CIVIL ENGINEERING

For

UPSC Engineering Services Examination,

State Engineering Service Examination & Public Sector Examination

(BHEL, NTPC, NHPC, DRDO, SAIL, HAL, BSNL, BPCL, NPCL, etc.)

SOIL MECHANICS

I.E.S MASTER

Institute for Engineers

IES/GATE/PSUs

Office: F-126, Katwaria Sarai, New Delhi - 110 016

Phone: [illegible]

Website: www.iesmaster.org, E-mail: ies_master@yahoo.co.in

Office: F-126, Katwaria Sarai, New Delhi - 110 016
Phone: 011-41013406, 7838813406, 9711853908
Website: www.iesmaster.org, E-mail: ies_master@yahoo.co.in

CONTENTS

CHAPTER

1

Origin of Soil and Soil Water Relationship

DEFINITION OF SOIL

The term 'soil' in soil engineering is defined as an unconsolidated material, composed of solid particles, produced by the disintergration of rocks. The void space between the particles may contain air, water or both. The soil particles may contain organic matter.

DEFINITION OF SOIL MECHANICS

- The term 'soil mechanics' was coined by Dr. Karl Terzaghi in 1925, who is also known as the father of soil mechanics.
- According to Terzaghi, 'Soil mechanics is the application of the laws of mechanics and hydraulics to engineering problems dealing with sediments and other unconsolidated accumulations of solid particles produced by the mechanical and chemical disintegration of rock, regardless of whether or not they contain an admixture of orginic constituents.
- Soil mechanics is, therefore, a branch of mechanics which deals with the action of forces on soil and with the flow of water in soil.

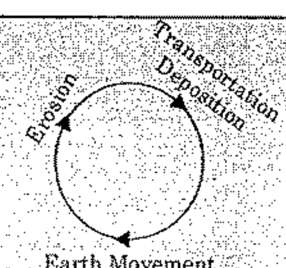
DEFINITION OF SOIL ENGINEERING

- Soil engineering in an applied science dealing with the applications of principles of soil mechanics to practical problems. It has a much wider scope than soil mechanics, as it deals with all engineering problems related with soils. It includes site investigations, design and construction of foundations, earth-retaining structures and earth structures.

DEFINITION OF GEOTECHNICAL ENGINEERING

- Geotechnical engineering is a broader term which includes soil engineering, rock mechanics and geology. Some times Geotechincal Engineering is used synonymously with Soil Engineering.

ORIGIN OF SOIL



The Geological Cycle

- In a broad sense, soil may be thought of as an incidental material in vast geological cycle which has been going on continuously for millions of years of geological time.
- The geological cycle consists of 3 phases, Erosion; Transportation and deposition & Earth Movement.

(a) Erosion Phase

- The cycle starts with the erosional phase in which there is degradation of exposed rock by weathering processes.
- The weathearing Processes may be
  - (i) Physical weathearing
  - (ii) Chemical Weathearing

Physical weathering :

- The physical weathering process may be.
  - (a) erosion of rocks caused by the action of wind, water, glaciers
  - (b) disintegration caused by alternate freezing and thawing in cracks in the rock.
- The resulted soil particles retain the same composition as that of parent rock.
- Particles of this type are described as being of '*bulky*' form.
- There shape can be indicated by terms such as *angular, rounded, flat* and *elongated*
- Gravel and sand fall into this group.
- The structural arrangement of these are described as *single grain*. Each particle are in direct contact with adjoining, Particle *without there being any bond between them.*
- State of the particles can be described as *dense, medium dense* or *loose*, depending on how they are packed together.

Chemical weathering :

- The chemical process results in changes in the mineral form of parent rock due to the action of water (especially if it contains acids or alkalies, oxygen and CO_2)
- Chemical weathering results in the formation of group of crystalline particles of colloidal size ($< 2\ \mu$) known as **clay mineral.**
- Most clay mineral particles have "*Plate-like*" form having a high specific surface (i.e. high surface area to mass ratio) with the result that their structure is *influenced significantly by surface forces.*
- If the products of rock weathering are still located at the place where they originated, they are called *residual soil.*
- Most residual soils are weakly bonded, they have widely varying void ratio. They contain angular rock fragments of varying sizes. **Residual soils have better engineering property.**

Transportation/Deposition

- In the second phase, the fragmented material is transported by agents such as wind, water or ice to new locations.
- Soil transported from their origin by wind, water, ice or any other agency and has been deposited is called *Transported soil.* They have generally small grain sizes, large amount of pores.
- Characteristics of soil such as Size of particle, Shape and roundness, Surface texture and, Degree of shorting are influenced by the agency of transportation.

- According to the transporting agency, soils are classified as:

  Alluvial deposit → deposited by river water.

  Lacustrine deposit → deposited by still water like lakes.

  Marine deposit → deposited by sea water.

  Aeolian deposit → transported by wind.

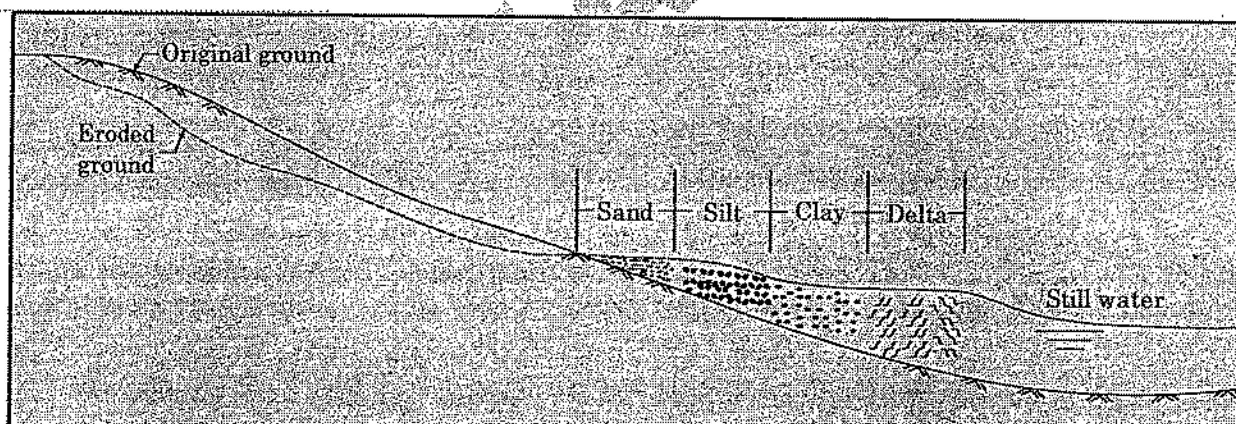
  Glacial deposit → Transported by ice.

- Air transported soil have small size (20-50 μ) and they are *homogeneous, highly porous* and *friable*.
- Loess is an Aeolian soil. They are formed in arid and semi arid regions.
- Alluvial soils are generally rounded and have considerable shorting.

According to transporting Agency soils are classified as:

Water transported Soils :

- Flowing water is one of the most important agents of transportation of soils. Swift running water carries a large quantity of soil either in suspension or by rolling along the bed. Water erodes the hills and deposits the soils in the valleys.
- The size of the soil particles carried by water depends upon the velocity. The swift water can carry the particles of large size such as boulders and gravels. With a decrease in velocity, the coarse particles get deposited. The finer praticles are carried further downstream and are deposited when the velocity reduces. A delta is formed when the velocity slows down to almost zero at the confluence with a receiving body of still water, such as a lake, a sea or an ocean



Alluvial Deposits.

- All type of soils carried and deposited by River water are known as alluvial deposits. Deposits made in lakes are called *lacustrine deposits*. Such deposits are laminated or varved in layers. *Marine deposits* are formed when the flowing water carries soils to ocean or sea.

Wind transported Soils :

- Soil particles are transported by winds. **The particle size of the soil depends upon the velocity of wind.** The finer particles are carried far away from the place of the formation. A dust storm gives a visual evidence of the soil particles carried by wind. Soils deposited by wind are known as *aeolian deposits*.
- Loess is a silt deposit made by wind. These deposits have low density and high compressibility. The bearing capacity of such soils is very low. The permeability in the vertical direction is large.

Glacier-Deposited Soils :

- Glaciers are large masses of ice formed by the compaction of snow. As the glaciers grow and move, they carry with them soils varying in size from fine grained to huge boulders. Soils get mixed with the ice and are transported far away from their original position.
- Drift is a general term used for the deposits made by glaciers directly or indirectly.
- Deposits directly made by melting of glaciers are called *till.*
- Deposits of glacial till are generally well-graded and can be compacted to a high dry density. These have generally high shearing strength.

Gravity-deposited soils :

- Soils can be transported through distances under the action of gravity. Rock fragments and soil masses collected at the foot of the cliffs or steep slopes had fallen from higher elevation under the action of the gravitational force. **Colluvial soils, such as talus,** have been deposited by the gravity. Talus consists of irregular, coarse particles.

Soils transported by combined action :

- Sometimes, two or more agen ts of transportation act jointly and transport the soil. For example, a soil particle may fall under gravity and may be carried by wind to a far off place. It might by picked up again by flowing water and deposited. A glacier may carry it still further.

REGIONAL SOIL DEPOSITS IN INDIA

- Soil deposits of India may be classified in the following five major groups :

Alluvial Deposits :

- A large part of north India is covered with alluvial deposits. The thickness of alluvium in the Indo-Gangetic and Brahmputra flood plains varies from a few metres to more than one hundred metres. Even in the peninsular India, alluvial deposits occur at some places.
- The distinct characteristics of alluvial deposits is the existence of alternating layers of sand, silt and clay.
- The thickness of each layer depends upon the local terrain and the nature of floods in the rivers causing deposition.
- The deposits are generally of low density and are liable to liquefaction in earthquake-prone areas.

Black Cotton Soils :

- A large part of central India and a portion of South India is covered with black cotton soils.
- These soils are residual deposits formed from basalt or trap rocks. The soils are quite suitable for growing cotton.
- Black cotton soils are clays of high plasticity. The contain essentially **the clay mineral montmorillonite.**
- The soils have high shrinkage and swelling characteristics.
- The shearing strength of the soils is extremely low.
- The soils are highly compressible and have very low bearing capacity. It is extremely difficult to work with such soils.

Lateritic Soils :

- Lateritic soils are formed by decomposition of rock, removal of bases and silica, and accumulation of iron oxidé and aluminum oxide.
- The presence of iron oxide gives these soils the characteristic red or pink colour.
- These are residual soils, **formed from basalt**.
- Lateritic soils exist in the central, southern and eastern India.
- The lateritic soils are soft and can be cut with a knife when wet. However, these harden with time.
- The plasticity of the lateritic soils decreases with depth as they approach the parent rock. These soils, especially those which contain iron oxide, have relatively high specific gravity.

Desert Soils :

- A large part of Rajasthan and adjoining states is covered with sand dunes. In this area, arid conditions exist, with practically little rainfall.
- Dune sand is uniform in gradation. The size of the particles is in the range of fine sand. The sand is non-plastic and highly pervious. As the sand is generally in loose condition, it requires densification to increase its strength.

Marine Deposits :

- Marine deposits are mainly confined along a narrow belt near the coast. In the south-west coast of India, there are thick layers of sand above deep deposits of soft marine clays.
- The marine deposits have very low shearing strength and are highly compressible. They contain a large amount of organic matter. The **marine clays are soft and highly plastic**.

NAMES OF VARIOUS TYPES OF SOILS

Bentonite : It is a type of clay with a very high percentage of clay mineral montmorillonite. It is a highly plastic clay, resulting from the decomposition of volcanic ash. It is highly water absorbent and has high shrinkage and swelling characteristics.

Black Cotton Soil : It is a residual soil containing a high percentage of the clay mineral montmorillonite. It has vey low bearing capacity and high swelling and shrinkage properties.

Boulders : Boulders are rock fragments of large size, more than 300 mm in size.

Calcareous soils : These soils contain a large quantity of calcium carbonate. Such soils effervesce when tested with weak hydrochloric acid.

Caliche : It is a type of soil which contains gravel, sand and silt. The particles are cemented by calcium carbonate.

Clay : It can be made plastic by adjusting the water content. It exhibits considerable strength when dry. Clay is a fine-grained choesive soil. The particle size is less than 0.002 mm.

Orbanic clay contains finely divided organic matter and is usually dark grey or black in colour. It has a conspicuous odour. Organic clay is highly compressible and its strength is very high when dry.

Cobbles : Cobbles are large size particles in the range of 80 mm to 300 mm.

Diatomaceous earth : Diatoms are minute unicellular marine organisms. Diatomaceous earth is a fine, light grey, soft sedimentary deposit of the silicious remains of skeletons of diatoms.

Dispersive clays : These are special type of clays which defloculate in still water. Such soils erode if exposed to low-velocity water. Susceptibility to dispersion depends upon the cations in the soil pore water.

Dune sands **:** These are wind-transported soils. There are composed of relatively uniform particles of fine to medium sand.

Expansive clays **:** These are prone to large volume changes as the water content is changed. These soil contain the mineral montmorillonite.

Fills **:** All man-made deposits of soil and waste-materials are called fills. These are the soil embankments raised above the ground surface. Engineering properties of fills depend upon the type of soil, its water content and the degree of compaction.

Gravel **:** Gravel is a type of coarse-grained soil. The particle size ranges from 4.75 mm to 80 mm. It is a cohesionless material.

Hardpans **:** Hardpans are types of soils that offer great resistance to the penetration of drilling tools during soil exploration. The soils are designated hardpans regardless of their particle size. These are generally dense, well-graded, cohesive aggregates of mineral particles. **Hardpans do not disintegrate when submerged in water.**

Humus **:** It is a dark brown, organic amorphous earth of the topsoil. It consists of partly decomposed vegetal matter. It is not suitable for engineering works.

Laterites **:** Laterites are residual soils formed in tropical regions. Laterites are very soft when freshly cut but become hard after long exposure. Hardness is due to cementing action of iron oxide and aluminium oxide. These soils are also called lateritic soils.

Loam **:** It is a mixture of sand, silt and clay.

Loess **:** It is a wind blown deposit of silt. It is generally of uniform gradation, with the particle size between 0.01 to 0.05 mm. It consists of quartz and feldspar particles, cemented with clacium carbonate or iron oxide. When wet, it becomes soft and compressible because cementing action is lost. A loess deposit has a loose structure with numerous root holes which produce vertical cleavage.

Marl **:** It is a stiff, marine calcareous clay of greenish colour.

Moorum **:** The word *moorum* is derived from a Tamil word, meaning powdered rock. It consists of small pieces of disintegrated rock or shale, with or without boulders.

Muck **:** It donotes a mixture of fine soil particles and highly decomposed organic matter. It is black in clour and of extremely soft consistency. It cannot be used for engineering works. The orginic matter is in an advanced stage of decomposition.

Peat **:** It is an organic soil having fibrous aggregates of macroscopic and microscopic particles. It is formed from vegetal matter under conditions of excess moisture, such as in swamps. It is highly compressible and not suitable for foundations.

Sand **:** It is a coarse-grained soil, having particle size between 0.075 mm to 4.75 mm. The particles are visible to naked eye. The soil is cohesionless and pervious.

Silt **:** It is a fine-grained soil, with particle size between 0.002 mm and 0.75 mm. The particles are *not visible* to naked eyes. Inorganic silt consists of bulky, equidimensional grains of quartz. It has little or no plasticity, and is cohesionless. Organic silt contains an admixture of organic matter. It is a plastic soil and is choesive.

Till **:** It is an unstratified deposit formed by melting of a glacier. The deposit consists of particles of different sizes, ranging from boulders to clay. The soil is generally well-graded. It can be easily densified by compaction. Till is also known as *boulder-clay*.

Tuff **:** It is a fine-grained soil composed of very small particles ejected from volcanoes during its explosion and deposited by wind or water.

Tundra **:** It is a mat of peat and shrubby vegetation that covers clayey subsoil in arctic regions. The deeper layers are permanently frozen and are called permafrost. The surface deposit is the active layer which alternately freezes and thaws.

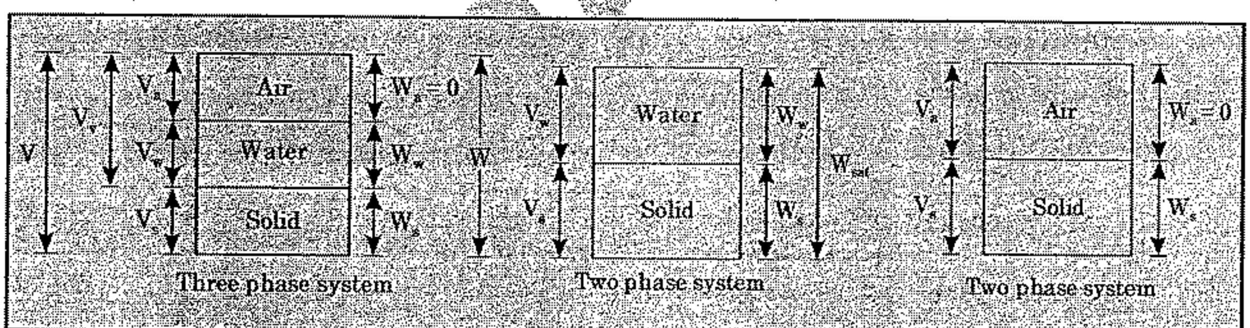
Varved clays **:** These are sedimentary deposits consisting of alternate thin layers of silt and clay. The thickness of each layer seldom exceeds 1 cm. These clays are the results of deposition in lakes during periods of alternately high and low waters.

Indurated Clay **:** Hardening of clay due to heat and pressure.

Note: **Lithification** : Process by which unconsolidated materials are converted into coherent solid rock as by compaction or cementation.

SOIL WATER RELATIONSHIP

- A soil mass consists of solid particles which form a porous structure. The voids in the soil mass may be filled with air, with water or partly with air and partly with water. The three constituents are blended together to form a complex material. However, for convenience, all the solid particles are segregated and placed in the lower layers of the three-phase diagram. Likewise, water and air particles are placed separately, as shown. The 3-phase diagram is also known as *Block diagram.*
- It may be noted that the three constituents cannot be actually segregated, as shown. A 3-phase diagram is a n artifice used for easy understanding and convenience in calculation.
- Soil can be either two-phase or three-phase composition.
- Fully saturated soil and fully dry soil are two phase system.
- Partially saturated soil are three phase system.



Where,

V_a = Volume of Air

V_w = Volume of water

V_s = Volume of Solid

V = Total volume of Soil mass.

V_v = Volume of Voids i.e. sum of volume of air and Volume of Water.

W_a = Weight of air i.e. equals to Zero.

W_w = Weight of water

W_s = Weight of Soil solids

W = Weight of Soil mass

Wsat = Saturated weight of soil mass.

IMPORTANT DEFINITIONS

Water Content (w)

$$\boxed{w = \frac{W_w}{W_s}; \quad w \geq 0}$$

- Water content or moisture content of a soil mass is defined as the ratio of weight of water to the weight of solids (dry weight) of the soil mass.

$$w = \frac{W_w}{W_s} \times 100$$

- It is denoted by the letter symbol w and is commonly expressed as a percentage. eg. 20%, 40% etc.
- The minimum value for water content is 0.
- There is no upper limit for water content.
- Generally fine grained soils have higher water content as compared to coarse grained soil.

Void ratio (e)

$$\boxed{e = \frac{V_V}{V_S}; \quad e > 0}$$

- Void ratio is defined as ratio of volume of void to the volume of solids.
- It is denoted by letter symbol **(e)** and generally expressed as a demical fraction eg 0.20, 0.45 etc.
- There is no upper limit of void ratio in soil suspension and in macro-porous soils like loess, V_v could be much greater than V_s.
- Void ratio of fine grained soil are generally higher than those of coarse grained soil.
- Size of void in coarse grained soil are generally larger than that in fine grained soil.

Porosity (n)

$$\boxed{n = \frac{V_V}{V}; \quad 100 > n > 0}$$

- Porosity is defined as ratio of volume of voids to the total volume. It is also known as percentage voids.
- Porosity is denoted by letter symbol **(n)** and is commonly expressed as a percentage.

$$V_v = V_a + V_w$$
$$V = V_a + V_w + V_s$$

- The porosity of soil can not exceed 100% hence it has a upper limit of 100%.
- Both porosity and void ratio are measure of denseness (or looseness) of soil.

Note: Porosity is defined with respect to V while void ratio is defined with respect to Vs. The total Volume V is a variable quantity. But, since solids are incompressible, Vs remains invariant in the total volume V of the soil. Thus in Engineering studies, void ratio serves as a useful parameter for representing change in volume under compression.

Degree of Saturation (S)

$$\boxed{S = \frac{V_w}{V_v};\quad 0 \le S \le 100}$$

- Degree of Saturation of a soil mass is defined as the ratio of the volume of water in the voids to the volume of voids.
- It is denoted by letter symbol S and is commonly expressed as a percentage.
- $V_v = V_a + V_w$
- For a fully saturated soil mass $V_v = V_w$, hence for a saturated soil mass S = 100%.
- For fully dry soil mass $V_w = 0$, hence for a fully dry soil S = 0%.
- For partially saturated soilmass degree of saturation varies between 0 – 100%, which is most common condition in nature.

Percentage Air Voids (n_a)

$$\boxed{n_a = \frac{V_a}{V} \times 100}$$

- Percentage air voids of a soil mass is defined as the ratio of the volume of air voids to the total volume of the soil mass.
- It is denoted by letter symbol (**n_a**) and commonly expressed as a percentage.

Air content (a_c)

$$\boxed{a_c = \frac{V_a}{V_v}}$$

- Air content of a soil mass is defined as a ratio of Volume of air voids to the total volume of voids. It is denoted by letter symbol **a_c** and commonly expressed as a percentage.

Bulk Unit weight (γ_t)

$$\boxed{\gamma_t = \frac{W}{V} = \frac{W_s + W_w}{V_s + V_w + V_a}}$$

- Bulk unit weight of soil mass is defined as the weight per unit volume of soil mass.
- It is denoted by letter or γ or γ_t and is generally expressed as $\frac{kN}{m^3}, \frac{N}{m^3}, \frac{kg-f}{cm^3}$

Note: Density (r) is mass per unit volume. Hence,

Unit weight of Solids (γ_s)

$$\boxed{\gamma_s = \frac{W_s}{V_s}}$$

- Unit weight of solids is the weight of soil solids per unit volume of solids alone. It is also called as "absolute unit weight" of a soil.
- It is denoted by letter γ_s

Unit weight of water (γ_w)

$$\gamma_w = \frac{W_w}{V_w}$$

- Unit weight of water is the weight per unit volume of water.
- It is denoted by letter symbol γ_w.

Note: The value of changes with temperature but usually we take = 9.81 kN/m3 which is at 4°C.

Dry Unit weight (γ_d)

$$\gamma_d = \frac{W_s}{V} = \frac{W_d}{V}$$

- Dry unit weight is defined as weight of soil solids (or weight of dry soil) per unit volume of soil. It is denoted by letter symbol γ_d it has the unit of kN/m³.
- Dry unit weight is used as a measure of denseness of soil. A high value of dry unit weight indicates that more solids are packed in unit volume of soil hence a more compact soil.

Saturated Unit weight (γ_{sat})

$$\gamma_{sat} = \frac{\text{Wt. of Saturated Soil}}{\text{Volume of Soil}}$$

- The saturated unit weight of soil is defined as bulk unit weight of soil mass in saturated condition.
- It is denoted by letter and γ_{sat} and has unit kN/m³.

Submerged (Bouyant) Unit Weight (γ_{sub})

$$\gamma_{sub} = \frac{(W_s)_{sub}}{V}$$

- Submerged unit weight is defined as submerged unit weight of soil solids per unit of total volume.
  It is denoted by symbol γ_{sub} and has unit kN/m³.
- When the soil exits below ground water i.e. in submerged condition, a boyant force acts on the soil solids.
- Hence it is obvious that net weight of saturated soil solids has been reduced and this reduced mass is known as Submerged Unit Weight or Bouyant Unit Wt.

$$\gamma' = \gamma_{sat} - \gamma_w$$

Note: Soil in Submerged condition will be in saturated state but soil in saturated condition need not to be submerged. For example, Soil mass below water table is in submerged as well as saturated condition where as soil mass in capillary zone is in saturated condition only.

Specific gravity of solids (G)

$$\boxed{G = \frac{\gamma_s}{\gamma_w}}$$

- The specific gravity of solids is defined as the ratio of the unit weight of solids (absolute unit weight of soil) to the unit weight of water. It is denoted by letter G and is a Unit less quantity.
- This is also known as "Absolute specific gravity" or "Grain specific gravity".

Mass Specific gravity of solids (G_m)

$$\boxed{G_m = \frac{\gamma_t}{\gamma_w}}$$

- Mass specific gravity is defined as the ratio of bulk unit weight of soil to unit weight of water.
- It is denoted by letter symbol G_m and is a unit less quantity.

Specific Volume (V_{SP})

$$\boxed{V_{SP} = \frac{V}{V_s}}$$

- It is defined as total volume of soil which contains unit volume of solids.
- It is denoted by letter V_{sp} and is being expressed as decimal fraction.

$$V_{SP} = \frac{V}{V_S} = \frac{V_V + V_S}{V_S} = \frac{V_V}{V_S} + 1 = 1 + e$$

$$\boxed{V_{SP} = 1 + e}$$

Relative density (D_r)

$$\boxed{(D_r) = \frac{e_{max} - e}{e_{max} - e_{min}}}$$

- The relative density is a parameter used in **sandy and gravelly soils.**
- The value of e_{min} & e_{max} represents the soil in very dense and loose conditions, respectively, and are determined by a standard laboratory test.
- Loose soil have low values of D_r. While dense soils have high values.
- The theoritically lowest possible value of D_r is 0% and highest theoritical possible value is 100%. Thus D_r, is often more useful than void ratio (e) because we can easily comare the field value to the lowest and highest possible values. According to Relative density the soil is classified as:

Relative density	Classification
0 — 15	Very loose
15 — 35	Loose
35 — 65	Medium dense
65 — 85	Dense
85 — 100	Very dense.

SOME IMPORTANT RELATIONSHIPS

Abrivations

w = Water Content

W = Total Weight of soil

V = Total volume of soil

W_w = Weight of water

V_w = Volume of water

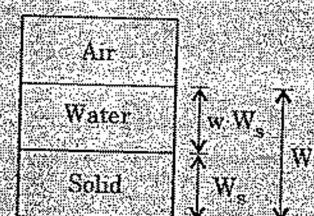
W_S = Weight of solid

V_S = Volume of solid

S = Degree of Saturation

V_V = Volume of voids

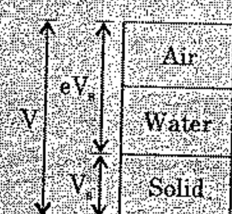
- $W_S = \dfrac{W}{1+w}$



$\Rightarrow W = W_S(1+w)$

$\Rightarrow \boxed{W_S = \dfrac{W}{1+w}}$

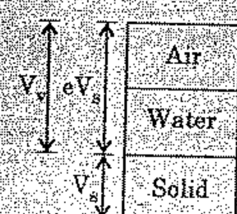
- $\boxed{V_S = \dfrac{V}{1+e}}$



$\Rightarrow V_S(1+e) = V$

$\Rightarrow \boxed{V_S = \dfrac{V}{1+e}}$

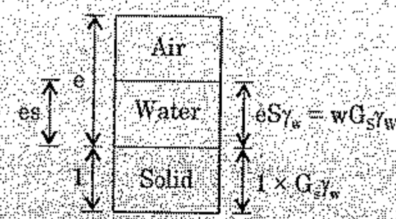
- $\boxed{n = \dfrac{e}{1+e}}$



$\Rightarrow n = \dfrac{V_v}{V} = \dfrac{eV_s}{V_S + V_v} = \dfrac{eV_S}{V_S + eV_S} = \dfrac{e}{1+e}$

$\Rightarrow \boxed{n = \dfrac{e}{1+e}}$

- $eS = wG_s$



$$e = \frac{V_v}{V_s} = \frac{V_v}{V_w} \times \frac{V_w}{V_s} = \frac{1}{S} \times \frac{\frac{W_w}{\gamma_w}}{\frac{W_S}{\gamma_S}}$$

$$e = \frac{1}{S} \times \frac{W_w}{W_s} \times \frac{\gamma_s}{\gamma_w} = \frac{1}{S} \times wG_S$$

$\Rightarrow$ $\boxed{eS = wG_S}$

$\Rightarrow$ $\boxed{eS = wG}$

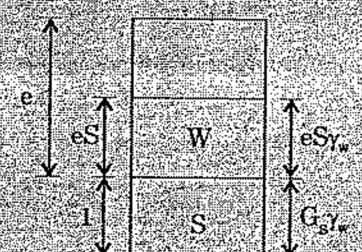
- $\gamma_t = \dfrac{G_s + Se}{1+e}\gamma_w$

$$\gamma_t = \frac{W}{V} = \frac{W_S + W_w}{V_S + V_V} = \frac{W_S\left(1 + \frac{W_w}{W_S}\right)}{V_S\left(1 + \frac{V_V}{V_S}\right)} = \frac{W_S(1+w)}{V_S(1+e)}$$

$$\gamma_t = \frac{G_S\gamma_w(1+w)}{1+e} = \frac{G_S + Se}{1+e}\gamma_w$$

$\Rightarrow$ $\boxed{\gamma_t = \dfrac{G_S + Se}{1+e}\gamma_w}$

Alternatively

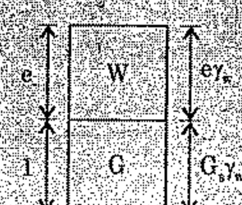


$$\gamma_t = \frac{W_S + W_W}{V_S + V_V}$$

$$= \frac{G_S\gamma_w + Se\,\gamma_w}{1+e}$$

$\Rightarrow$ $\boxed{\gamma_t = \dfrac{G_S + Se}{1+e}\gamma_w}$

- $\gamma_{sat} = \dfrac{G_s + e}{1+e}\gamma_w$



In the expression $\gamma_t = \dfrac{G_s + Se}{1+e}\gamma_w$,

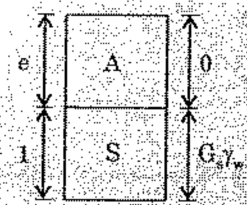
by putting S = 1 for saturated condition $\boxed{\gamma_{sat} = \dfrac{G_S + e}{1+e}\gamma_w}$

$$\gamma_{sat} = \frac{G_s\gamma_w + e\gamma_w}{1+e} = \frac{(G_s + e)\gamma_w}{1+e}$$

- $\gamma_d = \dfrac{G_s\gamma_w}{1+e}$

In the expression $\gamma_t = \dfrac{G_s + Se}{1+e}\gamma_w$,

by putting S = 0 for completely dry condition



$$\gamma_d = \frac{G_s\gamma_w}{1+e}$$

$$\Rightarrow \boxed{\gamma_d = \frac{G_s\gamma_w}{1+e}}$$

- $\gamma' = \dfrac{G_S - 1}{1+e}\gamma_w$

We know that γ' = submerged unit wt = $\gamma_{sat} - \gamma_w$

$$= \frac{G_s + e}{1+e}\gamma_w - \gamma_w = \frac{G_s - 1}{1+e}\gamma_w$$

$$\Rightarrow \boxed{\gamma' = \frac{(G_s - 1)}{1+e}\gamma_w}$$

- $\gamma_d = \dfrac{\gamma_t}{1+w}$

$$\gamma_t = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{W_S(1+w)}{V} = \gamma_d(1+w)$$

$$\Rightarrow \boxed{\gamma_d = \frac{\gamma_t}{1+w}}$$

- $\boxed{\gamma_d = \dfrac{\gamma_w G_S}{1+\dfrac{wG_S}{S}}}$

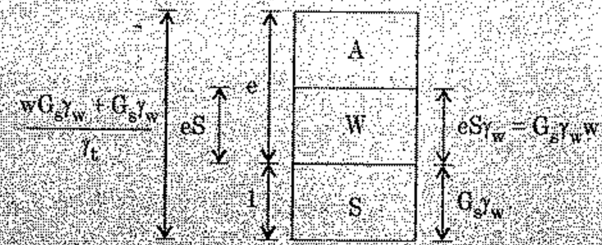
We know that $\gamma_d = \dfrac{G_s\gamma_w}{1+e} = \dfrac{G_s\gamma_w}{1+\dfrac{wG_s}{S}}$

- $1 - n_a = 1 - \dfrac{V_a}{V} = \dfrac{V_w + V_s}{V} = \dfrac{W_w}{\gamma_w V} + \dfrac{W_s}{G_s\gamma_w V}$

$$= \frac{\gamma_d}{G_s\gamma_w} + \frac{wW_S}{\gamma_w V} = \frac{\gamma_d}{\gamma_w}\left(w + \frac{1}{G_S}\right)$$

$$\Rightarrow \boxed{\gamma_d = \frac{(1-n_a)G_s\gamma_w}{1+wG_s}}$$

- $$S = \frac{w}{\frac{\gamma_w}{\gamma_t}(1+w) - \frac{1}{G}}$$



$$S = \frac{V_w}{V_v} = \frac{G_s\gamma_w \frac{w}{\gamma_w}}{\frac{G_s\gamma_w}{\gamma_d} - 1}$$

$$= \frac{wG_S}{\frac{G_S\gamma_w(1+w)}{\gamma_t} - 1} = \frac{w}{\frac{\gamma_w(1+w)}{\gamma_t} - \frac{1}{G}}$$

Summary

1. $W_S = \frac{W}{1+w}$
2. $V_S = \frac{V}{1+e}$
3. $e = \frac{n}{1-n}$
4. $n = \frac{e}{1+e}$
5. $e_S = WG_S$
6. $\gamma_t = \frac{G_S + Se}{1+e}\gamma_w$
7. $\gamma_{sat} = \frac{G_S + e}{1+e}\gamma_w$
8. $\gamma_d = \frac{G_S}{1+e}\gamma_w$
9. $\gamma' = \frac{G_S - 1}{1+e}\gamma_w$
10. $\gamma_d = \frac{\gamma_t}{1+w}$
11. $\gamma_d = \frac{G_S\gamma_w}{1+\frac{WG_S}{S}} = \frac{(1-n_a)\,G_S\gamma_w}{1+wG_S}$

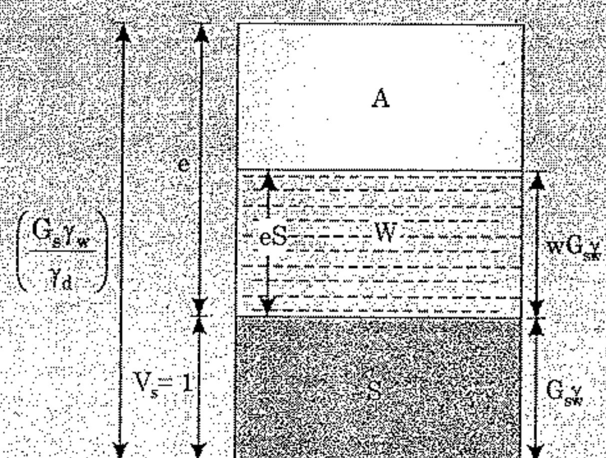
Example 1.

Prove that $$S = \frac{w}{\frac{\gamma_w}{\gamma_t}(1+w) - \frac{1}{G_s}}$$

Sol. Proof: We have $$S = \left(\frac{V_w}{V_V}\right)$$

$$= \left(\frac{V_w}{V - V_s}\right)$$

$$= \frac{\left(\frac{wG_s\gamma_w}{\gamma_w}\right)}{\left(\frac{G_s\gamma_w}{\gamma_d} - 1\right)}$$



$$= \frac{wG_s}{G_s\left[\frac{\gamma_w}{\gamma_d} - \frac{1.0}{G_s}\right]}$$

$$= \frac{w}{\frac{\gamma_w(1+w)}{\gamma_t} - \frac{1}{G_s}} \qquad \left[\because \gamma_d = \frac{\gamma_t}{1+w}\right]$$

$$S = \frac{w}{\frac{\gamma_w}{\gamma_t}(1+w) - \frac{1}{G_s}} \qquad \textbf{Proved.}$$

Example 2.

A sampler with a volume of 45 cm³ is filled with a soil sample, when the soil is poured into a graduated cylinder, it displaces 25 cm³ of water. What is the porosity and void ratio of the soil.

Sol. Here Total volume of soil is = 45 cm³

As we droped the soil in water the solid particles will displace water hence we get volume of soil $V_s = 25$ cm³

$$V_v = V - V_s = 45 - 25 = 20$$

$$e = \frac{V_v}{V_s} = \frac{20}{25} = 0.80$$

$$e = \frac{V_v}{V_s} = \frac{20}{25} = 0.80$$

$$n = \frac{V_v}{V} = \frac{20}{45} = 0.444$$

Example 3.

The void ratio and specific gravity of a sample of clay are 0.73 and 2.7 respectively. If the voids are 92% saturated, find the bulk density, the dry density and the water content.

What would be the water content for complete saturation, the void ratio remaining the same?

Sol. $e = 0.73$, $G = 2.7$, $S = 92\%$

$$S.e = w.G$$

$$w = \frac{0.92 \times 0.73}{2.7}$$

$$w = 0.24$$

$$\gamma_d = \frac{G}{1+e}\gamma_w$$

$$= \frac{2.7}{1+0.73} \times 9.81 = 15.31 \text{ kN/m}^3$$

$$\gamma_{sat} = \frac{G+e}{1+e}\gamma_w$$

$$= \frac{2.7+0.73}{1+.73} \times 9.81$$

$$= 19.449 \text{ kN/m}^3$$

Water content for full saturation at same void ratio

$$\Rightarrow \quad S.e = w.G$$

$$\Rightarrow \quad \frac{1 \times 0.73}{2.7} = w$$

$$\Rightarrow \quad w = 0.2703$$

$$w = 27.03\%$$

Example 4.

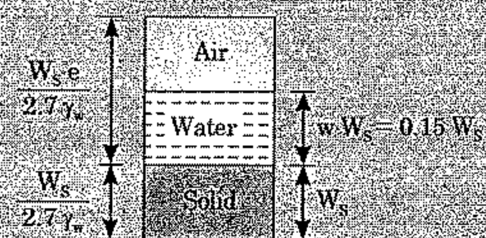
In a Proctor compaction test, the soil specimen of one of the observations had a bulk density of 19 kN/m³ with a moisture content of 15%. Find,

(a) Degree of saturation of the specimen if $G_S = 2.7$

(b) Additional moisture content required for saturating the soil specimen.

Sol.

$$\gamma_t = \frac{W_S + W_w + W_A}{V_S + V_V} = \frac{W_S + 0.15 W_S + 0}{\frac{W_S}{2.7\gamma_w} + \frac{W_S}{2.7\gamma_w}e}$$



$$\Rightarrow \quad \frac{1.15 W_S}{\frac{W_S}{2.7\gamma_w} + \frac{W_S}{2.7\gamma_w}e} = 19$$

$$\frac{1.15\gamma_w \times 2.7}{(1+e)} = 19$$

$$\Rightarrow \quad e = 0.603$$

$$\Rightarrow \quad e.S = wG$$

$$\Rightarrow \quad S = \frac{wG}{e} = \frac{0.15 \times 2.7}{0.603} = 0.6715$$

Moisture content req. to saturate soil

$$S.e. = w.G_S$$

For fully saturated soil, S = 1

$$w = \frac{e}{G_S}$$

$$w = \frac{e}{G_S} = \left(\frac{0.603}{2.7}\right) = 0.2333$$

$$= \frac{e}{2.7} = \frac{0.603}{2.7} = 0.2233$$

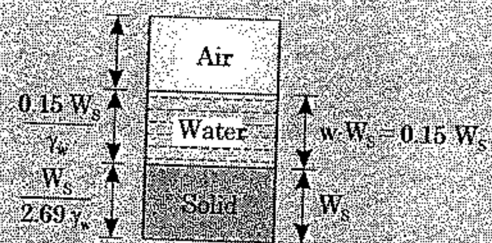
Additional moisture content $= 0.2233 - 0.15$

$$= 0.0733 = 7.33\%$$

Example 5.

A compacted cylindrical specimen 50 mm dia and 100 mm long is to be prepared from dry soil. If the specimen is required to have a water content of 15% and the percentage of air voids is 20, calculate the weight of soil and water required in the preparation of soil where sp. gravity = 2.69.

Sol.



$$\text{Final volume} = \frac{\pi}{4}(0.05)^2 \times 0.1 = 1.9625 \times 10^{-4}\ \text{m}^3$$

$$\frac{W_W}{W_S} = 0.15$$

$$\frac{V_a}{V} = 0.2$$

$$G_S = 2.69$$

$$\text{Volume of air} = 0.2 \times 1.9625 \times 10^{-4}\ \text{m}^3$$
$$= 3.925 \times 10^{-5}\ \text{m}^3$$

Volume of solids + Volume of water = Total Volume – Volume of Air.

$$\Rightarrow \quad \frac{W_S}{2.69\gamma_w} + \frac{0.15\,W_S}{\gamma_w} = (1.9625 \times 10 - 3.925) \times 10^{-5}\ \text{m}^3$$

$$\Rightarrow \quad W_S = 2.952\ \text{N}$$

$$\boxed{W_S = 300.9\ \text{gm.}} \quad \ldots(i)$$

$$W_W = (0.15 \times 300.9) = 45.135\ \text{gm}$$

Example 6.

The pavement of a road on a level ground is to be laid on a base course 400 mm thick, consisting of a coarse-grained gravel-sand mixture with good draining properties, placed evenly on an impervious subgrade. The porosity of the gravel-sand is 40% and the degree of saturation, 60%. There is a sudden downpour during construction work. Assuming that all water immediately infiltrates into the ground, calculate the rainfall in mm that would saturate the base course to its full thickness?

Sol. Consider a prism of gravel-sand soil 400 mm thick with a base area of 1 m².

$V = 0.4\ \text{m}^3$

$$V_v = nV = 0.4 \times 0.4 = 0.16\ \text{m}^3$$
$$V_w = S\,V_v = 0.6 \times 0.16 = 0.096\ \text{m}^3$$
$$V_a = V_v - V_w = 0.16 - 0.096 = 0.064\ \text{m}^3$$

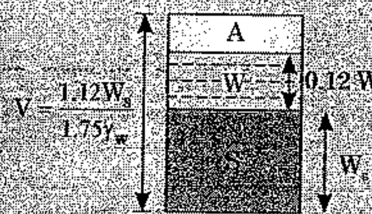
Water from rainfall has to fill this volume of air in order to saturate the soil. The required amount of rain fall is, therefore, equal to

$$\frac{0.064\text{m}^3}{1} = 0.064\text{m} = 64 \text{ mm}$$

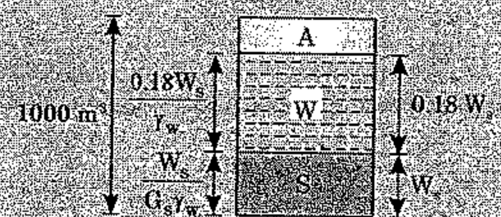
Example 7.

Soil is to be excavated from a barrow pit which has a density of 1.75 gm/cc and water content of 12%. The specific gravity of soil particles is 2.7. The soil is compacted so that water content is 18% and dry density is 1.65 gm/cc. For 1000 cum of soil in fill, estimate (i) quantity of soil to be excavated from the pit in cum, (ii) amount of water to be added. Also, determine the void ratio of the soil in borrow pit and fill.

Sol.



$w_c = 12\%$
$\gamma_t = 1.75\,\gamma_w$
Borrow Pit



$G_s = 2.7$
$w_c = 18\%$
$\gamma_d = 1.65\,\gamma_w$
Compacted Soil

For compacted soil

$$1.65\,\gamma_w = \frac{W_S}{1000}$$

$$1650\,\gamma_w = W_S$$

Amount of soil to be excavated from Borrow Pit

$$V = \frac{1.12\,W_S}{1.75\,\gamma_w} = \frac{1.12 \times 1650\,\gamma_w}{1.75\,\gamma_w} = 1056 \text{ m}^3$$

Amount of water to be added

$$(0.18 - 0.12)\,W_S = 0.06 \times 1650\,\gamma_w$$

$$= 99\,\gamma_w = 971.2 \text{ kN}$$

Void ratio in Borrow Pit, $e = \frac{V_V}{V_S} = \frac{V - V_S}{V_S} = \frac{V}{V_S} - 1$

$$= \frac{1.12\,W_S}{1.75\,\gamma_w \times \frac{W_S}{G_S\gamma_w}} - 1 = \frac{1.12\,G_S}{1.75} - 1 = 0.728$$

Void ratio in compacted soil

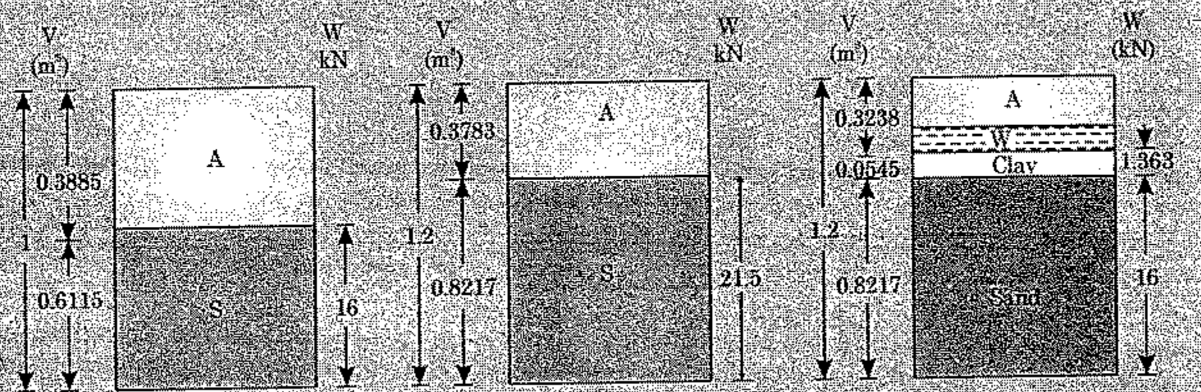
$$\frac{1000 - \frac{W_S}{G_S\gamma_w}}{\frac{W_S}{G_S\gamma_w}} = \frac{1000 \times G_S\gamma_w}{1650\,\gamma_w} - 1 = 0.06$$

Example 8.

The *in situ* dry unit weight of a medium to coarse sand used as subgrade for a highway, was 16 kN/m³. It was decided to improve the soil by mechanical stabilization. When 5.5 kN of a mixture of dry sand and silt was added to 1 m³ of this subgrade, the volume was increased by 20 per cent. How much reduction in porosity of the soil was achieved? Assume G_s as 2.67 for both

Further, 1.5 kN of clay, at a moisture content of 10 per cent was added to the above mixture such that no further increase in the volume of the subgrade resulted. Determine the further reduction in porosity that this addition of clay brought about. Assume G_s of clay particles as 2.55

Sol. Shows the phase diagrams corresponding to (a) the *in situ* condition; (b) condition after the addition of sand and silt, and (c) condition after the addition of clay.



(a) In situ (b) After the addition (c) After the addition of

In Fig. (a), $$Vs = \frac{W_s}{G_s\gamma_w} = \frac{16}{2.67\times 9.8} = 0.6115 \text{ m}^3$$

$$V_v = 1 - 0.6115 = 0.3885 \text{ m}^3$$

Porosity, $$n = \frac{V_v}{V} = \frac{0.3885}{1} \text{ or } 38.9\%$$

In Fig. (b), $$V_S = \frac{21.5}{2.67\times 9.8} = 0.8217 \text{ m}^3$$

$$V_v = 1.2 - 0.8217 = 0.3783 \text{ m}^3$$

Porosity $$n = \frac{Vv}{V} = 0.3153 \text{ or } 31.5\%$$

$$\text{Reduction in porosity achieved} = 38.85 - 31.53 = 7.3\%$$

In Fig. (c) $$W_s \text{ (clay)}, \frac{W_{clay}}{1+w_{clay}} = \frac{1.5}{1.1} = 1.363 \text{ kN}$$

$$V_s \text{ (clay)} = \frac{W_s}{G_s\gamma_w} = \frac{1.363}{2.55\times 9.8} = 0.0545 \text{ m}^3$$

$$V_v = 1.2 - 0.8217 - 0.0545 = 0.3238 \text{ m}^3$$

Porosity $$n = \frac{V_v}{V} = \frac{0.3238}{1.2} = 0.2698 \text{ or } 26.98\%$$

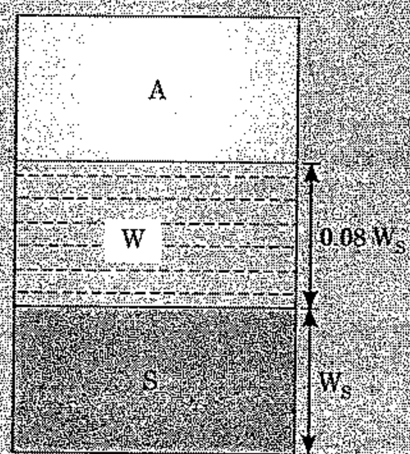
Further reduction in porosity form (b) = 31.53 − 26.98 = 4.6%

Example 9.

Earth is required to be excavated from borrow pits for building an embankment of height 6 m, top width 2 m and side slopes 1 : 1. The unit weight of undisturbed soil in wet condition is 18 kN/m³ and its natural water content is 8 per cent. The dry density required in the embankment is 20 kN/m³ with a water content of 10%. The specific gravity of soil solids is 2.70. Estimate the quantity of earth required to be excavated in the borrow area to construct one meter length of the embankment. If each truck has a capacity to carry 80 kN per trip, what is the number of truck loads required per meter length of embankment? What are the values of porosity and degree of saturation on the embankment?

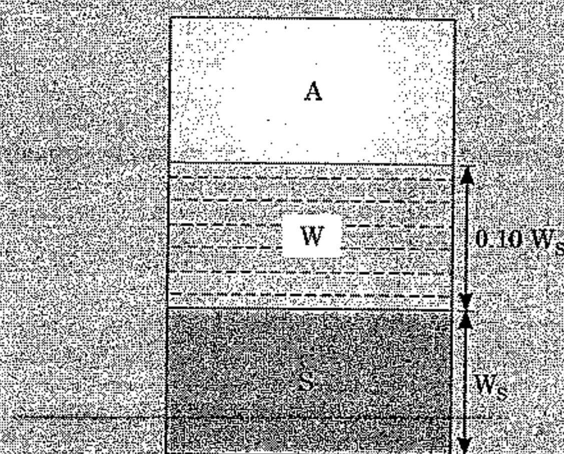
Sol.

For Borrow pit



γ_t = 18 kNm³
w_n = 8%

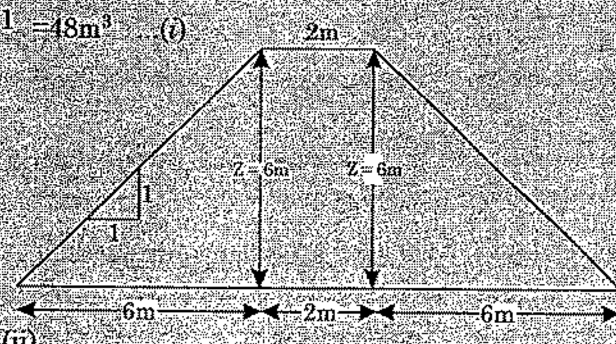
For Embankment



γ_d = 20 kNm³
w = 10%
G = 2.70

Vol. of the Soil for per meter length construction of Embankment

$$V = \left(\frac{2+14}{2}\right)\times 6\times 1 = 48\text{m}^3 \quad ...(i)$$



$\therefore$ Dry Density of Soil $\gamma_d = \left(\frac{W_s}{V}\right)$

$$\Rightarrow \quad 20 = \left(\frac{W_s}{48}\right)$$

$$\therefore \quad W_s = 960 \text{ kN} \quad ... (ii)$$

For Borrow pit Area,

$$\gamma_t = \left(\frac{W}{V}\right)$$

$$\gamma_t = \frac{W_s(1+w)}{V}$$

$$\Rightarrow \quad 18 = \frac{960(1+0.08)}{V}$$

$$\Rightarrow \quad V = 57.6\text{m}^3$$

Gross weight of Earth to be excavated from the Borrow Area

$$W = W_s(1+w)$$

$$= 960(1+0.008)$$

$$= 1036.8 \text{ kN} \qquad \text{...(iii)}$$

Capacity of truck = 80 kN per trip.

∴ No. of truck Req. per meter length of Embankment

$$= \left(\frac{1036.8}{80}\right)$$

$$= 12.96$$

$$= 13 \text{ trip}$$

(ii) Porosity of soil on the embankment

$$n = \left(\frac{V_w}{V}\right)$$

$$V_s = \left(\frac{W_s}{G\gamma_w}\right)$$

$$= \frac{960}{(2.70 \times 9.81)}$$

$$= 36.24 \text{ m}^3$$

$$V_V = (V - V_s) = (48 - 36.24)$$

$$= 11.76$$

$$\therefore \quad n = \left(\frac{11.76}{48}\right) = 0.245 = 24.5\%$$

(iii) Degree of Saturation on the embankment

$$= \left(\frac{V_w}{V_V}\right)$$

$$V_w = \left(\frac{0.10W_s}{\gamma_w}\right) = \left(\frac{0.10 \times 960}{9.81}\right) = 9.79 \text{ m}^3$$

$$S = \left(\frac{9.79}{11.76}\right) = 0.832 = 83.2\%$$

WATER CONTENT DETERMINATION

- Water content of soil is an important soil parameter which significantly influences the behaviour of soil, particularly of cohesive soils.
- It is important to quantify the state of soil immediately after it is received in the testing laboratory and just prior to commencing any other tests (example, shear strength test, compression test etc.).
- Water content and unit weight changes during transportation and storage. Hence it is important to determine it before carrying out any other tests.

- Water content determination is also important because some physical state properties are calculated using water content following the practical measurements of others e.g. dry unit weight from bulk unit weight.
- Water content is a quantitative measure of the wetness of a soil mass. The water content of a soil can determined to a high degree of precision, as it involves only mass which can be determined more accurately than volumes.
- The water content of a soil sample can be determined by any one of the following methods:

  (1) Oven Drying method(
  2) Torsion Balance method
  (3) Pycnometer method
  (4) Sand Bath method
  (5) Alcohol metbod
  (6) Calcium Carbide method
  (7) Radiation method

(1) Oven Drying method :

- The oven drying method is a standard laboratory method. This is a very accurate method.
- The soil sample is taken in a small, non-corridible, airtight container.
- The soil sample in container is then dried in over temperature 105-110°C for 24 hrs in laboratory. Above 110°C water of crystallisation may be lost.
- Water of crystallisation is the water in the molecular structure. It is lost only above 110°C.
- For soils containing significant amount of organic matter, a temperature of 60° to 80°C is recommended. At higher temperature, gypsum loses its water of crystalline and the organic soils tend to decompose and get oxidized.

$$\text{Water content} = \frac{W_w}{W_s} = \frac{W_2 - W_3}{W_3 - W_1}$$

W_1 = wt of container

W_2 = wt of container + wt of moist soil

W_3 = wt of container + wt of dry soil

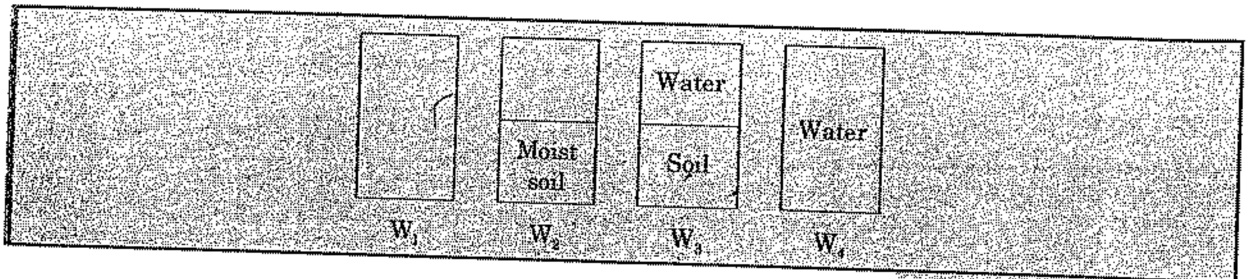
(2) Torsion Balance Method :

- The infra-red lamp and torsion balance moisture meter is used for rapid and accurate determination of the water content. The equipment has two main parts : (i) the infra-red lamp, and (ii) the torsion balance.
- Since drying and weighing occur simultaneously, the method is useful for soils which quickly re-absorb moisture after drying.

(3) Pycnometer method :

- A pycnometer is a glass jar of about 1 litre capacity and fitted with a brass conical cap by means of a screw-type cover. The cap has a small hole of 6 mm diameter at its apex.
- The pycnometer method for the determination of water content can be used only if the specific gravity of solid (G) particles is known.
- First, the weight of the empty pycnometer is determined (W_1) in the dry condition. Then the sample of moist soil, is placed in the pyonometer and its weight with the soil is determined

(W_2). The remaining volume of the pycnometer is then gradually filled with distilled water or kerosene. The entrapped air should be removed either by gentle heating and vigorous shaking or by applying vacuum. The weight of the pycnometer, soil and water is obtained (W_3) carefully. Lastly, the bottle is emptied, throughly cleaned and filled with distilled water or kerosene, and its weight taken (W_4).



$$W_W = W_2 - W_1 - W_S$$

$$w = \frac{W_W}{W_S} = \frac{W_2 - W_1}{W_S} - 1$$

$$W_4 - W_1 = W_3 - W_1 - W_S + \frac{W_S \gamma_w}{G_S \gamma_w}$$

$$W_S = \frac{(W_3 - W_4)\, G_S}{G_S - 1}$$

$$w = \frac{W_2 - W_1}{\dfrac{(W_3 - W_4)\, G_S}{(G_S - 1)}} - 1$$

$$w = \left[\left(\frac{W_2 - W_1}{W_3 - W_4}\right)\left(\frac{G_S - 1}{G_S}\right) - 1\right]$$

- Removal of entrapped air is difficult from cohessive soil. *Hence this method is more suited for cohesionless soil.*

(4) Sand Bath Method :

- Sand bath method is a field method for the determination of water content. The method is rapid, but not very accurate.
- A sand bath is a large, open vessel containing sand filled to adepth of 3 cm or more.
- The soil sample is taken in a tray. The sample is crumbled and placed loosely in the tray. A few pieces of white paper are also placed on the sample. The tray is weighed and the mass of wet sample is obtained.
- The tray is then placed on the sand bath. The sand bath is heated over a stove. Drying takes about 20 to 60 minutes, depending upon the type of soil.
- During heating, the specimen is turned with a palette knife. Overheating of soil should be avoided. The white paper turns brown when overheating occurs.
- The drying should be continued till the sample attains a constant mass.
- When drying is complete, the tray is removed from the sand bath, cooled and weighed. The water content is determined as : $\frac{W_w}{W_s} = \frac{W_{Final} - W_{initial}}{W_{Final} - W_{tray}}$

(5) Calcium Carbide Method :

- This method of the determination of water content makes use of the fact that when water reacts with calcium carbide ($Ca\ C_2$), acetylene gas ($C_2\ H_2$) is produced.

$$CaC_2 + H_2O = C_2H_2 + Ca\ (OH)_2$$

- The water content of the soil is determined indirectly from the pressure of the acetylene gas formed. The instrument used is known as *moisture tester*.
- The pressure of the acetylene gas produced acts on the diaphragm of the moisture tester. The quantity of gas is indicated on a pressure guage. From the calibrated scale of the pressure gauge, the water content (w_t) based on the total mass is determined. The water content (w) based on the dry mass is determined using the following formula:

$$w_t = \frac{W_w}{W} = \frac{W_w}{W_S + W_w}$$

$$W_W = w_t \times W_s + w_t \times W_w$$

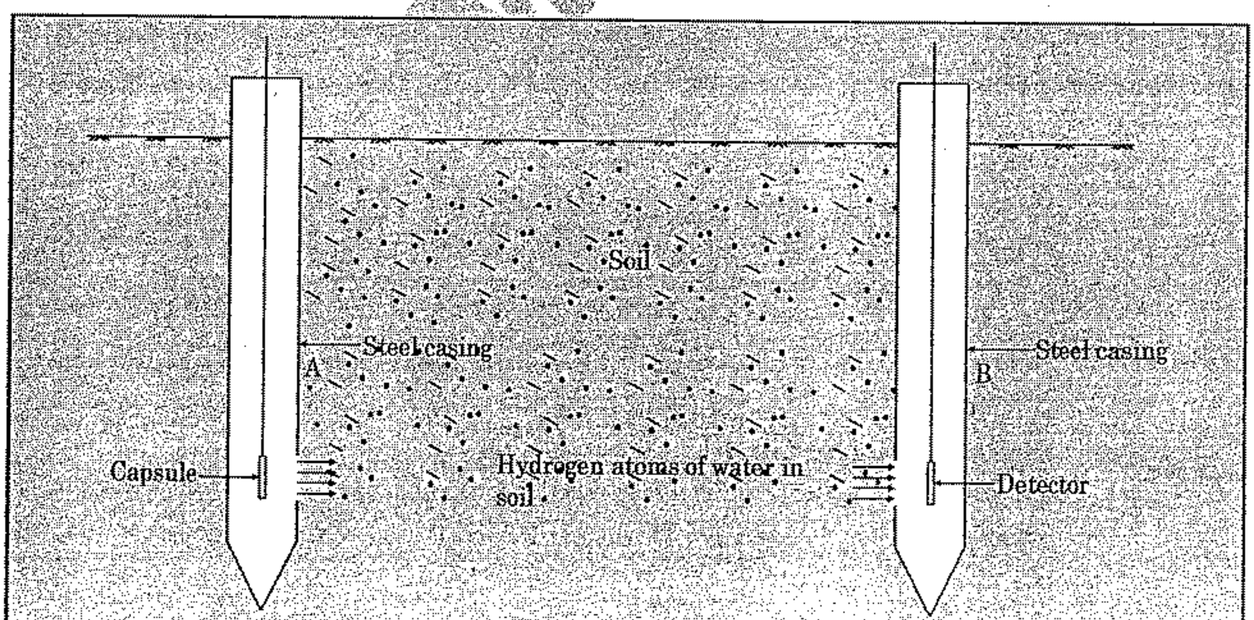
$$\Rightarrow \quad W_w\ (1 - w_t) = W_s \times w_t$$

$$\frac{W_w}{W_S} = \frac{w_t}{1 - w_t}$$

$$w = \frac{w_t}{1 - w_t} = \text{water content based on weight of solid.}$$

(6) Radiation Method :

- Radio-active isotopes are used for the determination of water content of soils. A device containing a radio-active isotopes material, such as cobalt 60, is placed in a capsule. It is then lowered in



Radiation Method

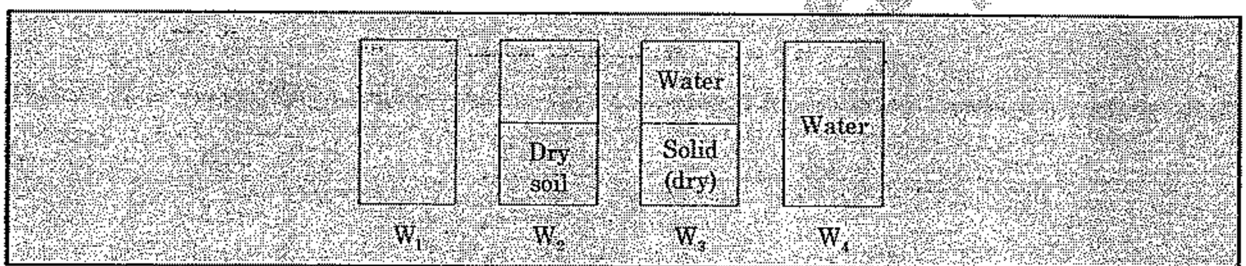
- A steel casing A, placed in a bore hole as shown in Fig. The steel casing has a small opening on its one side through which rays can come out. A detector is placed inside another steel casing *B*, which also has an opening facing that in casing *A*.

- Neutrons are emitted by the radio-active material. The hydrogen atoms in water of the soil cause scattering of neutrons. As these neutrons strike with the hydrogen atoms, they lose energy. The loss of energy is proportional to the quantity of water present in the soil. The detector is calibrated to give directly the water content.
- The method is extremely useful for the determination of water content of a soil in the in-situ conditions. The method should be very carefully used, as it may lead to radiation problems if proper shielding precautions are not taken.

SPECIFIC GRAVITY OF SOLID DETERMINATION (GS)

Pycnometer Method :

- This method is same as pycnometer method of water content determination with the difference that here dry soil sample is taken instead of moist soil sample as was taken in water content determination.



wt of solid = $(W_2 - W_1)$

wt of equivalent volume of water = $(W_4 - W_1) - (W_3 - W_2)$

$$\therefore G_s = \frac{\text{wt of solid}}{\text{wt of equivalent volume of water}}$$

$$G_s = \frac{W_2 - W_1}{(W_4 - W_1) - (W_3 - W_2)}$$

- Sp. gravity values are generally reported at 27°C.
- If $G_{sT°C}$ = Sp. gravity at T°C [if test temperature = T°]

Then, $$G_{s27°C} = \frac{G_{sT°C} \times \text{Sp. gravity of water at T°C}}{\text{Sp. gravity of water at 27°C}}$$

MEASUREMENT OF UNIT WEIGHT

- The Bulk Unit weight of the sample is total weight per Unit Volume of soil mass.
- The weight of soil sample can be determined to a high degree of precision in comparission to the volume of sample.
- The methods discussed below basically differ in procedure for the measurement of the Volume.
- Dry unit weight can be determined from the bulk unit weight by using the formula.

$$\gamma_d = \frac{\gamma_t}{1+w}$$

The following methods are generally used for determination of Bulk unit weight.

(1) Core Cutter Method.
(2) Water Displacement Method.
(3) Sand Replacement Method.
(4) Water Baloon Method.
(5) Radiation Method.

1. Core Cutter Method

- It is a field Method.
- A core cutter of known volume 1000 cm^3 is punched into ground and then core containing soil is taken out of the ground.
- Weight of core with soil sample and empty core is measured.
- A representative soil sample is taken for water content measurement.
- Bulk unit weight is calculated as below.

$$\gamma_t = \frac{W_2 - W_1}{V}$$

W_2 : Weight of core filled with soil.
W_1 : Weight of Empty core.
V : Volume of Core.

- Further γ_d is calculated from the γ_t & w (water content) values as :

$$\gamma_d = \frac{\gamma_t}{1 + w}$$

- The method is quite suitable for soft, fine grained soils.
- It cannot be used for stoney, gravelly soils & dry soil.
- The method is practicable only at the places where the surface of the soil is exposed and the cutter can be easily driven.

2. Water Displacement Method

- The volume of the specimen is determined in this method by water displacement. As the soil mass disintegrates when it comes in contact with water, the sample is coated with paraffin wax to make it impervious.
- A test specimen is trimmed to more or less a regular shape and weighed. It is then coated with a thin layer of paraffin wax by dipping it in molten wax.
- The specimen is allowed to cool and weighed. The difference between the two observations is equal to the mass of the paraffin.
- The waxed specimen is then immersed in a water-displacement container. The volume of the speciment is equal to the volume of water which comes out of the outflow tube.
- The actual volume of the soil specimen is less than the volume of the waxed specimen.
- The volume of the wax is determined from the mass of the wax peeled off from the specimen after the test and the mass density of wax.

Now, $$V = V_t - \frac{(M_t - M)}{\rho_p}$$

where,

V = volume of specimen.

V_t = volume of waxed specimen.

M_t = mass of waxed specimen,

M = mass of specimen.

ρ_p = mass density of paraffin (approximately 0.998 gm/m).

- A representative sample of the soil is taken from the middle of specimen for the water content determination.
- Once the mass, volume and the water content of the specimen have been determined, the bulk density and the dry density are found.

Example 10.

A sample of clay was coated with paraffin wax and its mass, including the mass of wax, was found to be 697.5 gm. The sample was immeresed in water and the volume of the water displaced was found to be 355 ml. The mass of the sample without wax was 690.0 gm, and the water content of the representative specimen was 18%.
Determine the bulk density, dry density, void ratio and the degree of saturation. The specific gravity of the solids was 2.70 and that of the wax was 0.89.

Sol.

$$\text{Mass of wax} = 697.5 - 690.0 = 7.5 \text{ gm}$$

$$\text{Volume of wax} = \frac{7.50}{0.89 \times 1.0} = 8.43 \text{ ml}$$

$$\text{Volume of soil} = 355.0 - 8.43 = 346.57 \text{ ml}$$

$$\text{Bulk density} = \frac{690}{346.57} = 1.99 \text{ gm/ml}$$

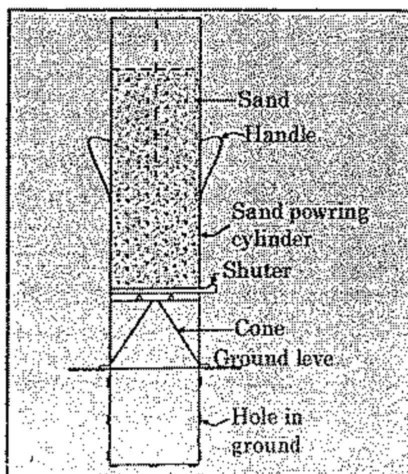
$$= \frac{1.99}{1+0.18} = \mathbf{1.69 \text{ gm/ml}}$$

$$1 + e = \frac{2.70 \times 1.0}{1.69} = 1.60 \quad \text{or} \quad e = \mathbf{0.60}$$

$$S = \frac{wG}{e} = \frac{0.18 \times 2.70}{0.60} = \mathbf{0.81\ (81\%)}$$

3. Sand Replacement Method

- This is a field method.
- A small area is excavated, and the excavated soil sample is precisely weighted.
- A calibrated cylinder containing sand is placed over the excavated area and the pit is filled with the sand.
- Volume of the pit is obtained from the calibrated cylinder.
- The bulk unit weight is calculated.
- A representative sample is tested for water content.
- γ_d is obtained from γ_t & w.
- It is suitable for gravelly, sandy and dry soil.



Sand Replacement method

Example 11.

During soil investigation for a residential complex site at Roorkee, the following observations were taken for the *in situ* unit weight measurement by sand replacement method:

Weight of excavated soil	=	761.25 g
Weight of sand + clyinder (W_1)	=	10500 g
Weight of sand + cylinder after pouring in the excavated hole and cone (W_2)	=	9450 g
Weight of sand + cylinder after pouring for the cone only (W_3)	=	9005 g
Volume of calibrating can	=	1000 cc
Weight of sand in calibrating can after pouring from cylinder	=	1500 g

Calculate the *in situ* unit weight of the soil.

Sol.

Weight of sand filling the excavated hole and the cone = $(W_1 - W_2)$

= (10500-9450)

= 1050 g

Weight of sand filling cone only = $(W_2 - W_3)$

= 9450-9005

= 445 g

Weight of sand filling hole = 1050–445

= 605 g

$$\text{Unit weight of sand} = \frac{1550}{1000} = 1.55 \text{ g/cc}$$

$$\text{Volume of the hole} = \frac{605}{1.55} = 390.32 \text{ cc.}$$

$$\textit{In situ} \text{ unit weight} = \frac{761.25}{390.32} = 1.95 \text{ g/cc}$$

4. Water Ballon method.

- In this method volume of the excavated pit is obtained by covering the hole with a pastic sheet and filling it with water. Weight of excavated sample is measured and γ_t is calculated.

5. Radiation Method.

- The bulk density of insitu soil can also be determined using the radiation method.
- Radiation methods for determination of bulk density of soil are quick and convenient and are gaining popularity.

INDEX PROPERTY OF SOIL

- Those properties which help to access the engineering behaviour of a soil and which assist in determining its classification accurately are termed as index properties. Index properties include indices which help in determining the engineering behaviour such as

(a) strength
(b) load-bearing capacity
(c) swelling and shrinkage
(d) settlement etc.

These properties may be relating to

1. individual soil grain
2. Aggregate soil mass

- The properties of individual particles can be determined from a remoulded, disturbed sample. These depend upon the individual grains their mineralogical composition, size and shape of grains and are independent of soil formation.
- The soil aggregate properties depend upon the mode of soil formation, soil history and soil structure. These properties should be determined from undisturbed samples or preferably from in situ tests.

Type of soil	Index property
Coarse soil	Particle size, Relative density, Grain shape
Fine soil	Attenberg's Limit & Consistency

GRAIN SHAPE

- Particularly useful in case of coarse grained soil. Soil grain are called bulky in case of sand & gravel. It is called flaky grains in case of sub-microscopic crystals of clay minearals. Clay mineral **kaolinite** is said to have needle shaped grains.
- Bulky grains of sand and gravel are classified as : angular, subangular, subrounded, rounded, well rounded.
- High angularity of soil grain leads to higher shearing strength
- Classification of bulky grain is done on the basis of sphericity where sphericity = $S = \frac{D_e}{L}$

  Where,

  D_e = dia of equivalent Spherical Particles.

  L = length of particle

i.e. $$\text{Volume} = V = \frac{4}{3}\pi\left(\frac{D_e}{2}\right)^3$$

$$V = \frac{\pi D_e^3}{6}$$

$$\Rightarrow \quad D_e = \left(\frac{6V}{\pi}\right)^{1/3}$$

GRAIN SIZE ANALYSIS

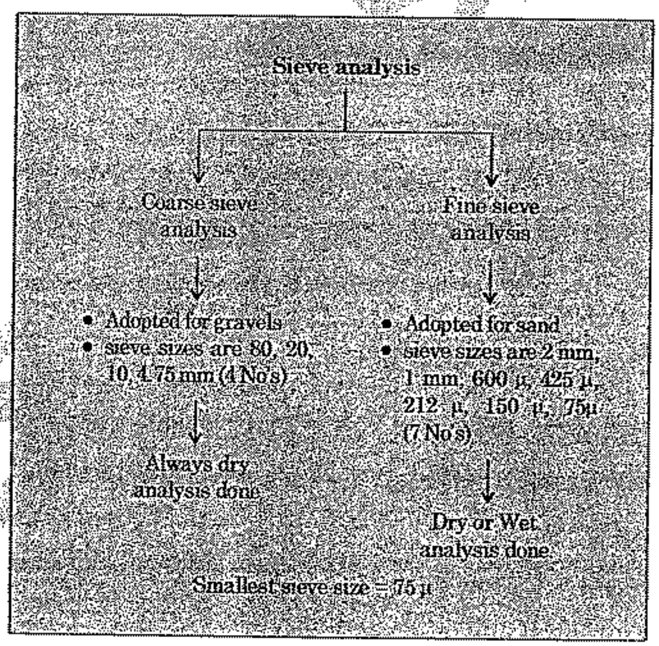
- Grain Size Analysis is a method of seperation of soils into different fractions based on the particle size. It is also called as Mechanical Analysis or Particle Size Analysis.

- It expresses quantitatively the proportions, by mass, of various sizes of particles present in a soil. It is shown graphically on a particle size distribution curve.

Silent Points

Particle Size	Type of Soil
> 300 mm	Boulder
300 – 80 mm	Cobbels

80 mm — 4.75 mm	gravel		Coarse Grained soil
4.75 mm — 2 mm	coarse sand	Sand	
2 mm — 0.475 mm	medium sand		
0.475 mm — 75 μ	fine sand		
75 μ — 2 μ	silt		Fine Grained soil
less then 2 μ	clay		



- Cobbles and Boulders have size greater than 80 & 300 mm respectivals (They are not grouped as soil)
- Properties of coarse grained soil (cohesionless) are to a greater extent *depend on grain size distribution.*
- Properties of fine grained soil depends little on grain size distribution. They rather depend on structure, shape of grain, geological origin etc.
- Interparticle forces are more important in case of fine grained soil.
- Grain size analysis helps in determining the gradation and uniformity of soil. This knowledge helps in construction of earth dam, embankments filters etc.

- In foundation design, however, compressibility, shear resistance etc are more important.
- Grain size analysis of *coarse grained soil* is carried out by sieve analysis.
- Fine grained soil are analysed by sedimentation analysis using hydrometer method or pipette method.

Fine sieve analysis

- Dry sieving
- Wet sieving

- The soil is sieved through a set of sieves.
- According to IS : 1498—1970. the sieves are designated by the size of square opening, in mm or microns (1 micron = 10^{-6} m = 10^{-3} mm).
- Sieves of various sizes ranging from 80 mm to 75 microns are available.
- All the sieves may not be required for a particular soil.
- The selection of the required number of sieves is done to obtain a good particle size distribution curve.
- The sieves are stacked one over the other, with decreasing size from the top to the bottom. Thus the sieve of the largest opening is kept at the top. A lid or cover is placed at the top of the largest sieve. A receiver, known as pan, wihch has no opening, is placed at the bottom of the smallest sieve.

PROCEDURE OF SIEVE ANALYSIS

- The soil sample to be tested is dried, clumps are broken if necessary, and the sample is passed through the series of sieves by shaking.
- The fractions retained on and passing 2 mm IS Sieve are tested separately.
- An automatic sieve-shaker, run by an electric motor, may be used; about 10 to 15 minutes of shaking is considered adequate.
- Larger particles are caught on the upper sieves, while the smaller ones filter through to be caught on one of the smaller underlying sieves.
- The material retained on any particular sieve should naturally include that retained on the sieves on top of it, since the sieves are arranged with the aperture size decreasing from top to bottom.
- The weight of material retained on each sieve is converted to a percentage of the total sample.
- The percentage material finer than a sieve size may be got by subtracting this from 100.
- The material passing the bottom-most sieve, which is usually the 75–μ sieve, is used for conducting sedimentation analysis for the fine fraction.
- If the soil is **clayey in nature** the fine fraction cannot be easily passed through the 75–μ sieve in the dry condition.
- In such a case, the material is to be washed through it with water (preferably mixed with 2 gm of sodium hexametaphosphate per litre), until the wash water is fairly clean.
- The material which passes through the sieve is obtained by evaporation. This is called 'wet seive analysis, and may be required in the case of cohesive granular soils'.
- The resulting data are conventionally presented as a "Grain-size distribution curve" plotted on semi-log co-ordinates, where the sieve size is on a horizontal 'logarithmic' scale, and the percentage by weight of the size smaller than a particular sieve-size is on a vertical 'arithmetic' scale.

- Logarithmic scales for the particle diameter gives a very convenient representation of the sizes because a wide range of particle diameter can be shown in a single plot.

