160 MPa
(c) 40 MPa
(d) 16 MPa

36. Prestressing anchorage unit using multiple wire cables exists in the

(a) Freyssinet system
(b) Lee-McCall system
(c) Gifford-Udall system
(d) Hoyer system

37. What is the limiting principal tensile stress in prestress uncracked concrete member of M25 grade?

(a) 1 MPa
(b) 1.5 MPa
(c) 2 MPa
(d) 2.5 MPa

38. At the time of initial tensioning, the maximum tensile stress immediately behind the anchorage should not exceed which one of the following?

(a) 0.50 × ultimate tensile stress
(b) 0.60 × ultimate tensile stress
(c) 0.70 × ultimate tensile stress
(d) 0.80 × ultimate tensile stress

39. High strength steel used in prestressed concrete can take how much maximum strain?

(a) 2%
(b) 3%
(c) 4%
(d) 6%

40. In pre-tensioned beams, which of the following losses is/are not considered?

1. Anchor loss
2. Shrinkage
3. Creep
4. Relaxation
5. Friction
6. Elastic shortening

Select the correct answer using the codes given below:

(a) 1, 2 and 3 only
(b) 4, 5 and 6 only
(c) 5 only
(d) 6 only

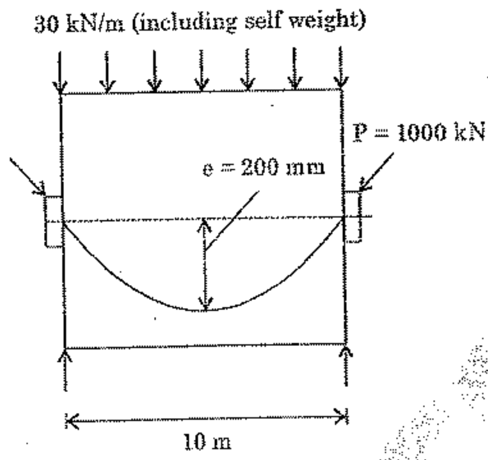
41. What is a tendon profile, in which the eccentricity is proportional to the bending moment caused by any loading on a rigidly supported indeterminate structure, at all cross-sections?

(a) Cable profile
(b) Resultant profile
(c) Concordant profile
(d) Reduced profile

42. Fully prestressed concrete beams

(a) resist all the working loads by prestress
(b) resist the full live load by prestress
(c) resist the part of the load by prestress
(d) resist only the dead loads

43. What is the net downward load to be considered for the analysis of the prestressed concrete beam provided with a parabolic able as shown in the figure below?



(a) 12 kN/m
(b) 13 kN/m
(c) 14 kN/m
(d) 15 kN/m

44. If a simply supported concrete beam, prestressed with a force of 2500 kN, is designed by load balancing concept for an effective span of 10 m and to carry a total load of 40 kN/m, the central dip of the cable profile should be

(a) 100 mm
(b) 200 mm
(c) 300 mm
(d) 400 mm

45. **IS:1343-1980** limits the minimum characteristic strength of pre-stressed concrete for post tensioned work and pretension work as

(a) 25 MPa, 30 MPa respectively
(b) 25 MPa, 35 MPa respectively
(c) 30 MPa, 35 MPa, respectively
(d) 30 MPa, 40 MPa, respectively

46. A concrete beam of rectangular cross-section of 200 mm × 400 mm is prestressed with a force 400 kN at eccentricity 100 mm. The maximum compressive stress in the concrete is

(a) 12.5 N/mm^2
(b) 7.5 N/mm^2
(c) 5.0 N/mm^2
(d) 2.5 N/mm^2

47. The percentage loss of prestress due to anchorage slip of 3 mm in a concrete beam of length 30 m which is post-tentioned by a tendon with an initial stress of 1200 N/mm^2 and modulus of elasticity equal to 2.1×10^5 N/mm^2 is

(a) 0.0175
(b) 0.175
(c) 1.75
(d) 17.5

48. A concrete beam of rectangular cross-section of size 120 mm (width) and 200 mm (depth) is prestressed by a straight tendon to an effective force of 150 kN at an eccentricity of 20 mm (below the centroidal axis in the depth direction). The stresses at the top and bottom fibres of the section are

(a) 2.5 N/mm^2 (compression), 10 N/mm^2 (compression)
(b) 10 N/mm^2 (tension), 2.5 N/mm^2 (compression)
(c) 3.75 N/mm^2 (tension), 3.75 N/mm^2 (compression)
(d) 2.75 N/mm^2 (compression), 3.75 N/mm^2 (compression)

49. A rectanguar concrete beam of width 120 mm and depth 200 mm is prestressed by pretensioning to a force of 150 kN at an eccentricity of 20 mm. The cross-sectional area of the prestressing steel is 87.5 mm^2. Take modulus of elasticity of steel and concrete as 2.1×10^5 MPa and 3.0×10^4 MPa respectively. The percentage loss of stress in the prestressing steel due to elastic deformation of concrete is

(a) 8.75 (b) 6.125
(c) 4.81 (d) 2.19

Answers

1. (b)	14. (b)	27. (a)	40. (c)
2. (a)	15. (d)	28. (b)	41. (c)
3. (d)	16. (c)	29. (c)	42. (c)
4. (a)	17. (d)	30. (b)	43. (c)
5. (d)	18. (d)	31. (c)	44. (b)
6. (a)	19. (b)	32. (a)	45. (d)
7. (c)	20. (d)	33. (b)	46. (a)
8. (b)	21. (c)	34. (c)	47. (c)
9. (d)	22. (d)	35. (c)	48. (a)
10. (c)	23. (a)	36. (a)	49. (b)
11. (c)	24. (a)	37. (a)	
12. (b)	25. (a)	38. (d)	
13. (a)	26. (d)	39. (c)	

10

Concrete Technology

INTRODUCTION

Concrete a composite man-made material, is the most widely used building material in the construction industry. It consists of a rationally chosen mixture of binding material such as lime or cement, well grade fine and coarse aggregates, water and admixtures (to produce concrete with special properties).

- The good concrete are which fulfils two criteria :

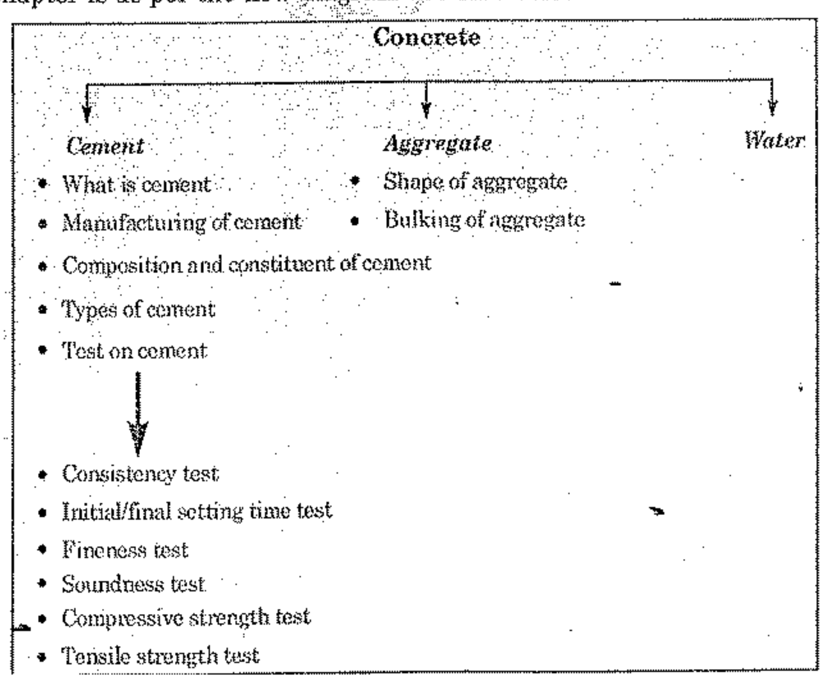
  (a) The concrete has to be satisfactory in its hardened state, and

  (b) Also in its fresh while being transported from the mixer and placed in the formawork.

- The requirements in the fresh state are that the consistency of the mix be such that it can be compacted by the means desired without excessive effort, and also that the mix be cohesive enough for the methods of transporting and placing used so as not produce segragation with a consequent lack of homogeneity of the finished product.

- The primary requirments of a good concrete in its hardened state are asatisfactory compressive strength and an adequate durability.

The flow of this chapter is as per the flow diagram describe below.



In this chapter we will discuss tbe constitutent material of concrete as discussed in above flow diagram of concrete one by one.

Cement: Cement is a material which has cohesive and adhesive properties in the presence of water.

Manufacturing Process

1. Portland cement is manufactured by grinding together 2 part of calcareous ($CaCO_3$) and one part of argillacious (clay and shals). (Dry process)
2. The mixture is burnt in a kiln at a temperature of about 1300°C to 1500°C where the material fuse and form a small Clinkers of 3 mm to 20 mm in size.
3. Then the clinker is cooled down using moderate cooling condition (It is observed that slow or quick cooling condition reduced the strength of the cement).
4. In moderate cooling condition temperature of clinker is brought down to 500°C in 15 min and in further 10 minute the temperature is brought, down to atmospheric.
5. Cooled, clinker is then mix with gypsum.
6. The mixture is ground to required fineness in ball mills to get the final product known as cement.
7. Gypsum is required to retard the setting time.

CHEMICAL COMPOSITION OF RAW MATERIALS

- The three constituents of hydraulic cements are lime, silica and alumina.
- In addition, most cements contain small proportions of iron oxide, magnesia, sulphur trioxide and alkalis.
- There has been a change in the composition of Portland cement over the years, mainly reflected in the increase in lime content and in a slight decrease in silica content.
- An increase in lime content beyond a certain value makes it difficult to combine completely with other compounds.
- Consequently, free lime will exist in the clinker and will result in an unsound cement. An increase in silica content at the expense of alumina and ferric oxide makes the cement difficult to fuse and form clinker.
- The approximate limits of chemical composition in cement are given in table.

Constituents of Potland Cement (Raw Material)

Table 10.1

Oxide	*Function*	*Composition (%)*
CaO	Controls strength and soundness. Its deficiency reduces strength and setting time	60-65
SiO_2	Gives strength. Excess of it causes slow setting.	17-25
Al_2O_3	Responsible for quick setting, if in excess, it lowers the strength.	3-8
Fe_2O_3	Gives colour and helps in fusion of different ingredients.	0.5-6
MgO	Imparts colour and hardness. If in excess, it causes cracks in mortar and concrete and unsoundness.	0.5-4
$Na_2O + K_2O$	These are residues, and if in excess cause efflorescence and cracking.	0.5-1.3
TiO_2		0.1-0.4
P_2O_5		0.1-0.2
SO_3	Makes cement sound.	1-2

Notes:

1. The rate of setting of cement paste is controlled by regulating the ratio $SiO_2/(Al_2O_3 + Fe_2O_3)$.
2. Where development of much heat of hydration is undesirable, the silica content is increased to about 21 per cent, and the alumina and iron oxide contents are limited to 6 per cent each.
3. Resistance to the action of sulphate waters is increased by raising further the silica content to 24 per cent and reducing the alumina and iron contents in 4 per cent each.
4. The variation in composition depends largely on the ratio of CaO to SiO_2 in the raw materials.

When this raw material put in kiln then it fuse and following compounds are formed and they are known as Bogue compound.

Table 10.2

The principal mineral compounds in Portland cement	*Formula*	*Name*	*Symbol*
1. Tricalcium silicate	$3CaO.SiO_2$	Alite	C_3S
2. Dicalcium silicate	$2CaO.SiO_2$	Belite	C_2S
3. Tricalcium aluminate	$3CaO.Al_2O_3$	Celite	C_3A
4. Tetracalcium alumino ferrite	$4CaO.Al_2.Fe_2O_3$	Felite	C_4AF

Tricalcium silicate is supposed to be the best cementing material and is well burnt cement.

- It is about 25-50% (normally about 40 per cent) of cement.
- It renders the clinker easier to grind, increases resistance to freezing and thawing, hydrates rapidly generating high heat and develops an early hardness and strength.
- However, raising of C_3S content beyond the specified limits increases the heat of hydration and solubility of cement in water.

Dicalcium silicate is about 25-40% (normally about 32 per cent) of cement.

- It hydrates and hardens slowly and takes long time to add to the strength (after a year or more).
- It imparts resistance to chemical attack.
- Raising of C_2S content renders clinker harder to grind, reduces early strength, decreases resistance to freezing and thawing at early ages and decreases heat of hydration.

Tricalcium aluminate is about 5-11% (normally about 10.5 per cent) of cement.

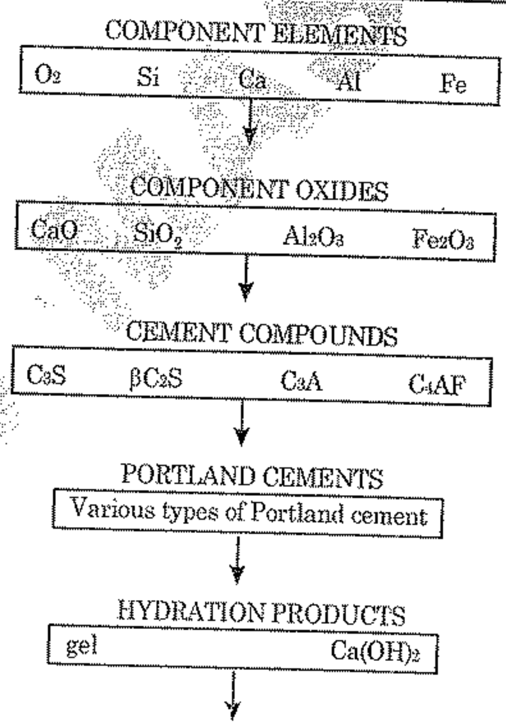
- It rapidly reacts with water and is responsible for flash set of finely grounded clinker.
- The rapidity of action is regulated by the addition of 2-3% of gypsum at the time of grinding cement.
- Tricalcium aluminate is responsible for the initial set, high heat of hydration and has greater tendency to volume changes causing cracking.
- Raising the C_3A content reduces the setting time, weakens resistance to sulphate attack and lowers the ultimate strength, heat of hydration and contraction during air hardening.

Tetracalcium alumino ferrite (C_4AF): It is comparatively inactive.

Hydration of cement: When water is added to cement. It produces a significant amount of heat. This is known as hydration and the liberated heat is called the heat of hydration. It depends upon.

1. Temperature at which hydration takes place. Higher the temperature rapid is the hydration.
2. Fineness of cement—Finner the cement rapid is the hydration.
3. Proportion of ingredients of cement—The reaction can be made rapid or slow by changing the proportions of the ingredients of the cement.

Note: For complete hydration w/c ratio needed is > 0.35 but < 0.45.



- *When water is added to cement and chemical reaction starts it is found that:*
  (a) C_3S and C_2S contribute most to the eventual strength.
  (b) Initial setting is due to tricalcium Aluminats (C_3A)

(c) C_3S hydrates quickly and contributes more to the early strength.

(d) C_2S strengthen the concrete from 7 days to 1 year.

(e) C_4AF is comparatively inactive.

Note: C_3A hydrates quickly generates much of heat and makes only a small contribution to the strength within the first 24 hr. C_3A generates maximum heat and is responsible for the most of the undesirable properties of concrete

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Cement having less C_3A will have higher ultimate St., less generationof heat and less cracking.

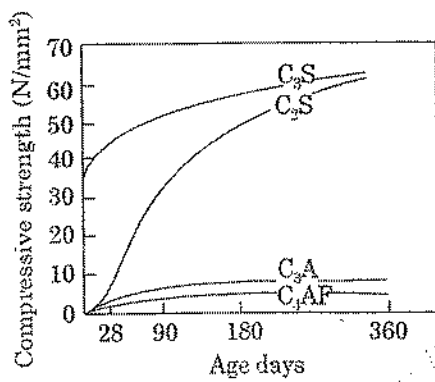


Fig. 10.1: Contribution of cement compounds to strength of cement.

WATER REQUIREMENT FOR HYDRATION

Water by weight of cement is required for complete hydration of Portland cement is combines chemically with the cement compounds and is known as *bound water.*

- Some quantity of water, about 15 per cent by weight of cement, is required to fill the cement gel pores and is known as *gel water.*
- Therefore, a total of 38 per cent of water by weight of cement is required to complete the chemical reaction.

TYPES OF CEMENT

1. **Ordinary Portland Cement:** It is a widely used cement for most of the work. It is suitable for general constuction works when there is no exposure to sulphates in the soil or in the ground water.
   - Three different grade of ordinary portland cement to produce high strength cement is
     33 grade of O.P.C.
     43 grade of O.P.C.
     53 grade of O.P.C.
2. **Rapid hardening Cement:** (Name it self shows that it hardened and attains its strength earlier than ordinary portand cement).
   - Rapid hardening cement is finer than ordinary portland cement and it contains more C_3S and less C_2S than the *OPC.*
   - 3 days strength of *RHC* is same of 7 days strength of *OPC.*
   - The main advantage of a rapid hardening cement is that shuttering may be removed much earlier, thus saving considerable time and expenses.

- Rapid hardening cement is also used for road work where it is necessary to open the road traffic with the minimum delay.

3. **Extra Rapid Hardening Cement:**

- In this cement 2% $CaCl_2$ is mixed with rapid hardening cement.
- This $CaCl_2$ imparts quick setting properties.
- While using this cement maximum time of 10 minute is available for mixing, transporting and placing the concrete and also this cement is used within one month.
- Extra rapid hardening cement used in cold weather because of large heat of evolution.

4. **Portland Blast Furnace Cement:**

- It is manufactured by mixing Portland cement clinker with granulated blast furnace slag (This slag is waste product obtained from blast furnace which contains oxide of lime, silica and alumina) and gypsum in suitable proportions and grinding the mixture to the required fineness.
- The proportion of the slag should not less than 25% and not more than 65% of the total mass of the mixture.
- It contains approximately 45% CaO and 35% silica.
- It is similar and cheaper than O.P.C. (and it can be replace it).
- It has low heat of hydration and is better resistant to soil and water containing excessive amounts of sulphates, alkalies.
- Because of its low heat evolution it can be used in mass concrete structure such as dams & foundations.

5. **Pozzolana Cement:**

- This is manufactured by grinding portland cement clinker with Pozzolana and required quantity of gypsum.

  (The Pozzolana are materials which at ordinary temperature react with lime in presence of water, resulting in cementing materials). Fly ash, burnt clay are used as Pozzolana.
- The proportion of Pozzolana may be 10-25% by weight of Pozzolana cement.
- This cement has higher resistance to chemical agencies and to sea water because of absence of lime.
- Advantage of this cement are reduction in cost, increased impermeability.

6. **High Alumina Cement:**

- It is non portland cement, it is manufactured by Melting mixture of aluminous and Calcareous Materials in Suitable proportion and grinding the resulting clinker to fine powder (which is black in colour).
- The raw materials used for its manufacturing are chalk and bauxide which is a special clay of extremely high alumina content.
- It is used where early removal of form work is required.
- Its rapid hardening properties arise from the presence of Calcium aluminate like. Calcium silicates in Portland cement.
- Its one day strength is equal to 28 days strength of ordinary Portland cement.
- It is recommended for sea and under water work.

Table 10.3: Composition of a typical high alumina cement

Composition	*Percentage*
Al_2O_3, TiO_2	43.5
Fe_2O_3, FeO, Fe_3O_4	13.1
CaO	37.5
SiO_2	3.8
MgO	0.3
SO_3	0.4
Insoluble material	1.2
Loss on ignition	0.2

7. Low Heat Portland Cement:

- It is Manufactured by reducing the % of C_3S and C_3A of ordinary Portland cement, because of this cement gets the strength at a slower rate and the heat of hydration is less. This will require long time curing and keeping forms for a long time.
- It is used in Mass Concrete Works, such as dams and retaining walls. (For mass construction work when OPC is used temperature may rises at its heightest level when time passes outer layer will cool and contract while the inner mas is still at a higher temp. This may leads to serious cracking).
- Heat generated in OPC at the end of 3 days = 80 cal/gm. While in low heat cement it is about 50 cal/gm of cement.

8. Sulphate resisting cement:

- It is similar to ordinary Portland cement except that it contains more silicates and less quantity of aluminates.
- It is used for under water structure particularly exposed to alkali actions.
- Soluble Sulphates like $MgSO_4$, $CaSO_4$ and Na_2SO_4 when present in ground reacts with cement and form *Sulpho-aluminates* which have expansive properties and so causes disintegration of concrete (Due to this nature it expand and this larger volume will create pressure on concrete which will result in cracks and finally disintegration of concrete).
- It is strongly recommended for structure in sea water, coastal areas and Marshy lands.
- This used in using of canal.