A Typical Results obtained from particle size analysis :

Sieve size	Total wt of soil taken	wt retained on a particular sieve (gram)	Cumulative wt retained	Cumulative % retained	% finer = 100 – cumulative % retained
80 mm		10	10	$\frac{10}{500}\times100=2$	98%
20 mm	500 g	165	175	35	65%
10 mm		100	275	55	45%
4.75 mm		85	360	72	28%
2 mm		60	420	84	16%
1 mm		20	440	88	12%
600 μ		40	480	96	4%
425 μ		10	490	98	2%
212 μ		5	495	99	1%
150 μ		3	498	99.6	0.4%
75 μ		1	499	99.8	0.2%

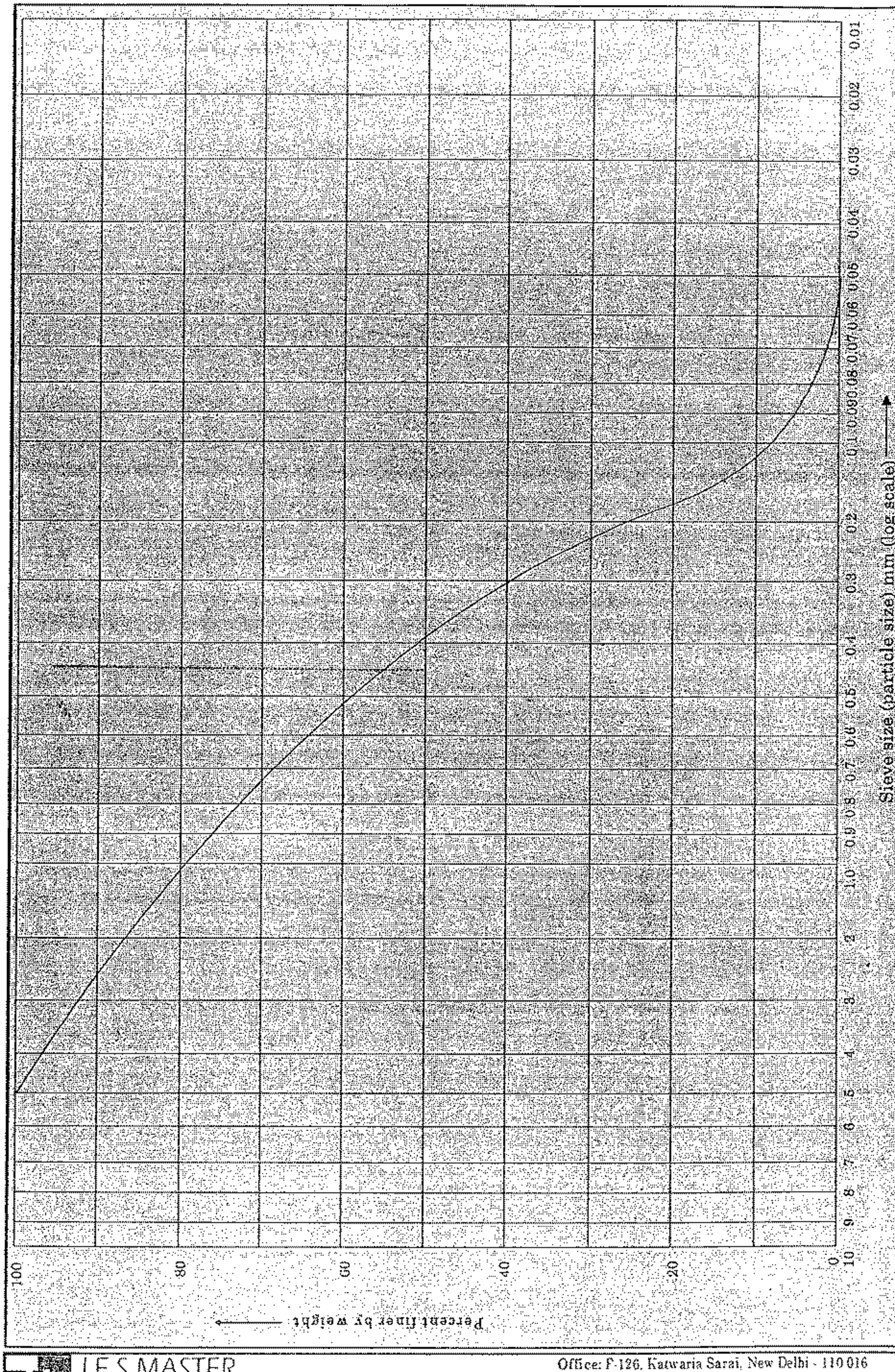
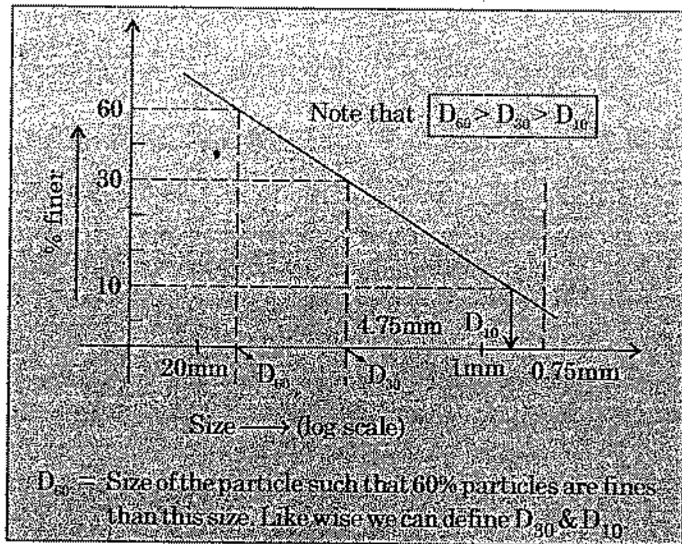
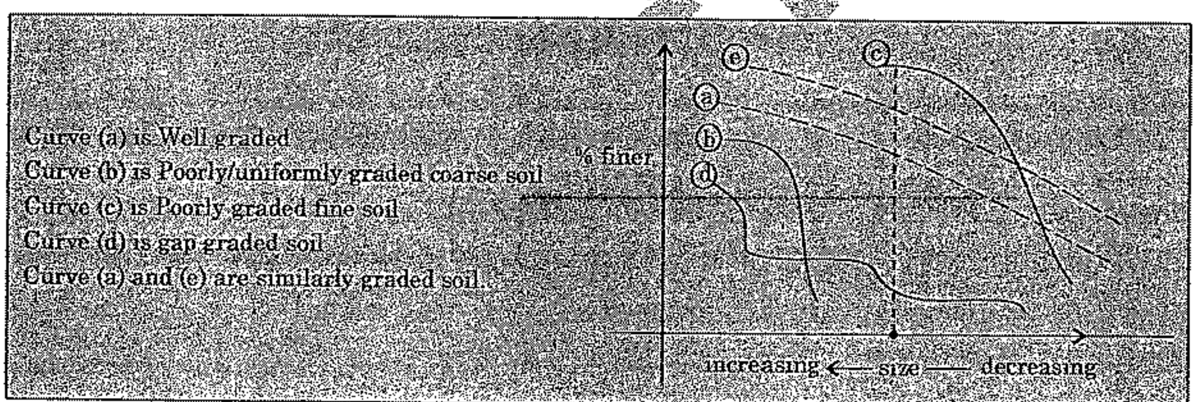


Fig. Particle-size distribution curve

The resulting grain size destribution curve is as shown below



- Actually sieve size is assumed to correspond to size of particle.
- The various types of curves obtained in sieve analysis are classified as follows



- Well graded mean soils of all sizes are present
- Poorly graded/uniformly graded means soil of predominantly one size is only present.
- Gap graded means some of the soil particle sizes are missing.
- Position of the curve indicates–**type of soil** where as shape of the curve indicates–**gradation**.
- As the slope of curve decreases–gradation increases.
- D_{10} = effective size of particle i.e. particle sizes which if present alone will cause the same effect as is caused by the soil.
- In filter analysis it was found that the size smaller than D_{10} affects the functioning of filter more than remaining 90% particle.
- Grain size distribution curve is used to find out the following shape parameters.

1. Co-efficient of uniformity (Cu) = $\dfrac{D_{60}}{D_{10}}$

2. Coefficient of curvature (C_C) = $\dfrac{D_{30}^2}{D_{60} \times D_{10}}$

If, Cu = 1 → Soil is perfectly uniformly graded. (curve will be vertical)

Cu > 4 → well graded gravel

Cu > 6 → well graded sand

$1 \le C_C \le 3$ → well graded soil

For well graded sand, Cu > 6, $1 \le C_C \le 3$

For well graded gravel, Cu > 4, $1 \le C_C \le 3$

Larger the value of Cu, larger is the range of particles is soil.

Example 12.

The results of a sieve analysis of a soil are given below:

Total mass of sample = 900 gm.

IS Sieve	20 mm	10 mm	4.75 mm	2 mm	1.0 mm	0.6 mm	4.25 μ	212 μ	150 μ	75 μ	Pan
Mass of soil retained (gm)	35	40	80	150	150	140	115	55	35	25	75

Draw the particle size distribution curve and hence determine the uniformity coefficient and the coefficient of curvature.

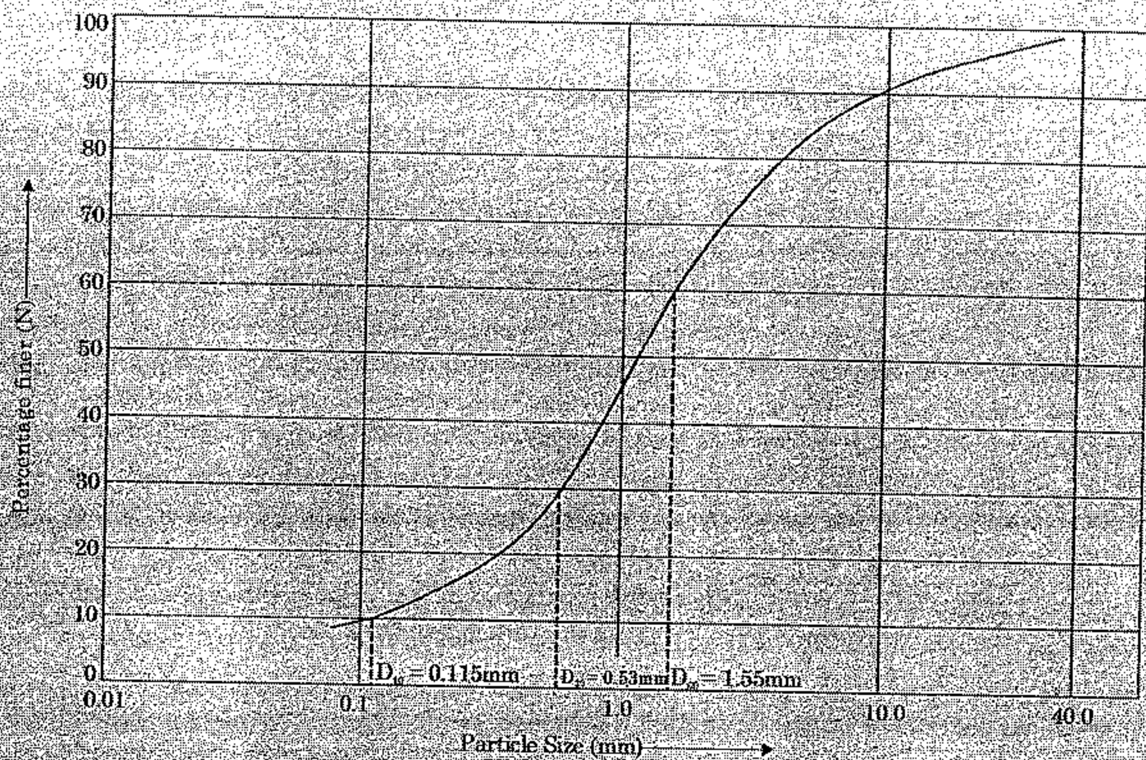
Sol. The calculations for percentage finer *N* than different sizes are shown in Table

IS Sieve	Mass retained	Percentage retained $= \frac{(2)}{(900)} \times 100$	Cumulative percentage retainde	Percentage Finer (N) = 100 – (4)
(1)	(2)	(3)	(4)	(5)
20 mm	35 gm	3.89	3.89	96.11
10	40	4.44	8.33	91.67
4.75	80	8.89	17.22	82.78
2.0	150	16.67	33.89	66.11
1.0	150	16.67	50.56	49.44
0.6	140	15.56	66.12	33.88
425 μ	115	12.78	78.90	21.10
212 μ	55	6.11	85.01	14.99
150 μ	35	3.89	88.90	11.10
75 μ	25	2.78	91.68	8.32
Pan	75	8.32	100.00	

$\Sigma = 900.0$ gm

The particle size distribution curve is shown in Fig.

From plot, D_{60} = 1.55 mm; D_{30} = 0.53 mm; D_{10} = 0.115 mm



$$C_u = \frac{D_{60}}{D_{10}} = \frac{1.55}{0.115} = 13.48$$

$$C_c = \frac{(D_{30})^2}{D_{60} \times D_{10}} = \frac{(0.53)^2}{1.55 \times 0.115} = 1.58$$

SEDIMENTATION ANALYSIS

- The soil particles less than 75–μ size can be further analysed for the distribution of the various grain-sizes of the order of silt and clay by 'sedimentation analysis' or 'wet analysis'.
- The soil fraction is kept in suspension in a liquid medium, usually water.
- The particles descend at velocities, related to their sizes, among other things.
- The analysis is based on 'Stokes Law'.
- As per this law, if a single sphere is allowed to fall in an infinite liquid medium without interference, its velocity first increases under the influence of gravity, but soon attains a constant value.
- This constant velocity, which is maintained indefinitely unless the boundary conditions change, is known as the 'terminal velocity'.
- The principle is obvious; coarser particles tend to settle faster than finer ones.
- By Stokes' law, the terminal velocity of the spherical particle is given by

$$v = \frac{(\gamma_s - \gamma_l)D^2}{18\mu}$$

- Stokes' Law is considered valid for particle diameters ranging from 0.2 to 0.0002 mm.
- For particle sizes greater than 0.2 mm, turbulent motion is set up and for particle sizes smaller than 0.002 mm, Brownian motion is set up. In both these cases Stokes' law is not valid.

The limitations of sedimentation analysis, based on Stokes' law, are as follows:

(i) The finer soil particles are never perfectly supherical. Their shape is flake-like or needle-like. However, the particles are assumed to be spheres, with equivalent diameters (the basis of equivalence being the attainment of the same terminal velocity as that in the case of a perfect sphere.)

(ii) Stokes' law is applicable to a sphere falling freely without any interference, in an infinite liquid medium. The sedimentation analysis is conducted in a one-litre jar, the depth being finite; the walls of the jar could provide a source of interference to the free fall of particles near it. The fall of any particle may be affected by the presence of adjacent particles; thus, the fall may not be really free. However, it is assumed that the effect of these sources of interference is insignificant if suspension is prepared with about 50 g of soil per litre of water.

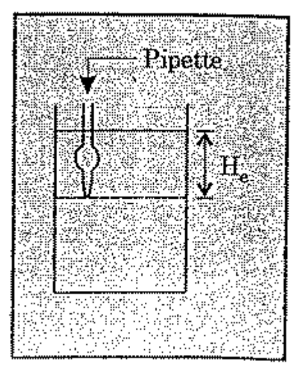
(iii) All the soil grains may not have the same specific gravity. However, an average value is considered all right, since the variation may be insignificant in the case of particles constituting the fine fraction.

(iv) Particles constituting to fine soil fraction may carry surface electric charges, which have a tendency to create 'flocs'. Unless these floces are broken, the sizes calculated may be those of the flocs. Flocs can be a source of erroneous results. A deflocculating agent, such as sodium silicate, sodium oxalate, or sodium hexametaphosphate, is used to get over this difficulty.

The general procedure for sedimentation analysis, which may be performed either with the aid of a pipette or a hydrometer is as follows:

- An appropriate quantity of an oven-dried soil sample, finer than 75–μ size, is mixed with a known volume (V) of distilled water in jar.
- The sample is pretreated with an oxidising agent and an acid to remove organic matter and calcium compounds.
- Addition of hydrogen peroxide on heating would remove organic matter. Treatment with 0.2 N hydrochloric acid would remove calcium compounds.
- Later, a deflocculating or a dispersing agent, such as sodium hexameterphosphate is added to the solution. The mixture is shaken thoroughly by means of a mechanical stirrer and the test is started, keeping the jar vertical.
- The soil particles are assumed to be uniformly distributed throughout the suspension, at the instant of commencement of the test.

PIPETTE METHOD



- It is a laboratory method. A pipette, sedimentation jar, and a number of sampling bottles are necessary for the test.
- The method consists in drawing off 10 ml samples of soil suspension by means of the sampling pipette from a standard depth of 10 cm at various time intervals after the start of sedimentation.
- The usual total time intervals at which the samples are drawn are 30 s , 1 min., 2 min., 4 min., 8 min., 5 min., 30 min., 1 h, 2h, and 4 h from the start of sedimentation.
- The pipette should be inserted about 20 seconds prior to the chosen instant and the process of sucking should not take more than 20 seconds.
- Each of the samples taken is transferred to a sampling bottle and dried in an oven.

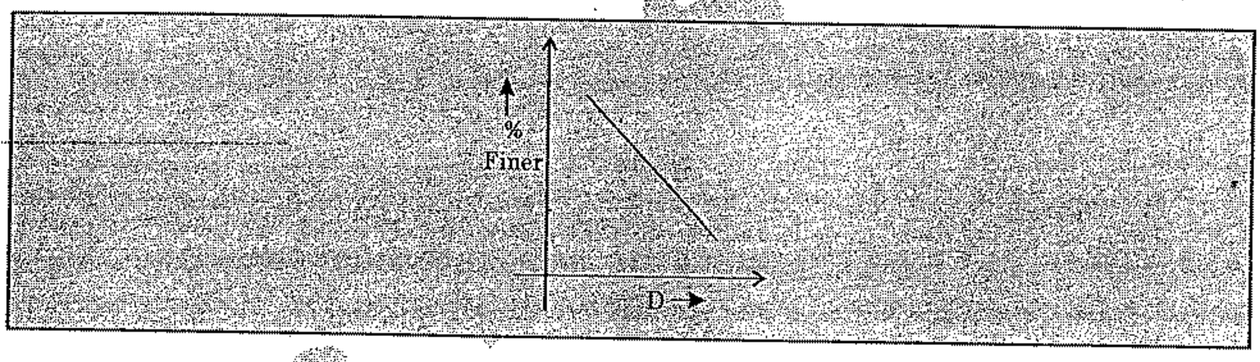
The concentration of all sized particles are present in a particular volume at time t = 0.

- This concentration is equal to weight of solid per cc in original soil suspension = x.
- If after time 't' sample is taken out in pipette from height H_e then all particles having settling velocity greater than H_e will have settled below height H_e.
- At this height at time 't' sample collected will have the same concentration of particles of settling velocity less than $\frac{H_e}{t}$ as was there in the original soil suspension.
- If settling velocity of particle size (D) = H_e/t then

$$\boxed{\%\text{ finer than D} = \frac{\text{wt of solid per cc at depth } H_e \text{ after time 't'}}{\text{wt of solid per cc in the original suspension}}}$$

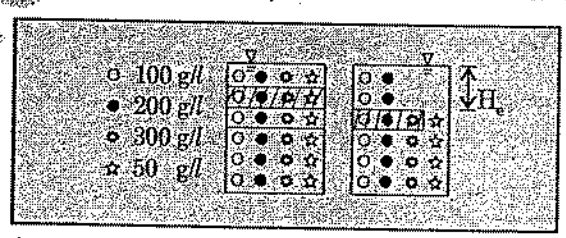
As, $$\boxed{\frac{(\gamma_s - \gamma_w)\,D^2}{18\,\mu} = \frac{H_e}{t}}$$

Find 'D' from this corresponding to various time t and plot grain size distribution curve.



In pipette method, sample is collected from height H_e at various time intervals i.e. H_e is fixed.

Note: • Let there be 4 types of particles in the sample. Let in the original soil suspension before the start of sedimentation the concentration of various particles be as shown below be as shown below



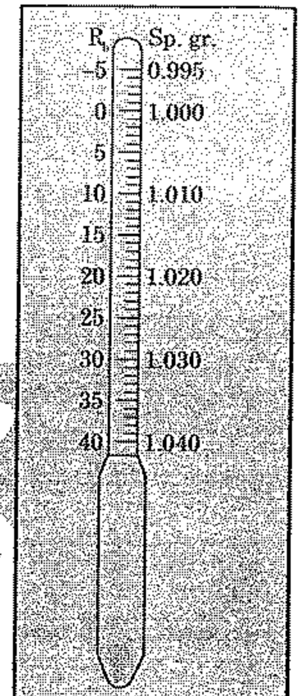
- If particle has settling velocity greater than H_e/t, then concentration of soil suspension collected at height He will be 600 g/l.

Thus, % finer than size ☆ = $\dfrac{\text{concentration of sample collected at time t from height } H_e}{\text{concentration of soil from original soil suspension}} \times 100 = \dfrac{600}{650} \times 100$

- If particle ☆ has size D, then % finer than size D is $\dfrac{600}{650} \times 100$. Where D is given by $\dfrac{H_e}{t} = \dfrac{(\gamma_s - \gamma_w)D^2}{18\mu}$)

HYDROMETER ANALYSIS

The hydrometer method differs from the pipette analysis in that the weights of solids per ml in the suspension at the chosen depth at chosen instants of time are obtained indirectly by reading the specific gravity of the soil suspension with the aid of a hydrometer.



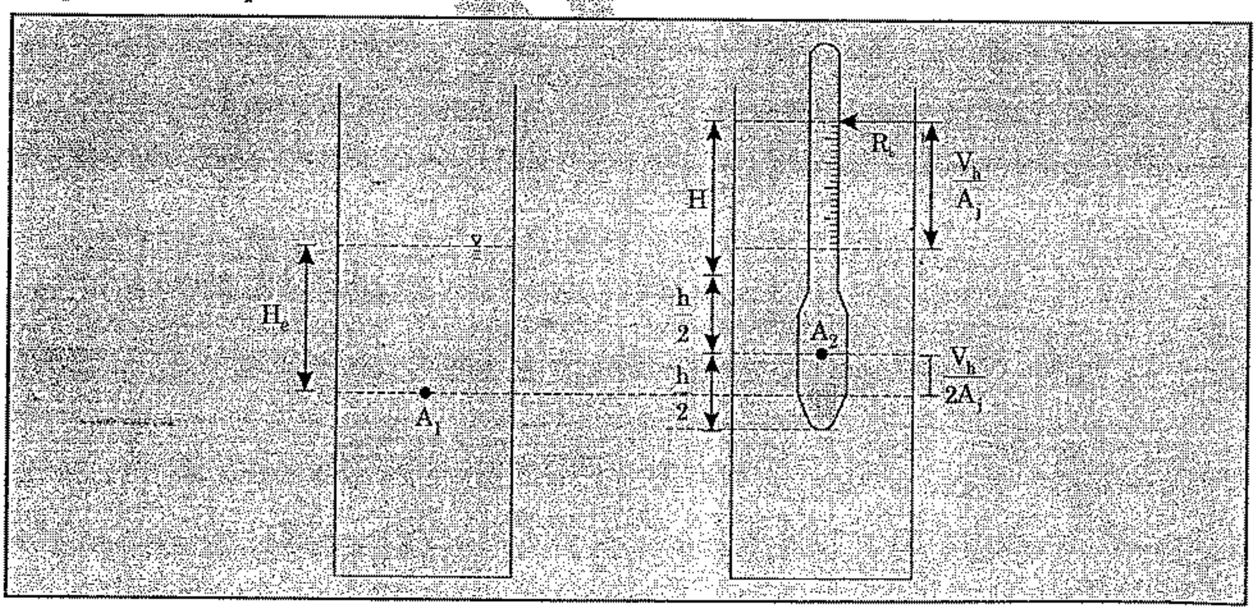
Hydrometer

- Hydrometer is a device which is used to measure the specific gravity of liquids. However, for a soil suspension, the particles start settling down right from the start, and hence the unit weight of the suspension varies from top to bottom.
- As time passes the hydrometer goes down because wt of hydrometer is balanced by the wt of liquid displaced by hydrometer. As solid concentration goes on reducing in water, more amount of liquid has to be displaced to balance the weight of hydrometer.

We have already discussed that % of finer than D $= \dfrac{\text{Wt. of solid per cc at depth } H_e \text{ after time t}}{\text{Wt. of solid per cc in original soil suspension}}$

Where D is given by $\boxed{\dfrac{(\gamma_s - \gamma_w)\,D^2}{18\,\mu} = \dfrac{H_e}{t}}$

Thus using hydrometer we have to measure density of soil suspension at a depth H_e after time t. Fig. (a) shows the height H_e. But due to its own volume, insertion of hydrometer will lead to displacement of liquid.



Thus, Point at A_1 occuptes the position A_2. A_2 is the centre of volume of hydrometer.

V_h = Volume of hydrometer
A_j = Area of Jar
h = length of bulb in hydrometer

- Volume of bulb can be thought of as the total volume of hydrometer.
- Hydrometer measures density of soil suspension at depth H_e which is given by

$$H_e = H + \frac{h}{2} + \frac{V_h}{2A_j} - \frac{V_h}{A_j}$$

$$\boxed{H_e = H + \frac{1}{2}\left(h - \frac{V_h}{A_j}\right)}$$

H will correspond to hydrometer reading $\mathbf{R_h}$, Thus, $\mathbf{H_e}$ will be related to R_h

Hydrometer is calibrated such that $1 + \frac{R_h}{1000}$ = Sp. gravity of soil suspension at depth H_e

$$1 + \frac{R_h}{1000} = \frac{\text{density of soil suspension at depth } H_e}{\text{density of water}}$$

$$1 + \frac{R_h}{1000} = \frac{\begin{pmatrix}\text{density of}\\ \text{water}\end{pmatrix} + \begin{pmatrix}\text{immerged wt of}\\ \text{solid per unit volume}\end{pmatrix}}{\text{density of water}}$$

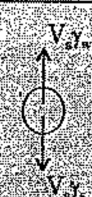
Note:

Density of soil suspension = $\dfrac{V_s\gamma_s + V_w\gamma_w}{V_s + V_w} = \dfrac{(V_s + V_w)\gamma_w + (\gamma_s - \gamma_w)V_s}{(V_s + V_w)}$

$$= \gamma_w + \frac{(\gamma_s - \gamma_w)V_s}{(V_s + V_w)} = \gamma_w + \frac{V_s\gamma_s}{V_s + V_s}\left(1 - \frac{\gamma_w}{\gamma_s}\right) = \gamma_w + W_d\left(1 - \frac{1}{G_s}\right)$$

W_d = wt of solid per unit volume

Density of soil suspension = Density of water + immerged wt of solid per unit volume



$V_s(\gamma_s - \gamma_w)$ = immersed wt = $V_s\gamma_s\left(1 - \dfrac{1}{G_s}\right)$

$\Rightarrow$ immerged wt unit volume = $\dfrac{V_s\gamma_s}{V_s + V_w}\left(1 - \dfrac{1}{G_s}\right)$

$$\Rightarrow \quad 1 + \frac{R_h}{1000} = 1 + \frac{\text{immerged wt of solid/CC}}{\text{wt density of water}}$$

$$1 + \frac{R_h}{1000} = 1 + \frac{W_d\left(1 - \frac{1}{G_s}\right)}{\gamma_w}$$

$$\Rightarrow \quad W_d = \frac{R_h \gamma_w G_s}{1000\,(G_s - 1)}$$

$$\%\text{ finer than D} = \frac{W_d}{W} \times 100$$

where,

W = weight of solid per unit volume in original soil suspension

$$\Rightarrow \quad \%\text{ finer than D} = \frac{R_h \gamma_w G_s}{(G_s - 1)\,1000} \times \frac{100}{W}$$

$$\boxed{\%\text{ finer than D} = \frac{R_h G_s \gamma_w}{(G_s - 1) \times 10\,W}}$$

Note:

$$\boxed{\frac{(\gamma_s - \gamma_w)\,D^2}{18\,\mu} = \frac{He}{t} = \frac{H + \frac{1}{2}\left(h - \frac{V_h}{A}\right)}{t}}$$

Corrections to Hydrometer Readings

The following three corrections are necessary:

1. Meniscus correction
2. Temperature correction
3. Deflocculating agent correction.

Meniscus Correction

The reading should be taken at the lower level of the meniscus. However, since the soil suspension is opaque, the reading is taken at the upper meniscus. Therefore, a correction is required to be applied to the observed reading. Since the hydrometer readings increases downward on the stem, **the meniscus correction (C_m) is obviously positive**.

The magnitude of the correction can be got by placing the hydrometer in distilled water in the same jar and noting the difference in reading at the top and bottom levels of the meniscus.

Temperature Correction

Hydrometers are usually calibrated at a temperature of 27°C. If the temperature at the time of conducting the test is different, a correction will be required to be applied to the hydrometer reading on this account. **If the temperature** at the time of test is **more** than that of calibration of the hydrometer, the observed reading will be less and the **correction (C_t) would be positive and vice versa.**

Defloculating Agent Correction

The addition of the **deflocculating agent increases the density of the suspension** and thus necessitates a correction (C_d) which is always negative.

A composite correction for all the above may be obtained by noting the hydrometer reading in a solution of the deflocculating agent at different temperatures. These with reversed sign give the composite correction.

The corrected hydrometer reading R_h may be got from the observed reading R_h' by applying the composite correction C:

$$R_h = R_h' \pm C$$

$$C = C_m - C_d \pm C_t$$

Example 13.

A sample of dry soil ($G_s = 2.68$) weighing 125 gm is uniformly dispersed in water to form a 1 litre suspension at a temperature of 28°C.

(i) Determine the unit weight of the suspension immediately after its preparation.

(ii) 10 cc of the suspension was removed from a depth of 20 cm beneath the top surface after the suspension was allowed to settle for 2.5 min. The dry weight of the sample in the suspension drawn was found to be 0.398 gm. Determine a single point on the particle size distribution curve corresponding to this observation. Given, at 28°C, viscosity of water = 8.36 millipoises and unit weight of water = 0.9963 gm/cc

Sol. (i) Volume of solids in the suspension $= \dfrac{125}{2.68} = 46.64$ cc.

Considering unit weight of suspension,

$$\text{Volume of solids present} = \frac{46.64}{1000} = 0.0466 \text{ cc}$$

$$\text{Volume of water present} = 1 - 0.0466 = 0.9534 \text{ cc}$$

$$\text{Weight of 0.466 cc of solids} = (0.0466)(2.68) = 0.1249 \text{ gm}$$

$$\text{Weight of 0.9534 cc of water at 28°C} = (0.9534)(0.9963) = 0.9499 \text{ gm}$$

$$\therefore \text{Total weight of 1 cc of suspension} = 0.1249 + 0.9499 = 1.0748 \text{ gm}$$

$$\text{Therefore, unit weight of suspension} = 1.0748 \text{ gm/cc}$$

(ii) We have, from Stokes' law,

$$v = \frac{\gamma_s - \gamma_w}{18\mu} \times D^2$$

or,

$$D = \sqrt{\frac{18\mu}{\gamma_s - \gamma_w}} \times \sqrt{v}$$

Let D be the diameter of the particles settled to a depth of 20 cm at t = 2.5 min. with a uniform velocity v.

$$v = \frac{Z}{t} = \frac{20}{(2.5)(60)} = 0.133 \text{ cm/sec}$$

$$\mu = 8.36 \text{ millipoises}$$

$$= \frac{8.36 \times 10^{-3}}{981} = 8.522 \times 10^{-6} \text{ gm-sec/cm}^2$$

$$\gamma_s = 2.68 \text{ gm/cc}, \ \gamma_w = 0.9963 \text{ gm/cc},$$

$$\therefore \quad D = \sqrt{\frac{(18)(8.522 \times 10^{-6})}{2.68 - 0.9963}} \times \sqrt{0.133} \text{ cm}$$

$$= 3.48 \times 10^{-3} \text{ cm} = 0.035 \text{ mm}$$

Again, at time t = 0, weight of solids present in 1 cc of suspension = 0.1249 gm.

$\therefore$ Weight of solids present in 10 cc of suspension = 1.249 gm.

At time t = 2.5 min, weight of solids present in 10 cc of suspension = 0.398 gm

$$\therefore \quad \% \text{ finer} = \frac{0.398}{1.249} \times 100 = 31.86\%$$

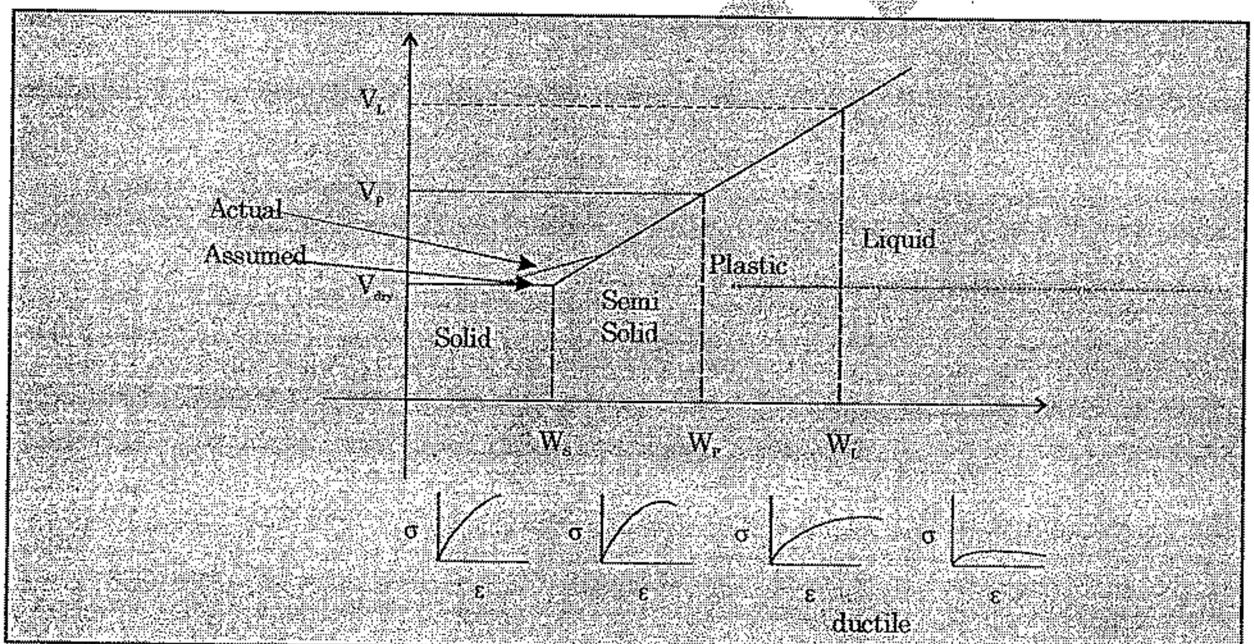
Hence the co-ordinates of the required point on the particle size distribution curve are :

$$D = 0.035 \text{ mm}$$

$$N = 31.86\%$$

CONSISTENCY LIMITS

- Consistency represents the relative ease with which a soil can be deformed.
- This term is mainly used for clayey soil and is related to water content i.e. how with change in water content the consistency of soil changes.
- Atterberg classified the consistency in 4-stages. Behaviour of soil is different in different stages.
  - Solid stage
  - Semi solid stage
  - Plastic stage
  - Liquid stage



W_L = liquid limt water content
W_P = Plastic limit water content
W_S = Shrinkage limit water content
V_L = Volume of soil at liquid limit
V_P = Volume of soil at plastic limit
V_{dry} = Volume of soil at shrinkage limit

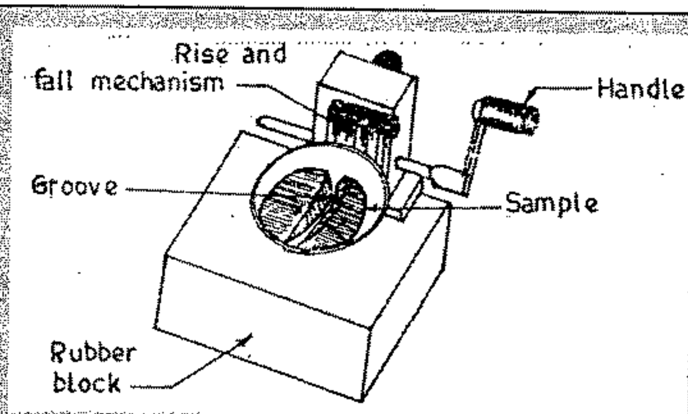
From the above figure

$$\frac{V_L - V_P}{W_L - W_P} = \frac{V_P - V_{dry}}{W_P - W_S}$$

Note: Naturally existing soil has water content between WL and WP.

LIQUID LIMIT

- Min water content of which soil have tendency to flow is called liquid limit water content.
- All soils at liquid limit will have similar shear strength (approx 2.7 kN/m²) which is negligible.
- Liquid limit is found out using
  (a) Casagrande's tool
  (b) Cone penetration
- **Casagrade's Tool :** Soil is taken and water is added and the soil is put inside Casagrande's apparatus. A groove of 2 mm size is cut and the apparatus is given blows over a rubber pad. No of blows required to close the 2 mm grove is noted. Water content at which 25 blows closes the groove is called **liquid limit.**



Casagrande's apparatus

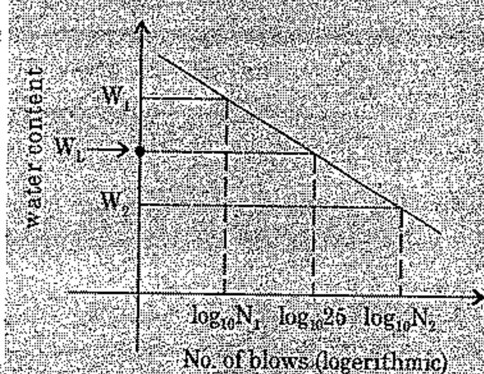
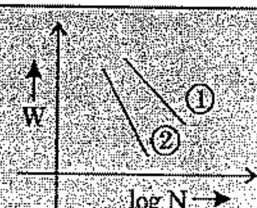


Fig. (α)

- Slope of the above curve is called flow index (I_f) and the curve is called **flow-curve.**

$$I_f = \frac{W_1 - W_2}{\log_{10} N_2 - \log_{10} N_1} = \frac{W_1 - W_2}{\log_{10} \frac{N_2}{N_1}}$$

$$\text{Flow index} \propto \frac{1}{\text{Shear strength}}$$



- Curve (1) has larger shearing strength
  Curve (2) has lower shearing strength
- Calculation of liquid limit using fig. (α) require two measurement however liquid limit can also be calculated from one-point method as discussed below

$$W_L = W_N\left(\frac{N}{25}\right)^x$$

W_N = water content at which N blows are required to close groove.

x varies between 0.086 – 0.121.

Example 14.

The results of a liquid limit test are given below:

No. of blows	48	38	29	20	14
Water content (%)	32.1	35.9	40.7	46.1	52.8

(a) Determine the liquid limit of the soil.

(b) If the plastic limit of the soil be 23%, find out the plasticity index, flow index and toughness index. Hence comment on the nature of the soil.

Sol. (a) From the Given Data:

A curve between the water content and the number of blows is plotted on a semi-log graph paper. Fig. (a) shows this w vs. $\log_{10}$ N curve. The water content corresponding to 25 blows, as obtained from the curve, is 43%. Hence the liquid limit of the soil is 43% *i.e.*, w_l = 43%

(b) Plasticity index, $I = w_l - w_p = 43\% - 23\% = 20\%$

Flow index, $$I_f = \frac{52.8 - 32.1}{\log_{10} 48/14} = 38.68\%$$

Toughness index $$I_t = \frac{I_P}{I_f} = \frac{20}{38.68} = 0.52$$

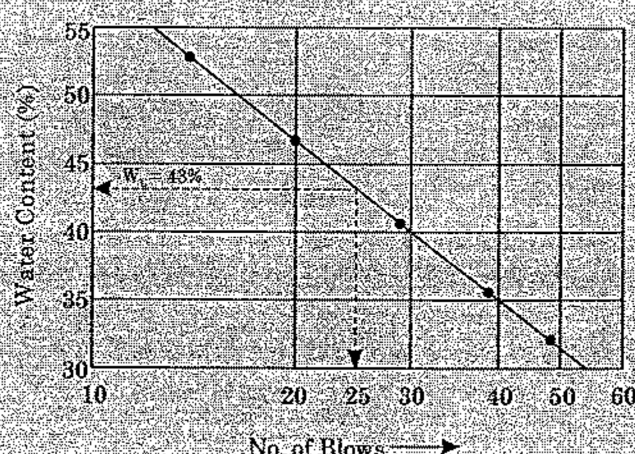


Fig. (a)

- As the plasticity index is greater than 17%, the soil is highly plastic in nature.
- As the toughenss index is less than 1, the soil is friable at liquid limit.

CONE PENETROMETEGR METHOD

The cup is placed below the cone, and the cone is gradually lowered so as to just touch the surface of the soil in the cup. The graduated scale is adjusted to zero. The cone is released, and allowed to penetrate the soil for 30 seconds. The water content at which the penetration is 25 mm is the liquid limit.

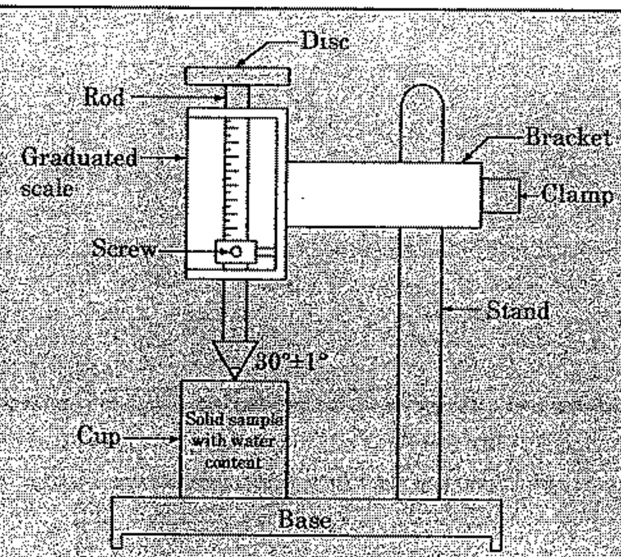


Fig. : Cone Penetrometer

The shear strength of soil at liquid limit, as determined by this method, is about 1.76 kN/m² which occurs when the penetration is 25 mm.

The cone penetrometer method has several advantages over the Casagrande method.

(1) It is easier to perform.

(2) The method is applicable to a wide range of soils.

(3) The results are reliable, and do not depend upon the judgment of the operator.

PLASTIC LIMIT

Min water content at which soil is in plastic stage is called plastic limit water content. At plastic limit water content, a soil when rolled into a thread of 3 mm starts to crumble.

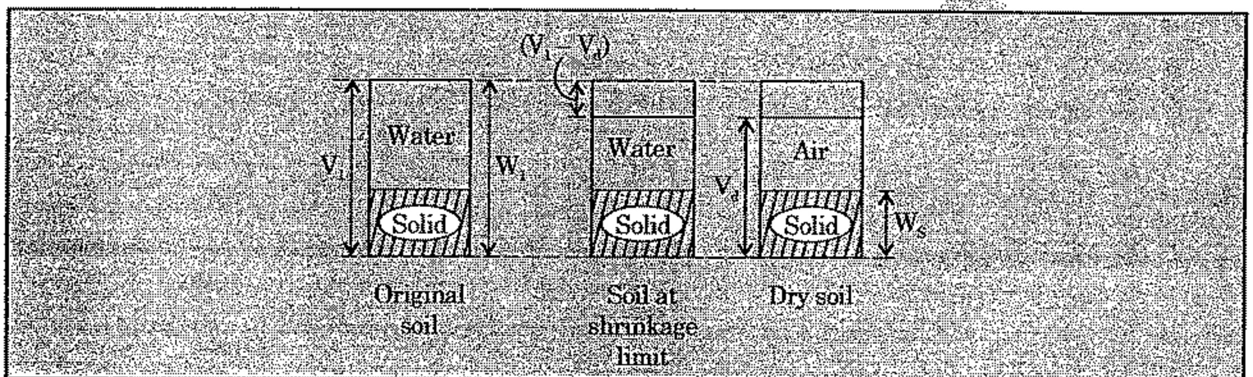
- For determination of the plastic limit of a soil, it is air-dried and sieved through a 425μ IS seive. About 30 gm of soil is taken in an evaporating dish. It is mixed throughly with distilled water till it becomes plastic and can be easily moulded with fingers.
- About 10 gm of the plastic soil mass is taken in one hand and a ball is formed. The ball is rolled with fingers on a glass plate to form a soil thread of uniform diameter.
- If the diameter of thread becomes smaller than 3 mm, without crack formation, it shows that the water content is more than the plastic limit.
- The soil is kneaded further. This results in the reduction of the water content, as some water is evaporated due to the heat of the hand.
- The soil is re-rolled and the procedure repeated till the thread crumbles.
- The water content at which the soil can be rolled into a thread of approximately 3 mm in diameter without crumbling is known as the palstic limit (PL or w_p).

- The shear strength at the palstic limit is about 100 times that at the liquid limit.

Shrinkage Limit

- That max water content at which further reduction in water content does not cause any reduction in the volume of soil sample.
- When water content is reduced below W_s, the particles are so closely spaced that volume reduction will not take place and the void space starts getting occupied by air instead of water.
- It is the min water content at which soil is saturated.

Determination of Shrinkage Limit



V_1 = Original soil volume
w_1 = Original wt. of soil
W_s = dry wt of soil
V_d = dry volume of soil

- (V_1), (W_1), (W_S) and (V_d) are known experimentally.

$$\text{Water content} = \frac{W_W}{W_S}$$

- At shrinkage limit, $W_W = W_1 - W_S - (V_1 - V_d)\,\gamma_w$
- Shrinkage limt $= \dfrac{(W_1 - W_S) - (V_1 - V_d)\,\gamma_w}{W_S}$
- Shrinkage limit test can be used to determine sp. gravity of solids

$$G_s = \frac{\gamma_s}{\gamma_w} = \frac{W_S}{V_S \gamma_w} = \frac{W_S}{\left[V_1 - \dfrac{(W_1 - W_S)}{\gamma_w}\right]\gamma_w} \qquad \Rightarrow \quad G_S = \frac{W_S}{V_1\gamma_w - (W_1 - W_S)}$$

- Once G_s is known, shrinkage limit can also be determined as follows

$$W_S = \frac{W_W}{W_S} = \frac{(V_d - V_s)\,\gamma_w}{W_S} = \frac{\left(V_d - \dfrac{W_S}{G_s\gamma_w}\right)\gamma_w}{W_S}$$

- Shrinkage ratio is defined as volume change in soil above shrinkage limit expressed as percentage of dry soil per unit change in water content above shrinkage limit.

$$\text{Shrinkage ratio} = \frac{\left(\dfrac{V_1 - V_2}{V_d}\right)\times 100}{w_1 - w_2} = R$$

$$R = \frac{(V_1 - V_2)/V_d}{w_1 - w_2} \text{ as fraction}$$

$$= \frac{(V_1 - V_2)/V_d}{\frac{(V_1 - V_2)\gamma_w}{W_S}} = \frac{W_S}{V_d \gamma_w} = \frac{\gamma_d}{\gamma_w}$$

$$\text{Thus, Shrinkage ratio } = \frac{\text{Dry density}}{\text{Density of water}}$$

- $R = \frac{\gamma_d}{\gamma_w}$ = mass specific gravity of the soil in dry state
- Volumetric Shrinkage = $\frac{V_1 - V_d}{V_d} \times 100$

$$R = \frac{\frac{V_1 - V_2}{V_d} \times 100}{w_1 - w_2}, \quad R = \frac{\frac{V_1 - V_d}{V_d} \times 100}{w_1 - w_S}$$

$$\Rightarrow \quad (w_1 - w_S) \times R = \text{Volumetric shrinkage}$$

where w_S = Shrinkage limit

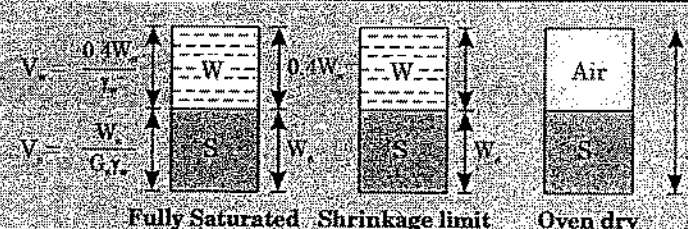
Example 15.

The mass specific gravity of a fully saturated specimen of clay having a water content of 40% is 1.88. On oven drying the mass specific gravity drops to 1.74. Calculate the specific gravity of clay and its shrinkage limit.

Sol. Given that, for fully saturated specimen [**Note :** As soil is fully saturated $V_v = V_w$]

$$G_m = \frac{\gamma_{soil}}{\gamma_w} = 1.88, \; w = 0.4$$

$$G_m \gamma_w = \gamma_{soil} = \frac{W_w + W_s}{V_w + V_s} = 1.88\,\gamma_w$$



$$\frac{0.4W_s + W_s}{\frac{0.4\,W_s}{\gamma_w} + \frac{W_s}{G_s\gamma_w}} = 1.88\,\gamma_w$$

Canceling W_s from numerator & denominator

$$\frac{1.4\,\gamma_w}{\left(\frac{1}{G_S}+0.4\right)} = 1.88\,\gamma_w$$

$$\frac{1}{G_S}+0.4 = \frac{1.4\,\gamma_w}{1.88\,\gamma_w}$$

$$\boxed{G_s = 2.9} \quad \text{...(i)}$$

Given that for oven dry sample

$$G_m = \frac{\gamma_d}{\gamma_w}$$

$$\gamma_d = G_m\gamma_w$$

$$\frac{W_s}{V_v+V_s} = G_m\gamma_w$$

$$\frac{W_S}{\frac{W_S}{G_S\gamma_w}+\frac{w_s.W_S}{\gamma_w}} = 1.74\,\gamma_w$$

$$\frac{1}{\frac{1}{G_S}+w_s} = 1.74$$

$$\frac{1}{G_S}+w_s = \frac{1}{1.74}$$

$$w_s = \frac{1}{1.74}-\frac{1}{2.9}$$

$$\boxed{w_s = 0.23}$$

Example 16.

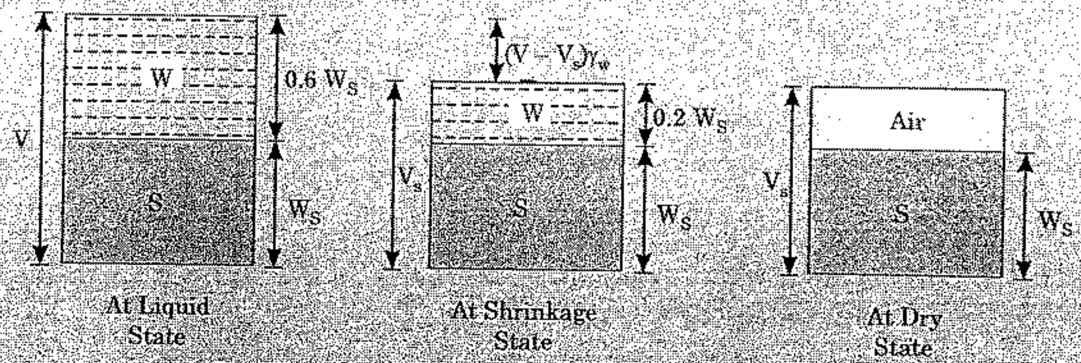
The values of liquid limit, plastic limit and shrinkage limit of a soil were reported as below:

$w_L = 60\%$, $w_P = 30\%$, $w_S = 20\%$

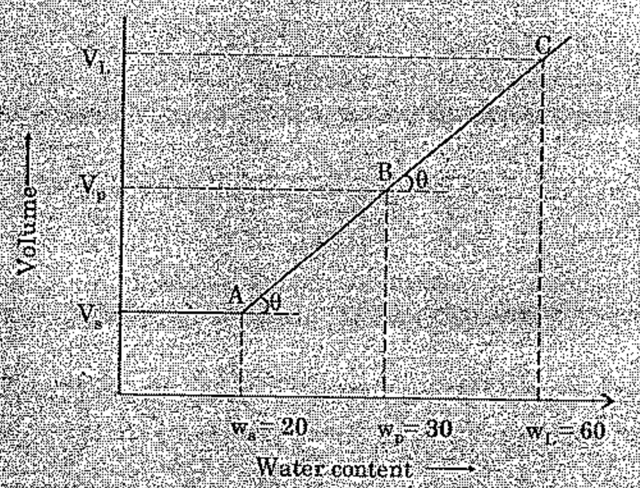
If a sample of this soil at liquid limit has a volume of 40 cc and its volume measured at shrinkage limit was 23.5 cc, determine the specific gravity of the solids. What is its shrinkage ratio and volumetric shrinkage?

Sol. Given that

at Liquid Limit	$w_l =$ 60%	$V_L =$ 40 cc
at Shrinkage Limit	$w_s =$ 20%	$V_s =$ 23.5 cc
at Plastic Limit	$w_p =$ 30%	



Equating the slope of AB and BC



Slope of AB = Slope of BC

$$\Rightarrow \quad \frac{V_p - V_S}{(w_p - w_s)} = \frac{(V_L - V_p)}{(w_L - w_p)}$$

$$\Rightarrow \quad \frac{V_p - 23.5}{(30-20)} = \frac{(40 - V_p)}{(60-30)}$$

$$\Rightarrow \quad \frac{(V_p - 23.5)}{10} = \left(\frac{40 - V_p}{30}\right)$$

$$\Rightarrow \quad 3V_p - 70.5 = 40 - V_p$$

$$\Rightarrow \quad V_p = \frac{70.5 + 40}{4} = 27.625 \text{ cc} \qquad \text{...(i)}$$

At the Shrinkage State

$$\Rightarrow \quad (W - W_s) - (V - V_s)\gamma_w = W_s w_s$$

$$(1.6W_s - W_s) - (40 - 235) \times 1 = 0.20W_s$$

$$0.6W_s - 0.2W_s = 16.5$$

$$W_s = \left(\frac{16.5}{0.4}\right) = 41.25 \qquad \text{...(ii)}$$

We know that
$$G_s = \left(\frac{\gamma_s}{\gamma_w}\right)$$

$$V_s = \left(\frac{w_s}{G_s \gamma_w} + \frac{0.2 w_s}{\gamma_w}\right)$$

$$23.5 = \frac{w_s}{\gamma_w}\left[\frac{1}{G_s} + 0.2\right] \qquad \text{... (iii)}$$

Put the value of W_S from (*ii*)

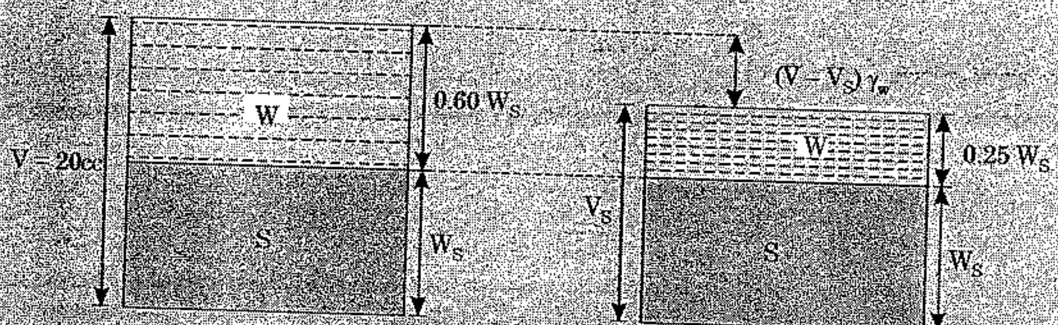
$$\Rightarrow \quad 23.5 = \frac{41.25}{1}\left[\frac{1}{G_s}+0.2\right]$$

$$\Rightarrow \quad G_s = 2.70$$

Example 17.

The Atterberg limits of a given soil are L.L = 60%, PL = 45% and S.L = 25%. The Specific gravity of Soil solids is 2.67. A sample of thin soil at liquid limit has a volume of 20 cc. What will be its final volume if the sample is brought to its shrinkage limit ?

Sol.



At L.L
G = 2.67

At S.L

$$\Rightarrow \quad w_s \times W_S = (W - Ws) - (V - Vs)\gamma_w$$

$$\Rightarrow \quad 0.25\,W_s = (1.60\,W_s - W_s) - (20 - V_s) \times 1$$

$$\Rightarrow \quad 0.25\,W_s = 0.60\,W_s - (20 - V_s) \times 1$$

$$\Rightarrow \quad 20 - V_s = 0.35\,W_s$$

$$V_s = (20 - 0.35\,W_s) \quad \text{...(i)}$$

At liquid limit

$$\Rightarrow \quad V = 20 = \left(\frac{W_s}{G\gamma_W}+\frac{0.60W_s}{\gamma_w}\right)$$

$$\Rightarrow \quad 20 = \frac{W_s}{\gamma_w}\left[\frac{1}{2.67}+0.60\right] \quad [\gamma_w = 1\text{ gm/cc}]$$

$$\therefore \quad W_s = 20.52\text{ gm,} \quad \text{...(ii)}$$

Put the value of (ii) in (i).

$$V_s = (20 - 0.35\,W_s)$$

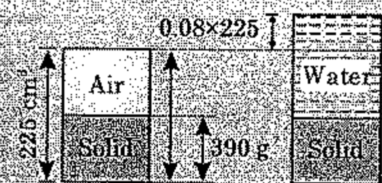
$$= (20 - 0.35 \times 20.52)$$

$$[\,V_s = 12.82\text{ cc}]$$

Example 18.

An Oven dry soil sample of volume 225 cm³ weights 390 g. If the specific gravity of soil is 2.72. Determine the void ratio and shrinkage limit. What water content will fully saturate the sample and cause an increase in volume equal to 8% of the original dry volume.

Sol.



$$\text{Oven dry } G_S = 2.72 = \frac{\gamma_d}{\gamma_w} = \frac{W_s}{V_s \times \gamma_w}$$

$$\text{Volume of solid} = \frac{390}{2.72 \times \gamma_w}$$

$$= \frac{390}{2.72 \times 1} = 143.38 \text{ cm}^3$$

$$\text{Volume of void} = 225 - 143.38 = 81.62 \text{ cm}^3$$

$$\Rightarrow \quad e = \frac{V_v}{V_s} = \frac{81.62}{143.38} = 0.569 \qquad \text{...(i)}$$

Shrinkage limit occurs if the voids are fully filled with water.

$$w_S = \frac{W_W}{W_S} = \frac{V_v \cdot \gamma_w}{W_S} = \frac{81.62 \times 1}{390} = 0.209 \qquad \text{... (ii)}$$

Water content which will saturate the soil sample and causes volume increase of 8%

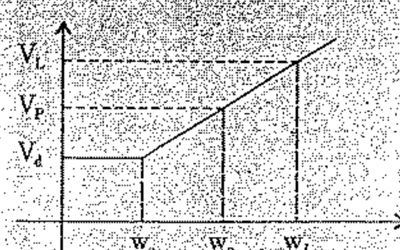
$$w = \frac{V_v \times 1.08\,\gamma_w}{W_S} = \frac{81.62 \times 1.08}{390} = 0.226 \qquad \text{... (iii)}$$

Example 19.

The plastic limit of a soil is 25% and its plasticity index is 8% when the soil is dried from its state at plastic limit, the volume change is 25% of its volume at plastic limit. Similarly the corresponding volume change from liquid limit to dry state is 34% of its volume at liquid limit. Determine the shrinkage limit and shrinkage ratio.

Sol. Data Given: $w_p = 25\%$; $I_P = w_l - w_p = 8\%$

$$\therefore \quad w_l = 33\%$$



$$\frac{V_L - V_P}{w_l - w_p} = \frac{V_P - V_d}{w_p - w_s}$$

$$V_d = (1-0.25)\,V_P = 0.75\,V_P$$
$$V_d = (1-0.34)\,V_L = 0.66\,V_L$$
$$\therefore \quad w_p - w_s = \frac{(w_l - w_p)(V_P - V_d)}{(V_L - V_P)}$$
$$= \frac{8\left(\frac{1}{0.75}-1\right)V_d}{\left(\frac{1}{0.66}-\frac{1}{0.75}\right)V_d}$$
$$w_p - w_s = 14.66$$
$$w_s = w_p - 14.66 = 25 - 14.66$$
$$\boxed{w_s = 10.33\%}$$

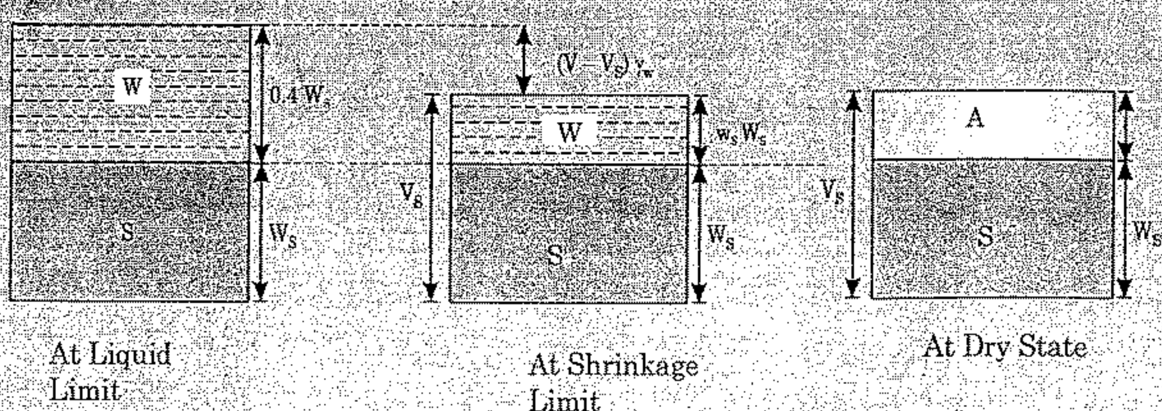
Shrinkage ratio

$$R = \frac{\frac{V_1 - V_2}{V_d}\times 100}{w_1 - w_2}$$
$$= \frac{\frac{V_L - V_d}{V_d}}{w_l - w_s}\times 100$$
$$= \frac{\left(\frac{1}{0.66}-1\right)}{33-10.33}\times 100$$
$$\boxed{R = 2.2724}$$

Example 20.

The mass specific gravity of a fully saturated specimen of clay having a water content of 40% is 1.88. On Oven drying the mass specific gravity drops to 1.74. Calculate the specific gravity of caly and its shrinkage limit.

Sol. Given than for fully saturated soil speciman



$$G_m = \frac{\gamma_{soil}}{\gamma_w} = 1.88$$

$$\gamma_w G_m = \gamma_{soil}$$

$$\gamma_{soil} = \frac{W_s + W_w}{V_s + V_v} \quad \text{... (i)}$$

$$[V_v = V_w]$$

As soil is fully saturated hence $V_v = V_w$ because all soils will be filled with water.

∴ Water content is 40%

$$w = \frac{W_w}{W_s}$$

$$0.4\, W_s = W_w$$

We know that

$$G_s = \frac{\gamma_s}{\gamma_w}$$

$$\gamma_w G_s = \frac{W_s}{V_s}$$

$$V_s = \frac{W_s}{\gamma_w G_s}$$

$$e = \frac{V_v}{V_s}$$

$$S.e = w.G$$

As soil is fully saturated

$$s = 1$$

$$e = w.G$$

$$\frac{V_v}{V_s} = w.G$$

$$Vv = w.\, G_s.\, V_s$$

$$V_v = 0.4G_s \frac{W_s}{G_s \gamma_w}$$

$$V_v = \frac{0.4\, W_s}{\gamma_w}$$

Put values of V_v and V_s in equation (1) we get

$$\frac{0.4W_s + W_s}{\frac{0.4W_s}{\gamma_w} + \frac{W_s}{G_s \gamma_w}} = 1.88\ \gamma_w$$

$$\frac{1.4Ws}{\frac{W_s}{\gamma_w}\left(0.4 + \frac{1}{G_s}\right)} = 1.88\ \gamma_w$$

$$\frac{1.4\gamma_w}{0.4 + \frac{1}{G_s}} = 1.88\ \gamma_w$$

$$0.4 + \frac{1}{G_s} = \frac{1.4}{1.88}$$

$$\frac{1}{G_s} = \frac{1.4}{1.88} - 0.4$$

$$\boxed{Gs = 2.9}$$

Given that for oven dry sample. .... (i)

$$G_m = \frac{\gamma_d}{\gamma_w}$$

$$\gamma_d = G_m \gamma_w \quad ...(ii)$$

We know that

$$\gamma_d = \frac{W_s}{V_s + V_v}$$

Let at shrinkage limit water content = W_s

$$\gamma_d = \frac{W_s}{\frac{W_s}{Gs\gamma_w} + \frac{w_s \cdot W_s}{\gamma_w}}$$

Put this value in equation (2)

$$\frac{W_s}{\frac{W_s}{G_s \gamma_w} + \frac{W_s \cdot W_s}{\gamma_w}} = G_m \gamma_w$$

$$\frac{\gamma_w}{\frac{1}{G_s} + W_s} = 1.74\,\gamma_w$$

$$w_s = \frac{1}{1.74} - \frac{1}{2.9}$$

$$\boxed{w_s = 0.23}$$

Plasticity Index

Range of consistency (water content) within which soil behaves as a plastic material is called Plasaticity Index.

$$I_P = w_L - w_P$$

- This property is due to the presence of clay minerals.
- Clay soils with high specific surface area and charged surfaces are able to bind/assimilate water molecule and the overall soil will still behave as a plastic solid. Such soil will have high plasticity index.
- Soils with smaller specific surface area will not be able to bind/assimilate water molecule and thus will have much smaller PI values.

Typical atterberg limits for soils are as given below in the table

Soil type	W_L	W_P	I_P
Sand	—	—	NP = non plastic
Silt	30-40	20-25	10-15
Clay	40-50	25-50	15-100

If plasticity index = 0, soil is non plastic.

I_P	Consistency
0	Non plastic
< 7	Low plastic
7-17	Medium plastic
> 17	Highly plastic

low plastic soil is used for embankment because easy to compact.

Note: Atterberg limit values for clay minerals with various adsorbed cations.

	Na^+		K^+		Ca^{++}		Mg^{++}	
	W_L	I_P	W_L	I_P	W_L	I_P	W_L	I_P
Kaolinite	29	1	35	7	34	8	39	11
Illite	41	27	81	38	90	50	83	44
Monmoiltonite	344	251	161	104	166	101	158	99

- Mont morillonite has largest specific surface area hence can assimilate more water
- Depending on minerals and adsorbed cation the atterberg limits change
- Soil with large I_P and W_L → Fat clay
- Soil with low I_P and W_L → Lean clay

Shrinkage Index

$$I_S = w_P - w_S$$

w_p = Plastic limit water content
w_s = Shrinkage limit water content

Consistency Index (I_C) or Relative Consistency

w = Natural Water Content
w_L = liquid limit water content
w_p = plastic limit water content

$$\boxed{I_C = \frac{w_L - w}{w_L - w_P}}$$

Liquidity Index (I_L)

$$\boxed{I_L = \frac{w - w_P}{w_L - w_P}}$$

Note: Consistency index = 1 at plastic limit and liquidity index = 1 at liquid limit

W/C	Solid	Semi Solid	Plastic	Liquid
0	w_S	w_P	w_L	
	$\boxed{I_L < 0}$	$\boxed{I_L < 0}$	$\boxed{0 < I_L \le 1}$	$\boxed{I_L > 1}$
	$\boxed{I_C < 1}$	$\boxed{I_C < 1}$	$\boxed{0 \le I_C \le 1}$	$\boxed{I_C < 0}$

$$I_C + I_L = 1$$

- **In Situ** behaviour of saturated fine grained soil deposits at its natural moisture content may be studied by their consistency index

  On the basis of I_C & I_L consistency ig given as under

I_C	I_L	Consistency	
> 1	< 0	Very stiff ⟶ Brittle	
1–0.75	0–0.25	Stiff	Plastic Range
0.75–0.5	0.25–0.5	Medium stiff	Plastic Range
0.5–0.25	0.5–0.75	Soft	Plastic Range
0.25–0	0.75–1.0	Very soft	Plastic Range
< 0	< 1.0	Liquid state ⟵ Liquid Range	

Toughness Index (I_t)

$$\boxed{I_t = \frac{I_P}{I_F} = \frac{\text{Plasticity index}}{\text{Flow index}}}$$

- It indicates the loss of shear strength with increase in moisture content
- It also indicates the shear strength of soil at plastic limit

  We know that,

  $w = I_f \log_{10}N + C$

  N = No. of blows and also $N \propto$ shear strength

  $W_L = -I_F \log_{10} N_L + C$

  $W_P = -I_F \log_{10} N_P + C$

  $W_L - W_P = I_F \log \dfrac{N_P}{N_L}$

  $W_L - W_P = I_F \log \dfrac{S_P}{S_L}$

$$\boxed{\frac{I_P}{I_F} = \log_{10} \frac{S_P}{S_L}}$$

  N_L = No. of blows corresponding to liquid limit

  N_P = No. of blows corresponding to plastic limit

  S_P = Shear strength at plastic limit

  S_L = shear strength at liquid limt = 2.7 kN/m²

- For most soil it is between 0–3

  $I_t < 1 \Rightarrow$ Friable soil, i.e. easily crushable soil at plastic limit

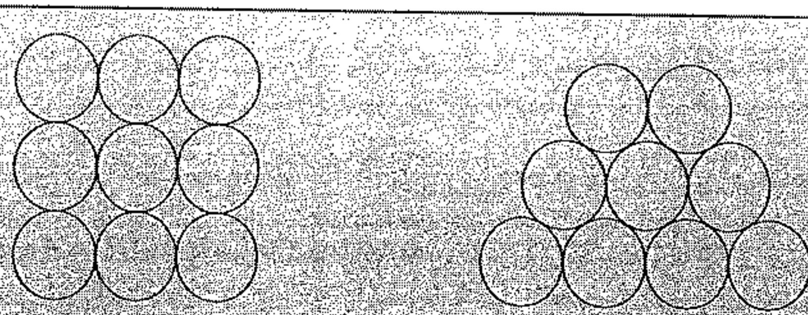
- For clayey soil, $1 \le I_t \le 3$

Relative Density or Density Index

Relative density of coarse-grained soil is equivalent of relative consistency of clayey soil.

Degree of denseness and coarsenessof natural deposits of coarse grained soil is measured by its relative density

$$D_r = \frac{e_{max} - e_{natural}}{e_{max} - e_{min}} \times 100$$



(a) Loose (b) Dense

Shapes in loosest and densest states.

Practically e_{max} and e_{min} are calculated by experiment

$e_{max} \rightarrow$ by dropping the coarse grained soil from a height

$e_{min} \rightarrow$ vibration

- Higher relative density mean higher shear strength and low compressibility

Relative density	Classification
< 15	very loose
15-35	loose
35-65	medium
65-85	dense
> 85	very dense

We know that

$$\gamma_d = \frac{\gamma_s}{1+e} = \frac{G_s\gamma_w}{1+e}$$

$$\Rightarrow \quad e = \frac{G_s\gamma_w}{\gamma_d} - 1$$

$$\Rightarrow \quad D_r = \frac{\left(\frac{G_s\gamma_w}{\gamma_d} - 1\right)_{max} - \left(\frac{G_s\gamma_w}{\gamma_d} - 1\right)_{natural}}{\left(\frac{G_s g_w}{\gamma_d} - 1\right)_{max} - \left(\frac{G_s\gamma_w}{\gamma_d} - 1\right)_{min}} \times 100$$

$$D_r = \frac{\frac{\gamma_w}{\gamma_{d\,min}} - \frac{\gamma_w}{\gamma_{d\,natural}}}{\frac{\gamma_w}{\gamma_{d\,min}} - \frac{\gamma_w}{\gamma_{d\,min}}} \times 100 = \left(\frac{\gamma_{d\,natural} - \gamma_{d\,min}}{\gamma_{d\,max} - \gamma_{d\,min}} \times \frac{\gamma_{d\,max}}{\gamma_{d\,nat}} \times 100\right)$$

$$\boxed{D_r = \frac{\gamma_{d\,max}}{\gamma_{d\,natural}}\left(\frac{\gamma_{d\,natural} - \gamma_{d\,min}}{\gamma_{d\,max} - \gamma_{d\,min}}\right) \times 100}$$

Example 21.

The unit weight of a sand back fill was determined by field measurements to be 1746 kg/m³. The water content at the time of test was 8.6 per cent, and the unit weight of the solid constitutents was 2.6 gm/cm³. In the laboratory the void ratios in the loosest and densest states were found to be 0.642 and 0.462, respectively. What was the relative density of the fill? Write the importance of this term.

Sol. Given

$\gamma_t = 1746\,\text{kg/m}^3$, $w = 8.6\%$, $\gamma_s = 2.6\,\text{gm/cm}^3$, $e_{max} = 0.642$, $e_{min} = 0.462$

$$\gamma_t = \frac{G(1+w)}{1+e}\gamma_w, \quad G = \frac{\gamma_s}{\gamma_w} \Rightarrow G = \frac{2.6}{1} = 2.6$$

$$1746 = \frac{2.6\times(1+0.086)}{1+e}\times 1000$$

$$1.617 = 1+e$$

$$e = 0.617$$

Relative Density

$$I_D = \frac{e_{max}-e}{e_{max}-e_{min}} = \frac{0.642-0.617}{0.642-0.462} = 0.1388$$

$$I_D = 13.88\%$$

As we know that for $I_D < 15\%$ soil is classified as loose soil which has low shear strength and highly compressible.

Example 22.

In order to find the relative density of a sand, a mould of volume 1000 ml was used. When the sand was dynamically compacted in the mould, its mass was 2.10kg, whereas when the sand was poured in loosely, its mass was 1.635 kg. If the in-situ density of the soil was 1.50 Mglm³, calculate the relative density. G = 2.70. Assume that the sand is saturated.

Sol.

$$(\rho_{max})_{sat} = \frac{2.10\times10^3}{1000} = 2.1\text{ g/ml}$$

$$(\rho_{min})_{sat} = \frac{1.635\times10^3}{1000} = 1.635\text{ g/ml}$$

As

$$\rho_d = 1.50\,\text{Mg/m}^3 = 1.50\text{ g/ml}$$

$$e = \frac{G\rho_w}{\rho_d} - 1 = \frac{2.70}{1.5} - 1 = 0.80$$

Now

$$(\rho_{max})_{sat} = \left(\frac{G+e_{min}}{1+e_{min}}\right)\rho_w$$

or

$$2.1 = \left(\frac{2.70+e_{min}}{1+e_{min}}\right)\times 1.0 \quad \text{or} \quad e_{min} = 0.545$$

Likewise

$$(\rho_{min})_{sat} = \left(\frac{G+e_{max}}{1+e_{max}}\right)\rho_w$$

The
vas
342
m.

or $$1.635 = \left(\frac{2.70+e_{max}}{1+e_{max}}\right)\times 1.0 \text{ or } e_{max} = 1.677$$

$$D_r = \frac{e_{max}-e}{e_{max}-e_{min}}\times 100$$

$$= \frac{1.677-0.80}{1.677-0.545}\times 100 = 77.47\%$$

THIXOTROPY

- It is that property of soil due to which loss of strength (shear strength) on remoulding can be regained if left undisturbed for some time.
- Increase in strength witb passage of time is due to tendency of clay soil to regain their chemical equillibrium with the reorientation of water molecule in absorbed layer.
- Clay soils have large thixotropy than single grained soil.

***Note:** If pile is driven into soil, it disturbs the soil. Frictional resistance of driven pile in clay soil immidiately after installation will be much less than its value after a month. This is due to thixotropy.*

Sensitivity (S_t)

ghly

Degree of disturbance achieved on remoulding in expressed by sensitivity. It is basically evident more in clays.

$$S_t = \frac{\text{Unconfined compressive strength of undisturbed soil}}{\text{Unconfined compressive strength for remoulded soil}}$$

nd
ed
he

$$\boxed{S_t = \frac{q_{u(\text{undisturbed})}}{q_{u(\text{remoulded})}}}$$

$S_t \to 1$	gravel and coarse sand, insensitive
2–4	normal sensitive
4–8	sensitivie (silt and clay)
8–16	extra sensitive
≥ 16	unstable or quick clay

Higher the sensitivity, higher is the thixotropic handening.

Note: Sensitivity is generally greater than 1. However it can be less than one also. Stiff clay having fissures and cracks have $St \le 1$. Since these weaknesses are present in undisturbed soil. It will be removed in remoulded soil.

Acitivity Number (A_C)

As per skempton, volume changes during swelling or shrinkage = f(I_P % clay fraction)

Activity no. is used to study the swelling behaviour

$$\text{Activity no. } (A_C) = \frac{I_P}{\text{\% of clay size particle (i.e. } < 2\,\mu) \text{ in soil}}$$

$A_C < 0.75 \rightarrow$ inactive
$0.75 < A_C < 1.25 \rightarrow$ Normal active
$A_C > 1.25 \rightarrow$ Active

- Active means more prone to volume change.

Kaolinite, $A_C < 0.35 \rightarrow$ inactive
Illite, $0.38 < A_C < 0.9 \rightarrow$ Normal active

- Montmoronolite, (Black cotton soil) $0.9 < A_C < 7.2$ Active.

General Relationships between Atterberg Limits and Engineering Properties

Characteristics	*Comparing soils at equal liquid limit with plasticity index increasing*	*Comparing soils with equal plasticity index with liquid limit increasing*
Dry strength	Increases	Decreases
Toughness near plastic limit	Increases	Decreases
Compressibility	About the same	Increases
Permeability	Decreases	Increases
Rate of volume change	Decreases	Increases

Example 23

Two caly soils have the following characteristics. Calculate their Activity values compare their Engineering Behaviours.

	Clay A	Clay B
w_L (%)	60	50
w_P (%)	25	30
I_P (%)	35	20
% of Finer than 0.002 mm size	25	40

Sol. We have,

$$\text{Activity of clay A} = \frac{I_P}{(\text{\%Finer } 0.002\text{mm Size})}$$

$$= \frac{35}{25} = \frac{7}{5} = 1.4 \quad \text{... (i)}$$

$$\text{Activity of clay B} = \frac{(I_P)_B}{(\text{\%Finer } 0.002 \text{ mm size})}$$

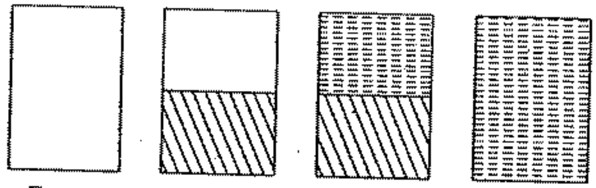
$$= \frac{20}{40} = 0.5 \quad \text{... (ii)}$$

∴ Since Activity of clay A is more than that of clay B.

- Therefore clay A is more likely to undergo high volume change.
- Clay A, has higher compressibility than that of B.
- But permeability and Rate of Volume change are smaller than that of clay B.

OBJECTIVE TYPE QUESTIONS

1. The given figure indicate the weights of different pycnometers :



Empty Pycnometer W_1 | Pycnometer + dry soil W_2 | Pycnometer + soil + water W_3 | Pycnometer + water W_4

The specific gravity of the soils is given by

(a) $\frac{W_2}{W_4 - W_2}$

(b) $\frac{W_1 - W_2}{(W_3 - W_4) - (w_2 - W_1)}$

(c) $\frac{W_2}{(W_3 - W_4)}$

(d) $\frac{W_2 - W_1}{(W_2 - W_1) - (W_3 - W_4)}$

2. A dry soil has mass specific gravity of 1.35. If the specific gravity of solids is 2.7, then the void ratio will be

(a) 0.5
(b) 1.0
(c) 1.5
(d) 2.0

3. A clay sample has a void ratio of 0.50 in dry state and specific gravity of solids = 2.70. Its shrinkage limit will be

(a) 12%
(b) 13.5%
(c) 18.5%
(d) 22%

4. A deposit of fine sand has a porosity n and specific gravity of soil solids is G. The hydraulic gradient of the deposit to develop boiling condition of sand is given by

(a) $i_c = (G - 1)(1 - n)$
(b) $i_c = (G - 1)(1 + n)$
(c) $i_c = \frac{G-1}{1-n}$
(d) $i_c = \frac{G-1}{1+n}$

5. Consider the following statements in relation to the given sketch :

Volume (cc)	Component	Weight (g)
0.2	Air	0
0.3	Water	0.3
0.5	Solids	1.0

1. Soil is partially saturated at degree of saturation = 60%
2. Void ratio = 40%
3. Water content = 30%
4. Saturated unit weight = 1.5 g/cc

Which of these statements is/are correct?

(a) 1, 2 and 3
(b) 1, 3 and 4
(c) 2, 3 and 4
(d) 1, 2 and 4

6. A fill having a volume of 1,50,000 cum is to be constructed at a void ratio of 0.8. The borrow pit soil has a void ratio of 1.4. The volume of soil required (in cubic meters) to be excavated from the borrow pit will be

(a) 1,87,500
(b) 2,00,000
(c) 2,10,000
(d) 2,50,000

7. Given that plasticity index (PI) of local soil = 15 and PI of sand = zero, for a desired PI of 6, the percentage of sand in the mix should be

(a) 70
(b) 60
(c) 40
(d) 30

8. A soil has mass unit weight γ, wate content 'w' (as ratio). The specific gravity of soil solids = G, unit weight of water = γ_w; S the degree of saturation of the soil is given by

(a) $S = \dfrac{1+w}{\dfrac{\gamma_w}{\gamma}(1+w) - \dfrac{1}{G}}$

(b) $S = \dfrac{w}{\dfrac{\gamma_w}{\gamma}(1+w) - \dfrac{1}{G}}$

(c) $S = \dfrac{(1+w)}{\dfrac{\gamma_w}{\gamma}(1+w) - \dfrac{1}{wG}}$

(d) $S = \dfrac{w}{\dfrac{\gamma_w}{\gamma}(1+w) - \dfrac{1}{wG}}$

9. If a soil sample of weight 0.18 kg having a volume of 10^{-4} m^3 and dry unit weight of 1600 kg/m^3 is mixed with 0.02 kg of water then the water content in the sample will be

(a) 30%
(b) 25%
(c) 20%
(d) 15%

10. If an unconfined compressive strength of 4 kg/cm^2 in the natural state of clay reduces by four times in the remoulded state, then its sensitivity will be

(a) 1
(b) 2
(c) 4
(d) 8

11. The plasticity index and the percentage of grain size finer than 2 microns of a clay sample are 25 and 15, respectively. Its activity ratio is

(a) 2.5
(b) 1.67
(c) 1.0
(d) 0.6

12. A soil sample having a void ratio of 1.3, water content of 50% and a specific gravity of 2.60, is in a state of

(a) Partial saturation
(b) Full saturation
(c) Over saturation
(d) Under saturation

13. **Assertion (A) :** A soil is at its liquid limit if the consistency index of the soil is equal to zero.

Reason (R) : The consistency index of a soil is defined as ratio of (liquid limit minus the natural water content) to (natural water content minus plastic limit).

14. Consistency as applied to cohesive soils is an indicator of its

(a) Density (b) Moisture content

(c) Shear strength (d) Porosity

15. While computing the values of limits of consistency and consistency indices, it is found that liquidity index, has a negative value.

Consider the following comments on this value :

1. Liquidity index cannot have a negative value and should be taken as zero.
2. Liquidity index can have a negative value. ?
3. The soil tested is in semisoild state and stiff.
4. The soil tested is in medium soft state.

Which of these statements are correct?

(a) 1 and 4 (b) 1 and 3

(c) 2 and 4 (d) 2 and 3

16. Match **List_I** (Unit/Test) with **List-II** (Purpose) and select the correct answer using the codes given below the lists :

List-I	List-II
A. Casagrande's apparatus	1. Determination of grain size distribution
B. Hydrometer	2. consolidation characteristics
C. Plate load test	3. Determination of consistency limits
D. Oedometer of soil	4. Determination of safe bearing capcity

Codes:

	A	B	C	D
(a)	1	3	2	4
(b)	1	3	4	2
(c)	3	1	2	4
(d)	3	1	4	2

17. What are the respective values of void ratio, porosity ratio and saturated density (in kN/m^3) for a soil sample which has saturation moisture content of 20% and specific gravity of grains as 2.6? (take density of water as 10 kN/m^3)

(a) 0.52, 1.08, 18.07

(b) 0.52, 0.34, 18.07

(c) 0.77, 1.08, 16.64

(d) 0.52, 0.34, 20.14

18. Embankment fill is to be compacted at a density of 18 kN/m^3. The soil of the borrow area is at a density of 15 kN/m^3. What is the estimated number of trips of 6 m^3 capcity truck for hauling the soil required for compacting 100 m^3 fill of the embankment? (Assume that the soil in the borrow area and that in the embankment are at the same moisture content)

(a) 14 (b) 18

(c) 20 (d) 23

19. The laboratory tests on the sample yielded the following results :

Plasticity index : 32%

Liquidity index : 0.15

Activity number : 1.58

Which of the following inferences can be drawn?

1. The soil is very stiff
2. The soil is medium soft
3. The soil is highly plastic
4. The soil is medium plastic
5. The soil is active

Select the correct answer using the codes given below :

(a) 1, 3 and 5 (b) 1, 3 and 4

(c) 2, 3 and 5 (d) 1, 2 and 4

20. What is caused by the addition of coarser particles like sand or silt to clay?

(a) Decrease in liquid limit and increase in plasticity index

(b) Decrease in liquid limit and no change in plasticity index

(c) Decrease in both liquid limit and plasticity index

(d) Increase in both liquid limit and plasticity index

21. Consider the following properties for clays X and Y :

S.No.	Properties	Clay X (%)	Clay Y (%)
1.	Liquid limit	42	56
2.	Plastic limit	20	34
3.	Natural water content	30	50

Which of the clays, X or Y, experiences larger settlement under identical loads; is more plastic; and is softer in consistency?

(a) X, Y and X (b) Y, X and X

(c) Y, X and Y (d) X, X and Y

22. A soil has liquid limit = 35, plastic limit = 20, shrinkage limit = 10 and natural moisture content = 25%. What will be its liquidity index, plasticity index and shrinkage index?

(a) 0.67, 15 and 25 (b) 0.33, 15 and 10

(c) 0.67, 25 and 15 (d) 0.33, 20 and 15

23. A specimen of clayey silt contains 70% slit size particles. It liquid limit = 40 and plastic limit = 20. In liquid limit test, at moisture content of 30%, required number of blows was 50. Its plasticity index, activity and consistency index will respectively be

(a) 20, 0.67 and 0.5 (b) 20, 1.5 and 2.0

(c) 30, 1.5 and 0.72 (d) 20, 0.286 and 0.38

24. Consider the following statements :
   1. Sensitivity of a natural soil deposit cannot be less than 1.0.
   2. A saturated loose sand deposit liquifies when water flows through it in upward direction under critical hydraulic gradient.
   3. A quick clay has very high sensitivity.

   Which of these statements are correct?

   (a) 1, 2 and 3 (b) 1 and 2 only
   (c) 1 and 3 only (d) 2 and 3 only

25. 1. At shrinkage limit, the soil remains fully saturated.
   2. The shear strength of all soils at liquid limit is the same.
   3. The shear strength of all soils at plastic limit is the same.

   Which of these statements is/are correct?

   (a) 1, 2 and 3 (b) 1 and 2 only
   (c) 2 and 3 only (d) 1 only

26. Which one of the following expresses the degree of disturbance of undisturbed clay sample due to remoulding?

   (a) Thixotropy (b) Dilatancy
   (c) Sensitivity (d) Plasticity

27. The relationship between water content (w%) and number of blows (N) in soils, as obtained from Casagrande's liquid limit device, is given by

   $w = 20 - \log_1 N$

   The liquid limit of soil is

   (a) 15.6% (b) 16.6%
   (c) 17.6% (d) 18.6%

28. Which one of the following is the water content of the mixed soil made from 1 kg of soil (say A) with water content of 100% and 1 kg of soil (say B) with water content of 50%?

   (a) 66% (b) 71%
   (c) 75% (d) 82%

29. By placing a soil sample at 105°C for 24 hours in an oven
   1. Hygroscopic moisture is lost
   2. Capillary water is lost
   3. Free water is lost
   4. Structural water is lost

   Which of these statements are correct?

   (a) 1, 2 and 4 (b) 3 and 4
   (c) 1, 2, 3 and 4 (d) 1, 2 and 3

30. Consider the following statements:
   1. Porosity is zero in fully saturated soil.
   2. The dry density of soil is less than the density of soil solids.
   3. The moisture content of a soil can be more than 100%.
   4. Drying of soil results in increasing the voids ratio.

Which of these statements is/are correct?

(a) 1 only
(b) 1 and 2
(c) 2 and 3
(d) 3 and 4

31. When is a soil mass said to have entered the solid phase?

(a) When loss in water is not accompanied by a corresponding reduction in the volume of soil mass.
(b) When reduction in the volume of the soil mass is nearly equal to the volume of water lost.
(c) When the soil mass becomes brittle.
(d) When the soil mass shows a small shearing resistance as the water content is reduced.

32. Match List-I (Cause) with List-II (Effect) and select the correct answer using the codes given below the lists:

List -I	List-II
A. Brownian movement	1. Soil structure
B. Soil grading	2. Flow index
C. Consistency	3. Coefficient of curvature
D. Stratification	4. Particle settlement

Codes:

	A	B	C	D
(a)	4	1	2	3
(b)	2	3	4	1
(c)	4	3	2	1
(d)	2	1	4	3

33. The relationship between water content (w%) and number of blows (N) in soils, as obtained from Casagrande's liquid limit device, is given by, w = 20 – log10 N

The liquid limit of soil is

(a) 15.6%
(b) 16.6%
(c) 17.6%
(d) 18.6%

34. A soil sample has 28 g of soil solids, 10 cm^3 of voids, 9 g of water and specific gravity of soil grains is 2.7. Consider the following statements in this regard:

1. The water content is $\frac{9}{28} \times 100\%$

2. The void ratio is $\frac{10 \times 2.7}{28}$

3. Degree of saturation is $\frac{9}{10 \times 27} \times 100$

4. The porosity is $\frac{10 \times 2.7}{(28 + 10 \times 2.7)}$

Which of these statements are correct?

(a) 1, 2 and 3
(b) 2, 3 and 4
(c) 1, 3 and 4
(d) 1, 2 and 4

35. A soil sample has natural moisture content 'w', void ratio 'e' and specific gravity of soils solids G_S. The bulk unit weight of soil g is given by (g_W is unit weight of water)

(a) $\frac{(1-W)G_S\gamma_W}{(1-e)}$
(b) $\frac{(1+W)G_S\gamma_W}{(1-e)}$
(c) $\frac{(1+W)G_S\gamma_W}{(1+e)}$
(d) $\frac{(1-W)G_S\gamma_W}{(1+e)}$

36. Which one of the following is the water content of the mixed soil made from 1 kg of soil (say A) with water content of 100% and 1 kg of soil (say B) with water content of 50%?

(a) 66 %
(b) 71%
(c) 75 %
(d) 82 %

37. By placing a soil sample at 105°C for 24 hours in an oven

1. hygroscopic moisture is lost
2. capillary water is lost
3. free water is lost
4. structural water is lost

Which of these statements are correct?

(a) 1, 2 and 4
(b) 3 and 4
(c) 1, 2, 3 and 4
(d) 1, 2 and 3

38. Soil samples A and B have void ratios of 0.5 and 0.7 respectively. If 1.5 m^3 of soil sample A and 1.7 m^3 of soil sample B are mixed to form sample C having a volume of 3.2 m^3, which one of the following correctly represents the porosity of sample C?

(a) 0.375
(b) 0.60
(c) 1.66
(d) 2.66

39. Following data are given for two soil samples A and B:

Sample A: Void ratio = 1, total volume = 1 m^3

Sample B: Void ratio = 2, total volume = 1 m^3

The two samples are mixed and compacted in a mould to a volume of 1 m^3. Which one of the following figures correctly indicates the porosity of the compacted soil?

(a) 17%
(b) 54%
(c) 60%
(d) 83%

40. Which one of the following soils is the Aeolian?

(a) Transported soil
(b) Residual soil
(c) Weathered soil
(d) Volcanic soil

41. When is a soil mass said to have entered the solid phase?
   (a) When loss in water is not accompanied by a corresponding reduction in the volume of soil mass.
   (b) When reduction in the volume of the soil mass is nearly equal to the volume of water lost.
   (c) When the soil mass becomes brittle.
   (d) When the soil mass shows a small shearing resistance as the water content is reduced.

42. Match List-I (Property of soil) with List-II (Laboratory equipment) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Grain size	1. Pycnometer
B. Specific gravity	2. Permeameter
C. Coefficient of permeability	3. Vane shear apparatus
D. Cohesion	4. Pipette
	5. Sand pouring cylinder

Codes:

	A	B	C	D
(a)	4	1	2	3
(b)	4	5	2	3
(c)	5	1	2	4
(d)	1	5	3	2

43. Loess deposits are formed by
   (a) physical disintegration of rock
   (b) constant blowing of wind from the same direction
   (c) vertical deposition of glacial till
   (d) chemical weathering of residual deposits

44. Consider the following statements:
   Clays which exhibit high activity
   1. contain montmorillonite
   2. contain kaolinite
   3. have a high silt content
   4. have a high plasticity index
   5. have a low plasticity index

   Which of these statements are correct?
   (a) 1, 3 and 5
   (b) 2, 3 and 5
   (c) 2 and 4
   (d) 1 and 4

45. Which one of the following correctly defines the term 'Activity' of clays?
   (a) $\dfrac{\text{Plastic index}}{\text{Percentage of clay}}$
   (b) $\dfrac{\text{Plastic limit}}{\text{Liquidity index}}$
   (c) $\dfrac{\text{Unconfined compression strength}}{\text{Cohesion}}$
   (d) $\dfrac{\text{Unconfined compression strength of remoulded sample}}{\text{Unconfined compression strength of undisturbed sample}}$

of soil

er lost.

ed.
answer

46. On which of the following factors does activity of a soil depend?

1. Plasticity
2. Minerals present
3. Clay content
4. Silt content

Select the correct answer using the codes given below:

(a) 1, 2 and 3 (b) 2, 3 and 4
(c) 1, 3 and 4 (d) 1, 2 and 4

47. Bentonite is a material obtained due to the weathering of

(a) limestone (b) quartzite
(c) volcanic ash (d) shales

48. Match List-I (Type of soil) with List-II (Feature) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Lacustrine	1. Transported by wind
B. Alluvial	2. Transported by running water
C. Aeolian	3. Deposited at the bottom of lakes
D. Marine	4. Deposited in sea water

Codes:

	A	B	C	D
(a)	1	4	3	2
(b)	3	2	1	4
(c)	3	4	1	2
(d)	1	2	3	4

49. Given for a sample of a river sand:

Void ratio at the densest state = 0.40

Void ratio at the loosest state = 1.20

Which one of the following correctly represents the relative density of a sample prepared with a void ratio of 1.0?

(a) 12.5% (b) 25%
(c) 75% (d) 87.5%

50. Match List-I (Soil) with List-II (Description) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Lacustrine	1. A glacial clay characterized by distinctly marked annual deposit of sediments
B. Peat	2. Part of glacial drift which is directly deposited by ice
C. Till	3. An organic soil formed of vegetational matter
D. Varved clay	4. A soil which is deposited in lakes

Codes :

	A	B	C	D
(a)	4	3	2	1
(b)	2	1	4	3
(c)	4	1	2	3
(d)	2	3	4	1

Instructions :

The following items consists of two statements, one labelled as ***'Assertion A'*** *and the other labelled as* ***'Reason R'****. You are to examine these two statements carefully and decide if the Assertion A and the Reason R are individually true and if so, whether the Reason is a correct explanation of the Assertion. Select your answers to the these items using the codes given below :*

(a) both A and R are true and R is the correct explanation of A

(b) both A and R are true but R is not a correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

51. **Assertion (A) :** The water content of inorganic soils is determined by heating the soil in an oven at a temperature of **105°** to 110°C.

    Reason (R) : The free water, absorbed water and structural water are all completely removed fromthe soil by heating it at **105°** to 110°C.

52. **Assertion (A) :** At the same voids ratio desiccated clay is stronger than saturated clay.

    Reason (R) : Desiccation impacts (induces) pre-compresive forces in the soil structure.

53. **Assertion (A) :** The aggregate physical propertids in coarse grained soil are a function of relative density and particle shape.

    Reason (R) : Single grained structure is formed when the soil grains settle out independently due to mass derived forces.

54. **Assertion (A):** The aggregate physical properties in coarse grained soil are a function of relative density and particle shape.

    Reason (R): Single grained structure is formed when the soil grains settle out independently due to mass derived forces.

ANSWERS

1. (d)	**2.** (b)	**3.** (c)	**4.** (a)	**5.** (b)	**6.** (b)	**7.** (b)	**8.** (b)	**9.** (b)	**10.** (c)
11. (b)	**12.** (b)	**13.** (c)	**14.** (c)	**15.** (d)	**16.** (d)	**17.** (d)	**18.** (c)	**19.** (a)	**20.** (c)
21. (c)	**22.** (b)	**23.** (a)	**24.** (d)	**25.** (b)	**26.** (c)	**27.** (d)	**28.** (b)	**29.** (d)	**30.** (c)
31. (a)	**32.** (c)	**33.** (d)	**34.** (d)	**35.** (c)	**36.** (b)	**37.** (d)	**38.** (a)	**39.** (a)	**40.** (a)
41. (a)	**42.** (a)	**43.** (b)	**44.** (d)	**45.** (a)	**46.** (a)	**47.** (c)	**48.** (b)	**49.** (b)	**50.** (a)
51. (c)	**52.** (a)	**53.** (b)	**54.** (b)						

Hints

2. $\gamma_d = \dfrac{G}{1+e}\gamma_w$

$G_m = \dfrac{\gamma_d}{\gamma_w}$

$\gamma_d = 1.35\gamma_w$

$1.35 = \dfrac{G}{1+e}$

$1+e = \dfrac{2.7}{1.35}$

$e = 1$

3. $S.e = w.G$

$W_s = \dfrac{1\times 0.5}{2.70}$

$W_s = 0.185$

4. $i_c = \dfrac{G-1}{1+e} \quad \therefore i_c = (G-1)(1-n)$

$e = \dfrac{n}{1-n} \Rightarrow 1+e = \dfrac{1}{1-n}$

5. $S = \dfrac{V_w}{V_v} = \dfrac{0.3}{0.5} = 0.60$

$e = \dfrac{V_v}{V_s} = \dfrac{0.5}{0.5} = 1$

$w = \dfrac{W_w}{W_s} = \dfrac{0.3}{1} = 0.3$

$\gamma_{sat} = \dfrac{\text{Wt of solid + Wt of water in Voids}}{\text{total volume}}$

$= \dfrac{1+0.5}{1} = 1.5\text{ gm/cc}$

6. $V_s = \dfrac{V}{1+e}$

$\dfrac{V_1}{1+e_1} = \dfrac{V_2}{1+e_2}$

$\dfrac{1{,}50{,}000}{1+0.8} = \dfrac{V_2}{1+1.4}$

$V_2 = 2{,}00{,}000$

7. Let total volume of mix be 100 unit

Let Sand be x unit out of 100 unit

$\therefore$ Soil will be having $(100-x)$ unit.

$\dfrac{(100-x)\times 15 + x\times 0}{100} = 6$

$-15x = 600 - 1500$

$x = \dfrac{-900}{-15} = 60$

9. $\text{Water content} = \dfrac{\text{wt. of water}}{\text{wt. of solid}}$

$= \dfrac{\text{Total wt.} - \text{wt. of solid}}{\text{wt. of solid}}$

$W = \dfrac{(\text{Total wt.} - \text{wt. of solid}) + \text{water added}}{\text{wt. of solid}}$

$= \dfrac{(0.18-0.16)+0.2}{0.16} = 0.25$

10. Sensitivity

$= \dfrac{\text{Unconfined compressive strength of undisturbed soil sample}}{\text{Unconfined compressive strength of Remoulded soil sample}}$

$= \dfrac{4}{1} = 4$

11. $\text{Activity} = \dfrac{I_p}{\%\text{clay}} = \dfrac{25}{15} = \dfrac{5}{3} = 1.67$

12. $S.\,e = W.G$

$S = \dfrac{0.5\times 2.6}{1.3} = 1$

17. $S.e = W.G$

i.e. $= 0.2 \times 2.6$

$e = 0.52$

$n = \dfrac{e}{1+e} = \dfrac{0.52}{1+0.52}$

$n = 0.342$

$\gamma_{sat} = \dfrac{G+S.e}{1+e}\gamma_w$

$= \dfrac{2.6+0.52}{1.52}\times 10 = \dfrac{3.12}{1.52}\times 10$

$\gamma_{sat} = 20.52\text{ kN/m}^3$

18. $\gamma_t = \dfrac{G(1+w)}{1+e}$

$$\gamma_{t_1} = 18 = \frac{G(1+w)}{1+e_1}$$

$$\gamma_{t_2} = 15 = \frac{G(1+w)}{1+e_2}$$

$$\frac{\gamma_{t_1}}{\gamma_{t_2}} = \frac{18}{15} = \frac{1+e_2}{1+e_1}$$

$$V_s = \frac{V_1}{1+e_1} = \frac{V_2}{1+e_2}$$

$$V_1 = 100$$

$$100 \times \frac{1+e_2}{1+e_1} = V_2$$

$$\frac{600}{5} = V_2$$

$$120 = V_2$$

$$\text{No. of trips} = \frac{120}{6} = 20\,\text{Nos.}$$

22. $I_L = \frac{w - w_p}{w_L - w_p} = \frac{25-20}{35-20} = 0.333$

$I_P = W_L - W_P = 35 - 20 = 15$

$I_S = W_P - W_S = 20 - 10 = 10$

23. $I_P = W_L - W_P = 40 - 20 = 20$

$\text{Activity} = \frac{I_P}{\%\text{of clay}} = \frac{20}{30} = 0.667$

$I_C = \frac{W_L - W}{W_L - W_P} = \frac{40-30}{40-20} = 0.5$

27. Given $W = 20 - \log_{10} N$

N for liquid limit = 25

$W = 20 - \log_{10} 25$

$W = 20 - 2\log_{10} 5$

$W = 20 - 2 \times 0.698$

$W = 20 - 1.4 = 18.6\%$

28.

Soil A	Soil B
(1 kg)	(1 kg)
W = 100%	W = 50%
$\therefore W_s = 500$ gm	$W_S = 666$ gm
$\therefore W_w = 500$ gm	$W_w = 333$ gm

Water content of missed soil

$$= \frac{\text{wt. of water}}{\text{wt. of solids}} = \frac{500+333}{500+666} = 0.7144$$

33. Given $w = 20 - \log_{10} N$

N for liquid limit = 25

$w = 20 - \log_{10} 25$

$w = 20 - 2\log_{10} 5$

$w = 20 - 2 \times 0.698$

$w = 20 - 1.4 = 18.6\%$

34. (1) Water content

$$w = \frac{\text{weight of water}}{\text{Weight of solid}} = \frac{9}{28} \times 100\%$$

(2) $Gs = \frac{\gamma_s}{\gamma_w}$

$\gamma_s = G_s \times \gamma_w \Rightarrow \gamma_s = 2.7 \times 1$

$\gamma_s = \frac{W_s}{V_s} \quad V_s = \frac{28}{2.7}$

$e = \frac{V_v}{V_s} = \frac{10}{28} \times 2.7 = \frac{10 \times 2.7}{28}$

(3) $S.e = W.G$

$S = \frac{W.G}{e}$

$= \frac{9}{28} \times 10 \times 2.7 \times \frac{28}{10 \times 2.7} = \frac{9}{10} \times 100$

(4) $n = \frac{e}{1+e} = \frac{\frac{10 \times 2.7}{28}}{1 + \frac{10 \times 2.7}{28}} = \frac{10 \times 2.7}{28 + 10 \times 2.7}$

36.

Soil A	Soil B
(1 kg)	(1 kg)
W = 100%	W = 50%
$\therefore W_S = 500$ gm $\therefore$	$= W_S = 666$ gm
$W_w = 500$ gm	$W_w = 333$ gm

Water content of mixed soil

$$= \frac{\text{weight of water}}{\text{weight of solid}} = \frac{500+333}{500+666} = 0.7144$$

$w = 71.44\%$

38. Soil A Soil B

$e = 0.5$ $e = 0.7$

$V = 1.5\ m^3$ $V = 1.7\ m^3$

$V_v = 0.5$ $V_v = 0.7$

$V_s = 1$ $V_s = 1$

$$\text{e of soil sample C} = \frac{0.5+0.7}{3.2} = \frac{1.2}{3.2} = 0.375$$

39. **Sample A** **Sample B**

$e = 1$ $e = 2$

$V = 1m^3$ $V = 1m^3$

$e = \frac{V_v}{V_s}$ $V_v = 0.5\ m^3$

$V_v = 0.5\ m^3$ $V_v = 0.666\ m^3$

$V_S = 0.5\ m^3$ $V_s = 0.333\ m^3$

As both the soil samplesa re mixed and compacted, therefore V_s of sample

$= V_{S_A} + V_{S_B}$, but volume of void will reduce

$V_s = 0.50 + 0.333 = 0.833$

As the sample is compacted to volume $1m^3$

$$\therefore \text{Porority} = \frac{V_v}{V_{Total}} = \frac{V_T - V_S}{V_T} = \frac{1-0.833}{1}$$

$= 0.17$

49. (b) $I_D = \frac{e_{max} - e}{e_{max} - e_{min}}$

$$I_D = \frac{1.20-1}{1.20-0.40} = \frac{0.20}{0.80} = 0.25$$

CHAPTER

2

Classification of Soil

INTRODUCTION

- A Soil classification means sorting of soil into groups which would show similar behaviour.
- Soil classification does not eleminate the need of exact analysis based on engineering properties.

Note: The soil classification system can be linked to classification of human beings into 12 zodiac signs done by an astrologer. Although general behaviour of a human being under a particular zodiac sign can be estimated from his zodiac sign, for complete prediction, his detailed horoscope is required.

- Generally soil classification is done on the basis of simple index properties. The most commonly used properties are Grain-Size distribution and Plasticity.
- Several classification systems were evolved by different organisations having a specific purpose.
- Few Important systems of classification are
  (1) Unified Soil Classification System (USCS)
  (2) American Assosiation of state highway and Transport officials (AASHTO) system.
  (3) Indian standard soil classification system (ISSCS)

Note: ISSCS is simillar to USCS with slight modifications.

1. The Unified Soil Classification System

- The Unified Soil Classification System (USCS), originally developed by Casagrande (1948), was intended for use in airfield construction.
- Later it was slightly modified to make it applicable to foundations, dams and other constructions.
- According to the USCS, the coarse-grained soils are classified on the basis of their grain-size distribution and the fine-grained soils (whose behaviour is controlled by plasticity) on the basis of their plasticity characteristics.
- All the soils are classified into four major groups, namely, coarse grained, fine grained, organic soils and peat.

2. Aashto Classification System

- American Association of State Highway and Transportation Official (AASHTO) Classification system is useful for classifying soils for highways.
- The particle size analysis and the plasticity characteristics are required to classify a soil.
- According to the AASHTO system, the soils are classified into eight groups: A-1 through A-7 with an additional group A-8 for peat or muck.

- The system includes several sub-groups. Soils within each group are evaluated according to the group index calculated from the empirical formula.

$$\boxed{\text{Group index, } GI = 0.2\,a + 0.005\,ac + 0.01\,bd}$$

Where,

a = that part of the percent passing the 75 μ sieve (–75 μ) greater than 35 and not exceeding 75, expressed as a positive whole number (range 1 to 40)

b = that part of the percent passing the 75 μ sieve greater than 15 and not exceeding 55, expressed as a positive whole number (range 1 to 40)

c = that part of liquid limit greater than 40 and not greater than 60, expressed as a positive whole number (range 1 to 20)

d = that part of the plasticity index greater than 10 and not exceeding 30 expressed as a positive whole number (range 1 to 20)

- While calculating GI, if any term becomes negative, it is dropped. The group index should be rounded off to the nearest whole number.
- If the computed value is negative, it is reported as zero.
- In general, the greater the group index value, the less desirable a soil is for highway construction within that subgroup. A group index of 0 indicates a good subgrade material, while a group index of 20 or more indicates a very poor subgrade meterial.
- The group index must be mentioned even when it is zero to indicate that the soil has been classified as per AASHTO system.

Indian Standard Classification System

- Indian Standard Classification (SC) system adopted by Bureau of Indian Standards is in many respects similar to the Unified Soil Classification (USC) system. But, there is one basic difference in the classification of fine-grained soils.
- The fine-grained soils in ISC system are subdivided into three categories of low, medium and high compressiblity instead of two categories of low and high compressibility in USC system.
- Soils are divided into three broad divisions:
  (1) Coarse-grained soils, when 50% or more of the total material by weight is retained on 75 micron IS sieve.
  (2) Fine-grained soils, when more than 50% of the total material passes 75 micron IS sieve.
  (3) If the soil is highly organic and contains a large percentage of organic matter and particles of decomposed vegetation, it is kept in a separate category marked as peat (P_t).

The basic soil classification is done on the basis of grain size.

Division of soil fraction on the basis of grain size (mm)

boulder	cobble	Coarse grained soil					Fine Grained	
		Garavel		sand			Silt	clay
		Coarse	fine	Coarse	Medium	Fine		
>300	300-80	80-20	20-4.75	4.75-2.0	2.0-0.425	0.425-075	0.075-0.002	<0.002

Basic Soil Components

Sl. No.	*Soil component*	*Symbol*	*Particle-size range and description*
	Boulders	None	Rounded to angular, bulky, hard, rock particle : average diameter more than 300 mm
	Cobbles	None	Rounded to angular, bulky, hard, rock particle: average diameter smaller than 300 mm but retained on 80 mm IS sieve
(1)	Coarse-grained soils		
	Gravel	*G*	Rounded to angular, bulky, hard, rock particle : passing 80 mm IS sieve but retained on 4.75 mm IS sieve
			Coarse : 80 mm to 20 mm
			Fine : 20 mm to 4.75 mm
	Sand	*S*	Rounded to angular, bulky, hard, rock particle : passing 4.75 mm IS sieve but retained on 75 micron
			Coarse : 2.0 mm to 2.0 mm
			Medium : 2.0 mm to 425 micron
			Fine : 425 micron to 75 micron
(2)	Fine-grained soils		
	Silt	*M*	Particles smaller than 75 micron : identified by behaviour, that is, slightly plastic or non-plastic regardless of moisture and exhibits little or no strength when air dried.
	Clay	*C*	Particles smaller than 75 micron: identified by behaviour, that is, it can be made to exhibit plastic properties within a certain range of moisture and exhibits considerable strength when air dried.
(3)	Organic matter	*O*	Organic matter in various sizes and stage of decomposition (no specific grain size)
(4)	Peat	*Pt*	Fibrous, spongy (no specific grain size)

Pre fix & Suffix used in soil classification

Soil type	Prefix	Sub group	Sufix
Gravel	G	Well graded	W
Sand	S	Poorly graded	P
Silt	M	Silty	M
Clay	C	Clayey	C
Organic	O	$w_L < 35$	L (*i.e.* low compressibility)
Peat	Pt	$35 < w_L < 50$	I (*i.e.* intermediate compressibility)
		$50 < w_L$	H (*i.e.* high compressibility)

CLASSIFICATION OF COARSE GRAINED SOIL

Classification of Coarse grained soil is done on the basis of

(1) Grain Size

(2) Gradation characterstics

(3) Percentage of fines present in soil by weight (fines means particle size < 75 μm).

Case —1 : When fines are less than 5%

GW Well graded gravel	GP Poorly graded gravel
1. More than half of coarse fraction > 4.75 mm 2. $C_u = \frac{D_{60}}{D_{10}} > 4$ 3. $C_c = \frac{D_{30}^2}{D_{60} \times D_{10}}$ $1 < C_C < 3$	Otherwise

SW Well graded sand	SP Poorly graded sand
1. more than 1/2 of coarse fraction < 4.75 mm 2. $C_u > 6$ 3. $1 < C_C < 3$	Otherwise

Case —2 : When fines are between 5–12%.

- This case is also known as borderline case and given dual symbol eg SW–SC
- The first part of the dual symbol indicates gradation (SW), where as second part indicates the nature of fines (SC)
- The soil having dual symbol SW – SC signifies well-graded sand with clay as fines which is ploted above A-line on Plasticity chart.

  The following Classification can occur.

  1. GW – GC → Well graded gravel containg clay as fines
  2. GP – GC → Poorly graded gravel containing clay as fines
  3. GW – GM → Well graded gravel containing Silt as fines
  4. GP – GM → Poorly graded gravel containing Silt as fines
  5. SW – SC → Well graded sand containing clay as fines
  6. SP – SC → Poorly graded sand containing clay as fines
  7. SW – SM → Well graded sand containing silt as fines
  8. SP – SM → Poorly graded sand containing silt as fines.

Case —3 : When fines are greater than 12%

(1) Gravel

(a) Fineness > 12%, & $I_p < 4\%$

GM → Silty Gravel

(b) Fineness > 12% & $I_p > 7\%$

GC → Clayey Gravel

(2) Sand

(a) Fineness > 12%, & $I_p < 4\%$

SM → Silty Sand

(b) Fineness > 12%, & $I_p > 7\%$

SC → Clayey Sand

Note: If IP is between 4-7 then again Dual symbols are used.

CLASSIFICATION OF FINE GRAINED SOIL

Classification of Fine Grained Soil is done on the basis of Plasticity chart.

1. First W_L & W_P are determined for seived fraction of fine grained soil.
2. If the limit plot above A-line, classify as clay.

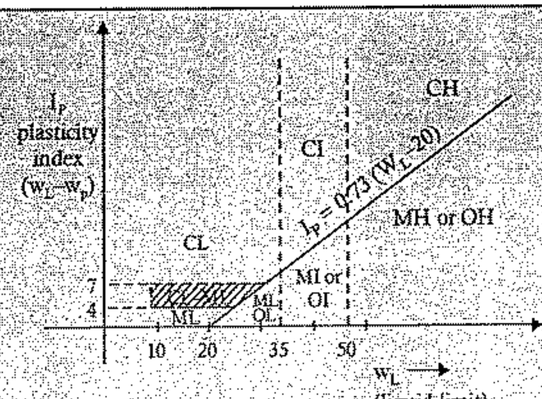
   Further,

- If, $35 > W_L$ classify as CL
- If, $35 < W_L < 50$ classify as CI
- If, $50 < W_L$ classify as CH

3. If the limit plots below A line, then we need to find out whether, soil is organic or inorganic. If its inorganic and below A line then it is silt (M).

   Further,

- If, $35 > W_L$ Classify as ML or OL
- If, $35 < W_L < 50$ Classify as MI or OI
- If, $50 < W_L$ Classify as MH or OH
- GM or SM if fines are silty i.e. limit plots below A-line on plasticity chart.
- GC or SC—limit plots above A line



Plasticity chart

- Dual symbol are used if limits plots between hatched zone *i.e.* IP between 4–7. Symbol = *CL– ML*
- If W_L & I_P falls closer to A-line, dual are used. Again if liquid limit falls closer to 35% or 50% dual symbol is used. Possible dual symbols are

CL – ML	ML – MI	CI – CH
CI – MI	MI – MH	OL – OI
CH – MH	CL – CI	OL – OH

Note: Dual symbols are also used when soil have equal percentage of coarse & fine grains.

(a) Determine whether the coarse-grained fraction is gravel (G) or sand (S)

(b) Determine w_L and w_P for the fine fraction.

(c) Depending on whether the limits plot above the A-line or below the A-line, classify as C or M.

(d) Based on w_L, classify as L, I or H

(e) Assign the dual symbol as per the data obtained GM-ML, GM-MI

$GM-ML$	$GC-CL$	$SM-ML$	$SC-CL$
$GM-MI$	$GC-CI$	$SM-MI$	$SC-CI$
$GM-MH$	$GC-CH$	$SM-MH$	$SC-CH$

Example 21.

Two clays A and B has following properties

	Clay A	Clay B
w_L	44	55
w_P	20	35
w_N	30	50

which of the clays A or B would experience layer settlement under identical loads. Which of the soil is more plastic which is softer in consistency.

Sol:

	A	B	
I_P	24	20	$\Rightarrow$ A is more plastic
I_C	0.58	0.25	$\Rightarrow$ B is softer inconsistency

But in this case for A

$$I_p = 24\ ;\ w_l = 44;$$

$$I_p = 0.73\,(w_l - 20)$$

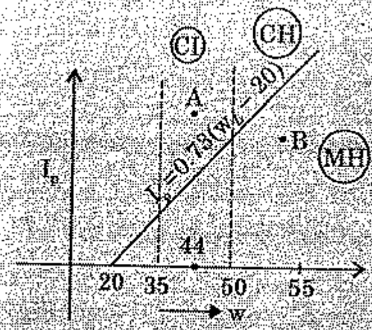
$$= 0.73\,(44 - 20) = 17.52$$

For B

$w_l = 55$; $I_p = 20$

$I_p = 0.73\,(w_l - 20)$

$= 0.73\,(55 - 20) = 25.55$



Therefore clay B lies in high compressible zone thus clay B is more compressible.

FIELD IDENTIFICATION PROCEDURE

- IS – 1498 – 1970 recomends field identification procedure through following tests.
  - (1) Visual examination
  - (2) Dilatancy test
  - (3) Toughness
  - (4) Dry Strength
  - (5) Organic Content and Colour
  - (6) Other Identification test

(1) Visual examination

- A representative sample of the soil is selected which is spread on a flat surface or in the palm.
- All particles larger than 80 mm are removed from the sample. Only the fraction of the sample smaller than 80 mm is classified.
- Sample is classified as coarse-grained or fine-grained by estimating the percentage by weight of individual particles which can be seen by the unaided eye.
- Soils containing more than 50 percent visible particles are coarse-grained soils, soils containing less than 50 percent visible particles are fine-grained soils.

Note: *If the soil is classified as fine grained soil then following test are performed.*

(2) Dilatancy (Reaction to Shaking)

- Take a small representative sample of a soil pat of the size of about 5 cubic centimetres and add enough water to just saturate it.
- Place the pat in the open palm of one hand and shake horizontally, striking vigorously against the other hand several times.
- Squeeze the pat between fingers.
- The appearance and disappearance of the water with shaking and squeezing is referred to as a reaction. This reaction is known as quick, if water appears and disappears rapidly; slow, if water appears and disappears slowly; and no reaction, if the water condition does not appear to change.
- Silt exibits quick reaction, where as clay shows no or slow reaction.

(3) Toughness (Consistency near Plastic limit)

- Dry the pat used in the dilatancy test by working and moulding, untill it has the consistency of putty.
- Time required to dry the pat is the indication of its plasticity.
- The Pat is rolled on a smooth surface or between the palms into a thread about 3 mm in diameter. Fold and reroll the thread repeatedly to 3 mm in diameter untill its moisture content is gradually reduced and 3 mm thread starts crumbling.
- The moisture content at this time is called the plastic limit and the resistance to moulding at the plastic limit is called the toughness.
- After the thread crumbles, lump the pieces together and continue the slight kneading action until the lump crumbles.
- If lump can still be moulded slightly drier than the plastic limit and high pressure is required to roll the thread between the palms of the hand, the soil is described as having high toughness.
- Medium toughness is exibited by a medium thead and a lump formed of the thread slightly below the plastic limit will crumble; while low toughness is exibited by a weak thread that breaks easily and cannot be lumped together when drier than the plastic limit.
- Highly organic clays exibits very weak and spongy feel at the plastic limit. Non-plastic soils cannot be rolled into-thread of 3 mm in diameter at any moisture content.

(4) Dry Strength (Crushing resistance)

- If dry pat can be easily powdered, then dry strength of soil is designated as low, medium if considerable finger pressure is required and high, if it can not be powdered at all.
- Clay of high plasticity exibits high dry strength, where as in organic silt exibits medium dry strength. Silty fine sand show low dry strength.
- Completely dry the prepared soil pat and measure its resistance to crumbling and powdering between fingers. This resistance is called as dry strength and is a measure of the palsticity of the soil and is influenced largely by the colloidal fraction content.

(5) Organic Content and Colour.

- Fresh wet organic soils usually have a distinctive odour of decomposed organic matter. This odour can be made more noticeable by heating the wet sample.
- Dark colour is another indication of presence of organic matter in soil.

(6) Other Identification test

(1) Acid test :

- Acid test using dilute hydrochloric acid (HCL) is basicly a test for the presence of calcium carbonate.
- Soils with high dry strength, a strong reaction indicates that the strength may be due to calcium carbonate as cementing agent rather than colloidal clay.

(2) Shine test :

- This is a quick additional test for determining the presence of clay.
- The test is performed by cutting a lump of dry or slightly moist soil with a knife.
- The shiny surface imparted to the soil indicates highly plastic clay, where as dull surface indicates silt or clay of low plasticity.

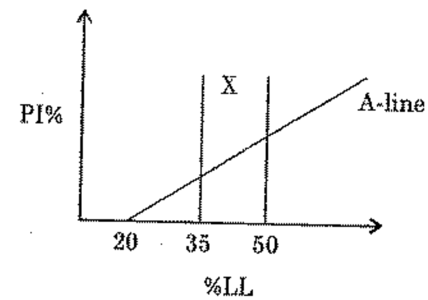
Note: Presence of High strength water soluble cementing materials eg :- $CaCO_3$ or Iron oxides may cause high dry strength. Non plastic soils, eg:- Caliche, Coral, Crushed lime stone or soils containing carbonaceous cementing agents may also have high dry strength. But this can be detected by the effervercence caused by the application of delated hydrochloric acid.

IMPORTANT POINTS

- Only particles of coarse grained soil can be seen by naked eye.
- Best soil for homogeneous rolled embankment dam—GC
- Best soil for core of rolled earthen dam—GC
- Best soil for shell of rolled earth dam—GW
- Best soil for foundation when seepage not important—GM
- Best soil for foundation when seepage important—GW
- Best soil for road surfacing—GC

OBJECTIVE TYPE QUESTIONS

1. The standard plasticity chart to classify fine grained soils is shown in the given figure.



The area marked X represents

(a) silt of low plasticity
(b) clay of high plasticity
(c) organic soil of medium plasticity
(d) clay of intermediate plasticity

2. Consider the following statements in the context of aeolian soils:
   1. The soil has low density and low compressibility
   2. The soil is deposited by wind
   3. The soil has large permeability

   Which of these statements are correct?

   (a) 1, 2 and 3
   (b) 2 and 3
   (c) 1 and 3
   (d) 1 and 2

3. If the proportion of soil passing 75 micron sieve is 50% and the liquid limit and plastic limit are 40% and 20% respectively, then the group index of the soil is

   (a) 3.8
   (b) 6.5
   (c) 38
   (d) 65

4. Given that coefficient of curvature = 1.4, D_{30} = 3 mm, D_{10} = 0.6 mm.

   Based on this information of particle size distribution for use as subgrade, this soil will be taken to be

   (a) Uniformly-graded sand
   (b) Well-graded sand
   (c) Very fine sand
   (d) Poorly-graded sand

5. In a soil specimen, 70% of particles are passing through 4.75 mm IS sieve and 40% of particles are passing through 75 μ IS sieve. Its uniformity coefficient is 8 and coefficient of curvature is 2. As per IS classification, this soil is classified as

   (a) SP (b) GP (c) SW (d) GW

6. Match **List-I** (Soil classification symbol) with **List-II** (Soil property) and select the correct answer using the codes given below the lists :

List-I	List-II
A. GW	1. Soil having uniformity coefficient > 6
B. SW	2. Soil having uniformity coefficient > 4
C. ML	3. Soil have low plasticity
D. CL	

Codes :

	A	B	C	D
(a)	1	2	4	3
(b)	2	1	3	4
(c)	2	1	4	3
(d)	1	2	3	4

7. Consider the following statements :

1. The minimum value of group index for a soil can be taken as zero
2. The maximum possible value of group index for a soil is twenty

Which is these statements is/are correct?

(a) Both 1 and 2
(b) 1 only
(c) 2 only
(d) Neither 1 nor 2

8. Match **List-I** (Property of soil) with **List-II** (Laboratory equipment) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Grain size	**1.** Pycnometer
B. Specific gravity	**2.** Permeameter
C. Coefficient of permeability	**3.** Vane shear apparatus
D. Cohesion	**4.** Pipette
	5. Sand pouring cylinder

Codes:

	A	B	C	D
(a)	4	1	2	3
(b)	4	5	2	3
(c)	5	1	2	4
(d)	1	5	3	2

9. Match **List-I** (Range of particle size) with **List-II** (Type of soil) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Less than 0.002 mm	**1.** Gravel
B. 0.075 mm to 0.002 mm	**2.** Sand
C. 80 mm to 4.75 mm	**3.** Cobble
D. 4.75 mm to 0.075 mm	**4.** Silt
	5. Clay

Codes:

	A	B	C	D
(a)	4	5	1	3
(b)	4	5	2	1
(c)	5	4	1	2
(d)	5	4	3	1

10. Match List-I with List-II and select the correct answer using the codes given below the lists (Notations have their usual meaning):

List-I	List-II
A. Find grained soil with $w_L = 60$, $I_p = 20$, $w_s = 8$	**1.** Expansive CH soil
B. Fine grained soil with $w_L = 60$, $I_p = 30$, $w_s = 8$	**2.** Non- expansive SC soil
C. Fine grained soil with, $w_L = 30$, I_p £ 4, $w_s = 20$	**3.** Expansive OH soil
D. Coarse grained sand with $w_L = 40$, $I_p = 15$, $w_s = 20$	**4.** Non-expansive ML soil

Codes:

	A	B	C	D
(a)	1	3	4	2
(b)	1	3	2	4
(c)	3	1	2	4
(d)	3	1	4	2

11. The engineering behaviour of clay is essentially different from that of a sand at the same void ratio. Which one of the following is not responsible for this behaviour?
(a) Particle size and shape
(b) Specific surface area
(c) Soluble salts
(d) Cation exchange capacity

12. On which of the following factors does activity of a soil depend?
1. Plasticity
2. Minerals present
3. Clay content
4. Silt content

Select the correct answer using the codes given below:
(a) 1, 2 and 3
(b) 2, 3 and 4
(c) 1, 3 and 4
(d) 1, 2 and 4

13. A soil mass contains 40% gravel, 50% sand and 10% silt. The soil can be classified as
(a) silty sandy gravel having coefficient of uniformity less than 60.
(b) silty gravelly sand having coefficient of uniformity equal to 10.
(c) gravelly silty sand having coefficient of uniformity greater than 60.
(d) gravelly silty sand and its coefficient of uniformity cannot be determined.

Statement for Linked Answer Questions 14 and 16:

Laboratory sieve analysis was carried out on a soil sample using a complete set of standard IS sieves. Out of 500 g of soil used in the test, 200 g was retained on IS 600m sieve, 250 g was retained on IS 500m sieve and the remaining 50 g was retained on IS 425m sieve.

14. The coefficient of uniformity of the soil is
(a) 0.9
(b) 1.0
(c) 1.1
(d) 1.2

15. The classification of the soil is
(a) SP
(b) SW
(c) GP
(d) GW

16. The laboratory test results of a soil sample are given below:

Percentage finer than 4.75 mm = 60

Percentage finer than 0.075 mm = 30

Liquid Limit = 35%

Plastic Limit = 27%

The soil classification is
(a) GM
(b) SM
(c) GC
(d) ML-MI

17. By using sieve analysis, the particle size distribution curve has been plotted for a particular soil. The coefficient of curvature C_c is given b

(a) $\frac{D_{30}}{D_{60} \times D_{10}}$

(b) $\frac{\sqrt{D_{30}}}{D_{60} \times D_{10}}$

(c) $\frac{D_{30}}{\sqrt{D_{60} \times D_{10}}}$

(d) $\frac{D_{30}^2}{D_{60} \times D_{10}}$

18. Consider the following statements:

1. Coarse-grained soils having fines (< 75 m in size) between 5% and 15%, have a dual symbol according to IS codes for soil classification.
2. At liquid limit, all soils have the same shearing strength.
3. Lower the shrinkage limit, greater is the volume change in a soil with change in water content.

Which of these statements are correct?

(a) 1 and 2 (b) 1 and 3 (c) 2 and 3 (d) 1, 2 and 3

19. Match **List-I** (Range of particle size) with **List-II** (Type of soil) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Less than 0.002 mm	**1.** Gravel
B. 0.075 mm to 0.002 mm	**2.** Sand
C. 80 mm to 4.75 mm	**3.** Cobble
D. 4.75 mm to 0.075 mm	**4.** Silt
	5. Clay

Codes:

	A	B	C	D
(a)	4	5	1	3
(b)	4	5	2	1
(c)	5	4	1	2
(d)	5	4	3	1

20. Match **List-I** (Type of soil) with **List-II** (Group symbol as per IS classification) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Well-graded gravel mixture with little or no fines	**1.** ML
B. Poorly graded sand or gravelly sand with little or no fines	**2.** CH
C. Inorganic silts and very fine sand	**3.** GW
D. Inorganic clay of high plasticity	**4.** SP

Codes:

	A	B	C	D
(a)	3	4	2	1
(b)	3	4	1	2
(c)	4	3	1	2
(d)	4	3	2	1

21. The best indication of the behavior of a deposit of sand under load can be obtained from its
(a) bulk density (b) dry density (c) relative density (d) grading

22. For distinguishing clays from silts in the field, a moist soil is rolled into a thread of 3 mm diameter. This test will indicate the
(a) dilatancy (b) dry strength
(c) wet and manipulated strength (d) toughness

23. Match **List-I** (Different types of soil) with **List-II** (Group symbol of IS classification) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Well-graded gravel sand mixture with little or no fines	**1.** ML
B. Poorly graded sands or gravelly sands with little or no fines	**2.** CH
C. Inorganic silts and very fine sands or clayey silts with low plasticity	**3.** GW
D. Inorganic clays of liquid limit over 50%	**4.** SP

Codes:

	A	B	C	D
(a)	3	1	4	2
(b)	3	4	1	2
(c)	2	4	1	3
(d)	2	1	4	3

24. Which one of the following is true in respect of a well-graded sand (C_u = coefficient of uniformity ; C_c = coefficient of curvature)?
(a) More than 50% of the soil grains are greater than 4.75 mm size and $C_u > 6$ and $C_c < 1.0$
(b) More than 50% of the soil grains are greater than 0.075 mm size and $C_u > 6$ and $C_c > 1.0$
(c) More than 50% of the soil grains pass through 4.75 mm sieve and $C_u < 6$ and $1.0 < C_c < 3.0$
(d) More than 50% of the coarse grains pass through 4.75 mm sieve but retained in 0.075 mm sieve and $C_u > 6$ and $1.0 < C_c < 3.0$

ANSWERS

1. (d)	2. (b)	3. (b)	4. (b)	5. (c)	6. (b)	7. (a)	8. (d)	9. (c)	10. (d
11. (a)	12. (a)	13. (d)	14. (a)	15. (b)	16. (d)	17. (c)	18. (c)	19. (b)	20. (a)
21. (c)	22. ~~(a)~~ (c)	23. (b)	24. (d)						

Hints

4.
$$C_c = \frac{D_{30}^2}{D_{10} \times D_{60}}$$
$$1.4 = \frac{3}{0.6 \times D_{60}}$$
$$D_{60} = \frac{3}{1.4 \times 0.6}$$
$$D_{60} = 3.57$$
$$Cu = \frac{D_{60}}{D_{10}} = \frac{3.57}{0.6} = 5.95$$
$C_u \approx 6$ C_c = 1–3 Well graded Sand

CHAPTER

3

Clay Mineral and Soil Structure

INTRODUCTION

- Normally it is expected that engineering properties of soil are greatly influenced by their mineralogical composition. But as a matter of fact it is only partically true.
- In case of Coarse grained soil engineering properties are hardly effected by Mineralogical composition.
- As discussed earlier coarse grained soil are formed due to physical forces and retain their mineralogical composition, which is same as the composition of the original rock. There is no bonding between the particles.
- On the other hand, the behaviour of fine grained soils, depends to a large extent on the nature and characterstics of minerals present. Most significant properties of clay depends upon the type of mineral.
- Chemical weathering results in formation of group of crystalline particles of colloidal size ($<2\mu$) known as clay mineral. These clay mineral imparts cohession and plasticity.
- Most clay mineral particles have plate-like form having high specific surface. The formation of clay structure is greatly affected by surface bonding force i.e. surface forces.

Note: *Because of larger surface area, more water is attached to clay than coarse grained soil. Water content of coarse grained soil is smaller as compared to clay for same void ratio. Grain Size distribution and grain shape influence the engineering behaviour of granular soils and hardly affect the behaviour of clay soils.*

$$\text{Specific surface} = \frac{\text{Surface area}}{\text{Volume (mass)}}$$

SOIL STRUCTURE

- Soil structure means the geometrical arrangement of soil particles in a soil mass, relative to each other and the forces acting between them to hold them together in their positions.
- The concept is further extended to include the mineralogical composition of the grains, the electrical properties of the particle surface, the physical characteristics, ionic composition of the pore water, the interactions among the solid particles, pore water and the adsorption complex.
- In coarse grained soil structure is goverened by gravity forces where as in clayey soil by surface forces are predominant.

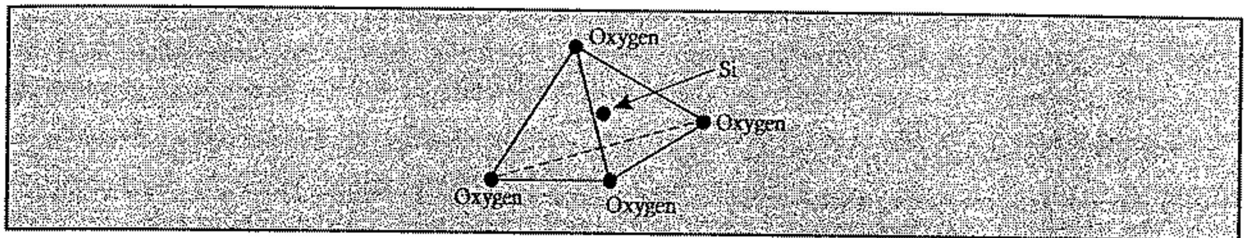
Structure of Clay Mineral

- It has been observed that, Clay soil is made up of many crystal sheets which have a repeating atomic structure.

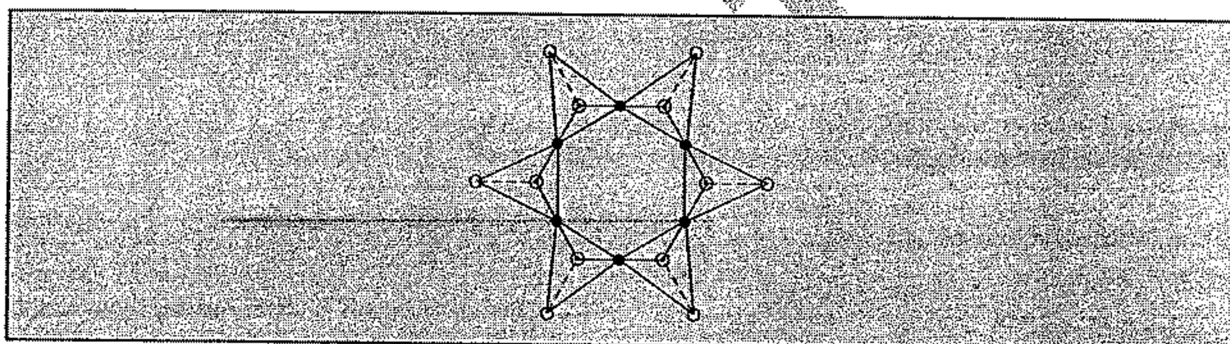
- Atomic structure of clay minerals are built of two fundamental crystal sheets
  - (*a*) tetrahydron or Silica sheet
  - (*b*) Octahydral or Alumina Sheet
- The mode of staking of these sheets, the nature of bonding forces and different metallic ions in the crystal lattice go on to make different clay minerals.

Tetralydial unit

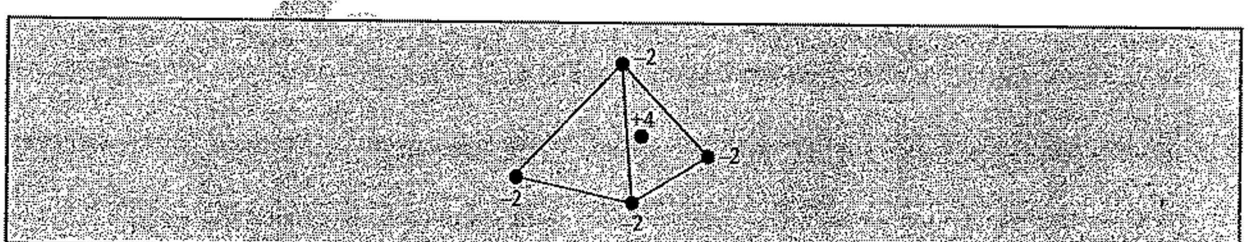
- An tetrahedral unit has four oxygen atoms placed at the tip of tetrahedron enclosing a silicon atom.



- Hexagonal opening at Base, and each O_2 at base is common to two units.
- Hence Three oxygen at Base are each common to two units.



- Net charge on one unit = $-\frac{2}{2}+\frac{(-2)}{2}+\frac{(-2)}{2}-2+4=-1$

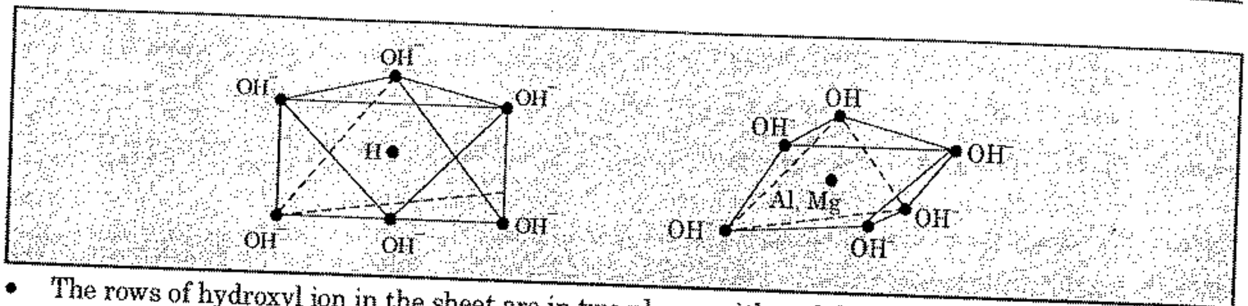


- The sheet like structure of Tetrahydral unit is represented as

Symbolic representation of Silica Sheet

OCTAHYDRAL UNIT

- An Octahydral unit has six hydroxyl ion (OH^-) at the tip of an octahyderon enclosing an aluminium or magnesium or some other Metallic atom.



- The rows of hydroxyl ion in the sheet are in two planes with each hydroxyl ion in one unit common to three octahydral units.
- Hence, the negative charge of -1 of each hydroxyl ion charge is divided into three parts. Therefore single negative charge is shared by three units and each OH^- contributes $-\frac{1}{3}$ charge on one unit.
- Net charge on single octahedral unit is $6OH^-$ contributes $\frac{-1}{3}\times 6 = 2$ negative charge on single unit.
- If *Al* is present at centre (positive charge) contributed by *Al*, there is +3 Net charge = $3-2=+1$
- Octahydral sheet are represented by
- If the atom at the centre is Al, sheet is called gibbsite sheet.
- If the atom at the center is Mg, sheet is called Brucite sheet.

Isomorpbous Substitution

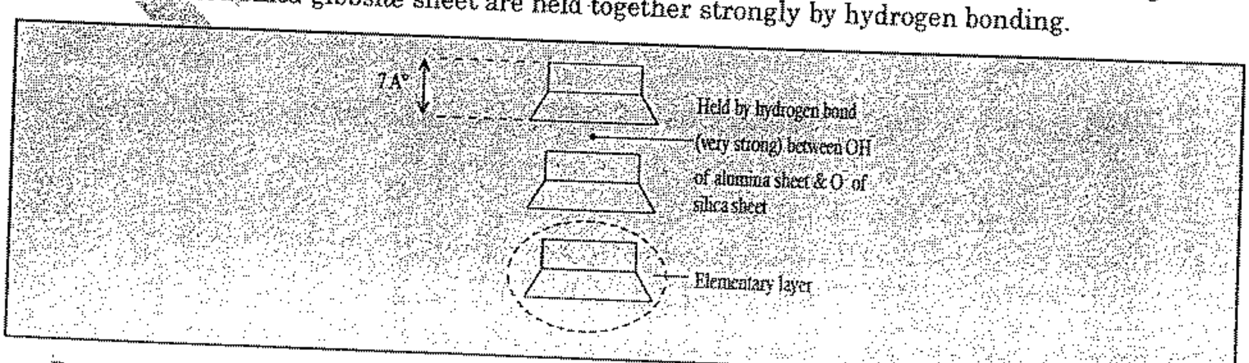
- Frequently in mineral lattice, metallic ions of one kind may be substituted by other metallic ion of a lower value but of the same physical size. Such a substitution is called isomorphous substitution.
- Isomorphous substitution may lead to a different clay minerals with different physical properties.
- For example, a Si^{4+} atom is replaced by an Al^{3+} in a tetrahedral unit.

VARIOUS CLAY MINERALS

- Clay minerals continues to change from one form to other due to weathering. Depending upon various stages of occurance they are named as follows.

1. Kaolinite

- Kaolinite consists of structure based on single sheet of silica combined with single sheet of gibbsite. The combined silica-gibbsite sheet are held together strongly by hydrogen bonding.



- Bonding due to hydrogen bond is very strong and mineral is stable.
- Water can not easily enter between the structural units and cause swelling.

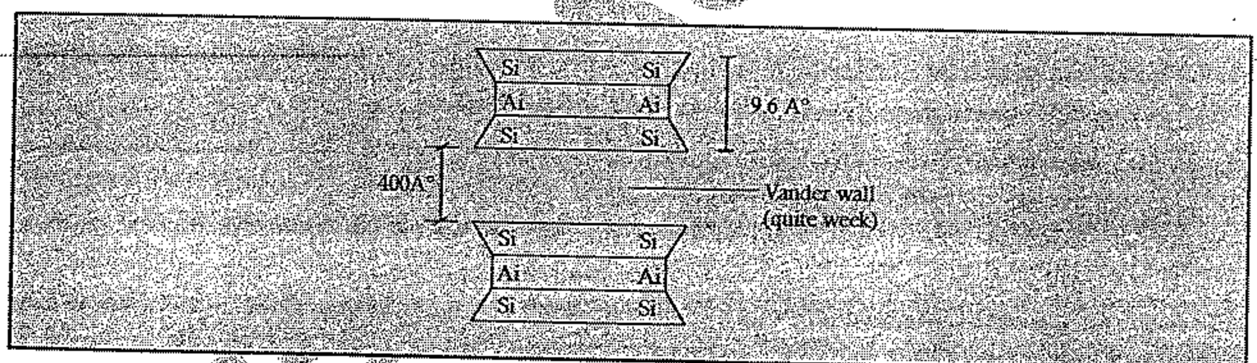
- It is least active of the clay minerals.
- It is often called as 1:1 clay mineral. Because of stalking of one layer of each of the two basic sheets produce the mineral.
- There is little or no isomorphous substitution in kaolinite.
- It is commonly found in sedimentary and residual soil.
- Found in old deposits. Also in highly weathered soil with good drainge.

Note: 1A° = 10^{-10}m

2. Halloysite:

- It is a clay mineral of kaolinite group
- Successive Structural Units are separated by water molecule.
- Halloysite particles are tubular in shape in contrast to platy shape of kaolinite particles.
- When airdried, they may convert to Kaolinite.
- Hence although engineering properties of Halloysite is different from kaolinite in natural condition, the two may be alike when halloysite is air dried.
- Halloysite & Kaolinite clays are used for making chinaware.
- Kaolinite clay is also used as an intestinal absorbant in anti–diarrhoeal medicine.

3. Montmorillonite

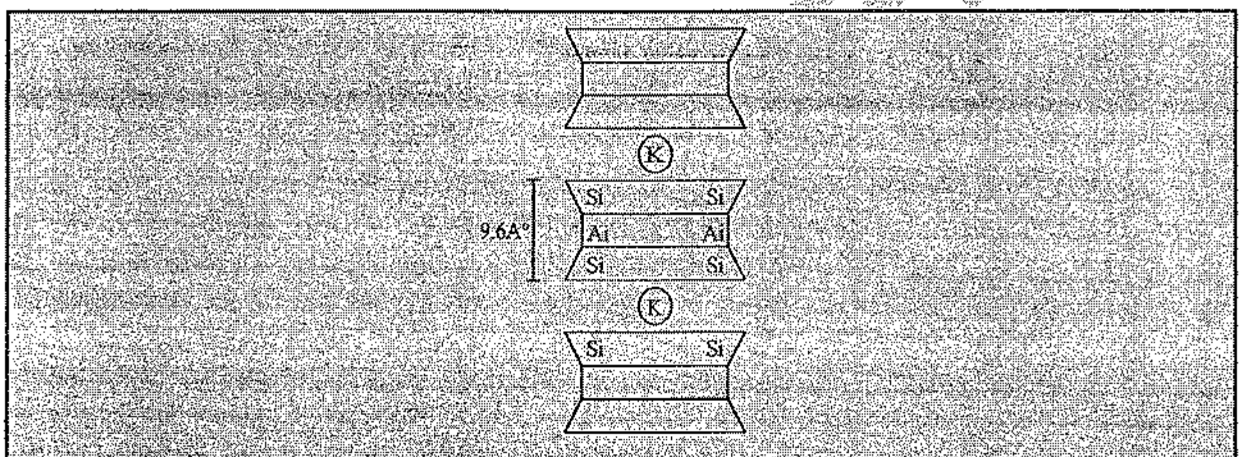


- It Also called Smectite.
- Montmorillonite is 2:1 clay mineral as it is composed of 2 silica sheet and 1 Alumina (Gibbsite) sheet.
- Montmorillonite has largest specific surface among clay mineral.
- Montmorillonite has large amount of water and other exchangeable ions can easily enter between the layers causing the layers to be separated. Because of this property it is susceptable to substantial volume changes.
- Bentonite is a Montmorillonite clay.
- Montmorillonite is found in black cotton Soil.
- Bentonite is used as drilling mud because Montmorillonite is highly plastic and has little interal friction.
- Montmonillonite is common in residual soils derived from volcanic ash,
- It is common in sediments of arid areas. It is often mixed with clay mica, results from the weathing of volcanic rock under poor drainage.

- Its excessive swelling capacity may seriously endanger the stability of overlying structure and road pavements.

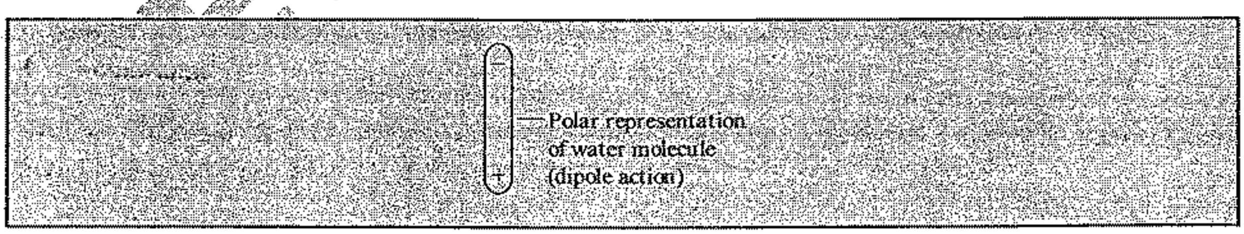
4. Illite

- Illite is a 2:1 clay mineral.
- Illite has substantial amount of isomorphous substitution in the form of Al in place of Si.
- K+ bonds the two negative surfaces of silica sheet tightly.
- Actually the hexagonal hole in silica sheet is filled with approximately dome sized pottassium ion.
- It is common in Stiff clays & in lacustrine soft clay.
- This potassium ion bonds the layers more firmly than montmorillonite. Thus it is much less susceptible to cleavage or spliting.
- Illite does not swell much in presence of water as montmorillonite. But it swells more than kaolinite.

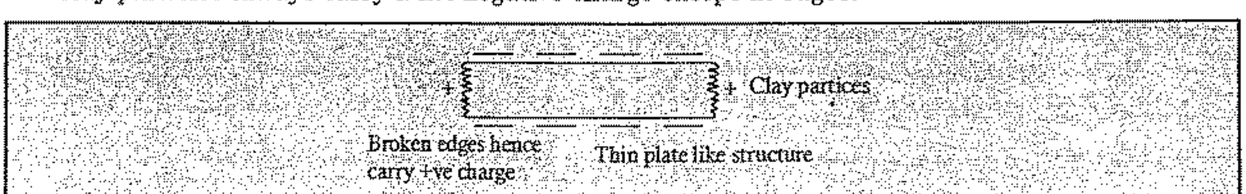


CLAY WATER RELATIONSHIP

- Normally clayey soils are associated with water and its properties are significantly influence with the presence of water. Where as granular soils are not sensitive to the amount of water present.
- Water is a dipolar molecule. The centre of gravity of +ve & –ve charges do not coincide. Hence it is treated as a bar magnet.



- Clay particles always carry a net negative charge except at edges.

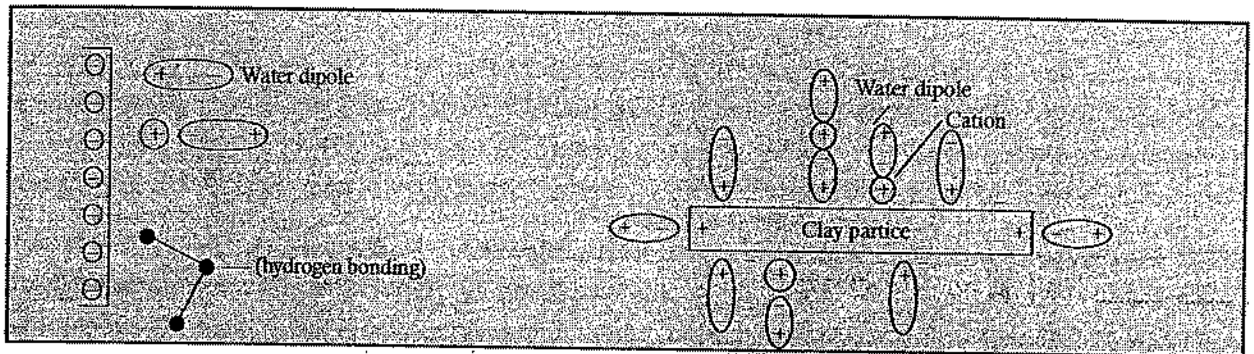


- Reason for charge accumumlation

1. Isomorphous substitution by lower valancy cation.
2. Breakage of particles
3. Dissociation of hydroxyl (OH^-) radical into hydrogen ions.

General Mechanism of Clay water Interaction

- Due to the attraction of +ve and –ve charges, close to the clay surfac results in formation of **adsorbed layer of water** and **double diffused layer.**



Following are the ways by which water is held to the clay surface:

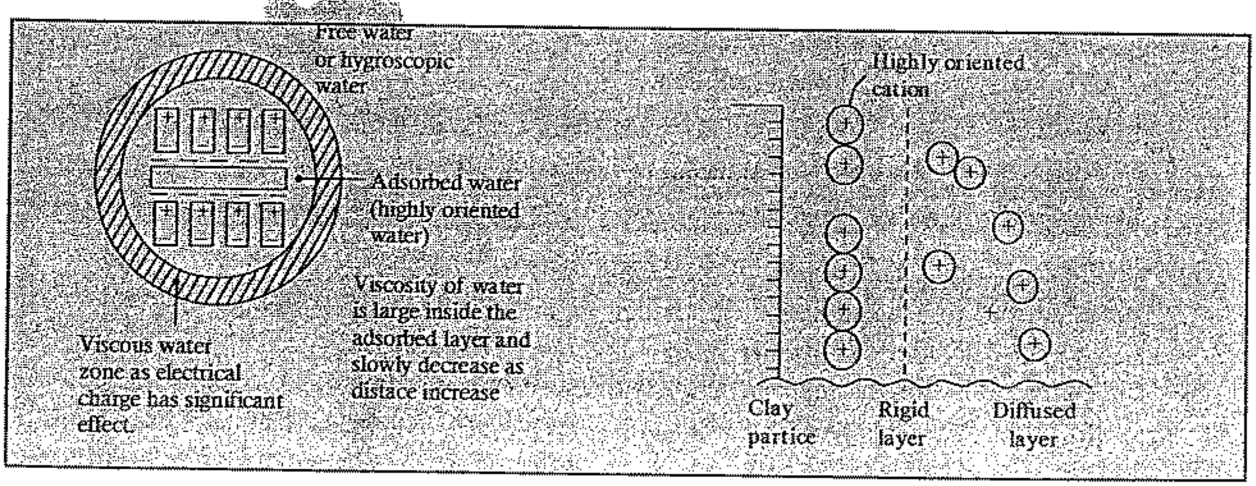
(a) Attraction between negatively charged faces of clay and +ve ends of dipoles (electrostatic attraction).

(b) Attraction between cations in the water and negatively charged ends of dipoles.

(c) Hydrogen atom of the water molecule is attracted to the oxygen or hydroxyl ion on the surface of clay crystals.

Absorbed Water

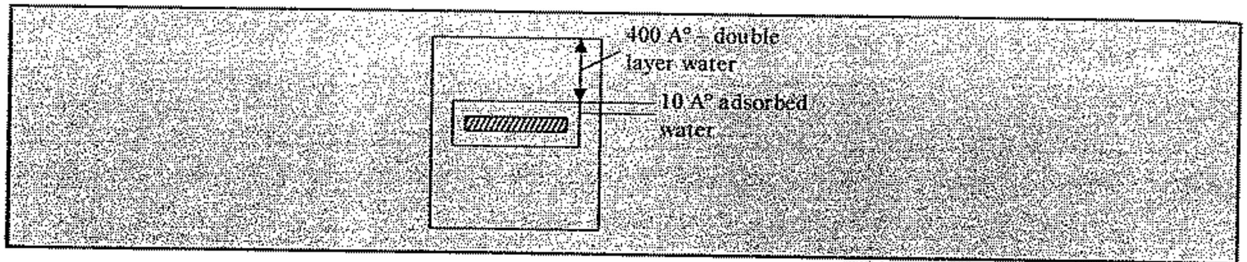
- Water held on the surface of particle by powerful forces of electrical attraction and virtually in a solid state this layer is very thin (10A°).
- Adsorbed water can not be removed by oven drying at 105-110°C and may therefore be considered to be part of the solid grain. Heating at 105–110°C will change clay soil structure.



- Cations distribute themselves around the negatively charged surface of clay particles with the greatest density near the surface.
- Beyond this, there is an outer layer which is attracted to lesser degree and is more mobile. This layer which extends up to the limit is called as diffused double layer and its density decreases with

the increasing distance from the surface. Ultimately the concentration becomes equal to that in water in the void space.

- Rigid layer and diffused layer combinely is called Electrical double layer.
- The oriented water is considered to affect the behaviour of clay particles when subjected to stress because if comes between the particle surface.



- For a given particle, the thickness of the cation layer depends mainly on the valency and can contraction of cations.
- An increase in valency (due to cation on exchange) or an increase in concentration will lead to decrease in the layer thickness.
- Increase in temperatures also decreases the thickness.

Note: Lesser no. of larger valence cation will be required for surface charge clay particle.

CATION EXCHANGE CAPACITY

- The ability of clay particles to absorb ions on its surface or edges is called base or cation exchange capacity. Which is a function of the mineral structure of the clay and the size of the particles.

$$\text{CEC} = f\,[\text{Mineral structure, size of particles}]$$

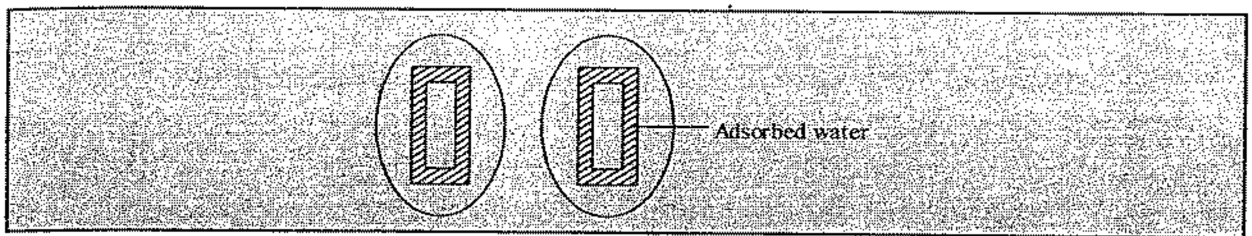
- Cation are termed as exchangable because one cation can easily be exchanged with the other.
- Behaviour of clay soil is dependent on the exchangable cation in the clay.
- Different clay have different change deficiencies and have varying ability to adsorb exchangable cations.
- The base exchange capacity is expressed in terms of the weight of a cation which may be held on the surface of 100 g of dry soil material. It is measured in milliequivalent (meq) per 100 of dry soil where 1 meq is 1 mg of hydrogen or the portion of any ion which will combine with or displace 1 mg of hydrogen.
- Montmorillonite has 10 times base exchange capacity as compared to Kaolinite. CEC of Illite is in between Kaolinite and Montmorillonite.
- Depositional environment, weathiring action, leaching etc. will govern the kind of ions present in a clay.
- Marine clay has predominantly Na & Mg cations.
- Valance of the cations is the basic factor in the process of replacment of exchange.
- Higher valency cations can easily replace lower valency cation.
- Replacement ability of various cations are as follows:

$$Al^{3+} > Ca^{2+} > Mg^{2+} > K^{+} > H^{+} > Na^{+} > Li^{+}$$

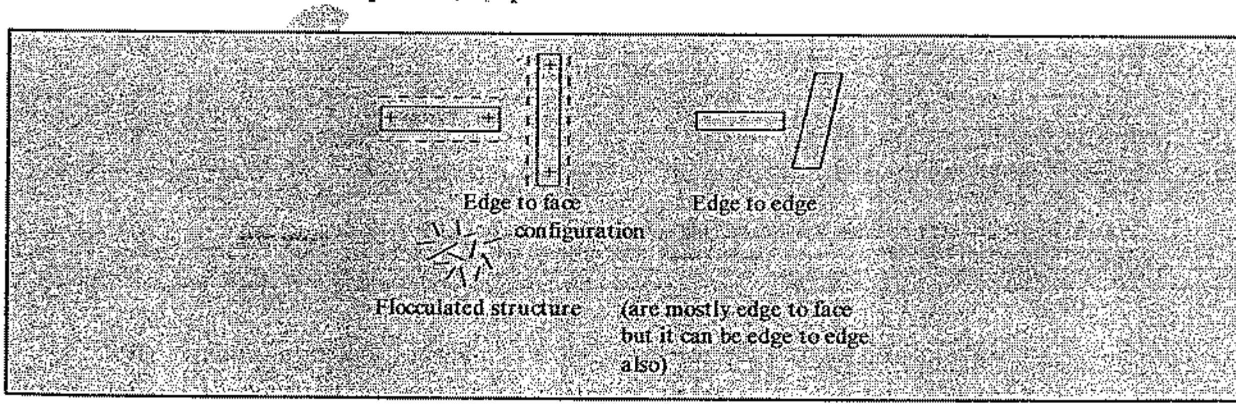
- This principle of cation exchange can be used with advantage in many practical situations, as for example, in the stabilisation of sodium clay soil by using lime. Here, calcium ions replace the sodium ions by virtue of their superior replacing power and reduces the swellings of sodium montmorillonite, because the adsorbed layer of water would be thinner when the valence of the cation is larger.