9. Super Sulphated Cement:

- It is manufactured by grinding together a mixture of 80-85% of granulated blast furnace slag and 10-15% of calcium sulphate and about 5% of portland cement clinker.
- The cement is highly resistant to sulphate attack.
- Because of low heat of evolution it is usefull for mass concrete works. It is also usefull for foundation works where aggressive chemical conditions exist.

10. Quick Setting Portland Cement

The quantity of gypsum is reduced and small percentage of aluminium sulphate is added. it is ground much finer than ordinary Portland cement.

Properties: Initial setting time = 5 minutes
Final setting time = 30 minutes

Use: It is used when concrete is to be laid under water or in running water.

TEST ON CEMENT

Consistency Test

This test determines the quantity of water required to produce a cement paste of standard consistency for the use in other test. Vicat apparatus used for this purpose.

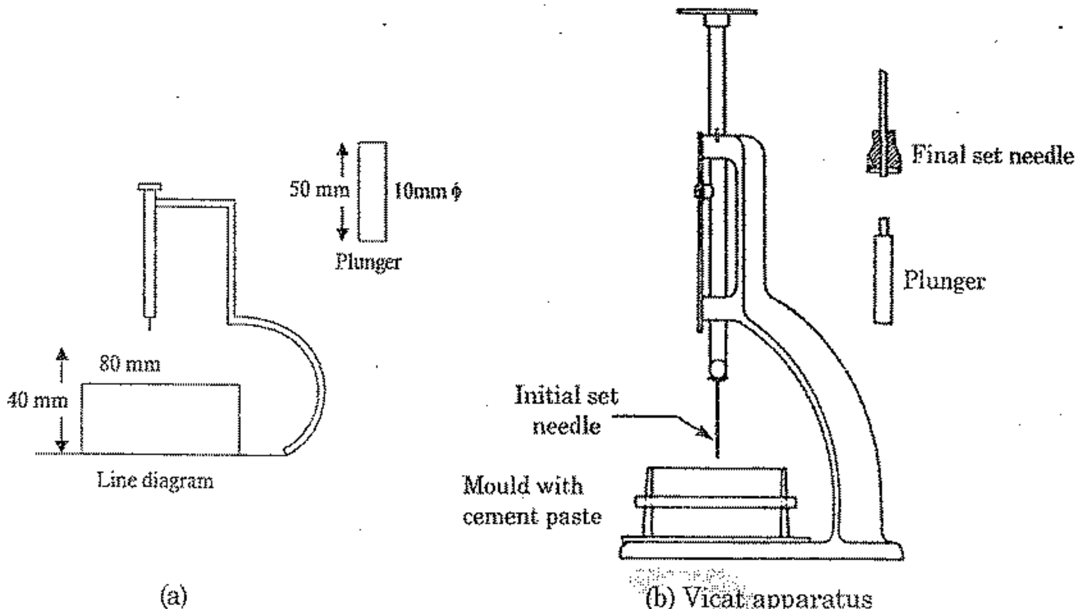


Fig. 10.2

The consistency of cement pastes is defined as that consistency which will permit the vicat plunger 50 mm long and having 10 mm dia to penetrate to a point 5 mm-7 mm from the bottom of the vicat mould.

Procedure

- Take 400 gm cement and add to it 30% water on a glass plate.
- Mix throughly and fill the mould of vicat apparatus.
- The interval from the time of adding water to the dry cement until commencing to fill the mould is known as time of gauging and must be not less than 3 minute and not more than 5 minute.
- Lower the plunger gently to touch the surface of the test block and quickly release it, allowing it to sink into the paste and note down the settlement of the Plunger.
- The settlement of the plunger should be 5 mm-7mm from the bottom of the mould. If not repeat the procedure using fresh cement and other percentage of water until the described penetration of the plunger is obtained.
- Consistency of Cement paste is expressed as the amount of water as a percentage by mass of the dry cement denoted by p.

$$p = \frac{m_2}{m_1} \times 100 = \text{\% of water or standard consistency}$$

m_1 = mass of the cement taken

m_2 = mass of water added when the plunger has a penetration of 5 mm to 7 mm from the bottom of the mould.

TEST FOR SETTING TIME

Objective

- When water is added to cement and mixed properly, the chemical reaction soon starts and the cement paste remains plastic for a short period. During this period it is possible to remix the paste and this period is called initial setting time.
- It is assumed that no hardening will start in this period. As the time passes, the reaction is continued and cement begins to harding and time elapse at the times of mixing water to hardened is known as final setting time.

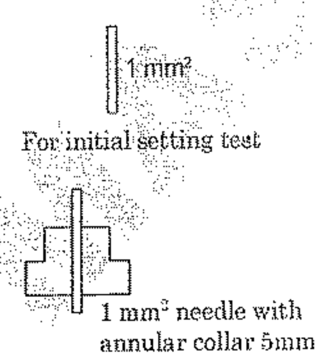
Why Initial and Final Setting Test is done?

Initial setting time test in done because:

- Concrete once placed should not be disturbed till the initial setting has taken place.
- There must be sufficient time for placing of second batch otherwise this may disturb the first batch of concreting.
- The transportation of concrete from the place where concrete is prepared to the placing of concrete required some finite time for that also initial setting time test is required.
- Final setting time test is done because the concrete should achieve the desired strength as early as possible so that the shuttering can be removed and reused.

Procedure

Needle used for setting time



For initial setting test

For final setting test

Fig. 10.3

- Mix 400 gm of cement with 0.85p percentage of water where p is the consistency of standard cement paste.
- Start the stop watch at the instant when water is added to cement.
- Fill the vicat mould with this paste and smooth off the surface of the paste making it level with the top of mould.
- Attach 1 mm × 1 mm square cross-section needle to the vicat rod.
- Lower the needle gently near the surface of the block and release it quickly allowing it to penetrate into the test block.

- Note whether the needle pierces completely, if so wait for a while and drop the needle at a fresh place.
- Repeat the procedure till the needle fails to pierce the block for 5 ± 0.5 mm measured from the bottom of the mould.
- The period elasping between the time when water is added to the cement and the time at which the needle fails to pierce the test by 5 ± 0.5 mm is taken as initial setting time.
- The test temp. is 27% ± 2°C.

Final Setting Times

- Replace the needle (1 mm^2) by the needle of 1 mm^2 c/s with 5 mm dia annular collar and release it on the same test block as done in initial setting time.
- Note the time when needle makes an impression, but the attachment fails to do so.
- The time elasping between the time when water is added to cement and the time at which needle makes an-impression but the attachment fails to do so is taken of the final setting time.

Note: (1) Initial setting time for *OPC* $\nless$ 30 min.

Final setting times for *OPC* $\ngtr$ 10 hr. (600 min).

(2) Initial setting times for quick setting cement = 5 min and for final setting time = 30 min.

Flash Set

As soon as the water is added to cement, the reaction between cement and water immediately starts if gypsum is not added while grinding of clinker. This is called flash setting. However, this is not acceptable since mixing the ingredients and placing the concrete to the requires some time.

False Set

- False set is the name given to the abnormal premature stiffening of cement within a few minutes of mixing with water.
- It differs from flash set in that no appreciable heat is evolved, and remixing the cement paste without addition of water restores plasticity of the paste unitl it sets in the normal manner and without a loss of strength.
- Some of the causes of false set are to be found in the dehydration of gypsum when integround with too hot a clinker : hemihydrate ($CaSO_4 \frac{1}{4} H_2O$) or anhydrite ($CaSO_4$) are formed and when the cement is mixed with water these hydrate to form needle shaped crystals of gypsum. Thus what is called 'plaster set' takes place with a resulting stiffening of the paste.

Note: 1. The change from fluid to rigid state of a cement paste is termed as setting of cement during this cement acquires some straingth

2. The gain of strength of set cement paste is known as hardening of cement.
3. The setting of cement is purely a conventional one. It is not related to the setting and hardening of actual concrete.

Soundnes Test

- Soundness is the ability of cement to maintain a constant volume. The presence of excess gypsum or free lime and magnisia in the cement shall be limited as these undergo a large change in volume.

- Concrete once placed should not undergo the large change in volume as this produces cracks in the concrete.
- If the cement shows expansion after hardening, it is called unsound cement which may cause cracks and destruction of structure.

  (The hydration of free lime is accelerated by boiling. Which causes the expansion of cement). It is this expansion of lime which is one of the cause of cracking of cement concrete.

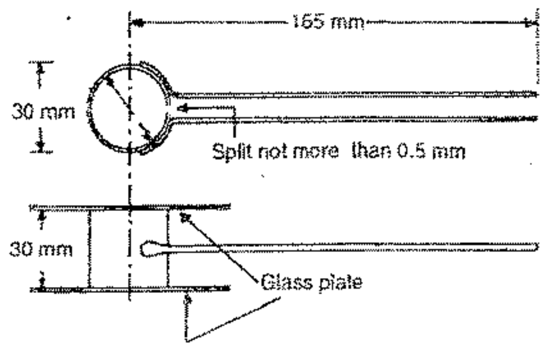


Fig. 10.4: Le-chatelier apparatus.

Procedure

- Take about 100 g of cement.
- Mix it with 0.78 p% of water where p is the consistency of standered cement paste.
- Gauge the paste thoroughly.
- The mould is placed on a glassplate, and filled with the cement paste.
- The mould is covered with another piece of glass sheet and a small weight is placed over the glass sheet.
- The whole assembly is immediately place in water bath having a temperature of 27° + 0°C and kept there for 24 hours.
- The assembly is taken out after 24 hr. and the distance separating the indicator point is measured.
- Mould is then immersed in a water bath.
- The water of the bath is brought to boiling point with the mould submerged in 25-30 min and kept boiling for 3 hr.
- Mould is taken out from water and allowed to cool.
- Again distance between the points is measured.
- The difference between the two measurement represents the expansion of cement and that $\ngtr$ 10 mm.

Note: Autoclave expansion test is done to find expansion due to Magnesia and this $\ngtr$ 0.8%.

Compressive Strength Test

The objective of this test is to determine the strength of cement as represented by compressive strength test on mortar cubes.

- Prepare a mixture of cement and standard (Ennore) Sand in proportion of 1:3 by mass (generally 200 gm cement and 600 gm sand)
- Place it on a non porous plate and mix it dry with trowel for one minute.

- Add water equal to $\left(\frac{p}{4}+3\right)$% of combined mass of cement and sand where p is the percentage of water required to produce a paste of standard consistency.
- Place the mortar in a cubical mould having 50 cm^2 surface area (this is equal to 70.6 × 70.6 × 70.6 mm cube) in two equal layer. Each layer shall be tamped 20 times in about 8 second to ensure the elimination of entrapped air.
- A number of cubes in the similar manner are cast.
- Place the mould in a damp condition for 24 hr.
- Remove the specimen from moulds and immerse them in water.
- The cube shall be tested at 3 days. 7 day and 28 days.
- A minimum of three specimens shall be tested on their sides, the load being applied at the rate of 3.5 N/mm^2/minute.
- The mould shall be placed in testing machine such that the testing load is applied on finished surface of the cube.
- For O.P.C. of grade 33 the crushing strength

  $\nless$ 16 N/mm^2 at 3 day

  $\nless$ 22 N/mm^2 at 7 day

  $\nless$ 33 N/mm^2 of 28 days.
- Temperature of test is 27° ± °C and Relative humidity 90% for 24 hr.

Tensile Strength Test

Object of this test is to determine the tensile strength of cement mortar.

- Prepare a mixture of cement and standard sand inproportion 1:3 by mass (generally 250 gm cement and 750 gm sand).
- Place it on a non porous plates and mix it dry with trawel for one min.
- Add water equal to $\left(\frac{p}{5}+2.5\right)$% of combined mass of cement and sand when p is the % of water required to produce a paste of standard consistency.
- This motar is placed in Briquettes of cross-sectional area of 6.45 cm^2 and mortar is tested.
- Twelve specimens are prepared; Six of which are tested after 3 days of curing and the other six are tested after 7 days.
- The Tensile strength at the end of 3 days $\nless$ 20 kg/cm^2

  7 days at $\nless$ 25 kg/cm^2
- Test in done on 90% humidity and 27° ± 1°C.

Note: Tensile strength of cement mortar = 10 – 15% of compressive strength of mortar.

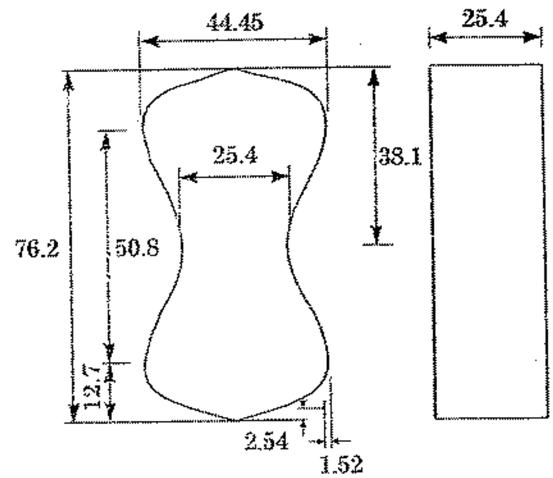


Fig. 10.5: Dimensions of standard Briquette.

Finenes

- The objetive of this test is to check the proper grinding of cement the rate of hydration of cement depends upon its fineness of particles.
- Finenes of cement paste is calculated using **air permeability** method.
- Fineness of cement is measured in terms of its specific surface.

Cement types	Specific surface area ≮
OPC	2250 cm^2/gm
R.H.C.	3250 cm^2/gm
Low heat	3200 cm^2/gm

Note: 1. Too fine cement is susceptible to air set and deteriorates earlier. So too fine cement is not good for use.

2. If cement is not uniformly fine, the concrete made out of it will have poor workability and will require a large quantity of water for mixing. Also bleeding (flowing of water above surface) of concrete can occur.

ADMIXTURE

Suitable materials known as admixtures which may be added to concrete mix, Just before or during the mixing to modify one or more properties of the concrete in the plastic or hardened states as desired.

The objective of admixture are:

- To increase the rate of strength development at early ages.
- To retard the initial setting times.
- To increase the workability without changing the water content
- To increase the resistance to freezing and thawing, vinsol resins an air entertainment admixture which is used for this purpose which cause air to be incorporated in the form of minute tiny bubbles in concrete during mixing to increase the workability and resistance to freezing and thawing.

DIFFERENT TYPE OF ADMIXTURE

(1) Mineral Admixtures

(a) Fly ash

- This increase impermeability because it is finer than the cement particals.
  This is by product of wooden thermal power plant and is produced daily in large quantities.
- These require very less water to wet their surface.
- Same work which is obtained by OPC is achieved by lower water/cement ratio.

(2) Chemical Admixtures

(a) Accelerating Admixture

A substance which increases the rate of strength development or reduce the setting time. $CaCl_2$, Sodium silicate, Sodium chloride, Calcium nitrite and calcium nitrate are used as accelerator.

- Berium chloride acts as an acclerator only under warm conditions
- Most frequently used accelerator is calcium chloride ($CaCl_2$)

(b) Retarding Admixture

A substance which delays the setting time of cement paste. Gypsum, Tartaric acid, sugar are used as retarding admixture.

- A small quantity of sugar about 0.05 per cent of the mass of cement will act as an acceptable retarder: the delay in seting of concrete is about 4 hours.
- A large quantity of sugar, say 0.2 to 1 percent of the mass of cement, will virtually prevent the setting of cement.
- When sugar is used as a controlled set retarder, the early strength of concrete is severely reduced but beyound about 7 days, there is an increase in strength of several percent compared with a non-retarded mix. This is probably due to the fact that delayed setting produces a denser hydrated cement gel.

Note : Retarders tend to increase the plastic shrinkage because the duration of the plastic stage is extended, but drying shrinkage is not affected.

(c) Water Reducing Admixture

A substance which either increases workability of freshly mixed mortar or concrete without increasing water-cement ratio or maintains workability with reduced water-cement ratio. These are:

- Lignosulphonic acid and its salts.
- Formaldehyde derivatives
- Hydroxylated carboxylic acids
- Calcium lignosulphonate

(d) Air-entraining Admixture

A substance which causes air to be entrapped in the form of tiny bubbles in motor or concrete, during mixing to increasing its workability and resistance to freezing and thawing. Example: Vinsol resin, using of Aluminium powder.

(e) Super Plasticizing Admixture

- A substance which imports very high workability with a large decrease in water content (at least 20%) for a given workability.

  The resulting improvement in workability can be exploited in two ways. by

  (a) By producing concrete with a very high workability or concrete with a very high strength

  (b) Concrete of normal workability but with an externely high strength owing to a very substantial reduction in the water/cement ratio.

- A high range water reducing admixture is also reffered to as a superplasticizer and these are sulfonated melamine formaldehyde condensates; sulfonated naphthalene-formaldehyde condensaties; modified lignosulfonates; and others such as sulfonic-acid esters and carbohydrate esters.
- The role of superplasticizer it to disperse, the particles remove air bubbles, example: Sulphonated melanin formaldehyde and to retard setting.

Note: The admixture essentially work as surface active agent, reduce surface tension of water and disperse cement particles more effectively thus lowering the water requirement.

AGGREGATES

The aggregates consists of about 75% of volume of concrete and they greatly influence the properties of concrete.

- Aggregate give body to the concrete reduce the shrinkage effect of cement and make the concrete durable.
- Generally rounded shape aggregate is used because shape of aggregate affects the workability of concrete.
- To achieve the best possible strength, concrete should be as dense as posible *i.e.* it should contain minimum void. Voids are greatly influenced by the shape of aggregate.
- The rounded particles can be packed to produce a concrete with 33% void means 67% of the volume of concrete is occupied by the aggregate.
- The Rounded particly produce smoother mix for a given water/cement ratio.
- On the other hand, angular or flaky particles reduce the workability and demand more cement and water to give the specified strength of concrete mix.
- Not more than 10-15% of flaky particles should be used in concrete.

Bulking of Sand

Due to the presence of moisture content, aggregates bulk in volume. The moisture particles form a thin film around the aggregates and exert surface tension. This keeps the particles away from each other and thus aggregates bulk in volume.

- This bulking in volume generally negligible in the case of coarse aggregates. It has great importance in case of fine aggregates or sand.
- For sand the volume goes on increasing until the moisture content is about 8% by the mass of sand.
- Bulking increases with fineness of the aggregates.
- The Bulking of sand may be as large as 30-40%.

- With further addition of moisture content the thin film of water coated round the sand start disappearing and the volume of sand begins to decrease till finally at 25 - 30% of moisture content. The volume of sand returns to its original volume when it is dry.

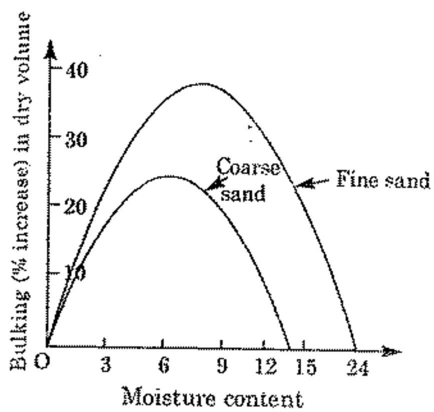


Fig. 10.5

Fineness Modulus

In order to ensure the presence of all sizes of particles, the property of aggregate called fineness modulus is defined. The fineness modulus of an aggregate is an index number which is roughly proportional to the average size of the particles in the aggregate.

Fineness Modulus for Different Type of Sand

Fine sand = 2.2 – 2.6

Medium sand = 2.6 – 2.9

Coarse sand = 2.9 – 3.2

A sand having fineness modulus more than 3.2 will be unsuitable for making satisfactory concrete.

The fineness modulus is the sum divided by 100 of the commulative percentage mas which is retained on each of the ten Sieves specified by I.S. code.

Sieves are 150 mμ, 300 mμ, 600 mμ, 1.18 mm, 2.36 mm, 4.75 mm, 10 mm, 20 mm, 40 mm, 80 mm and larger is required increasing in the ratio of 2 : 1.

Note : Fineness modulus of 4.00 can be interpreted to mean that the fourth sieve, i.e. the average size of particle is 1.18 mm.

Water

Water is required for proper chemical action and amount of water required is about (25 – 35%) of the weight of cement used. water/cement = 0.4

PERMISIBLE LIMITS FOR IMPURITIES IN WATER

Impurity	Permissible Limits
Organic	200 mg/l
Inorganic	3000 mg/l
Sulphates	400 mg/l
Chlorides	2000 mg/l for plain concrete work
	500 mg/l for reinforced concrete work
Suspended matter	2000 mg/l

CONCRETE

Concrete is a carefully proportioned mixture of cement, fine aggregate, coarse aggregate and water. Sometimes to modify the physical properties of concrete a variety of admixture may be added. The preparation of concrete consists of the following operations:

1. Proportioning of ingredients
2. Measurement of materials
3. Mixing and placing of concrete
4. Compaction
5. Curing.

(1) Proportioning of Ingredient

Proportioning of ingredients means determining the relative amounts of ingredients (Cement, FA, CA to get the required strength of concrete.