Clay Particle Interaction

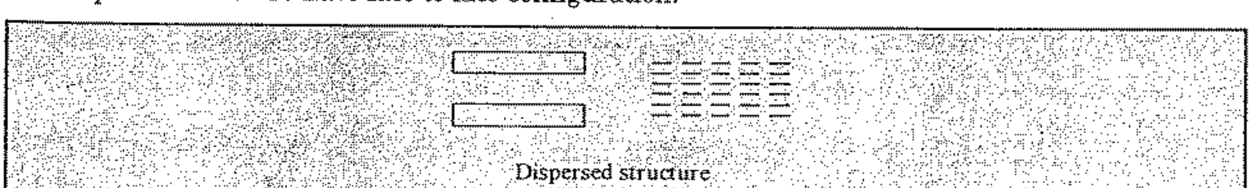
- Soil structure of clay depends on the forces of attraction and repulsion between the soil particles. But there is an adsorbed layer of water between two particles. Hence particles interact through adsorbed layer.



- Thus factors such as nature of ions present, their concentration, size etc. affects the soil structure.
- The repulsive forces between clay particles are due to similar charges in clay particles. This force comes into play when particles comes closer to such an extent that adsorbed water layer starts interacting.
- Attractive forces between clay particles are due to vanderwall forces, various types of linkage such as hydrogen bond etc.
- Force of repulsion and attraction both act between clay mineral particles.
- Repulsion occurs between like charges of the double layer of cation and the force of repulsion depending is on the characteristics of layers.
- An increase in cation valency or concentration will result in decrease in repulsive force and vice versa.
- Attraction between clay particles is due to Vanderwall forces forces which are independent of double layer.
- If two particles come closer and interact such that Total energy decreases then net force is attractive. This leads to Flocculated structure.
- If however, net force is repulsive, Dispersed structure will form.



- Flocculated structure (are mostly edge to face but it can be Edge to edge also).
- Dispersed structure have face to face configuration.



Tendency to flocculate depends on

1. Concentration of electrolytes
2. Valency of ions
3. Temperature.

- Dispersed structure with face to face configuration has lower void ratio as compared to floculated structure which has large void ratio.
- Marine clay has flocculated structure Lacustrine clay has dispersed structure.
- Sodium clay can absorb more water than Ca clay because adsorbed layer thickness in calcium clay is smaller (diffused layer being thinner, there by tendency to adsorb water being less). Thus Liquid limit of sodium clay is larger as compared to calcium-clay.
- As sodium clay can adsorb more water its swelling property is more. If Na is replaced by calcium, tendency to swellingis less. This is used in soil stabilization.

Note: (1) Decrease in layer thickness, decrease repulsion.

[redacted]

Clay Mineral	Grain size	Base exchange capacity	I_P	Dry strength	Active tip
Montmorillomite	min	largest	max	max	largest
Illite	intermediate	intermediate	intermediate	intermediate	intermediate
Kaolinite	max	least	min	min	least

SOIL STRUCTURE

- As discussed earlier in this chapter soil structure is the geometrical arrangemet of the soil particles in a soil mass.
- Generally the soil structure is influenced by the

(1) Particle size distribution
(2) Gravity Forces. } Coarse grained Soil

(3) Type of clay mineral
(4) Surface force. } Clay Soil

Following are the few structure usually found in soil.

Single Grained Structure :

- Chosession less soils such as Gravel and Sand.
- Plastics are in contact with each other under the influence of Gravitational forces.
- Under Shock & Vibration they show little settlement as these get densified from loose state.
  Ex : Marbels filled in a box.

Honey Combed Structure :

- Fine Sand and Silt.
- The particles are held in position by mutual attraction due to cohession but they do not possess plasticity.
- Soils in honey combed structure are in loose state. Under shocks & vibrations, structure collopses and show large deformation.

Flocculated Structure :

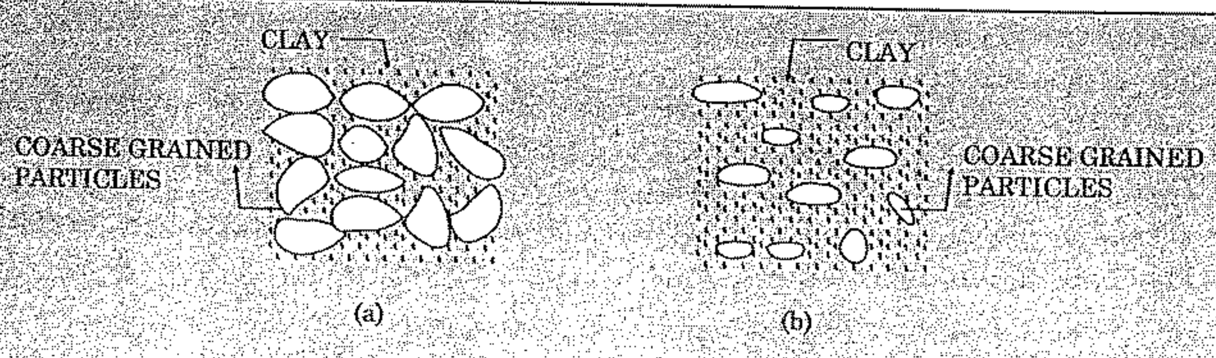
- As discussed earlier this type of structure occurs in clayey soil.
- Soils with a flocculent structure are light in weight and have a high void ratio and water content. These soils are quite strong and can resist external forces because of a strong bond due to attraction between particles.
- These soils are insensitive to vibrations.
- These soils in a flocculated structure have a low compressibility, a high permeability and a high shear strength.

Dispersed Structure :

- Dispersed structure develops in clays that have been reworked or remoulded. The particles develop more or less a parallel orientation.
- Clay deposits with a flocculent structure when transported to other places get remoulded. Remoulding converts the edge-to-face orientation to face-to-face orientation. The dispersed structure is formed in nature when there is a net repulsive force between particles.
- Soils in dispersed structure generally have a low shear strength, high compressibility and low permeability. Remoulding causes a loss of strength in a cohesive soil. Due to remoulding, the chemical equilibrium of the particles and associated adsorbed ions and water molecules within the double layer is disturbed.
- The soil regains strength as a result of re-establishing a degree of chemical equilibrium. This phenomenon of regain of strength with the passage of time, with no change in water content, is known as thixotropy.

Coarse-grained Skeleton :

- A coarse-grained skeleton is a composite structure which is formed when the soil contains particles of different types. When the amount of bulky, cohesionless particles is large compared with that of fine-grained clayey particles, the bulky grains are in particle-to-particle space between the bulky grains is occupied by clayey particles, known as binders.
- The bulky grains are deposited first during sedimentation and the binder is subsequently deposited.
- If soil structure is not disturbed, a coarse-grained skeleton can take heavy loads without much deformations. When the structure is disturbed, the load is transferred from the coarse-grained particles to clayey particles, and the supporting power and the stability of the soil is considerably reduced.



Coarse Grained Skelton Clay Matrix

Clay-Matrix Structure :

- Clay-matrix structure is also a composite structure formed by clay particles is very large as compared with bulky, coarse-grained particles. The clay forms a matrix in which bulky grains appear floating without touching one another.
- Soils with a clay-matrix structure have almost the same properties as clay. Their behaviour is similar to that of an ordinary clay deposit. They are more stable, as disturbance has very little effect on the soil formation with a clay-matrix structure.

OBJECTIVE TYPE QUESTIONS

1 Consider the following clay minerals :

1. Kaolinite 2. Illite 3. Montmorillonite

Which one of the following is the correct sequence of the minerals given above in the increasing order of their grain size?

(a) 3-2-1 (b) 1-3-2 (c) 1-2-3 (d) 3-1-2

2. Consider the following statements :

1. Peat and muck are organic soils
2. Peat is an inorganic soil whereas muck is an organic soil
3. Indurated clay is a type of clay which does not soften under prolonged wetting.

Which of these statements is/are correct?

(a) 1, 2 and 3 (b) 2 only (c) 3 only (d) 1 and 3 only

3. Honey-combed structure is found in

(a) Gravels (b) Coarse sands
(c) Fine sands and silts (d) clay

4. The weakest bond in soils is

(a) Ionic bond (b) Covalent bond
(c) Hydrogen bond (d) Secondary valance bond

5. An octahedral unit has

(a) Four negative charges (b) Three negative charges
(c) One negative (d) No negatie charge

6. In illite mineral, the bond between structural units is

(a) Hydrogen bond (b) Potassium ion bond
(c) Water molecules bond (d) Covalent bond

7. The plasticity characteristics of clay are due to

(a) Adsorbed water (b) Free water (c) Capillary water (d) None of above

8. In fine sands and silts, the most common type structure is

(a) Single grained (b) Honey comb
(c) Flocculated (d) Dispersed

9. What is the type of soil structure having arrangement of soil particles with a 'face-to-face' or parallel orientation generally recognized as?

(a) Honeycomb structure (b) Single-grained structure
(c) Flocculent structure (d) Dispersed structure

10. Which one of the following is the most active clay material?

(a) Na-illite (b) Na-kaolinite
(c) Na-montmorillonite (d) Ca-montmorillonite

ANSWERS

1. (a) 2. (d) 3. (c) 4. (d) 5. (b) 6. (b) 7. (a) 8. (b) 9. (b) 10. (d)

CHAPTER

4

Soil Compaction

INTRODUCTION

- Compaction of soil is the process of increasing the unit wt of soil by forcing the soil solids into a dense state and reducing the air voids.
- Compaction leads to increase in shear strength and helps impove the stability and bearing capacity of soil. It also reduces the compressibility and permeability of soil.
- This is achieved by applying static or dynamic loads to the soil.
- Compaction is measured quantitatively in terms of dry unit wt (γ_d) of the soil.
- Difference between compaction and consolidation are as tabulated below.

Compaction	Consolidation
1. Instantaneous phenomenon	1. Time dependent Phenomenon
2. Soil always partially saturated	2. Soil is completely Saturated
3. Densification due to reduction in the volume of air voids at a given water content.	3. Volume reduction is due to expulsion of pore water from voids.
4. Specific compaction techniques are used.	4. Consolidation occurs on account of a load placed on the soil.

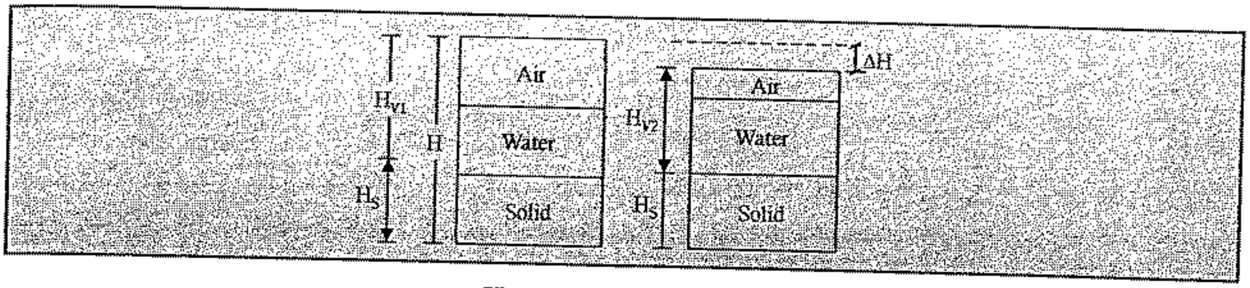
Why do we need to compact soil:

1. Max shear strength occurs at min. void ratio.
2. Large air voids if left, may lead to compaction under working loads causing settlement of the structure during service or may get filled with water which reduces the shear strength.
3. Increase in water content is also accompanied by swelling and loss of shear strength with time.

Thus, the various advantages of compaction are:

1. Settlement can be reduced or prevented.
2. Soil strength increases and stability can be improved.
3. Load carrying capacity of the pavement subgrade can be improved.
4. Undesirable volume changes (by frost action, swelling shrinkage) may be controlled.

General formula for settlement due to compaction:



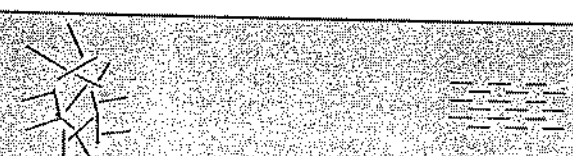
$$H_{V1} = e_o H_s \quad ; \; e_0 = \text{initial void ratio}$$
$$H_{V2} = e_f H_s \quad ; \; e_f = \text{final void ratio}$$
$$\Delta H = H_{V1} - H_{V2}$$
$$\frac{\Delta H}{H} = \frac{(e_o - e_f) H_s}{H_s + e_o H_s} = \frac{(e_o - e_f)}{1 + e_o}$$
$$\boxed{\frac{\Delta H}{H} = \frac{e_o - e_f}{1 + e_o}}$$

COMPACTION OF COHESIONLESS SOIL

- A cohesionless soil can be in various states like
  1. Loose angular soil
  2. dense angular soil
  3. honey-combed state
- Soil in loose and honey-combed state must be compacted, because loose sand when subjected to vibratory loading may lead to large settlement. For example liquifaction may ocur in loose-sand.
- Cohesionless soils are compacted by vibration.
- Medium and fine sands do not get compacted easily when moist because of shear strength developed by capillary forces.
- Dry and submerged sand can be compacted by vibrations.
- Static loading cause very little compaction of loose sand.

COMPACTION OF COHESIVE SOIL

- Clays cannot be compacted by vibration.
- Shaking or vibration does not change the volume of clay.
- In compacting clays, position of the particles is changed by forcing the contact points along adjacent surface to a more parallel position with reduced voids.



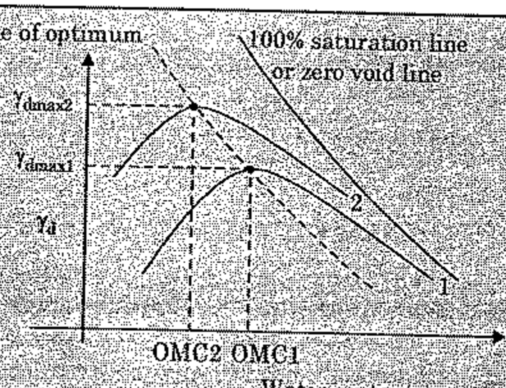
Loose structure of clay before composition (edge to face or edge to edge configuration of clay particles)

Dense structure after compaction (face to face configuration)

- The application of static pressure causes

PROCTOR TEST

- As per proctor, a definite relationship exist between the soil moisture content and the degree of dry density to which a soil may be compacted.
- For a specific amount of compaction energy applied on the soil, there is one moisture content termed as per factor optimum moisture content (OMC) at which a particular soil attain maximum dry density.



- Maximum dry unit weight obtained is a function of compactive effort and method of compaction for a particular type of soil.
- Compactive effort is a measure of mechanical energy applied to soil mass.
- Typical values of γ_{dmax} = 16 – 20 kN/m^3
- Typical values of OMC = 10 – 20%

STANDARD PROCTOR TEST

- Weight of hammer = 2.495 kg (5.5 *lb*)
- Height of fall = 12″ = 304.8 mm
- Volume of mould = 944 cc (1/30 *cft*)
- Compacted in 3 layers with 25 blows in each layer.
- Test is conducted at various moisture content and the curve as shown above is obtained.
- Compactive energy (E) applied per unit volume

$$\boxed{E = \frac{NnWh}{V}} \quad \ldots\ldots\ldots$$

Where,

N = no. of blows per layer ; n = no. of layers
w = weight of Hammer ; h = height of fall
V = volume of mould.

- Standard Proctor Compactive effort per unit volume = $\dfrac{25 \times 3 \times 2.5 \times 0.305 \times 9.81}{944 \times 10^{-6}} \dfrac{J}{m^3}$

$$= 594.29 \text{kJ/m}^3$$

MODIFIED PROCTOR TEST

- Weight of hammer = 4.54 kg (10 *lb*)
- Height of fall = 18″ = 457.2mm

- Volume of mould = 944 cc
- Compacted in 5 layers with 25 blows in each layer.
- Modified Proctor Compactive effort per unit volume $= \frac{25\times5\times4.54\times0.4572\times9.81}{944\times10^{-6}}\frac{J}{m^3}$
$= 2696.31$ kJ/m^3

IMPORTANCE OF PROCTOR TEST

Before starting compaction of soil in field we must know the compaction characteristic of soil. Proctor test gives idea about the compaction charaterstics of soil.

1. It gives the density that must be achieved in the field.
2. Provides the moisture range that allows for minimum compactive effort to achive required density.
3. Provides data on the behaviour of the material in relation to various moisture content. (*i.e.* how permeability etc. will he affected by m/c).

LIGHT COMPACTION TEST (IS : 2720, PART VII-1974)

- Weight of hammer = 2.6 kg
- Height of fall = 310 mm
- Volume of mould = 1000 cc
- Compacted in 3 layers with 25 blows in each layer.
- Light Compaction Test Compactive effort per unit volume $= \frac{25\times3\times2.6\times0.31\times9.81}{1000\times10^{-6}}\frac{J}{m^3}$
$= 593.014$ kJ/m^3

HEAVY COMPACTION TEST (IS : 2720, PART VIII-1983)

- Weight of hammer = 4.9 kg
- Height of fall = 450 mm
- Volume of mould = 1000 cc
- Compacted in 5 layers with 25 blows in each layer.
- Heavy Compaction Test Compactive effort per unit volume $= \frac{25\times5\times4.9\times0.45\times9.81}{1000\times10^{-6}}\frac{J}{m^3}$
$= 2703.88$ kJ/m^3

Example 1.

Compare the compactive energy used in the IS heavy compaction test with that of the IS light compaction test.

Sol. Compactive energy in IS heavy compaction test

$$= \frac{4.9(\text{kgf})\times0.45(\text{m})\times5(\text{layers})\times25(\text{blows / layer})}{10^3\times10^{-6}(\text{m}^3)}$$

$= 27562.5$ kgf m/m^3

Compactive energy in IS light compaction test

$$= \frac{2.6(\text{kgf}) \times 0.31(\text{m}) \times 3(\text{layers}) \times 25(\text{blows/layer})}{10^3 \times 10^{-6}(\text{m}^3)}$$

$$= 60450 \text{ kgf m/m}^3$$

IS heavy compaction test uses 4.56 times the compactive energy that is used in the IS light compaction test.

Example 2.

The compaction of an embankment is carried out in 300 mm thick lifts (layers). The rammer used for compaction has the foot of area 0.05 sq m. The energy developed per drop of the rammer is 40 kg m. Assuming 50 percent more energy in each pass over the compacted area due to overlap, calculate the number of passes required to develop compactive energy equivalent to IS light compaction for each layer.

Sol.

Compactive energy as per IS light compaction test

$$= \frac{2.6(\text{kgf}) \times 0.31(\text{m}) \times 3(\text{layers}) \times 25}{10^3 \times 10^{-6}(\text{m}^3)}$$

$$= 60450 \text{ kgf m/m}^3$$

Compactive energy per drop provided by the rammer per cum of the soil

$$= \frac{40}{0.05 \times 300 \times 10^{-3}}$$

$$= 2666.67 \text{ kgf m/m}^3$$

However, in each pass over a layer, the energy supplied will be 1.5 times this value on account of overlap of rammer footprints. If n is the number of passes required to develop compactive energy equivalent to IS light compaction.

$$n \times 1.5 \times 2666.67 = 60,450$$

$$n = 15.11 \text{ say } 16$$

FACTORS AFFECTING COMPACTION

- Factors affecting compaction
  - Moisture content
  - Soil type
  - Compactive effort
    - Nature of effort (i.e. method of compaction)
    - Amount of effort

1. Moisture Content (Lubrication theory)

- At low water content soil is stiff and offers more resistance to compaction. As moisture content increases, a film of water surrounds the soil particles and which tends to lubricate the particles and make them easier to be worked around, hence they come more closer and become dense.
- This phenomenon occurs upto OMC.

- Beyond OMC water starts taking the space which would otherwise have been taken by soil.
- As γ_w is smaller than γ_s, the dry unit weight reduces.

LAMBE EXPLANATION

- In case of cohesive soil, there is an attractive force namely, Vanderwall forces between two soil particles and a repulsive force due to double layer of adsorbed water. While the attractive force essentially remains same, the repulsive force is more if adsorbed layer thickness is more.
- If Net force is attractive flocculated structure results. However if net force is repulsive, dispersed structure results.
- At low water content, the double layer is not fully developed, i.e. repulsive force is small and net force is attractive. This makes it difficult for the particles to move closer when compactive effort is applied and a low dry unit weight is the consequence.
- As the water content increases, the double layer expands and repulsion between particles increase. The particles can easily slide over one another and get packed more easily resulting into higher dry density.
- The double layer expansion is complete at OMC, the addition of water does not cause any further expansion of double layer but water starts to occupy the space which otherwise would have been occupied by soil grains. Hence a decrease in dry unit wt. occurs beyond OMC
- This theory also explains why compaction curve is not inverted v-shape in soil which are not cohesive and plastic in nature.
- The dry unit weight can also be related to the water content and degree of Saturation.

$$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{G\gamma_w}{1+\frac{wG}{S}} \qquad \text{(i)}$$

- For a given moisture content, max dry unit weight is obtained when no air voids are left. When zero air voids are achieved at a particular moisture content, degree of saturation become 1. Hence at various moisture content Zero air void density can be achived by putting $S = 1$ in

$$\gamma_d = \frac{G\gamma_w}{1+\frac{wG}{S}} \cong \gamma_d = \frac{G\gamma_w}{1+wG} \qquad \text{... (ii)}$$

- When this is plotted we get zero void line.

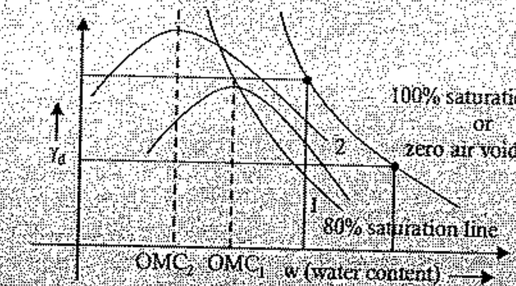
Zero Air Void density :

- The maximum dry unit weight that can be achived for a given water content by applying compaction.
- But zero air void density is very tough to achive in reality because no matter how much the compaction effort is, some air voids will remain with in the soil.
- This density is different from the case when water content is increased to saturate the soil.
- Zero air void line and 100% saturation line at various m/c are same.

γ_d can also be calculated as: $\gamma_d = \frac{(1-n_a)G\gamma_w}{1+wG}$

for $n_a = 0\%$ equation becomes identical to (i) equation

- It should be noted that 90% saturation line and 10% air void line are different
- 90% Saturation line $\gamma_d = \frac{G\gamma_w}{1+\frac{\omega G}{0.9}}$
- 10% air void line $\gamma_d = \frac{(1-0.1)\gamma_w^{G_S}}{1+wG}$



Example 3.

The results of a laboratory proctor test are shown below:

No.of Test	1	2	3	4	5	6
Wt.of Mould &Soil (Kg)	3.526	3.711	3.797	3.906	3.924	3.882
Water content (%)	8.33	10.40	12.23	16.20	17.92	20.39

The mould is 12.7 cm. high and has an internal diameter of 10 cm., The weight of the Empty mould is 1.89 kg.

(i) Plot the moisture content vs. dry density curve and determine the optimum moisture content andthe max dry density.

(ii) Plot the zero air void curve and 10% air void curve. Given G = 2.68

Sol. Data given H = 12.7 cm

D = 10 cm.

The weight of empty mould = 1.89 kg.

G = 2.68

$$\text{volums of the mould } V = \frac{\pi}{4} D^2 \times H$$

$$= \frac{\pi}{4} \times (30)^2 \times 12.7$$

$$= 997 \text{ cc}$$

In tbe first test, weight of soil = (3.526 – 1.89)

= 1.636 kg.

= 1636 gm.

$$\text{Bulk density } \gamma = \left(\frac{W}{V}\right) = \left(\frac{1636}{997}\right)$$

$$= 1.64 \text{ gm/cc}$$

$$\gamma_d = \left(\frac{\gamma_t}{1+w}\right) = \frac{1.64}{(1+0.0833)} = 1.515 \text{ gm/cc}$$

The dry density γ_{ds} of the soil corresponding to the zero air void condition.

$$\gamma_{ds} = \frac{G\gamma_w}{(1+wG)}$$

$$= \frac{2.68\times1}{1+(0.0833\times2.68)}$$

$$= 2.19\text{gm/cc}$$

When the soil has 10% air void, its digree of saturation is 90%

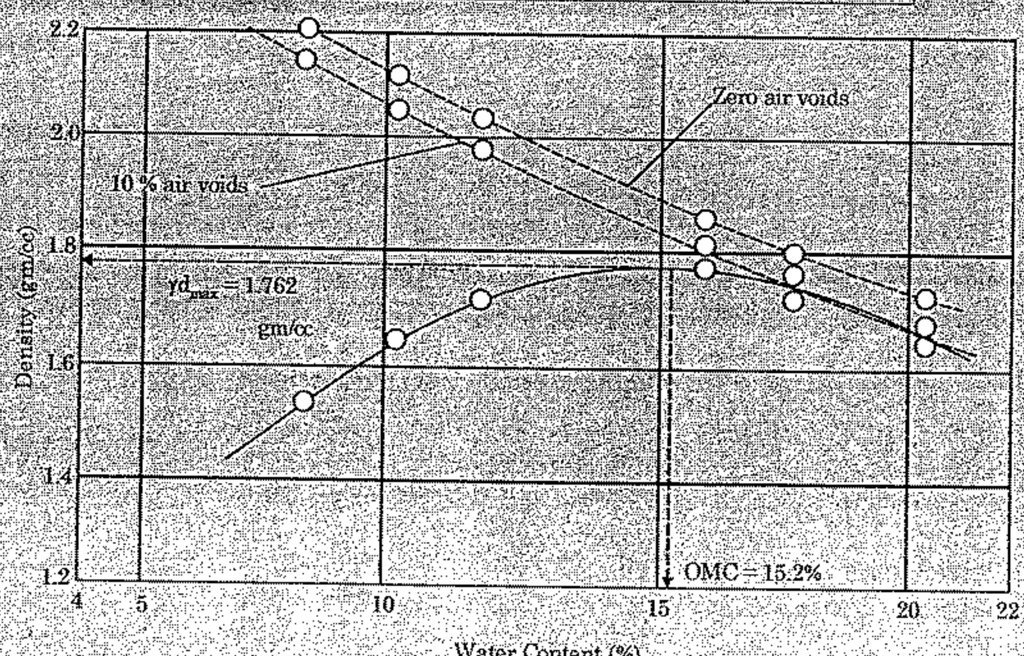
$$e = \frac{wG}{S} = \left(\frac{(0.0833\times2.68)}{0.9}\right)$$

$$e = 0.248$$

$$\gamma_{d90} = \frac{2.68\times1.0}{(1+0248)} = 2.147 \text{ gm/cc}$$

Similarly, dry densities c/p to actual proctor test, the zero air void condition and the 10% air void condition are computed for the remaining five tests.

No.of Test	1	2	3	4	5	6
Wt. of Mould & Soil (Kg)	3.526	3.711	3.797	3.906	3.924	3.882
Water content (%)	8.33	10.40	12.23	16.20	17.92	20.39
Bulk density(gm / cc)	1.641	1.826	1.913	2.022	2.040	1.998
Dry density(λd)	1.515	1.654	1.705	1.740	1.730	1.660
Dry density for zero air voids(γds)	2.190	2.096	2.018	1.869	1.810	1.733
Dry density for 10% air void rdgo(gm / cc)	2.147	2.046	1.964	1.808	1.747	1.667



From the curve, we find

optimum moisture content (OMC) = 15.2%

$(\gamma_d)_{max} = 1.76$ gm/cc

Example 4.

The optimum moisture content of a soil is 16.5% and its maximum dry density is 1.57 gm/cc. The specific gravity of solids is 2.65. Determine

content

(i) The degree of saturation and % of air ~~voids~~ of the soil at OMC.

(ii) The theoretical dry density at OMC corresponding to zero air voids.

Sol. Data given

$$\text{OMC} = 16.5\%$$

$$(\gamma_d)_{max} = 1.57 \text{ gm/cc}$$

$$G = 2.65$$

Now We have $\gamma_d = \left(\dfrac{G\gamma_w}{1+e}\right)$

$$\Rightarrow \quad 1.57 = \frac{(2.65 \times 1)}{1+e}$$

$$\Rightarrow \quad e = \left(\frac{2.65}{1.57} - 1\right)$$

$$e = 0.688$$

$$S = \left(\frac{wG}{e}\right) = \left(\frac{0.165 \times 2.65}{0.688}\right)$$

$$= 0.635$$

$$S = 63.5\% \qquad \text{... (i)}$$

Hence, the required degree of saturation = 63.5%

% of air ~~void~~ = 100 − 63.5

(content) = 36.5%

(ii) At zero air void the soil is fully saturated

$$S = 1$$

$$e = \left(\frac{wG}{S}\right) = \left(\frac{0.155 \times 2.65}{1}\right) = 0.437$$

$$\gamma_d = \left(\frac{G\gamma_w}{1+e}\right)$$

$$= \frac{2.65 \times 1}{(1+0.437)}$$

Therefore the theoretical dry density at OMC for zero air void = 1.844 gm/cc.

Example 5.

In order to determine the relative density of a sand sample, its natural moisture content and bulk density were determined in the field and were found to be 7% and 1.61 gm/cc respectively. Sample of this soil were then compacted in a proctor's mould of 1/30 cft. Capacity, at the loosest and the densest states.

The following data were obtained.

Wt of empty mould = 2100 gm

Wt. of mould + soil in the loosest state = 3363.6 gm

Wt. of mould + soil in the densest state = 3857.4 gm

Moisture content of the sample used in tests = 11%

Determine the relative density of sand.

Sol. Data given : Vol. of the mould $= \frac{1}{30}$ cft

$$= \frac{(12)^3(2.54)^3}{30}$$

$$= 943.89 \text{ cc.}$$

<u>In the loosest state</u>

$$\text{Bulk density} = \frac{(3363.6-2100)}{943.89}$$

$$= 1.339 \text{ gm/cc}$$

$$\text{Min. Dry density } (\gamma_d)_{min} = \left(\frac{\gamma}{1+w}\right) = \frac{1.339}{(1+0.11)}$$

$$= 1.206 \text{ gm/cc}$$

In the densest state

$$\text{Bulk density} = \frac{(3857.4-2100)}{943.89}$$

$$= 1.862 \text{ gm/cc}$$

$$(\gamma_d)_{max} = \frac{1.862}{(1+0.11)}$$

$$= 1.677 \text{ gm/cc}$$

$$\text{In-situ density of the soil} = 1.61 \text{ gm/cc}$$

$$w = 7\%$$

$$\text{In-situ dry density } (\gamma_d) = \frac{1.61}{(1+0.07)}$$

$$= 1.505 \text{ gm/cc}$$

$$\text{Relative density } R_D = \frac{(\gamma_d)_{max}}{\gamma_d}\left(\frac{\gamma_d-(\gamma_d)_{min}}{(\gamma_d)_{max}-(\gamma_d)_{min}}\right)\times 100$$

$$= \left(\frac{1.677}{1.505}\right)\times\frac{(1.505-1.206)}{1.677-1.206)}\times 100$$

$$= 70.74\%$$

Example 6.

The maximum dry density of a sample by the light compaction test is 1.78 g/ml at an optimum water content of 15%. Find the air voids and the degree of saturation. G = 2.67.

Sol.

$$\rho_d = \frac{G\rho_w}{1+e} = \frac{G\rho_w}{1+(wG/S)}$$

or

$$1.78 = \frac{2.67\times 1.0}{1+(0.15\times 2.67/S)}$$

$$1.78+\frac{0.713}{S} = 2.67 \quad \text{or} \quad S = 0.801 \quad \text{or} \quad \mathbf{80.1\%}$$

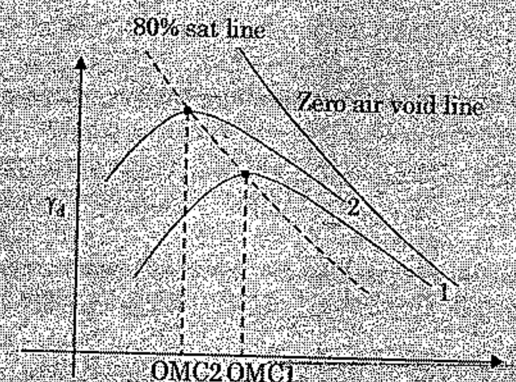
Now $\rho_d = \frac{(1-n_a)G\rho_w}{1+wG} = \frac{(1-n_a)\times 2.67\times 1.0}{1+0.15\times 2.67} = 1.78$

or $n_a = 0.066$ or 6.6%

$$(\gamma_d)theomax = \frac{G\rho_w}{1+wG} = \frac{2.67\times 1.0}{1+0.15\times 2.67} = \mathbf{1.91\ g/ml}$$

COMPACTIVE EFFORT

- For a given type of compaction, the higher the compactive effort, the higher the maximum dry unit weight and lower is the OMC.



Curve-1→ Lower compactive effort

Curve-2→ Large compactive effort

- As the water content increases, the influence of compactive effort on dry unit weight tends to diminish.
- Margin of increase of dry density is very less at low moisture content.
- Margin of increase of dry density is insignificant in wet soil.
- Line of optimus is parallel to zero air void line.
- If implies that max density that can be achieved at a particular compactive effort has a particularly fixed difference with the maximum theoretical possible density at that moisture content.
- Degree of saturation at OMC remains almost the same in all cases.

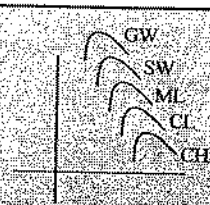
$$S = \frac{V_w}{V_v} = \frac{V_w}{V_w + V_a} = \frac{1}{1+\frac{V_a}{V_w}} \qquad \left(i.e.\ \frac{V_a}{V_w} \text{ ratio is almost fixed at } OMC\right)$$

Note: That if large compactive effort is applied on Wet side of optimum, density will not increase significantly. However pore water pressure may build up, causing following effects.

(1) Slope instability.

(2) The excess pore water building up may later dissipate causing consolidation settlement.

TYPICAL COMPACTION CURVES FOR DIFFERENT SOILS



Points to be noted

1. Coarse grained soil, well graded, compacted to high γ_d specially if they contain some fines. However if the quantity of fines is excessive γ_d decreases.
2. Poorly graded or uniform sand lead to lowest dry unit wt values.
3. In clay soil max dry unit wt tend to decrease as plasticity increases.
4. Heavy clay with high plasticity has very low dry unit wt and very high OMC.

Example 7.

Work out the theoretical maximum dry density for a soil sample having specific gravity of 2.7 and OMC = 16% also explain the difference in OMC values in case of proctor test and modified proctor test for cohesive soils and granular soil.

Sol. Data given : $G = 2.7,\ w = OMC = 16\%$

$\gamma_w = 9.81\ kN/m^3$

We know that

$$\gamma_d = \left(\frac{G\gamma_w}{1+e}\right) \quad \ldots(i)$$

$$eS = wG$$

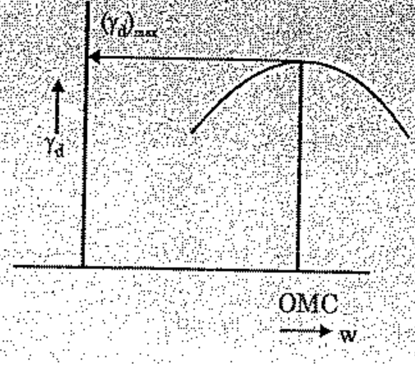
$$= \left(\frac{wG}{S}\right) \quad \ldots(ii)$$

From (i) & (ii)

$$\text{We have } \gamma_d = \frac{G\gamma_w}{\left(1+\frac{wG}{S}\right)} \quad \ldots(iii)$$

For a given water content, the theoretical maximum value of dry unit weight for a compacted soil is obtained, when no air vaoids are left

i.e when S = 1.

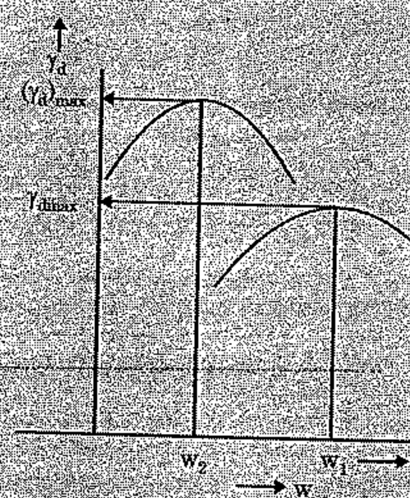


$$(\gamma_d)_{max} = \frac{G\gamma_w}{(1+wG)}$$

$$= \frac{2.7 \times 9.81}{(1+0.16\times 2.7)}$$

$$= 18.49 \text{ kN/m}^3 \quad \text{... (iv)}$$

- The only difference between proctor test and modified proctor test is that the latter gives more compactive effort than the former.
- The dry density increases with the compactive effort but the water content decreases along with it.
- The difference betwen OMC values of cohesive soils and granular soil is that cohesive soils have large value of OMC than granular soils.



COMPACTION OF SAND

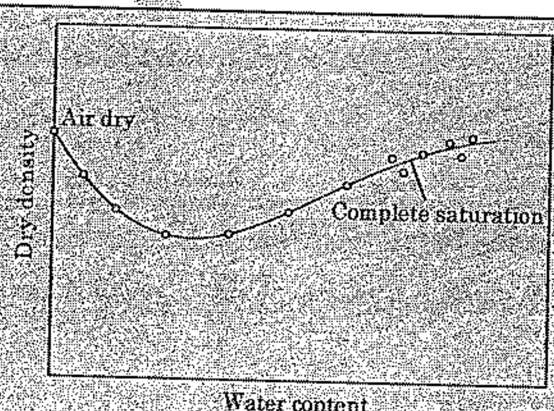


Fig. : Typical compaction curve for cohesion less sands.

- The typical inverted *V* shape of a compaction curve applies only to soils possessing some amount of plasticity. In the case of pure sandy soils, the trend observed is somewhat different.
- Above figure, shows the variation of dry unit weight of sand with moisture content.
- Initially, there is, in fact, a decrease in dry unit weight with increase in water content. This is due to the capillary tension in pore water which prevents soil particles coming closer. This phenomenon is known as *bulking of sand*.

- Maximum bulking occurs at a water content of about 4 to 5 per cent.
- With increase in water content, the dry unit begins to increase as the menisci are broken and the particles are able to move and adopt a closer packing.
- The maximum dry unit weight results when the soil is fully saturated.
- As the water content is increased further, there is a fall in dry unit weight again.
- Coarse-grained soils do not adsorb water, the way fine-grained soils do and hence the Lambe's double layer theory is not applicable to them.
- Maximum dry unit weight is obtained, when the soil is either dry or completely saturated. Therefore, sands are usually compacted either in a dry state or in a saturated state by flooding with water.
- In any case, the difference in dry unit weight at different states is so small that compaction water content is not a very significant factor in sandy soils.

EFFECT OF COMPACTION ON PROPERTIES OF SOILS

1. Soil structure :

- The water content at which the soils is compacted plays an important role in the engineering properties of the soil. Soils compacted at a water content less than the optimum water content generally have a flocculated structure, regardless of the method of compaction.
- Soils compacted at a water content more than the optimum water content usually have a dispersed structure at point A on the dry side of the optimum, the water content is so low that the attractive forces are more predominant than the repulsive forces. This results in a flocculated structure.
- As content is increased beyond the optimum, the repulsive forces increase and the particles get oriented into a dispersed structure.
- If the compactive effort is increased, there is a corresponding increase in the orientation of the particles and higher dry densities are obtained, as shown by the upper curve.

2. Permeability :

- The permeability of a soil depends upon the size of voids.
- There is an improved orientation of the particles and a corresponding reduction in the size of voids which cause a decrease in permeability.
- If the compactive effort is increased, the permeability of the soil decreases due to increased dry density and better orientation of particles.

3. Swelling :

- A soil compacted dry of the optimum water content has high water deficiency and more random orientation of particles. Therefore, it imbibes more water than the sample compacted wet of the optimum, and has, therefore, more swelling.

4. Pore water pressure :

- A sample compacted dry of the optimum has low water content.
- The pore water pressure developed for the soil compacted dry of the optimum is therefore less than that for the same soil compacted wet of the optimum.

5. Shrinkage :

- Soils compacted dry of the optimum shrink less on drying compared with those compacted wet of the optimum.
- The soils compacted wet of the optimum shrink more because the soil particles in the dispersed structure have nearly parallel orientation of particles and can pack more efficiently.

6. Compressibility :

- The flocculated structure developed on the dry side of the optimum offers greater resistance to compression than the dispersed structure on the wet side. Therefore, the soils on the dry side are less compressible.
- It increase with an increase in the degree of saturation.
- The compressibility of a soil compacted on the wet side of the optimum is also influenced by the method of compaction. If the compaction is of kneading or impact type, it creates a more dispersed structure with a corresponding increase in the compressibility.
- If the compaction causes very large stresses, the compressibility increases due to breakdown of the structure and greater orientaion of the particles.

7. Stress-Strain relationship :

- The soils compacted dry of the optimum have a steeper stress-strain curve than those on the wet side.
- The modulus of elasticity for the soils compacted dry of the optimum is therefore high. Such soils have brittle failure like dense sands or over-consolidated clays.
- The soils compacted wet of the optimum have relatively flatter stress-strain curve and a corresponding lower value of the modulus of elasticity.

8. Shear Strength :

- At a given water content, the shear strength of the soil increases with an increase in the compactive effort till a critical degree of saturation is reached.
- With further increase in the compactive effort, the shear strength decreases.
- The shear strength of the compacted soils depends upon the soil type, the moulded water content, drainage conditions, the method of compaction, etc.

COMPACTION IN THE FIELD

- Laboratory compaction tests are usually utilised to specify the compacted dry unit weight to be attained in the field. Depending upon the size of equipment to be used in the field.
- Indian Standard (light or heavy compaction) test may be used.
- Since the control in the field cannot be as strict as in the laboratory, the specifications usually require attainment of 90 to 95 per cent of dry unit weight attained in the laboratory.
- Different types of soils in the field can be compacted by various methods, e.g., rolling, ramming (by impact) and vibrations.
- The various types of rollers are : smooth wheel rollers, pneumatic tyred rollers and sheepsfoot rollers. Ramming equipment can be the impact type, internal combustion type or the pneumatic type.
- Vibrating unit can be mounted on plates or rollers. Vibration may be induced by rotating an unbalanced mass or by a pulsating hydraulic system.
- The selection of the equipment and the procedure of compaction depends on the characteristics of the soil to be compacted. The compaction achieved in the field would depend on
  (i) thickness of the lift (layer)
  (ii) type of roller,
  (iii) number of passes of the roller, and
  (iv) intensity of pressure on the soil.

SUITABILITY OF COMPACTION EQUIPMENT

Type of equipment	*Suitability for soil type*	*Nature of project*
Rammers or tampers	All soils	In confined areas such as fills behind retaining walls, basement walls, etc. Trench fills.
Smooth wheeled rollers	Crushed rocks, gravels, sands	Road construction, etc.
Pneumatic tyred rollers	Sands, gravels silts, clayey soils	Base, sub-base and embankment compaction for highways, airfields, etc, Earth dams
Sheepsfoot rollers	Clayey soil	Core of earth dams
Vibratory rollers	Sands	Embankments for oil storage tanks, etc.

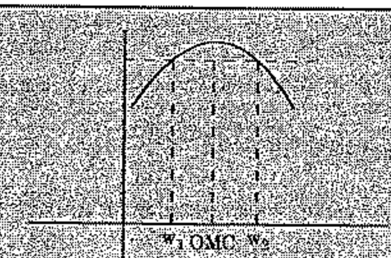
COMPACTION SPECIFICATIONS AND FIELD CONTROL

- Earthwork specifications can be classified into two categories, namely, end-product specification and method specifications.
- Under the first group, a certain relative compaction or per cent compaction is specified.
- Relative compaction is defined as the ratio of the field dry unit weight, $\gamma_{d\,(\text{field})}$ to the laboratory maximum dry unit weight $\gamma_{(d\max)}$ as per specified standard test, namely Indian standard light or heavy compaction test.

$$\text{Relative compaction} = \frac{\gamma_d(\text{field})}{\gamma_d(\max)}$$

1. End Product Specifications

- These are used mostly for building fondations and highways. Under these specifications, it is the specified relative compaction which is important, the method for achieving this is not so important.



- Same γ_d can be obtained at two moisture content and it is easier to compact soil with wet optimum moisture content.
- But this results in lower shear strength than compacting the soil on dry side of optimum moisture content.
- Along with shear strength, permeability and shrinkage characterstic will also get effected. Therefore range of water content should also be specified in addition to the percentage of relative compaction.

Comparison of Dry of Optimum with Wet of Optimum Compaction

Property	*Dry of Optimum*	*Wet of Optimum*
Structure after compaction	Flocculated (random)	Dispersed (oriented)
Water deficiency	More	Less
Permeability	More, isotropic	Less, anisotropic ($k_H > K_v$)
Compressibility		
at low stress	Low	Higher
at high stress	High	Lower
Swellability	High	Low
Shrinkage	Low	High
Stress-strain behaviour	Brittle; high peak higher elastic modulus	Ductile, no peak, lower elastic modulus
Construction pore water pressure	Low	High
Strength (undrained) as moulded,	High	High
Strength (undrained) as moulded,	High	Much lower
	somewhat higher if swelling prevented	Low

Selection of Compaction Water Content

Project	*Compaction water content*	*Reason*
Core of an earth dam	Wet of optimum	To reduce permeability and prevent cracking in core
Homogeneous earth dam	Dry of optimum	To have a stronger soil and to prevent build-up of high pore water pressure
Subgrade of pavement	Wet of optimum	To limit volume changes

2. Method Specifications

- In this method, type of weight of roller, no. of passes and thickness layer are specified. This method requires better quality control at site by both contractor and client engineer.
- As this method is expensive and requires prior knowledge of borrow soils and its compaction results, due to this reason this method is usually preffered only for large projects.

FIELD CONTROL TEST

Field Control Test

Destructive	Non Destructive
Requires Excavation & Removal of Material	Indirect methods are used.
(1) Sand Replacement Method (2) Baloon Method (3) Core Cutter Method	Nuclear methods.
• It is relatively slow process • Loss costly	It is relatively quick process • It is highly costly.

MISLANEOUS METHODS OF COMPACTION

1. Vibro Floatation Method :

- Vihro Floatation Method is used for compacting thick deposit of loose, sandy soils upto 30 m depth.
- Vibrafloat is a 2m diameter cylinder tube, fitted with water jets at top and bottm. It has a rotating eccentric mass which develops a horizontal vibratory motion.
- Vibrofloat is sunk into loose sand using the lower jet. As the lower jet is operatated quick sand condition is developed which causes reduction in shear strength and vibrofloat settles further due to its own mass the till the required depth is achived.
- As the desired depth is reached, vibration is activated, this causes the some compaction of sand in horizontal direction upto a radius of 1.5 m.
- Vihrofloat is slowly-slowly pulled up.
- Additional sand is continually dropped into the void space created around the vibrofloat and compacted.
- The spacing of hloes is usually kept 2-3 m on a grid pattern.
- Relative density achived is 70%.
- Vihrofloatation is not useful for cohessive soil.

2. Tera Probe Metbod

- It can be used up to a depth of 20 m. In this method an open ended pipes, about 75 cm diameter with a vibratory pile drive is used.
- The vibratory pile driver gives vertical vihration to teraprobe and it goes down.
- After reaching the required depth the tera probe is gradually raised upward while the vibrodriver continues to operate. Thus the soil within and around terra prohe is densified.
- The spacing of holes is kept 1.5m.
- This method is faster but less effecttive than Vibrofloatation method. as Zone of influence and relative density achived is less.

3. Compaction by Pounding

- It is very effective for compacting loose sandy soils and fine grained soil.
- A heavy mass is dropped from a large height on the ground surface.
- While using Pounding we have to take care that vibrations do not get transfer to adjacent buildings.

4. Compaction By Explosives

- Buried explosives are sometimes used to densify the cohesionless soils.
- The shock wave and vibrations produced by explosives are somewhat similar to that produced by vibratory compaction equipment.
- This method is quite effective when the cohesionless soil is fully saturated.
- The shock waves cause liquefaction of sand, which is followed by densification.
- In partially saturated cohesionless soils, compressive stresses develop due to capillary action and prevent the soil particles from taking closer positions.
- This method is not effective for partially saturated soils.
- The depth upto which the blast is effective is limited to about 25 m.
- This uppermost zone of the soil upto a depth of about 1 m gets displaced in a random manner and is, therefore, not properly densified. This zone should be compacted using the conventional methods by rollers.

5. Compaction Piles

- Cohesionless soils can be densified by constructing compaction piles.
- A capped, pipe pile is driven into the soil. The soil surrounding the pile is compacted due to vibrations caused during driving. The pile is then extracted and the hole formed is backfilled with sand.

Example 8.

In a Proctor compaction test, the soil specimen of one of the observations had a bulk density of 19 kN/m³ with a moisture content of 15%. Find,

(a) degree of saturation of the specimen if G = 2.7

(b) additional moisture content required for saturating the soil specimen.

Sol.

$$\gamma_t = 19 \text{ kN/m}^3 \qquad w = 15\% \qquad G = 2.7$$

$$\gamma_t = \frac{G(1+w)}{1+e}\gamma w$$

$$19 = \frac{2.7(1+0.15)}{1+e} \times 9.81$$

$$e = 0.603$$

$$Se = w\ G$$

$$S = \frac{0.15 \times 2.7}{0.603}$$

$$S = 0.671$$

Water content for full saturation

$$w = \frac{e}{G}$$

$$w = 0.2485$$

Additionla water content req for full saturation = 24.85 − 15 = 9.85%

Example 9.

A sample of soil was prepared by mixing a quantity of dry soil with 10% by mass of water. Find the mass of this wet mixture required to produce a cylindrical, compacted specimen of 15 cm diameter and 12.5 cm deep and having 6% air content. Find also the void ratio and the dry density of the specimen if G = 2.68.

Sol. : Air content, $a_c = V_a/V_v = 0.06$

or $V_a = 0.06\ V_v$, Hence $V_w = 0.94 V_v$

Thus $$V_a = 0.06\left(\frac{V_w}{0.94}\right) = 0.638\ V_w$$

Volume of specimen $(V) = \pi/4 \times (15)^2 \times (12.5) = 2208.9 \text{ ml}$

Now, with usual notations, $V = V_s + V_w + V_a$

or $2208.9 = V_s + V_w + 0.0638\ V_w = V_s + 1.0638\ V_w$

Writing volume terms of mass.

$$2208.9 = \frac{M_s}{(2.68\times1.0)} + 1.0638\left(\frac{M_w}{1.0}\right)$$

Substituting $M_w = 0.10\,M_s$,

$$2208.9 = \frac{Ms}{(2.68)} + 1.0638\times0.1M_s$$

or $M_s = 4606.54$ gm, $M_w = 460.65$ gm

Mass of wet soil, $M = M_s + M_w = 4606.54 + 460.65 = 5067.19$

Bulk density, $\rho = \frac{M}{V} = \frac{5067.19}{2208.9} = 2.294$ gm/ml

Dry density, $\rho_d = \frac{\rho}{1+w} = \frac{2.294}{1+0.10} = 2.085$ gm/ml

Therefore, $e = \frac{G\rho_w}{\rho_d} - 1$

$$= \frac{2.68\times1.0}{2.085} - 1 = \mathbf{0.285}$$

Example 10.

During the construction of an Embankment, the density attained by field compaction was investigated by the sand jar method. A test pit was excaveted in the newly compacted soil and was filled-up by pouring sand. The following were the observations.

Weight of the soil excaveted from pit = 2883 gm.

Weight of sand req. to fill the pit = 2356 gm.

Bulk density of sand = 1.52 gm/cc

Miosture content of embankment soil = 16%

Determine the dry density of the compacted soil.

Sol. Data given : Bulk density of sand = 1.52 gm/cc

Weight of sand = 2356 gm

The volume of the sand req. to fill up the pit

$$V = \left(\frac{W}{\gamma}\right) = \left(\frac{2356}{1.52}\right) = 1550 \text{ cc}$$

Vol. of the pit = 1550 cc

But, weight of the soil excaveted from the pit = 2883 gm.

$\therefore$ In-situ bulk density of soil

$$\gamma_t = \left(\frac{2883}{1550}\right) = 1.86 \text{ gm/cc}$$

$\therefore$ In-situ dry density of the soil $\gamma_d = \left(\frac{\gamma_t}{1+w}\right)$

$$\gamma_d = \frac{1.86}{(1+0.16)} = 1.60 \text{ gm/cc}$$

Example 11.

The rock content in a fill is 80% by dry weight. The rock can be compacted to a minimum void ratio of 0.73. The maximum dry unit weight to which the soil fraction can be compacted is 1.63 gm/cc. What is the maximum dry density to which the fill can be compacted? Given specific gravity of rock = 2.56.

Sol. Data given :

$$(e_{min})_{rock} = 0.73$$

Therefore, when the rock present in the fill is compacted to the densest state, its dry unit weight is given by

$$(\gamma_d)_{max} = \left(\frac{G\gamma_w}{1+e}\right)$$

$$= \frac{(2.56 \times 1)}{(1+0.73)}$$

$$= 1.48 \text{ gm/cc}$$

$$\text{For the soil } (\gamma_d)_{max} = 1.63 \text{ gm/cc}$$

Let us now consider 1 gm of the given fill. According to the question, the weight of rock and soil present in the fill are 0.8 gm and 0.2 gm respectively.

$$\text{Vol. of 0.8 gm of rock} = \frac{0.8}{1.48} = 0.54 \text{ cc}$$

$$\text{and vol. of 0.2 gm of dry soil} = \frac{0.2}{1.63} = 0.123 \text{ cc}$$

$$\text{Total vol. of 1 gm. of fill} = (0.54 + 0.123)$$

$$= 0.663 \text{ cc}$$

$$\text{Dry unit weight of the fill} = \frac{\text{Dry weight}}{\text{Vol.}}$$

$$= \frac{1}{(0.663)} = 1.508 \text{ gm/cc}$$

Example 12.

In a proctor compaction test, the soil specimen of one of the observation had a bulk density of 19kN/M³ with moisture content of 15% find

(i) Degree of saturation of the specimen if G = 2.7

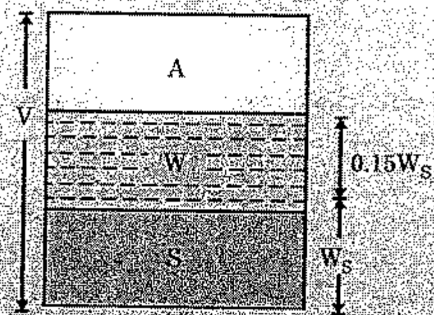
(ii) Additional moisture content req. For saturating the soil specimen.

Sol. Data given :

$$\gamma_t = 19 \text{ kN/cm}^3$$

$$w = 15\%$$

$$\text{We know that } \gamma_d = \left(\frac{\gamma_t}{1+w}\right)$$



$$\gamma_t = \frac{(G+eS)\gamma_w}{(1+e)} \quad \text{...(i)}$$

$$eS = wG \quad \text{...(ii)}$$

$$\gamma_t = \frac{(G+wG)\gamma_w}{1+e}$$

$$e = \frac{G(1+w)\,\gamma_w}{\gamma_t} - 1$$

$$e = \frac{2.7(1+0.15)\times 9.81}{19} - 1$$

$$e = 0.603$$

Degree of saturation $S = \left(\frac{wG}{e}\right)$

$$S = \left(\frac{0.15\times 2.7}{0.603}\right)$$

$$S = 67.15\%$$

(ii) For degree of saturation, $S = 1$

$$eS = wG$$

$\Rightarrow$ $$e = \left(\frac{wG}{S}\right)$$

$\Rightarrow$ $$w = \frac{eS}{G}$$

$$= \frac{0.603\times 1}{2.7}$$

$$w = 0.2233 \text{ or } 22.33\%$$

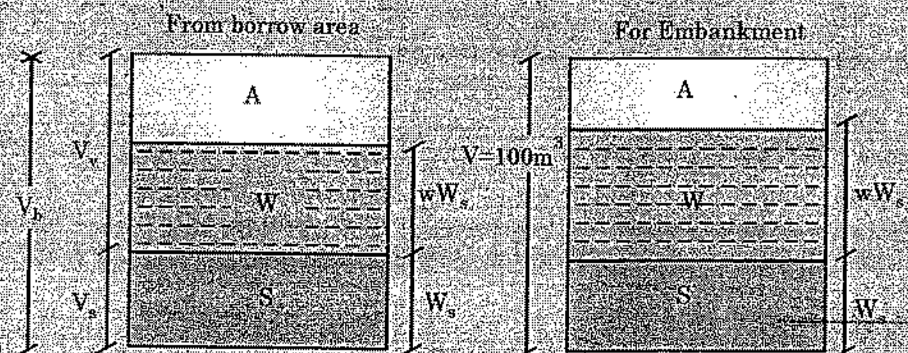
$\therefore$ Additional water content req= (22.33 – 15)

= 7.33%

Example 13.

It is required to construct an embankment by compacting a soil excavated from nearly borrow areas. The optimum moisture content and the corresponding dry density of this soil were determined in the laboratory and were found to be 22.5% and 1.66 gm/cc respectively. However the natural moisture content and bulk density of the soil were 9% and 1.78 gm/cc respectively find out the quantity of soil to be excavated and the quantity of water to be added to it. For every 100m³ of finished embankment.

Sol. The embankment should be constructed by compacting the soil obtained from borrow area at the optimum moisture content and the corresponding maximum dry density. but the natural moisture content of the existing soil is less then its OMC. Hence a certain amount of water is to be added to the soil before the compaction.



For embankment Data given $(\gamma_d)_{max} = 1.66$gm/cc

$$OMC = w = 22.5\%$$

$$(\gamma_d)_{max} = \left(\frac{W_s}{V}\right) = 1.66 \text{ gm/cc}$$

$$= 1.66 \text{ t/m}^3$$

$$\Rightarrow \quad W_s = (1.66 \times 100)$$

$$= 166 \text{ tonn} \quad \text{... (i)}$$

$$\text{The weight of water } W_w = wW_s$$

$$= (0.225 \times 166)$$

$$= 37.35 \text{ tonn.}$$

For borrow pit area

$$\gamma_t = \text{bulk density} = 1.78 \text{ gm/cc} = 1.78 \text{ t/m}^3$$

$$w = 9\%$$

Therefore,
$$\gamma_t = 1.78 = \frac{Ws(1+w)}{(V_b)}$$

$$\Rightarrow \quad V_b = \frac{166(1+0.09)}{1.78}$$

$$= 101.65 \text{ m}^3 \quad \text{... (ii)}$$

$\therefore$ Weight of water available from this soil

$$W_w = (W_s \times w)$$
$$= (166 \times 0.09)$$
$$= 14.94 \text{ tonn.}$$

Therefore quantity of water to be added

$$= (37.35 - 14.94)$$
$$= 22.41 \text{ tonn.}$$
$$\gamma_w = 1 \text{ gm/cc}$$
$$= 10^{-6} t / cc = (10^{-6} \times 1000) \text{ t/lit}$$
$$= 10^{-3} \text{ t/lit}$$

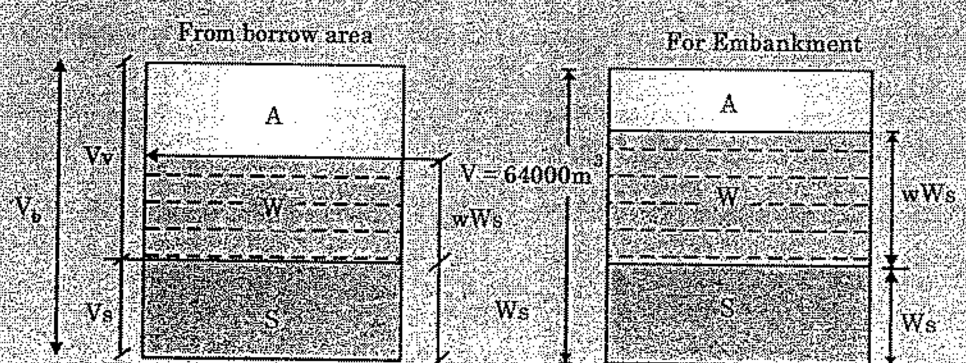
$\therefore$ Volume of water to be added $= \dfrac{\text{Weight of water}}{\text{Density of water}}$

$$= \frac{22.41}{10^{-3}}$$
$$= 22430 \text{ lit} \quad \text{...(iii)}$$

Example 14.

It is required to construct an embankment having a total volume of 64000 m³. The required soil is to be collected from borrow pits. It was found that the existing soil has a moisture content of 14% void ratio of 0.63 and sperific gravity of solids of 2.68. Laboratory test indicate that the OMC and maximum dry density of the soil are 19.5% and 1.72 gm/cc respectively. The soil is to be carried from the borrow pit to the construction site by trucks having average net carrying capacity of 5.5 tonn. Determine the total no. of trips the truck have to make for constructing the entire embankment. Also find out the quantity of water to be added to the borrowed soil before compaction.

Sol.



$w = 14\%$
$e = 0.63$
$G = 2.68$

$OMC = w = 19.5\%$
$(\gamma_d)_{max} = 1.72$ gm/cc

Note: While constructing the embankment this soil has to be compacted at moisture content of 19.5% and at a dry density of 1.72 t/m³

For Embankment

$$\text{OMC} = w = 19.5\%$$

$$(\gamma_d)_{max} = 1.72 \text{ gm/cc}$$
$$= 1.72 \text{ t/m}^3$$

$$(\gamma_d)_{max} = 1.72 = \left(\frac{W_s}{V}\right)$$

$$\Rightarrow \quad 1.72 = \frac{W_s}{(64000)}$$

$$W_s = 110080 \text{ tonn} \quad \text{...(i)}$$

$$\text{The req wt. of water} = W_w = (wW_s)$$
$$= (0.195 \times 110080)$$
$$= 21465.6 \text{ tonn}$$

For Borrow pit area

In situ dry density of the soil

$$\gamma_d = \left(\frac{G\gamma_w}{1+e}\right)$$
$$= \frac{2.68 \times 1}{(1+0.63)} = 1.64 \text{ t/m}^3$$

In situ bulk unit weight

$$\gamma_t = \gamma_d (1 + w)$$
$$= 1.64 \times (1 + 0.14) = 1.87 \text{ t/m}^3$$

$\therefore$ Total volume of excavation to be made $V_b = \left(\frac{W_s}{\gamma_d}\right)$

$$= \left(\frac{110080}{1.64}\right)$$
$$= 67122 \text{ m}^3$$

$\therefore$ Gross weight of this soil $W = (\gamma_t \times V)$

$$= 1.87 \times 67122$$
$$= 125518 \text{ tonn.}$$

$$\text{No. of tripe to be made} = \left(\frac{125518}{505}\right)$$
$$= 22821.48$$
$$= 22822 \text{ trip} \quad \text{... (ii)}$$

$\therefore$ Wt. of water available $W_w = W_s w$

$$= (0.14 \times 110080)$$
$$= 15411.2 \text{ tonn.} \quad \text{... (iii)}$$

$\therefore$ Quantity of water to be added before compaction

$$= (21465.6 - 15411.2)$$
$$= 6054 \text{ tonn}$$

OBJECTIVE TYPE QUESTIONS

1. For conducting a Standard Proctor Compaction Test, the weight of hammer (P in kg), the fall of hammer (Q in mm), the number of blows per layer (R) and the number of layers (S) required are respectively

	P	Q	R	S
(a)	5.89	550	50	3
(b)	4.89	450	25	3
(c)	3.60	310	35	4
(d)	2.60	310	25	3

2. Sheep-foot rollers are recommended for compacting
(a) granular soils
(b) cohesive soils
(c) hard rock
(d) any type of soil

3. In a standard proctor test, 1.8 kg of moist soil was filling the mould (volume = 944 cc) after compaction. A soil sample weighing 23 g was taken from the mould and ovendried for 24 hours at a temperature of 110°C. Weight of the dry sample was found to be 20 g. Specific gravity of soil solids is G = 2.7. The theoretical maximum value of the dry unit weight of the soil at that water content is equal to
(a) 4.67 kN/m^3
(b) 11.5 kN/m^3
(c) 16.26 kN/m^3
(d) 18.85 kN/m^3

4. Match **List-I** (Roller type) with **List-II** (Soil type) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Pneumatic roller	1. Cohesive and granular soils
B. Smooth wheeled roller	2. Plastic soils of moderate cohesion
C. Sheep foot roller	3. Cohesionless soils
D. Vibratory roller	4. Silty soils of low plasticity

Codes:

	A	B	C	D
(a)	4	2	1	3
(b)	3	1	2	4
(c)	4	1	2	3
(d)	3	2	1	4

5. In a compaction test if the compacting effort is increased, it will result in
(a) increase in maximum dry density and OMC
(b) increase in maximum dry density but OMC remains unchanged
(c) increase in maximum dry density and decrease in OMC
(d) no change in maximum dry density but decrease in OMC

below the lists:

List-I	List-II
A. Vibratory rollers	1. To compact soils in confined areas and at corners
B. Sheep foot rollers	2. To compact road and railway embank-ments of sandy soils
C. Frog hammers	3. To density sandy soils over a large area and to a larger depth
D. Vibrofloats	4. To compact clayey soil fills

Codes:

	A	B	C	D
(a)	4	2	1	3
(b)	4	2	3	1
(c)	2	4	1	3
(d)	2	4	3	1

7. Soil is compacted at which one of the following when a higher compactive effort produces highest increase in dry density?
   (a) Optimum water content
   (b) Dry side of the optimum moisture content
   (c) Wet side of the optimum moisture
   (d) Saturation moisture content

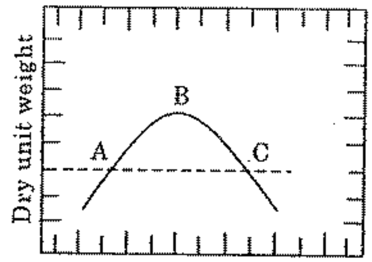
8. The following soils are compacted at the same compactive effort in the field. Which one of the following is the correct sequence in the increasing order of their maximum dry density?
   (a) Silty clay – Clay – Sand – Gravel sand clay mixture
   (b) Sand – Gravel sand clay mixture – Silty clay – Clay
   (c) Clay – Silty clay – Sand – Gravel sand clay mixture
   (d) Sand – Gravel sand clay mixture – Clay – Silty clay

9. In laboratory compaction tests, the optimum moisture content of soil decreases
   (a) with increase of compaction energy and with decrease of coarse grains in the soil
   (b) with decrease of compaction energy and with increase of coarse grains in the soil
   (c) with increase of both compaction energy and coarse grains in the soil
   (d) with decrease of both compaction energy and coarse grains in the soil

10. Consider the following:
    1. Increase in shear strength and bearing capacity
    2. Increase in slope stability
    3. Decrease in settlement of soil
    4. Decrease in permeability

    Which of the above with respect to compaction of soil is/are correct?
    (a) 1 only (b) 1 and 2 only
    (c) 2 and 3 only (d) 1, 2, 3 and 4

11. Points A, B and C correspond to three compaction states of a silty soil on the compaction curve given below.



Which one of the following is correct in respect of permeability of the soil for states A, B and C?

(a) A > B, B < C
(b) A < B, B > C
(c) A = C > B
(d) A < B = C

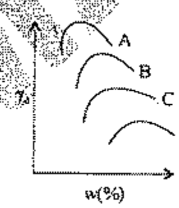
12. At what value of saturation does the zero air voids curve in a compaction test represent the dry density?

(a) 0%
(b) 80%
(c) 100%
(d) 50%

13. Compaction of an embankment is carried out in 500 mm thick layers. The rammer used for compaction has a foot area of 0.05 m^2 and the energy imparted in every drop of rammer is 400 N-m. Assuming 50% more energy in each pass over the compacted area due to overlap, the number of passes required to develop compactive energy equivalent to Indian Standard light compaction for each layer would be

(a) 10
(b) 16
(c) 20
(d) 26

14. The results (curves A, B, C and D) of four compaction tests on different soils are shown in the graph.



Tests:

1. Silty sand, modified test
2. Silty sand, standard test
3. Fat clay, modified test
4. Fat clay, standard test

Curves A, B, C and D correspond respectively to tests

(a) 1, 3, 2 and 4
(b) 1, 2, 3 and 4
(c) 2, 1, 3 and 4
(d) 2, 1, 4 and 3

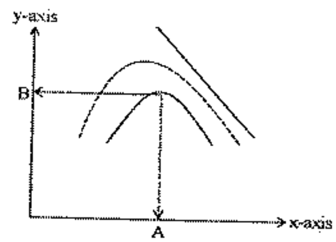
15. Given below are methods of compaction:

1. Vibration technique
2. Flooding the soil
3. Sheeps foot roller
4. Tandem roller
5. Heavy weights dropped from a height

The methods suitable for cohesionless soils include

(a) 1, 2 and 3
(b) 2, 3 and 4
(c) 1, 2 and 5
(d) 3, 4 and 5

16. The standard compaction curve obtained from a laboratory test is shown in the figure:



The dotted compaction curve of the same soil (shown in the figure) will be obtained if

(a) compactive effort is decreased
(b) moisture content is reduced with same compactive effort
(c) moisture content is increased with same compactive effort
(d) compactive effort is increased

17. Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

List-I	List-II
A. Optimum moisture content	1. Compaction of cohesive soil
B. Vibratory rollers	2. Compaction of granular soil
C. Zero air void line	3. Maximum dry density
	4. Relative density
	5. 100% saturation

Codes:

	A	B	C
(a)	4	1	3
(b)	3	2	5
(c)	4	1	5
(d)	3	2	4

18. Match **List-I** (Type of soils) **List-II** (Compaction parameters) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Sand	1. OMC = 18%, gdry max = 17 kN/m3
B. Sandy clay	2. OMC = 14%, gdry max = 18.9 kN/m3
C. Silty clay	3. OMC = 15%, gdry max = 17.4 kN/m3
D. Heavy clay	4. OMC = 10%, gdry max = 20.5 kN/m3

Codes:

	A	B	C	D
(a)	2	4	3	1
(b)	2	4	1	3
(c)	4	2	3	1
(d)	4	2	1	3

19. Consider the following methods:
1. Core cutter method
2. Sand replacement method
3. Proctor's needle method
4. Field vane shear method

Which of these methods enable control of field compaction?

(a) 1, 2 and 3
(b) 1, 2 and 4
(c) 1, 3 and 4
(d) 2, 3 and 4

20. Vibratory rollers are suitable for compacting

(a) organic soil
(b) clays
(c) sands and gravels
(d) clayey silts

21. Why are sheep foot rollers more effective in compacting clayey soils?

(a) There is differential expulsion of water under the roller
(b) Contact pressure is high
(c) Roller speed is high
(d) Drum width is large

22. Consider the following statements:
1. When compacted dry of optimum, a cohesive soil will get a flocculated structure.
2. When compacted dry of optimum, a cohesive soil becomes more sensitive.

Which of these statements is/are correct?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

Instructions :

*The following items consists of two statements, one labelled as **'Assertion A'** and the other labelled as **'Reason R'**. You are to examine these two statements carefully and decide if the Assertion A and the Reason R are individually true and if so, whether the Reason is a correct explanation of the Assertion. Select your answers to the these items using the codes given below :*

(a) both A and R are true and R is the correct explanation of A
(b) both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

23. Assertion (A): For a given soil, the optimum moisture content increases with the increase in compactive effort.

Reason (R): Higher the compactive effort, higher is the dry density at the same moisture content.

24. Assertion (A): Optimum moisture content obtained from Proctor's compaction test represents the water content at which the soil is fully saturated.

Reason (R): Presence of water facilitates rearrangement of soil grains under given compactive effort, thereby reducing the voids in between the soil grains.

ANSWERS

1. (d)	2. (b)	3. (d)	4. (c)	5. (c)	6. (c)	7. (b)	8. (c)	9. (c)	10. (d)
11. (a)	12. (c)	13. (d)	14. (b)	15. (c)	16. (d)	17. (b)	18. (c)	19. (a)	20. (c)
21. (b)	22. (a)	23. (d)	24. (d)						

Hints

3. $\gamma_{bulk} = 1.8$ kg $\quad V = 944$ cc

$$w = \frac{wt. of\ water}{wt. of\ solid} = \frac{23-20}{20} = \frac{3}{20} = 0.15$$

$$\gamma_t = \frac{G+S.e}{1+e}\gamma w$$

$$\gamma_{theo\ max} = \frac{G+(1+w)}{1+w.G.}\gamma_w \qquad S.e. = w.G$$

$$= \frac{2.7(1+.15)}{1+0.15\times 2.7}\times 9.81$$

$$= \frac{2.7\times 1.15}{1.405}\times 9.81 = 21.679$$

$$\gamma_d = \frac{\gamma_t}{1+w} = \frac{21.679}{1.15} = 18.85 \text{ kN}$$

CHAPTER

5

Effective Stress, Capillarity and Permeability

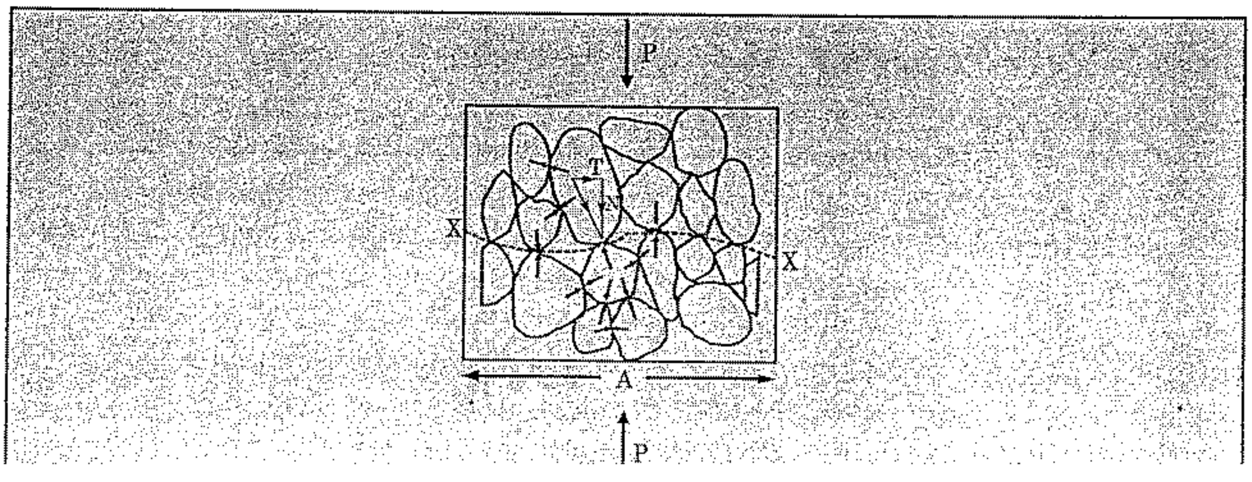
INTRODUCTION

- Water may influence the nature of the mineral surface chemically and consequently affect the bonding forces between adjacent soil grain. This kind of interaction between the soil solid and water is called **chemical interaction.**
- The other kind of interaction is a **physical interaction between solid and water.**
- Volume of the soil skeleton as a whole can change due to rearrangement of soil particles into new positions mainly by **rolling and sliding** due to forces acting between particle. This physical interaction is studied when we study the **effective stress concept.**
- Effective stress concept was developed by Terzaghi.
- Effective stress concept applies to a fully saturated soil and relates three types of stress.
  - (i) Total stress
  - (ii) Neutral stress (Pore Pressure)
  - (iii) Effective Stress.

TOTAL STRESS, PORE WATER PRESSURE AND EFFECTIVE STRESS

Total stress

- Total stress (σ) on a plane with in a soil mass is the force per unit area of soil mass transmitted in normal direction across a plane.



$\therefore$ Total stress (σ) $= \dfrac{P}{A}$

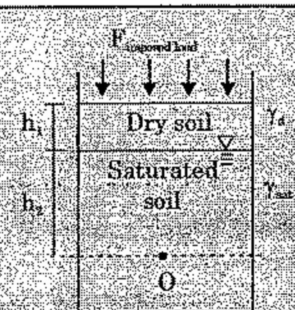
where,

P = Force on plane X–X for weight above plane X–X.

A = Area of c/s of soil mass.

- When there is imposed load on soil mass as shown below, the total stres value at point O is given by

$$\text{Total stress at 'O'} = \frac{F + \gamma_d . A . h_1 + \gamma_{sat} . A h_2}{A}$$

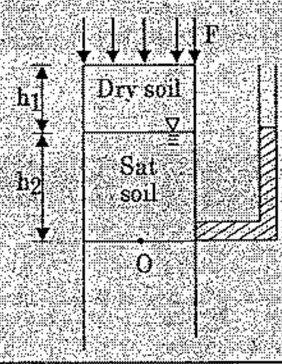


$$\sigma = \frac{F}{A} + \gamma_d h_1 + \gamma_{sat} h_2$$

- Total stress is a physical parameter which can be measured by suitable arrangement, such as by **pressure cell**.

Pore water pressure (u)

- It is the Pressure of water filling the void space between solid particles.

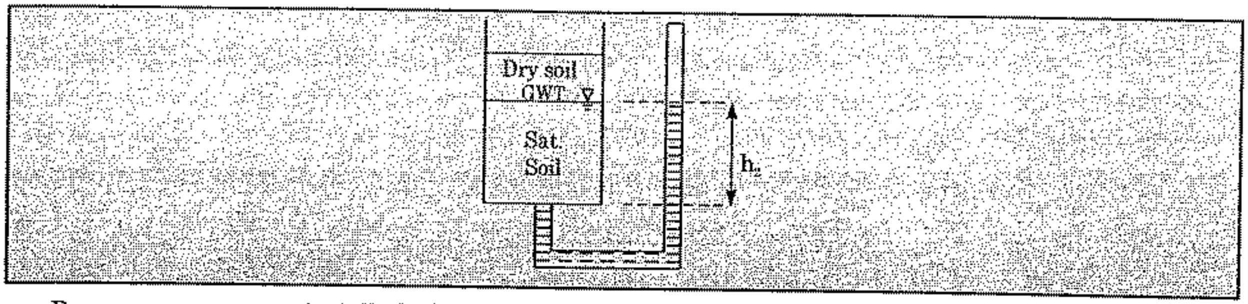


$\boxed{u = \gamma_w h_2}$ (A case when force 'F' has been acting on soil since long ago)

- Pore water pressure is also called as neutral stress because is acts on all sides of the particles, but does not cause particles to press against adjacent particle. It has no shear component.
- Pore water pressure at any point can be measured by inserting a stand pipe at the point under investigation and observing the height upto which the water rises in the stand pipe. (Thus it is a measurable quantity).
- Pore water pressure is measured using a Peizometer or a stand pipe.

EFFECT OF EXTERNAL LOAD ON PORE WATER PRESSURE

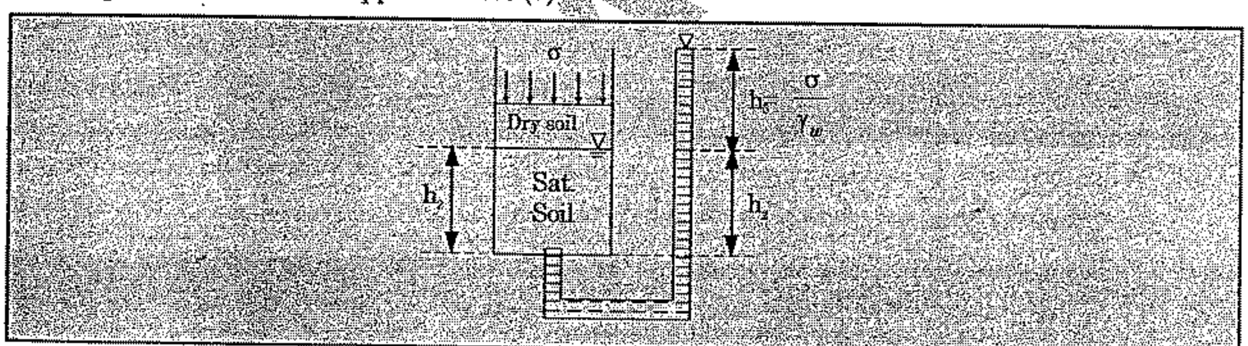
Case I : Pore water pressure before any loading is applied



- Pore water pressure intially before the application of loading is constant at value governed by the position of water table ($\gamma_w h_2$). This intial pressure is called **static pore water pressure.** As shown in the figure above the water level rises in the stand pipe upto the same level as the Ground Water Table.

Case II : Pore water Pressure immeadiately after the application of external stress.

- As the external load (total stress) is applied, soil particles immediately tries to occupy a new position closer together.
- But as water and soil are incompressible and if soil is **laterally confined**, this rearrangement will not be possible without expulsion of pore water.
- Thus as water is resisting the particle rearangement, Pore Water Pressure is increased above the static pore water pressure.
- At t = 0, i.e. just after application of load all the external load is taken up by pore water. This pore water pressure above static value is called **excess pore water pressure,** whose magnitude is equal to the external applied stress (σ)



Total Pore water pressure = Static Pore water pressure + Excess Pore water pressure

$$u = \gamma_w h_2 + \sigma$$

or
$$u = (h_2 + h_3)\gamma_w$$

Where,
$$h_3 = \frac{\sigma}{\gamma_w}$$

- Initially no part of the stress increment is taken up by the soil particles. This is possible for a fully saturated soil which is laterally confined such that volume changes occur entirely due to deformation of soil in vertical direction. (This condition will practically be met when thickness of soil layer is small compared to its area)
- However, if lateral strain is possible than from the starting itself, some part of the stress increment is taken up by the soil particles an hence, initial excess pore water pressure will be lesser than the increment of external load.

Case III : Long time after application of external load.

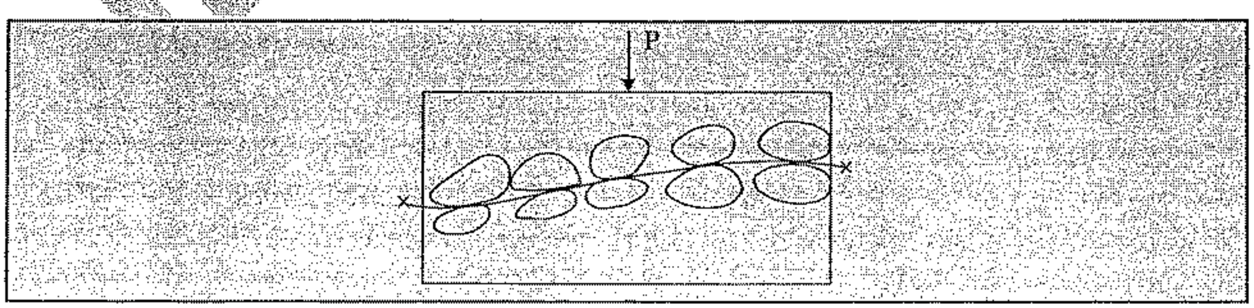
- Increase in Pore water pressure above static value causes pressure gradients and results in transient flow of pore water towards drainage face. This flow continues so long as the pore water pressure has not fallen down to ($h_2\gamma_w$) i.e hydrostatic pore pressure.
- Reduction of excess pore water pressure as drainage takes place is called **dissipation**.
- When dissipation is complete, soil is said to be in **drained condition**.
- Prior to dissipation, with the excess pore water pressure at its initial value, the soil is said to be in undrained condition.
- Drained condition means that there is no stress induced pressure in the pore water.
- Soil remains saturated throughout the process of dissipation.
- Thus as pore water pressure dissipates, external forces are transferred to the soil grains, resulting into rearrangement of soil grains and increase in effective stress.
- When all excess pore water has dissipated the effective stress increases by magnitude of stress increment.
- When soil is subjected to reduction in total normal stress, soil does not rebound to the enitial condition of reduced normal stress.
- This is because particle rearangement due to total stress increase is largely irreversible.
- If the soil is having clay, there can be a significant expansion. As a result, pore water pressure will be reduced and excess pore water pressure will become negative.
- Pore water pressure will gradually increase to its static value, flow taking place into the soil, effective normal stress decreases and volume increases. This process is called **swelling**.

EFFECTIVE STRESS

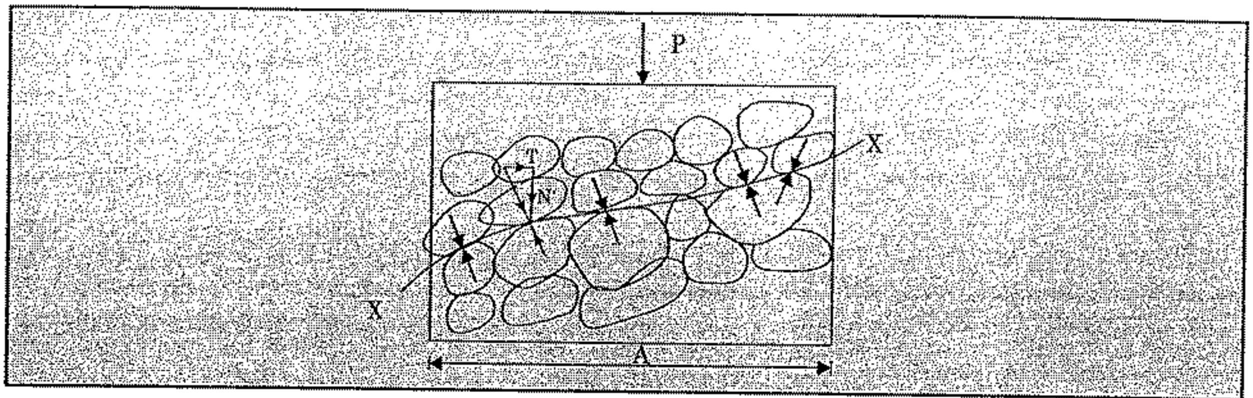
- Effective stress is defined as equal to the total stress (σ) minus the netural stress (or pore water pressure (u)). It is represented by symbol ($\bar{\sigma}$).

$$\bar{\sigma} = \sigma - u$$

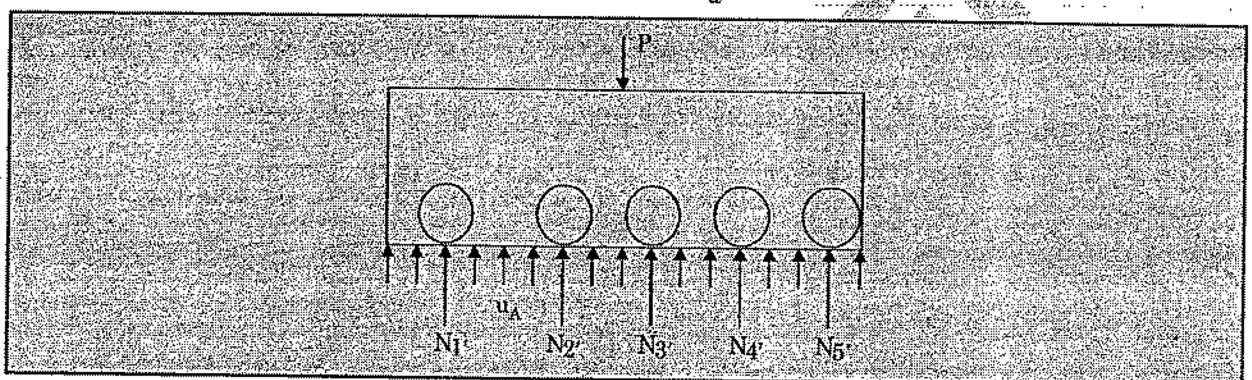
- Consider a fully saturated soil mass. Let us take a wavy plane X – X passing through the points of contacts of soil particles in the soil mass.
- For the sake of ease in analysis, wavy plane XX can be considered as a horizontal plane. Because the size of particles is very small and on macroscopic scale, this wavy plane can not be distinguished from a horizontal plane.



- Under the application of external load, a inter particle force gets developed between the two soil grains, at every point of contact, at the same time pore water pressure builds up. Further, these interparticle forces can be resolved into components normal and tangential direction as shown in the figure below.



- The total normal force acting on the soil mass is combinely resisted by the normal componet of interparticle forces (e.g N'_1) acting at the point of contacts of soil grains, and Second force is equal to Pore water pressure multiplied by area of pores (A_w).



- Equilibrium Equation in the direction normal to the horizontal plane is given by

$$P = \Sigma N' + u.A_w$$

$$\frac{P}{A} = \frac{\Sigma N}{A} + u \cdot \frac{A_w}{A}$$

- As the area of contact of solids is very small in comparision to total area (Area of contact of solid being approximately 3% of total cross sectional area). We can assume that pore water pressure acts over entire area A.

$$\therefore \quad \frac{P}{A} = \frac{\Sigma N'}{A} + U$$

$$\therefore \quad \sigma = \bar{\sigma} + u$$

$$\Rightarrow \quad \bar{\sigma} = \text{effective stress} = \frac{\Sigma N'}{A}$$

Total stress = effective stress + Pore water pressure

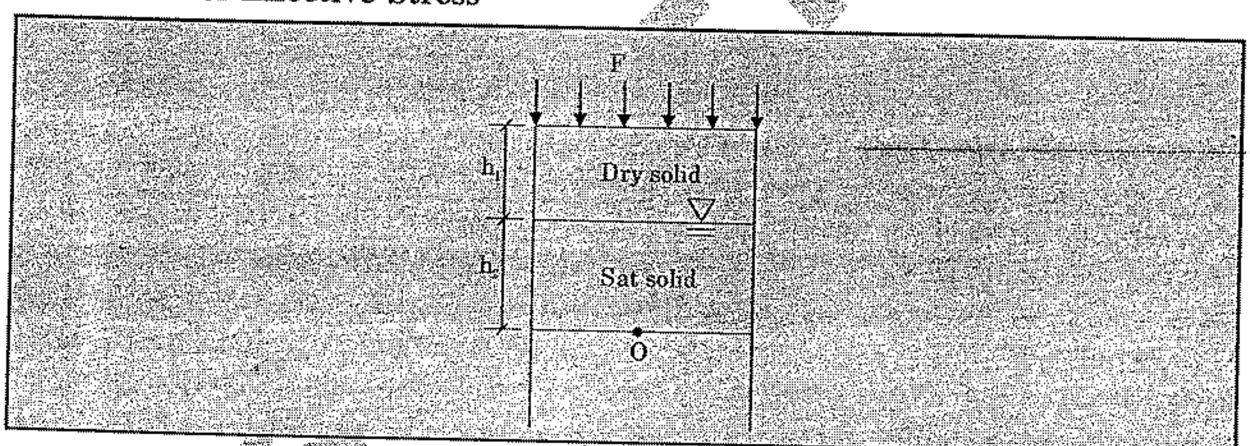
$$\therefore \quad \boxed{\bar{\sigma} = \sigma - u}$$

SILENT POINTS ABOUT EFFECTIVE STRESS

- It should be noted that some times effective stress is also called as intergranunal stress $\left(\frac{\text{Intergranular force}}{\text{Contact Area}}\right)$ which is actually not true.

- The actual contact stress or intergranular stress is very large, as the contact area between the particles is very small. Effective stress is an imaginary parameter which is the sum of the contact forces divided by the **gross area**. This is the reason why effective stress can not be measured, hence it is not a physical parameter.
- Effective stress is a function of normal force acting at the contact points of grains and pore water pressure.
- Effective stress can only be computed by subtracting pore water pressure (or neutral stress) from Total stress, both of which are physical parameters.
- Even though Effective stress is not a measurable quantity, it is a vey important parameter in Soil mechanics, because Effective stress is a parameter on which **compressibility, consolidation, settlement, sbear stress and bearing capacity** depends. These parameters do not depend on total stress directly.
- In the case of clay mineral particles in a soil mass, mineral crystals are not in direct contact, since they are surrounded by adsorbed layers of water. But it is observed that inter-granular forces can be transmitted through the adsorbed layer water.
- Tbe principal of effective stress is valid for coarse grained soils and clayey soils both.

Calculation of Effective Stress



$$\bar{\sigma} = \frac{F}{A} + \gamma_d h_1 + \gamma_{sat} h_2 - \gamma_w h_2$$

$$\boxed{\bar{\sigma} = \frac{F}{A} + \gamma_d h_1 + \gamma_{sub} h_2}$$

Where,

γ_{sub} = submerged unit weight of soils

CAPILLARITY IN SOILS

Ground water can exists in two forms

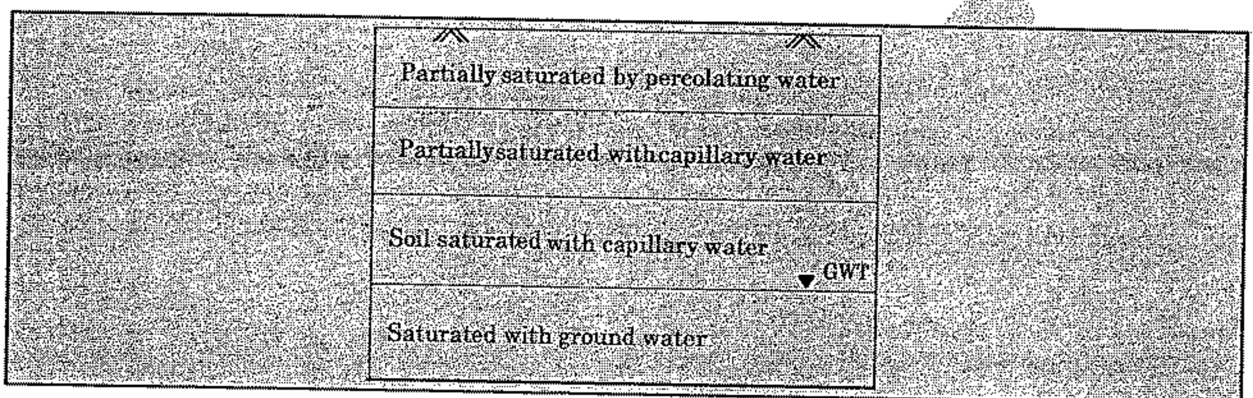
(i) **Pheratic or Gravitational water –**

- This water is subjected to gravitational forces, and saturates the voids completely. Commonly it is represented by Ground water table.
- At ground water table the pressure is atmospheric and below which pressure is in hydrostatic condition.

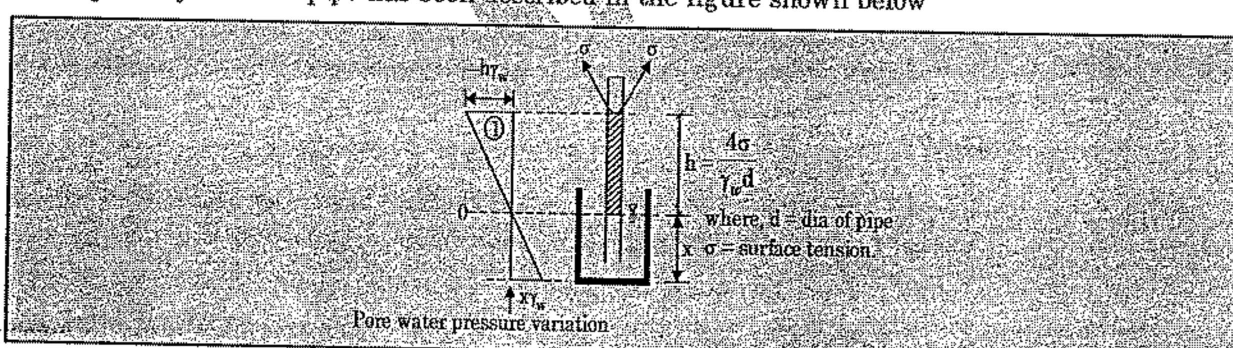
(ii) **Capillary water :**

- If the water contained in the pores of soil was only subjected to gravitational forces then soil above ground water table would have been perfectly dry.
- But in reality it is observed that soil is fully saturated upto certain height above water table and partially saturated upto some more height. This is because of capillary phenomenon in soils.
- Capillary water is held above the water table by Surface tension.
- Pressure in the capillary Zone is negative (–ve) and it does not contribute to hydrostatic pressure below Ground water table.

Presence of water in various zones of soil is depicted as below



- Due to the effect of capillary rise there is increase in unit weight of the soil upto the height of capillary rise (owing to presence of water in void spaces)
- Capillarity rise in a pipe has been described in the figure shown below



- If soil pores are treated as capillary tube, the capillary rise in it can be predicted on the basis of capillary tube analogy.
- Normally σ = 73 m N/m for pure water in clean glass tube and $\gamma_w = 9.8$ kN/m^3

Hence, for pure glass tube rise in water is given by

$$\therefore \quad \boxed{h = \frac{0.03}{d(mm)}}$$

Where,

d = dia of the tube in mm.

h = capillary rise in meter.

- It has been shown that if we take 'd' as 0.2 D_{10}, we get a good result for capillary rise in sands and silts.

$$\boxed{h = \frac{0.03}{0.2D_{10}}}$$

Where,

D_{10} = Effective size of the particle in mm.

- Other emperical formula used for capillary rise is as given below

$$\boxed{h_{C(cm)} = \frac{C}{eD_{10}}}$$

where

e = void ratio

D_{10} = effective size of particle in cm.

C = emperical constant having value between 0.1 – 0.5 cm^2

- Capillary rise is a function of **pore size.**
- Two soils having same D_{10} can have different capillary size depending on soil structure and fabric geological history.
- As due to capillary rise, pressure of water is (–) ve (i.e., pore water pressure is (–)ve,) effective stress $\bar{\sigma} = \sigma - u$ will become

$$\boxed{\bar{\sigma} = \sigma - (-u_C) = \sigma + u_C}$$

Where,

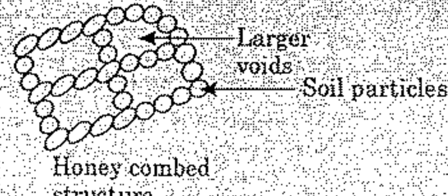
$u_C = (-)h\gamma_w$

h = height of capillary rise above water table.

σ = Total stress

$\bar{\sigma}$ = Effective stress

- Thus due to capillary rise, effective stress in capillary zone increases.
- Due to increase in effective stress, shear strength of soil also increases.
- Negative pressure of water (capillary pressure) held above water table results in attractive forces between particles it is called as **soil suction.**
- Capillary moisture in fine sand and silt allows unsupported excavation to be made because of stability it provides by virtue of induced shear strength.
- Bulking of sand also occurs because of capillary. Capillarity producess apparant cohesion which holds the particles in clusters, enclosing honey combs.



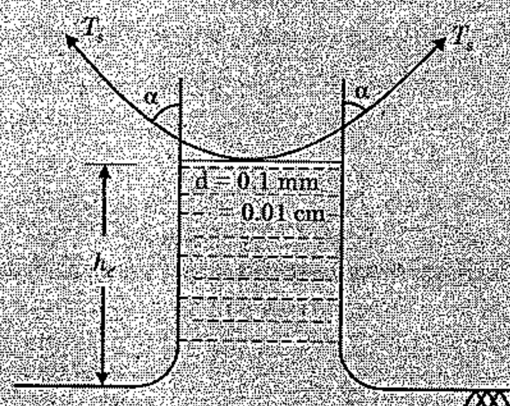
- As moisture content increases (*i.e.* saturation occurs) menisci are broken and appreciable volume

Example 1

A capillary glass tube of 0.1 mm internal diameter is immersed vertically in a beaker full of water. Assuming the tube to be perfectly clean and wet, determine the height of the capillary rise of water in the tube when the room t_{temp} is 20°C. Given at 20°C unit weight of water = 0.9980 gm/cc and surface tension = 72.8 degree/cm.

What is meant by degree?

Sol.



For equilibrium

$$(T_s \cos\alpha) \times \pi d = \left(\frac{\pi}{4} d^2 h_c \times \gamma_w\right) g$$

$$\Rightarrow \qquad h_c = \frac{4T_s \cos\alpha}{(d\,\gamma_w g)}$$

Assuming the tube to be perfectly clean and wet, $\cos\alpha = 1$

$$= \frac{4 \times 72.8}{(0.998 \times 0.05 \times 981)}$$

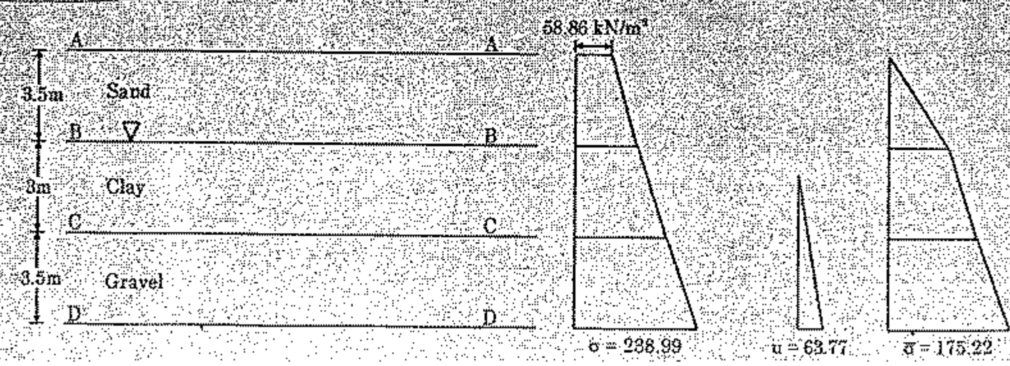
$$[h_c = 29.74 \text{ cm}]$$

Example 2

?? Won't this condition affect the solution

A soil profile consists of a surface layer of sand 3.5 m thick ($\rho = 1.65$ Mg/m³), an intermediate layer of clay 3 m thick ($\rho = 1.95$ Mg/m³) and the bottom layer of gravel 3.5 m thick ($\rho = 1.925$ Mg/m³). The water table is at the upper surface of the clay layer. Determine the effective pressure at various levels immediately after placement of a surcharge load of 58.86 kN/m² to the ground surface.

Sol. :



Section A — A $\sigma = 4 \times 19.5 + 2 \times 18.5 = 115.0 \text{ kN/m}^2$

$u = 7 \times 10.0 = 70.0 \text{ kN/m}^2,\ \bar{\sigma} = 58.86 \text{ kN/m}^2$

Section B — B $\sigma = 58.86 + 3.5\,(1.65 \times 9.81) = 115.51$

$u = 0,\ \bar{\sigma} = 115.51 \text{ kN/m}^2$

Section C — C $\sigma = 115.51 + 3 \times (1.95 \times 9.81) = 172.90$

$u = 3 \times 9.81 = 29.43,\ \bar{\sigma} = 172.90 - 29.43 = 143.47 \text{ kN/m}^2$

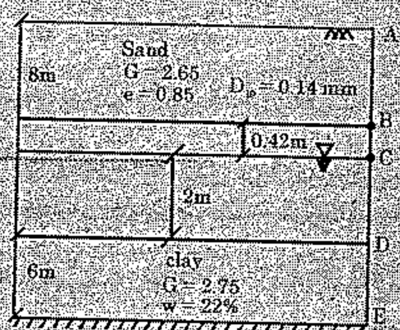
Section D — D $\sigma = 172.90 + 3.5\,(1.925 \times 9.81) = 238.99$

$u = 6.5 \times 9.81 = 63.77,\ \bar{\sigma} = 238.99 - 63.77 = 175.22 \text{ kN/m}^2$ 0

Example 3 → verify the solution

At a site the subsoil consists of a 8 m thick layer of dry sand ($G = 2.65$, $e = 0.85$, $D_{10} = 0.14$ mm) Which is under lain by a 6 m thick clay layer ($G = 2.75$, $w = 22\%$) below which there Exists a thick layer of hardpan. The W.T is located at a depth of 6m below the G.L plot the distribution of total, neutral and Effective stress.

Sol.



Soil Profile

Height of capillary rise in the sand layer

$$h_c = \frac{C}{(eD_{10}} = \frac{0.5}{0.85 \times (.014)} \text{ (Assuming C = 0.5 cm}^2\text{)}$$

42.0207

$$= \cancel{42.1} \text{ cm} = 0.42 \text{ m.}$$

Hence the sand will be saturated upto 0.42m above the water table for the sand layer,

$$\gamma_{sat} = \frac{(G+e)}{(1+e)} = \frac{(2.65+0.85)\times 1.0}{(1+0.85)}$$

$$= 1.89 \text{ t/m}^3$$

$$\gamma_d = \left(\frac{G\gamma_w}{1+e}\right) = \frac{2.65 \times 1.0}{(1+0.85)} = 1.43 \text{ t/m}^3$$

As the clay layer submerged below wate, it is saturated

Therefore, we have $eS = wG$

$$\Rightarrow \quad e = \frac{0.22 \times 2.75}{(1.0)} = 0.605$$

$$\gamma_{sat} = \frac{(2.75+0.605)\times 1}{(1+0.605)} = 2.09 \text{ t/m}^3$$

At Z = 0, the total neutral and effective stress are all equal to zero.

At B (Z = 5.58m),

$$\sigma = \text{Total stress}$$
$$= \gamma_d h_1 = 1.43 \times 5.58$$
$$= 7.98 \text{ t/m}^2$$

$$\text{Neutral stress } u = -h_c\gamma_w$$
$$= -0.42 \times 1 = -0.42 \text{ t/m}^2$$

$$\text{Effective stress } \sigma' = (\sigma - u)$$
$$= 7.98 - (-0.42) = 8.40 \text{ t/m}^2 \quad \text{---- (i)}$$

At C (Z = 6m)

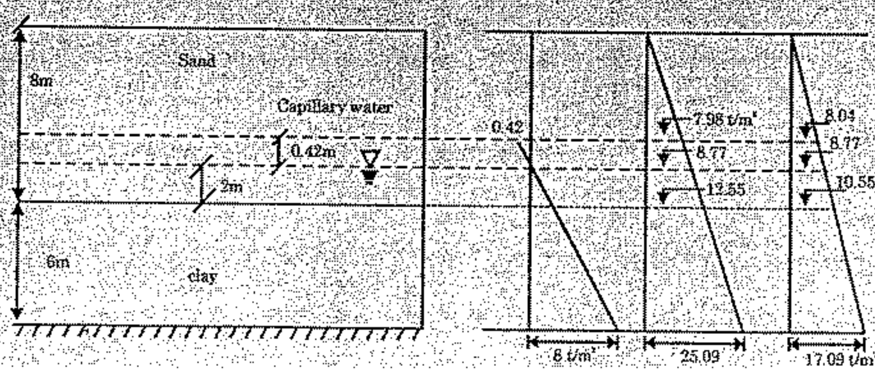
$$\sigma = (1.43 \times 5.58 + 1.89 \times 0.42)$$
$$= 8.77 \text{ t/m}^2$$
$$u = 0$$
$$\sigma' = (8.77 - 0) = 8.77 \text{ t/m}^2 \quad \text{---- (ii)}$$

At D (Z = 8m)

$$\sigma = (1.43 \times 5.58 + 1.89 \times 2.42)$$
$$u = h_w\gamma_w$$
$$= 2 \times 1 = 2 \text{ t/m}^2$$
$$\sigma' = (\sigma - u)$$
$$= 12.55 - 2 = 1055 \text{ t/m}^2 \quad \text{---- (iii)}$$

At E (Z = 14.0 m)

$$\sigma = (1.43 \times 5.58) + (1.89 \times 2.42) + (2.09 \times 6)$$
$$= 25.09 \text{ t/m}^2$$
$$u = h_w\gamma_w = 8 \times 1 = 8.0 \text{ t/m}^2$$
$$\sigma' = (\sigma - u) = (25.09 - 8.0) = 17.09 \text{ t/m}^2$$



Example 4

A layer of saturated clay 4 m thick is overlain by sand 5 *m* deep, the water table is 3 m below the surface.

$\gamma_{sat\ clay} = 19\ kN/m^3$, $\gamma_{sat\ sand} = 20\ kN/m^3$.

γ_t of sand above water table = 17 KN/m^3

(a) Plot the total stress, effective stress against depth.

(b) If sand to a height of 1 m above water table is saturated with capillary water, how are the above stress affected.

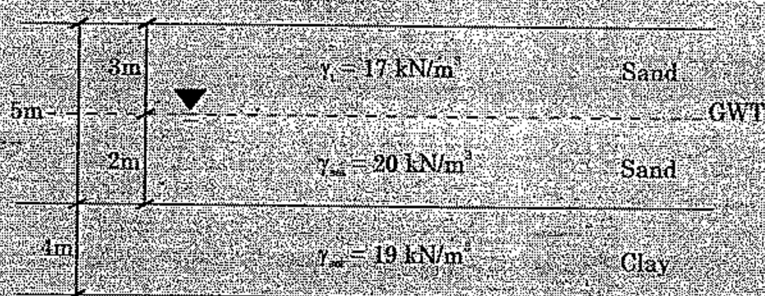
(c) If in 1st case a 4 m deep sand layer of $\gamma_t = 20\ kN/m^3$ is placed over the surface. Find the effective vertical stress at the centre of clay layer.

(i) Immediately after the fill has been placed.

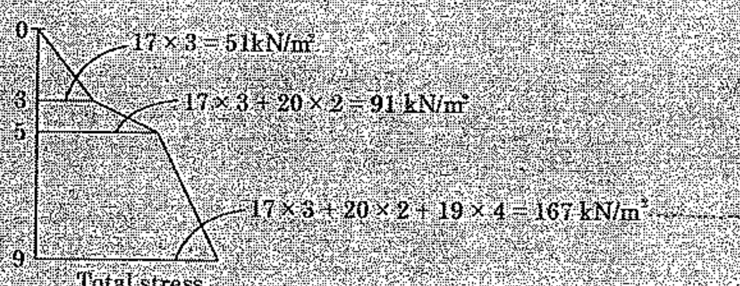
(ii) Many years after the fill has been placed.

Sol.

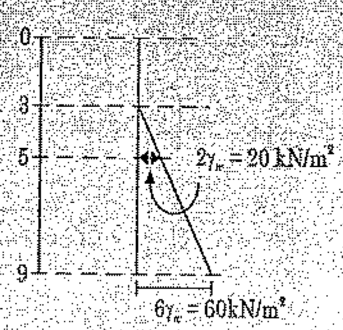
Case I. When 4 m of clay is below 5 m deep sand. Water table is at 3 m from ground.



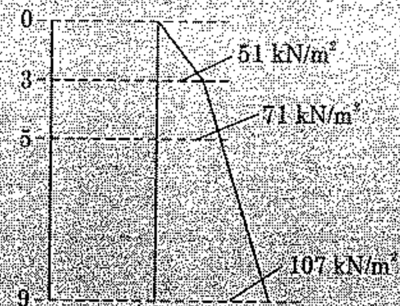
Total Stress



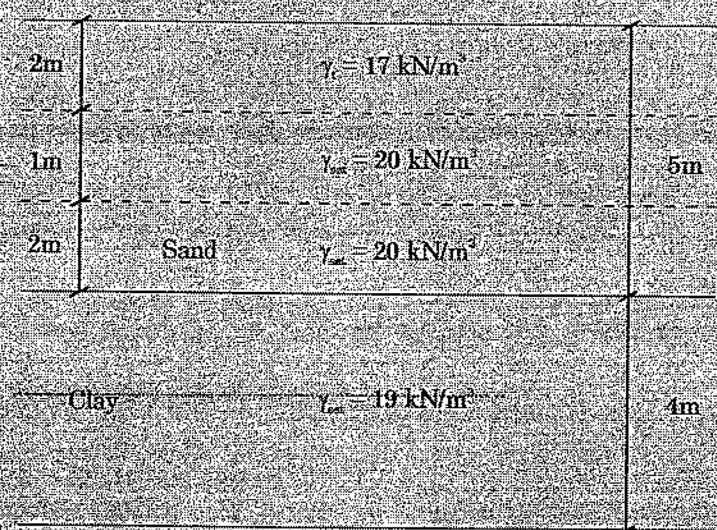
Pore water pressure



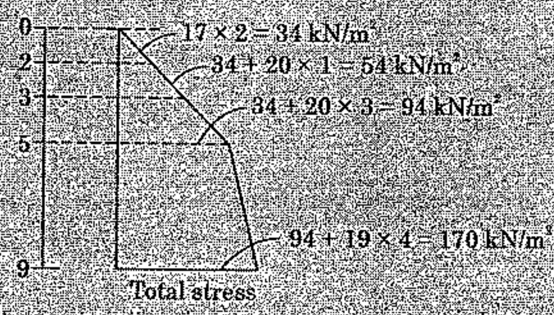
Effective stress



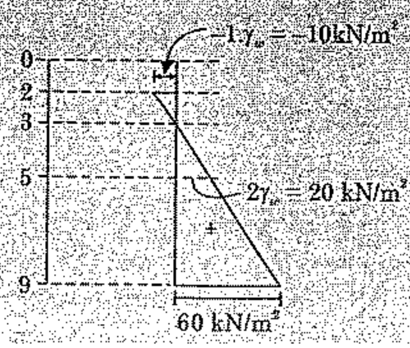
Case II. If there is capillary rise



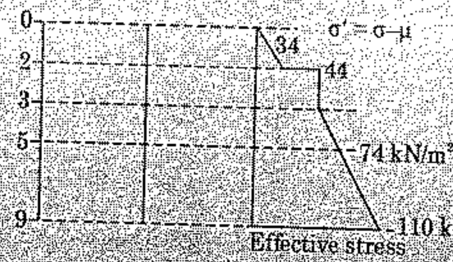
Total stress



Pore water pressure



Effective stress



Case III. 4 m of sand layer of $\gamma_t = 20$ kN/m³ placed over the 5m of sand. (Imediately after the Sand layer is placed)

- Immediately after application of vertical loading all of the stress is taken up by pore water. Hence pore water pressure incresses by $20 \times 4 = 80$ KN/m²

Immediately after application of vertical loading total stress increased by 80 KN/m²

$\Rightarrow$ Effective stress increase, $80 - 80 = 0$

Hence effective stress at 7 m below original surface = original value

$$= 17 \times 3 + 20 \times 2 + 19 \times 2 - 4\gamma_w$$
$$= 51 + 40 + 38 - 40$$
$$= 89 \text{ KN.m}^2$$

Case IV. Many years after the 4 m of sand layer is placed.

Long time after the load is placed, the pore water pressure will dissipate completely and all the load will be carried by the soil grains hence pore water pressure will corresponds to original water table position.

Total stress will get increase by 80 kN/m² (i.e. $\gamma_t \times 4$).

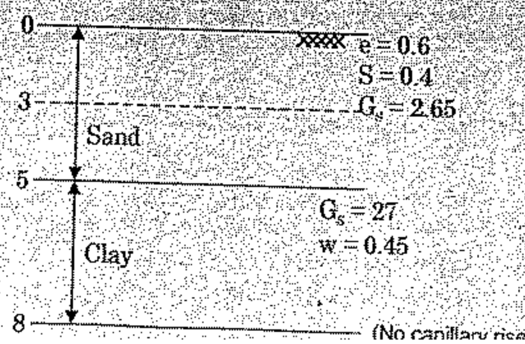
$\therefore$ Net effective stress at the center of clay will be

$\Rightarrow$ Effective stress $= 51 + 40 + 38 + 80 - 4\gamma_w = 89 + 80 = 169$ KN/m²

Example 5

1. Find total stress, pore water pressure and effective stress at various depth.
2. When water table rises upto ground water find change in effective stress at 8 m
3. Find effective stress at 8 m if water table lowered by 2 m.
4. Find effective stress at 8 m if water table rises 1 m above ground level.

take $\gamma_w = 10$KN/m³



Sol. Bulk unit weight of sand above water table

$$= \frac{G+Se}{1+e}\gamma_w$$

$$= \frac{2.65+0.4\times0.6}{1.6}\times10$$

$$= 18.06 \text{ kN/m}^3$$

Bulk unit weight of sand saturated

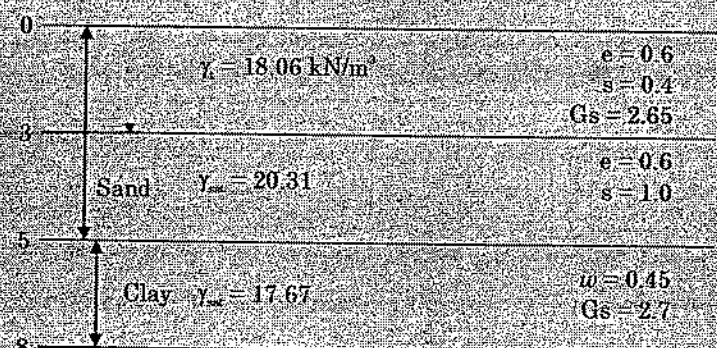
$$= \frac{G+e}{1+e}\gamma_w = \frac{2.65+0.6}{1.6}\gamma_w$$

$$= 20.31 \text{ kN/m}^3$$

Bulk unit weight of clay $= \frac{G(1+w)}{1+wG}\gamma_w = \frac{2.7\times1.45}{1+0.45\times2.7}\gamma_w$

$$= 17.67 \text{ kN/m}^3$$

Case I



at elevation 0 m from top

$$\sigma = 0$$

$$u = 0$$

$$\bar{\sigma} = \sigma - u = 0$$

at elevation 3 m from top

$$\sigma = 3\times\gamma_t = 3\times18.06 = 54.18 \text{ kN/m}^2$$

$$u = 0\times\gamma_w = 0$$

$$\bar{\sigma} = \sigma - u = 54.18 \text{ kN/m}^2$$

At elevation 5m from ground level.

$$\sigma = 3\times\gamma_{t\ sand} + 2\times\gamma_{sat\ sand}$$

$$= 3\times18.06 + 2\times20.31 = 94.80 \text{ kN/m}^2$$

$$u = 2\times\gamma_w = 20 \text{ kN/m}^2$$

$$\bar{\sigma} = \sigma - u = 94.80 - 20 = 74.80 \text{ kN/m}^2$$

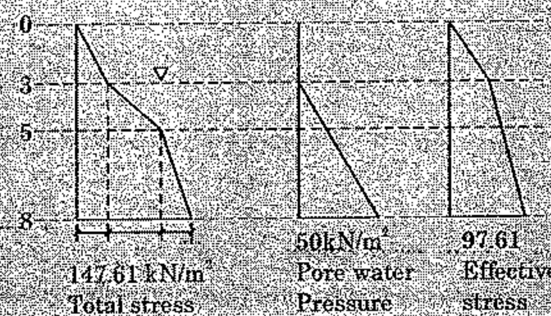
at elevation 8m from ground level.

$$\sigma = 3 \times \gamma_{t\ sand} + 2 \times \gamma_{sat\ sand} + 3 \times \gamma_{sat\ clay}$$

$$= 3 \times 18.06 + 2 \times 20.31 + 3 \times 17.67 = 147.81 \text{ kN/m}^2$$

$$u = 5 \times \gamma_w = 5 \times 10 = 50 \text{ kN/m}^2$$

$$\bar{\sigma} = \sigma - u = 147.81 - 50 = \mathbf{97.81 \text{ kN/m}^2}$$



Case II When water table rises up to Ground below

Total stress, Pore water pressure & effective stress at 8 m below ground.

$$\sigma = \gamma_{sat\ sand} \times 5 + \gamma_{sat\ sand} \times 3$$

$$u = 20.31 \times 5 + 17.67 \times 3 = 154.56$$

$$u = 8 \times \gamma_w = 8 \times 10 = 80\ \gamma_w$$

$$\bar{\sigma} = \sigma - \mu = 154.56 - 80 = 74.56 \text{ kN/m}^2$$

Due to rise in water table upto ground level we observed that total stress and pore water pressure both got increased but there was net fall in effective stress of 23.25 kN/m²

Case III If water table is lowered by 2 m.

Total stress Pore water pressure and Effective stress at 8 m below ground level.

$$\sigma = 3 \times \gamma_{t\ sand} + 2 \times \gamma_{sat\ sand} + 3 \times \gamma_{sat\ clay}$$

$$= 3 \times 18.06 + 2 \times 20.31 + 3 \times 17.67 = 147.81 \text{ kN/m}^2$$

$$u = 4 \times \gamma_w = 4 \times 10 = 40 \text{ kN/m}^2$$

$$\bar{\sigma} = \sigma - u = 147.81 - 40 = 107.81 \text{ kN/m}^2$$

Here we observed that as water table is lowered total stress remained unchanged but pore water pressure got decreased. Effective stress got increased by 20 kN/m² (by same amount pore pressure got decrease.)

Case IV When water table rises 1 m above ground level.

Total stress, Pore water pressure and effective stress 8 m below ground level.

$$\sigma = 1 \times \gamma_w + 3 \times \gamma_{t\ sand} + 2 \times \gamma_{sat\ sand} + 3 \times \gamma_{sat\ clay}$$

$$= 1 \times 10 + 3 \times 18.06 + 2 \times 20.31 + 3 \times 17.67 = 157.81 \text{ kN/m}^2$$

$$u = 9 \times \gamma_w = 9 \times 10 = 90 \text{ kN/m}^2$$

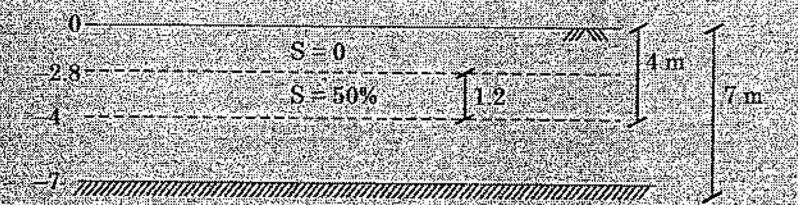
$$\bar{\sigma} = \bar{\sigma} - u = 157.81 - 90 = 67.81 \text{ kN/m}^2$$

We can observed that as water table rises above ground level there is a equal amount of increase in total stress and pore water pressure.

Example 6

A granular soil deposit is 7 m deep over an impermeable layer. The ground water table is 4 m below the ground surface. The deposit has a zone of capillary rise of 1.2 m with a saturation of 50%. Plot variation of total stress pore water pressure and effective stress with the depth of deposit, e = 0.6 & Gs = 2.65.

Sol.



$$\gamma_d = \frac{G}{1+e}\gamma_w = \frac{2.65}{1+0.6} \times 9.81 = 16.24\,\text{kN}/m^2$$

$$\gamma_{sat} = \frac{G+e}{1+e}\gamma_w = \frac{2.65+0.6}{1+0.6} \times 9.81 = 19.93\ \text{kN/m}^2$$

$$\gamma_t = \frac{G+Se}{1+e}\gamma_w = \frac{2.65+0.5\times0.6}{1+0.6} \times 9.81 = 18.09\ \text{kN/M}^2$$

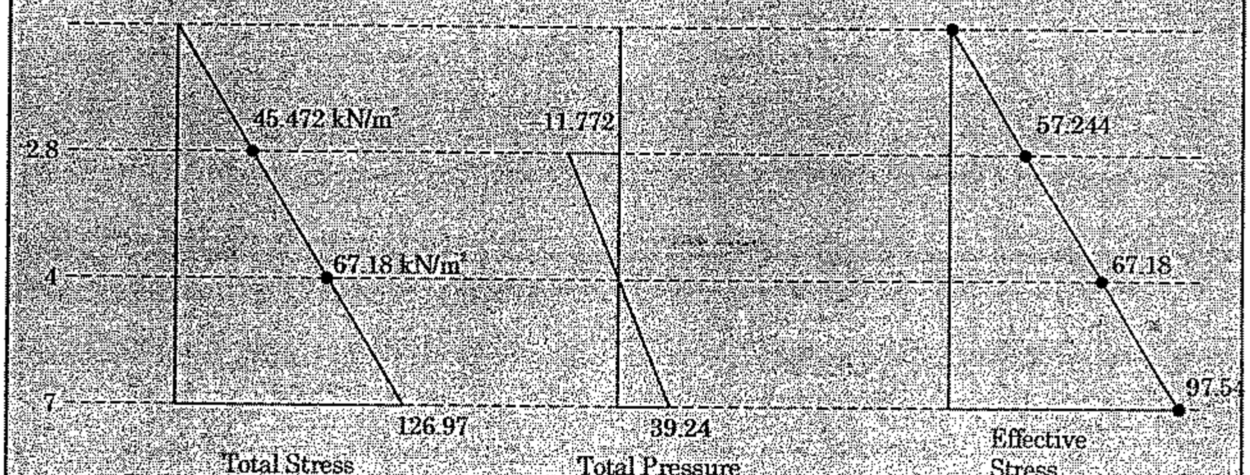
Total Stress

$$\sigma_{0\text{-}0} = 0$$

$$\sigma_{-2.8} = 2.8 \times \gamma_d = 2.8 \times 16.24 = 45.472\ \text{kN/m}^2$$

$$\sigma_{-4} = 2.8 \times \gamma_d = 1.2 \times \gamma_t = 2.8 \times 16.24 + 1.2 \times 18.09 = 67.18\ \text{kN/m}^2$$

$$\sigma_{-7} = 2.8 \times \gamma_d = 1.2 \times \gamma_t = 3 \times \gamma_{sat} = 2.8 \times 16.24 + 1.2 \times 18.09 + 3 \times 19.93 = 126.97\ \text{kN/m}^2$$



Pore water Pressure

$$u_0 = 0$$

$$u_{-2.8} = -1.2 \times \gamma_w = -1.2 \times 9.81 = -11.772\ \text{kN/m}^2$$

$$u_{-4} = 0$$

$$u_{-7} = 3 \times \gamma_w = 3 \times 9.81 = 29.43\ \text{kN/m}^2$$

Effective stress

$$\bar{\sigma}_0 = \sigma - u = 0 - 0 = 0$$

$$\bar{\sigma}_{-2.8} = \sigma - u = 45.472 - (-11.772) = 57.244 \text{ kN/m}^2$$

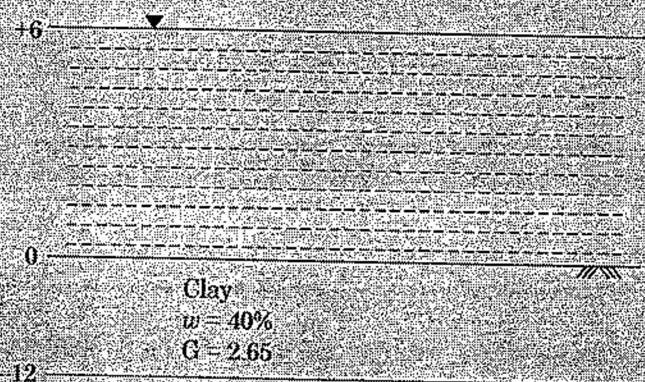
$$\bar{\sigma}_{-4} = \sigma - u = 67.18 - 0 = 67.18 \text{ kN/m}^2$$

$$\bar{\sigma}_{-7} = \sigma - u = 126.97 - 29.43 = 97.54 \text{ kN/m}^2$$

Example 7

Compute the total, effective and pore pressure at a depth of 12m below the bottom of lake 6 m deep. The bottom of lake consists of soft clay with a thickness of more than 15m. The average water content for the clay is 40% and specific gravity of soils may be assumed as 2.65. Assume that the lake is filled with water up to the top.

Sol.



$$\gamma_{sat} = \frac{G+e}{1+e} \times \gamma_w \qquad (S.e = w.G)$$

$$\gamma_{sat} = \frac{G+w.G}{1+w.G} \times \gamma_w = \frac{2.65+0.4\times 2.65}{1+0.4\times 2.65} = 17.66$$

Total stress at depth 12 m below bottom of lake

$$\sigma = 6 \times \gamma_w + 12 \times \gamma_{sat} = 6 \times 9.81 + 12 \times 17.66 = 270.78 \text{ kN/m}^2$$

Pore pressure at depth of 12 m below bottom of lake

$$= \sigma - u = 270.78 - 176.58 = 94.2 \text{ kN/m}^2$$

Example 8

A deposit of sand has porosity of 35% and G = 2.7. The soil is dry in top 1.5 m depth, it has 15% moisture content in next 1.8 m depth and it is submerged below it. Find:

(i) effective pressure at a depth of 8 m below the ground level.

(ii) change in effective pressure if the water table suddenly drops to a level of 6 m below the ground level.

(iii) shear strength of the soil on horizontal plane at 8 m depth for both the positions of ground water table if $\phi = 30°$.

Sol. Given G = 2.7, n = 35% w = 15%

$$e = \frac{n}{1-n} = \frac{0.35}{1-0.35} = 0.54$$

we have to calculate various densities of sand

(1) $\gamma_d = \frac{G}{1+e}\gamma_w = \frac{2.7}{1+0.54}\times 9.81 = 17.20 \text{ kN/m}^3$

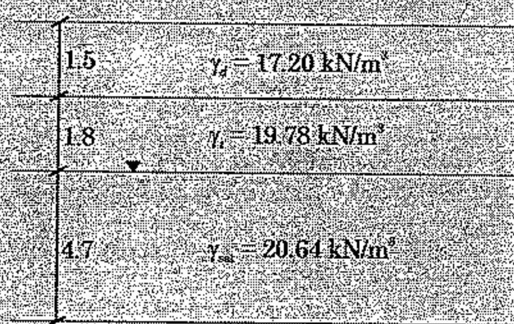
(2) γ_t with w = 15% $\Rightarrow \gamma_t = \frac{G(1+w)}{1+e}\gamma_w = \frac{2.7\times 1.15}{1.54}\times 9.81 = 19.78 \text{ kN/m}^3$

$$\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w$$

$$= \frac{2.7+0.54}{1.54}\times 9.81$$

$$= 20.64 \text{ kN/m}^3$$

Case I



Effective stress at the depth of 8 m = Total Stress – Pore water Pressure

Total stress = $\gamma_d \times 1.5 + \gamma_t \times 1.8 + \times \gamma_{sat}$ 4.7

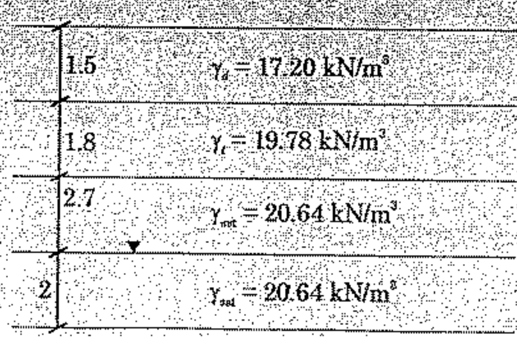
= 17.20 × 1.5 + 19.78 × 1.8 + 20.64 × 4.7

= 158.412 kN/m³

Pore Pressure = 4.7 × γ_w = 46.107 kN/M³

Effective Stress = $\sigma - u$ = 158.412 – 46.107 = 112.305 kN/m³

Case II



at elevation of 8 m from top

$$\sigma = 1.5\gamma_d + 1.8\gamma_t + 2.7\gamma_{sat} + 2 \times \gamma_{sat}$$

$$= 1.5 \times 17.20 + 1.8 \times 19.78 + 2.7 \times 20.64 + 20.64 \times 2$$

$$= 158.412 \text{ kN/m}^3$$

$$= 2\gamma_w = 2 \times 10 = 19.62$$

$$\bar{\sigma} = \sigma - u = 158.421 - 19.62 = 138.801 \text{ kN/m}^3$$

Hence we can observe that due to the fall in water table effective stress increased.

Case III

Shear strength of soil is given as

$$S = C + \bar{\sigma} \tan\theta$$

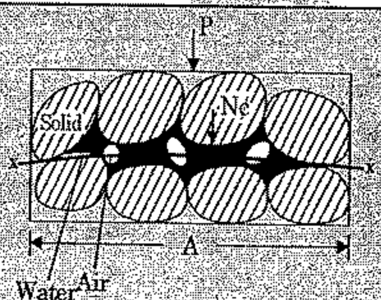
For sand $C = 0,\ \theta = 30°$ (given)

Case I $\Rightarrow \bar{\sigma} \tan 30° = 112.305 \tan = 64.839 \text{ kN/m}^2$

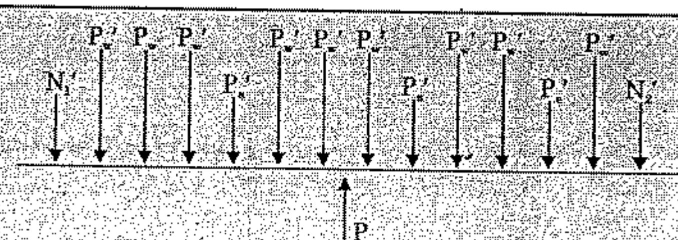
Case II $\Rightarrow \bar{\sigma} \tan 30° = 138.801 \tan 30 = 80.137 \text{ kN/m}^2$

EFFECTIVE STRESS IN PARTIALLY SATURATED SOIL

- In a partially saturated soil, soil mass is in a 3 phase system.



- Thus there are 3 measureable parameter in partially saturated soil system : total stress, pore air pressure and pore water pressure.
- Let us consider a wavy plane **x–x** as show in the diagram which passes through the point of contact of grains.
- Further as explained earlier wavy plane can be considered as a horizontal plane, and forces are resolved in the normal direction to this horizontal plane.



Where,

N' = grain to grain contact force

P_w' = force acting on plane due to pore water pressure.

P_a' = Force acting on plane due to pore air pressure.

- Assuming x is the fraction of total area A over which pore water pressure acts. Thus Pore air pressure acts over fraction $(1 - x)$ of total area A. Neglecting the area of point of contact as it is very small to the total area.

$$P = \Sigma N' + u_w x A + u_a(1 - x)A$$

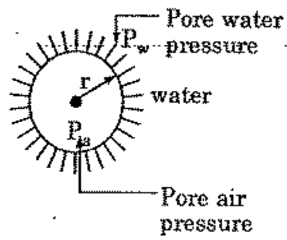
$$\frac{P}{A} = \frac{\Sigma N'}{A} + u_w x + u_a(1 - x)$$

$$\frac{P}{A} = \frac{\Sigma N'}{A} + u_a + (u_w - u_a)x$$

$$\sigma = \bar{\sigma} + u_a + (u_a - u_w)x$$

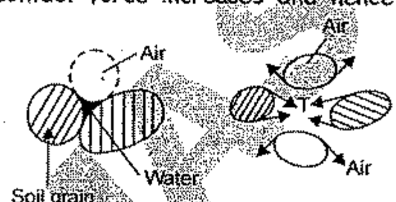
$$\bar{\sigma} = \sigma - u_a + (u_a - u_w)x$$

Note: Pore air pressure will be more than pore water pressure due to surface tension.



$$\boxed{\Delta P = P_a - P_w = \frac{2\sigma}{r}}$$

Due to tension force 'T' particle contact force increases and hence shear strength of soil increases.



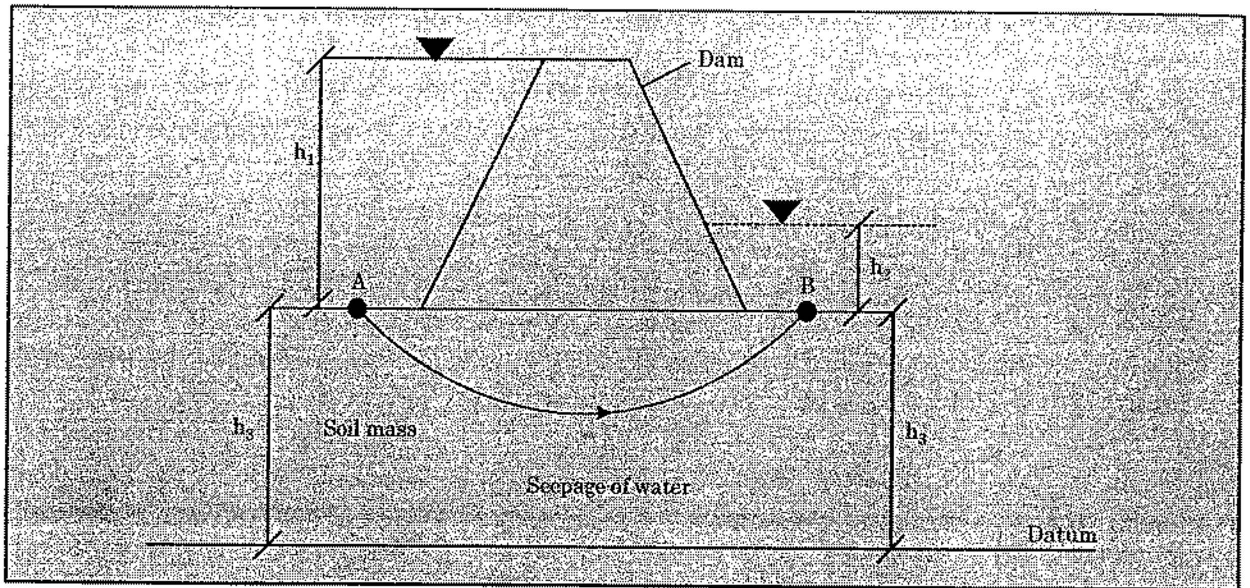
INFLUENCE OF SEEPAGE ON EFFECTIVE STRESS

- Before discussing the effect of seepage on effective stress, let us first of all discuss the various types of energy or head available for fluid flow in a medium.
- There are three types of head in fluid flow.

1. Velocity head $= \dfrac{V^2}{2g}$
2. Pressure head $= \dfrac{P}{\gamma_w}$
3. Datum head $= Z$

- Total head at any point = Velocity head + Pressure head + Datum head.
- We know that laminar flow occurs during the seepage in soil and velocity is very small during laminar flow. Hence the contribution of velocity head is almost negligible so we can drop this term from the total head equation.
- Therefore, for seepage analysis we can say that Total head = Pressure head + datum head.
- Pressure head is measured by a piezometer and the height of water column raised in a piezometer represents the pressure head. In other words Pressure head is pore water pressure per unit weight at any point. i.e $\left(\dfrac{u}{\gamma_w}\right)$

- Elevation head is the vertical distance between the point under consideration and the datum plane.



$$\therefore \quad \text{Total head at Point A} = \text{Pressure head + Elevation head.}$$

$$= h_1 + h_3$$

$$\text{Total head at Point B} = h_2 + h_3$$

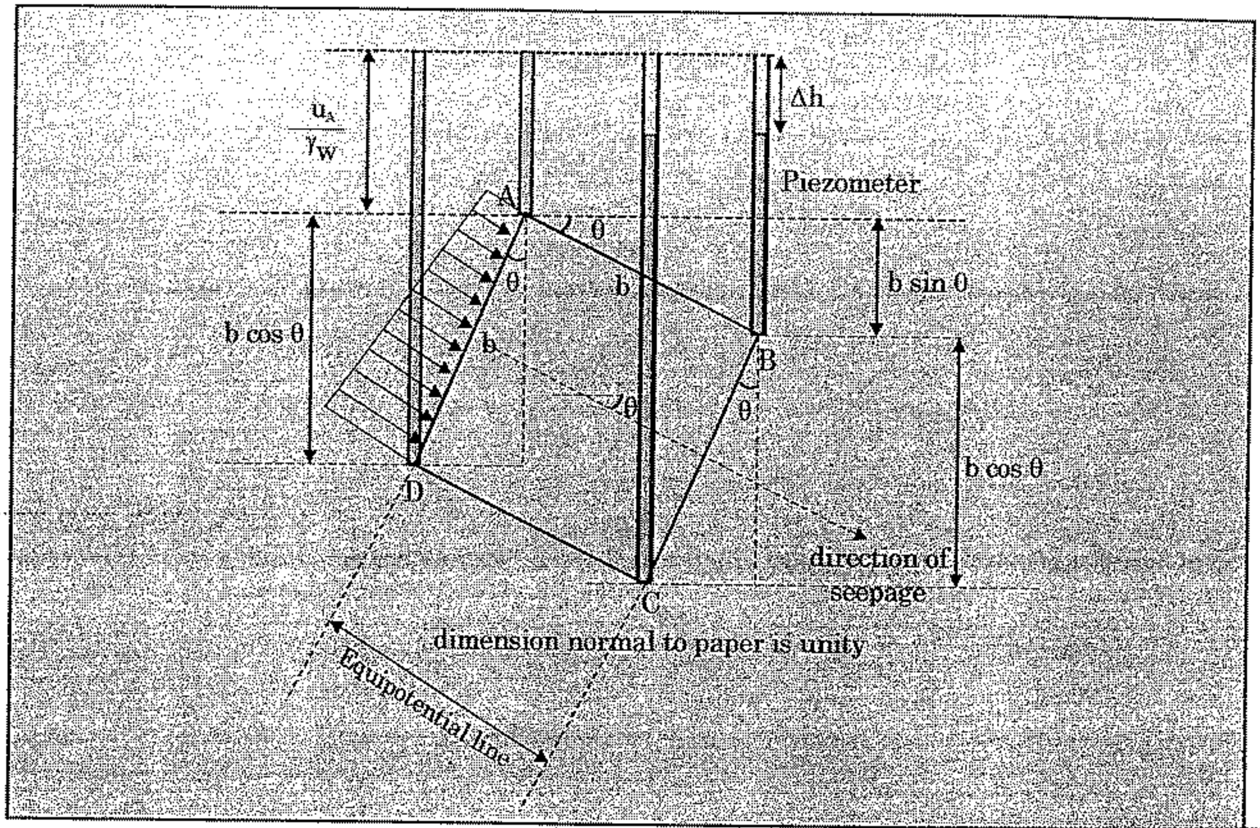
$$\text{Total head at A} - \text{Total head at B} = (h_1 + h_3) - (h_2 + h_3)$$

$$= h_1 - h_2$$

- This difference in total head between points A & B is termed as Head loss (h_L)
- Rate of change of Head loss w.r.t the distance between points of consideration is termed as **hydraulic gradient (*i*)**

$$\boxed{i = \frac{h_L}{L}}$$

- We all are aware with this fact that flow of water takes place from high head (high total head) to low head (low total head).
- When water is seeping through the soil, **total head is dissipated as viscous friction, producing a frictional drag in the direction of flow on the soil particles.**
- This drag force results in seepage force on soil mass which acts in the **direction of flow.**
- Component of this seepage force acting vertically upwards will therefore reduce vertical effective stress component from the static values.
- Component of this seepage force acting vertically downwards will increase the vertical effective stress component from the static value.
- To illustrate the calculation of seepage force and to make you conversent with the application of energy equation, following derivation has been done.
- Consider a soil mass ABCD square in shape of dimension **b**. Direction of seepage is such that it makes an angle θ with the horizontal (parallel to AB and normal to AD).



- Drop of Total head between AD and BC = Δh
- Let pore water pressure at A = u_A
- Difference of pressure between A and D is due to difference of elevation.
- Difference of pressure between A and B or A and C is due to difference of elevation head and due to loss of 'Total head'.

Writing energy equation between A and B

$$\frac{u_A}{\gamma_w} + 0 = \frac{u_B}{\gamma_w} - b\sin\theta - \Delta h$$

$$\Rightarrow \quad \boxed{u_B = u_A + \gamma_w(b\sin\theta - \Delta h)}$$

$$\frac{u_A}{\gamma_w} + 0 = \frac{u_c}{\gamma_w} - b\sin\theta - b\cos\theta + \Delta h$$

$$\Rightarrow \quad \boxed{u_c = u_A + \gamma_w(b\cos\theta + b\cos\theta - \Delta h)}$$

$$\frac{u_A}{\gamma_w} + 0 = \frac{u_D}{\gamma_w} - b\cos\theta$$

$$\Rightarrow \quad \boxed{u_D = u_A + \gamma_w b\cos\theta}$$

u_A, u_B, u_C & u_D are pore water pressure

$$\Rightarrow \quad u_B - u_A = \gamma_w (b\sin\theta - \Delta h)$$

$$u_C - u_D = \gamma_w (b\sin\theta - \Delta h)$$

$$u_D - u_A = \gamma_w b\cos\theta$$

$$u_C - u_B = \gamma_w b\cos\theta$$

Note: Note That difference of pore water pressure is the difference of Piezometric head

$\Rightarrow$ Net force on the element due to seepage only $= \gamma_w \Delta hb$ (in the direction of seepage)

$\Rightarrow$ seepage force $= \gamma_w \Delta hb$

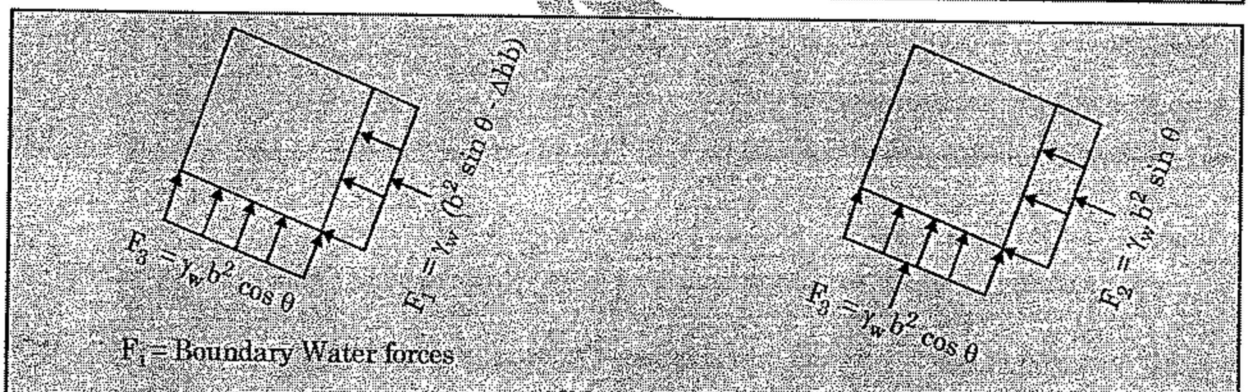
$$\frac{\text{Seepage force}}{\text{Volume}} = \frac{\gamma_w \Delta hb}{b \times b \times 1} = \gamma_w \frac{\Delta h}{b}$$

$$\frac{\Delta h}{b} = \text{hydraulic gardient } (i)$$

$$\boxed{\frac{\text{Seepage force}}{\text{Volume}} = i\gamma_w} \text{ in the direction of seepage.}$$

Note that hydraulic gradient is measured w.r. to total head.

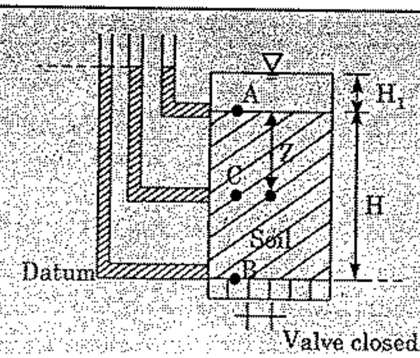
Resultant body force on soil mass = vector summation of effective weight + seepage force



Net force on element with seepage **Net force without seepage**

ANALYSIS OF SEEPAGE FORCE ON EFFECTIVE STRESS

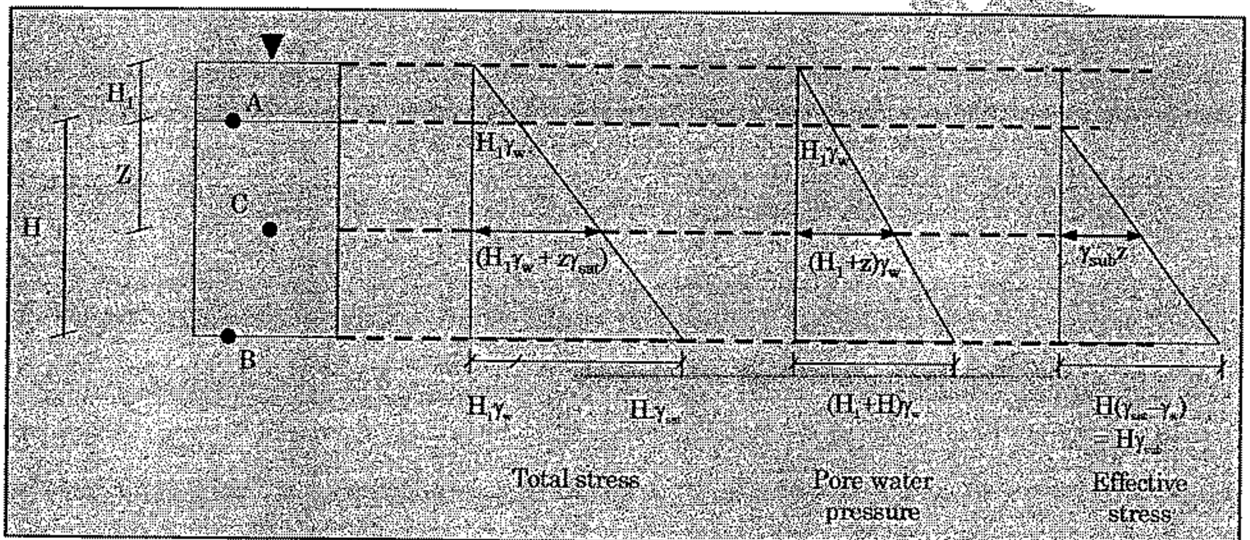
Hydrostatic Condition (No flow condition) :



- The above figure show a tank filled with submerged soil, with no seepage occuring because value of base is closed. Hence total head of water remains same at all point A, B and C

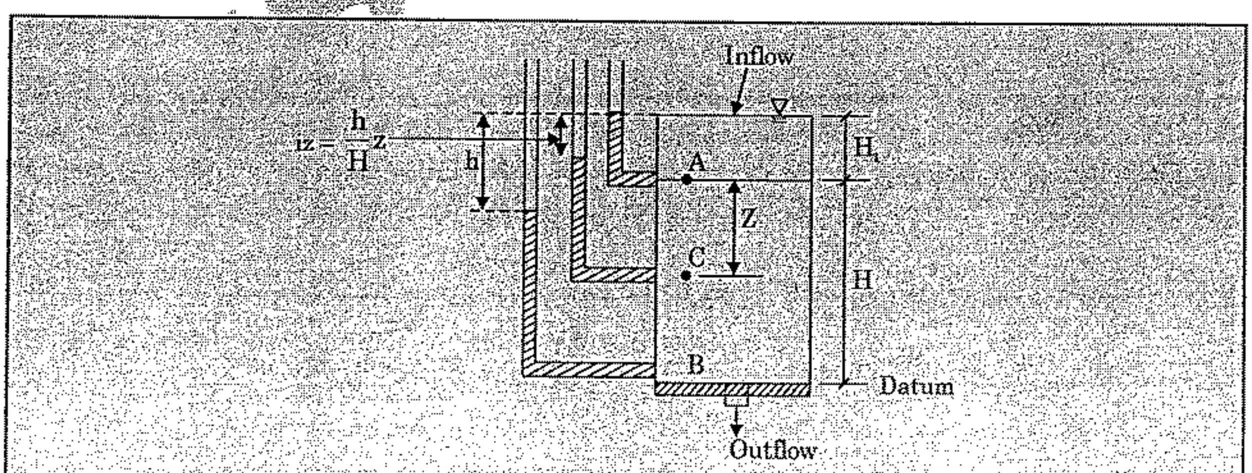
Points head	Pressure head	Elevation head	Total head
A	H_1	H	H_1+H
B	H_1+H	0	H_1+H
C	H_1+z	$H-z$	H_1+H

- The variation of total stress (σ), pore water pressure (u) and effective stress ($\bar{\sigma}$) with no flow conditions are plotted as shown below.



- Normally datum is taken at tail water level
- Note that effective stress ($H\gamma_{sub}$) does not change if water level is above the top level of soil mass.

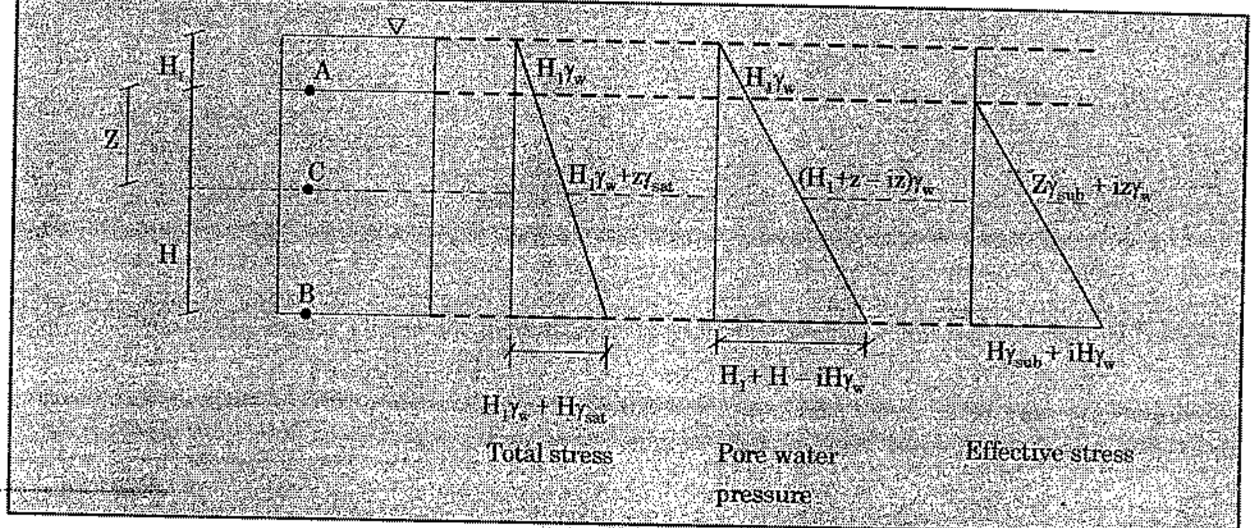
Downward flow condition :



- As show in the above figure, the valve at the bottom of the tank is open and downward seepage is allowed. A constant level of water in the soil is matained by adjusting water supply at top and outflow at bottom.

Points/Heads	PH	EH	TH
A	H_1	H	H_1+H
B	H_1+H-h	0	$(H_1+H)-h$
C	$Z+H_1\frac{-h}{H}z$	$H-z$	$(H_1+H)-\frac{h}{H}z$

- The variation of total stress (σ), pore water pressure (u) and effective stress ($\bar{\sigma}$) with downward flow condition is as shown below.