This can be done by two way:

(i) Design Mix.

(ii) Nominal Mix.

(i) **Design Mix:** In design mix the proportion of ingredients of concrete to obtain a desired strength can be foundout by laboratory method > M 20 design mix.

(ii) **Nominal Mix:** In this method proportion of ingradients of concrete can be choose on the arbitrary method M 5, M 7.5, M 10 and M 20 are the nominal mix.

METHODS OF PROPORTIONING CONCRETE

(1) Maximum Density Method

It is used as a theoretical approach to determine the grading of aggregate to obtained maximum density. It is given by Fuller's

$$P_d = 100\sqrt{\frac{d}{D}}$$

d = Maximum size of fine aggregate.

D = Maximum size of coarse aggregate.

P_d = % by weight of particle size finer than d in total mixture.

Example 1

Maximum size of fine aggregate = 20 mm

Maxi. size of C.A. = 40 mm

Then the % of particles sizes smaller than 20 mm is?

Sol:

$$P_{20} = 100\sqrt{\frac{20}{40}} = 70.71\%$$

Fineness Modulus Method: It is given by

$$P_m = \left(\frac{A-C}{A-B}\right) \times 100$$

P_m = Proportion of fine aggregate to mixed aggregate.

A = Fineness modulus of the C.A.

B = Fineness modulus of the F.A.

C = Mix of coarse and fine aggregate

(2) Measurement of Materials

(i) ***Mass batching:*** 1 bag of cement contains 50 kg mass of cement.

(ii) ***Volume batching:*** Volume of 50 kg cement shall be taken of 34.5 lit. For volume batching boxes of size 300 × 300 × 380 mm are used. Size is in cubical shape and equal to 50 kg cement.

- Generally mass batching is prefered to avoid bulking of sand while carrying volume batching.

(3) Mixing and Placing of Concrete

The ingredients of concrete should be thoroughly mixed such that the cement paste is coated to the surface of all aggregates and a uniform mass is obtained. This can be done by

- The mixed concrete should be transported to the place of laying as early as possible.
- During transport care should be taken to see that seggregation does not take place and the concrete should be placed before its starts setting.
- If seggregation does occur during unloading, the concrete should be remixed before placing.

(4) Compaction

After concrete is placed at the desired location, the next step in the process of concrete production is its compaction. Compaction consolidates fresh concrete within the moulds or frameworks and around embedded parts and reinforcment steel. Compaction of the concrete is the process to get rid of the entrapped air and voids.

- Compaction is extremely important as 5% of void can give a loss of 30% of strength, 10% of void can give a loss of 60% in strength, 25% of voids can give a loss of 90% in strength.

- Other properties of concrete like durability, impermeability etc. also greatly depends on the compaction of concrete

Compaction may be done by
- Hand tool
- Vibrator

Hand Tool

- This method of compaction is used for small and unimportant jobs. However, this method is extremely useful for thin elements such as slabs, and for members with congested reinforcements.
- This method can be used for mixes with any workablity except for very fluid or very plastic mix. Hand compaction is achieved by rodding ramming, or temping.

Compaction by Vibration

- This is the most common and widely used method of compacting concrete for any structural element.
- The vibrations imparted to the fresh concrete reduce the internal friction between the particles of concrete by setting the particles in motion and thus produce a dense and compact mass.
- On vibration, the concrete mix gets fluidize and the internal friction between the aggregate particles reduces, resulting in entrapped air to rise to the surface.
- Onlosing entrapped air the concrete gets denser.
- The various types various types of vibrators in use are needle, formwork, table or platform, and surface vibrators.

Needle Vibrator

These are also knows as immersion, internal, or poker vibrator. Needle vibrator can be used for any type of concrete work.

Formwork Vibrator

These are also known as external or shutter vibrator. These are generally used under the following circumstances:

- Compaction of concrete is required to be done in a very thin or very densely congested reinforced section.
- In addition to internal vibration, compaction is required to be done specially in the cover area where at times needle or pocker vibrator is unable to do satisfactory compaction.
- Compaction of very stiff concrete is required to be done because such concrete cannot be compacted by internal vibrator.

Note: Fromwork vibrators are used for concreting columns, thin walls and precast units. These are rigidly clamped to the formwork, causing it to vibrate and consequently transfer the vibrations to concrete.

Surface Vibrator

- These are also known as screed board vibrators. Surface vibrators are used for floor and roof slabs and pavement surfaces.
- These are effective only up to a thickness of 150 mm of concrete but can be used up to 250 mm.
- Surface vibrators cause movement of fine particles to the top and hence aid the finishing operation. The operating frequency is 4,000 cycles per minute.

Note: Compaction of concrete through vibrator is very imp. however over-vibration makes the concrete non-homogeneous.

(5) Curing

The proces by which the loss of water from concrete is prevented is known as curing.

OR

The process by which keeping the concrete surface wet is known as curing.

Different method of curing:

(*i*) Moist curing

(*ii*) Membrane curing

(*iii*) Steam curing

(1) Moist Curing

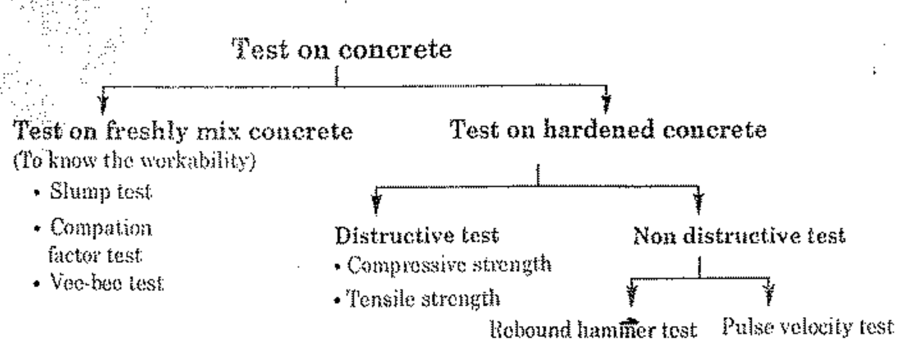
- Explosed surface of concrete shall be kept continuously in a damp or wet condition for at least seven days from the date of placing of concrete.
- 7 days in minimum curing period however longer curing is beneficial.

(2) Membrane Curing

- In this concrete may be coated with approved curing compounds.
- The compounds should be applied to all exposed surfaces of concrete as soon as possible ofter the concrte has set.
- This method is used where water is in short supply or is hot weather conditions.

(3) Steam Curing

- It is done under atmospheric pressure or under pressure.
- Steam curing under pressure is very costly as it requires pressure chambers.
- Half an hours strength of concrete Steam cured under pressure, can be equal to 28 days strength of similar concrete.
- Steam curing reduces the bond strength of concrete and shall be used only in exceptional cases.
- Steam curing is used only for achieving 50 – 75% of the strength in a short period.
- High pressure steam curing is also referred to as autoclaving.
- Stream curing should be applied to concretes made with Portland cement.
- Hitgh-alumina and supersulfated cements would be adversely affected by the high temperature.
- High pressure steam curing reduces efflorescence as there is no lime left to be leached out.



WORKABILITY

- Workability is the ease with which a concrete can be mixed, placed and compacted so that a dense concrete is obtained.
- In wide sence workability is difined as the amount of useful internal work necessary to produce full compaction.

FACTOR AFFECTING WORKABILITY

(a) **Water Content** : The fludity of concrete increases with water content. At site the normal practice is to increase the water content to make the concrete workable which lowers strength. In controlled concrete this cannot be resorted and even in uncontrolled concrete this should be the last choice. However, in case if more water is added due to any reason the cement content should be proportionately increased.

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(b) **Mix Proportions:** Aggregate-cement ratio influences the workability to a large extent. The higher the ratio leaner will be the concrete. In a lean concrete, paste available for lubrication of per unit surface area of aggregates will be less and hence the workability is reduced.

(c) **Size of Aggregate:** For big size aggregate the total surface area to be wetted is less, also less paste is required for lubricating the surface to reduce internal friction. For a given water content big size aggregate give high workability.

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(d) **Shape of Aggregate:** For a given water content, round and cubical shape aggregates are more workable than rough, angular or flaky aggregates, because the former type of aggregates requires less cement past for lubrication as these have less surface area and lesser voids. In case of round aggregates frictional resistance is also small so less lubrication is required. For this reason river sand and gravel provide greater workability than crushed sand and aggregates.

(e) **Surface Textures:** A rough surface aggregate will have more surface area than a smooth round textured aggregate. Hence, lagger will be more workable for the reasons discussed above.

:h of

(f) **Grading of Aggregates** : Properly graded aggregates are more workable. It is so because such a mix will have least voids and thus excess cement paste will be available as lubricant. This also prevents segregation.

(g) **Admixtures:** Air entrained concrete is more workable. It is so because air forms bubbles, on which the aggregates slide past each other increasing the workability. Another factor is that air entraining agents are surface active and they reduce the internal friction between the aggregates.

Seggregation

- Seggregation is said to occur when the constituent material of concrete try to separate out from eachother, producing concentration of Coarser Material at one place and finer material at other place in concrete. Such concrete. contains large voids and is less durable.
- Seggregation occurs due to poor grading of aggregates (*i.e.* large difference in sizes of particles), over vibration and dropping the concrete from above a certain height. Which should be avoided.

Bleeding of Concrete

- When the unreacted water in the mix tends to rise to the surface of fresly placed concrete is known as Bleeding of concrete.

- Due to bleeding of concrete continuous capillary pores are formed which provides a clear and straight access to chemicals and deleterious materials in concrete and lower the strength of concrete.

Assesment of Workability

There are numbers of test is available to find the workability, some important test are

(*i*) Slump test
(*ii*) Compaction factor test
(*iii*) Vce-Bee Consistometer text.

Slump Test

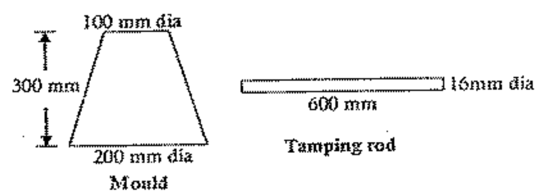


Fig. 10.16

Objective

- Slump test is used to fined the consistency of concrete. Which can be done either in laboratory or at sites.
- Slump test is not suitable method for very wet or very dry and stiff mix concrete

Procedure

- The mould is placed on a smooth, Hz. rigid and non-absorbant surface.
- The mould is then filled in four layers, each approximately 1/4 of the height of the mould.
- Each layer is tamped 25 times by the tamping rod taking care to distribute the strokes evenly over the cross-section.
- The mould is removed from the concrete immediately by raising it slowly and carefully in VL direction.
- This allow the concrete to subside due to lack of support.
- This subsidence is referred as slump of concrete
- The difference in level between the height of the mould and that of the highest point of the subsided concrete is measured.
- The difference in height in mm is taken as slump of concrete.

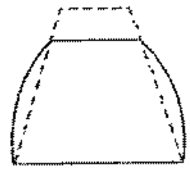
  Slump is usually 25 – 75 mm for low workability.

  50 – 100 mm for medium work.

  100 – 150 high workability.
- For Reinforced concrete medium workability is used.

Typical Slump Patterns are:

1. **True Slump:** In this case concrete. Subsided evenly

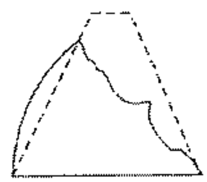


True slump

(a)

2. **Shear slump:**

- In this one half of the concrete slides down. In this shear slump is measured as a difference in ht. b/w the height of the mould and average value of subsidence.

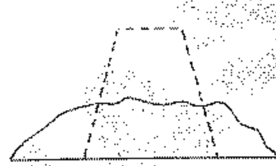


Shear slump

(b)

- If the sample shows shear slump experiment shall be repeated and if in the repeated test also shows shear slump, the slump should be measured and recorded as a shear slump.
- Shear slump shows poor grading of aggregate.

3. **Collapse**



(c) This shows leaner mix.

Fig. 10.7

Note: 1. Slump test is not a true guide of workability as the same slump can be obtained for diff. workabilities of concrete.
2. Too high or too low slump gives immediate warning.

COMPACTION FACTOR TEST

Objective

- The degree of workability of concrete is measured in terms of internal energy required to compact the concrete thoroughly.
- This test is developed at the Road Research Laboratory U.K. is more. Precise and sensitive than the slump test.
- It is generally useful for concrete. mixes of very low workability.

Procedure

- The sample of concrete to be tested is placed in the upper hopper up to the brim.
- Trap door is opened so that the concrete falls into the lower hopper.

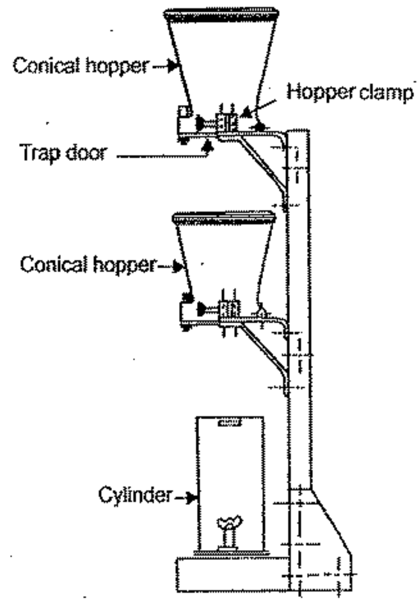


Fig. 10.8

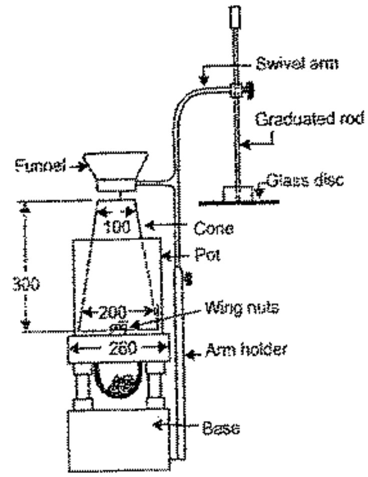
- Then the trap door of the lower hopper is opened and the concrete is allowed to fall into the cylinder.
- In the case of dry mix, it is likely that the concrete may not fall on opening the trap door. In such a case a slight poking by a rod may be required to set the concrete in motion.
- The excess concrete remaining above the top level of the cylinder is then cut off with the help of plane blades supplied with the apparatus.
- The outside of the cylinder is wiped clean.
- Concrete is filled up exactly up to the top level of the cylinder.
- It is weighed to the nearest 10 gm. and this wt is known weight of partially compacted concrete.
- The cylinder is emptied and then refilled with the concrete from the same sample in layers (5 cm)
- The layers are heavily formed so as to obtained full compaction.
- The weighed nearest to 10 gm and this weight is known of weight of fully compacted concrete.

$$CF = \frac{\text{Weight of partially compacted concrete}}{\text{Weight of fully compacted concrete}}$$

Vee-Bee Test

Objective

In this test the consistency of concrete is measured in terms of V.B. Second a Vee-Bee consistometers is used for this time.



Vee-Bee Consistometer

Fig. 10.9

- This consistometer consists of a vibrating table, metal pot, a standard iron rod, sheet metal.
- Placing the slump concrete inside the sheet metal cylindrical pot of the consistometer.
- The glass disc attached to the swivel arm is turned and placed on the top of the concrete in the pot.
- The electrical vibrator is then switched on and simultaneously a stop watch started.
- The vibration is continued till such a time as the conical shape of the concrete disappears and the concrete. assumes a cylindrical shape.
- Immediately when the concrete fully assumes a cylindrical shape the stop watch is switched off.
- The time required for the shape of concrete to change from slump concrete shape to cylindrical shape in second is known as vee bee Degree.
- It is useful for the very dry concrete whose slump value can not be measured by slump test.

Degree of Workability	*Consistency*	*Slump (mm)*	*Compacting factor*	*Vee-Bee degree (sec)*	*Characteristics*	*Uses*
Extremely low	Moist earth	0	0.65–0.7	>20	Particles of coarse aggregate in the concrete are adhesive, but concrete does not clot, risk of segregation.	Precast paving sbs
Very low	Very dry	0–25	0.7–0.8	12–20	Concrete has the consistency of very stiff porridge, forms a stiff mound when dumped, and barely tends to shake or roll itself to form an almost horizontal surface when conveyed for a long time in, say, a wheel-barrow.	Roads (power vibrator)
Low	Dry	25–50	0.8–0.85	6–12	Concrete has the consistency of stiff porridge, forms a mound when dumped, and shakes or rolls itself to form a horizontal surface when conveyed for a long time in, say, a wheel-barrow.	Mass concreting light reinforced section, roads (hand vibrator)
Medium	Plastic	50–100	0.85–0.95	3–6	Concrete can be shaped into a ball between the palms of the hands, and adheres to the skin.	Flat slabs, heavily reinforced section, RCC sections (manual vibrator)
High	Semi-fluid	100–175	0.95–1	0–3	Concrete cannot be rolled into a ball between the plams of the hands, but spreads out even though slowly and without affecting the cohesion of the constituents so that segregation does not occur.	RCC with congested reinforcement (can not be vibrated)

TEST ON HARDENED CONCRETE

Compressive Test

- Cement, fine aggregate and coarse aggregate (up to 38mm) to be used for making concrete are brough to room temperature (preferably 27±3°C before commencing the test.
- The ingredients are weighed in the ratio to be used in the field and are mixed by hand mixing or by machine mixing.
- First, the cement and sand are mixed throughly till uniform colour is achived.
- The coarse aggregate are then added and mixed till these are distributed uniformly throughout the mix.
- The water is then added and the entire batch mixed untill the concrete appears to be homogeneous and has the desired consistency.
- The test specimens recommended are 150 × 150 × 150 mm cubes or cylinders of 150 mm diameter and 300mm height.
- The mixed concrete is filled into the moulds in layers of 50 mm to achive full compaction. Each layer of mix so placed is tamped with bar, 16 mm in diameter and 600 mm long, 35 times or with a vabrator.
- The test specimens are stored at a temperature of 27 + 2°C and 90% humidity for $24 \pm \frac{1}{2}$ hour from the time of addition of water to the dry ingredients.
- After this period the specimens are removed from the moulds and placed in water and kept there unit taken out just prior to test.
- Normally, the recognized age of test of specimens is 7 and 28 days.
- At least three specimens, preferably from different batches, are tested at each selected age.
- The specimens should be tested immediately after taking out them from water with surface water wiped off.
- The specimen is placed between the platens of the compression testing machine with the care that the axis of specimen is aligned with the centre of throust of the spherically seated platen.
- The compression testing machine should be able to apply gradual load of 14N/mm^2/minute unit the specimen is crushed.

$$\text{Compressive Strengtrh} = \frac{\text{Maximum Load}}{\text{Cross sectional Area}}$$

- The average of the three vaues is taken as the compressive strength of concrete of the batch, provided the individual variation is not more than + 15% of the average.

  Note: IS code also recommends use of cubes of size 100 × 100 × 100 mm provided the aggregate size does not exceed 19 mm.

  Cylinder *strength* = 0.8 Cube strength.

Flexure Test

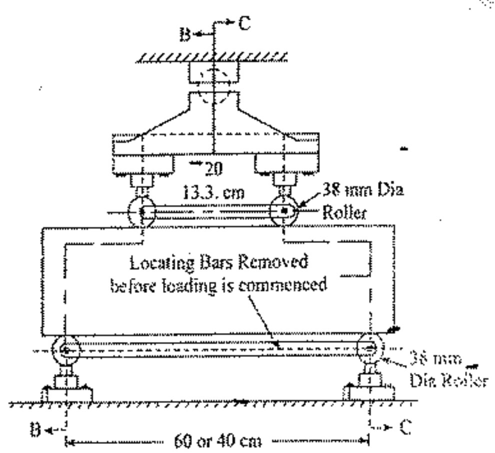
- The flexural tensile strength test is performed to estimate the tensile load at which concrete may crack.
- This is an indirect thest for assessing the tensile strength of concrete. The test consists in determining the tensile strength at failure or the modulus of rupture.
- The ingredients of concrete are mixed as explained in the compression strength.

- The concrete is filled in the mould of size 150 × 150 ×700 mm and compacted with the temping bar weighing 2 kg, 400 mm long and with a ramming face 25 mm square.
- The specimen to be tested is placed in the testing machine on two 38 mm diameter rollers with a c/c distance of 600 mm.
- The load is applied through two similar rollers mounted at the third points, i.e., spacied at 200 mm c/c. The specimen are stored in water at a temperature of 27 +3°C for 48 hours before testing and are tested in wet condition.
- The load is applied without shock and increasing continuously at a rate of 0.7 N/mm²/minute unit the specimen fails.

$$\text{Modulus of rupture} = \frac{pl}{bd^2}(a > 200\text{mm})$$

$$= \frac{3pa}{bd^2}(280\text{mm} < a > 170\text{mm})$$

where, a is the distance between the line of fracture and the nearest support, b and d are width and depth of specimen, l is the length of the span on which the specimen is supported, and p is the maximum load applied to the specimen.



Arrangement for Loading of Flexural Test Specimen

Fig. 10.10

Notes : (i) Is code also recommends use of specimens of size 100 × 100 ×500 mm provided the largest nominal size of aggregate size does not exceed 19 mm.

(ii) Concrete made with crushed rock aggregate tends to have a higher transverse strength than concrete of the same compressive strength made with sand and gravel; thus if a concrete is required for its transverses strength, economical mix may be possible if crushed rock aggregate is used.

Split Tensile Strength Test (IS : 5816)

- Some of the other methods to estimate the tensile strength of concrete are briquette test (direct method) and split tensile strength test (indirect method). Direct methods may not reflect the correct tensile strength because of the paractical difficulties involved (e.g., application of uniaxial tensile load) in the test. This has lead to the development of a number of indirect methods to determine tensile strength of which splitting tests are most common.

- The test consists in applying a compressive force to the concrete specimen in a way that the specimen fails due to induced tensile stresses in the specimen.
- The speciemen is made of cylindrical shape with the diameter not less than four times the maximum size of coarse aggregate and not less than 150 mm. The length of cylinder varies from one to two diameters.
- Normally the test cylinder is 150 mm diameter and 300 mm long. The test consists of applying compressive line loads along the opposite generators of the concrete cylinder placed with its axis horizontalbetween the platens as shown in figure.
- The load is applied at a rate so as to produce a tensile splitting stress of about 2.0 N/mm2/minute until the resistance of the specimen to the increasing load breaks down and no greater load can be sustained.
- The specimen fails finally by splitting along the loaded diameter. The maxmum load applied is recorded. The splitting tensile strength is given by.

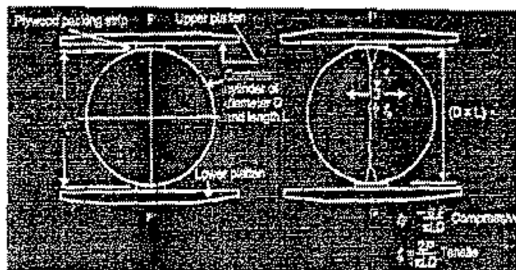


Fig. 10.11

$$T = \frac{2P}{\pi DL} = \frac{0.637P}{DL}$$

where P = maximum load in N applied to the specimen

D = diameter of specimen in mm

L = length of the specimen in mm

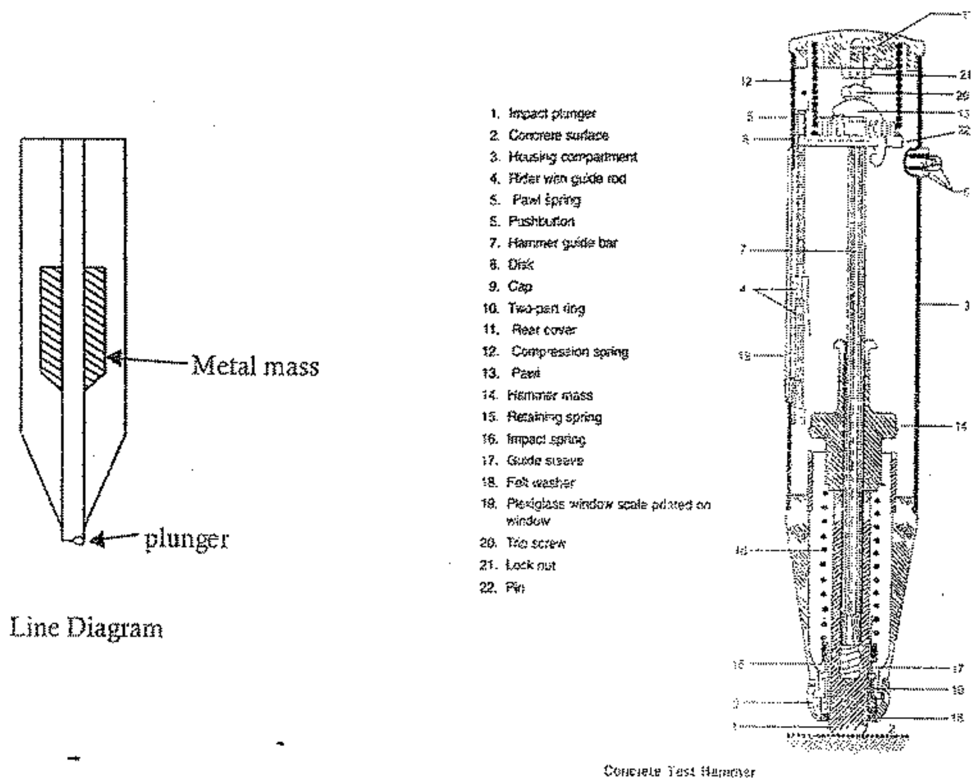
Note: The split cylinder strength is used for estimating the shear strength of beams with unreinforced webs.

Non Destructive Tests: Many times, it is necesary to find the strength of hardened concrete which is already acting as a structural members. For this two type of that is performed.

1. Rebound Hammer Test

- In this test a rebound hammer is used.
- The test is based on the principal that the rebound of on elastic mass depends on the hardness of the surface against which the mass is pressed (impinges).
- The hammer consist of a plunger connected with a spring driven metal mass.
- The plunger is held against at 90° to the smooth concrete. surface and pressed.
- This will impart a fixed amount of energy.
- Upon release the metal mass rebounds, and the plunger being still in contact with concrete.
- The distance-travelled by the Metal mass noted on a scale which gives an indication of the concrete strength.

- Larger is the rebound, higher is the strengthof concrete.



(a) (b)

Fig. 10.12

2. Ultrasonic Pulse Velocity Test

Objective : This test is based on the principal that the velocity of sound in a solid Material is a function of the Square root of the ratio of its modulus of elasticity 'E' to its density ρ.

$$V = \sqrt{\frac{E}{\rho}}$$

If the velocity of sound through a solid media is known, it is possible to correlate it with the strength of the solid media.

Test Procedure

- An ultrasonic pulse apparatus consists of a transmitter and a receiver which are held against two faces of concrete.
- The apparatus generates pulses of ultrasonic frequency which are transmitted through concrete by the transmitter.
- The other face, the receiver receives the pulses and the apparatus records them.
- The velocity of the pulses is found which is correlated to the strength of the concrete.
- Higher the velocity of pulses, greater is the strength of concrete.

General condition	Pulse Velocity m/sec
Excellent	Above 5000
Good	4000 – 5000
Questionable	3000 – 4000
Poor	2000 – 3000
Very poor	below – 2000

Need of Non-Destructive Testing

Information about the in-situ strength of concrete may be required for both under construction as well as existing concrete structures. For new concrete structures under construction, N.D.T. may be required under following circumstances:

- None-compliance of the material supplied in terms of work specimen test results or other specified requirements.
- Uncertainties concerning the level of workmanship involved in construction operations afecting the hardened properties of the in-situ concrete.
- Quality control of construction.
- Monitoring of strength development in relation to formwork removal, curring, prestressing, load application or similar purposes.

Existing Concrete Structure Require an Examination under the Following Cirumstances

- Deterioration of the concrete due to factors such a external or internal chemical attack, fire explosion and other environmental effects.
- To make assessment of the load carrying capacity of an existing structure for change of ownership or insurance purpose or in relation to a proposed change of use or alternation.

Advantage of Non-destructive Testing

Non-destructive methods have following distinct advantages over the prevalent destructive methods of testing.

- The measurement can be done on concrete in-situ and thus representative samples are not required.
- In destructive method of testing the change in quality of concrete has to be studied on a long-term basis with respect to curing or deterioration due to certain causes.
- A large number of specimens are required which could be tested to destruction, at various ages.
- Since it cannot be guaranteed that all specimens are of the same quality, the results obtained may not be very reliable.
- Non-desitructive testing makes its possible to study the variation in quality of concrete with time and external influences.
- In N.D.T. method the concrete is not loaded to destruction. Its quality is judged by measuring certain of its physical properties, which are related to its quality.
- In N.D.T. there is no wastage of material as in destructive methods of testing.

Target Mean Strength

For designing the concrete mix of a given characteristic strength, the statisticcal methods are used. It can be seen from the accepted Gaussian curve for analysis of cube test results that there is only 5 percent probability that a result would fall below $\bar{x} - 1.65\,\sigma$ or above $\bar{x} + 1.65\,\sigma$.

The characteristic strength of concrete f_{ck} can be defined as

$$f_{ck} = f_m - 1.65\,\sigma$$

wher f_m = the mean strength

and σ = standard deviation.

Therefore while designing a concrete mix, the target should be to be get a mean strength or target strength of $f_t = f_{ck} + 1.65\,\sigma$, where f_t is the target mean strength.

Shrinkage

The volumetric contraction of the concrete per unit volume due to drying when concrete sets is known as shrinkage.

- The shrinkage is irreversible process.
- It is to assumed 0.003 is absence of all the data (shrinkage strain).
- Shrinkage is a long process and continues for many years, particularly for mass concrete.
- Shrinkage and shrinkage strain near the surface of the member is more than the shrinkage of internal particles, as external particles are open to atmosphere.
- Because of this differential shrinkage self balancing force are set up within the concrete resulting in shrinkage cracks.
- To avoid shrinkage cracks one can provide shrinkage steel.
- For analysis purpose it is assumed that half of the shrinkage take place during the first month and about three-quarter of shrinkage takes in the first six month.
- Shrinkage is not related to application of loads.
- Shrinkage produces tensile cracking in any structural member which is restrained and these cracks permits water to enter which deteriorates reinforcement, and hence reduce shear strength.

To Minimize Shrinkage

- Use dence non porous aggregate, low water/cement ratio and produce well completed, low slump and well cured concrete
- Add shrinkage steel to concrete to reduce hair cracks formed due to shrinkage visible to naked eye.
- Use construction and expansion joint to control the location of cracks.

Creep

Creep is that property of concrete by which it continues to deform with time under sustained loading or stress total deformation of one element can be in two parts.

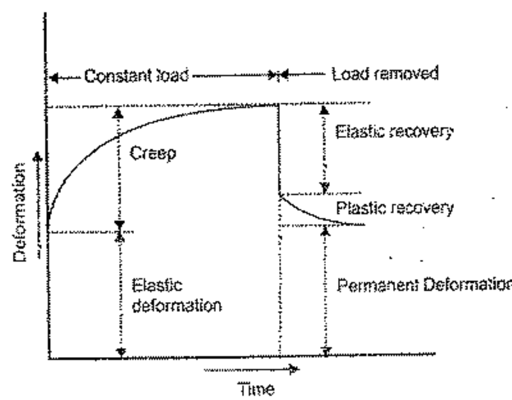
- an initial, instantaneous deformation at an application of load.
- a time deformation due to creep.

- Creep deformation is a long process, about 75% of the ultimate creep deformation occurs during first year and continues at a slower rate for many year.
- It is assumed that creep deform are completed after 2 to 3 year.

Negative Effects of Creep

Deformation of the concrete member are 2 to 3 times the initial deformations and this effects can be reduced by

- Using high strength concrete
- Delaying the application of finishes, partions wall.
- Adding reinforcement.
- Steam curing under pressure.



Deformation of Hardened Concrete

Fig. 10.13

WATER CEMENT RATIO

- Water cement ratio is the ratio of volume of water mixed in concrete to volume of cement used.
- The strength and workability of concrete depends to a great extent on the amount of water used.
- For a given proportion of the materials, there is an amount of water which gives the greatest strength.
- Amount of water less than this optimum water decreases the strength and about 10 percent less may be insufficient to ensure complete setting of cement. Similarly, more than the optimum water increases the workability but decreases the strength.
- The use of an excessive amount of water not only produces low strength but increases shrinking, and decreases density and durability.
- According to Abram's water cement ratio law, for any given conditions of test, the strength of a workable concrete mix is dependent only on the water-cement ratio.
- Lesser the water-cement ratio in a workable mix, greater will be its strength.
- From Abram's law, it follows that provided the concrete is fully compacted, the strength is not affected by aggregate shape, type or surface texture, or the aggregate grading, the workability and the richness of the mix.
- Concrete vibrated by efficient mechanical vibrators requires less water-cement ratio, and hence have more strength. Sometimes, plasticising agents may be mixed to increase the workability of the mix.
- For such concrete, therefore, water-cement ratio is reduced, resulting is an increase in the strength.

SOME SPECIAL CONCRETE

Ferro-Cement Concrete

- Ferrocement is a composite material consisting of cement (OPC or PPC or any other), coarse sand and wire mesh.
- The closely speced wire mesh is impregnated with rich cement mortar mix. the wire mesh consists of 0.5 mm to 2 mm diameter wire at 5 mm to 50 mm c/c spacing.
- The cement mortar consists of 1 : 2 or 1 : 3 cement sand with water - cement ratio verying from about 0.4 to 0.5. the natural aggregate size varies from 2 mm to 10 mm depending upon the mesh size.
- The ferro cement elements are usually of the order of 10 to 50 mm in thickness with 2 to 5 mm clear cover to the wire mesh.
- Once the mortar is set, the moist curing is done for about 10 to 14 days.
- The common types of steel wire mesh used in ferro cement are square, rectangular, hexagonal and expended metal mesh.
- These meshes may be woven, twisted or welded. This composite material exhibits two important properties : higher elasticity and higher resistance to cracking.
- The basic advantage of using this material is that concrete can undergo large strains in the vicinity of the reinforcement without cracking and the magnitude of strains will depend upon the distribution and sub division of the reinforcement throughout the mass of the concrete.

Steel Fiber Reinforced Concrete

- Fiber have been in use since ancientr times to strengthen the weak materials.
- Straw, rice husk and bamboo were used to reinforce sum dried bricks, and mud roofs and walls.
- Even today, in rural India these fibers are being used by individul users.
- The fibers may be produced ffrom steel, plastic, glass or other natural materials in various shapes and sizes.
- During the past one decade, a considerable research has been carried out in the field of steel fiber reinforced concrete (SFRC).
- The basic advantage of using steel fibers is to gain an increase in the tensile, flexural, fatigue and impactr strengths of concrete.
- The increase may be as high as 50%.
- The fibers may be a wire like, crimped, hooked or a flat and are described by a parameter called aspect ratio.
- The diameter of fibers varies from 0.25 mm to 0.75 mm.
- The aspect ratio (length/diameter) varies from 30 to 150. the length varies from about 25 mm to 50 mm.
- The amount of steel fibers may very from 0.25% to 2.5% by volume of concrete.
- About 0.75% to 1% steel fibers are more common. The addition of steel fibers reduces the workability of concrete.
- Hence it becomes necessary to make use of super plasticizers. The ultimate tensile strength of steel should not be less than 350 MPa.

- Typical glass fibers (chopped strand) have diameters of 0.005 to 0.015 mm hut these fibers may be bonded together to produce glass fiber elements with diameters of 0.013 to 1.3 mm.
- Typical plastics such as nylon, polypropylene, polyethylene, polyester and rayon have been made into fibers with diameter of 0.02 to 0.38 mm.

Heavy Weight Concrete

- These concretes made with specially selected heavy weight aggregate to obtain a density higher than 3000 kg/m^3.

Light Weight Concrete :

- These concretes made with artificially produced aggregate to obtain a density less than 1800 kg/m^3. Its strength can be as high as 50 MPa.

Foam Concrete :

- It is also referred to as aerated concrete. It is a very light weight concrete consisting of cellular structure with bubbles of gas made with the help of a suitable admixture.

Polymer concrete :

- The mechanical properties and durability of concrete can be improved by filling the pores, voids and cracks by using polymers with concrete.
- Polymer concrete composities are obtained by processing polymeric materials with some or all of the ingredients of the cement concrete composition.
- A polymer concrete can be classified depending upon the process by which the polymeric materialic materials are incorporated in concete.

(a) Polymer concrete or resin concrete (PC)
(b) Polymer impregnated concrete (PIC)
(c) Latex modified concrete (LMC)

Styrene, methyl – mathacrylic, vinyl chloride, epoxy polyvinyl acetate, styene butadine copolymer latex etc. are commonly used polymers. The polymer concrete is very effective in repairing damaged concrete structural membbers.

Note: 1. The ratio of split tensile strength of concrete and compressive strength is $\frac{2P}{\pi DL} / f_{ck}$. But for practical purposes split of tensile the concrete is taken as Flexural strength $\left(0.7\sqrt{f_{ck}}\right)$

2. Split tensile strength $\left(\frac{2P}{\pi DL}\right)$, Flexural strength $\left(0.7\sqrt{f_{ck}}\right)$, cylinder strength, cube strength are in the order of increasing strength
3. Cube strength = 0.8 cylinder strength
4. The strength of cement concrete at 7 days = 2/3 strength of cement concrete at 28 days.
5. The requirement of individua test results in N/mm^2 of characteristic compressive strength as per IS : 456-2000, for M15 > f_{ck}^{-3} . and for M20 or above > f_{ck}^{-4} .

Some Important Points

- **What are the constituent materials of plain concrete?**

  The constituent materials of plain concrete are cement, sand (fine aggregate), gravel (coarse aggregate), water and mineral admixtures in some special cases.

- **Define characteristic strength f_{ck} of concrete.**

  Characteristic strength of concrete is defined as the compressive strength of 150 mm size cube at 28 days and expressed in N/mm^2 below which not more than five per cent of the test results are expected to fail.

- **How and when the characteristic compressive strength f_{ck} is determined?**

  Characteristic compressive strength is determined by conducting compressive strength tests on specified number of 150 mm concrete cubes at 28 days after casting. It is expressed in N/mm^2.

- **What do the symbols M and 20 mean for grade M 20 concrete?**

  The symbol *M* refers to mix and the number 20 indicates that the characteristic strength f_{ck} of grade M 20 N/mm^2.

- **Express the relation between flexural strength (f_{cr}) and characteristic compression strength f_{ck} of concrete.**

  The generally accepted relation is: $f_{cr} = 0.7\sqrt{f_{ck}}$ where f_{cr} and f_{ck} are in N/mm^2.

- **Express the short term static modulus E_c in terms of the characteristic compressive strength f_{ck} of concrete.**

  The suggested expression is : $E_c = 5000\sqrt{f_{ck}}$ where E_c and f_{ck} are in N/mm^2.

- **State the approximate value of total shrinkage strain of concrete to be taken for the design purpose and mention the relevant clause no. of IS code.**

  As per cl. 6.2.4.1 of IS 456:2000, the approximate value of total shrinkage strain of concrete is to be taken as 0.0003.

- **Define workability of concrete.**

  Workability of concrete is the property which determines the ease and homogeneity with which concrete can be mixed, placed, compacted and finished.

- **Differentiate between design mix and nominal mix concrete.**

  In design mix, the proportions of cement, aggregates (sand and gravel), water and mineral admixtures, if any are actually determined by actual design to have a desired strength. In nominal mix, however, these proportions are nominally adopted.

- **What are the different types of batching in mixing the constitutent materials of concrete and name the type of batching to be adopted for different materials?**

  Mass and volume are the two types of batching. The quantities of cement, aggregates (sand and gravel) and solid admixtures shall be measured by mass batching. Liquid admixtures and water are measured either by mass or volume batching.

- **Differentiate between steel bars and rods.**

Bars are steel bars of diameter up to 12 mm which are coiled during transportation. Rods are steel bars of diameter greater than 12 mm and cannot he coiled. They are transported in standard lengths.

- **What are the criteria of properly mixed concrete and how to achieve them?**

Properly mixed concrete will have uniform distribution of materials having uniform colour and consistency.

These are achieved by mixing the constitutent materials in a mechanical mixer at least for two minutes or such time till those qualities are achieved.

- **What should be the expected strength of concrete structure at the time of removal of formwork?**

The concrete at the time of removing the formwork should have strength of at least twice the stress that it may be subjected to at the time of removal of formwork.

- **Name the sameple tests to he performed for checking the strength of concrete.**

The main test to be performed is 150 mm cube strength at 28 days made of fresh concrete and cured. Additional tests should also be conducted on 150 mm cubes at 7 days and beam tests to determine modulus of rupture at 3 to 7 days. There should be a least 3 or more samples of such specimens to represent the entire concrete work. Each sample should have at least three specimens for conducting each of the above-mentioned tests.

- **When is it essential to conduct standard core test?**

Standard core tests are needed if the inspection of concrete work raises doubt regarding the grade of concrete either due to poor workmanship or unssatisfactory cube strength results performed following standard procedure.

- **When do you consider core test results as satisfactory?**

The core test results are considered satisfactory if:

(i) the average equivalent cube strength of the cores is at least 85 per cent of the cube strength of the grade of concrete at that age, and

(ii) each of the individual cores has strength of at least 75 per cent of the concrete cube strength at that age.

- **What are to he done for unsatisfactory core test results?**

Load tests are to be conducted for the flexural members and analytical investigations are to be performed for non-flexural members.