Where

i = hydraulic gradient

$$i = \frac{\text{head loss occured during seepage}}{\text{Length over which head loss occured}}$$

$$i = \frac{h}{H}$$

Note: Compared to no-flow condition, effective stress increase by $iz\gamma_w$.

$$\text{i.e.,}\quad \frac{\text{Seepage force}}{\text{Area}} = \frac{(i\gamma_w)\times z.A}{A} = iz\gamma_w$$

- It is observed that downward seepage increases the effective stress by $iz\gamma_w$ and which is equal to seepage pressure.

$$\text{Seepage pressure at C} = iz\gamma_w$$

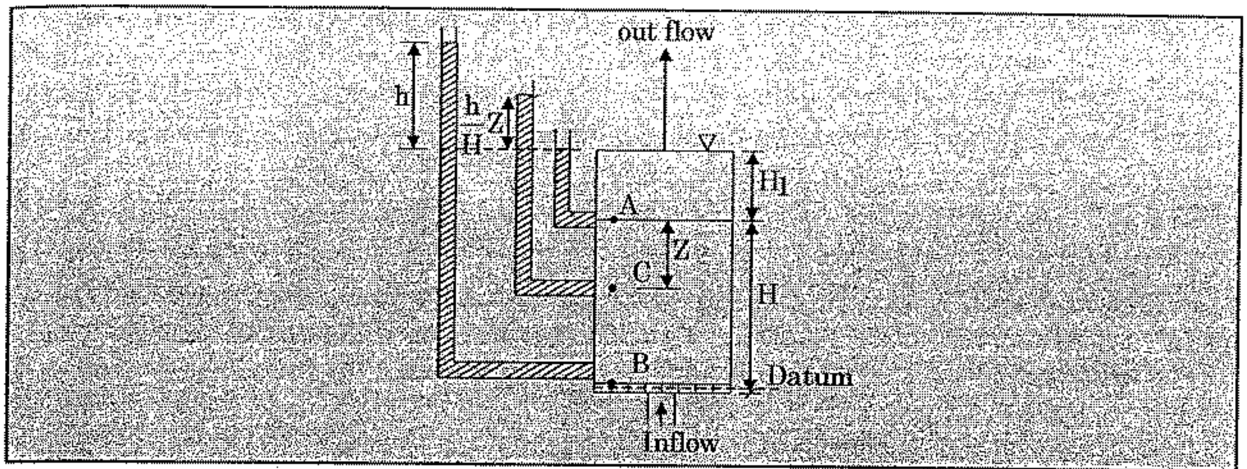
$$\text{Seepage pressure at B} = iH\gamma_w$$

$$\text{Effective stress at C} = \frac{zA\gamma_{sub} + (i\gamma_w)zA}{A}$$

$$= z\gamma_{sub} + iz\gamma_w$$

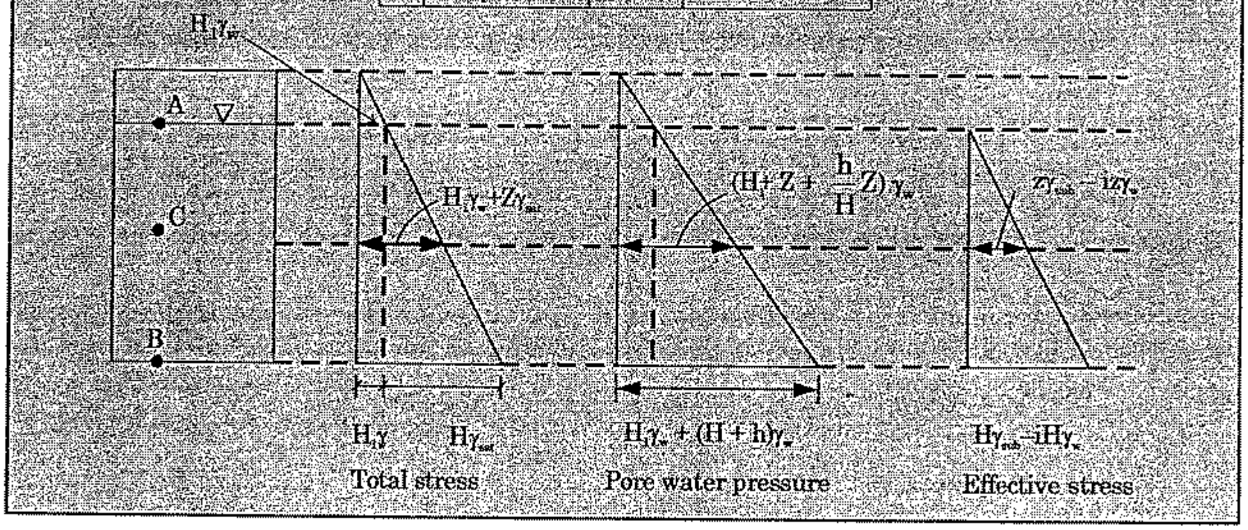
$$\text{For downward seepage, Effective stress} = \frac{\text{Bouyant weight of soil + seepage force}}{\text{Area}}$$

3. **Upward flow Condition**



- As shown in the above figure, valve at the bottom of tank is open and upward seepage is allowed. Water level at the top of tank is maintained constant by regulating. The supply of water at top and flow at bottom.
- The variation fo total stress (σ), Pore water pressure (u) and effective stress ($\bar{\sigma}$) with upward flow condition is as shown below.

	PH	EH	TH
A	H_1	H	$H_1 + H$
B	$H_1 + H + h$	0	$(H_1 + H) + (h)$
C	$H_1 + Z + \frac{hz}{H}$ $= H_1 + z + iz$	$H - Z$	$H_1 + H + iz$



Note: Compared to no flow condition, effective stress decreases by $iz\gamma_w$

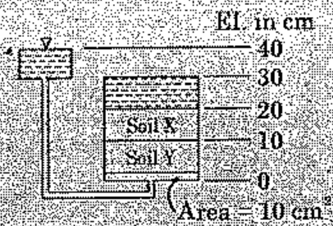
$$i.e., \quad \frac{\text{Seepage force}}{\text{Area}} = \frac{i\gamma_w Z.A}{A} = i\gamma_w.z$$

Hence it is observed that upward seepage decreases the effective stress by $iz\gamma_w$ and which is equal to seepage pressure.

$$\text{For upward seepage, Effective stress} = \frac{\text{Bouyant weight} - \text{seepage force}}{\text{Area}}$$

Example 9

If in Fig. soil X has a permeability of 4×10^{-3} cm/s and the head lost in soil Y is 9 times the head lost in soil X.



(a) What is the permeability of flow per hour?

(b) What is quantity of flow per hour?

(c) To what elevation would water rise in a piezometer inserted in soil Y at El. 5 cm? What is the pressure head at this point?

Sol.

$$h_{L(Y)} = 9h_{L(X)}$$

$\therefore$ Total head loss during flow = 10 cm (from Fig.)

$$\therefore \quad h_{L(Y)} + h_{L(X)} = 10$$

$$10\,h_L(x) = 10$$

or $\quad h_L(x) = 1$ cm

and $\quad h_{L(Y)} = 9$ cm

(a) For vertical flow, velocity of flow is constant in soil X and soil Y.

$$V_x = V_y = v$$

$$V_y = k_y\, i_y = k_x\, ix$$

$$k_y \times \frac{9}{10} = 4 \times 10^{-3} \times \frac{1}{10}$$

$$k_y = 4.4 \times 10^{-4} \text{ cm/s}$$

(b) $q = (k_x \times i_x \times A) = 4 \times 10^{-3} \times \frac{1}{10} \times 10 \text{ cc/s or } 14.4 \text{ cc/hr}$

(c) El. of water in peizometer at El. 0 cm in soil Y = 40 cm (El. 0 m is assumed as datum)

El. of water in piezometer inserted at El. 10 cm in soil Y = 40 − 9 = 31 cm (total head at El. 10 cm)

Elevation of water in piezometer inserted at El. 5 cm = 35.5 cm (total head at El. 5 cm)

$\therefore$ Pressure head at El. 5 cm = 35.5 − 5.0 = 30.5 cm

QUICK SAND CONDITION

- In case of upward seepage flow, if the upward seepage force becomes equal to the bouyant weight of soil the effective stress in the soil becomes zero.
- In other words by increasing the head difference h, it is possible to reach a condition where the effective stress in the soil becomes Zero.

$$\therefore \quad H\gamma_{sub} - i\,H\,\gamma_w = 0$$

$$\Rightarrow \quad H\gamma_{sub} = iH\gamma_w$$

$$\text{or} \quad i = \frac{\gamma_{sub}}{\gamma_w} = i_{cr}$$

- This hydraulic gradient is also called as Critical hydraulic gradient represented by a symbol i_{cr}.

$$\therefore \quad \boxed{\text{Critical hydraulic gradient} = i_{cr} = \frac{\gamma_{sub}}{\gamma_w}}$$

- When upward flow is taking place at critical hydraulic grading, a soil such as sand looses all its shearing strength because effective stress become zero.
- Effective stress zero in sand means contact force between grains becomes zero.
- This condition is called quick sand condition or boiling of sand because surface of sand looks as if its boiling.
- Note that quick sand is not a sand its a hydraulic condition.
- Effective stress throughout the soil bcomes zero.

  $z\gamma_{sub} - iz\gamma_w = 0$: effective stress at z location becomes zero.

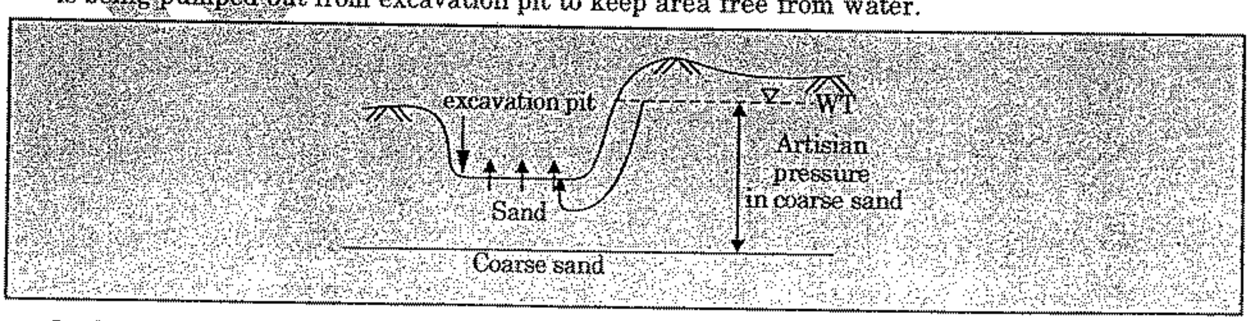
  $H\gamma_{sub} - iH\gamma_w = 0$: effective stress at H location becomes zero.

as $i_{cr} = \dfrac{\gamma_{sub}}{\gamma_w}$, hence

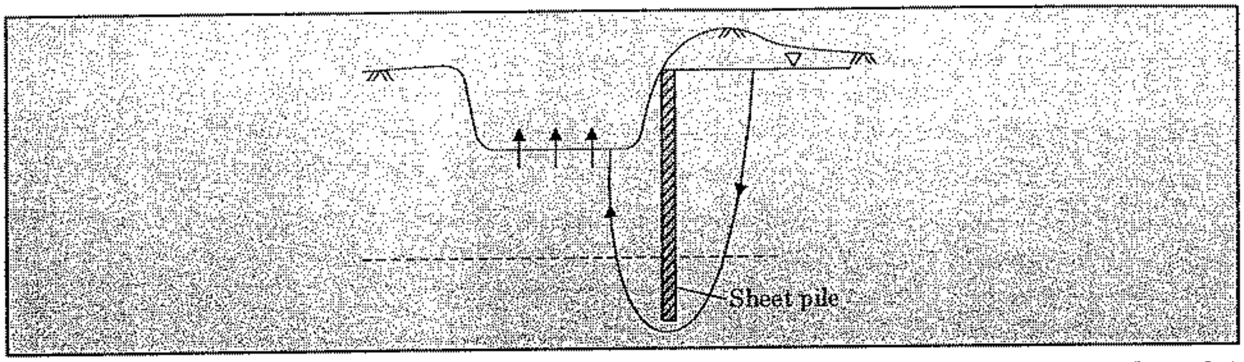
- $$i_{cr} = \frac{\dfrac{(G_S - 1)}{1+e}\gamma_w}{\gamma_w}$$

$$\boxed{\frac{G_S - 1}{1+e} = i_{cr}}$$

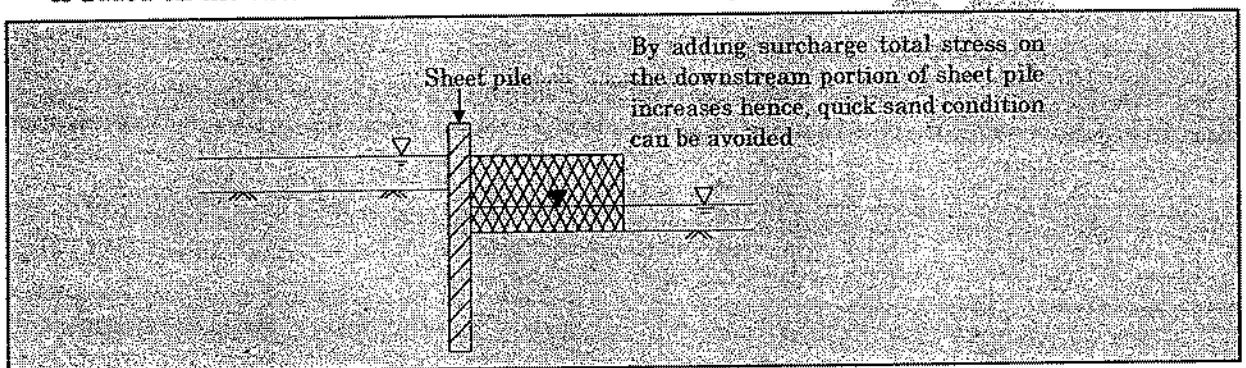
- For usual range of void ratio ($e = 0.6 - 0.7$) in sand and with specific gravity of ($G_S = 2.65$) $i_{cr} \simeq 1$
- At critical hydraulic gradient, quick sand condition occurs in sand but not in clay because in clay cohesion exist. Thus even if $\bar{\sigma}$ becomes zero, $\tau = c + \bar{\sigma}\tan\phi \neq 0$.
- In practice, quick sand condition occur when excavation is being made below water table and water is being pumped out from excavation pit to keep area free from water.



- In the above case quick sand condition can be prevented by lowering of water table at site before excavation or by increasing the upward flow length by providing a sheet pile wall as show in the figure below because as the length of flow increase i decrease $\left[i = \frac{h}{L}\right]$

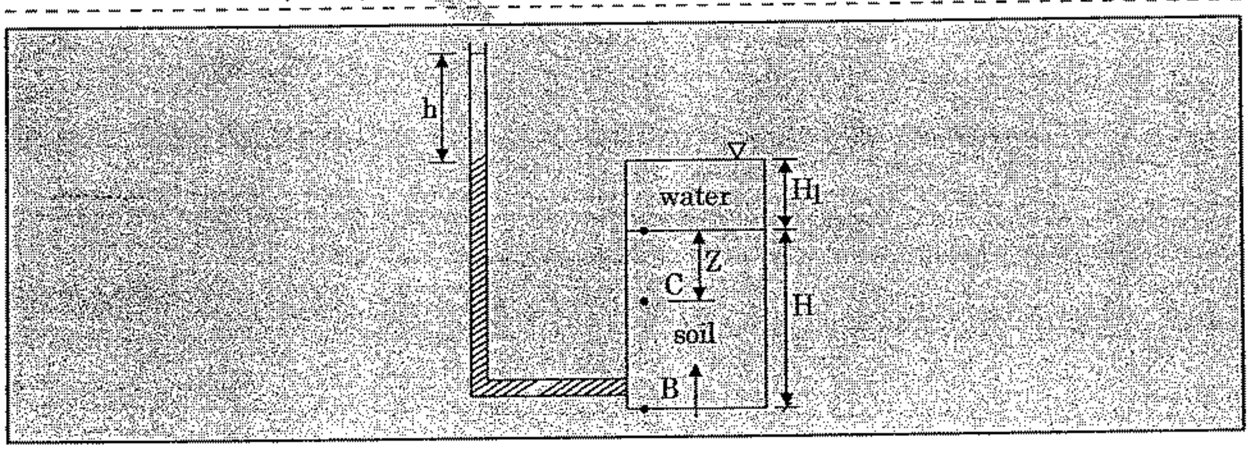


- Another method adopted at sites to avoid quick sand condition is a surchage (i.e. additional weight) is added on the excavation side to increase the net weight of the soil mass.



- Quick sand condition generally does not occur in coarse sand. because in equation $Q = KiA$, for i to be more, Q has to be much more because K is large for coarse sand. This large seepage discharge (Q) is generally not available in the field condition.

Note: i_{cr} can also be calculated with the knowledge that at icr contact force between soil is lost. Net downward force (due to soil + water) = Net upward force due to water under.
i.e. downward force = upward pore water pressure.



At Point B

$$H_1\gamma_w + H\gamma_{sat} = (H_1 + H + h)\gamma_w$$
$$= (H_1 + H)\gamma_w + (iH\gamma_w)$$

or
$$H(\gamma_{sat} - \gamma_w) = iH\gamma_w$$

$$i = \frac{\gamma_{sub}}{\gamma_w}$$

or

At Point C

$$H_1\gamma_w + z\gamma_{sat} = H_1\gamma_w + iz\gamma_w + z\gamma_w$$

$$z(\gamma_{sat} - \gamma_w) = iz\gamma_w$$

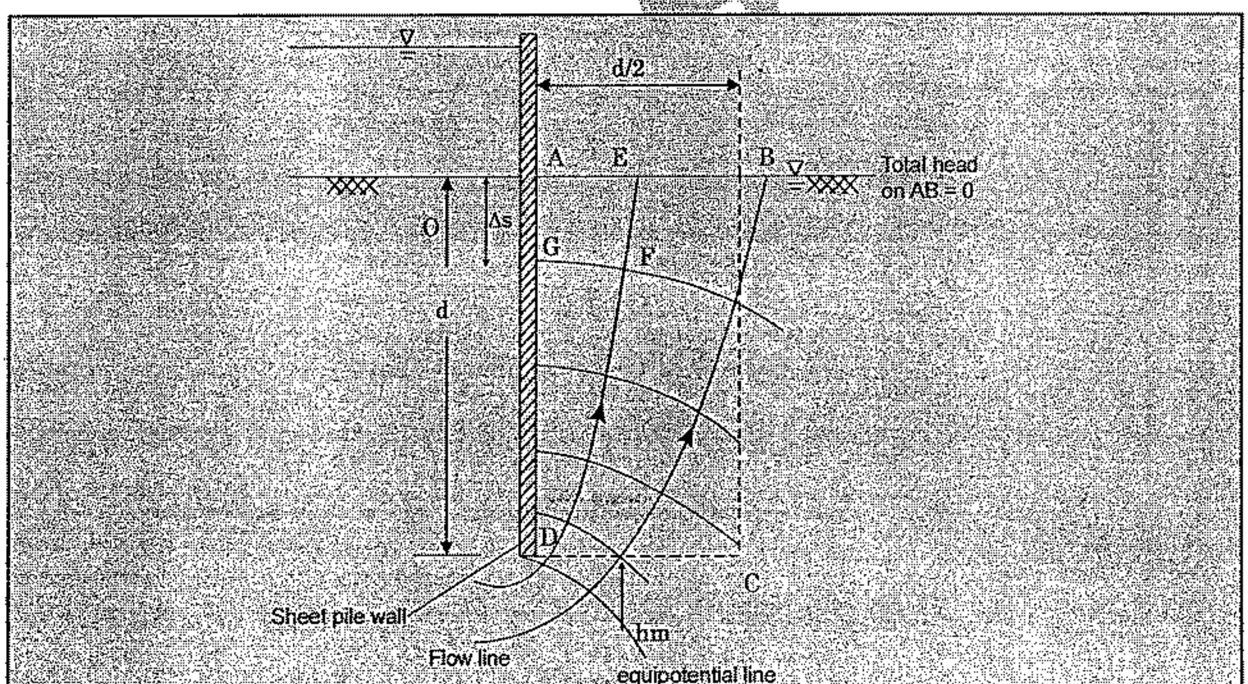
$$\boxed{i = \frac{\gamma_{sub}}{\gamma_w}}$$

Note: That instead of working in terms of total downward weight we can work in terms of total downward effective weight. But this downward effective weight has to be balanced by seepage pressure upwards for finding of i_{cr} i.e Hgsub = iHgw

$$\boxed{i = \frac{\gamma_{sub}}{\gamma_w}}$$

High artisian pressure in coarse sand can cause quick sand condition.

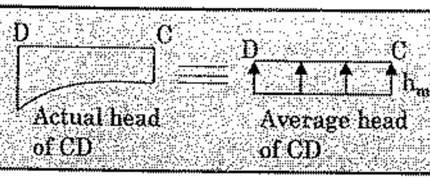
CONDITION ADJACENT TO SHEET PILE



- High upward hydraulic gradient may be experienced in soil adjacent to down stream face of a sheet pile wall. Faliure is likely to occur with in a soil mass of appropriate dimension $d \times \frac{d}{2}$ adjacent to sheet pile as shown above figure (*ABCD*). Failure 1st shows in the form of a rise or **heave** at the surface, associated with an expansion of the soil which results in increased permeability of soil mass.
- This in turn leads to increased flow, surface boiling. In case of sands this condition leads to complete failure of soil mass.
- FOS with respect to boiling heaving is analysed as follows

Approach A:

Average Total head on CD = hm



Total head on AB = 0

Average hydraulic gradent = $\frac{h_m}{d} = i_m$

If i_{cr} = critical hydraulic gradient than

$$\boxed{F.O.S = \frac{i_{cr}}{i_m}} \quad ..(i)$$

Aproach B :

(b) F.O.S. safety can also be determined w.r. to boiling at surface.

$$\boxed{\text{exit hydraulic gradient} = i_e = \frac{\Delta h}{\Delta s}}$$

where, Δh = drop of total head between equipotential *GF* and *AE*.

ΔS = dimension of flow net field *AE FG*

Hence,
$$\boxed{F.O.S = \frac{i_{cr}}{i_e}} \quad ...(ii)$$

Equation (*i*) and (*ii*) give almost the same result.

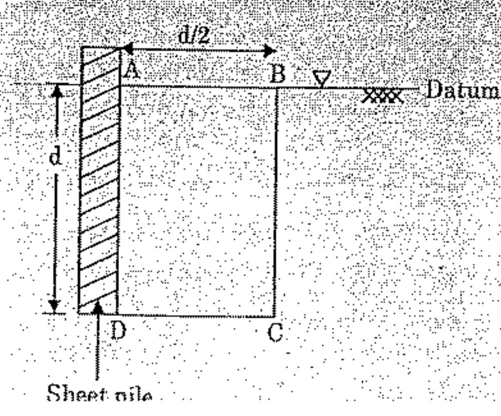
- If factor of saftey against heaving is considered inadequate, embedded length of sheet pile '*d*' can be increased or a surcharge load in term of a filter may be placed on the surface *AB*.

  If the effective weight of filter per unit area $= W'$

then
$$\boxed{\frac{\gamma_{sub} d + W'}{h_m \gamma_w} = \frac{\gamma_{sub} d + w'}{\text{seepage pressure}} = F.O.S}$$

Example 10

Check safety against boiling



Sol. Total downward force $= \left(\frac{d}{2} \times d \times 1\right) \cdot \gamma_{sat}$

Total upward pore water pressure $= u_{CD}$

writing the energy equation between AB & CD

$\frac{u_{CD}}{\gamma_w} + (-d) = 0 + 0 + h_m$, where h_m = head lost between AB & CD

$$u_{CD} = (h_m + d)\gamma_w$$

If total downward force become equal to total upward force boiling starts in sand or in other words if net body force becomes zero, boiling starts

$$\text{Net body force} = \frac{\gamma_{sat} d^2}{2} - u_{CD} \cdot \frac{d}{2}$$

$$= \frac{\gamma_{sat} d^2}{2} - (h_m + d) \times \frac{d}{2} \times \gamma_w$$

$$= \frac{\gamma_{sat} d^2}{2} - \frac{\gamma_w d^2}{2} - \frac{h_m d}{2}\gamma_w$$

$$= \frac{\gamma_{sub} d^2}{2} - \frac{1}{2} h_m d \gamma_w$$

$$\text{F.O.S} = \frac{\frac{1}{2}\gamma_{sub} d^2}{\frac{1}{2} h_m d \gamma_w} = \frac{\gamma_{sub}/\gamma_w}{\frac{h_m}{d}} = \frac{i_{cr}}{i_m}$$

Note: That we could have directly use the

$$\text{FOS} = \frac{\gamma_{sub} d}{\text{seepage pressure}} = \frac{\gamma_{sub} d}{i\, d\gamma_w} = \frac{\gamma_{sub} d}{\frac{h_m}{d} d\gamma_w} = \frac{\gamma_{sub}/\gamma_w}{\frac{h_m}{d}} = \frac{i_{cr}}{i}$$

The net body force can also be calculated as

$$\text{Effective weight of ABCD} = \frac{\gamma_{sub} d^2}{2}$$

$$\text{Average hydraulic gradient } P_m = \frac{h_m}{d}$$

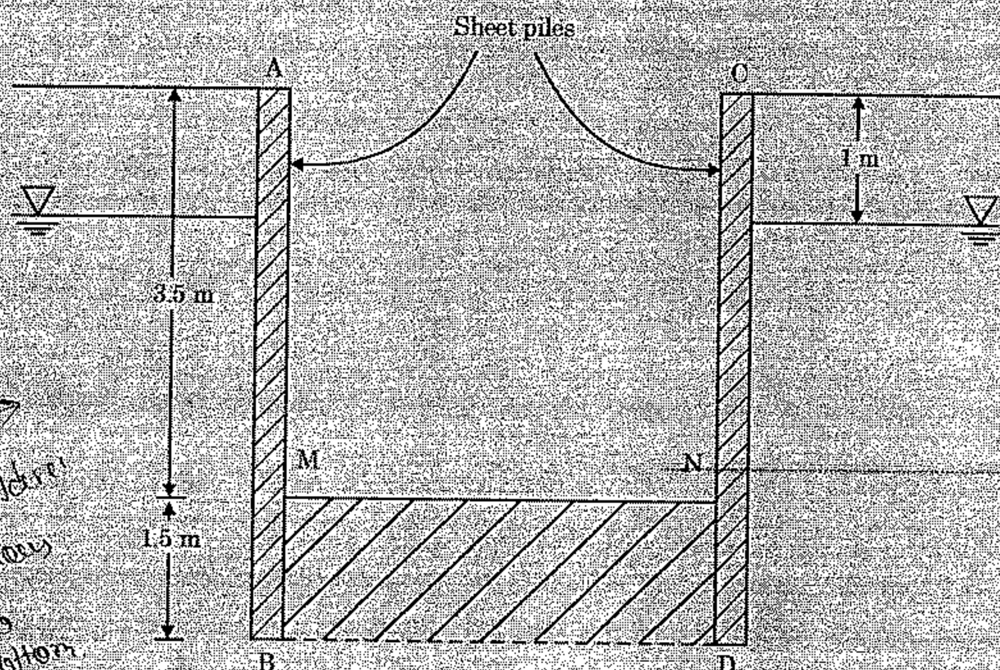
$$\Rightarrow \quad \text{Net force on body} = \frac{\gamma_{sub} d^2}{2} - \frac{h_m \gamma_w d}{2}$$

$$\text{F.O.S.} = \frac{\gamma_{sub} d^2/2}{h_m \gamma_w d/2} = \frac{\gamma_{sub}/\gamma_w}{\frac{h_m}{d}} = \frac{i_{cr}}{i_m}$$

Example 11 → How to calc using Upward wt. = Down ward wt. approach

It is reqired to excavate a long trench in a sand deposit upto a depth of 3.5 m below G.L. The sides of the tench should be vertical and are to be supported by steel sheet piles driven upto 1.5 m below the bottom of the trench the G.W.T is at 1 m below G.L. In order to have dry working area, water accumulated in the trench will be continuosly pumped out. If the sand has a void ratio of 0.72 and the specific gravity of solids be 2.66, check whether quick sand condition is likely to occur. If so what remedial measures would you suggest?

Sol.



Why no con considered when soil flows from top to Bottom

There will be an upward flow of water through the soil mass

The differential head which will cause this flow

$$h = 2.5 \text{ m}$$

Again thickness of the soil mass through which the flow occurs

$$L = MB = ND = 1.5 \text{ m}$$

∴ Hydraulic Gradient $\quad i = \left(\frac{h}{L}\right) = \left(\frac{2.5}{1.5}\right) = 1.67 \quad \ldots \text{(i)}$

∴ Critical hydraulic gradient $\quad i_c = \left(\frac{G-1}{1+e}\right) = \frac{(2.66-1)}{(1+0.72)} = 0.965 \quad \ldots \text{(ii)}$

From (i) and (ii) $i > i_c$,

Therefore, quick condition phenomenon will occurs

The following remedial measures can be recommended:

(i) The depth of embedment of sheet piles below the bottom of the trench should be increased. This will increase the thickness of the soil layer through which water perlocates and hence will reduce the hydraulic gradient.

Let x be the required depth of sheet piles below the bottom of the trench, which gives a factor of safety of 1.5 against quick condition

$$\therefore \quad i = \left(\frac{h}{L}\right) = \left(\frac{2.5}{x}\right) \quad \text{...(i)}$$

$$F.S = \frac{i_c}{i} = 1.5$$

$$\Rightarrow \quad i = \left(\frac{i_c}{1.5}\right) = \left(\frac{0.965}{1.5}\right) = 0.643 \quad \text{...(ii)}$$

From (i) and (ii) we have

$$\frac{2.5}{x} = 0.643$$

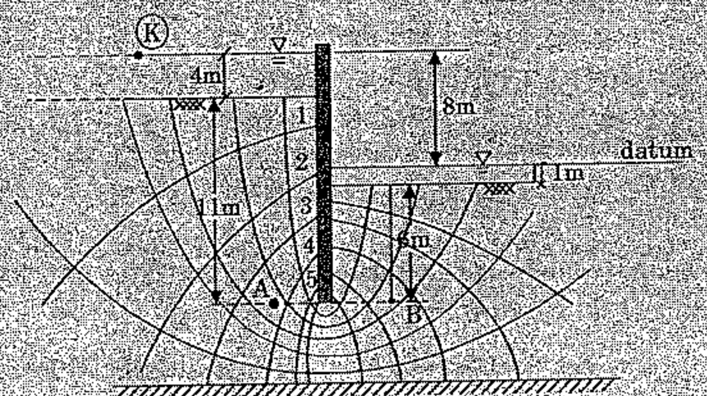
$$\Rightarrow \quad x = \left(\frac{2.5}{0.643}\right) = 3.89 \text{ m}$$

(ii) Alaternatively, water table at the site may be lowered by any suitable dewatering method. This will reduce the differential head and hence the hydraulic gradient will be reduced.

EFFECTIVE STRESS CALCULATION USING FLOW NET

Example 12

A flow net for seepage under a sheet pilewall is shown below. Saturated unit weight of soil is γ_{sat} = 20kN/m^3. We have to determine the effective stress at A & B. It is given that No. of equipotential drop at A = 3.8 and that and B = 8.5. There are 12 equipotential drop between u/s and d/s face of sheet pilewall. Determine values of effective vertical stress at A and B?



Sol. No. of equipotential dropupto A = 3.8

No. of equipotential drop upto B = 8.5

Total stress at A = $4\gamma_w + 11\gamma_{sat}$

$$= 4 \times 9.81 + 11 \times 20$$

$$= 259.24 \text{ KN/m}^2$$

head loss upto 3.8 equipotential drop

$$= \frac{3.8}{12} \times 8 = 2.533m$$

writing energy equation between up stream end (K) and point (A), we have

$$0 + 8 = -7 + \frac{u_A}{\gamma_w} + 2.533$$

$$\Rightarrow \quad u_A = (15 - 2.33)\,\gamma_w$$

$$u_A = 122.3 \text{ KN/m}^2$$

Effective stress $= \sigma_A - u_A = 259.24 - 122.3 = 136.94$ kN/m^2

Similarly effective stress at B = Total stress at B – Pore water Pressure at B.

Let pore water pressure at B = u_B

$$\Rightarrow \quad \frac{u_B}{\gamma_w} - 7 = 1 - 1 + \frac{(12 - 8.5) \times 8}{12}$$

$$\frac{u_B}{\gamma_w} = 9.33$$

$$u_B = 9.33\gamma_w$$

$\therefore$ effective stress at B = Total stress at B – Pore water Pressure at B.

$$= (1 \times \gamma_w + 6 \times \gamma_{sat}) - 9.33\gamma_w$$

$$= 129.81 - 9.33\,\gamma_w$$

$$= 38.28 \text{ KN/m}^2$$

Alternative approach

Effective stress = $\dfrac{\text{Boyant } wt \pm \text{vertical component of seepage force}}{\text{Area}}$

Vertical component of seepage is considered *+ve* for downward flow and *–ve* if flow is upward.

Vertical component of seepage force at any point is equal to $iz\gamma_w A$,

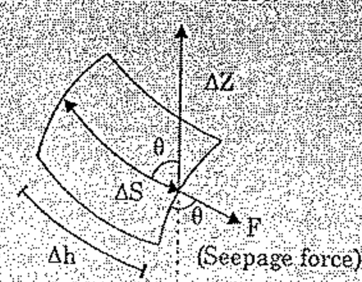
where

i = hydraulic gradient at that point

z = vertical depth of soil upto that point

Note: Calculation of vertical component of seepage force at any point.

Let us consider a element of soil mass at A in the flow net. The direction of seepage is shown below in th diagram which can be observed from the flow net.



Note: That seepage is not vertical hence we should work on vertical comonent of seepage force.

$$\frac{\text{Vertical component of Seepage force}}{\text{volume}} = F\cos\theta$$

$$= \left(\frac{\Delta h}{\Delta s}\right)\gamma_w \cos\theta$$

$$= \frac{\Delta h \gamma_w}{\Delta z}$$

Because

$$\Delta z = \Delta s/\cos\theta$$

$\Rightarrow$ Vertical component of seepage force $= \dfrac{\frac{\Delta h}{\Delta z} \times \Delta z \times A\gamma_w}{A} = i(\Delta z)\gamma_w$

Where,

i has been calculated as $\dfrac{\text{Head loss upto that point}}{\text{vertical distance } \Delta z}$

$\Rightarrow$ Effective stress $= (20 - 9.81) \times 11 + \dfrac{[(3.8/12)\times 8]}{11} \times 11\gamma_w = 136.942 \text{ kN/m}^2$

Bouyant weight of soil at B $= (20 - 9.81) \times 6 \times A = 61.14A$

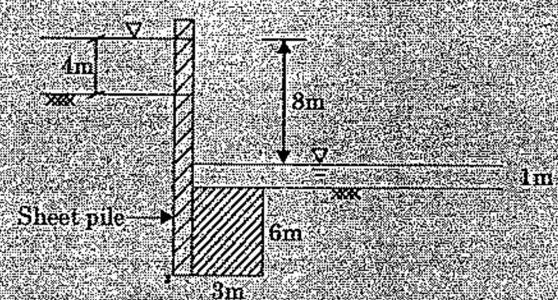
Vertical component of seepage force $= iz\gamma_w \times A = \dfrac{(12-8.5)\times 8}{12\times 6} 6\gamma_w A = 22.89A$

Flow at B is upward

$\Rightarrow$ effective stress at B $= \dfrac{(61.14 - 22.89)A}{A} = 38.25 \text{ kN/m}^2$

Example 13

In the above problem find F.O.S against failure by heaving adjacent to d/s face of sheet pile.



→ Why F.O.S is not calc using eff wt seepage force

Sol.

$$\text{F.O.S.} = \frac{i_c}{i_m}, \quad i_m = \frac{h_m}{6}$$

$$\text{Av. value of } h_m = \frac{6}{12}\times 8 + \frac{3.5}{12}\times 8$$

$$H_m = \frac{4 + 2.33}{2} = 3.16 \text{ m}$$

$$i_m = \frac{3.16}{6} = 0.528$$

$$\Rightarrow \quad \text{F.O.S} = \frac{\gamma_{sub}/\gamma_w}{0.528} = \frac{(20 - 9.81)/9.81}{0.528} = 1.967$$

Example 14

1.5 m layer of soil is subjected to an upward seepage head load of 1.95 m. What depth of coarse sand will be required above this soil to provide a factor of safety of 1.5 against piping. Coarse soil has sp greavity 2.67 and porosity = 30%.

Sol.

$$i_{cr} = \frac{G-1}{1+e} = \frac{G-1}{1+\frac{n}{1-n}} = \frac{2.67-1}{1.428} = 1.169$$

$$i_{possible} = \frac{1.169}{1.5} = 0.78$$

$$(1.5 + x) \times 0.78 = 1.95$$

$$\Rightarrow \quad x = 1 \text{ m}$$

Alternative approach:

$$\frac{(1.5+x)\gamma_{sub}}{1.95\gamma_w} = 1.5$$

$$\frac{(1.5+x)\left(\frac{G-1}{1+e}\right)\gamma_w}{1.95\gamma_w} = 1.5$$

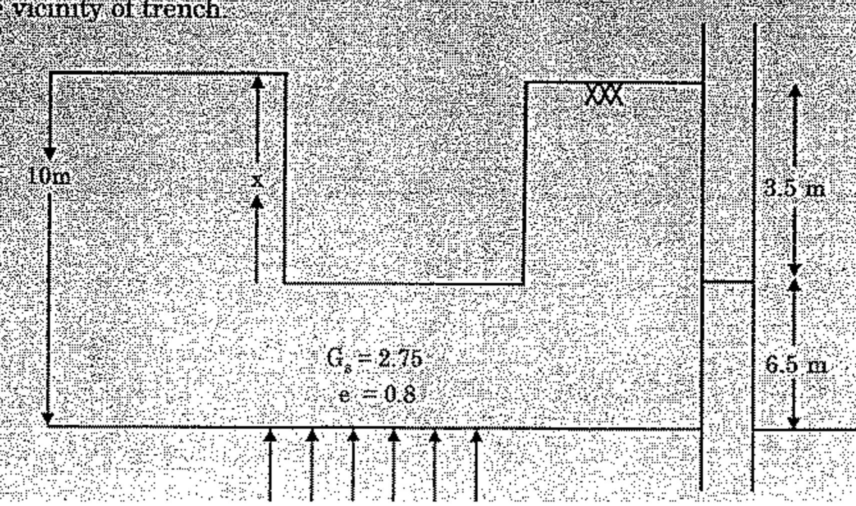
$$\Rightarrow \quad x = 1\text{m}$$

Example 15

Foundation trench to be excavated in a stratum of dense sand 10 m thick underlain by a bed of coarse sand. In a trial bore hole, the G.W.T. is observed to rise an elevation of 3.5 m below ground surface, before excavation, the coarse sand layer is under artesion pressure determine the depth upto which an excavation can be carried out, without the danger of water becoming unstable under the artesion pressure assume the before excavation no flow is taking place. $G_s = 2.75$ e = 0.8.

(ii) If the excavation to be carried out safely to a depth of 8m, how much should W.T. lowered in the vicinity of trench.

Sol.



For stability total downward weight = total upward pore water pressure.

$(10 - x)\ (\gamma_{sat})_{sand}$

Let us assume that dense sand layer is saturated in that case

Down ward weight = $(10 - x)\ (\gamma_{sat})_{sand}$

Under no flow condition upward pressure = 6.5m head of water.

Hence for instability $(10 - x)\ (\gamma_{sat})_{sand} = 6.5\ \gamma_w$

$$(\gamma_{sat})_{sand} = \frac{(G_s + e)}{(1+e)}\gamma_w$$

$$= \frac{(2.75+0.8)}{1.8}\times 10 = 19.72\ kN/m^2$$

$$\boxed{x = 6.704m.}$$

When depth of excavation becomes more than 6.70 m. In stability occurs. Hence the upward pressure hhas to be reduced, under no flow condition

$$2\ \gamma_{sat} = (6.5\text{-}y)\ \gamma_w$$

$$\boxed{y = 2.556m.}$$

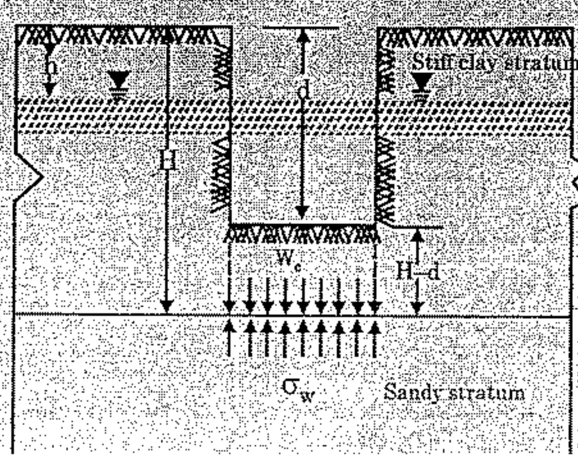
W.T. will be lowered by = 2.556

Example 16

A large excavation is made in a stiff clay whose saturated unit weight is 17.27 kN/m². When the depth of excavation reaches 7.5 m, cracks appear and water begins to flow upward to bring sand to the surface. Subsequent borings indicated that tbe clay is underlain by sand at a depth of 11m below tbe original ground surface.

What is the depth of the water table outside the water table outside the excavation below the original ground level?

Sol. Making an excavation in the clay creates a hydraulic gradient between the top of the sand layer and the bottom of the excavation. As a consequence, water starts seeping in an upward direction from the sand layer towards the excavated floor. Because the clay has a very low permeability, flow equilibrium can only be reaced after a long period of time. The solution must be considered over a short time intervail.



The floor of the excavation at depth d is stable only if the water pressure σ_w at the top of the sand layer at a depth of 11 m is counter balanced by the saturated weight W per unit area of the clay above it disregarding the shear strength of the clay.

Let H = total thickness of clay layer = 11m, d = depth of excavation in clay = 7.5 m, h = depth of water table from ground surface, γ_{sat} = saturated unit weight of the clay.

Let (H – d) = 11 – 7.5 = 3.5 m, the thickness of cflay strata below the bottom of the trench.

$$W_c = \gamma_{sat} \times (h - d)$$
$$= 17.27 \times 3.5 = 60.24 \text{ kN/m}^2$$
$$\sigma_w = \gamma_w (H - h) = 9.81 (11 - h) \text{ kN/m}^2$$

cracks may develop when $W_c = \sigma_w$

or $\quad 60.24 = 9.81 (11 - h) \;\text{ or }\; h = 11 - \dfrac{60.24}{9.81} = 4.84\text{m}$

Example 17

A 12 m thick layer of relatively impervious saturated clay lies over a gravel aquifer. Piezometer tubes introduced to the gravel layer show an artesian pressure condition with the water level standing in the tubes 3m above the top surface of the clay stratum. The properties of the clay are e = 1.2 and G = 2.7.

Determine (a) the effective stress at the top of the gravel strata, (b) the depth of excavation that can be made in the clay stratum without bottom heave.

Sol. (a) At th top of the gravel stratum

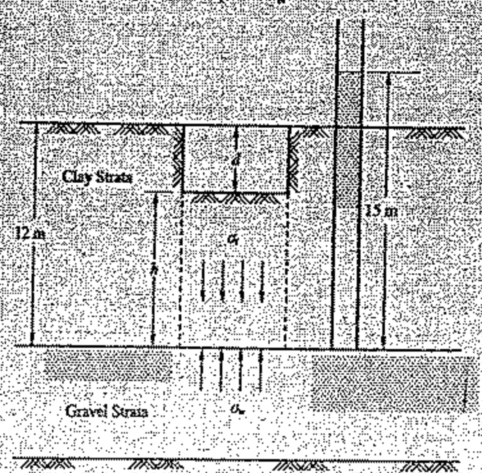
$$\sigma_t = 12 \times 17.39 = 2.8.\ 68 \text{ kN/m}^2$$

The pore water pressure at the top of gravel strata

$$u_w = 9.81 \times 15 = 147.15 \text{ kN/m}^2$$

The effective stress at the top of gravel strata

$$\sigma' = \sigma_t - u_w = 208.68 - 147.15 = 61.53 \text{ kN/m}^2$$



(b) If an excavation is made into the clay stratum as shown in Fig. Ex. 8.12, the depth must be such that

$$\sigma_t = u_w$$

Let the bottom of the excavation be h m above the top of gravel layer. Now the downward pressure acting at the top of the gravel layer is

$$\sigma_1 = \gamma_t h = 17.39h \text{ kN/m}^2$$

$$u_w = 147.15 \text{ kN/m}^2$$

Now,
$$17.39h = 147.15$$

or
$$h = \frac{147.15}{17.39} = 8.46 \text{ m} = 8.5 \text{ m}$$

Depth of excavation,
$$d = 12 - 9.30 = 2.7 \text{ m}$$

This is juust the depth of excavation with a factor of safety F = 1.0. If we assume a minimum

$$F_s = 1.10$$

$$h = \frac{147.15 \times 1.1}{17.39} = 9.3 \text{ m}$$

$$\text{Depth of excavation} = 12 - 9.30 = 2.7 \text{ m}$$

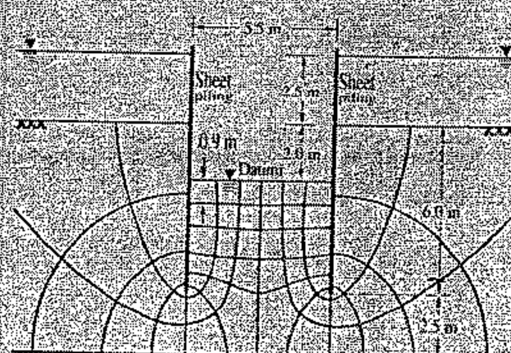
Example 18

The lines of sheet piles were driven in a river bed as shown in Fig. Ex. 7.2. The depth of water over the river bed is 2.5m. The trench level within the sheet piles is 2.0m below the river bed. The water level within the sheet piles is kept at trench level by resorting to pumping. If a quantity of water flowing into the trench from outside is 0.30 m³/ hour per foot length of sheet pile, what is the hydraulic coefficient of permeability of sand? What is the hydraulic gradient immediately below the trench bed?

Sol. Figure Ex. 7.2 gives the flow net and other details. The differential head between the bottom of trench and the water level in the river is 4.5 m.

Number of channels = 6

Number of equipotential drops = 10



$$q = kh\frac{N_f}{N_d}$$

or
$$0.30 = 4.5 \times \frac{6}{10} \times k$$

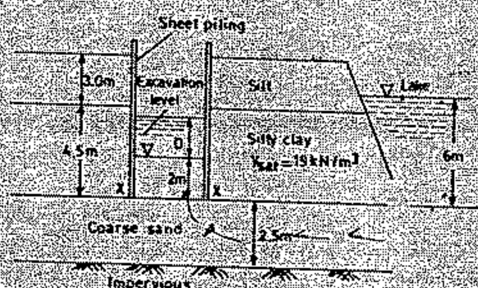
or
$$k = \frac{0.30 \times 10}{4.5 \times 6} \times \frac{1}{60 \times 60} = 3.08 \times 10^{-5} \text{ m/s}$$

The distance between the last two equipotentials given is 0.9 m. The calculated hydraulic gradient is

$$i = \frac{\Delta h}{\Delta s} = \frac{4.5}{10 \times 0.9} = 0.50$$

Example 19

An excavation is to be made in subsoil conditions shown in Figure below. Assuming that the head loss in the coarse sand layer at X-X to be 15 per cent, write an expression for the effective stress at level X-X in terms of D. If boiling condition is to be avoided, what should be the minimum depth of water that has to be left in the pit?



Sol. We know that there is 15% of head loss at section XX, therefore head available at section XX is 5.1 m.

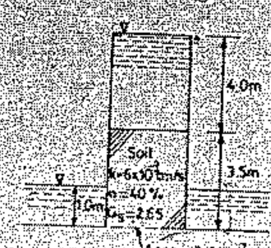
$$\text{Effective stress at} = D\gamma_w + 2\gamma_{sat} - 5.1\,\gamma_w$$
$$= 9.81\,D + 2 \times 19 - 5.1 \times 9.81$$
$$= 9.81\,D - 12.04\ \text{kN/m}^2$$

To calculate the minimum depth of water required above the excavation, we will take effective stress to be zero.

$$\bar{\sigma} = 9.81\,D - 12.04 = 0$$
$$D = 1.23\ \text{m}$$

Example 20

Shows a set-up a water filter. Determine the amount of water that can be filtered in a day. Draw the total, neutral and effective stress distribution diagrams.



Sol. Given, $n = 0.4$, $G_s = 2.65$

$$e = \frac{n}{1-n} = \frac{0.4}{1-0.4} = \frac{0.4}{0.6} = 0.667$$

$$\gamma_{sat} = \frac{G+e}{1+e}\gamma_w = \frac{2.65+0.667}{1+0.667} \times 9.81 = 19.52 \text{ kN/m}^2$$

Calculation for effective stress

(a) At depth 0 m

$$\bar{\sigma} = 0$$
$$u = 0$$
$$\bar{\sigma} = \sigma - u = 0$$

(b) At depth (–) 4.0 m

$$\sigma = 4\gamma_w = 4 \times 9.81 = 39.24 \text{ kN/m}^2$$
$$u = 4 \times \gamma_w = 4 \times 9.81 = 39.24 = 0 \text{ kN/m}^2$$
$$\bar{\sigma} = \sigma - u = 39.24 - 39.24 = 0 \text{ kN/m}^2$$

(c) At depth (–) 7.5 m

$$\sigma = 4\gamma_w + 3.5\,\gamma_{sat} = 4 \times 9.81 + 3.5 \times 19.52 = 107.56 \text{ kN/m}^2$$
$$\bar{\sigma} = \sigma - u = 107.56 - 9.81 = 97.75 \text{ kN/m}^2$$

From Darcy's law

$$q = k\,i\,A$$
$$q = k\frac{\Delta h}{L}A = 6 \times 10^{-5} \times \frac{6.5}{3.5} \times 3.14$$
$$= 3.49 \times 10^{-4}$$
$$\text{Discharge in a day} = q \times 86400$$
$$= 3.49 \times 10^{-4} \times 86400 = 30.15 \text{ m}^3\text{/sec}$$

PERMEABILITY

- Permeability is the ease with which water can flow through any medium.
- In Soil mechanics, Permeability of soil is a soil property which describes quantitatively, the ease with which water flows through soil.
- Permeability is a very important engineering property of soils.
- Knowledge of permeability is essential in a number of soil engineering problems, such as settlement of buildings, yield of wells, seepage through and below the earth structures. It controls the hydraulic stability of soil masses.
- The permeability of soils is also required in the design of filters used to prevent piping in hydraulic structures and subgrade drainage, rate of consolidation of compressible soils and many other aspects.

DARCY'S LAW

- In one dimensional flow, discharge through fully saturated soil is given by Darcy.

$$\boxed{q = kiA \text{ or } V = ki}$$

where,

q = discharge

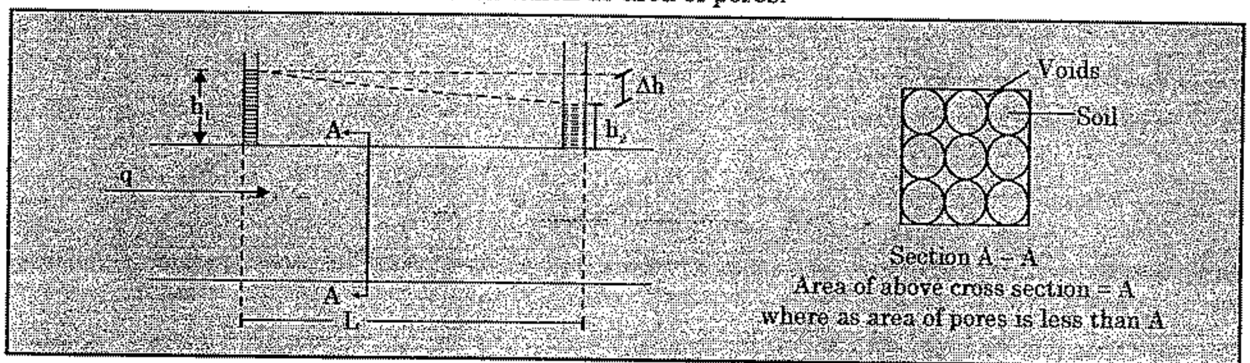
A = Cross section area of soil corresponding to flow 'q'

k = Coefficient of permeability.

i = hydraulic gradient $= \dfrac{\Delta h}{L} = \dfrac{\text{Loss of head}}{\text{length}}$

V = superficial velocity of flow or discharge velocity.

- The above velocity of flow is not the true velocity of flowbecause water flow only through the pore, hence area of flow should have been taken as area of pores.



- The above law assumes that the flow of water takes place through the whole cross section of the soil but in reality water flows through the void present between the soil particles. Hence it is correct to call this velocity as superficial velocity .
- Superficial velocity is also called as velocity of flow or discharge velocity.
- **Actual velocity/seepage velocity** (V_s) is given by

$$V_S = \frac{q}{A_V},\ A_v = \text{area of voids.}$$

But

$$q = A.V$$

$$AV = A_V V_S$$

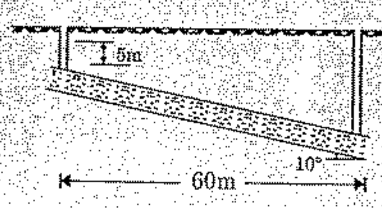
$$V_S = \frac{AV}{A_V} = \frac{V}{A_V/A} = \frac{V}{n}$$

$$\boxed{V_s = \frac{V}{n}},\ n = \text{Porosity of soil}$$

- Since $n < 1$, V_S is always greater than V
- But For the sake of convenience, in engineering practice, 'V' is used instead of "V_S".

Example 21

Fig. shows an aquifer inclined at 10° to the horizontal. The difference of water levels in two observation wells at a horizontal distance of 60 m is 5 m. Determine the discharge through the aquifer per unit width if k = 0.7 mm/sec. The depth of aquifer normal to the direction of flow is 2.951 m.



Sol. Length of aquifer between two observation wells

$= 60/\cos 10° = 60.926$ m

$$\text{Hydraulic gradient} = h/L = \frac{5.0}{60926} = 0.082$$

From Darcy's law, discharge per unit width,

$$q = k\,i\,A$$
$$= 0.7 \times 10^{-3} \times 0.082 \times (2.95 \times 1) = 0.169 \times 10^{-3}\ m^3/sec$$
$$= \textbf{0.169 lit/sec.}$$

COEFFICIENT OF PERMEABILITY

- Coefficient of Permeability can be defined as Superficial velocity of flow (or velocity of flow) which would occur under a unit hydraulic gradient.
- Permeability is usually expressed in the unit of velocity.
- Typical values of Pearmeability are as listed in the table below

Soil type	Coefficient of Permeability cm/sec	Drainage Characterstics
Gravel	>1	Pervious
Sand	$1 - 10^{-3}$	Pervious
Silt	$10^{-3} - 10^{-6}$	Slightly Pervious
Clay	$<10^{-6}$	Impervious

- Coeficient of permeability divided by porosity is called coefficient of percolation (K_p)

$$\boxed{K_p = \frac{K}{n}}$$

Where,

K_p = Coefficient of percolation

K = Coefficient of permeability

n = Porosity

DETERMINATION OF COEFFICIENT OF PERMEABILITY

- The coefficient of permeability of a soil can be determined using the following methods.
  - (a) *Laboratory Methods* : The coefficient of permeability of a soil sample can be determined by the following methods :
    - *(i)* Constant-head permeability test.
    - *(ii)* Variable-head permeability test.
- The instruments used are permeameters. The former test is suitable for relatively more pervious soils, (sand) and the latter for less pervious soils (clay).

(b) ***Field Methods*** : The coefficient of permeability of a soil deposit in-situ conditions can be determined by the following fields methods.

(i) Pumping-out tests.

(ii) Pumping-in tests.

- The pumping-out tests influence a large area around the pumping well and give an overall value of the coefficient of permeability of the soil deposit. The pumping-in test influences a small area around the hole and therefore gives a value of the coefficient of permeability of the soil surrounding the hole.

(c) ***Indirect Methods*** **:** The coefficient of permeability of the soil can also be determined indirectly from the soil parameters by

(i) Computation from the particle size and its specific surface.

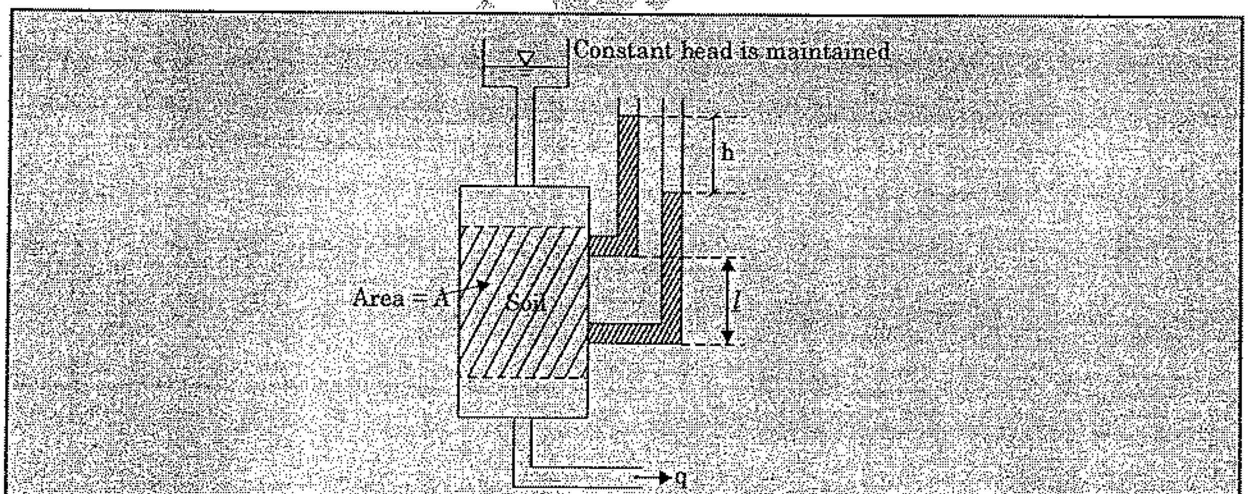
(ii) Computation from the consolidation test data.

- The first method is used if the particle size is known. The second method is used when the coefficient of volume change has been determined from the consolidation test on the soil.

(d) ***Capillarity-Permeability test*** **:** The coefficient of permeability of an unsaturated soil can be determined by the capillarity–permeability test.

CONSTANT HEAD PERMEABILITY TEST

- Coefficient of permeability for **coarse soil is** determined by means of **constant-head permeability test.**
- Degree of saturation of soil should be 100%



We know that,

$\Rightarrow$

$$\bar{\sigma} = \text{effective stress} = \frac{\Sigma N'}{A}$$

$$q = k\,i\,A$$

$$q = k\frac{h}{L}A$$

$$k = \frac{qL}{Ah}$$

where,

q = discharge collected in time t

L = distance between manometer taping points

A = Cross sectional area of sample.

h = difference in manometer levels.

Example 22

A sample of coarse sand is tested in a constant head permeater. The sample is 20 cm. high and has a diameter of 8 cm. Water flows through the soil under a constant head of 1m for 15 minutes. The mass of discharged water was found to be 1.2 kg. Determine the coefficient of permeability of the soil.

Sol. We have, for constant head permeabilty test.

$$q = kiA$$

$$= k\left(\frac{h}{L}\right)A$$

$$\frac{Q}{t} = k\left(\frac{h}{L}\right)A$$

$$k = \left(\frac{QL}{hAt}\right) \quad ...(i)$$

Data given

Mass of discharged water =1200 gm

∴ Vol. of discharged water =1200 cc

$$t = 15 \text{ min} = 15 \times 60$$

$$= 900 \text{ sec}$$

$$\text{Head of water } h = 1 \text{ m} = 0100 \text{ cm}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}\times(8)^2$$

$$= 50.26 \text{ cm}^2$$

$$L = \text{Length of soil} = 20 \text{ cm}$$

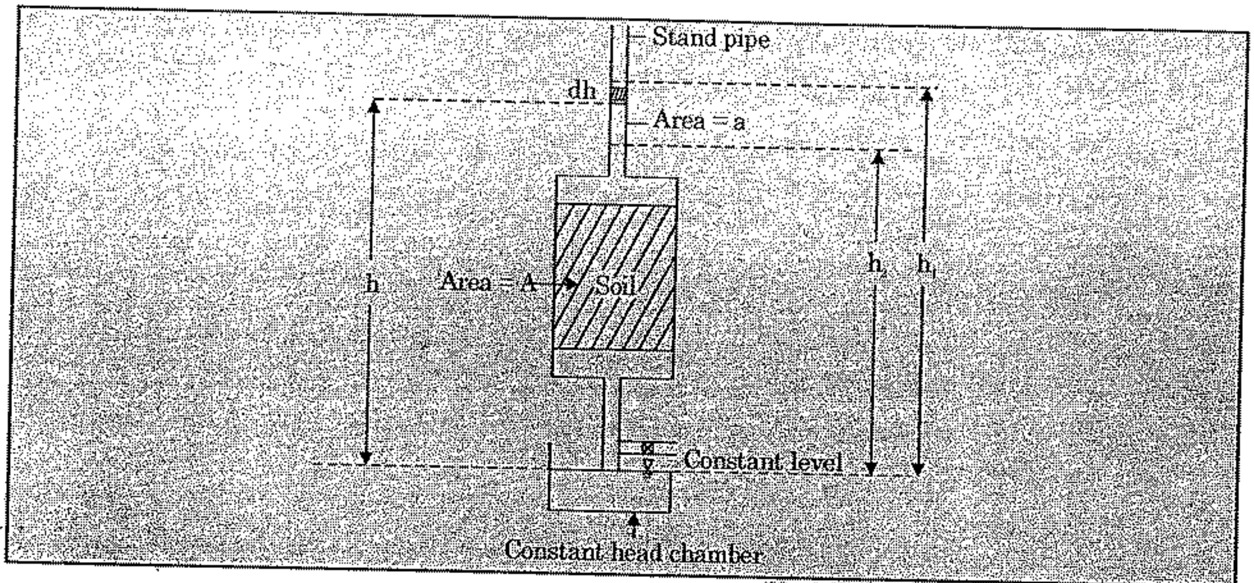
$$K = \left(\frac{QL}{hAt}\right)$$

$$= \frac{1200\times 20}{(100\times 50.26\times 900)}$$

$$= 5.306 \times 10^{-3} \text{ cm/sec}$$

Variable Head or Falling Head Permeability Test

- For fine soils, falling head method is used.
- In the case of fine soils, **undisturbed specimens are generally tested** and the cylinder containing soil sample is used as a sampling tube itself for the test.



- The test is started by allowing the water in the stand pipe to flow through the sample to the constant-head chamber due to which the water level in the stand pipe falls.
- The time required for the water level to fall from a known initial head (h_1) to a known final head (h_2) is determined.
- The head is measured with reference to the level of water in the constant- head chamber.

Let us consider the instant at which the head is h. For the infinitesimal small time dt, the head falls by height $\boldsymbol{dh}$. Let the discharge through the sample be q. From continuity of flow.

$$-a\,dh = q\,dt$$

Where a is cross-sectional area of the standpipe.

$$a\,dh = -(A \times k \times i) \times dt$$

$$a\,dh = -Ak \times \frac{h}{L} \times dt$$

$$\frac{Akdt}{aL} = \frac{-dh}{h}$$

Integrating,

$$\frac{Ak}{aL}\int_{t_1}^{t_2} dt = -\int_{h_1}^{h_2} \frac{dh}{h}$$

$$\frac{Ak}{aL}(t_2 - t_1) = \log_e (h_1/h_2)$$

$$\boxed{k = \frac{aL}{At}\log_e(h_1/h_2)}$$

where, $t = (t_2 - t_1)$, the time interval during which the head falls from h_1 to h_2.

$$\boxed{k = \frac{2.30\,aL}{At}\log_{10}(h_1/h_2)}$$

Note: • Sometimes, the permeability test is conducted using the consolidometer instead of the permeameter mould. The fixed-ring consolidometer is used as a variable-head permeameter by attaching a stand pipe to base.

- Reliability of laboratory method, depend on the extent to which the test specimen are representative of the soil mass as a whole. But it is difficult to simulate original soil structure in test sample. Hence more reliable results are obtained from in-situ methods.

Example 23

A falling head permeability test was carried out on a 15 cm long of silty clay. The diameter of the sample and the stand pipe were 9.8 cm and 0.75 cm respectively. The water level in the stand pipe was observed to fall from 70 cm. to 45 cm. In 12 minutes. Determine

(i) The coefficient of permeability of the soil in m/day

(ii) Height of water level in stand pipe after 20 minutes

(iii) time requred for water level to drop to 10 cm.

Sol. For a falling head permeabilty test

We have $$K = \frac{aL}{At}\log_e\left(\frac{h_1}{h_2}\right)$$

Data given $$a = \frac{\pi}{4}\times(0.75)^2 = 0.442\ \text{cm}^2$$

$$A = \frac{\pi}{4}(9.8)^2 = 75.43\ \text{cm}^2$$

L =15 cm, t = 12 min = 12 × 60 = 720 sec

$$h_1 = \ \text{cm},\quad h_2 = 45\ \text{cm}$$

$$K = \frac{aL}{At}\log_e\left(\frac{h_1}{h_2}\right)$$

$$= \frac{0.442\times15}{(75.43\times720)}\log_e\left(\frac{60}{45}\right)$$

$$= 3.51\times10^{-5}\ \text{cm/sec} = \frac{3.51\times10^{-5}}{\left(\frac{1}{86400}\right)} = 0.03\text{m/day}$$

(ii) Set h be the head at the end of another 20 minutes

Again $$K = \frac{aL}{At}\log_e\left(\frac{h_1}{h_2}\right)$$

$$3.51\times10^{-5} = \frac{0.442\times15}{75.43\times(1200)}\log_e\left(\frac{45}{h}\right)$$

$$\Rightarrow \quad \frac{45}{h} = e^{0.479}$$

$$\Rightarrow \quad h = \left(\frac{45}{1.62}\right) = 27.86\ \text{cm}$$

(iii) Set t be the time required for the head drop from 45 cm to 10 cm.

$$\therefore \quad t = \frac{aL}{AK}\log\left(\frac{h_1}{h_2}\right) = \frac{0.442\times15}{(75.43\times3.51\times10^{-5})}\log_e\left(\frac{45}{10}\right) = 3766.45\ \text{sec}$$

Example 24

In a falling head permeability test on a sample 12.2 cm high and 44.41 cm^2 in cross-sectional area, the water level in a standpipe of 6.25 mm internal diameter dropped from a height of 75 cm to 24.7 cm in 15 minutes. Find the coefficient of permeability.

Sol.:

$$k = 2.3\frac{al}{At}\log_{10}\frac{h_1}{h_2}$$

$$a = \frac{\pi \times 0.625^2}{4} = 0.307 \text{ cm}^2$$

$$t = 15 \times 60 = 900 \text{ s}$$

$$k = 2.3 \times 0.307 \times \frac{12.2}{44.41 \times 900}\log_{10}\frac{75}{24.7} = 1.04 \times 10^{-4} \text{ cm/s}$$

Example 25

A sandy layer 10 m thick overlies an impervious stratum. The water table is in the sandy layer at a depth of 1.5 m below the ground surface. Water is pumped out from a well at the rate of 100 litres per second and the drawndwon of the water table at radial distances of 3.0 m and 25.0 m is 3.0 and 0.50 m, respectively. Determine the coefficient of permeability.

Sol.:

$$k = \frac{q}{\pi(z_2^2 - z_1^2)}\log_e(r_2/r_1)$$

In this case, $z_2 = 8.50 - 0.50 = 8.0\text{m}$ and $z_1 = 8.50 - 3.0 = 5.50$ m

Therefore,
$$k = \frac{100 \times 10^{-3}}{\pi[(8)^2 - (5.50)^2]}\log_e(25/3)$$

$$= 0.002 \text{ m/sec} = 2 \text{ mm/sec.}$$

FIELD TEST METHODS

- The laboratory methods for the determination of the coefficient of permeability, as discussed before, do not give correct results.
- The samples used are generally disturbed and do not represents the true in-situ structure.
- For more accurate, representative values, the field tests are conducted. The field tests may be in the form of pumping out test where in the water is pumped out from the wells drilled for this purpose.
- The other type of the field tests are pumping-in tests, wherein the water is pumped into the drilled holes,

1. Pumping out Test

- For large engineering projects, it is the usual practice to measure the permeability of soils by pumping-out tests.
- This method is extremely useful for a homogeneous, coarse grained deposits.
- In this test, the soil deposit over a large area is influenced, and therefore the results represent an overall coefficient of permeability of a large mass of soil. However, the test is very costly and can be justified only for large projects.

- Water is pumped out from the main well at a constant rate until the water levels in the observation wells (at least two in number) become steady or constant, indicating a steady state of flow.
- The flow quantity and the levels in the observation wells are noted. One of the two basic flow conditions will apply, namely **unconfined flow** or **confined flow**.
- Perforated casing is required to support the sides of well.

***Note:* Aquifers** : *A soil deposit which is pervious in nature and allows extraction of water. These are of two types unconfined and confined aquifer.*

Aquiclude : *Aquiclude is a soil formation such as clay which contains water, but which is not capable of transmitting or supplying adequate quantity of water.*

Aquifuse : *It is a soil mass of rock or an impervious formation which neither transmit nor stores any water.*

- The coefficient of permeability of the soil can be found out using the equations developed below separately for unconfined aquifer and confined aquifer.

UNCONFINED AQUIFER

- The aquifer is underlain by an impermeable stratum and the test well extends to the bottom of the permeable stratum.
- In its original state, it is assumed that the ground water is at rest and pumping generates a radial flow of water towards the filter well and as a result, the water table assumes a curved surface called drawdown water table.
- Consider the flow through an elementary cylinder of soil having radius r, thickness dr and height h. Hydraulic gradient (from outside to inside)

$$i = \frac{dh}{dr}$$

Area of flow, $$A = 2\pi r h$$

From Darcy's law, $$q = k\,i\,A$$

$$\therefore \quad q = k\frac{dh}{dr}2\pi r h$$

$$\frac{dr}{r} = 2\frac{\pi}{q}k h\,dh$$

The expression for the coefficent of permeability can be derived making the following assumptions, known as Dupit's assumption.

(1) The flow is laminar and Darcy's law is valid.

(2) The soil mass is isotropic and homogeneous.

(3) The well penetrates the entire thickness of aquifer such that water does not come into wall from below.

(4) The flow is steady.

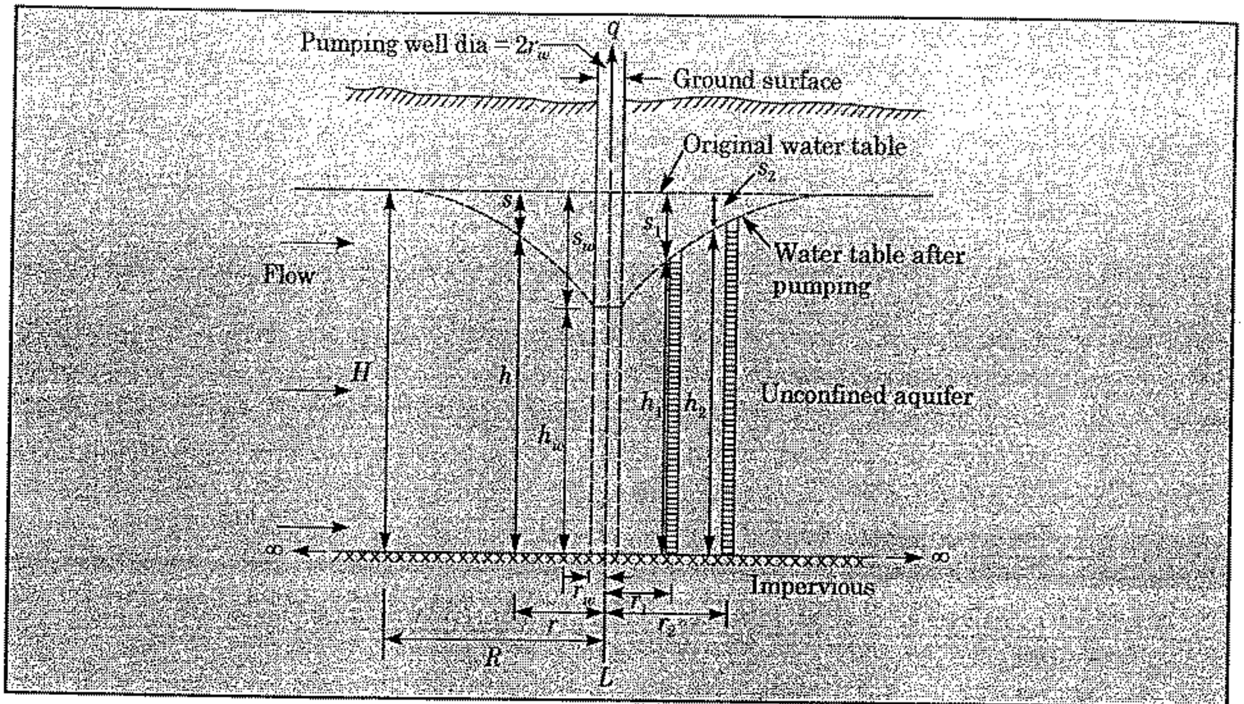
(5) The coefficient of permeability remains constant throughout.

(6) The flow towards the well is radial and horizontal.

(7) Natural ground water regime remains constant.

(8) Hydraulic gradient at any distance 'r' from the centre of well is assumed to be constant with depth and is equal to the slope of water table.

$$\boxed{i = \frac{dh}{dr}}$$



$$\frac{dr}{r} = 2\frac{\pi}{q}k\,h\,dh$$

$$q\int_{r_1}^{r_2}\frac{dr}{r} = 2\pi k\int_{h_1}^{h_2} h\,dh$$

$$\boxed{k = \frac{2.303\,q\log_{10}\frac{r_2}{r_1}}{\pi(h_2^2 - h_1^2)}}$$

- Here q is constant.
- Though two observation wells are sufficient for computing k, higher accuracy will be obtained by observing water table in more than two boreholes.
- Values of h_1 and h_2 can be determined by measuring the drawdown at the two observation wells.
- An approximate value of the coefficient of permeability can be determined if the radius of influence (R) is known or is estimated. The circle of influence, over which the effect of pumping is observed, extends to a very large area.
- In fact, it gradually merges asymptotically to the water table. The radius of influence varies between 150 to 300 m. According to Sichardt, it can be found using the relation.

$$R = 3000\,d\sqrt{k}$$

where, R = radius of influence (m), d = drawdown in the well (m)

k = coefficient of permeability (m/sec)

Example 26

Determine the coefficient of permeability of a confined aquifer 5 m thick which gives a steady discharge of 20 litres/sec through a well of 0.3 m radius. The height of water in the well which was 10 m above the base before pumping dropped to 8 m. Take the

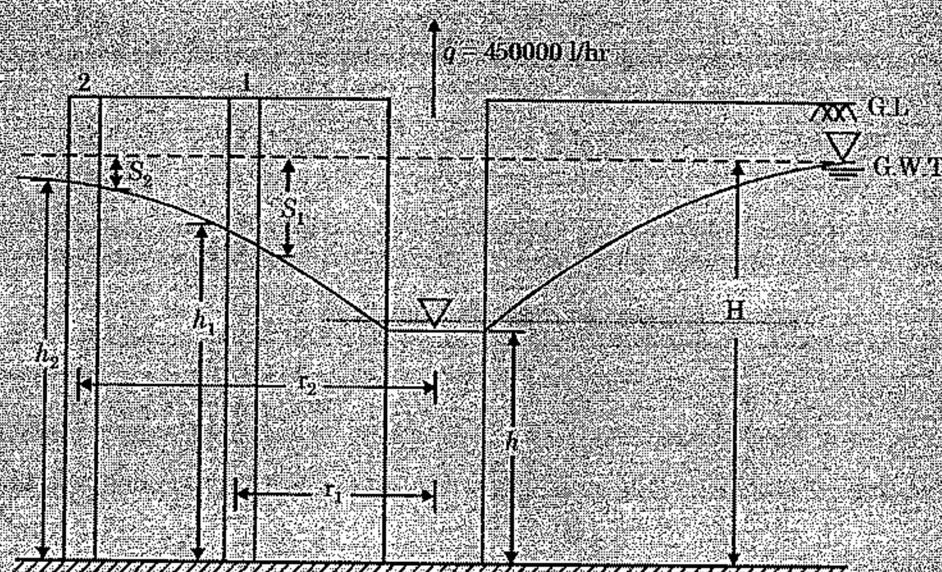
Sol.
$$k = \frac{q \log_e (R/r)}{2\pi b(D-h)}$$

or
$$k = \frac{0.020 \log_e (300/0.3)}{2\pi \times 5 \times (10-8)} = 0.0022 \text{ m/sec.}$$

Example 27

A well is fully penetrated into a 16 m thick layer of sand which is underlain by a rock layer water is pumped out of the well at contant rate of 450000l/hr. The water level in two observation wells situated at 15 m and 30 m from the test well are found to be 3.7m and 2.6 m respectively below the G.L. Determine the coefficient of permeability of the soil.

Sol.



Data given

$$r_1 = 15 \text{ m} = 1500 \text{ cm}$$
$$r_2 = 30 \text{ m} = 3000 \text{ cm}$$
$$h_1 = (16 - 3.7) \text{ m}$$
$$= 12.30 \text{ m} = 1230 \text{ cm}$$
$$h_2 = 16 - 2.6$$
$$= 13.4 \text{ m} = 1340 \text{ cm}$$
$$Q = 450000 \text{ litre/hr}$$
$$= 450000 \times \frac{10^3}{3600} = 125000 \text{ cc/sec}$$

$$K = \frac{Q \log_e \left(\frac{r_2}{r_1}\right)}{\pi(h_2^2 - h_1^2)} = \frac{125000 \times \log_e \left(\frac{3000}{1500}\right)}{\pi[(1340)^2 - (1230)^2]}$$

$$K = 0.095 \text{ cm/sec}$$

Example 28

A pumping out test was carried out in the field, in order to determine the average coeffcient of permeability of a 18 m thick sand layer. The ground water tabel is located at a depth of 2.2 m below the ground level. A Steady state was reached when the discharge from the well was 21.5 lit/sec. At this stage, the drawdown in the test well was 2.54 m, while the drawdowns in two observation well situated at 8 m and 20 m from the test well were found to be 1.76 m and 1.27 m respectively. Determine:

(i) Co-efficient of permeability of the sand layer in m/day

(ii) Radius of influence of test well

(iii) Effective size of the sand

Sol. For the unconfined aequifer we have

$$K = \frac{Q\log\left(\frac{r_2}{r_1}\right)}{\pi(h_2^2-h_1^2)} \quad \text{...(i)}$$

Data given

$$Q = 21.5 \text{ lit/sec}$$

$$= \frac{21.5\times10^{-3}}{\left(\frac{1}{86400}\right)} = 1857.6 \text{ m}^3\text{/day}$$

$$r_1 = 8 \text{ m } r_2 = 20 \text{ m}$$

∴ Height of the water table above the base of the well

$$H = (18-2.2) \text{ m} = 15.8 \text{ m}$$

Drawdown in the observation wells

$$S_1 = 1.76 \text{ m } S_2 = 1.27 \text{ m}$$

$$h_1 = (H - S_1) = (15.8 - 1.76) = 14.04 \text{ m}$$

$$h_2 = (H - S_2) = (15.8 - 1.27) = 14.53 \text{ m}$$

$$K = \frac{Q\log_e\left(\frac{r_2}{r_1}\right)}{\pi[(h_2)^2-(h_1)^2]}$$

$$= \frac{18576.\log_e\left(\frac{20}{8}\right)}{\pi[(014.53)^2-(14.04)^2]} = 38.70 \text{ m/day}$$

(ii) The Radius of influence line is given by

$$R = 3000\, S\sqrt{K}$$

$$K = \frac{38.70\text{m}}{(86400)\text{sec}} = 4.48\times10^{-4} \text{ m/sec}$$

$$S = 2.54 \text{ m},$$

$$R = 161.29 \text{ m}$$

(iii) The effective size can be determined from Allen Hazen's formula

$$K = CD_{10}^2$$

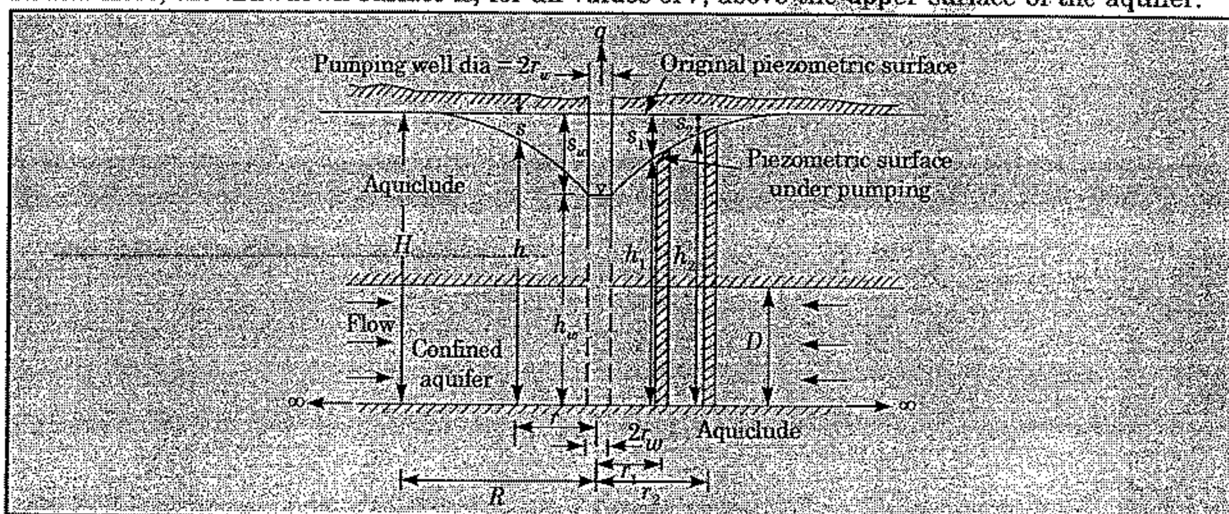
$$D_{10} = \sqrt{\frac{K}{C}}$$

Assuming $C = 100\ \text{cm}^{-1}\ \text{sec}^{-1}$

$$D_{10} = \sqrt{\frac{4.48\times10^{-4}}{100}} = 2.12\times10^{-3}\ \text{cm} = 0.0212\ \text{mm}$$

CONFINED AQUIFER

A confined flow condition occurs when the aquifer is confined both above and below by impermeable strata. Here, the drawdown surface is, for all values of r, above the upper surface of the aquifer.