- **Name the acceptable non-destructive tests to be performed on structures.**

The acceptable non-destructive tests are ultrasonic pulse velocity, rebound hammer, probe penetration, pull out and maturity.

Practice Objective Questions PART-I: Cement

1. Match List-I with List-II and select the correct answer using the codes given below the lists:

	List-I		List-II
A.	fineness of cement	1.	Le-Chatelier apparatus
B.	Setting time	2.	Vicat's needle
C.	Soundness	3.	Air permeability apparatus
D.	Workability	4.	Slump cone

Codes:

	A	B	C	D
(a)	1	2	3	4
(b)	3	1	4	2
(c)	3	2	1	4
(d)	1	4	3	2

2. If 'p' is the standard consistency of cement, the amount of water used in conducting the initial setting time test on cement is
(a) 0.65p (b) 0.85p
(c) 0.6p (d) 0.8p

3. For complete hydration of cement the water-cement ratio needed is
(a) less than 0.25
(b) more than 0.25 but less than 0.35
(c) more than 0.35 but less than 0.45
(d) more than 0.45 but less than 0.60

4. Match List-I (Type of cement) with List-II (Characteristics) and select the correct answer using the codes given below the lists:

List-I
A. Air entraining portland cement
B. Low-heat portland cement
C. Hydrophobic portland cement
D. Rapid hardening portland cement

List-II
1. Suitable for very large structures
2. Unsuitable for very large masses of concrete
3. Greater resistance to frost attack
4. Safe storage under unfavourable

Codes:

	A	B	C	D
(a)	4	2	1	3
(b)	3	4	1	2
(c)	3	1	4	2
(d)	4	1	2	3

5. Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

List-I

A. Vicat's needle

B. Michaeli's compound lever apparatus

C. Le Chatelier's apparatus

D. Turbidimeter

List-II

1. Setting time

2. Specific surface

3. Tensile strength

4. Soundness

Codes:

	A	B	C	D
(a)	1	2	3	4
(b)	1	3	4	2
(c)	2	4	3	1
(d)	3	4	1	2

6. Blast furnace slag has approximately

(a) 45% calcium oxide and about 35% silica

(b) 50% alumina and 20% calcium oxide

(c) 25% magnesia and 15% silica

(d) 25% calcium sulphate and 15% alumina

7. Gypsum is used as an admixture in cement grouts for

(a) accelerating the setting time

(b) retarding the setting time

(c) increasing the plasticity

(d) reducing the grout shrinkage

8. Which of the following pairs in respect of Ordinary Portland Cement (OPC) are correctly matched?

1. Initial settime time ... 30 minutes

2. Final setting time ... 10 hours

3. Normal consistency ... 10%

Select the correct answer using the codes given below:

(a) 1, 2 and 3 (b) 2 and 3

(c) 1 and 2 (d) 1 and 3

9. High alumina cement is produced by fusing together a mixture of

(a) limestone and bauxite

(b) limestone, bauxite and gypsum

(c) limestone, gypsum and clay

(d) limestone, gypsum, bauxite, clay and chalk

10. Consider the following statements:
High early strength of cement is obtained as a result of

1. fine grinding
2. decreasing the lime content
3. burning at higher temperatures
4. increasing the quantity of gypsum

Which of these statements are correct?
(a) 1 and 2 (b) 1 and 3
(c) 2, 3 and 4 (d) 1, 3 and 4

11. The temperature range in a cement kiln is
(a) 500° to 1000°C (b) 1000° to 1200°C
(c) 1300° to 1500°C (d) 1600° to 2000°C

12. Before testing setting time of cement one should test for
(a) soundness (b) strength
(c) fineness (d) consistency

13. Consider the following statements:

1. Tests on cement paste to determine initial and final setting times are done at normal consistency condition.
2. Low heat cement has a high percentage of tricalcium aluminate.
3. High early strength portland cement contains a larger percentage of tricalcium silicate and a lower percentage of dicalcium silicate.

Which of these statements are correct?
(a) 1 and 2 (b) 1 and 3
(c) 2 and 3 (d) 1, 2 and 3

14. Match **List-I** (Property of cement) with **List-II** (Testing apparatus) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Specific gravity	**1.** Blain's apparatus
B. Setting time	**2.** Le Chatelier's flask
C. Soundness	**3.** Compressometer
D. Fineness	**4.** Autoclave
	5. Vicat's apparatus

Codes:

	A	B	C	D
(a)	3	5	1	2
(b)	2	5	1	4
(c)	2	5	4	1
(d)	5	3	4	1

15. Consider the following oxides:

1. Al_2O_3 2. CaO 3. SiO_2

The correct sequence in increasing order of their percentage in an ordinary portland cement is

(a) 2, 1, 3 (b) 1, 3, 2
(c) 3, 1, 2 (d) 1, 2, 3

16. Match **List-I** (Cement) with **List-II** (Characteristic) and select the correct answer using the codes given below the lists:

List-I		List-II	
A.	High alumina cement	**1.**	High early strength
B.	Blast furnace cement	**2.**	Gypsum free cement
C.	Quick setting cement	**3.**	Selenetic cement
D.	Rapid hardening cement	**4.**	Used in mass concrete work
		5.	Used in chemical factories and mines

Codes:

	A	B	C	D
(a)	5	4	2	1
(b)	4	3	2	1
(c)	5	4	3	2
(d)	4	5	1	2

17. The fineness of cement is tested by

(a) air-content method (b) air-permeability method
(c) Le-Chatelier apparatus (d) Vicat's apparatus

18. The test on cement designed to accelerate the slaking process of the ingredient of cement and to determine the resulting expansion in a short time is

(a) setting time test (b) soundness test
(c) normal consistency test (d) accelerated test

19. Consider the following statements:
High Alumina Cement (HAC)

1. has high early compressive strength and high heat of hydration than OPC-43 grade
2. is not suitable to be used in cold regions

Which of these statements is/are correct?

(a) 1 alone (b) 2 alone
(c) Both 1 and 2 (d) Neither 1 nor 2

20. Consider the following statements:
When cement is tested for setting time; on gauging it shows quick setting. This phenomenon known as "Flash set" of cement is due to the presence of high

1. tricalcium aluminate (C_3A) in cement
2. alkalies in cement
3. tricalcium silicate (C_3S) in cement

Which of these statements are correct?

(a) 1, 2 and 3 (b) 2 and 3

(c) 1 and 2 (d) 1 and 3

21. For marine works, the best suited cement is

(a) low heat portland cement (b) rapid hardening cement

(c) ordinary portland cement (d) blast furnace slag cement

22. Match **List-I** (Type of cement) with **List-II** (Characteristics) and select the correct answer using the codes given below the lists:

List-I

A. Ordinary portland cement

B. Rapid hardening cement

C. Low heat cement

D. Sulphate resistant cement

List-II

1. The percentage of C_3S is maximum and is of the order of 50%
2. The percentage of C_2S and C_3S are the same and of the order of 40%
3. Reacts with silica during burning and causes particles to unite together and development of strength
4. Preserves the form of brick at high temperature and prevents shrinkage

Codes:

	A	B	C	D
(a)	2	4	1	3
(b)	3	1	4	2
(c)	2	1	4	3
(d)	3	4	1	2

23. Match **List-I** (Type of cement) with **List-II** (Property/Characteristic) and select the correct answer using the codes given below the lists:

List-I

A. High strength portland cement

B. Super sulphated cement

C. High alumina cement

D. Rapid hardening portland cement

List-II

1. Should not be used with any admixture
2. Is extremely resistant to chemical attack
3. Gives a higher rate of heat development during hydration of cement
4. Has a higher content of tricalcium silicate

Codes:

	A	B	C	D
(a)	3	2	1	4
(b)	4	1	2	3
(c)	3	1	2	4
(d)	4	2	1	3

24. Four main oxides present in ordinary portland cement are: CaO, Al_2O_3, SiO_2 and Fe_2O_3. Identify the correct ascending order of their proportions in a typical composition of OPC.
(a) Al_2O_3, Fe_2O_3, CaO, SiO_2 (b) Al_2O_3, CaO, Fe_2O_3, SiO_2
(c) Fe_2O_3, Al_2O_3, SiO_2, CaO (d) Fe_2O_3, SiO_2, Al_2O_3, CaO

25. The proper size of mould for testing compressive strength of cement is
(a) 7.05 cm cube (b) 10.05 cm cube
(c) 15 cm cube (d) 12.05 cm cube

26. The specific gravity of commonly available ordinary portland cement is
(a) 4.92 (b) 3.15
(c) 2.05 (d) 1.83

27. A quick-setting cement has an initial setting time of about
(a) 50 minutes (b) 40 minutes
(c) 15 minutes (d) 5 minutes

28. Consider the following statements:
Low percentage of C_3S and high percentage of C_2S in cement will result in
1. higher ultimate strength with less heat generation
2. rapid-hardening
3. better resistance to chemical attack
Which of these statements are correct?
(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1, 2 and 3

29. Match **List-I** (Type of cement) with **List-II** (Characteristics) and select the correct answer using the codes given below the lists:

List-I
A. Rapidly hardening cement
B. Low heat portland cement
C. Portland pozzolana cement
D. Sulphate resisting cement

List-II
1. Lower C_3A content than that in OPC
2. Contains pulverised fly ash
3. Higher C_3S content than that in OPC
4. Lower C_3S and C_3A contents than that in OPC

Codes:

	A	B	C	D
(a)	1	2	4	3
(b)	3	4	2	1
(c)	1	4	2	3
(d)	3	2	4	1

30. Match **List-I** (Apparatus) with **List-II** (Purpose) and select the correct answer using the codes given below the lists:

List-I

A. Le-chatelier

B. Vicat needle with annular collar

C. Vee-Bee

D. Briquettes test machine

List-II

1. Workability of concrete

2. Soundness of cement

3. Tensile strength of cement

4. Final setting time of cement

Codes:

	A	B	C	D
(a)	1	3	2	4
(b)	2	4	1	3
(c)	1	4	2	3
(d)	2	3	1	4

31. Match **List-I** (Job requirement) with **List-II** (Type of cement binder) and select the correct answer using the codes given below the lists:

List-I

A. High early strength

B. Lining for canals

C. Frost and acid resistance

D. Marine structure

List-II

1. Pozzolanio cement

2. Rapid hardening cement

3. Sulphate resisting cement

4. High alumina cement

Codes:

	A	B	C	D
(a)	1	4	3	2
(b)	2	3	4	1
(c)	1	3	4	2
(d)	2	4	3	1

32. As per specifications, the initial setting time of ordinary portland cement should not be less than

(a) 10 minutes (b) 20 minutes

(c) 30 minutes (d) 60 minutes

33. An cements, generally the increase in strength during a period of 14 days to 28 days is primarily due to

(a) C_3A (b) C_2S

(c) C_3S (d) C_4AF

34. Ultimate strength of cement is influenced by which one of the following?
(a) Tricalcium silicate (b) Dicalcium silicate
(c) Tricalcium aluminate (d) Tetracalcium alumino-ferrite

35. Match **List-I** (Composition of raw material used in manufacture of cement) with **List-II** (Component of raw material) and select the correct answer using the codes given below the lists:

List-I	**List-II**
A. 25%	**1.** Silica
B. 65%	**2.** Calcium oxide
C. 5%	**3.** Aluminum oxide
D. 5%	**4.** Ferrous and magnesium oxides

Codes:

	A	B	C	D
(a)	1	2	3	4
(b)	4	3	2	1
(c)	1	3	2	4
(d)	4	2	3	1

36. Match **List-I** (Equipment) with **List-II** (Property) and select the correct answer using the codes given below the lists:

List-I
A. Briquette testing machine
B. Le Chatelier apparatus
C. Vicat apparatus

List-II
1. Compressive strength
2. Consistency
3. Soundness
4. Tensile strength

Codes:

	A	B	C
(a)	1	3	2
(b)	1	2	3
(c)	4	2	3
(d)	4	3	2

37. What is the requirement of water (expressed as % of cement w/w) for the completion of chemical reactions in the process of hydration of OPC?
(a) 10 to 15% (b) 15 to 20%
(c) 20 to 25% (d) 25 to 30%

38. If P is the percentage of water required for determination of normal consistency of vement, then percentage of water to be added for determination of initial setting time is
(a) 0.70 P (b) 0.75 P
(c) 0.80 P (d) 0.85 P

Practice Objective Questions Part-II: Cement Concrete

1. The split tensile strength of M 15 grade concrete when expressed as percentage of its compressive strength is
   (a) 10 to 15% (b) 15 to 20%
   (c) 20 to 25% (d) 25 to 30%

2. The approximate ratio between the strengths of cement concrete at 7 days and at 28 days is
   (a) 3/4 (b) 2/3
   (c) 1/2 (d) 1/3

3. Modulus of elasticity of M 25 concrete as determined by formula of **IS:456** is
   (a) 1,24,500 MPa (b) 90,125 MPa
   (c) 28,500 MPa (d) 16,667 MPa

4. **Assertion (A):** For identical mix, the cube compressive strength of concrete obtained from 15 cm cube is higher than 15 cm × 30 cm cylinder compressive strength.
   Reason (R): Cube compressive strength is higher than the cylinder compressive strength because of its higher contact area under the load.

5. **Assertion (A):** Workability of concrete is improved by air entraining agent.
   Reason (R): Air entraining agent increases concrete strength.

6. Tensile strength of concrete is measured by
   (a) direct tension test in the universal testing machine
   (b) applying compressive load along the diameter of the cylinder
   (c) applying third point loading on a prism
   (d) applying tensile load along the diameter of the cylinder

7. The approximate ratio of strength of 15 cm × 30 cm concrete cylinder to that of 15 cm cube of the same concrete is
   (a) 1.25 (b) 1.00
   (c) 0.85 (d) 0.50

8. If in a concrete mix the fineness modulus of coarse aggregate is 7.6, the fineness modulus of fine aggregate is 2.8 and the economical value of the fineness modulus of combined aggregate is 6.4, then the proportion of fine aggregate is
   (a) 25% (b) $33\frac{1}{3}\%$
   (c) 50% (d) $66\frac{2}{3}\%$

9. General shrinkage in cement concrete is caused by
   (a) carbonation
   (b) stressed due to external load
   (c) drying with starting with a stiff consistency
   (d) drying with starting with a wetter consistency

10. While concreting in cold weather where frosting is also likely, one uses
   (a) high quality portland cement with minimum additives
   (b) high alumina cement with calcium chloride additives
   (c) portland cement together with calcium chloride additives
   (d) a mixture of high alumina cement and portland cement

11. The modulus of elasticity (E) of concrete is given by
   (a) $E = 1000\ f_{ck}$
   (b) $E = 5700\ \sqrt{f_{ck}}$
   (c) $E = 5500\ \sqrt{f_{ck}}$
   (d) $E = 10,000\ \sqrt{f_{ck}}$

12. Consider the following statements:
   The addition of surfactants in the concrete mix results in
   1. increase in the water-cement ratio
   2. decrease in the water-cement ratio
   3. increase in the strength of concrete
   4. decrease in the curing duration
   5. increase in the density of concrete
   Which of these statements are correct?
   (a) 1, 3 and 4
   (b) 2, 3 and 5
   (c) 3, 4 and 5
   (d) 1, 4 and 5

13. Consider the following statements:
   Higher water-cement ratio in concrete result in
   1. stronger mix
   2. better workable mix
   3. a weak mix
   4. less bleeding
   Which of these statements are correct?
   (a) 1 and 2
   (b) 2 and 3
   (c) 3 and 4
   (d) 1 and 4

14. A splitting tensile test is performed on cylinder of diameter D and length L. If the ultimate load is P, then splitting tensile strength of concrete is given by
   (a) $\dfrac{P}{\pi DL}$
   (b) $\dfrac{2P}{\pi DL}$
   (c) $\dfrac{4PL}{\pi D^3}$
   (d) $\dfrac{4PD}{\pi L^3}$

15. Match **List-I** (Admixtures) with **List-II** (Chemicals) and select the correct answer using the codes given below the lists:
   List-I
   A. Water reducing admixture
   B. Air-entraining agent
   C. Superplasticiser

D. Accelerator

List-II

1. Sulphonated melanin formaldehyde
2. Calcium chloride
3. Lignosulphonate
4. Neutralised vinsol resin

Codes:

	A	B	C	D
(a)	2	4	1	3
(b)	1	3	4	2
(c)	3	4	1	2
(d)	3	4	2	1

16. To make one cubicv meter of 1 : 2 : 4 by volume concrete, the volume of coarse aggregates required is

(a) 0.95 m^3 (b) 0.85 m^3
(c) 0.75 m^3 (d) 0.65 m^3

17. Batching refers to:

(a) controlling the total quantity at each batch
(b) weighing accurately, the quantity of each material for a job before mixing
(c) controlling the quantity of each material into each batch
(d) adjusting the water to be added in each batch according to the moisture content of the materials being mixed in the batch

18. Consider the following strengths of concrete:

1. Cube strength
2. Cylinder strength
3. Split-tensile strength
4. Modulus of rupture

The correct sequence in increasing order of these strength is

(a) 3, 4, 2, 1 (b) 3, 4, 1, 2
(c) 4, 3, 2, 1 (d) 4, 3, 1, 2

19. The role of superplasticizer in a cement paste is to

(a) disperse the particles
(b) disperse the particles and to remove air bubbles
(c) disperse the particles, remove air bubbles and to retard setting
(d) retard setting

20. Consider the following statements:

Shrinkage of concrete depends upon the

1. relative humidity of the atmosphere
2. passage of time
3. applied stress

Which of these statements is/are correct?

(a) 1 and 2
(b) 2 and 3
(c) 1 alone
(d) 1, 2 and 3

21. Consider the following statements:
The effect of an air entrainment in concrete is to
1. increase resistance to freezing and thawing
2. improve workability
3. decrease strength

Which of these statements is/are correct?
(a) 2 and 3
(b) 1 and 3
(c) 1 alone
(d) 1, 2 and 3

22. As per IS Code of Practice, concrete should be cured at
(a) 5°C
(b) 10°C
(c) 27°
(d) 40°C

23. The correct sequence of workability test(s)/ method(s) in the order of their application from low to high workability is
(a) slump test, compacting factor test and vee-bee consistometer.
(b) compacting factor test, vee-bee consistometer and slump test.
(c) vee-bee consistometer, slump test and compacting factor test.
(d) vee-bee consistometer, compacting factor test and slump test.

24. While testing the compressive strength of cement concrete, the correct standard conditions (viz., temperature, age, humidity and size of the specimen) to be maintained as per I.S. are
(a) 27 ± 3°C, 28 days, 90% and 15 cm^3
(b) 26 ± 2°C, 21 days, 80% and 15 cm^3
(c) 25 ± 1°C, 14 days, 75% and 15 cm^3
(d) 27 ± 3°C, 7 days, 70% and 10 cm^3

25. Which one of the following statements is correct?
(a) Bulking of sand always decreases with increase in the quantity of water.
(b) The quantity of water in ordinary concrete should be 5% by weight of aggregate.
(c) While mixing by weight, bulking effect of sand is not taken into account.
(d) River sand is also known as standard sand.

26. The ratio of direct tensile strength to that of modulus of rupture of concrete is
(a) 0.25
(b) 0.5
(c) 0.75
(d) 1.0

27. The maximum bulking of sand is likely to occur at a moisture content of
(a) 5%
(b) 8%
(c) 11%
(d) 14%

28. Which one of the following types of concrete is most suitable in extreme cold climates?
(a) Air-entrained concrete
(b) Ready mix concrete
(c) Vacuum concrete
(d) Coarse concrete

29. Match **List-I** (Workability test) with **List-II** (Measurements) and select the correct answer using the codes given below the list:

List-I		**List-II**	
A.	Slump test	1.	300-500 mm
B.	Compacting factor	2.	75-125 mm
C.	Vee-bee test	3.	0.80 to 0.98
D.	Flow test	4.	0 to 10 sec

Codes:

	A	B	C	D
(a)	2	4	3	1
(b)	1	3	4	2
(c)	1	4	3	2
(d)	2	3	4	1

30. Consider the following statements:
Curing of concrete by steam under pressure
1. increases the compressive strength of concrete
2. reduces the shear strength of concrete
3. increases the speed of chemical reaction.

Which of these statements is/are correct?
(a) 1, 2 and 3 (b) 1 alone
(c) 2 and 3 (d) 3 alone

31. Which one of the following aggregates gives maximum strength in concrete?
(a) Rounded aggregate (b) Elongated aggregate
(c) Flaky aggregate (d) Cubical aggregate

32. Consider the following statements:
Ultrasonic pulse velocity test to measure the strength of concrete is
1. used to measure the strength of wet concrete
2. used to obtain estimate of concrete strength of finished concrete elements
3. a non-destructive test

Which of these statements are correct?
(a) 1, 2 and 3 (b) 2 and 3
(c) 1 and 2 (d) 1 and 3

33. Consider the following statements:
Pozzolana used as an admixture in concrete has the following advantages:
1. It improves workability with lesser amount of water.
2. It increases heat of hydration and so sets the concrete quickly.
3. It increases resistance to attack by salts and sulphates.
4. It leaches calcium hydroxide.

Select the correct answer using the codes given below:
(a) 1, 2, 3 and 4 (b) 1, 2 and 4
(c) 1 and 3 (d) 2, 3 and 4

34. The length of time for which a concrete mixture will remain plastic is usually more dependent on
   (a) the setting time of cement than on the amount of mixing water and atmospheric temperature
   (b) the atmospheric temperature than on the amount of mixing water and the setting time of cement
   (c) the setting time of cement and the amount of mixing water than on atmospheric temperature
   (d) the amount of mixing water used and atmospheric temperature than on the setting time of cement

35. **Assertion (A):** Addition of 5% to 6% of moisture content by weight increases the volume of dry sand from 18% to 38%.

    Reason (R): Bulking of sand is caused due to surface moisture on sand particles.

36. Match **List-I** (Admixture) with **List-II** (Action in concrete) and select the correct answer using the codes given below the lists:

    List-I

    A. Calcium lignosulphonate
    B. Aluminium powders
    C. Tartaric Acid
    D. Aluminium sulphate

    List-II

    1. Anti-bleeder
    2. Retarder
    3. Air entrainer
    4. Water reducer

    Codes:

	A	B	C	D
(a)	3	2	1	4
(b)	4	3	2	1
(c)	3	4	1	2
(d)	4	2	3	1

37. Bleeding of concrete leads to which of the following?
    1. Drying up of concrete surface.
    2. Formation of pores inside.
    3. Segregation of aggregate.
    4. Decrease in strength.

    Select the correct answer using the codes given below:

    (a) 1 only (b) 1 and 2
    (c) 1 and 3 (d) 2 and 4

38. Match **List-I** (Material characteristics) with **List-II** (Property of concrete) and select the correct answer using the codes given below the lists:

    List-I

    A. Water cement ratio
    B. Water content
    C. Minimum cement content
    D. Segregation

List-II

1. Durability
2. Compressive strength
3. Stability of mix
4. Workability

Codes:

	A	B	C	D
(a)	4	1	3	2
(b)	2	4	3	1
(c)	4	1	2	3
(d)	2	4	1	3

39. Stress-strain curve of concrete is

(a) a perfect straight line up to failure

(b) straight line up to 0.002% strain value and then parabolic up to failure

(c) parabolic up to 0.002% strain value and then a straight line up to failure

(d) hyperbolic up to 0.002% strain value and then a straight line up to failure

40. Consider the following statements:

Ultrasonic pulse velocity test is

1. used to measure the strength of wet concrete
2. used to obtain estimate of oncrete strength of finished concrete elements
3. a destructive test
4. a non-destructive test

Which of these statements are correct?

(a) 1, 2 and 3 (b) 2 and 3

(c) 2 and 4 (d) 1 and 3

41. The use of super-plasticizers as admixture

(a) increases compressive strength of concrete

(b) permits lower water cement ratio, thereby strength is increased

(c) reduces the setting time of concrete

(d) permits lower cement content, thereby strength is increased

42. Match **List-I** (Aggregate) with **List-II** (Effect) and select the correct answer using the codes given below the lists:

List-I

A. Rounded aggregates

B. Crushed aggregates

C. Flaky aggregates

D. Irregular aggregates

List-II

1. Reduce workability appreciably because of a high ratio of surface area to volume
2. Require more water than rounded aggregates and give strength lesser than crushed aggregates
3. Give concrete of higher compressive strength due to development of stronger aggregate-mortar bond
4. Require lesser amount of water and cement paste for a given workability

Codes:

	A	B	C	D
(a)	1	2	4	3
(b)	1	3	4	2
(c)	4	3	1	2
(d)	4	2	1	3

43. Consider the following statements:

For increasing the workability of concrete, it is necessary to

1. increase the quantity of cement
2. decrease the quantity of sand
3. alter the proportion of fine and coarse aggregates
4. decrease the quantity of water
5. use angular aggregate

Which of these statements are correct?

(a) 1, 2, 3, 4 and 5 (b) 2, 4 and 5
(c) 2 and 3 (d) 1 and 5

44. Slump and compaction factors are two different measures of workability of concrete. For a slump of 0 to 20 mm, what is the equivalent range of compaction factor?

(a) 0.50-0.70 (b) 0.70-0.80
(c) 0.80-0.85 (d) 0.85-0.92

45. The values of slump commonly adopted for the various concrete mixes are given below:

1. Concrete for road works : 20 to 28 mm
2. Ordinary RCC work : 50 to 100 mm
3. Columns retaining walls : 12 to 25 mm
4. Mass concrete : 75 to 175 mm

Which of the pairs given above are correctly matched?

(a) 1, 3 and 4 (b) 1 and 2
(c) 3 and 4 (d) 2 and 4

46. Consider the following pairs:

1. Hand compaction of heavily reinforced sections: Low workability (0-25 mm slump)
2. Concreting of shallow sections with vibrations: High workability (125-150 mm slump)
3. Concreting of lightly reinforced sections like pavements : Low workability (5-50 mm slump)
4. Concreting of lightly reinforced section by hand or heavily reinforced sections with vibration: Medium workability (25-75 mm slump)

Which of the pairs given above are correctly matched?

(a) 1 and 2 (b) 2 and 3
(c) 3 and 4 (d) 1 and 3

47. The fineness modulus of fine aggregate is 2.78 and of coarse aggregate is 7.82 and the desired fineness modulus of mixed aggregate is 6.14. What is the amount of fine aggregate to be mixed with one part of coarse aggregate?

(a) 55% (b) 50%
(c) 45% (d) 40%

48. Which one of the following correctly expresses the split tensile strength of a circular cylinder of length L and diameter D, subject to a maximum load of P?

(a) $\frac{P}{\pi DL}$ (b) $\frac{P}{2\pi DL}$
(c) $\frac{2P}{\pi DL}$ (d) $\frac{2P}{4\pi DL}$

49. Consider the following statements regarding the phenomenon of bulking of sand:
1. It is due to film of water around sand particles.
2. It is due to capillary action.
3. It is more in finer sands.

Which of these statements is/are correct?
(a) 1, 2 and 3 (b) 1 and 3
(c) 2 and 3 (d) 1 and 2

50. On which one of the following factors, does strength of concrete depend primarily?
(a) Quality of coarse aggregate
(b) Quality of fine aggregate
(c) Fineness of cement
(d) Water-cement ratio

51. Why is super plasticizer added to concrete?
1. To reduce the quantity of mixing water.
2. To increase the consistency.
3. To reduce the quantity of cement.
4. To increase resistance to freezing and thawing.

Select the correct answer using the codes given below:
(a) 1, 2 and 4 (b) 1, 3 and 4
(c) 2 and 4 (d) 4 only

52. On which of the following is the working principle of concrete hammer for non-destructive test based?
(a) Rebound deflections
(b) Radioactive waves
(c) Ultrasonic pulse
(d) Creep-recovery

53. Smaller size of aggregates in a concrete mix
(a) provides larger surface area for bonding with the mortar matrix which increases the compressive strength and reduces the stress concentration at the mortar-aggregate interface
(b) is economical as the concrete mix is more dense
(c) requires lesser cement for a particular water-cement ratio and hence is economical
(d) is beneficial as these aggregates can pass through the reinforcement bars more easily?

54. What is the correct sequence of operations involved in concrete production?
(a) Batching—mixing—handling—transportation
(b) Mixing—batching—handling—transportation
(c) Transportation—handling—mixing—batching
(d) Handling—transportation—mixing—batching

55. What is the approximate ratio of the strength of cement concrete at 7 days to that at 28 days curing?
(a) 0.40
(b) 0.65
(c) 0.90
(d) 1.15

56. Which one of the following properties of cement concrete is ascertained by conducting compaction factor test?
(a) Bulk density
(b) Compressive strength
(c) Modulus of rupture
(d) Workability

57. The mix design for pavement concrete is based on the
(a) flexural strength
(b) characteristic compressive strength
(c) shear strength
(d) bond strength

58. In what context is the slump test performed?
(a) Strength of concrete
(b) Workability of concrete
(c) Water-cement ratio
(d) Durability of concrete

59. Which one of the following is employed to determine strength of hardened existing concrete structure?
(a) Bullet test
(b) Kelly ball test
(c) Rebound hammer test
(d) Cone penetrometer

60. Which one of the following is the correct expression for the target mean strength f_t of concrete mix?
(a) $f_t = kf_{ck} + S$
(b) $f_t = f_{ck} + kS$
(c) $f_t = f_{ck} + S$
(d) $f_t = kf_{ck} + k$
where f_{ck} is characteristic strength, k is probability factor and s is standard deviation.

61. Which one of the following is the correct range of fineness modulus of medium sand usable in preparing cement mortar?
(a) 1.5 to 2.2
(b) 2.6 to 2.9
(c) 2.9 to 3.2
(d) 5.5 to 6.5

62. What is the percentage of the fine aggregate of fineness modulus 2.6 to be combined with coarse aggregate of fineness modulus 6.8 for obtaining combined aggregate of fineness modulus 5.4?
(a) 30%
(b) 40%
(c) 50%
(d) 60%

63. Match **List-I** (Admixture) with **List-II** (Action in concrete) and select the correct answer using the codes given below the lists:

List-I

A. Calcium Lignosulphonate

B. Aluminium powders

C. Tartaric acid

D. Sodium silicate

List-II

1. Accelerators

2. Retarder

3. Air entrainer

4. Water reducer

Codes:

	A	B	C	D
(a)	1	3	2	4
(b)	4	3	2	1
(c)	4	2	3	1
(d)	1	2	3	4

64. Which factors influence the workability of concrete without sacrificing strength?

1. Fine aggregate
2. Quantity of mixing water
3. Maximum size of coarse aggregate
4. Shape of coarse aggregate

Select the correct answer using the codes given below:

(a) 1 only (b) 2 only

(c) 1 and 2 (d) 3 and 4

65. The workability of concrete can be increased by which of the following?

1. Increasing the quantity of coarse aggregate without altering the total aggregate quantity.
2. Decreasing the quantity of coarse aggregate and at the same time correspondingly increasing the quantity of fine aggregate.
3. Using round aggregate.

Select the correct answer using the codes given below:

(a) 1 and 3 only (b) 1 and 2 only

(c) 2 and 3 only (d) 1, 2 and 3

66. Consider the following statements regarding cement concrete:

1. Bleeding indicates deficiency of coarser materials in the mix.
2. Segregation generally indicates poor aggregate grading.

Which of these statements is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

67. Consider the following statements:

Curing of concrete by steam under pressure

1. Increases the compressive strength of concrete

2. Reduces the shear strength of concrete
3. increases the speed of chemical reaction

Which of these statements is/are correct?

(a) 1, 2 and 3 (b) 1 only
(c) 2 and 3 only (d) 3 only

68. Match the sequence of determination of components of a concrete mix as per Indian Standard method of mix design and select the correct answer using the codes given below the lists:

List-I	List-II
A. Cement content	1. First step
B. Aggregate content	2. Second step
C. Water content	3. Third step
D. Water-cement ratio	4. Fourth step

Codes:

	A	B	C	D
(a)	1	2	4	3
(b)	3	2	4	1
(c)	3	4	2	1
(d)	1	4	2	3

69. Consider the following statements:

Ultrasonic pulse velocity test to measure the strength of concrete is

1. used to measure the strength of wet concrete.
2. used to obtain estimate of concrete strength of finished concrete elements.
3. a destructive test.
4. a non-destructive test.

Which of these statements is/are correct?

(a) 2 only (b) 1 and 3
(c) 2 and 4 (d) 3 and 4

70. Consider the following statements about the air entraining admixture in concrete:

1. Improve workability
2. Improve durability
3. Reduce segregation during placing
4. Decrease concrete density

Which of these statements are correct?

(a) 1, 2, 3 and 4 (b) 1 and 2 only
(c) 2 and 3 only (d) 3 and 4 only

71. Consider the following statements:

Admixtures are added to concrete to

1. increase its strength.
2. reduce heat of hydration.
3. delay the setting of cement.
4. reduce water-cement ratio.

Which of these statements is/are correct?

(a) 1 only (b) 1 and 2

(c) 2 and 3 (d) 3 and 4

72. Consider the following statements:
    1. Strength of concrete cube is inversely proportional to water-cement ratio.
    2. A rich concrete mix gives a higher strength than a lean concrete mix since it has more cement content.
    3. Shrinkage cracks on concrete surface are due to excess water in mix.

    Which of these statements is/are correct?

    (a) 1, 2 and 3 (b) 1 and 2 only

    (c) 2 only (d) 2 and 3 only

73. Consider the following statements:

    Entrainment of air in concrete is done so as to
    1. increase the workability.
    2. increase the strength.
    3. increase the resistance to freezing and thawing.

    Which of these statements is/are correct?

    (a) 1, 2 and 3 (b) 1 only

    (c) 1 and 3 only (d) 3 only

74. A cement bag contains 0.035 cubic meter of cement by volume. How many bags will one tonne of cement comprise?

    (a) 16 (b) 17

    (c) 18 (d) 20

Conventional Questions

1. What are the initial and final setting times of cement? How are they experimentally determined? Briefly explain the roles of gypsum and calcium chloride in cement.

2. Explain the purpose of conducting soundness test of cement. Describe the apparatus and method of test with the help of neat sketches. What are the permissible limits of observation in the test?

3. Name the four important constitutents of cement and state the role of each in achieving its properties.

4. Describe the hydration of Portland cement and outline the ways in which the Vicat apparatus and the Le Chatelier apparatus can be used to assess the properties of fresh and hardened pastes.

5. Explain how sulphate resisting cement and rapid harening Portland cement differ from OPC and the specific circumstances in which these cements would be used.

6. Explain how do the Portland pozzolana cement and super sulphate cement differ from OPC. Under what specific circumstances these cements would be used?

7. List out the products of hydration and their influence on the properties of cement.

8. Explain pozzolanic action.

9. Which are the four important compounds formed during the setting action of cement (four principal minerals in ordinary Portland cement)? Mention their relative proportions expressed as percentages and also functions of these compounds.

10. List the various laboratory tests for assessing the quality of cement and their importance.

11. Describe the procedure to list the soundness of cement. Name the constituents causing soundness.

12. Explain the factors affecting the strength of concrete. What do you understand by **"controlled concrete"** and **"workability of concrete"**?

13. Explain how would you determine the following properties of concrete:

    (i) compressive strength

    (ii) Tensile strength

    (iii) Modulus of elasticity

14. Explain the factors affecting the strength of cement concrete.

15. Explain workability of concrete. Describe one method of determining workability of concrete.

16. Explain the following:

    (a) Bulking of sand

    (b) Fineness modulus

    (c) Water cement ratio

17. What is understood by compacting factor? Describe the method of determining the compacting factor of fresh concrete. What does it indicate?

18. What do you mean by water cement ratio? Describe the test to measure the workability of concrete in the field.

19. List the principal requirements of aggregate for concrete. Define fineness modulus of an aggregate and explain its significance.

20. Explain the phenomenon of bulking of sand and its significance. How is it determined?

21. What is menat by "workability of concrete"? List the methods used for measurement of workability of concrete. Clearly indicate the aspect of workability measured by each method.

22. Describe in brief the rebound hammer method on non-destructive testing of concrete.

23. Define workability of concrete. What are the factors affective workability of concrete?

24. Explain how bulking of fine aggregates take place and how it is taken care of in the field. Also explain the method of preparation of bulking chart in the laboratory.

25. Write about various modulii of elasticity of plain cement concrete. Which values are used in design. What are the factors affecting modulus of elasticity of concrete?

Answers PART-I

1. (c)	11. (c)	21. (d)	31. (b)
2. (b)	12. (b)	22. (b)	32. (c)
3. (c)	13. (b)	23. (c)	33. (c)
4. (c)	14. (c)	24. (c)	34. (b)
5. (b)	15. (b)	25. (a)	35. (a)
6. (a)	16. (a)	26. (b)	36. (d)
7. (b)	17. (b)	27. (d)	37. (d)
8. (c)	18. (b)	28. (c)	38. (d)
9. (a)	19. (a)	29. (b)	
10. (b)	20. (d)	30. (b)	

Answers PART-II

1. (b)	20. (a)	39. (c)	58. (b)
2. (b)	21. (d)	40. (c)	59. (c)
3. (c)	22. (c)	41. (b)	60. (b)
4. (c)	23. (d)	42. (c)	61. (b)
5. (c)	24. (a)	43. (c)	62. (c)
6. (b)	25. (c)	44. (b)	63. (b)
7. (c)	26. (b)	45. (b)	64. (d)
8. (b)	27. (a)	46. (c)	65. (c)
9. (d)	28. (a)	47. (b)	66. (b)
10. (b)	29. (d)	48. (c)	67. (a)
11. (b)	30. (a)	49. (b)	68. (c)
12. (b)	31. (d)	50. (d)	69. (c)
13. (b)	32. (b)	51. (a)	70. (a)
14. (b)	33. (c)	52. (a)	71. (d)
15. (c)	34. (d)	53. (a)	72. (a)
16. (b)	35. (a)	54. (a)	73. (c)
17. (c)	36. (b)	55. (b)	74. (d)
18. (a)	37. (d)	56. (d)	
19. (c)	38. (d)	57. (a)	

Additional IS 456 Recommendation

Cl. 5.3.3 Size of Aggregate

- The nominal maximum size of coarse aggregate should be as large as possible within the limits specified but in no case greater than one-fourth of the minimum thickness of the member, provided that the concrete can be placed without difficulty so as to surround all reinforcement thoroughly and fill the corners of the form.
- For most work, 20 mm aggregate is suitable. Where there is no restriction to the flow of concrete into sections, 40 mm or larger size may be permitted.
- In concrete elements with thin sections, closely spaced reinforcement or shall cover, consideration should be given to the use of 10 mm nominal maximum size.

Cl. 5.3.3.1

For heavily reinforced concrete members as in the case of ribs of main beams, the nominal maximum size of the aggregate should usually be restricted to 5 mm less than the minimum clear distance between the main bars or 5 mm less than the minimum cover to the reinforcement whichever is smaller.

CL. 5.4 WATER

(a) To neutralize 100 ml sample of water, using phenolphthalein as an indicator, it should not require more than 5 ml of 0.02 normal NaOH.

(b) To neutralize 100 ml sample of water, using mixed indicator, it should not require more than 25 ml of 0.02 normal H_2SO_4.

(c) Permissible limits for solids shall be as given in Table 1.

Table 1: Permissible limit for solids (*Clasuse* 5.4)

Sl. No.	*Tested as per Max*	*Permissible limit,*
(i) Organic	IS 3025 (Part 18)	200 mg/l
(ii) Inorganic	IS 3025 (Part 18)	3000 mg/l
(iii) Sulphates (as SO_3)	IS 3025 (Part 24)	400 mg/l
(iv) Chlorides (as Cl) for concrete not containing embedded steel and 500 mg/l for reinforced concrete work	IS 3025 (Part 32)	2,000 mg/l
(v) Suspended matter	IS 3025 (Part 17)	2000 mg/l

Cl. 5.4.1.2

- Average 28 days compressive strength of at least three 150 mm concrete cubes prepared with water proposed to be used shall not be less than 90 percent of the average of strength of three similar concrete cubes prepared with distilled water.

- The cubes shall be prepared, cured and tested in accordance with the requirements of IS 516.

Cl. 5.4.1.3

- The initial setting time of test block made with the appropriate cement and the water proposed to be used shall not be less than 30 min and shall not differ by ± 30 min from the initial setting time of control test block prepared with the same cement and distilled water.
- The test blocks shall be prepared and tested in accordance with the requirements of IS 4031 (Part 5).

Cl. 5.4.2

The *pH* value of water shall be not less than 6.

Cl. 5.5.1

Admixture, if used shall comply with IS-9103.

Cl. 5.6.3

The modulus of elasticity of steel shall be taken as 200 kN/mm².

Cl. 6.1.1

The characteristic strength is defined as the strength of material below which not more than 5 percent of the test results are expected to fall.

Table 2: Grades of concrete (*Clause* 6.1, 9.2.2, 15.1.1 and 36.1)

Group	Grade designation	Specified characteristic compressive strength of 150 mm cube at 28 days in N/mm²
(1)	(2)	(3)
Ordinary Concrete	M 10	10
	M 15	15
	M 20	20
Standard Concrete	M 25	25
	M 30	30
	M 35	35
	M 40	40
	M 50	50
	M 55	55
High Strength Concrete	M 60	60
	M 65	65
	M 70	70
	M 75	75
	M 80	80

Notes:

1. In the designation of concrete mix M refers to the mx and the number to the specified compressive strength of 150 mm size cube at 28 days, expressed in M/mm².
2. For concrete of compressive strength greater than M 55, design parameters given in the standard may not be applicable and the values may be obtained from specialized literatures and experimental results.