According to Darcy's law

$$q = k\,i\,A,\quad A = 2\,\pi\,r\,D,\ q = 2\pi rD.K.\frac{dh}{dr}$$

$$\frac{qdr}{r} = 2\pi\,Dk\,dh$$

Integrating

$$q\int_{r_1}^{r_2}\frac{dr}{r} = 2\pi\,Dk\int_{h_1}^{h_2}dh$$

$$q\log_e\frac{r_2}{r_1} = 2\pi kD(h_2 - h_1)$$

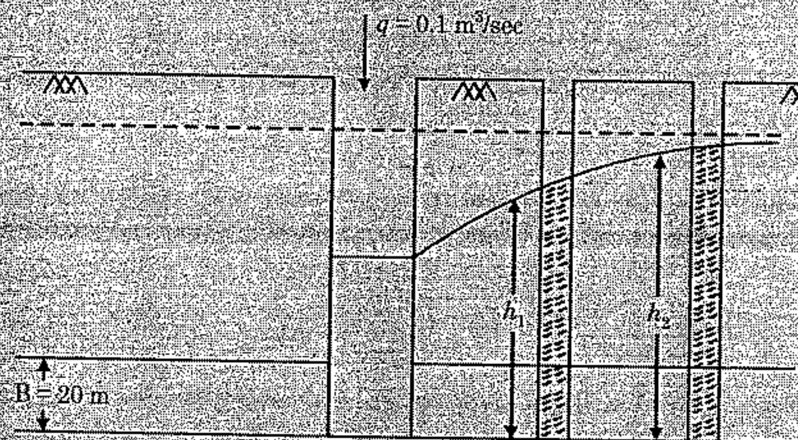
$$k = \frac{q\ \log_e r_2/r_1}{2\pi\,D(h_2 - h_1)}$$

$$\boxed{k = \frac{2.303\,q\log_{10}\left(\frac{r_2}{r_1}\right)}{2\pi D(h_2 - h_1)}}$$

Example 29

An acquifer of 20 m average thickness is overlain by an impermeable layer of 30 m thickness. A test well of 0.5 m diameter and two observation wells at a distance of 10 m and 60 m from the test well are drilled through the acquifer. After pumping at a rate of 0.1 m³/sec for a long time, the following draw down are stabilized in these wells: First observation well, 4m; second observation well, 3m. Show the arrangment in a diagram. Determine the coefficient of permeability and drawn down in the test well. State the validity of darcy law.

Sol.



Data given: $S_1 = 4\text{m} \quad S_2 = 3\text{m}$

We known for confined acquifer

$$K = \frac{q\log_e\left(\frac{r_2}{r_1}\right)}{2\pi B(h_2 - h_1)} \qquad [\because \quad T = kB]$$

$$\Rightarrow \quad q = \frac{2\pi T\,(S_1 - S_2)}{\log_e\left(\frac{r_2}{r_1}\right)} \qquad h_2 = H - S_2;\quad h_1 = H - S_1$$

$$\Rightarrow \quad 0.1 = \frac{2\pi T\,(4-3)}{\log_e\left(\frac{60}{10}\right)}$$

$$T = 0.0285 \text{ m}^2/\text{sec}$$

$$K = \left(\frac{T}{B}\right) = \frac{0.0285}{20} = 1.426\times10^{-3} \text{ m/sec}$$

$\Rightarrow$ Again

$$q = \frac{2\pi T\,(S_w - S_1)}{\log_e\left(\frac{r_1}{r_w}\right)}$$

$$r_w = \frac{0.5}{2} = 0.25$$

$$\Rightarrow \quad 0.1 = \frac{2\pi \times 0.0285\,(S_w - 4)}{\log_e\left(\frac{10}{0.25}\right)}$$

$$S_w = 6.06 \text{ m}$$

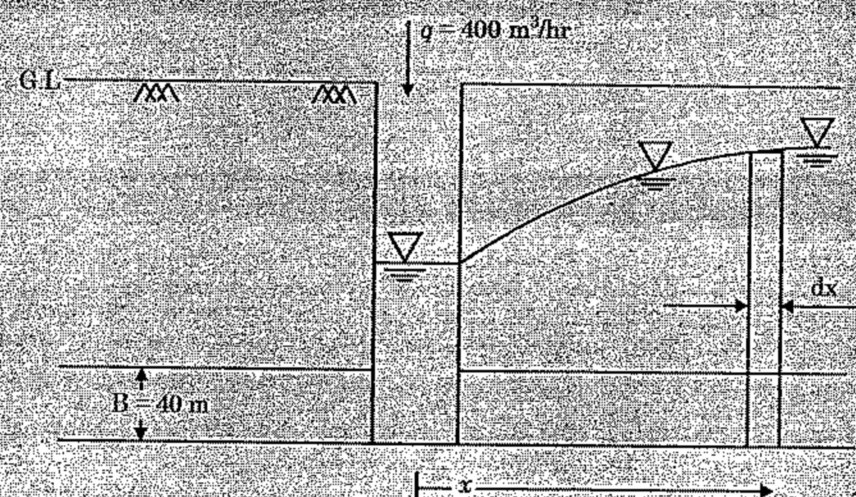
Darcy's law is valid only for saturated soils in which laminar flow condition prevails.

Example 30

A fully penetrated well of radius 0.2 m pumps at a constant rate of 400 m³/hr from a confined aquifer of sand of thickness 40 m and porosity 0.25 considering only the microscope flow velocity how long it will take a pollutant to reach the well if introduced into the acquifer, 100 m from it.

Sol. Data given

B = 40 m, n = 0.25, R = 100 m, r = 0.25, q = 400 m³/hr



We know that $V_S = \left(\frac{dx}{dt}\right)$ ... (i)

$\because$ q is constant

$$q = VA$$

$$q = V \times (2\pi x)\,B$$

$\because$ But seepage velocity $V_S = \left(\frac{V}{n}\right)$

$$q = V_S n\,(2\pi x)B$$

$$q = \left(-\frac{dx}{dt}\right) n \times (2\pi\,Bx)$$

$$\Rightarrow \quad q\int_0^t dt = -2\pi n B\int_R^r x\,dx$$

$$\Rightarrow \quad qt = \frac{2\pi\,Bn}{2}(R^2 - r^2)$$

$$\therefore \quad t = \frac{\pi n B}{q}(R^2 - r^2)$$

$$t = \frac{\pi \times 0.25 \times 40}{400}[(100)^2 - (0.2)^2]$$

$$t = 785.36 \text{ hrs.} \quad \text{or} \quad 32.72 \text{ days}$$

Thus pollutant will reach in 32.72 days.

PUMPING IN TEST

- Pumping-in tests are conducted to determine the coefficient of permeability of an individual stratum through which a hole is drilled. These tests are more economical than the pumping-out test.
- However, the pumping-out tests give more reliable values than that given by pumping-in tests.
- The pumping-in tests give the value of the coefficient of permeability of stratum just close to the hole, whereas the pumping-out tests give the value for a large area around the hole.
- There are basically two types of pumping-in tests :

  (1) Open-end tests,

  (2) Packer tests.

FACTORS AFFECTING PERMEABILITY

(1) *Particle size* :

- The coefficient of permeability of a soil is proportional to the square of a representative particle size. Empirical formulae was proposed by Allen Hazen.

$$k = CD_{10}^2$$

where,

k = permeability of soil, cm/sec

D_{10} = effective size of particle, mm

C = constant, range is (0.4 – 1.02), usually C = 1.

- An approximate equation $k = 100\,D_{10}^2$ is sometimes used, where k and D_{10} are in cm/s and cm respectively.
- Above equation holds good for $k \le 10^{-3}$ cm/s.

(2) *Effect of Void ratio* : According to Taylor

$$k \propto \frac{e^3}{1+e}$$

$$k = c\frac{e^3}{1+e^3}$$

Where, C = shape factor.

$$\boxed{\frac{k_1}{k_2} = \frac{C_1 \frac{e_1^3}{1+e_1}}{C_2 \frac{e_2^3}{1+e_2}}}$$

- C_1 and C_2 are shape factors that depends of manner of placing of grains and the shape characteristic of pores.
- For sand 'C' changes only slightly with void ratio i.e. $C_1 = C_2$

$\therefore$ For sand $\dfrac{k_1}{k_2} = \dfrac{e_1^3/1+e_1}{e_2^3/1+e_2}$

Note: Sometime $\dfrac{k_1}{k_2} = \dfrac{e_1^2}{e_2^2}$ used for sand.

For Silt and clays the relationship between e & k varies as $\dfrac{\log_{10} k_1}{\log_{10} k_2} = \dfrac{e_1}{e_2}$

---

(3) *Effect of Permeant (its viscosity and temperature)* :

- Permeability of soil also depends up on the density (or unit weight) of fluid and its viscosity.

$k \propto \dfrac{\gamma}{\mu}$

Where,

k = Permeability of Soil

γ = Unit weight of fluid, generally the fluid is water.

μ = Viscosity of the fluid.

$$\frac{k_1}{k_2} = \frac{\gamma_{w_1}}{\gamma_{w_2}} \cdot \frac{\mu_2}{\mu_1}$$

- It should be noted that 'k' is more affected by change in viscosity than change in unit weight, because unit weight of water does not change much over large range of temperature.

$\therefore \dfrac{k_1}{k_2} = \dfrac{\mu_2}{\mu_1}$

- From the above expression greater is the viscosity lower is the permeability.
- According to IS : 2720 Part 17, Permeability of soil shall be expressed at 27°c.

$$k_{27} = \frac{\mu_T}{\mu_{27}} k_T$$

where,

k_{27} = value of k at 27°C

k_T = value of k at T°C

μ_{27} = viscosity of permeant at 27°C

μ_T = viscosity of permeant at T°C

(4) *Shape of Particles* :

- Permeability of Soil is inversly proportional to the Specific Surface area.

$$k \propto \frac{I}{S_A}$$

- For the same void ratio, the soils with angular particles are less permeable than those with rounded particles as Angular particle have greater specific surface area in comparision to rounded particles.
- In a natural deposit, the void ratio for a soil with angular particles may be greater than that for rounded particles, and the soil with angular particles ...

(5) ***Degree of Saturation*** : The permeability of partially saturated soil is less than that of fully saturated soil.

(6) ***Effect of Soil Fabric :***

- In clays, influence of soil fabric on permeability is very important.
- Two clays at same void ratio, one having dispersed structure and other having flocculated structure have different permeability.
- Dispersion causes permeability to decreases basically because of reduction in size of void available for flow.

(7) ***Impurities in Water*** : Any foreign matter in water has a tendency to plug the flow passage and reduce the effective voids and hence the permeability of soils decreases.

(8) ***Effect of Adsorbed cation on Clay Mineral Surface*** : In clayey soils, the cation adsorbed on the surface of clay mineral also has influence on clay mineral.

- Depending on the type of cation adsorbed on the surface of Montmorillonite clay mineral, permeability increases in the following fashion.

$$K^+ < Na^+ < H^+ < Ca^{2+}$$

Simillarly for Kaolinite Mineral

$$Na^+ < K^+ < Ca^{2+} < H^+$$

Note: Na-clays have smaller permeability. Hence in clay core of earthen dam, soil is treated with salt water to inhibit seepage.

(9) ***Effect of Effective Stress*** : As effective stress increases, void ratio decreases and consequently permeability decreases.

Note: $k > 10^{-3}$ → determined by constant head method i.e. sand
$10^{-7} < k < 10^{-3}$ → determined by falling head
$k < 10^{-7}$ → determined from consolidation formula.

COFFICIENT OF PERMEABILITY OF INDIRECT METHODS.

(1) Kozeny – Carman Equation :

$$k = \frac{1}{C_S}\frac{\gamma_w}{\mu} \times \frac{1}{{S_A}^2} \times \frac{e^3}{1+e}$$

where,

k = Coefficient of Permeability

γ_w = Unit weight of water

μ = Coefficient of Viscosity

C_S = Shape factor Coefficient

e = Void ratio

S_A = Specific Surface area i.e. surface area per unit volume of solid.

Note: Specific surface area for spherical particle, $S = \dfrac{\pi D^2}{\pi D^3/6} = \dfrac{6}{D}$, If particle size is between a & b then Specific Surface area = $\dfrac{6}{\sqrt{ab}}$.

(2) Allen Hazen's Formula : The coefficient of permeability of a soil is proportional to the square of a representative particle size.

$$K = CD_{10}^2$$

(3) Loudon's Formula :

According to Loudon.

$$\log_{10}(KS_A^2) = a + bn$$

Where

K = Coefficient of permeability

S_A = Specific Surface area

n = Porosity

a & b = Constants

(4) Consolidation Equation : This method is suitable for fine grained soil whose $k < 10^{-7}$ cm/sec. for which permeabeality test can not be conducted in laboratory.

$$K = C_v \gamma_w m_v$$

where,

K = Permeability of soil

C_v = Coefficient of consolidation

γ_w = Unit weight of water

m_v = Coefficient of volume compressibility

Example 31

Estimate the value of coefficient of permeability for a uniform graded sand of size D_{10} = 0.15 mm obtained from sieve analysis. G = 2.67

Sol. We know that

$$k = C\ D_{10}^2$$

where C is a constant whose value is 1

$$k = 1 \times (0.15)^2$$

$$k = 2.25 \text{ mm/sec.}$$

Example 32

The Natural ground water table at a site is located at a depth of 2 m below the ground level. Laboratory tests reveal that the void ratio of the soil in 0.85 while the grain size corresponding to 10% fines is 0.05 mm. Determine the depth of the zone of saturation below G.L assume $c = 0.3$ cm².

Sol. Data given:

$$e = 0.85$$

$$D_{10} = 0.05 \text{ mm} = 0.05 \text{ cm}$$

$$C = 0.3 \text{ cm}^2$$

∴ The height of capillary rise of water is given by

$$h_c = \left(\frac{C}{eD_{10}}\right)$$

$$= \frac{0.3}{(0.85\times 0.005)} = 70.59 \text{ cm}$$

$$= 0.706 \text{ m}$$

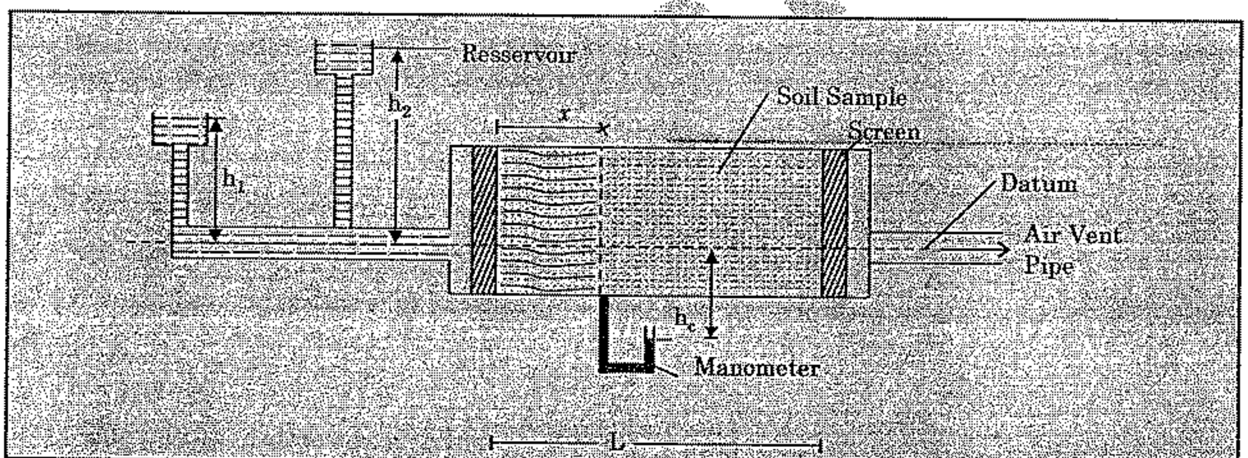
Hence depth of saturation below G.L.

$$= (2 - 0.706)$$

$$= 1.294 \text{ m}$$

CAPILLARITY PERMEABILITY TEST

- The coefficient of permeability for unsaturated soil can be determined by capillarity permeability test.
- A sample of partially saturated soil is placed in a 35 cm long and 4 cm diameter glass tube with screens fixed at both ends.



- Initially valve connecting to higher reservoir is closed and valve of the lower reservoir is opened.
- As the valve of lower reservoir is opened, capillary action in the soil starts drawing water into it and wetted surface advances towards the open end.
- Let the capillary head be h_c over the distance x from the left end screen. This negative capillary head causes increase in total head.

∴ $$h = h_1 + h_c$$

- Assuming uniform hydraulic gradient over the entire length x, the velocity is given by Darcy's law.

$$V = ki = k\frac{(h_1 + h_c)}{x}$$

- The wetted surface moves further with a seepage velocity V_s we know that $V_s = \frac{V}{n}$
- But for partially saturated soil above equation is modified by taking actual saturated porosity. as $(S \times n)$. Where S = degree of saturation.

$$V_s = \frac{k}{\alpha}\cdot\left(\frac{h_1 + h_c}{}\right.$$

$$V_S = \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{k}{Sn}\left(\frac{h_1 + h_c}{x}\right)$$

$$x\,d\,x = \frac{k}{Sn}(h_1 + h_c)dt$$

$$\int_{x_1}^{x_2} xdx = \frac{k}{Sn}(h_1 + h_c)\int_{t_1}^{t_2} dt$$

$$\boxed{\frac{x_2^2 - x_1^2}{t_2 - t_1} = \frac{2k}{Sn}(h_1 + h_c)}$$

- Actually capillary head is not known accurately, there fore we can not calculate the valve of k. To over come this difficulty we close the valve connecting low reservoir and open the valve connecting high reservoir and perform the same procedure. Therefore we get a second equation also. In time t_3 to t_4 let x_3 & x_4 be the distance measured from left end.

$$\therefore \quad \boxed{\frac{x_4^2 - x_1^2}{t_2 - t_1} = \frac{2k}{Sn}(h_1 + h_c)}$$

- The values of k and h_c can be obtained from above two equations.

Example 33

A capillary permeability test was conducted in two stages under a head of 60 cm and 180 cm, respectively, at the entry end. In the first stage, the wetted surface moved from 1.5 to 7 cm in minutes. In the second stage, it advanced from 7 cm to 18.5 cm in 24 minutes. The degree of saturation at the end of the test was 85% and the porosity was 35%. Determine the capillary head and the coefficient of permeability.

Sol. Given

Case I

head $h_1 = 60$ cm

$S = 85\%$

$n = 35\%$

$x_2 = 7$ cm

$x_1 = 1.5$ cm

$t_2 - t_1 = 7$ min

Case II

head $h_2 = 180$ cm

$x_4 = 18.5$ cm

$x_3 = 7.0$ cm

$S = 85\%$

$n = 35\%$

$t_4 - t_1 = 24$ min

Acc to capillary Permeability test

Case I

$$\frac{x_2^2 - x_1^2}{t_2 - t_1} = \frac{2k}{Sn}(h_1 + h_c)$$

$$\frac{7^2 - 1.5^2}{7} = \frac{2k}{0.85 \times 0.35}(60 + h_c) \quad \text{.... (i)}$$

Case II

$$\frac{x_4^2 - x_3^2}{t_4 - t_3} = \frac{2k}{Sn}(h_2 + h_c) \quad \text{.... (ii)}$$

$$\frac{18.5^2 - 7^2}{24} = \frac{2k}{0.85 \times 0.35}(180 + h_c) \quad \text{..... (iii)}$$

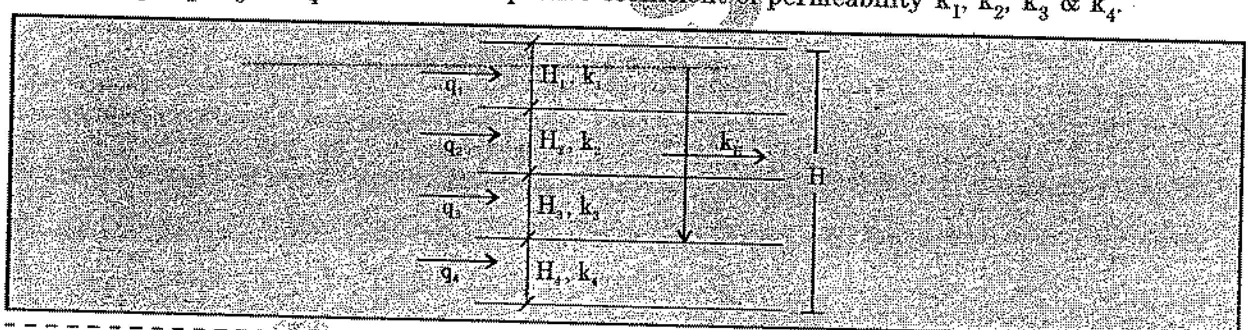
From solving (i) & (ii) we get

$$h_c = 84.64 \text{ cm}$$

$$k = 6.81 \times 10^{-3} \text{ cm/min.}$$

PERMEABILITY OF STRATIFIED SOIL

- Actually in field soil is present in the form of different stratum which has different Permeabilities.
- For the calculation of discharge average values of permeabilities are found out for the whose deposit.
- There are two such average values : the average horizontal coefficient of permeability, (k_H) when the flow is parallel to the strata and the average vertical coefficient of permeability, (k_V) when the flow is normal to the strata.
- Consider a section of stratified soil as shown in figure below of varying thickness of each stratum eg : H_1, H_2, H_3 & H_4. with their respective coefficient of permeability k_1, k_2, k_3 & k_4.



Note: It is assumed that within each stratum, the permeability is same for both horizontal and vertical flow.

(1) *Horizontal Flow (Parallel to bedding plane)* :

- Flow is taking place through all the layers at same time hence hydraulic gradient is same in each layer.

$$i_1 = i_2 = i_3 = i_4 = i \text{ (constant)}$$

Average discharge velocity over the soil mass can be written as.

$$V = k_H i = \frac{1}{H}(V_1H_1 + V_2H_2 + V_3H_3 + V_4H_4)$$

$$= \frac{1}{H} k_1i_1H_1 + k_2i_2H_2 + k_3i_3H_3 + k_4i_4H_4$$

$$k_H i = \frac{(k_1H_1 + k_2H_2 + k_3H_3 + k_4H_4)}{H} i$$

$$\boxed{k_H = \frac{(k_1H_1 + k_2H_2 + k_3H_3 + k_4H_4)}{H_1 + H_2 + H_3 + H_4}}$$

(2) *Vertical flow (Normal to bedding Plane)* :

- Let i_1, i_2, i_3 & i_4 be the hydraullic gradient in different layers of thickness H_1, H_2, H_3 & H_4 respectively.
- Let the total head loss be h over the total thickness of soil stratum H.
- Each layer having head loss h_1, h_2, h_3 & h_4. Then constant velocity of flow is given by.

$$V = k_V \frac{h}{H} = k_1 i_1 = k_2 i_2 = k_3 i_3 = k_4 i_4$$

- Head loss in any layer is given by.

$$i_n = \frac{h_n}{H_n}$$

$$\because \quad Q = k_iA = k_1 i_1 A = k_2 i_2 A = k_3 i_3 A = k_4 i_4 A$$

$$\therefore \quad ki = k_1 i_1 = k_2 i_2 = k_3 i_3 = k_4 i_4$$

$$\frac{K.h}{H} = \frac{k_1 h_1}{H_1} = \frac{k_2 h_2}{H_2} = \frac{k_3 h_3}{H_3} = \frac{k_4 h_4}{H_4}$$

$$h_1 + h_2 + h_3 + h_4 = h$$

$$h\left(\frac{k H_1}{H k_1} + \frac{k H_2}{H k_2} + \frac{k H_3}{H k_3} + \frac{k H_4}{H k_4}\right) = h$$

$$\boxed{k_V = \frac{H}{\frac{H_1}{k_1} + \frac{H_2}{k_2} + \frac{H_3}{k_3} + \frac{H_4}{k_4}}}$$

Note: k_H is always greater than k_V.

Example 34

What will be the ratio of average permeability in the horizontal direction to that in the vertical direction for a soil deposit consisting of three horizontal layers if the thickness and permeability of the second layer are twice those of the first and those of the third layer twice those of the second?

Sol. Given.

	Thickness	Permeability
layer 1	x	$K_1 = y$
layer 2	$2x$	$K_2 = 2y$
layer 3	$4x$	$K_3 = 4y$

$$K_H = \frac{K_1 x + K_2 . 2x + k_3 . 4x}{x + 2x + 4x}$$

$$K_H = \frac{yx + 4yx + 16yx}{7x} = \frac{21}{7}y$$

$$K_V = \frac{x + 2x + 4x}{\frac{x}{K_1} + \frac{2x}{K_2} + \frac{4x}{K_3}}$$

$$K_V = \frac{7x}{\frac{x}{y} + \frac{2x}{2y} + \frac{4x}{4y}}$$

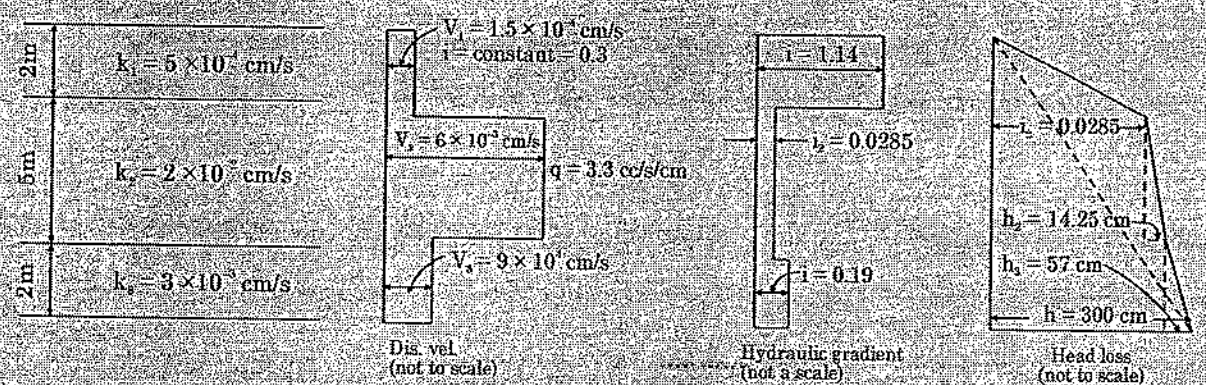
$$K_V = \frac{7xy}{3x}$$

$$K_V = \frac{7}{3}y$$

$$\frac{K_H}{K_V} = \frac{21/7\,y}{7/3\,y} = \frac{63}{49} = \frac{9}{7}$$

Example 35

A stratified soil deposit is shown in Fig. along with the coefficients of permeability of the individual strata. Determine the ration of k_H to k_V. Assuming an average hydraulic gradient of 0.3 in both horizontal and vertical seepage, find (*i*) discharge value and discharge velocities in each layer for horizontal flow, and (ii) hydraulic gradient and loss in head in each layer for vertical flow.



(a) Horizontal flow (b) Vertical flow

Sol. Average coefficient of permeability for horizontal flow,

$$k_H = \frac{k_1 H_1 + k_2 H_2 + k_3 H_3}{H}$$

$$= \frac{5\times10^{-4}\times200 + 2\times10^{-2}\times500 + 3\times10^{-3}\times300}{1000} = 1.1\times10^{-2}\ \text{cm/s}$$

Average coefficient of permeability for vertical flow,

$$k_V = \frac{H}{\frac{H_1}{k_1} + \frac{H_2}{k_2} + \frac{H_3}{k_3}} = \frac{1000}{\frac{200}{5\times10^{-4}} + \frac{500}{2\times10^{-2}} + \frac{300}{3\times10^{-3}}} = 1.9\times10^{-3}\ \text{cms}$$

$$\therefore \quad \frac{k_H}{k_V} = \frac{1.1\times10^{-2}}{1.9\times10^{-3}} = 5.79$$

(i) Horizontal flow

$$q = k_H i A = 1.1 \times 10^{-2} \times 0.3 \times 10 \times 10^2 = 3.3 \text{ cc/s/cm width}$$

$$v_1 = k_1 i = 5 \times 10^{-4} \times 0.3 = 1.5 \times 10^{-4} \text{ cm/s}$$

$$v_2 = k_2 i = 2 \times 10^{-2} \times 0.3 = 6.0 \times 10^{-3} \text{ cm/s}$$

$$v_3 = k_3 i = 3 \times 10^{-3} \times 0.3 = 9.0 \times 10^{-4} \text{ cm/s}$$

(for horizontal flow, the gradient of flow is the same but the velocity of flow is different for different layers).

$$v = k_V i = v_1 = k_1 i_1 = v_2 = k_2 i_2 = v_3 = k_3 i_3$$

(For vertical flow, velocity of flow is the same because of continuity of flow, but gradients are different for different layers)

$$v = 1.9 \times 10^{-3} \times 0.3 = 5.7 \times 10^{-4} \text{ cm/s}$$

$$i_1 = \frac{5.7\times10^{-4}}{5\times10^{-4}} = 1.14;$$

$$i_2 = \frac{5.7\times10^{-4}}{2\times10^{-2}} = 0.0285;$$

$$i_3 = \frac{5.7\times10^{-4}}{3\times10^{-3}} = 0.19$$

h_1 (head loss in layer 1) $= i_1 H_1 = 1.14 \times 200 = 228$ cm

Similarly,

$$h_2 = 0.0285 \times 500 = 14.25 \text{ cm}$$

and

$$h_3 = 0.19 \times 300 = 57 \text{ cm}$$

Check : Total head loss $= 228 + 14.25 + 57 = 299.25$ cm ≈ 0.3 $(i) \times 1000$ (H)

If we have to determine i_c only, then what role does depth of sand come in picture

OBJECTIVE TYPE QUESTIONS

1. A uniform sand stratum 2.5 m thick has a specific gravity of 2.62 and a natural void ratio of 0.62. The hydraulic head required to cause quick sand condition in the sand stratum is
   (a) 0.5 m
   (b) 1.5 m
   (c) 2.5 m
   (d) 3.5 m

2. Which one of the following equations correctly gives the relationship between the specific gravity of soil grains (G) and the hydraulic gradient (i) to initiate 'quick' condition in a sand having a void ratio of 0.5?
   (a) $G = 0.5i + 1$
   (b) $G = i + 0.5$
   (c) $G = 1.5i + 1$
   (d) $G = 1.5i - 1$

3. The following data were obtained when a sample of medium sand was tested in a constant head permeameter:
   Cross-section area of sample : 100 cm^2
   Hydraulic gradient : 10
   Discharge collected : 10 cc/s
   The coefficient of permeability of the sand is
   (a) 0.1 m/s
   (b) 0.01 m/s
   (c) 1×10^{-4} m/s
   (d) 1×10^{-8} m/s

4. A stratified soil deposit has three layers of thicknesses : $z_1 = 4$, $z_2 = 1$, $z_3 = 2$ units and the corresponding permeabilities of $k_1 = 2$, $k_2 = 1$ and $k_3 = 4$ units, respectively. The average permeability perpendicular to the bedding planes will be
   (a) 4
   (b) 2
   (c) 8
   (d) 16

5. Consider the following statements:
   1. Coarse sand is more than a million times permeable than a high plasticity clay.
   2. The permeability depends on the nature of soil and not on properties of liquid flowing through soil.
   3. If a sample of sand and a sample of clay have the same void ratio, both samples will exhibit the same permeability.
   4. Permeability of soil decreases as the effective stress acting on the soil increases.

   Which of these statements are correct?
   (a) 1 and 2
   (b) 1 and 3
   (c) 1 and 4
   (d) 2 and 3

6. In a falling head permeability test on a soil, the time taken for the head to fall from h_0 to h_1 is t. The test is repeated with same initial head h_0, the final head h' is noted in time t/2. Which one of the following equations gives the relation between h', h_0 and h_1?
   (a) $h' = h_0/h_1$
   (b) $h' = \sqrt{h_0/h_1}$
   (c) $h' = h_0 h_1$
   (d) $h' = \sqrt{h_0 h_1}$

7. Consider the following statements:
   1. Organic matter increases the permeability of a soil
   2. Entrapped air decreases the permeability of a soil.

   Which of these statements is/are correct?

   (a) 1 only (b) 2 only
   (c) Both 1 and 2 (d) Neither 1 nor 2

8. In a 6m thick stratum of fine sand having submerged density of 11 kN/m^3, quicksand condition occurred at a depth of 4.2 m of excavation. What is the depth of lowering of groundwater table required for making an excavation 5 m deep? Take density of water as 10 kN/m^3

   (a) 3.85 m (b) 1.68 m
   (c) 1.1 m (d) 0.897 m

9. In a typical deposit of submerged soil, the approximate depth at which the intergranular pressure is equal to 50 kN/m^2 is

   (a) 2.5 m (b) 5 m
   (c) 7.5 m (d) 10 m

10. The water table at a location is at the ground surface and the saturated unit weight of the soil is 20 kN/m^3. If due to heavy precipitation, the water level rises to 2 m above the ground level, the increase in the vertical effective stress at a point 2 m below the ground surface will be

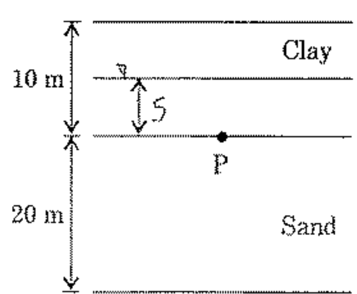
    (a) 40 kN/m^2 (b) 20 kN/m^2
    (c) 10 kN/m^2 (d) 0

11. Consider the following statements:
    1. In a sedimentary soil deposit permeability in the horizontal direction is greater than that in the vertical direction.
    2. Permeability of two sand specimens having equal void ratio will be always same.

    Which of these statements is/are correct?

    (a) 1 only (b) 2 only
    (c) Both 1 and 2 (d) Neither 1 nor 2

12. A 10m thick clay layer is underlain by a sand layer of 20 m depth (see figure below). The water table is 5 m below the surface of clay layer. The soil above the water table is capillary saturated. The value of γ_{sat} is 19 kN/m^3. The unit weight of water is γ_w. If now the water table rises to the surface, the effective stress at a point P on the interface will



    (a) increase by $5\gamma_w$ (b) remain unchanged
    (c) decrease by $5\gamma_w$ (d) decrease by $10\gamma_w$

13. Consider the following factors pertaining to flow through soil:

    1. Hydraulic gradient
    2. Grain size
    3. Void ratio
    4. Cross-sectional area of the sample

    Of these, the factors affecting permeability include

    (a) 1 and 4
    (b) 2 and 3
    (c) 1, 2 and 3
    (d) 2, 3 and 4

14. A sample of clay and a sample of sand have the same specific gravity and void ratio. Their permeabilities would differ because

    (a) their porosities would be different
    (b) their degrees of saturation would be different
    (c) their densities would be different
    (d) the size ranges of their void would be different

15. A particular soil sample is subjected to test for the determination of permeability coefficient in two separate constant head permeameters, whose specifications are as under:

	Permeameter A	Permeameter B
Diameter of sample	D	2 D
Length of sample	2L	L

If the tests on both the permeameters are conducted with equal head of water applied on the samples, then the ratio of amount of water discharged through the permeameters A and B during a period of one hour will be

(a) 4.000
(b) 1.000
(c) 0.250
(d) 0.125

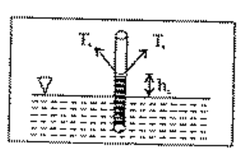
16. Consider the following statements regarding the principle of effective stress:

    1. Contact stress between soil grains is called the effective stress.
    2. It is not possible to physically measure the effective stress.
    3. The equation is not strictly applicable to a partially saturated soil.

    Which of these statements are correct?

    (a) 2 and 3
    (b) 1 and 2
    (c) 1 and 3
    (d) 1, 2 and 3

17. The capillary rise h_c in a pipe of diameter 'd' shown in the given figure is



(a) $\frac{4T_S}{\gamma_W d}$
(b) $\frac{2T_S}{\gamma_W d}$
(c) $\frac{T_S}{2\gamma_W d}$
(d) $\frac{T_S}{4\gamma_W d}$

18. The water level in a lake is 5 m above the bed. The saturated unit weight of the lake bed soil is 20 kN/m^3. The unit weight of water is 10 kN/m^3. The effective vertical stress at 5 m depth below the lake bed is

(a) 50 kN/m^2
(b) 75 kN/m^2
(c) 100 kN/m^2
(d) 150 kN/m^2

19. Consider the following statements:
Capillary water in soils

1. causes negative pore water pressure.
2. reduces effective pressure
3. increases bearing capacity
4. reduces bearing capacity

Which of these statements are correct?

(a) 1 and 3
(b) 1 and 4
(c) 2 and 3
(d) 2 and 4

20. A fully saturated capillary zone of thickness 2 m exists above water table in a fine silty sand deposit. What is the pore water pressure at 1.5 m above the water table?

(a) 5 kN/m2
(b) –15 kN/m2
(c) –5 kN/m2
(d) 15 kN/m2

21. Consider the following statements :
In soils the coefficient of permeability, K, varies

1. directly with the square of particle size
2. inversely with the square of particle size
3. inversely with the square of specific surface area

Which of these statements is/are correct?

(a) 1 and 3
(b) 3 only
(c) 2 and 3
(d) 1 only

22. Which of the following occurs when the ground water table arises above the ground surface?

(a) An equal increase in pore water pressure and total stress
(b) An equal decrease in pore water pressure and total stress
(c) A decrease in pore water pressure but an increase in total stress
(d) An increase in pore water pressure but a decrease in total stress

Instructions :

The following items consists of two statements, one labelled as ***'Assertion A'*** *and the other labelled as* ***'Reason R'****. You are to examine these two statements carefully and decide if the Assertion A and the Reason R are individually true and if so, whether the Reason is a correct explanation of the Assertion. Select your answers to the these items using the codes given below :*

(a) both A and R are true and R is the correct explanation of A
(b) both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

23. **Assertion (A):** In compacted fills the permeability in horizontal direction is less than the permeability in vertical direction.

Reason (R): By compaction of fills stratifications are formed

24. **Assertion (A):** Lowering of water table results in settlement of the ground surface.

Reason (R): Lowering of water table causes decrease in effective weight of soil between the original and final positions of the water table.

25. **Assertion (A):** When a saturated soil mass is subjected to consolidation, its volume at any instant is related to the total stress.

Reason (R): Total stress is equal to the sum of the effective stress and pore water pressure.

26. **Assertion (A):** Quick sand is not a type of sand but it is a condition arising in a sand mass.

Reason (R): When the upward seepage pressure becomes equal to the pressure due to submerged weight of a soil, the effective pressure becomes zero.

27. **Assertion (A):** The possibility of quicksand condition occurring is more on the downstream of a weir on permeable foundation.

Reason (R): Seepage lines are directed upwards at the downstream of such a weir.

28. **Assertion (A):** Effective vertical stress at some depth below a river bed is unaffected by the water depth in the river.

Reason (R): Equal amounts of increase in total stress and pore pressure will not change the effective stress.

29. **Assertion (A):** Permanent lowering of GWT results in settlements.

Reason (R): Increase in effective stress results in settlement of soils.

ANSWERS

1. (c)	**2.** (c)	**3.** (c)	**4.** (b)	**5.** (c)	**6.** (d)	**7.** (b)	**8.** (b)	**9.** (b)	**10.** (d)
11. (a)	**12.** (c)	**13.** (b)	**14.** (d)	**15.** (d)	**16.** (a)	**17.** (a)	**18.** (a)	**19.** (a)	**20.** (b)
21. (a)	**22.** (a)	**23.** (d)	**24.** (c)	**25.** (d)	**26.** (a)	**27.** (a)	**28.** (a)	**29.** (a)	

Hints

1. $i_c = \dfrac{G-1}{1+e} = \dfrac{2.62-1}{1+0.62}$

$i_c = 1$

2. $i_c = \dfrac{G-1}{1+e} = \dfrac{G-1}{1.5}$

$1.5i + 1 = G$

3. $k = \dfrac{qL}{Ah} = \dfrac{10}{100 \times 10} = 10^{-2}$ cm/sec

$= 10^{-4}$ m/sec

4. $k_v = \dfrac{H_1 + H_2 + H_3}{\dfrac{H_1}{k_1} + \dfrac{H_2}{k_2} + \dfrac{H_3}{k_3}}$; $k_v = \dfrac{4+1+2}{\dfrac{4}{2} + \dfrac{1}{1} + \dfrac{2}{4}}$

$k_v = \dfrac{7}{3.5} = 2$ units

8. $0.8 \times \gamma_{sat} = x \times 10$

$\dfrac{0.8 \times 21}{10} = x$

$x = 1.68$

9. $\bar{\sigma} = \gamma_{sub} \times x$

$\dfrac{50}{10} = x = 5$

15. $q \propto \dfrac{A}{L}$

CHAPTER

6

Seepage through Soil

INTRODUCTION

- Seepage is a process in which liquid leaks through a porous medium from high head to towards low head.

  Although flow velocity of water during seepage is very small (as this flow is Laminar) but in reality it poses many problems such as

  (1) Losses of water Reservoir
  (2) Reduction in effective weight of soil.
  (3) Uplift pressure below dam.
  (4) Piping failure.

- To find the solution of above disscussed problems and many more it becomes important to calculate the seepage flow.
- Seepage flow is calculated with the help of flownets. Where flownet is a graphical representation of path taken by water particle and head variation along the path.
- The concept of flow net is based on Laplace's equation of contunity.

Note: Only two dimensional seepage flow is in our course so we will restrict our discussion to 2-D flow. Where as seepage flow is generally 3-D flow. But analysis of 3-D flow is complex and out of our syllabus.

LAPLACE EQUATION – 2 D FLOW CONDITIONS

Following Assumptions are made in 2D flow.

1. Soil mass is fully saturated and darcy's law is valid.

$$v_x = K_X i_X = K_X \frac{dh}{dx} \quad \text{... (1)}$$

$$v_y = K_Y i_Y = \frac{K_y dh}{dy} \quad \text{... (2)}$$

2. Soil mass is homogeneous and isotropic
3. Both soil grains and pore fluid are incompressible.
4. Steady state condition exist (i.e. flow condition does not change with time)

From the continuity equation

$$\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} = 0$$

By puting (1) & (2) in above equation

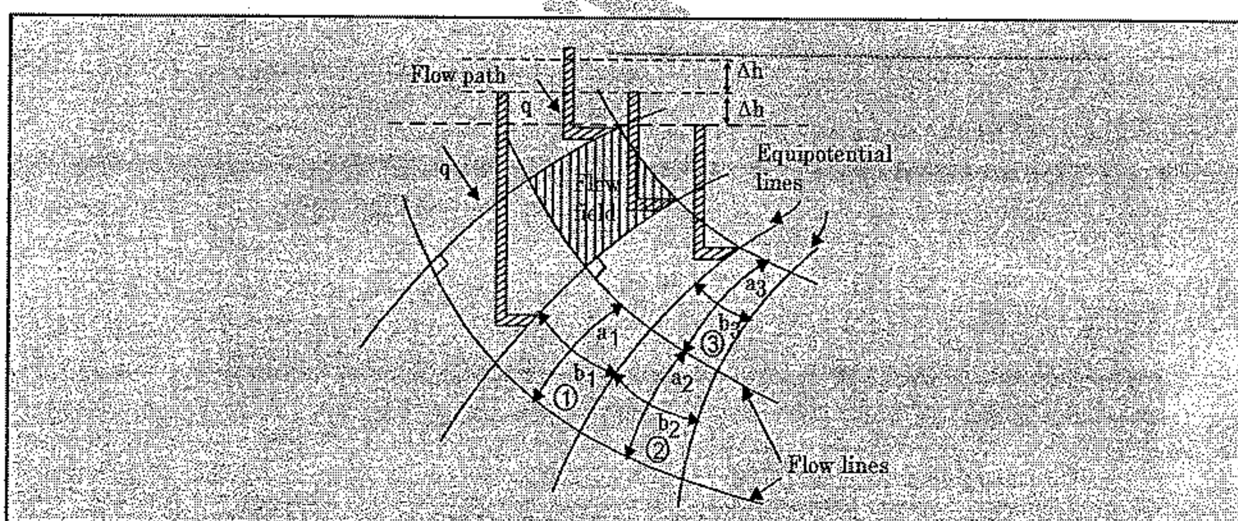
$$\boxed{K_X \frac{\partial^2 h}{\partial X^2} + K_y \frac{\partial^2 h}{\partial y^2} = 0}$$

- For isotropic soil $K_x = K_y = K$

$$\boxed{\frac{\partial^2 h}{\partial X^2} + \frac{\partial^2 h}{\partial y^2} = 0} \text{ Laplace equation}$$

- General solution of Laplace equation yields two sets of curve which are orthogonal to each other of these one set of curve is known as **flow lines** and other set is known as **equipotential lines.**
- Flow lines is a line which indicates direction of flow of the water particle.
- Equipotential line is the **line joining points of equal total head.**
- These two sets of curves **flow lines** and **equipotental lines** form a **flow net** which is a graphical representation of how the hydraulic energy is dispated as water flows through a pervious medium.

Properties and use of flow net



1. Flow lines and equipotential lines are orthogonal to each other in case of isotropic soil.
2. Space between two adjacent flowlines is called flow channel or flow path.
3. The figure formed in flow net between two adjacent flowlines and adjacent equipotential line is called flow field.
4. All flow fields are elemantry squares (linear or curvilinear)

i.e.,
$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$

5. Head loss through each successive equipotential line is equal $\Delta h_1 = \Delta h_2 = \Delta h_3 = \Delta h$

6. Discharge through each flow channel is constant

i.e, $\Delta q_1 = \Delta q_2 = \Delta q_3 = \Delta q$

Consider flow fields (1), (2) & (3) as shown in the diagram above.

According to darcy's law

$$Q = K\,i\,A$$

Consider flow per unit width perpendicular to the plane of the section (normal to the plane of paper)

$$\therefore \quad \Delta q_1 = K\frac{\Delta h_1}{b_1}\times a_1\times 1$$

$$\Delta q_2 = K\frac{\Delta h_2}{b_2}\times a_2\times 1$$

$$\Delta q_3 = K\frac{\Delta h_3}{b_3}\times a_3\times 1$$

Flow fields 1 & 2 are with in same flow channel Hence, $\Delta q_1 = \Delta q_2$. Because there can be no flow across the flow lines.

$\because$ Flow field 2 & 3 are with in same equipotential lines thus, $\Delta h_2 = \Delta h_3$

We know that $$\frac{a_2}{b_2} = \frac{a_3}{a_3}$$

$$\therefore \quad q_2 = q_3 = \Delta q_1$$

- Thus, it is observed that, if all the flow fields constructed in a flow net are elementry square, discharge and head loss though the flow field will be constant

CALCULATION FOR TOTAL DISCHARGE THROUGH A FLOW NET

- We know that discharge through a flow channel is

$$\Delta q = k\frac{\Delta h}{b}\times(a\times 1)$$

- Let total head loss through the flow net be H and N_d be the number of equipotential drop, N_d = No. of equipotential lines -1,

Then, $$\Delta q = k\frac{H}{N_d}\times\frac{a}{b}$$

- If N_f is the number of flow channels in a flow net where N_f = (No. of flow lines — 1)
Then total discharge through a flow net will be

$$Q = \Delta q \times N_f$$

$$Q = k\times\frac{H}{N_d}\times N_f\times\frac{a}{b}$$

$$Q = KH\frac{N_f}{N_d}\times\frac{a}{b}$$

- Flow net is generally drawn such that $\frac{a}{b} = 1$

Hence, $\boxed{q = KH\frac{N_f}{N_d}}$

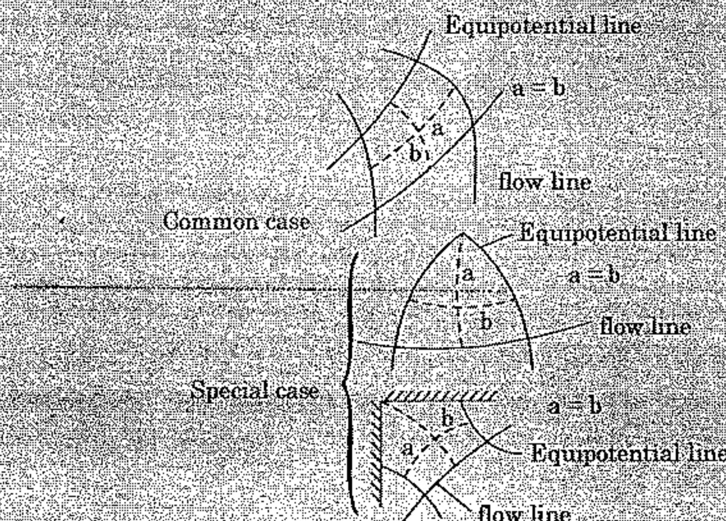
Note: If however, $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} = n$

In that case,

$$q = KH\frac{N_f}{N_d} \times n$$

Elementry square is not a real square, it only means that opposite side of square are equal i.e. a = b.

Infact, many of the flow fields are far from resembeling square, but they still conform to definition of elemantry square.



- From the above discussion we can conclude that rate of flow is a function of

(i) Permeability (k).
(ii) Headloss (H).
(iii Shape Factor $\left(\frac{N_f}{N_d}\right)$.

SILENT POINTS ABOUT FLOW NET

(i) Shape factor is only function of boundary condition.
(ii) Flow Net will not change if Permeability (k) of soil changes i.e. Soil is changed.
(iii) Flow net will not change if head loss during flow is changed.
(iv) In both the above cases (i.e. (ii) & (iii)) the quantity of seepage will be different.

(v) Flow net is unique for a given boundary condition and if the boundary, condition does not change $\frac{N_f}{N_d}$ will not change.

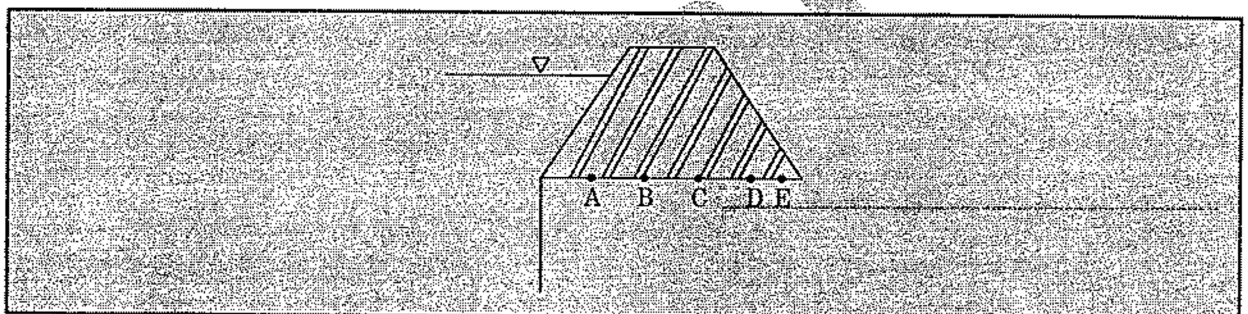
(vi) Note that even if no. of flow lines or no. of equipotential lines are changed and if boundary condition does not change, $\frac{N_f}{N_d}$ will not change.

USE OF FLOW NET

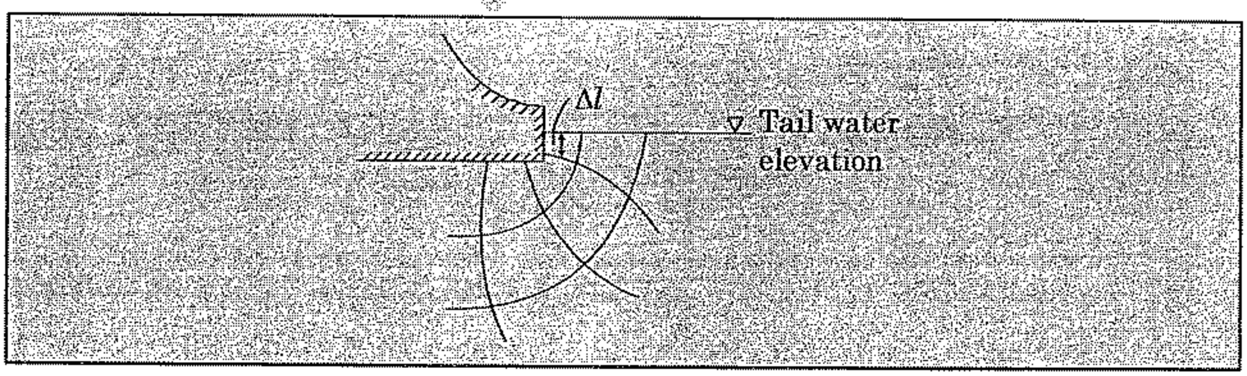
1. Seepage calculation, $\boxed{q = Kh\frac{N_f}{N_d}}$

2. **Uplift pressure calcualtion: Uplift pressure** at any point is the **pore water pressure** acting vertically upward **due to residual pressure head at that point.**

3. **Uplift pressure** along the base of a **masonary dam** can be effectively reduced by providing **vertical cut off wall at the u/s** end of the base of dam. Vertical cut off wall increases the flow length and much of the head will, have been lost upto A, B, C, D etc. Hence pressure at A, B, C, D reduces.



4. Calculation of exit gradient and determination of piping.



- Hydraulic gradient of flow will be max adjacent to the toe because here square is smallest in size.
- When upward hydraulic gradient approaches unity, boiling condition can occur, leading to erosion of soil and **piping.**
- Piping begins near the downstream toe but may lengthen Progressively towards the u/s side as seeping water gradually washes away more and more soil particles, leaving voids or pipes in the soil. Piping may work its way backwards along the base of the dam or along a bedding plane in the soil strata where the resistance is minimum.

- If piping is not stopped it may result in catastrophic collapse.
- FOS against piping is taken conservatively as 6. Hydraulic gradient at exit is called exit hydraulic gradient (i_e)
- F.O.S. = $\frac{i_C}{i_e}$, when i_C = critical hydraulic gradient
- Exit gradient can be reduced by providing **vertical cut off walls** at the downstream end of the base of the dam.
- Another kind of piping failure is caused by heave which occurs on the downstream of hydraulic structure in which upward acting seepage force becomes greater than the downward acting submerged weight of soil. Which can be prevented by providing a graded filter over the affected soil mass area.
- By this F.O.S. is increased which is given as below.

$$\text{F.O.S. against heave} = \frac{\text{Submerged weight of soil prism + Effective weight of filter}}{\text{Total seepage pressure}}$$

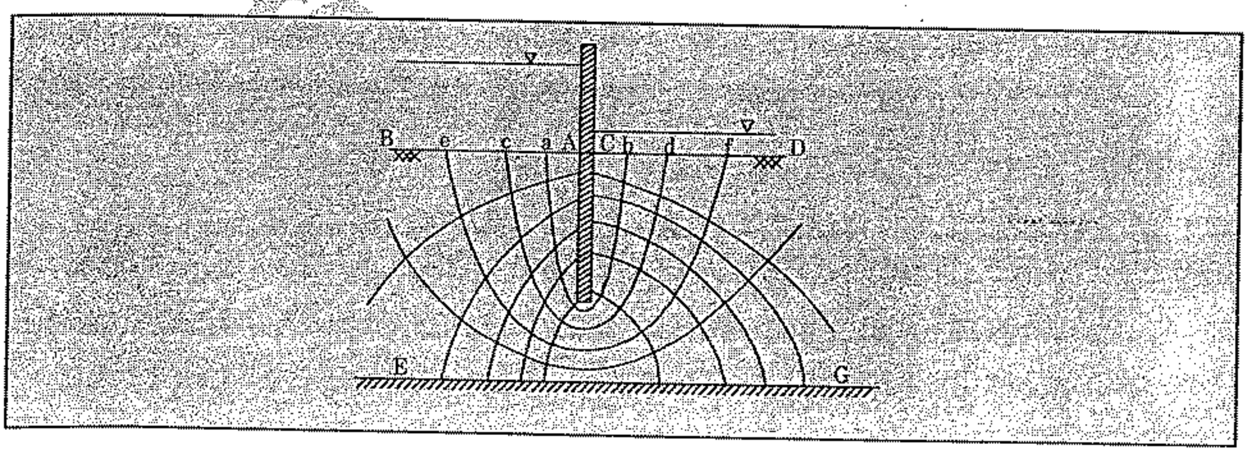
4. **Pore water pressure measurement:** Pore water pressure at any point can be calculated by working out the energy equation between the point at which pore water pressure is to be calculated and the point of known pressure and elevations.

METHODS OF OBTAINING FLOW NET

1. Analytical method
2. Electrical flow analogy
3. Capillary flow analogy
4. Sand model
5. Graphical method.

FLOW NET FOR CONFINED FLOW

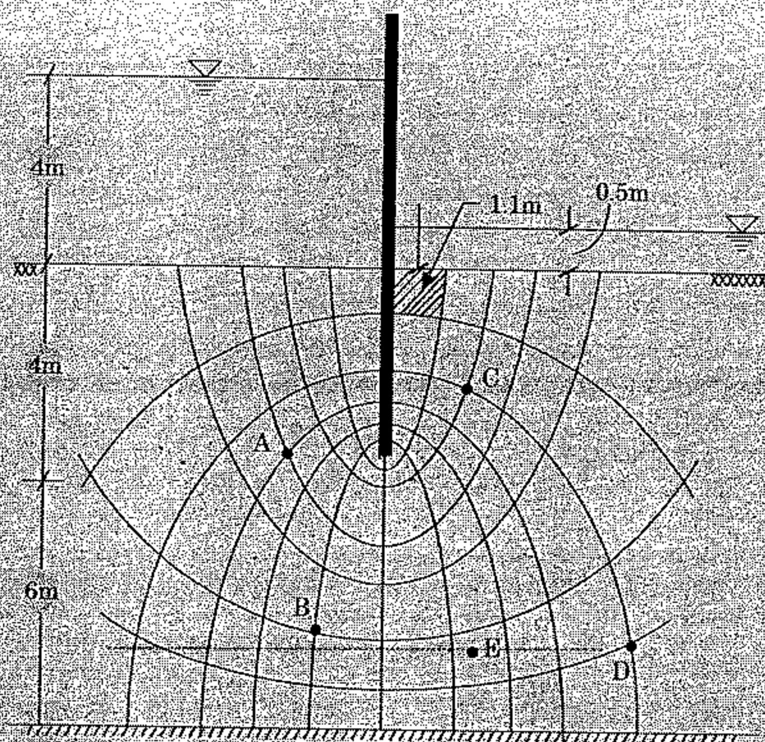
- When flow space is completely defined by boundary conditions, we get confined flow flownet.



- AB → equipotential line
- CD → Equipotential line
- ab → Flow line
- ef → Flow line
- Example of confined flow flonet are flow under sheet pile, Concrete or Masonary dams.

Example 1.

A single row of sheet piles is driven upto a depth of 4m In a Bed of clean sand having coefficient of permeability k = 0.002 cm/sec. An impereable layer of very stiff clay Exists at a depth of 10m. below the G.L. The sheet pile wall has to retrain water upto 4m above G.L. The height of water level on down stream side is 0.5m.



Determine,

(i) The Quantity of Seepage loss considering unit width of the sheet piles

(ii) The Piezometric heads at the points A, B, C, D, and E

(iii) The Exit gradient

(iv) Factor of safety against piping. Given G = 2.67, e = 0.95.

Sol. (i) Data given,

$$k = 0.002 \text{ cm/sec}$$

$$= \frac{0.002\times10^{-2}}{\left(\frac{1}{86400}\right)} = 1.728 \text{ m/day}$$

Head available $H = 4 - 0.5 = 3.5\text{m}$.

N_f = No. of Flow channels = 7

N_d = No. of potential drop = 12

q = Quantity of seepage loss per unit width of sheet piles

$$= kh\left(\frac{N_f}{N_d}\right) = 1.72 \times 3.5 \times \left(\frac{7}{12}\right)$$

$$= 3.53 \text{ m}^3\text{/day}$$

(ii) Initial piezometric head at the ground level on upstream side = 4m.

$$\therefore \quad \text{Head drop} = \frac{\text{Head Diff}^n}{(\text{No. of head drops})}$$

$$= \frac{(4-0.5)}{12} = 0.2917\text{m}.$$

$\therefore$ No. of Head drop at the point A = 3.

$$\therefore \quad \text{Head loss at A} = 3 \times (10.2917)$$

$$= 0.875\text{m}.$$

$$\therefore \quad \text{Residual head at A} = (\text{Initial head} - \text{head lon}) = (4 - 0.875)$$

$$= 3.125\text{m}.$$

Similarily the pizometric head At B, C and D are Pizeometric head at B

$$= 4 - (5 \times 0.2917)$$

$$= 2.542\text{m}.$$

at $\quad C = (4 - 10 \times (0.2917)$

$$= 1.083 \text{ m}.$$

at $\quad D = 4 - 10 \times 0.2917 = 1.083\text{m}.$

The point E lies In betweenthe 5^{th} and 6^{th} flow lines.

Hence the piezometric head at E should be obtained by linear Interpolation.

$$\text{Avg. no. of Head drop at E} = \left(\frac{7+8}{2}\right) = 7.5.$$

$$\text{Piezometric head at E} = 4 - (7.5 \times 0.2917) = 1.812\text{m}.$$

(iii) Avg. length of the smallest flow element near the downstream End = 1.1m.

$$\text{Exit gradient } i_e = \left(\frac{\Delta h}{l}\right) = \frac{0.2917}{1.1} = 0.265.$$

(iv) The Critical Hydraulic Gradient

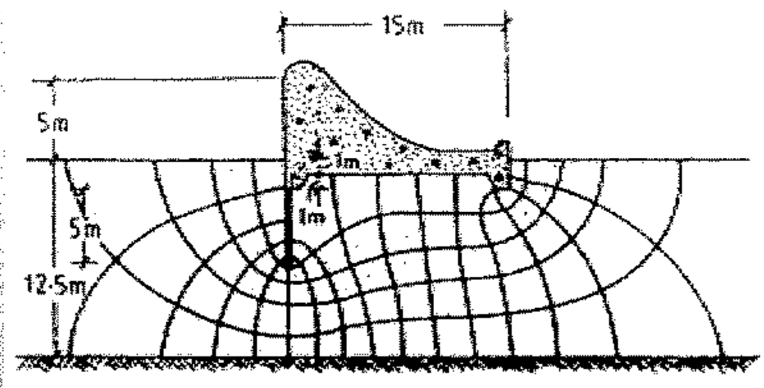
$$i_c = \left(\frac{G-1}{1+e}\right) = \frac{(2.67-1)}{1+0.95)} = 0.856$$

$$\therefore \quad \text{F.O.S Against Piping} = \frac{i_c}{i_e} = \left(\frac{0.856}{0.265}\right) = 3.23.$$

Example 2.

A concrete weir of 15m. Length has to retain water upto 5m above G.L. The cross section of the weir is shown in Figure. The foundation soil consists of a 12.5m thick stratum of sand having k = 0.015 cm/sec. In order to reduce the seepage loss, a 5m deep vertical sheet pile cut off wall is placed at the Bottom of the upstream face of the weir.

Determine the quantity of seepage loss that will occur in one day. If the width of weir be 55m. Also determine the F.O.S against piping If the soil has G = 2.65 and e = 1.08.



Sol. : From the Figure,

$$N_f = \text{No. of Flow channels} = 5$$
$$N_d = \text{No. of Head drop} = 16$$
$$K = 0.015 \text{ cm/sec} = \frac{0.015 \times 10^{-2}}{\left(\frac{1}{86400}\right)} = 12.96 \text{ m/day.}$$
$$H = 5\text{m.}$$
$$\therefore \quad q = KH\left(\frac{N_f}{N_d}\right)$$
$$= 12.96 \times 5 \left(\frac{5}{16}\right) = 20.25 \text{ m}^3\text{/day/m.}$$

$\therefore$ Total Quantity of seepage loss per day

$$= q \times \text{Width}$$
$$= 20.25 \times 55 = 1113.75 \text{ m}^3\text{/day.}$$

The Avg. length of smallest flow element adjascent to the weir = 1.2m.

$\therefore$ Exit Gradient $\quad i_e = \frac{\Delta h}{l} = \left(\frac{H}{N_d \times l}\right) = \left(\frac{5}{16 \times 1.2}\right) = 0.26$

Critical Hydraulic gradient

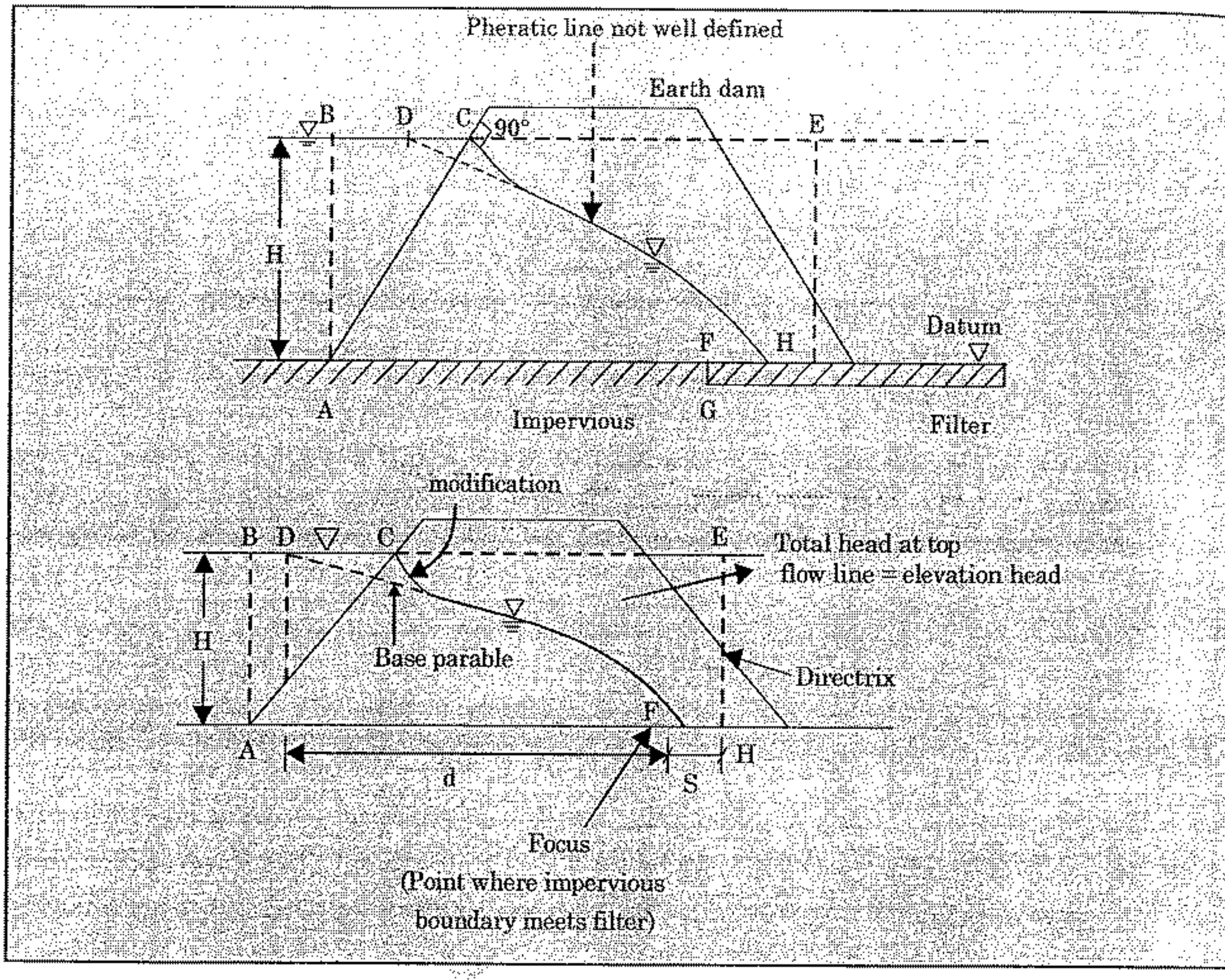
$$i_c = \left(\frac{G-1}{1+e}\right) = \left(\frac{2.65-1}{1+1.08}\right) = 0.79.$$

$\therefore$ Factor of safety against piping $= \frac{i_c}{i_e} = \frac{0.79}{0.26} = \mathbf{3.04}$

UNCONFINED FLOW

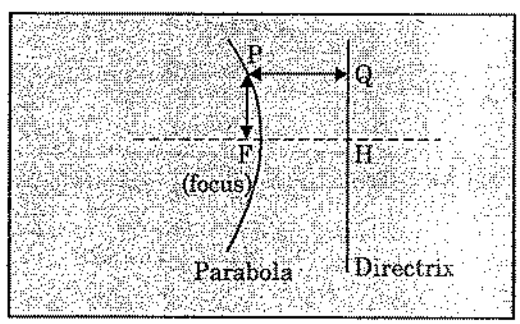
- Some times position of top boundary of flow lies is not clear even though geometrical boundaries of the pervious soil mass is known to us. Hence we can say that flow space is not completely defined. Seepage though earthen dam is a nice example of Unconfined flow. The biggest problem we face in these cases is to draw the top flow line of flow net. Which will be at atmospheric pressure.
- Sketching of top flow line (Pheratic surface) is the first step in the drawing of flow net for seepage under Unconfined flow.

- Consider a homogenous isotropic earthen dam resting on an impermeable foundation as shown below.



Bonudary Conditions

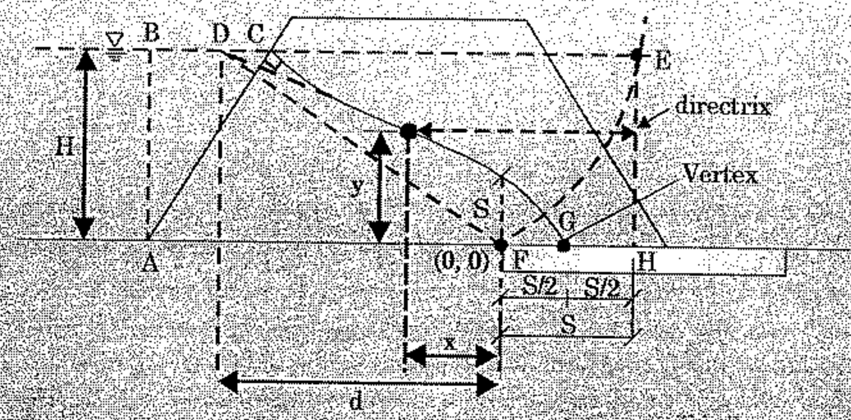
- AC — Boundary Equipotential line because total head is constant along its Bouyandary at head = H



- FH — Boundary equipotential line at zero head.
- AF — Boundary flow lines.
- Thus, three boundary conditions are known.
- The problem is to located **Top surface line**.
- Top flow line must start at C, at right angles to AC.
- According to casagrand flow lines and equipotential line are two sets of parabola with point 'F' as common focus for both sets. Top parabola is called base parabola.
- One of the Property of parabola is that distance of a point on parabola from focus is equal to distance of that point on parabola from directrix.

Prodecure

1. CD = 0.3BC is drawn 0.3 times of BC from point C.



2. An Arc FE is drawn with DF as radius and D as centre.
3. It cuts the original water surface line at E and a perpendicular is droped from point E to the filter surface and cuts the filter at H and (E H = directrix)
4. Vertex of parabola is midway between focus and directrix
5. S is calculated as follows
6. If (x, y) is a point on parabola then, from the property of parabola we can write that.

$$x + S = \sqrt{x^2 + y^2}$$

$$\text{At } \boxed{x = 0,\ y = S}$$

7. Also the Point D is on parabola, hence from the relation given below calculate S.

$$\Rightarrow \qquad \boxed{(d + S) = \sqrt{d^2 + H^2}}$$

8. Thus parabola is drawn and pheratic line is obtained.

Seepage calculation is done as follows.

$$\sqrt{x^2 + y^2} = x + S$$

$$x^2 + y^2 = x^2 + S^2 + 2\,x\,S$$

$$y = \sqrt{S^2 + 2xS}$$

$$\frac{dy}{dx} = \frac{S}{\sqrt{S^2 + 2xS}}$$

$$\text{At } x = 0, \quad \frac{dy}{dx} = 1$$

- If is assumed that the gradient of the top flow line can represent the average gradient on the vertical.

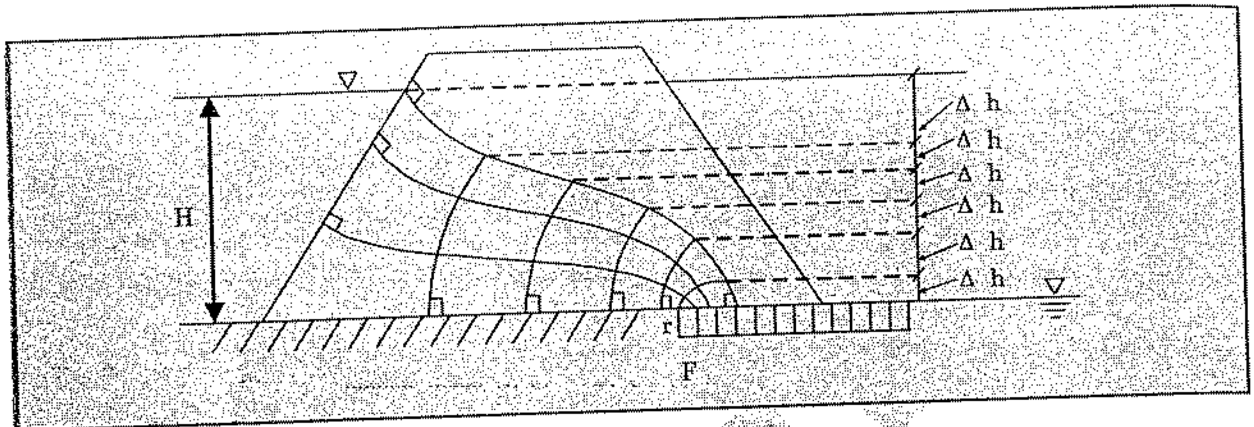
$$\text{At } F,\ dy/dx = 1 \text{ and } y = S$$

$$q = Ki.A$$

$$q = K.1 \times S$$

- q = KS gives Seepage through embankment dam provided with horizontal drain.

- However, after flow net is completely drawn, we can use the formula. $\boxed{q = KH\frac{N_f}{N_D}}$



SEEPAGE IN ANISOTROPIC SOIL

- We know that, for 2-D flow

$$K_x\frac{\partial^2 h}{dx^2} + K_z\frac{\partial^2 h}{dz^2} = 0$$

- When $K_x = K_{\text{horizontal}}$ and $K_z = K_{\text{vertical}}$, as shown in the figure

$$\frac{\partial^2 h}{\frac{K_z}{K_x}\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

- From the above equation we can note that, when soil is anisotropic with respect to permeability i.e. $k_x \neq k_z$, the flow and equipotential line are not necessarily to be orthogonal as because $\frac{K_z}{K_x} \neq 1$, and we can not get the acutal form of laplace equation which is $\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$

- But if we replace

$$x_t = x\sqrt{\frac{K_z}{K_x}}$$

Then,

$$\partial \chi_t^2 = \partial x^2\frac{K_z}{K_x}$$

$$\boxed{\frac{\partial^2 h}{\partial x_t^2} + \frac{\partial^2 h}{\partial z^2} = 0}$$

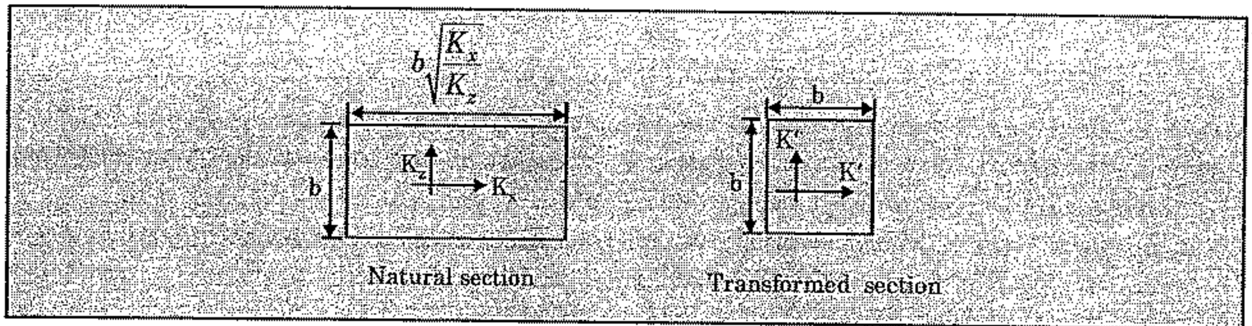
- Thus, new flow line and equipoentianl lines are drawn on tranformed section with x-distance changed to $x\sqrt{\frac{K_z}{K_x}}$ while keeping the vertical dimension constant.