Cl. 6.2.1.1

For concrete of grade M 30 and above, the rate of increase of compressive strength with age shall be based on actual investigations.

Cl. 6.2.2 Tensile Strength of Concrete

The flexural and splitting tensile strengths shall be obtained as described in IS 516 and IS 5816 respectively. When the designer wishes to use an estimate of the tensile strength from the compressive strength, the following formula may be used:

$$\text{Flexural strength } f_{cr} = 0.7\sqrt{f_{ck}} \text{ N/mm}^2$$

where f_{ck} is the characteristic cube compressive strength of concrete in N/mm².

Cl. 6.2.3 Elastic Deformation

The modulus of elasticity is primarily influenced by the elastic properties of the aggregate and to a lesser extent by the conditions of curing and age of the concrete, the mix proportions and the type of cement. The modulus of elasticity is normall related to the compressive strength of concrete.

Cl. 6.2.3.1

The modulus of elasticity of concrete can be assumed as follows:

$$E_c = 5000\sqrt{f_{ck}}$$

where

E_c is the short term static modulus of elasticity in N/mm².

Actual measured values may differ by ± 20 present from the values obtained from the above expression.

Cl. 6.2.4.1

In the absence of test data, the approximate value of the total shrinkage strain for design may be taken as 0.0003 (for more information, *see* IS 1343).

Cl. 7.1.1

In the 'very low' category of workability where strict control is necessary, for example pavement quality concrete, measurement of workability by determination of compacting factor will be more appropriate than slump (*see* IS 1199) and a value of compacting factor of 0.75 to 0.80 is suggested.

The factors influencing durability include:

(a) The environment;

(b) the cover to embedded steel;

(c) the type and quality of constituent materials;

(d) the cement content and water/cement ratio of the concrete;

(e) workmanship, to obtain full compaction and efficient curing; and

(f) the shape and size of the member.

For the very high sulphate concentrations in Class 5 conditions, some form of lining such as polyethylene or polychloroprene sheet; or surface coating based on asphalt, chlorinated rubber, epoxy; or polyurethane materials should also be used to prevent access by the sulphate solution.

Cl. 8.2.5.2 Chlorodes in concrete

- Whenever there is chloride in concrete there is an increased risk of corrosion of embedded metal. The higher the chloride content, or if subsequently exposed to warm moist conditions, the greater the risk of corrosion.
- All constituents may contain chlorides and concrete may be contaminated by chlorides from the external environmental.
- To minimize the chances of deterioration of concrete from harmful chemical salts, the levels of such harmful salts in concrete coming from concrete materials, that is, cement, aggregates water and admixtures, as well as by diffusion from the environment should be limited.

Cl. 8.2.5.3 Sulphates in concrete

- Sulphates are present in most cements and in some aggregattes; excessive amounts of water-soluble sulphate from these or other mix constituents can cause expansion and disruption of concrete.
- To prevent this, the total water-soluble sulphate content of the concrete mix, expressed as SO_3, should not exceed 4 percent by mass of the cement in the mix.
- The 4 percent limit does not apply to concrete made with supersulphated cement complying with IS 6909.

Cl. 8.2.5.4 Alkali-aggregate reaction

- Some aggregates containing particular varieties of silica may be susceptible to attack by alkalis (Na_2O and K_2O) originating from cement or other sources, producing an expansive reaction which can causes cracking and disruption of concrete.
- Damage to concrete from this reaction will normally only occur when all the following are present together:
  (a) A high moisture level, within the concrete;
  (b) A cement with high alkali content, or another source of alkali;
  (c) Aggregate containing an alkali reactive constituent.

Cl. 8.2.8 Concrete in Sea-water

- Concrete in sea-water or exposed directly along the sea-coast shall be at least M 20 grade in the case of plain concrete and M 30 in case of reinforced concrete.
- The use of slag or pozzolana cement is advantageous under such conditions.

Cl. 8.2.8.1

Special attention shall be given to the design of the mix to obtain the densest possible concrete; slab, broken brick, soft limestone, soft sandstone, or other porous or weak aggregates shall not be used.

Cl. 8.2.8.3

- No construction joints shall be allowed within 600 mm below low water-level or within 600 mm of the upper and lower planes of wave action.

- Where unusually sevear conditions or abrasion are anticipated, such parts of the work shall be protected by bituminous or silico-fluoride coatings or stone facing bedded with bitumen.
- Design mix concrete is preferred to nominal mix.
- If design mix concrete cannot be used for any reason on the work for grades of M 20 or lower, nominal mixes may be used with the permission of engineer-in-charge, which, however, is likely to involved a higher cement content.

Cl. 9.2.2

- The target mean strength of concrete mix should be equal to the characteristic strength plus 1.65 times the standard deviation.

(a) ***Number of test results of samples***—The total number of test strength of samples required to constitute an acceptable record for calculation of standard deviation shall be not less than 30. Attempsts should be made to obtain the 30 samples, as early as possible, when a mix is used for the first time.

CL. 9.3 NOMINAL MIX CONCRETE

Nominal mix concrete may be used for concrete of M 20 or lower.

Cl. 10.3.3

Dosages of retarders, plasticisers and superplasticisers shall be restricted to 0.5, 1.0 and 2.0 percent respectively by weight of cementitious materials and unless a higher value is agreed upon between the manufacturer and the constructor based on performance test.

CL. 11.3 STRIPPING TIME

Forms shall not be released until the concrete has achieved a strength of at least twice the stress to which the concrete may be subjected at the time of removal of formwork.

Type of Formwork Striking Formwork	*Minimum Period before*
(a) Vertical formwork to columns, walls, beams	16-24 h
(b) Soffit formwork to slabs (Props to be refixed immediately after removal of formwork)	3 days
(c) Soffit formwork to beams (Props to be refixed immediately after removal of formwork)	7 days
(d) Props to slabs:	
(1) Spanning up to 4.5 m	7 days
(2) Spanning over 4.5 m	14 days
(e) Props to beams and arches:	
(1) Spanning up to 6 m	14 days
(2) Spanning over 6 m	21 days

Cl. 12.3.1Tolerances on Placing of Reinforcement

Unless otherwise specified by engineer-in-charge, the reinforcement shall be palced within the following tolerances:

(a) for effective depth 200 mm or less $\pm$ 10 mm

(b) for effective depth more than 200 mm $\pm$ 15 mm

Cl. 12.3.2 Tolerance for Cover

Unless specified otherwise, actual concrete cover should not deviate from the required nominal cover by $^{+10}_{\ 0}$ mm.

CL. 12.5

- Where reinforcement bars upto 12 mm for high strength deformed steel bars and up to 16 mm for mild steel bars are bent aside at construction joints and afterwards bent back into their original positions, care should be taken to ensure that at no time is the radius of the bend less than 4 bar diameters for plain mild steel or 6 bar diameters for deformed bars.
- Care shall also be taken when bending back bars, to ensure that the concrete around the bar is not damaged beyond the band.

CL. 13.2

As a general guidance, the maximum permissible free fall of concrete may be taken as 1.5 m. Where high shear resistance is required at the construction joints, shear keys may be provided.

Cl. 14.2.1

When it is necessary to deposit concrete under water, the methods, equipment, materials and proportions of the mix to be used shall be submitted to and approved by the engineer-in-charge before the work is started.

Cl. 14.2.2

- Under-water concrete should have a slump recommended in Table 7.1 of IS 456.
- The water-cement ratio shall not exceed 0.6 and may need to be smaller, depending on the grade of concrete or the type of chemical attack.
- For aggregates of 40 mm maximum particle size, the cement content shall be at least 350 kg/m^3 of concrete.

CL. 15.4 TEST RESULTS OF SAMPLE

The test results of the sample shall be the average of the strength of three specimens. The individual variation should not be more than $\pm$ 15 percent of the average. If more, the test results of the sample are invalid.

CL. 16.2: FLEXURAL STRENGTH

When both the following conditions are met, the concrete complies with the specified flexural strength.

(a) The mean strength determined from any group of four consecutive test results exceeds the specified characteristic strength by at least 0.3 N/mm^2.

(b) The strength determined from any test result is not less than the specified characteristic strength less 0.3 N/mm^2

Table 11 Characteristics compressive strength compliance requirement (*Clauses 16.1 and 16.3*)

Specified grade	Mean of the group of 4 Non-overlapping consecutive Test results in N/mm²	Individual Test results in N/mm²
(1)	(2)	(3)
M 15	$\geq f_{ck}$ + 0.825 × established standard deviation (rounded off to nearest 0.5 N/mm²) or f_{ck} + 3 N/mm², whichever is greater	$\geq f_{ck}$ -3 N/mm²
M 20 or above	$\geq f_{ck}$ + 0.825 × established standard deviation (rounded off to nearest 0.5 N/mm²) or f_{ck} + 4 N/mm², whichever is greater	$\geq f_{ck}$ -4 N/mm²

Cl. 17.6.2

The structure should be subjected to a load equal to full dead load of the structure plus 1.25 times the imposed load for a period of 24 h and then the imposed load shall be removed.

Cl. 17.6.3

- The deflection due to imposed load only shall be recorded. If within 24 h of removal of the imposed load the structure does not recover at least 75 percent of the deflection under superimposed load, the test may be repreated after a lapse of 72 h.
- If the recovery is less than 80 percent, the structure shall be deemed to be unacceptable.

Cl. 17.6.3.1

If the maximum deflection in mm, shown during 24 h under load is less than $40l^2/D$, where l is the effective span in m; and D, the overall depth of the section in mm.

Note: If the maximum deflection in mm, shown during 24 h under load is less than 40 l^2/D where l is the effective span in m; and D is the overall depth of the section in mm, it is not necessary for the recover to be measured.

CL. 19.3 IMPOSED LOADS, WIND LOADS AND SNOW LOADS

Imposed loads, wind loads and snow loads shall be assumed in accordance with IS 875 (Part 2), IS 875 (Part 3) and IS 875 (Part 4) respectively.

CL. 19.4 EARTHQUAKE FORCES

The earthquake forces shall be calculated in accordance with IS 1893.

Cl. 19.5.1

In ordinary buildings, such as low rise dwellings whose lateral dimension do not exceed 45 m, the effects due to temperature fluctuations and shrinkage and creep can be ignored in design calculations.

CL. 20.5 LATERAL SWAY

Under transient wind load the lateral sway at the top should not exceed $H/500$, where H is the total height of the building. For seismic loading, reference should be made to iS 1893.

Cl. 21.3.1

Additional measures such as application of fire resistant finishes, provision of fire resistant false ceilings and sacrificial steel in tensile zone, should be adopted in case the nominal cover required exceeds 40 mm for beams and 35 mm for slabs, to give protection against spalling.

CL. 22.1 GENERAL

All structures may be analyzed by the linear elastic theory to calculate internal actions produced by design loads.

CL. 22.2 EFFECTIVE SPAN

Unless otherwise specified, the effective span of a member shall be as follows:

(a) *Simply supported beam or slab*—The effective span of a member that is not built integrally with its supports shall be taken as clear span plus the effective depth of slab or beam or centre to centre of supports, whichever is less.

(b) *Continuous Beam or Slab*—In the case of continuous beam or slab, if the width of the supports is less than 1/12 of the clear span, the effective span shall be as in 22.2 (a). If the supports are wider than 1/12 of the clear span or 600 mm whichever is less, the effective span shall be taken as under:

   (i) For end span with one end fixed and the other consinuous or for intermediate spans, the effective span shall be the clear span between supports;

   (ii) For end span with one end free and the other continuous, the efffective span shall be equal to the clear span plus half the effective depth of the beam or slab or the clear span plus half the width of the discontinuous support, whichever is less;

   (iii) In the case of spans with roller or rocket bearing, the effective span shall always be the distance between the centres of bearings.

(c) *Cantilever*—The effective length of a cantilever shall be taken as its length to the face of the support plus half the effective depth except where it forms the end of a continuous beam where the length to the centre of support shall be taken.

(d) *Frames*—In the analysis of a continuous frame, centre to centre distance shall be used.

CL. 23.0 EFFECTIVE DEPTH

Effective depth of a beam is the distance between the centroid of the area of tension reinforcement and the maximum compression fibre, excluding the thickness of finishing material not placed monolithically with the member and the thickness of any concrete provided to allow for wear. This will not apply to deep beams.

Cl. 23.1.1 General

A slab which is assumed to act as a compression flange of a T-beam or L-beam shall satisfy the following:

(a) The slab shall be case integrally with the web, or the web and the slab shall be effectively bonded together in any other manner; and

(b) If the main reinforcement of the slab is parallel to the beam, transverse reinforcement shall be provided; such reinforcement shall not be less than 60 percent of the main reinforcement at mid span of the slab.

Cl. 23.1.2: Effective Width of Flange

IN thé absence of more accurate determination, the effective width of flange may be taken as the following but in no case greater than the breadth of the web plus half the sum of the clear distances to the adjacent beams on either side.

(a) For T-beams, $b_f = \frac{l_0}{6} + b_w + 6D_f$

(b) For L-beams, $b_f = \frac{l_0}{12} + b_w + 3D_f$

(c) For isolated beams, the effective flange width shall be obtained as below but in no case greater than the actual width:

$$\text{T-beam, } b_f = \frac{l_0}{\left(\frac{l_0}{b}\right)+4} + b_w$$

$$\text{L-beam, } b_f = \frac{0.5\, l_0}{\left(\frac{l_n}{b}\right)+4} + b_w$$

where

b_f = effective width of flange,

l_0 = distance between points of zero moments in the beam,

b_w = breath of the web,

D_f = thickness of flange, and

b = actual width of the flange.

Note: For continuous beams and frames, 'l_0' may be assumed as 0.7 times the effective span.

CL. 23.2 CONTROL OF DEFLECTION

The deflection of a structure or part thereof shall not adversely affect the appearance or efficiency of the structure of finishes or partitions. The deflection shall generally be limited to the following:

(a) The final deflection due to all loads including the effects of temperature, creep and shrinkage and measured from the as-cast level of the supports of floors, roofs and all other horizontal 0.4% members, should not normally exceed span/250.

(b) The deflection including the effects of temperature, creep and shrinkage occurring after erection of partitions and the application of 0.28% finishes should not normally exceed span/350 or 20 mm whichever is less.

Cl. 23.2.1

The vertical deflection limits may generally be assumed to be satisfied provided that the span to depth ratios are not greater than the values obtained as below:

(a) Basic values of span to effective depth ratios for spans up to 10 m:

Cantilever 7

Simply supported 20

Continous 26

(b) For spans above 10 m, the values in (a) may be multiplied by 10/span in metres, except for cantilever in which case deflection calculations should be made.

(c) Depending on the area and the stress of steel for tension reinforcement, the values in (a) or (b) shall be modified by multiplying with the modification factor obtained as per Fig. 4.

(d) Depending on the area of compression reinforcement, the values of span to depth ratio be further modified by multiplying with the modification factor obtained as per Fig. 5.

CL. 23.3 SLENDERNESS LIMITS FOR BEAMS TO ENSURE LATERAL STABILITY

A simply supported or continuous beam shall be so proportioned that the clear distance between the lateral restraints does not exceed 60 b or $\frac{250\,b^2}{d}$ whichever is less, where d is the effective depth of the beam and b the breadth of the compression face midway between the lateral restraints.

For a cantilever, the clear distance from the free end of the cantilever to the lateral restraint shall not exceed 25 b or $\frac{100\,b^2}{d}$ which ever is less.

CL. 24.1 GENERAL

The provisions of 23.2 for beams apply to slabs also.

Notes:

1. For slabs spanning in two-directions, the shorter of the two spans should be used for calculating the span of effective depth ratios.
2. For two-way slabs of shorter spans (up to 3.5 m) with mild steel reinforcement, the span to overall depth ratios given below may generally be assumed to satisfy vertical deflection limits for loading class up to 3 kN/m^2.

   Simply supported slabs 35

   Continuous slabs 40

For high strength deformed bars of grade Fe 415, the values given above should be multiplied by 0.8.

Note: The most commonly used elastic methods are based or Pigeaud's or Wester-guard's theory and the most commonly used limit state of collapse method is based on Johansen's yield-line theory.

CL. 25.1 DEFINITIONS

Cl. 25.1.1

Column or strut is a compression member, the effective length of which exceeds three times the least lateral dimension.

Cl. 25.1.2 Short and Slender Compression Members

A compression member may be considered as short when both the slenderness ratios $\frac{l_{ex}}{D}$ and $\frac{l_{ey}}{b}$ are less than 12:

where

l_{ex} = effective length in respect of the major axis,

D = depth in respect of the major axis,

l_{ey} = effective length in respect of the minor axis, and

b = width of the member.

It shall othwerwise be considered as a slender compression member.

Cl. 25.1.3 Unsupported Length

The unsupported length, l, of a compression member shall be taken as the clear distance between end restraints except that:

(a) in flat slab construction, it shall be clear distance between the floor and the lower extremity of the capital, the drop panel or slab whichever is the least.

(b) in beam and slab construction, it shall be the clear distance between the floor and the underside of the shallower beam framing into the columns in each direction at the next higher floor level.

(c) in columns restrained laterally by struts, it shall be the clear distance between consecutive struts in each vertical plane, provided that to be an adequate support, two such struts shall meet the columns at approximately the same level and the angle between vertical planes through the struts shall not vary more than 30° from a right angle. Such struts shall be of adequate dimensions and shall have sufficient anchorage to restrain the member against lateral deflection.

(d) in columns restrained laterally by struts or beams, with brackets used at the junction, it shall be the clear distance between the floor and the lower edge of the braket, provided that the brakcet width equals that of the beam strut and is at least half that of the column.

CL. 25.3 SLENDERNESS LIMITS FOR COLUMNS

Cl. 25.3.1

The unsupported length between end restraints shall not exceed 60 times the least lateral dimension of a column.

where

b = width of that cross-section, and

D = depth of the cross-section measured in the plane under consideration.

CL. 25.4 MINIMUM ECCENTRICITY

All columns shall be designed for minimum eccentricity, equal to the unsupported length of column/500 plus lateral dimensions/30, subject to a minimum of 20 mm. Where bi-axial bending is considered, it is sufficient to ensure that eccentricity exceeds the minimum about one axis at a time.

CL. 26.1 GENERAL

Reinforcing steel of same type and grade shall be used as main reinforcement in a structural member. However, simultaneous use of two different types or grades of steel for main and secondary reinforcement respectively is permissible.

Cl. 26.1.1

Bars may be arranged singly, or in pairs in contact, or in groups of three or four bars bundled in contact. Bundled bars shall be a enclosed within stirrups or ties. Bundled bars shall be tied together to ensure the bars remaining together. Bars larger than 32 mm diameter shall not be bundled, except in columns.

Cl. 26.1.2

The recommendations for detailing for earthquake-resistant construction given in IS 13920 should be tkane into consideration, where applicable (see also IS 4326).

Cl. 26.2.1 Development Length of Bars

The developement length L_d is given by

$$L_d = \frac{\phi\sigma_s}{4\tau_{bd}}$$

where

ϕ = nominal diameter of the bar,

σ_s = stress in bar at the section considered at design load, and

τ_{bd} = design bond stress given in **26.2.1.1**.

Notes:

1. The development length includes anchorage values of hooks in tension reinforcement.
2. For bars of sections other than circular, the development length should be sufficient to develop the stress in the bar by bond.

For deformed bars conforming to IS 1786 these values shall be increased by 60 percent.

For bars in compression, the values of bond stress for bars in tension shall be increased by 25 percent.

Cl. 26.2.1.2 Bars bundled in Contact

The developement length of each bar of bundled bars shall be that for the individual bar, increased by 10 percent for two bars in contact, 20 percent for three bars in contact and 33 percent for four bars in contact.

Cl. 26.2.2.1 Anchoring bars in tension

(a) Deformed bars may be used without end anchorages provided development length requirement is satisfied. Hooks should normally be provided for plain bars in tension.

(b) *Bends and hooks*—Bends and hooks shall conform to IS 2502

(1) ***Bends***—The anchorage value of bend shall be taken as 4 times the diameter of the bar for each 45° bend subject to a maximum of 16 times the diameter of the bar.

(2) ***Hooks***—The anchorage value of a standard U-type hook shall be equal to 16 times the diameter of the bar.

Cl. 26.2.2.2 Anchoring bars in compression

The anchorage length of straight bar in compression shall be equal to the development length of bars in compression as specified in **26.2.1**. The projected length of hooks, bends and straight lengths beyond bends if provided for a bar in compression, shall only be considered for development length.

Cl. 26.2.2.5 Bearing Stresses at bends

The bearing stress in concrete for bends and hooks described in IS 2502 need not be checked. The bearing stress inside a bend in any other bend shall be calculated as given below:

$$\text{Bearing stress} = \frac{F_{bt}}{r\phi}$$

where

F_{bt} = tensile force due to design loads in a bar or group of bars,

r = internal radius of the bend, and

ϕ = size of the bar or, in bundle, the size of bar of equivalent area.

For limit state method of design, this stress shall not exceed $\dfrac{1.5 f_{ck}}{1+2\phi/a}$ where f_{ck} is the characteristic cube strength of concrete and a, for a particular bar or group of bars in contact shall be tkane as the centre to centre distance between bars or groups of bars perpendicular to the plane of the bend; for a bar or group of bars adjacent to the face of the member a shall be taken as the cover plus size of bar (ϕ). For working stress method of design, the bearing stress shall not exceed $\dfrac{f_{ck}}{1+2\phi/a}$.

Cl. 26.2.3 Curtailment of Tension Reinforcement in Flexural Members

Cl. 26.2.3.1

For curtailment, reinforcement shall extend beyond the point at which it is no longer required to resist flexure for a distance equal to the effective depth of the member or 12 times the bar diameter, whichever is greater except at simple support or end of cantilever. In addition **26.2.3.2** to **26.2.3.5** shall also be satisfied.

Note: A point at which reinforcement is no longer required to resist flexure is where the resistance moment of the section, considering only the continuiing bars, is equal to the design moment.

Cl. 26.2.3.2

Flexural reinforcement shall not be terminated in a tension zone unless any one of the following conditions is satisfied:

(a) The shear at the cut-off point does not *exceed* two-thirds that permitted, including the shear strength of web reinforcement provided.

(b) Stirrup area in excess of that required for shear and torsion is provided along each terminated bar over a distance from the cut-off point equal to three-fourths the effective depth of the member. The excess stirrup area shall be not less than 0.4 bs/f_y, where b is the breadth of beam, s is the spacing and f_y is the characteristic strength of reinforcement in N/mm². The resulting spacing shall not exceed $d/8\,\beta_b$ where β_b is the ratio of the area of bars cut-off to the total area of bars at the section, and d is the effective depth.

(c) For 36 mm and smaller bars, the continuiing bars provide double the area required for flexure at the cut-off point and the shear does not exceed three-fourths that permitted.

Cl. 26.2.3.3 Positive moment reinforcement

(a) At least one-third the positive moment reinforcement in simple members and one-fourth the positive moment reinforcement in continuous members shall extend along the same face of the member into the support, to a length equal to $L_d/3$.

(b) When a flexural member is part of the primary lateral load resisting system, the positive reinforcement required to be extended into the support as described in (a) shall be anchored to develop its design stress in tension at the face of the support.

(c) At simple supports and at points of inflection, positive moment tension reinforcement shall be limited to a diameter such that L_d computed for f_d by **26.2.1** does not exceed

$$\frac{M_1}{V} + L_0$$

where

M_1 = moment of resistance of the section assuming all reinforcement at the section to be stressed to f_d;

$f_d = 0.87\,f_y$ in the case of limit state design and the permissible stress σ_{st} in the case of working stress design;

V = shear force at the section due to design loads;

L_0 = anchorage beyond the centre of the support and the equivalent anchorage value of any hook or mechanical anchorage at simply supported and at a point of inflection, L_0 is limited to the effective depth of the members or 12ϕ, whichever is greater, and

ϕ = diameter of bar.

The value of M_1/V in the above expression may be increased by 30 percent when the ends of the reinforcement are confined by a compressive reaction.

Cl. 26.2.3.4 Negative moment reinforcement

At least one-third of the total reinforcement provided for negative moment at the support shall extend beyond the point of inflection for a distance not less than the effective depth of the member of 12ϕ or one-sixteenth of the clear span whichever is greater.

Cl. 26.2.3.5 Curtailment of bundled bars

Bars in a bundle shall terminate at different points spaced apart by not less than 40 times the bar diameter except for boundles stopping at a support.

Cl. 26.2.5.1 Lap splices

(a) Lap splices shall not be used for bars larger than 36 mm; for larger diameters, bars may be welded (*see* 12.4); in case where welding is not practicable, lapping of bars larger than 36 mm may be permitted, in which case additional spirals should be provided around the lapped bars.

(b) Lap splices shall be sondiered as staggered if the centre to centre distance of the splices is not less than 1.3 times the lap length calculated as described in (c).

(c) Lap length including anchorage value of hooks for bars in flexural tension shall be L_4 (*see* **26.2.1**) or 30ϕ whichever is greater and for direct tension shall be $2L_4$ or 30ϕ whichever is greater. The straight length of the lap shall not be less than 15ϕ or 200 mm. The following provisions shall also apply:

Where lap occurs for a tension bar located at:

(1) top of a section as cast and the minimum cover is less than twice the diameter of the lapped bar, the lap length shall be increased by a factor of 1.4.

(2) corner of a section and the minimum cover to either face is less than twice the diameter of the lapped bar or where the clear distance between adjacent laps is less than 75 mm or 6 times the diameter of lapped bar, whichever is greater, the lap length should be increased by a factor of 1.4.

Where both condition (1) and (2) apply, the lap length should be increased by a factor of 2.0.

Note: Splices in tension members shall be enclosed in spirals made of bars not less than 6 mm diameter with pitch not more than 100 mm.

(d) The lap length in compression shall be equal to the development length in compression, calculated as described in **26.2.1**, but not less than 24 ϕ.

(e) When bars of two different diameters are to be spliced, the lap length shall be calculated on the basis of diameter of the smaller bar.

(f) When splicing of welded wire fabric is to be carried out, lap splices of wires shall be made so that overlap measured between the extreme cross wires shall be not less than the spacing of cross wires plus 100 mm.

(g) In case of bundled bars, lapped splices of bundled bars shall be made by splicing one bar at a time; such individual splices within a bundle shall be staggered.

Cl. 26.2.5.2 Strength of welds

The following values may be used where the strength of the weld has been provided by tests to be at least as great as that of the parent bar.

(a) *Splices in compression*—For welded splices and mechanical connection, 100 percent of the design strength of joined bars.

(b) *Splices in tension*

(1) 80 percent of the design strength of welded bars (100 percent if welding is strictly supervised and if at any cross-section of the member not more than 20 percent of the tensile reinforcement is welded).

(2) 100 percent of design strength of mechanical connection.

Cl. 26.2.5.3 End-bearing splices

End bearing splices shall be used only for bars in compression. The ends of the bars shall be square cut and concentric bearing ensured by suitable devices.

Cl. 26.3.2 Minimum distance between individual bars

The following shall apply for spacing of bars:

(a) The horizontal distance between two parallel main reinofrcing bars shall usually be not less than the greatest of the following:

(1) The diameter of the bar if the diameters are equal,

(2) The diameter of the larger bar if the diameters are unequal, and

(3) 5 mm more than the nominal maximum size of coarse aggreagate.

Note: This does not preclude the use of larger size of aggregates beyond the congested reinforcement in the same member; the size of aggregates may be reduced around congested reinforcement to comply with this provision.

(b) Greater horizontal distance than the minimum specified in (a) should be provided whereever possible. However, when needle vibrators are used the horizontal distance between bars of a group may be reduced to two-thirds the nominal maximum size of the coarse aggregate, provided that sufficient space is left between groups of bars to enable the vibrator to be immersed.

(c) Where there are two or more rows of bars, the bars shall be vertically in line and the minimum vertical distance between the bars shall be 15 mm, two-thirds the nominal maximum size of aggregate or the maximum size of bars, whichever is greater.

Cl. 26.3.3 Maximum distance between bars in tension

Unless the calculation of crack widths shows that a greater spacing is acceptable, the following rules shall be applied to flexural members in normal internal or external conditions of exposure.

(b) *Slabs*

(1) The horizontal distance between parallel main reinforcement bars shall not be more than three times the effective depth of solid slab or 300 mm whichever is smaller.

(2) The horizontal distance between parallel reinforcement bars provided against shrinkage and temperature shall not be more than five times the effective depth of a solid slab or 450 mm whichever is smaller.

Cl. 26.4.1 Nominal Cover

Nominal cover is the design depth of concrete cover to all steel reinforcements, including links. It is the dimension used in design and indicated in the drawings. It shall be not less than the diameter of the bars.

Cl. 26.4.2.1

However for a longitudinal reinforcing bar in a column nominal cover shall in any case not be less than 40 mm, or less than the diameter of such bar. In the case of columns of minimum dimension of 200 mm or under, whose reinforcing bars do not exceed 12 mm, a nominal cover of 25 mm may be used.

Cl. 26.4.2.2

For footings minimum cover shall be 50 mm.

CL. 26.5 REQUIREMENTS OF REINFORCEMENT FOR STRUCTURAL MEMBERS

Cl. 26.5.1 Beams

Cl. 26.5.1.1 Tension reinforcement

(a) *Minimum reinforcement*—The minimum area of tension reinforcement shall be not less than that given by the following:

$$\boxed{\frac{A_s}{bd} = \frac{0.85}{f_y}}$$

where

A_s = minimum area of tension reinforcement,

b = breadth of beam or the breadth of the web of T-beam,

d = effective depth, and

f_y = characteristic strength of reinforcement in N/mm².

(b) *Maximum reinforcement*—The maximum area of tension reinforcement shall not exceed 0.04 bD.

Cl. 26.5.1.2 Compression reinforcement

The maximum area of compression reinforcement shall not exceed 0.04 bD. Compression reinofrcement in beams shall be enclosed by stirrups for effective lateral restraint. The arrangement of stirrups shall be as specified in 26.5.3.2.

Cl. 26.5.1.3 Side face reinforcement

Where the depth of the web in a beam exceeds 750 mm, side face reinforcement shall be provided along the two faces. The total area of such reinforcement shall be not less than 0.1 percent of the web area and shall be distributed equally on two faces at a spacing not exceeding 300 mm or web thickness whichever is less.

Cl. 26.5.1.4: Transverse reinforcement in beams for shear and torsion

The transverse reinforcement in beams shall be taken around the outer-most tension and compression bars. In T-beams and I-beams, such reinforcement shall pass around longitudinal bars located close to the outer face of the flange.

Cl. 26.5.1.5: Maximum spacing of shear reinforcement

The maximum spacing of shear reinforcement measured along the axis of the member shall not exceed 0.75 d for vertical stirrups and d for inclined stirrups at 45, where d is the effective depth of the section under consideration. In non case shall the spacing exceed 300 mm.

Cl. 26.5.1.6: Minimum shear reinforcement

Minimum shear reinforcement in the form of stirrups shall be provided such that:

$$\frac{A_{sv}}{bs_v} \geq \frac{0.4}{0.87\, f_y}$$

where

A_{sv} = total cross-sectional area of stirrup legs effective in shear,

s_v = stirrup spacing along the length of the member,

b = breadth of the beam or breadth of the web of flanged beam, and

f_y = characteristic strneght of the stirrup reinforcement in N/mm² which shall not be taken greater than 415 N/mm².

Where the maximum shear stress calculated is less than half the permissible value and in members of minor structural importance such as lintels, this provision need not be complied with.

Cl. 26.5.1.7 Distribution of torsion reinforcement

When a member is designed for torsion (*see* **41** or **B-6**) torsion reinforcement shall be provided as below:

(a) The transverse reinforcement for torsion shall be rectangular closed stirrups placed perpendicular to the axis of the member. The spacing of the stirrups shall not exceed the least of x_1, $\frac{x_1 + y_1}{4}$ and 300 mm, where x_1 and y_1 are respectively the short and long dimensions of the stirrup.

(b) Longitudinal reinforcement shall be placed as close as is practicable to the corners of the cross-section and in all cases, there shall be at least one longitudinal bar in each corner of the ties. When the cross-sectional dimension of the member exceeds 450 mm, additional longitudinal bars shall be provided to satisfy the requirements of minimum reinforcement and spacing given in **26.5.1.3**.

Cl. 26.5.1.8

Reinforcement in flanges of T-and L-beams shall satisfy the requirements in **23.1.1** (b). Where flanges are in tension, a part of the main tension reinforcement shall be distributed over the effective flange width or a width equal to one-tenth of the span, whichever is smaller. If the effective flange width exceeds one-tenth of the span, nominal longitudinal reinforcement shall be provided in the outer portions of the flange.

Cl. 26.5.2 Slabs

The rules given in **26.5.2.1** and **26.5.2.2** shall apply to slabs in addition to those given in the appropriate clauses.

Cl. 26.5.2.1 Minimum reinforcement

The mild steel reinofrcement in either direction in slabs shall not be less than 0.15 percent of the total cross-sectional area. However, this value can be reduced to 0.12 percent when high strength deformed bars or welded wire fabric are used.

Cl. 26.5.2.2.2 Maximum diameter

The diameter of reinforing bars shall not exceed one-eight of the total of the slab.

Cl. 26.5.3 Columns

Cl. 26.5.3.1 Longitudinal reinforcement

(a) The cross-sectional area of longitudinal reinforcmene,t shall be not less than 0.8 percent nor more than 6 percent of the gross cross-sectional area of the column.

Note: The use of 6 percent reinforcement may involved practical difficulties in placing and compacting of concrete; hence lower percentage is recommended. Where bars from the columns below have to be lapped with those in the column under consideration, the percentage of usually not exceed 4 percent.

(b) In any column that has a larger cross-sectional area than that required to support the load, the minimum percentage of steel shall be based upon the area of concrete required to resist the direct stress and not upon the actual area.

(c) The minimum number of longitudinal bars provided in a column shall be four in rectangular columns and six in circular columns.

(d) The bars shall not be less than 12 mm in diameter.

(e) A reinforced concrete column having helical reinforcement shall have at least six bars of longutudinal

(g) Spacing of longitudinal bars measured along the periphery of the column shall not exceed 300 mm.

(h) In case of pedestals in which the longutudinal reinforcement is not taken in account in strength calculations, nominal longutidnal reinforcement not less than 0.15 percent of the cross-sectional area shall be provided.

Note: Pedestal is a compression member, the effective length of which does not exceed three times the least lateral dimension.

(1) Transverse reinforcement is provided for the outer-most row in accorance with **26.5.3.2**.

(c) *Pitch and diameter of lateral ties*

(1) *Pitch*—The pitch of transverse reinofrcement shall be not more than the least of the following distances:

(i) The least lateral dimension of the compression members;

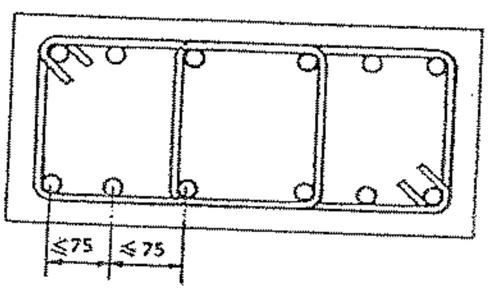
(ii) Sixteen times the smallest diameter of the longitudinal reinforcement bar to be tied; and

(iii) 300 mm

(2) *Helical reinforcement*

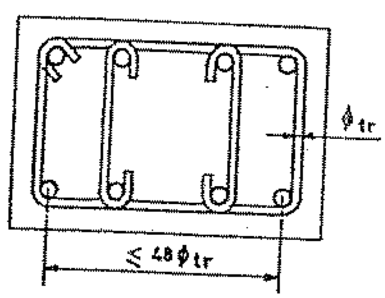
(1) *Pitch*—Helical reinforcement shall be of regular formation with the turns of the helix spaced evenly the its ends shall be anchored properly by providing one and a helf extra turns of the spiral bar. Where an increased load on the column on the strength of the helical reinofrcment is allowed for, the pitch of helical turns shall be not more than 75 mm, nor more than one-sixth of the core diameter of the column, nor less than 25 mm, nor less than three times the diameter of the steel bar forming the helix. In other cases, the requirements of **26.5.3.2** shall be complied with.

(2) The diameter of the helical reinforcement shall be in accordance with **26.5.3.2** (c) (2).



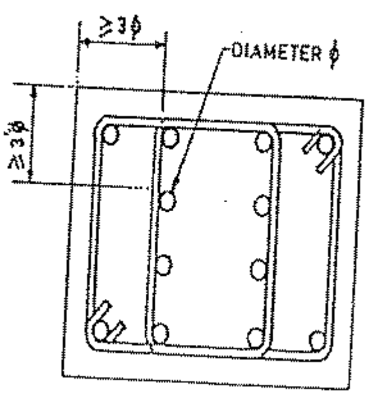
All dimensions in millimetres.

FIG. 8



All dimensions in millimetres.

FIG. 9



All dimensions in millimetres.

FIG. 10

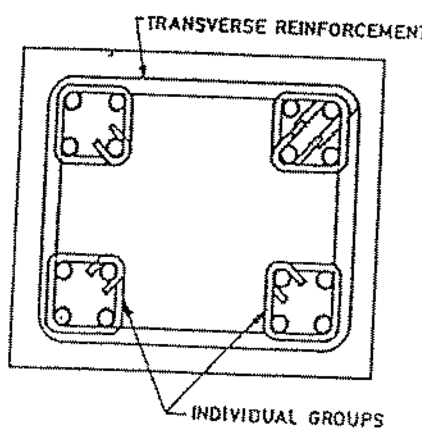


FIG. 11

CL. 29 DEEP BEAMS

CL. 29.1 GENERAL

(a) A beam shall be deemed to be a deep beam when the ratio of effective span to overall depth, $\frac{l}{D}$ is less than:

(1) 2.0 for a simply supported beam; and

(2) 2.5 for a continuous beam.

(b) A deep beam complying with the requirements of 29.2 and 29.3 shall be deemed to satisfy the provisions for shear.

CL. 29.2 LEVER ARM

The lever arm z for a deep beam shall be determined as below:

(a) For simply supported beams:

$$z = 0.2\,(l + 2D) \qquad \text{when } 1 \le \frac{l}{D} \le 2$$

or

$$z = 0.16\,l \qquad \text{when } \frac{l}{D} < 1$$

(b) For continuous beams:

$$z = 0.2\,(l + 1.5\,D) \qquad \text{when } 1 \le \frac{l}{D} \le 2.5$$

or

$$z = 0.5\,l \qquad \text{when } \frac{l}{D} < 1$$

where l is the effective span taken as centre of centre distance between supports or 1.15 times the clear span, whichever is smaller, and D is the overall depth.

Cl. 31.2.1 Thickness of Flat Slab

The minimum thickness of slab shall be 125 mm.

CL. 31.6 SHEAR IN FLAT SLAB

Cl. 31.6.1

The critical section for shear shall be at a distance $d/2$ from the periphery of the column/capital/drop panel, perpendicular to the plane of the slab where d is the effective depth of the section.

Cl. 31.6.3 Permissible Shear Stress

31.6.3.1

When shear reinforcement is not provided, the calculated shear stress at the critical section shall not exceed $k_s \tau_c$.

where, $k_s = (0.5 + \beta_c)$ but not greater than 1, β_c being the ratio of short side to long side of the column/capital; and

$\tau_c = 0.25\sqrt{f_{ck}}$ in limit state method of design, and $0.16\sqrt{f_{ck}}$ in working stress method of design.

The minimum thickness of walls shall be 100 mm.

Cl. 32.2.2 Eccentricity of Vertical Load

The design of a wall shall take account of the actual eccentricity of the vertical force subject to a minimum value of 0.05 t.

The ratio of effective height to thickness, H_{we}/t shall not exceed 30.

Cl. 32.2.4 Effective Height

The effective height of a braced wall shall be taken as follows:

(a) Where restrained against rotation at both ends by

(1) floors	$0.75\, H_w$ or
(2) intersecting walls or similar members whichever is the lesser.	$0.75\, L_1$

(b) Where not restrained against rotation at both ends by

(1) floors	$1.0\, H_w$ or
(2) intersecting walls or similar members whichever is the lesser.	$1.0\, L_1$

where

H_w = the unsupported height of the wall.

L_1 = the horizontal distance between centres of lateral restraint.

CL. 32.5 MINIMUM REQUIREMENTS FOR REINFORCEMENT IN WALLS

The reinforcement for walls shall be provided as below:

(a) the minimum ratio of vertical reinforcement to gross concrete area shall be:

(1) 0.001 2 for deformed bars not larger than 16 mm in diameter and with a characteristic strength of 415 N/mm² or greater.

(2) 0.001 5 for other types of bars.

(3) 0.001 2 for welded wire fabric not larger than 16 mm in diameter.

(b) Vertical reinforcement shall be spaced not farther apart than three times the wall thickness nor 450 mm.

(c) The minimum ratio of horizontal reinforcement to gross concrete area shall be:

(1) 0.002 0 for deformed bars not larger than 16 mm in diameter and with a characteristic strength of 415 N/mm² or greater.

(2) 0.002 5 for other types of bars.

(3) 0.002 0 for welded wire fabric not larger than 16 mm in diameter.

(d) Horizontal reinforcement shall be spaced not farther apart than three times the wall thickness nor 450 mm.

Note: The minimum reinforcement may not always be sufficient to provide adequate resistance to the effects of shrinkage and temperature.

Cl. 32.5.1

For walls having thickness more than 200 mm, the vertical and horizontal reinforcement shall be provided in two grids, one near each face of the wall.