- Value of coefficient of permeability for transformed section $\boxed{K' = \sqrt{K_x . K_z}}$

$$\boxed{q = \sqrt{K_x . K_z} . H\frac{N_f}{N_d}}$$

$$\boxed{q = K' H \frac{N_F}{N_D}}$$

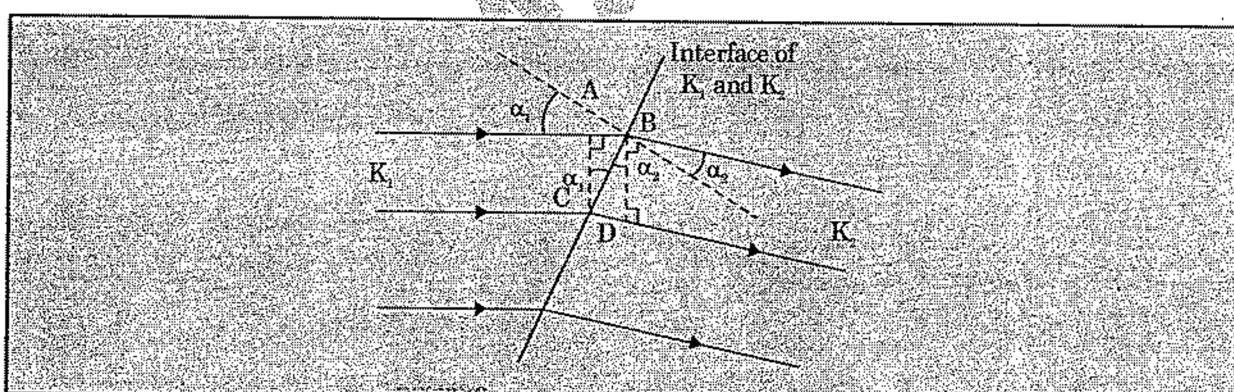
Note: By taking a flow field in which flow is taking place horizontally K_x will apply



$$\Delta q = K_x \times \frac{\Delta H}{b\sqrt{\frac{K_x}{K_z}}} \times b \times 1 = K' \frac{\Delta H}{b} \times b \times 1$$

$$\boxed{\sqrt{K_x K_z} = K'}$$

FLOW THROUGH NON-HOMOGENEOUS SECTION



- AC and BD are equipotential line let head loss between AB and CD would be same say ΔH.
- From continuity equation same rate of flow exists in the flow channels in the both the soil.

$$q = K_1 . \frac{\Delta H}{AB} . AC \times 1 = K_2 . \frac{\Delta H}{CD} \times BD \times 1$$

$$\frac{K_1}{K_2} = \frac{BD/CD}{AC/AB} = \frac{BD}{CD} \times \frac{AB}{AC} = \frac{AB/AC}{CD/BD}$$

$$\boxed{\frac{K_1}{K_2} = \frac{\tan \alpha_1}{\tan \alpha_2}}$$

- If $K_1 > K_2$, then $\alpha_1 > \alpha_2$; flow gets deflected towards normal
- If $K_1 < K_2$, then $\alpha_1 < \alpha_2$; flow gets deflected away from normal

PREVENTION OF EROSION

To prevent possibility of **erosion and piping** two approaches are used.

1. **Control of seepage and seepage force :** By providing cut off wall, increasing flow path by providing impervious blanket.
2. **Use of protective filter :** Use of protective filter **prevents erosion** and **reduees uplift pressure.** A protective filter consists of one are more layer of coarse grained material placed over a less pervious soil called the base.
   - A filter will prevent migration of finer particels but without inhibiting the flow of seepage water, so there is hardly any loss of head. This ensures that with in the filter itself, seepage forces are reduced.
   - If voids in the filter are much larger than the fine grains of the protected material (base), these grains are likely to be washed into the voids of the filter material and would ultimately obstruct the free flow.
   - On the otherhand if voids are too small, seepage forces are likely to develop to unaceptable levels. Both of the situation is to be avoided.
   - To achieve this, filter must have grain sizes that satisfy certain requirements.

Filter specifications.

1. $\dfrac{D_{15}(\text{filter})}{D_{85}(\text{Protected material})} < 5$

It ensures that no significant invasion of particles from the protected material to the filter. (Governs the upper limit of grain size of filter mater.)

2. $4 < \dfrac{D_{15}(\text{filter})}{D_{15}(\text{Protected material})} < 20$

It ensures that sufficient head is lost in filter without build up of seepage pressure (specifies the lower limit of material)

3. $\dfrac{D_{50}(\text{filter})}{D_{50}(\text{Protected material})} < 25$

This is the additional guideline for the selection of material.

Note: Provision of down stream cut off instead of upstream cutoff

effect on

(1) See page: There will be no change in $\dfrac{N_f}{N_d}$ ratio, it is only the pattern that will get changed.

(2) uplift pressure: Uplift pressure will be much greater, even greater than the case when no cutt off was provided. This is because the no. of equipotential drop is much less.

(3) exit gradient: This would be consideralIy reduced because size of elementary square would be larger in this case.

Example 3.

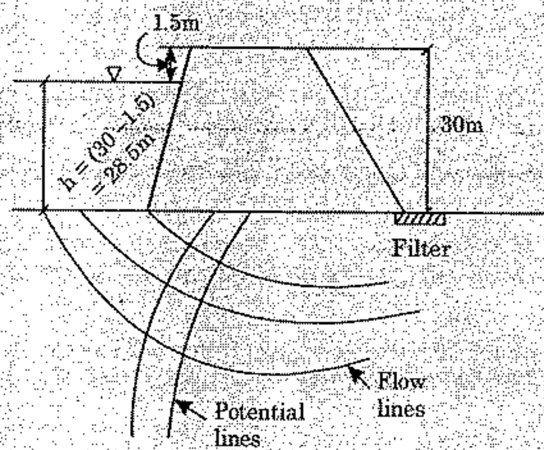
A Homogeneous Earth Dam, 30 m high, has a free board of 1.5m. A Flownet was con contructed and the following Remlts were noted:

No. of Potential Drops = 12

No. of Flow channels = 3.

The Dam has a 18m long horizontal filter at its downsbeam end. Calculate the Seeepage lon sown the dam per day if the width of dam be 200m and the coefficient of cumebility of the soil be 3.55×10^{-4} cm/sec.

Sol. Data given



$$H = 30\text{m}.$$

$$\text{Free Board} = 1.5\text{m}.$$

$\therefore$ Therefore Head availabe

$$h = (30 - 1.5) = 28.5\text{m}.$$

$$k = 3.55 \times 10^{-4} \text{ cm/sec.}$$

$$\text{Total width} = 200\text{m}.$$

$\therefore$ The Quantity of Seepage loss across unit width of the dam

$$q = kh \times \left(\frac{N_f}{N_d}\right)$$

$$\because \quad k = \frac{3.55 \times 10^{-4} \times 10^{-2}}{\left(\frac{1}{86400}\right)} = 0.3067 \text{ m/day}$$

$$N_f = 3, \; N_d = 12$$

$$\therefore \quad q = 0.3067 \times (28.5) \times \left(\frac{3}{12}\right)$$

$$= 2.185 \text{ m}^3/\text{m}.$$

As the Downstream end is provided with a long horizontal filter, the downstream side should be dry.

$\therefore$ The Quantity of seepage loss per day across the entire width of the Dam

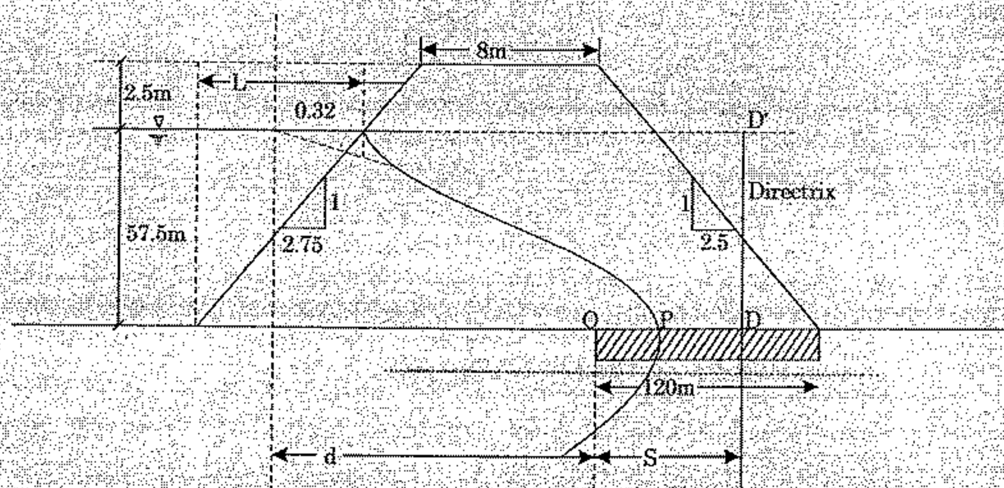
$$= (2.185 \times 200) = 437\text{m}^3$$

Example 4.

Calculate the seepage through an earth dam resting on an Impervious foundation. The relevant data are given below

Height of the dam = 60.0m
Unpstream slope = 2.75 : 1 (H:V)
Downstream slope = 2.50 : 1 (H:V)
Free board = 2.5m
Creßt width = 8.0m
Length of Dramage Blanket = 120.0m
Cofficient of permeability of the Embankrnist material in
X — direction = 8×10^{-7} m/sec
Y — direction = 2×10^{-7} m/sec

Sol.



Data given,

$$k_x = 8 \times 10^{-7} \text{ m/sec}$$

$$k_y = 2 \times 10^{-7} \text{ m/sec}$$

$$\therefore \quad d' = d\sqrt{\left(\frac{k_y}{k_x}\right)} \quad \text{... (i)}$$

From the Figure

$$L = (57.5 \times 2.75)$$
$$= 158.125 \text{ m.}$$
$$H = 60 - 2.5 = 57.5\text{m.}$$
$$d = (0.3 \times 158.125 + 2.75 \times 2.5 + 8 + (2.5 \times 60 - 120)$$
$$= 92.3125\text{m.}$$

$$\therefore \quad d' = d\sqrt{\left(\frac{k_y}{k_x}\right)}$$

$$= 92.3125\sqrt{\frac{2\times10^{-7}}{8\times10^{-7}}} = \frac{92.3125}{2} = 46.156\text{m.}$$

From the property of parabola

We have, $\quad d' + S' = \sqrt{(d'^2 + H^2)}$

$\therefore$ $S' = \sqrt{(46.153)^2 + (57.5)^2} - 46.156 = 27.58$ m.

$\therefore$ Now $K_{eq} = \sqrt{k_x k_y}$

$= \sqrt{(8\times10^{-7}\times2\times10^{-7}} = (4\times10^{-7})$

$\therefore$ Seepage discharge per unit length of dam

$q = (k_{eq}.S\cancel{c})$

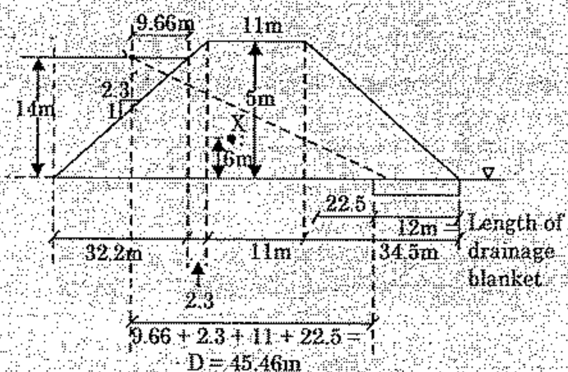
$= (4\times10^{-7}\times27.58) = 1.103\times6^{-5}$ m³/sec.

Example 5.

The section of a homogeneous earth dam is shown below

$K_X = 8\times10^{-7}$ cm/s, $K_z = 3.6\times10^{-7}$ cm/s. Estimate seepage through the dam section.

If the no of equipotential drops upto point 'X' is 3 and total no of equipotential drop = 18, Find pore water pressure at X.



Sol.

$$\sqrt{\frac{K_z}{K_x}} = \sqrt{\frac{3.6\times10^{-7}}{8\times10^{-7}}} = 0.67$$

$$D' = D\times\sqrt{\frac{k_z}{k_x}} = 45.46\times0.67$$

$$D' = 30.4582$$

$$S = \sqrt{D'^2 + H^2} - D'$$

$$= \sqrt{(30.4582)^2 + (14)^2} - 30.4582 = 3.063$$

$$q = K'S = \sqrt{K_X K_z}\,S$$

$$= 5.366\times10^{-7}\times3.063 \text{ m}^3/\text{s/m}$$

$$= 16.438\times10^{-9} \text{ m}^3/\text{s/m}$$

Total head = 14 m

Total head at $x = 14 - \frac{3}{18}\times14 = 11.67$ m

Total head = Pressure head + elevation head

$$\Rightarrow \quad 11.67 = \frac{P_X}{\gamma_w} + 6$$

$$\Rightarrow \quad \frac{P_x}{\gamma_w} = 5.67 \text{ m}$$

$$P_X = 5.67\gamma_w = 56.7 \text{ KN/m}^2$$

OBJECTIVE TYPE QUESTIONS

1. A flownet is drawn to obtain
   (a) seepage, coefficient of permeability and uplift pressure
   (b) coefficient of permeability, uplift pressure and exit gradient
   (c) exit gradient, uplift pressure and seepage quantity
   (d) exist gradient, seepage and coefficient of permeability

2. To provide safety against piping failure, with a factor of safety of 5, what should be the maximum permissible exit gradient for soil with specific gravity of 2.5 and porosity of 0.35?
   (a) 0.155 (b) 0.167 (c) 0.195 (d) 0.213

3. Due to rise in temperature, the viscosity and unit weight of percolating fluid are reduced to 70% and 90% respectively. Other things being constant, the change in coefficient of permeability will be
   (a) 20.0% (b) 28.6% (c) 63.0% (d) 77.8%

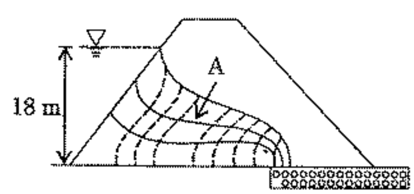
4. Match **List-I** (Flow type) with **List-II** (Flow characteristics) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Transient flow	**1.** Seepage flow is a function of time
B. Turbulent flow	**2.** Hydraulic gradient varies with square of velocity
C. Steady-state flow	**3.** Flow at low velocity
D. Laminar flow	**4.** Governing equation in 2-D is $k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} = 0$

Codes:

	A	B	C	D
(a)	1	2	4	3
(b)	3	2	1	4
(c)	1	2	3	4
(d)	2	1	4	3

5. In the schematic flownet shown in the given figure, the hydraulic potential at point A is



Schematic only

   (a) 5 m of water (b) 12 m of water
   (c) 15 m of water (d) 25 m of water

6. A flownet constructed to determine the seepage through an earth dam which is homogeneous but anisotropic, gave four flow channels and sixteen equipotential drops. The coefficients of permeability in the horizontal and vertical directions are 4.0×10^{-7} m/s and 1×10^{-7} m/s respectively. If the

storage head was 20 m, then the seepage per unit length of the dam (in m^3/s) would be

(a) 5×10^{-7} (b) 10×10^{-7}
(c) 20×10^{-7} (d) 40×10^{-7}

7. To make certain that the backfill material is more pervious than the soil to be drained, the relationship used is

(a) $(D_{15})_{filter} \le 5\ (D_{85})_{protected\ soil}$
(b) $(D_{15})_{filter} \ge 5\ (D_{85})_{protected\ soil}$
(c) $(D_{15})_{filter} \le 5\ (D_{15})_{protected\ soil}$
(d) $(D_{15})_{filter} \ge 5\ (D_{15})_{protected\ soil}$

8. Consider the following statements:

1. Constant head permeameter is best suited for determination of coefficient of permeability of highly impermeable soils.
2. Coefficient of permeability of a soil mass decreases with increase in viscosity of the pore fluid.
3. Coefficient of permeability of soil mass increases with increase in temperature of the pore fluid.

Which of these statements are correct?

(a) 1 and 2 (b) 1 and 3
(c) 2 and 3 (d) 1, 2 and 3

9. The configuration of flow nets depends upon

(a) the permeability of the soil
(b) the difference in the head between upstream and downstream sides
(c) the boundary condition of flow
(d) the amount of seepage that takes place

10. Consider the following statements:
Phreatic line in an earth dam is

1. elliptical in shape
2. an equipotential line
3. the topmost flow line with zero water pressure
4. approximately a parabola

Which of these statements is/are correct?

(a) 1, 2 and 3 (b) 2, 3 and 4
(c) 3 and 4 (d) 1 alone

11. Match **List-I** (Processes) with **List-II** (Governing laws/equations) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Flow of water in soil	**1.** Boussinesq's equation
B. Flow of water through pipe	**2.** Darcy's law
C. Sedimentation of soil particles in water	**3.** Poiseuille's equation
	4. Skempton's equation
	5. Stoke's law

Codes:

A B C

(a)	2	4	5
(b)	2	3	5
(c)	1	3	5
(d)	2	3	4

12. A flownet of a coffer dam foundation has 6 flow channels and 18 equipotential drops. The head of water lost during seepage is 6 m. If the coefficient of permeability of foundation to 4×10^{-5} m/min, then the seepage loss per metre length of dam will be
    (a) 2.16×10^{-2} m³/day
    (b) 6.48×10^{-2} m³/day
    (c) 11.52×10^{-2} m³/day
    (d) 34.56×10^{-2} m³/day

13. Match **List-I** (Type of strata below foundation) with **List-II** (Type of foundation movement) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Sand	**1.** Practically no movements
B. Heterogeneous landfill	**2.** Immediate settlements
C. Black cotton soil	**3.** Large relative settlements
D. Hard rock	**4.** Heaving of foundations

Codes:

	A	B	C	D
(a)	2	3	1	4
(b)	2	3	4	1
(c)	3	2	1	4
(d)	3	2	4	1

14. Match **List-I** (Unit) with **List-II** (Purpose) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Graded filter	**1.** To reduce seepage of water through body of earth dam
B. Lime treatment	**2.** To reduce water seepage through foundation below dam
C. Impervious clay core	**3.** To stabilize black cotton soils
D. Curtain grouting	**4.** To drain water without losing fines from the soil

Codes:

	A	B	C	D
(a)	4	3	1	2
(b)	1	2	4	3
(c)	4	2	1	3
(d)	1	3	4	2

15. From a flow net which of the following information can be obtained?
    1. Rate of flow
    2. Pore water pressure
    3. Exit gradient
    4. Permeability

Select the correct answer using the codes given below:

(a) 1, 2, 3 and 4 (b) 1, 2 and 3 only
(c) 2, 3 and 4 only (d) 1 only

16. For a sheet pile wall constructed in a soil having effective grain size = 0.1 mm, the difference of the upstream and downstream water levels is 3 m. If the flow net drawn for the problem yields 2 as the ratio of number of head drops to number of flow channels, then what is the discharge in unit of $m^3/s/m$ length of sheet pile wall?

(a) 3.0×10^{-4} (b) 3.0×10^{-2}
(c) 1.5×10^{-5} (d) 1.5×10^{-2}

17. Consider the following statements:

1. The hydraulic head at a point in the soil includes piezometric head as well as datum head.
2. Piping in soil occurs when effective pressure becomes equal to zero.
3. Piping in soil occurs when soil is highly porous.

Which of these statements is/are correct?

(a) 1, 2 and 3 (b) 1 and 2 only
(c) 2 and 3 only (d) 2 only

18. Which one of the following represents the correct relationship between seepage pressure (p_s), unit weight of water (γ_w) and hydraulic gradient (i) inside an earth dam?

(a) $p_s = i/\gamma_w$ (b) $p_s = i \times \gamma_w$
(c) $p_s = i^2 \times \gamma_w$ (d) $p_s = \gamma_w/i$

19. In a permeability test conducted on a soil with e = 0.50 the discharge velocity was found to be 2.4 $\times 10^{-1}$ cm/s. The seepage velocity is ....

(a) 7.2×10^{-1} cm/s (b) 4.8×10^{-1} cm/s
(c) 3.6×10^{-1} cm/s (d) 1.6×10^{-1} cm/s

20. Match **List-I** (Cause) with **List-II** (Effect) and select the correct answer using the codes given below the lists:

List-I		List-II	
A.	Water present in the soil above the water table	**1.**	Increase in effective stress
B.	Upward seepage flow	**2.**	No change in effective stress
C.	Downward seepage flow	**3.**	Water is in a state of tension
D.	Fluctuation of water level above ground level	**4.**	Decrease in effective stress

Codes:

	A	B	C	D
(a)	3	4	1	2
(b)	3	2	1	4
(c)	2	3	1	4
(d)	1	4	3	2

21. The process by which a mass of saturated soil caused by external forces to suddenly loose its shear strength and to behave as a fluid is called

(a) piping (b) slide
(c) quick condition (d) liquefaction

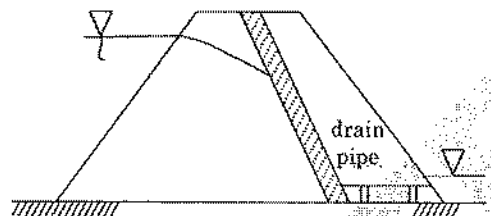
22. Critical hydraulic gradient i_c is given by (where G = specific gravity of solids, e = void ratio)

(a) $\frac{G+1}{1-e}$ (b) $\frac{G-1}{1+e}$

(c) $\frac{G+1}{1+e}$ (d) $\frac{G-1}{1-e}$

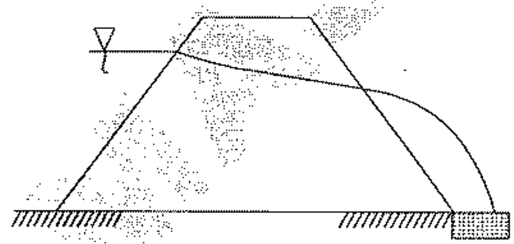
23. Phreatic lines for different types of drainage/filter arragements are shown in figure-A, B and C.

figure:(A)



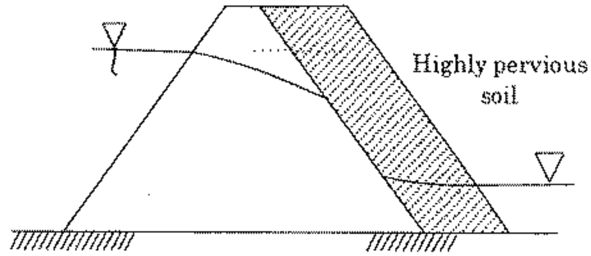
Homogeneous earth dam with chimney drain

figure:(B)



Homogeneous earth dam when the downstream slope forms in itself a medium for discharge and the horizontal filter is outside the toe

figure:(C)



A zoned earth dam

The phrealtic line is correctly shown in figure(s)

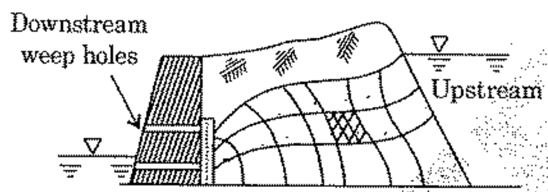
(a) A and B (b) A and C

(c) B alone (d) B and C

24. The foundation soil under the toe of a dam has a void ratio 'e'. The specific gravity of the soil solids is G. Factor of safety against piping is to be taken as 2.5. The maximum permissible upward exit gradient is given by

(a) $i = 2.5\left(\frac{G-1}{1+e}\right)$
(b) $i = 2.5\left(\frac{1+e}{G-1}\right)$
(c) $i = 0.4\left(\frac{1+e}{G-1}\right)$
(d) $i = 0.4\left(\frac{G-1}{1+e}\right)$

25. Consider the flownet shown in the following figure:



The ratio of the number of flow channels to the number of potential drops is

(a) 3/8
(b) 3/7
(c) 4/7
(d) 4/8

26. A 1.2 m layer of soil is subjected to an upward seepage head of 1.8 m. A layer of coarse sand is laid above the soil layer to attain a factor of safety of 2.0 against piping. Both the soil and coarse sand have the same values of G = 2.67 and e = 0.67. There is neglible head loss in the sand layer. The required depth of the coarse sand layer is

(a) 0.9 m
(b) 1.2 m
(c) 2.4 m
(d) 3.6 m

27. Consider the following statements regarding flownet representing flow through soil below a concrete dam:

1. The flownet will not alter if the level of reservoir is raised.
2. The flownet will not alter if the soil medium is altered.
3. The flownet would not alter if the upstream and downstream water levels were to be interchanged.

Which of these statements are correct?

(a) 1 and 2
(b) 1 and 3
(c) 2 and 3
(d) 1, 2 and 3

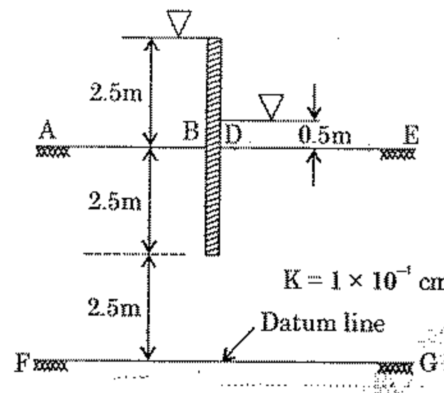
28. Consider the following statements regarding the flow nets:

1. Flow lines and equipotential lines always intersect one another at right angles irrespective of the permeability characteristics.
2. For an isotropic soil, the spacing of lines is inversely proportional to the hydraulic gradient.
3. For an isotropic soil and a flow net of approximate squares, the potential drop is the same through each field.
4. For an anisotropic soil having greater horizontal coefficient of permeability, a flow net of approximate squares can be constructed by reducing the vertical dimensions of the flow domain to a certain scale.

Which of these statements are correct?

(a) 2, 3 and 4
(b) 2 and 3
(c) 1, 2 and 4
(d) 1 and 3

29. A sheet pile is driven into a sandy bed as shown in the given figure.



The head along the upstream bed AB and the head along the downstream bed DE, respectively, are

(a) 7.5 m and 5.5 m
(b) 2.5 m and 0.5 m
(c) 5 m and 3 m
(d) 5 m and 5.5 m

30. Consider the following statements:

1. Seepage force is applied by flowing water to the soil skeleton through frictional drag.
2. The magnitude of seepage force per unit volume of soil at any point is equal to γ_w/i, where γ_w is the unit weight of water and 'i' is the hydraulic gradient at that point.
3. In a soil mass subjected to upward flow of water, quick condition develops when pore pressure is equal to the total stress.

Which of these statements is/are correct?

(a) 1 and 2
(b) 1 and 3
(c) 1 only
(d) 2 and 3

31. The flow net for an earthen dam with 30 m water depth consists of 25 potential drops and 5 flow channels. If the discharge per meter length of dam is 0.00018 m^3/s, then what is the coefficient of permeability of dam materials?

(a) 3×10^{-3} cm/s
(b) 6×10^{-3} cm/s
(c) 3×10^{-2} cm/s
(d) None of these

32. Which one of the following represents the correct relationship between seepage pressure (p_s), unit weight of water (γ_w) and hydraulic gradient (i) inside an earth dam?

(a) $p_s = i/\gamma_w$
(b) $p_s = i \times \gamma_w$
(c) $p_s = i - \gamma_w$
(d) $p_s = \gamma_w/i$

33. Consider the following statements:

1. Hydraulic gradient required to initiate 'quick' condition is independent of the ratio of volume of voids to volume of solids, in a soil mass.
2. Initiation of piping under hydraulic structures can be prevented by increasing the length of flow path of water.

Which of these statements is/are correct?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

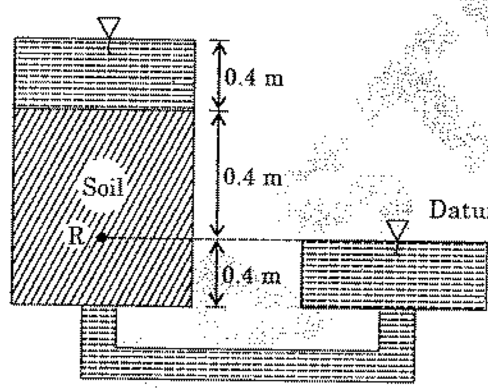
34. A unit volume of a mass of saturated soil is subjected to horizontal seepage. The saturated unit weight is 22 kN/m^3 and the hydraulic gradient is 0.3. The resultant body force on the soil mass is

(a) 1.98 kN
(b) 6.6 kN
(c) 12.36 kN
(d) 22.97 kN

(Note: Body force = vector summation of effective (Bouyant) wt and seepage pressure.

Common Data for Questions 35 and 36

Water is flowing through the permeability apparatus as shown in the figure. The coefficient of permeability of soil is 'k' m/s and the porosity of the soil sample is 0.50.



35. The total head, elevation head and pressure head in metres of water at the point R shown in the figure are

(a) 0.8, 0.4, 0.4
(b) 1.2, 0.4, 0.8
(c) 0.4, 0, 0.4
(d) 1.6, 0.4, 1.2

36. What are the discharge velocity and seepage velocity through the soil sample?

(a) k, 2 k
(b) $\frac{2}{3}k, \frac{4}{3}k$
(c) 2k, k
(d) $\frac{4}{3}k, \frac{2}{3}k$

37. During seepage through an earthen mass, the direction of seepage is

(a) parallel to the equipotential lines
(b) perpendicular to the streamlines
(c) perpendicular to the equipotential lines
(d) along the direction of gravity

38. A discharge Q is occurring through a soil sample of length L under a head H. When the head is doubled and length reduced to half, the discharge will become

(a) 4Q (b) 2Q (c) 1.5Q (d) 0.5Q

Instructions :

*The following items consists of two statements, one labelled as **'Assertion A'** and the other labelled as **'Reason R'**. You are to examine these two statements carefully and decide if the Assertion A and the Reason R are individually true and if so, whether the Reason is a correct explanation of the Assertion. Select your answers to the these items using the codes given below :*

(a) both A and R are true and R is the correct explanation of A

(b) both A and R are true but R is not a correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

39. Assertion (A): Flow net is dependent on the permeability of soil through which flow is taking place.
Reason (R): The flow net is useful in finding the discharge.

ANSWERS

1. (c)	**2.** (c)	**3.** (b)	**4.** (a)	**5.** (b)	**6.** (b)	**7.** (d)	**8.** (c)	**9.** (c)	**10.** (b)
11. (b)	**12.** (c)	**13.** (b)	**14.** (a)	**15.** (b)	**16.** (c)	**17.** (d)	**18.** (b)	**19.** (a)	**20.** (a)
21. (d)	**22.** (b)	**23.** (b)	**24.** (d)	**25.** (b)	**26.** (c)	**27.** (d)	**28.** (b)	**29.** (a)	**30.** (b)
31. (a)	**32.** (b)	**33.** (b)	**34.** (c)	**35.** (c)	**36.** (a)	**37.** (c)	**38.** (a)	**39.** (b)	

Hints

2. $e = \frac{n}{1-n} = \frac{0.35}{0.65} = 0.538$

$i_c = \frac{G-1}{1+e} = \frac{2.5-1}{1.538}$; $i_c = 0.98$

$F.O.S = \frac{i_c}{i_e}$; $i_e = 0.196$

3. $k \propto \frac{\gamma_w}{\mu}$

$k' = \frac{c \times .90 \times \gamma_w}{0.7\mu} = 1.286k$

6. $k = \sqrt{k_H \times k_v}$

$k = 2 \times 10^{-7}$

$q = k \times H \times \frac{N_F}{N_d}$

$q = 2 \times 10^{-7} \times 20 \times \frac{4}{16} = 10 \times 10^{-7}$

12. $q = k \times H \times \frac{N_F}{N_d}$

$= 4 \times 10^{-5} \times 6 \times \frac{6}{18} = 8 \times 10^{-5}\ m^3/min$

$= 11.52 \times 10^{-2}\ m^3/day$

16. $k = C\,D_{10}^2$

$k = 1 \times .1^2 = 10^{-2}$ mm/sec

$q = k \times H \times \frac{N_F}{N_d} = 10^{-5} \times 3 \times \frac{1}{2}$

$= 1.5 \times 10^{-5}\ m^3/sec/m$

19. $n = \frac{e}{1+e} = \frac{0.5}{1.5} = \frac{1}{3}$

$v_s = \frac{v}{n} = 3 \times 2.4 \times 10^{-1} = 7.2 \times 10^{-1}$ cm/sec

26. $i = \frac{G-1}{1+e} = \frac{2.67-1}{1+0.67} = 1$

$$F.O.S = \frac{i_c}{i_m}$$

$$2 \times i_m = i_c \;;\; i_m = \frac{1}{2}$$

$$\frac{h}{l} = \frac{1}{2} \;;\; l = 3.6$$

Layer of sand = 3.6 − 1.2 = 2.4

29. Head at AB

$$= 2 + \frac{p}{\gamma_w} = 2.5 + 5 = 7.5$$

Head at d/s AB

$$= 2 + \frac{p}{\gamma_w} = 2.5 + 3 = 5.5$$

31. $q = kh\dfrac{N_f}{N_d}$

$$\frac{1.8 \times 10^{-4}}{30 \times \frac{5}{25}} = 3 \times 10^{-5} \text{m/sec}$$

$$= 3 \times 10^{-3} \text{cm/sec}$$

34. Seepage force = $i\ \gamma_w = 0.3 \times 10$

Body force = $\sqrt{12^2 + 9} = 12.36\text{kN}$

35. E.L = 0

$$P.H = 0.8 - \frac{0.8}{2} = 0.4$$

T.H = 0 + 0.4 = 0.4

36. $v = ki$

$$v = k \times \frac{0.4}{0.4} \;;\; v = k$$

$$v_s = \frac{k}{n} = 2k$$

CHAPTER

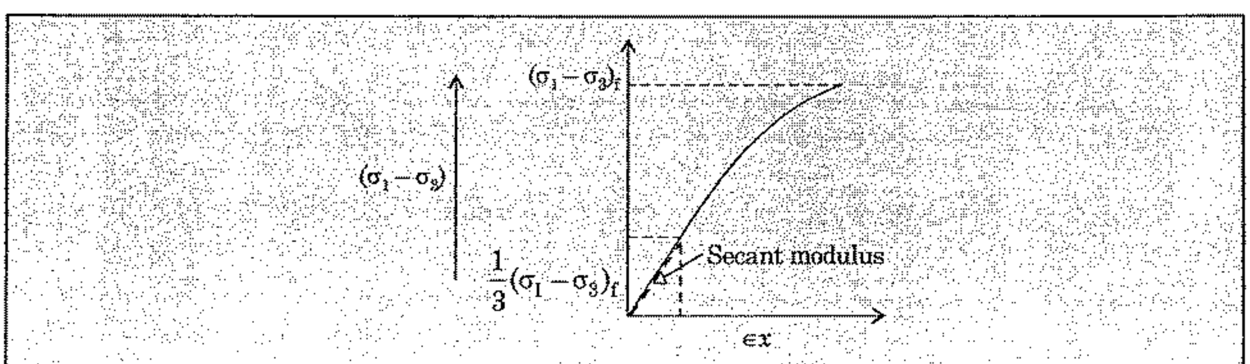
7

Vertical Stresses

INTRODUCTION

- Stresses are induced in a soil mass due to weight of overlying soil and due to the applied loads.
- These stresses are required to design a foundation such that the shear stress on any stratum of soil below it does not exceed, after providing Factor of safety for bearing capacity of soil.
- Further the vertical stresses transmitted to the soil layers below the foundation will lead to vertical deformation in the soil, causing foundation settlement.
- This settlement, again should not be allowed to exceed the permissible settlement.
- Hence the knowledge of distribution of stresses with in a soil mass, induced by loads applied on the surface of soil, is a prerequisite for foundation design.
- The stress induced in soil due to applied loads depend upon its stress-strain characterstics. The stress strain behaviour of soil is extremely complex and it depends upon a large number of factors, such as drainage conditions, water content, void ratio, rate of loading, the load level, and the stress path.
- Generally the stress strain relationship is assumed to be linear, and fortunately these results are good enough for the problems usually encountered in practice.
- Theory of Elasticity is used to determine the stresses in soil mass.
- The main stress-strain parameters required for the application of elastic theories are modulus of Elasticity (E) and Poisson's ratio (μ).

MODULUS OF ELASTICITY



- Modulus of Elasticity (E) can be determined in the laboratory by conducting a triaxial compression test.
- The stress strain curve is plotted between the deviator stress ($\sigma_1-\sigma_3$) and the axial strain ($\in_1$).

1. **For saturated, cohessive soil :** Unconsolidated undrained (UU) test or unconfined compression test is performed.
2. **For Cohession less soil :** Consolidated drained (CD) test is performed.

- The value of modulus of elasticity is generally taken as the secant modulus (1/2 to 1/3) of the peak stress. Sometimes, instead of secant modulus, the initial tangent modulus or the tangent modulus at (1/2 to 1/3) of the peak stress is also used.

POISSION'S RATIO

- For elastic material generally poission's ratio varies from 0 – 0.5
- For Undrained conditions, the value of poission's ratio is 0.50.
- For drained condition value is less than 0.50.
- It is difficult to acertain the exact value of poission's ratio; As soil being not a purely elastic material sometimes the value of μ comes more than 0.50.
- The effect of poission's ratio on the computed stress is not much and hence approximate value can be used without much error.

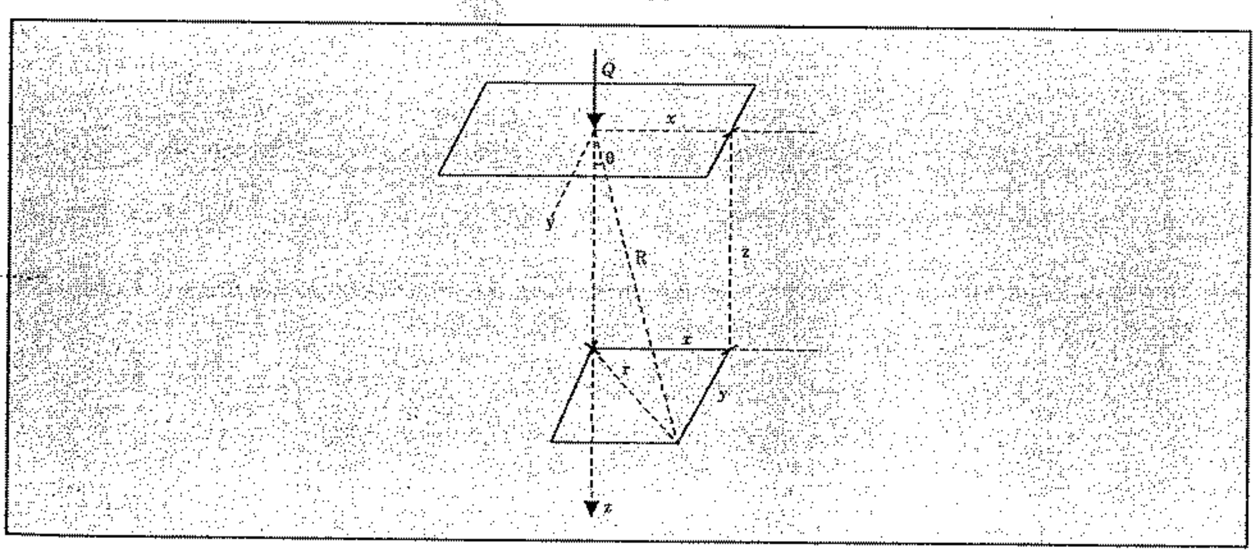
VERTICAL STRESSES DUE TO CONCENTRATED LOAD

1. Boussinesq Equation :

- 1885, Boussinesq gave the theoretical solutions for the stress destribution in an elastic medium subjected to concentrated load on its surface.

Assumptions

1. Soil mass is elastic.
2. Soil is homogenous and isotropic.
3. Soil is Semi Infinite.
4. Soil is weightless and unstresed before the application of load.



- Using logarithmic stress functions for the solution of elasticity problem, Boussinesq proved that the polar stress σ_R at any point P(x, y, z) is given by.

$$\sigma_R = \frac{3}{2\pi}\frac{Q\cos\theta}{R^2}$$

Where,

R = Polar distance between the origin O and point P.

θ = angle which OP makes with the vertical

$$R = \sqrt{x^2 + y^2 + z^2}$$

$$R = \sqrt{r^2 + z^2} \qquad \text{where } r^2 = x^2 + y^2$$

$$\sin\theta = \frac{r}{R}, \quad \cos\theta = \frac{z}{R}$$

$$\therefore \quad \sigma_z = \sigma_R \cos^2\theta$$

$$\sigma_z = \frac{3}{2\pi}\frac{Q\cos\theta}{R^2}\cos^2\theta$$

$$\sigma_z = \frac{3}{2\pi}\frac{Q}{R^2}\cos^3\theta$$

$$\sigma_z = \frac{3}{2\pi}\frac{Q}{R^2}\times\left(\frac{z}{R}\right)^3$$

$$\sigma_z = \frac{3}{2\pi}\frac{Q}{z^2}\frac{z^5}{R^5}$$

$$\sigma_z = \frac{3}{2\pi}\frac{Q}{z^2}\left[\frac{z^5}{(r^2+z^2)^{5/2}}\right]$$

$$\boxed{\sigma_z = \frac{3}{2}\frac{Q}{z^2}\left[\frac{1}{\left(1+\left(\frac{r}{z}\right)^2\right)^{5/2}}\right]}$$

$$\therefore \quad \boxed{\sigma_z = K_B\frac{Q}{z^2}}$$

Where,

$$K_B = \frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

K_B is a number and is a function of $\frac{r}{z}$ and is called the Boussinesq's Influence factor

Values of Boussinesq's Influence factor

$\frac{r}{z}$	Value
0	0.4775
1	0.0844
2	0.0085
3	0.0015
10	0.0000

- It should be noted that the vertical stress at a certain depth z is dependent only on the $\frac{r}{z}$ ratio and independent of the material. i.e. it is independent of E and μ.
- But the solution has been derived assuming that the soil is linear elastic.
- The intensity of vertical stress just below the point load $\left(\frac{r}{z}=0\right)$ is given by $\sigma_z = 0.4775\frac{Q}{z^2}$
- The vertical stress decreases rapidly with an increase in $\frac{r}{z}$ ratio, actually at $\frac{r}{z}=5.0$ or more vertical stress becomes extreemly small and is neglected.
- Boussinesq equation can be applied to actual field problems, provided the distance Z is measured from the point of application of load.
- Boussinesq's equation can be used for negative upward load i.e. vertical stress decrease due to an excavation.

Note: The field measurements indicate that actual stresses are generally smaller than the theoritical value obtained from the Boussinesq's equation. Hence it provides conservative value and is commonly used in Soil engineering problems.

Example 1.

A concentrated load of 2000 kN is applied at the ground surface. Determine the vertical stress at a point P which is 6m directly below the load. Also calculate the vertical stress at a point R which is at a depth of 6m but at a horizontal distance of 5m from the axis of the load.

Sol : We know that,
$$\sigma_z = \frac{3Q}{2\pi z^2}\times\frac{1}{[1+(r/z)^2]^{5/2}}$$

Point P, r/z = 0,
$$\sigma_z = \frac{3\times 2000}{2\pi(6)^2}\times\frac{1}{[1+0]^{5/2}} = 26.53 \text{ kN/m}^2$$

Point R, r/z = 5/6,
$$\sigma_z = \frac{3\times 2000}{2\pi(6)^2}\times\frac{1}{[1+(5/6)^2]^{5/2}} = 7.1 \text{ kN/m}^2$$

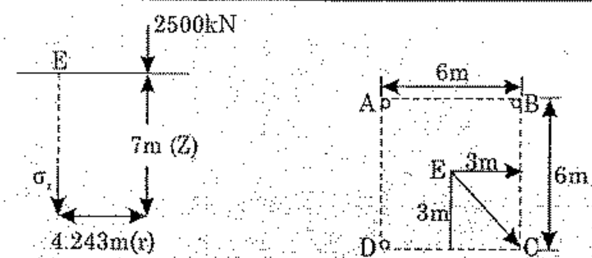
Example 2.

An elevated structure with a total weight of 10,000 kN is supported on a tower with 4 legs. The legs rest on piers located at the corners of a square 6m on a side. What is the vertical stress increment due to this loading at a point 7m beneath the centre of the structure?

Sol. Weight transferred to each pier = 2500 kN. The load can be approximated to a point load acting at the corners of a square of 6m side. The vertical stress is to be calculated at 7m depth. Horizontal distance r from each of the load is equal to $\sqrt{(3^2+3^2)}=\sqrt{18}=4.243\text{m}$.

$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

$$Q = 2500 \text{ kN},\ z = 7\text{m},\ r = 4.243\text{m}$$



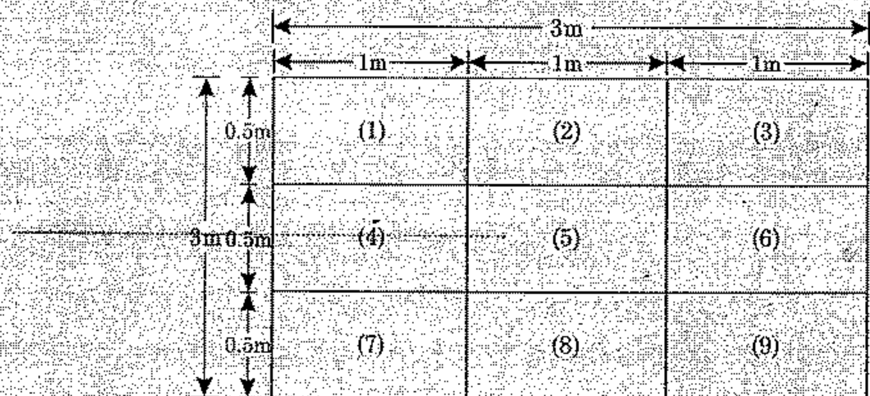
Hence vertical stress increase due to the total weight is equal to

$$\sigma_{z1} = 4 \times \sigma_z$$

$$\sigma_{z1} = \frac{4\times3\times2500}{2\pi\times7^2}\left[\frac{1}{1+(4.243/7)^2}\right]^{5/2} = 4 \times 11.143 = 44.57 \text{ kN/m}^2$$

Example 3.

A rectangular foundation 3.0 × 1.50m carries a uniform load of 40 kN/m². Determine the vertical stress at P which is 3m below the ground surface. Use equivalent point load method.



Sol. Divide the loaded area into 9 small areas of size 0.5m × 1.0m.

∴ Load on each area = 40 × (1.0 × 0.5) = 20kN

The stresses at point P can be determined using Boussinesq's solution for 9 point loads.

For loads (1) and (4), $r = \sqrt{1.5^2+(0.25)^2} = 1.521$, $r/z = 0.507$

For loads (2), (3), (5), (6), $r = \sqrt{0.5^2+0.25^2} = 0.559$, $r/z = 0.186$

For loads (8) and (9), $r = \sqrt{(0.75)^2+(0.5)^2} = 0.901$, $r/z = 0.300$

For load (7), $r = \sqrt{(1.5)^2+(0.75)^2} = 1.677$, $r/z = 0.559$

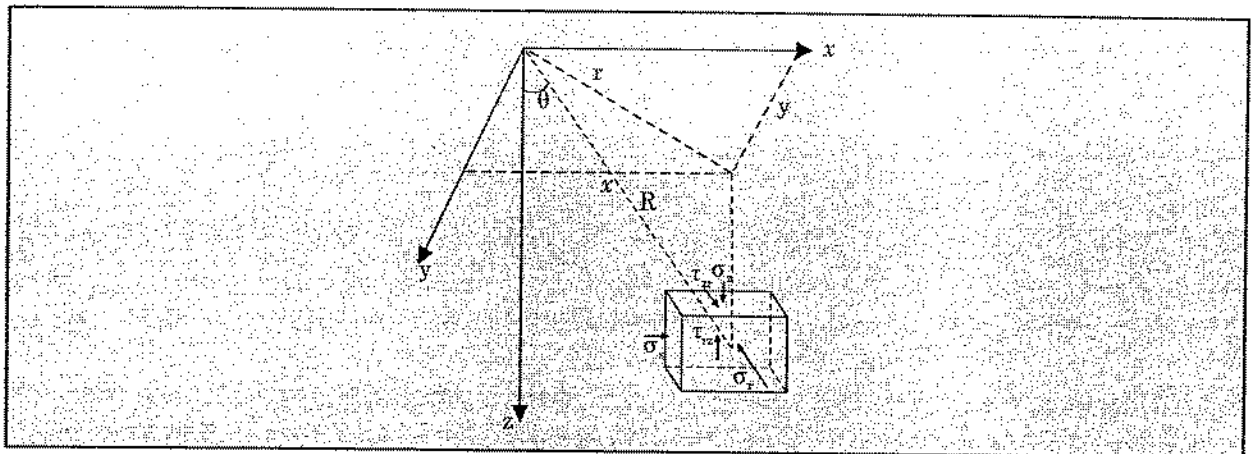
$$\sigma_z = \Sigma\frac{3Q}{2\pi(z)^2}\times\frac{1}{[1+(r/z)^2]^{5/2}}$$

$$\sigma_z = \frac{3\times20}{2\pi(3)^2}\times\left[\frac{2}{[1+(0.507)^2]^{5/2}}+\frac{4}{[1+(0.186)^2]^{5/2}}+\frac{2}{[1+(0.30)^2]^{5/2}}+\frac{1}{[1+(0.559)^2]^{5/2}}\right]$$

or $\sigma_z = 1.061\ [1.129 + 3.674 + 1.612 + 0.507] = 7.34 \text{ kN/m}^2$

Shear Stress and Horizontal Stress due to a concentrated load.

- Equations for Horizontal radial stress σ_r and the Horizontal circumferential stress σ_s are also available.



- Consider a elementary stress Block, indicating all components.

$$\sigma_r = \frac{Q}{2\pi}\left[\frac{3zr^2}{R^5} - \frac{(1-2\mu)}{R(R+z)}\right]$$

$$\sigma_s = \frac{Q}{2\pi}(1-2\mu)\left[\frac{1}{R(R+z)} - \frac{Z}{R^3}\right]$$

$$\tau_{rz} = \frac{3}{2\pi}\frac{Q}{z^2} \cos^4\theta \sin\theta$$

$$\tau_{rz} = \frac{3}{2\pi}\frac{Q}{z^2} \frac{Z^4}{R^4} \times \frac{r}{R}$$

where,

$$\text{Cos}\,\theta = \frac{Z}{R} \qquad \sin\theta = \frac{r}{R}$$

$$\tau_{rz} = \frac{3Q}{2\pi}\frac{rz^2}{(r^2+z^2)^{5/2}}$$

$$\tau_{rz} = \frac{3}{2\pi}\frac{Qr}{Z^3}\left[\frac{1}{\left(1+\frac{r}{z}\right)}\right]^{5/2}$$

$$\tau_{rs} = \tau_{zs} = 0$$

Note: *Except for the vertical stress, the other components of normal stresses depends on the stress deformation of the material.*

- Only σ_z is required for settlement computation.

2. Westergaard's Solution

- Westergaard gave theoritical solution for vertical stress distribution due to concentrated point load in 1938.

Assumptions

1. Soil mass is an-isotropic where as. Boussinesq assumed that soil mass is isotropic. But natural deposited soil are seldom isotropic.
2. Water deposited sedimentary soils, which are quite common in occurance, are formed by deposition, alternately, of horizontal layers of silts and clays.
3. Soil mass is elastic.
4. Soil is Semi Infinite.
5. Soil mass is divided into horizontal sheets of negligible thickness, closely spaced, and infinite rigidity in horizontal direction. That allows only vertical movement and prevents soil mass as a whole from undergoing any lateral strain.
6. Poission ratio equal to Zero.

- According to Westergaard, the vertical stress at a point P at a depth Z below the concentrated load Q is given by.

$$\sigma_z = \frac{C/2\pi}{\left[C^2 + \left(\frac{r}{CZ}\right)^2\right]^{3/2}} \frac{Q}{Z^2}$$

Where,

C depends upon the Poission ratio (μ) and is given by

$$C = \sqrt{(1-2\mu)/(2-2\mu)}$$

for $\mu = 0 \quad C = \frac{1}{\sqrt{2}}$

$$\therefore \quad \boxed{\sigma_z = \frac{1}{\pi\left[1+2\left(\frac{r}{Z}\right)^2\right]^{3/2}} \frac{Q}{Z^2}}$$

Where, $\boxed{\sigma_z = k_w \frac{Q}{Z^2}}$

k_w is Westergaards influence factor which is a function of $\frac{r}{Z}$.

K_w	r/z
0	0.318
1	0.0613
2	0.0118
3	0.0038

Note: $K_W = K_B$ @ $\frac{r}{z} = 1.5$

$K_W > K_B$ @ $\frac{r}{z} > 1.5$

$K_W < K_B$ @ $\frac{r}{z} < 1.5$

Westergaards results are more close to field conditions but Boussinesq results are used for calculation because they provide conservative results.

VERTICLE STRESS DUE TO LINE LOAD

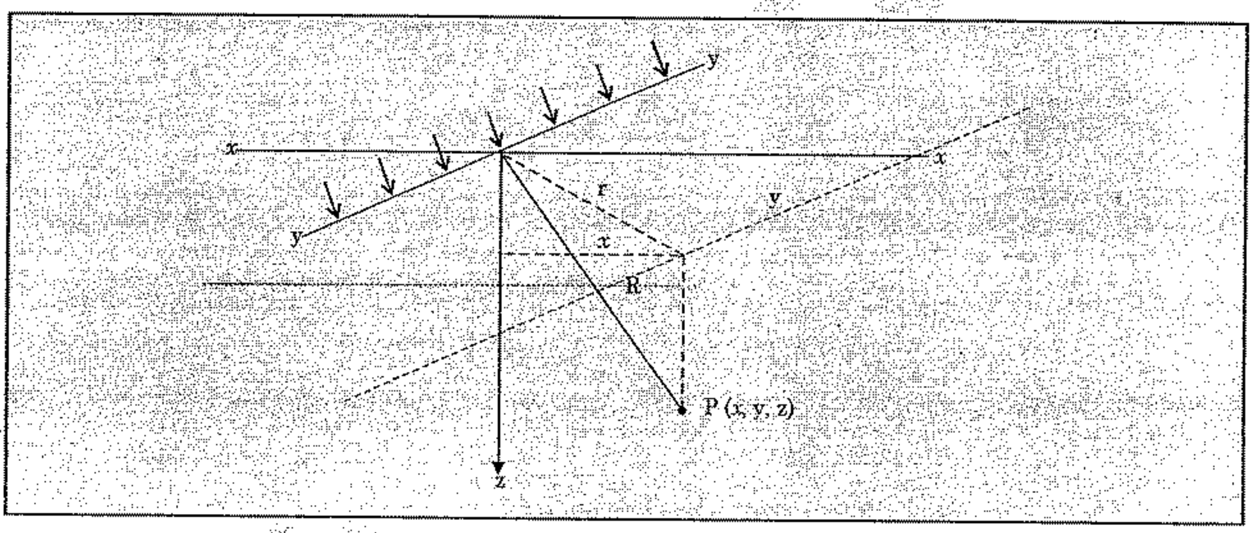
- The vertical stresses in a soil mass due to vertical line load can be obtained using Boussinesq's Solution.
- If the line load of intensity q/length, parallel to y axis on the surface of a Semi Infinite elastic medium, the vertical stress σ_z at a point O.
- Let us consider the load acting on a small element of length dy.
- The load can be taken as a point load of intensity qdy.
- Boussinesq's equation can be applied to determine the vertical stresses at $P(x, y, z)$.

$$\Delta\sigma_z = \frac{3}{2\pi} \times q\,dy \frac{z^3}{(r^2 + z^2)^{5/2}}$$

The vertical stress due to the line load extending from $-\infty$ to ∞

$$\int_{-\infty}^{+\infty} \Delta\sigma_z = \frac{3}{2\pi}\, qz^3 \int_{-\infty}^{+\infty} \frac{dy}{(r^2 + z^2)^{5/2}}$$

where, $r = x^2 + y^2$



$\therefore$
$$\sigma_{zz} = \frac{3q\,z^3}{2\pi} \int_{-8}^{+\infty} \frac{dy}{(x^2 + y^2 + z^2)^{5/2}}$$

Let, $u^2 = x^2 + z^2$

Let, $y = u \tan\theta$

$dy = u \operatorname{Sec}\theta\, d\theta$

$$\sigma_z = \frac{3qz^3}{2\pi} \int_0^{\pi/2} \frac{u \sec^2\theta}{u^5 \sec^5\theta}\, d\theta$$

$$\sigma_z = \frac{3qz^3}{2\pi u^4} \int_0^{\pi/2} \cos^3\theta\, d\theta$$

Let, $t = \operatorname{Sin}\theta$

$\therefore$ $dt = \cos\theta\, d\,\theta$

$$\sigma_z = \frac{3qz^3}{\pi u^4} \int_0^1 (1 - t^2)\,dt$$

$$\sigma_z = \frac{3qz^3}{\pi u^4}\left[1-\frac{1}{3}t^3\right]_0^1$$

$$\sigma_z = \frac{3qz^3}{\pi u^4}\times\frac{2}{3} = \frac{2qz^3}{\pi(x^2+z^2)^2}$$

$$\sigma_z = \frac{2q}{\frac{\pi}{z^3}(x^2+z^2)^2} = \frac{2q}{\pi z\left(\frac{x^2}{z^2}+1\right)^2}$$

$$\boxed{\sigma_z = \frac{2q}{\pi z}\left[\frac{1}{1+\left(\frac{x}{z}\right)^2}\right]^2}$$

- When the point P lies vertically below the line load, $x = 0$, at depth z.

  Then, $\boxed{\sigma_z = \frac{2}{\pi}\frac{q}{z}}$

Example 4.

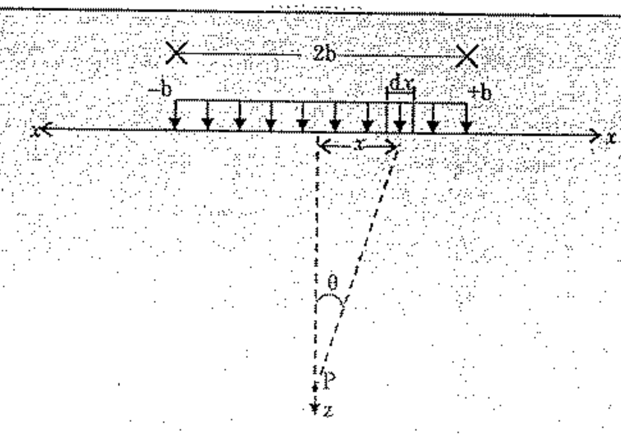
There is a line load of 120 kN/m acting on the ground surface along y-axis. Determine the vertical stress at a point P which has x and z coordinates as 2m and 3.5m, respectively.

Sol : We know that, $\sigma_z = \frac{2q}{\pi z}\left[\frac{1}{1+(x/z)^2}\right]^2$

At point P, $\sigma_z = \frac{2\times 120}{\pi\times 3.5}\left[\frac{1}{1+(2/3.5)^2}\right]^2$ = **12.40 kN/m²**

VERTICAL STRESS DUE TO STRIP LOAD

- The expression for Vertical stress at any point P under a strip load can be developed from the equation of line load.
- The expression will depend on whether the point P lies below the center of the strip load or not.
  Case I. Point P below the center of the strip.



- Let a strip load of a width 2b and intensity q.
- Let load acting on a small element dx at a distance x from the center of the load.
- This small load of intensity q dx can be considered as a line load of intensity q.

$$\Delta\sigma_z = \frac{2q\,dx}{\pi z}\left[\frac{1}{1+\left(\frac{x}{z}\right)^2}\right]^2$$

The stresses due to entire strip load is obtained as.

$$\sigma_z = \frac{2q}{\pi z}\int_{-b}^{+b}\frac{1}{\left[1+\left(\frac{x}{z}\right)^2\right]^2}dx$$

$$\frac{x}{z} = \tan u$$

$\therefore$
$$dx = z\sec^2 u\,du$$

$$\sigma_z = \frac{2q}{\pi z}\times 2\int_0^\theta \frac{z\,\mathrm{Sec}^2 u}{(1+\tan^2 u)^2}du$$

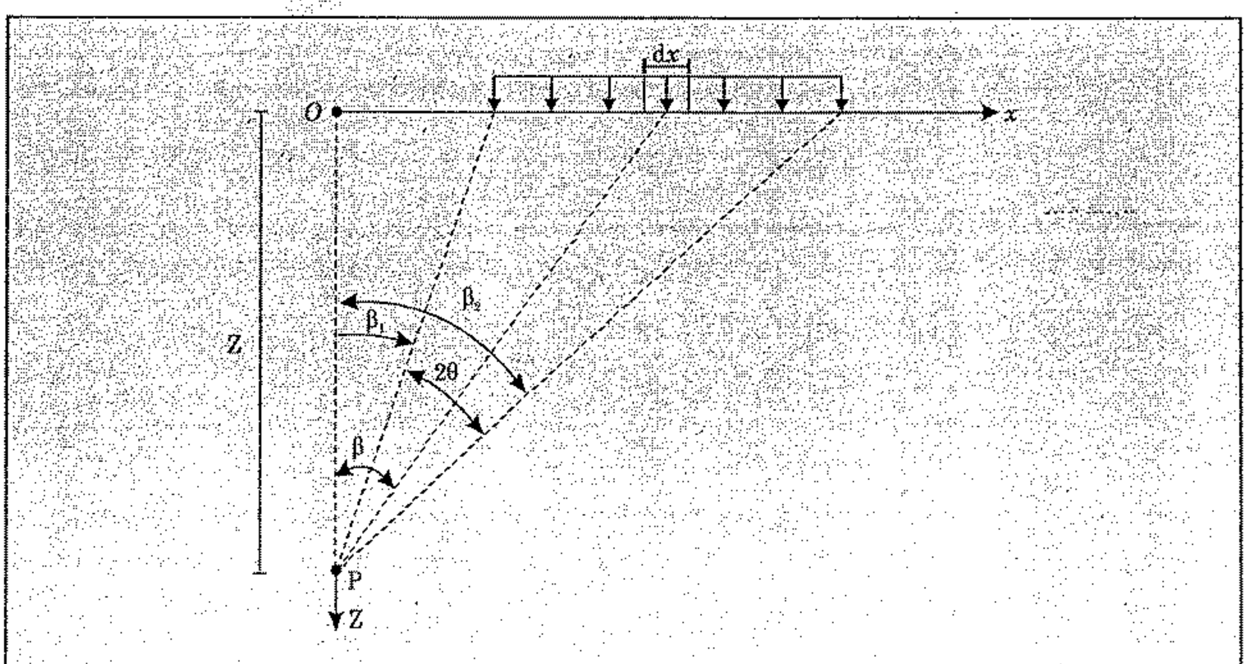
where,
$$\theta = \tan^{-1}\left(\frac{b}{z}\right)$$

$$\sigma_z = \frac{4q}{\pi}\int_0^\theta \cos^2 u\,du$$

$$\sigma_z = \frac{4q}{\pi}\int_0^\theta\left(\frac{1+\cos 2u}{2}\right)du$$

$$\boxed{\sigma_z = \frac{q}{\pi}(2\theta+\sin 2\theta)}$$

Case II. Point P not below the center of stirp



$$\sigma_z = \frac{2qdx}{\pi z}\left[\frac{1}{1+\left(\frac{x}{z}\right)^2}\right]^2$$

$$x = z \tan \beta$$

$$dx = z \, Sec^2 \beta \, d \beta$$

$$\Delta\sigma_z = \frac{2q(zSec^2\beta)d^\beta}{\pi z}\left[\frac{1}{1+\tan^2\beta}\right]^2$$

$$\Delta\sigma_z = \frac{2q}{\pi}\cos^2\beta d\beta$$

$$\sigma_z = \frac{q}{\pi}\int_{\beta_1}^{\beta_2}(1+\cos 2\beta)d\beta$$

$$= \frac{q}{\pi}\left[\beta+\frac{1}{2}Sin2\beta\right]_{\beta_1}^{\beta_2}$$

$$\sigma_z = \frac{q}{\pi}\left[(\beta_2-\beta_1)+(Sin\beta_2 Cos\beta_2 - Sin\beta_1 Cos\beta_1)\right]$$

Put, $\beta_2 - \beta_1 = 2\theta$

$$\sigma_z = \frac{q}{\pi}\left[2\theta+(Sin\beta_2 Cos\beta_2 - Sin\beta_1 Cos\beta_1)\right]$$

If, $(\beta_1 + \beta_2) = 2\phi$ then,

$$Sin\beta_1 Cos\beta_2 - Sin\beta_1 Cos\beta_2 = Sin\ 2\phi\ Cos\ 2\phi$$

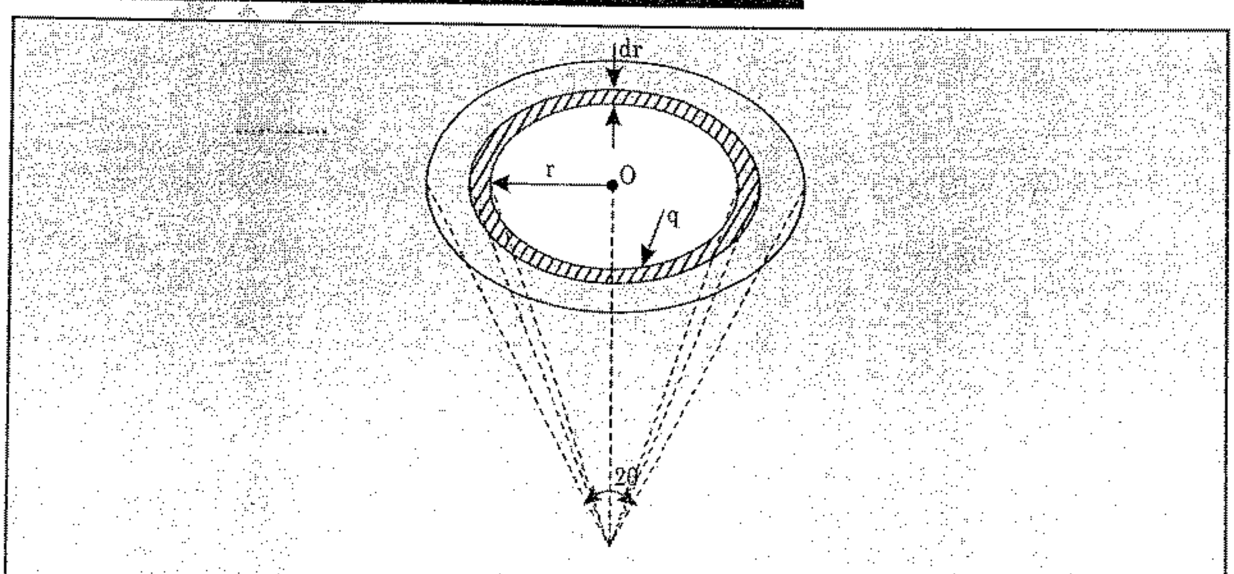
$\therefore$

$$\boxed{\sigma_z = \frac{q}{\pi}\left[2\theta + Sin2\theta\, Cos2\phi\right]}$$

If the point O is directly below the center of the strip, i.e. $\beta = 0$.

$$\boxed{\sigma_z = \frac{q}{\pi}\left[2\theta + Sin2\theta\right]}$$

VERTICAL STRESS UNDER A CIRCULAR AREA



- Consider a uniform load of intersity q acting over a circular area of radius R on the surface of a semi-infinite soil mass.
- Boussineq's Equation can be used to determine σ_z.
- The load on the elementary ring of radius R and width *dr* is equal to q $(2\pi r)$ dr.
- The load acts at a constant radial distance r from the point P.

$$\Delta\sigma_z = \frac{3(q\times 2\pi r dr)}{2\pi}\times\frac{1}{z^2}\times\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

$$\sigma_z = 3qz^3\int_0^R \frac{r dr}{(r^2+z^2)^{5/2}}$$

Let, $u = r^2 + z^2$

$\therefore$ $du = 2\pi r\ dr$

$\therefore$ $$\sigma_z = 3qz^3\int_{z^2}^{(R^2+z^2)}\frac{du}{2u^{5/2}}$$

$$\sigma_z = \frac{3}{2}qz^3\left(\frac{-2}{3}\right)\left[u^{-3/2}\right]_{z^2}^{R^2+z^2}$$

$$\sigma_z = -qz^3\left[\frac{1}{(R^2+z^2)^{3/2}}-\frac{1}{(z^2)^{3/2}}\right]$$

$$\sigma_z = qz^3\left[\frac{1}{z^3}-\frac{1}{(R^2+z^2)^{3/2}}\right]$$

$$\boxed{\sigma_z = q\left[1-\left(\frac{1}{1+\left(\frac{R}{z}\right)^2}\right)^{3/2}\right]}$$

$$\boxed{\sigma_z = I\,q}$$

Where, I is the influence Value.

$\frac{R}{z}$ can be written in terms of the angle 2θ subtended at point P by the load.

Let, $\tan\theta = \frac{R}{z}$

$$I = \left[1-\left(\frac{1}{1+\tan^2\theta}\right)^{3/2}\right]$$

$$I = \left[1-(\cos^2\theta)^{3/2}\right]$$

$$I = \left[1-\cos^3\theta\right]$$

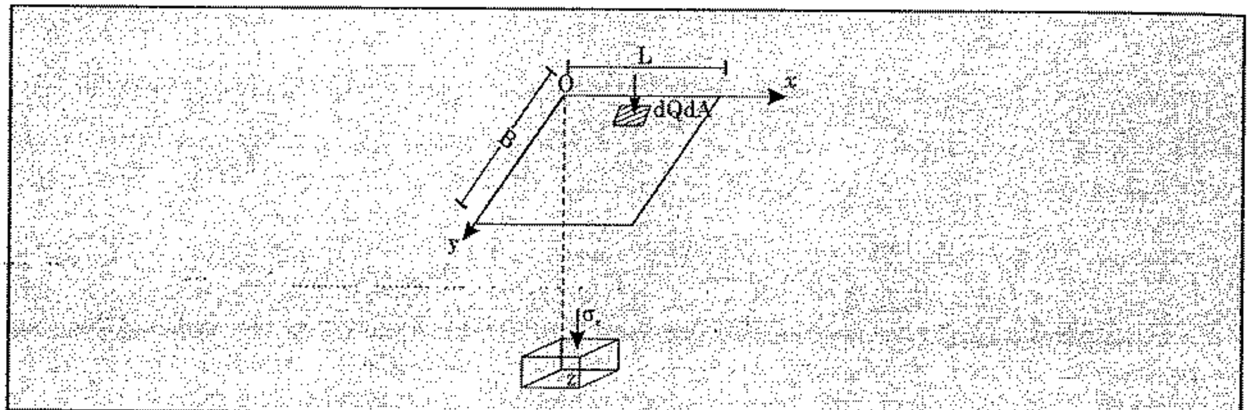
- From the above result it is observed that as $\theta = 90°$, $I = 1$, i.e. for a very large uniformly loaded area in comparission to depth z, the vertical stress at a point P is approximately equal to q.

VERTICAL STRESS BELOW THE CORNER OF A RECTANGULAR

- The vertical stress under a corner of rectangular area with a uniformly distributed load of intensity q can be obtained from Boussinesq's equation of vertical stress. The stress at depth z is given by,

$$dQ = qdA = q.dx.\ dy$$

$$\Delta\sigma_z = \frac{3}{2\pi}(q\,dx\,dy)z^3 \quad \frac{1}{(x^2+y^2+z^2)^{5/2}}$$



$$\sigma_z = \frac{3qz^3}{2\pi}\int_0^L\int_0^B \frac{q\,dx\,dy}{(x^2+y^2+z^2)^{5/2}}$$

- Newmark derived the following equation by considering, $m = \frac{B}{z}$ and $n = \frac{L}{z}$

$$\sigma_z = \frac{q}{2\pi}\left[\frac{mn}{\sqrt{m^2+n^2+1}}\times\frac{m^2+n^2+2}{m^2+n^2+m^2n^2+1}+\text{Sin}^{-1}\left(\frac{mn}{\sqrt{m^2+n^2+m^2n^2+1}}\right)\right]$$

- The values of m & n can be interchanged without effecting the σ_z.

I_N = Newmarks Influence Coefficient

$$I_N = \frac{1}{2\pi}\left[\frac{mn}{\sqrt{m^2+n^2+1}}\times\frac{m^2+n^2+2}{m^2+n^2+m^2n^2+1}+\text{Sin}^{-1}\left(\frac{mn}{\sqrt{m^2+n^2+m^2n^2+1}}\right)\right]$$

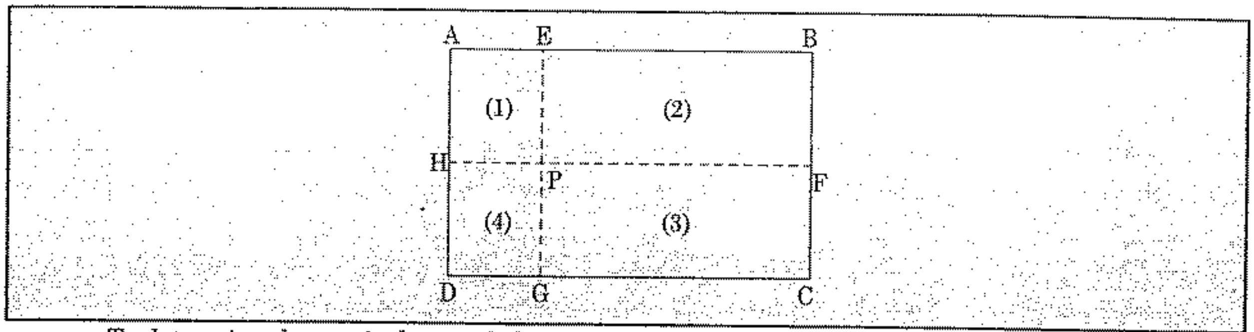
or

$$I_N = \frac{1}{4\pi}\left[\frac{2mn(m^2+n^2+1)^{1/2}}{m^2+n^2+m^2n^2+1}\times\frac{m^2+n^2+2}{m^2+n^2+1}+\tan^{-1}\frac{2mn(m^2+n^2+1)^{1/2}}{m^2+n^2+1-m^2n^2}\right]$$

VERTICAL STRESS AT ANY POINT UNDER A RECTANGULAR AREA

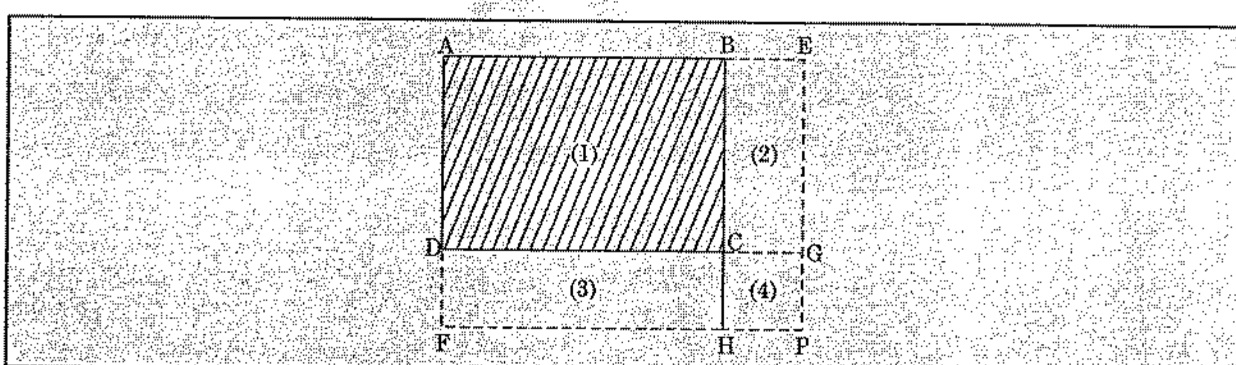
- If the point where vertical stress is to be determined does not fall below the corner of rectangular area then we use principle of superposition to determine stress at that point.

(1) Point any where below tbe rectangular area



- To determine the vertical stress below any point *within* a rectangular loaded area but not below a corner.
- The area is divided into four rectangles such that, the point under consideration is made the common corner of each rectangle.
- The corresponding influence values and then the vertical stresses are determined.
- The sum of the four stresses thus found is the vertical stress σ_z at a depth z below the point is question.
- The given rectangle is subdivided into 4 small rectangles AEPH, EBFP, HPGD and PFCG, each having one corner at P. The vertical stress at P due to the given rectangular load is equal to that from the four small rectangles.
  $\sigma_z = q\,[(I_N)_1 + (I_N)_2 + (I_N)_3 + (I_N)_4]$
  where,
  $(I_N)_1$, $(I_N)_2$, $(I_N)_3$ and $(I_N)_4$ are Newmark's influence factors.

(2) Point Outside the loaded Area.



- When the point is at some depth z outside the loaded area, we can fabricate uniformly loaded rectangles, all of which having corners above the point where the vertical stress is desired; the required value is then obtained by adding or subtracting their stress contributions, as necessery, by considering the actual area of loading to be the algebraic sum of the fabricated rectangles.

  Area of rectangle ABCD = Area of rectangle AEPF – Area of rectangel BEPH
  – Area of rectangle DGPF + Area of rectangle CGPH

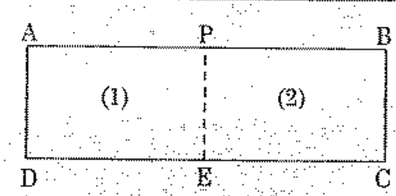
- The last rectangle CGPH is given plus sign because this area has been deducted twice, once in rectangle BEPH and once in DGPF. The stress at P due to a load on rectangel ABCD is given by

$$\sigma_z = q\,[(I_N)_1 - (I_N)_2 - (I_N)_3 - (I_N)_4]$$

where,

$(I_N)_1$, $(I_N)_2$, $(I_N)_3$ and $(I_N)_4$ are the influence coefficients.

(3) Point below the edge of Loaded Area.



- If point P is below the edge of the loaded area ABCD, the given rectangle is divided into two small rectangles APED and PBCE.

$$\sigma_z = q\,[(I_N)_1 + (I_N)_2 - (I_N)_3]$$

where,

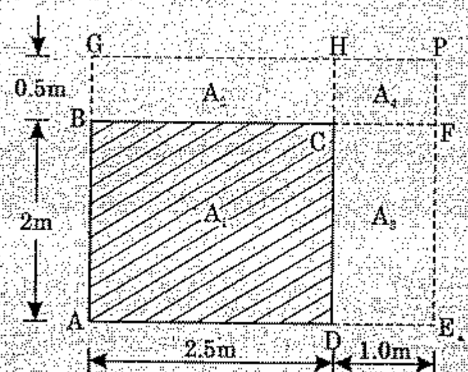
$(I_N)_1$ and $(I_N)_2$ are influence coefficients for rectangels APED and PBCE.

Example 5.

A rectangular loaded area 2m × 2.5m carries a load of 80 kN/m². Determine the vertical stress at point P located outside the loaded area at a depth of 2.5m.

Sol : We know that, $\sigma_z = [(I_N)_1 - (I_N)_2 - (I_N)_3 + (I_N)_4]$

$$m = \frac{L}{z},\ n = \frac{B}{z}$$



For rectangle AEPG, $m = 3.50/2.50 = 1.40,\ n = 2.50/2.50 = 1.00$

and $(I_N)_1 = 0.1914$

Area A_2 $m = 3.5/2.5 = 1.40,\ n = 0.5/2.5 = 0.20,\quad (I_N)_2 = 0.0589$

Area A_3 $m = 2.5/2.5 = 1.0,\ n = 1.0/2.50 = 0.40,\quad (I_N)_3 = 0.1013$

Area A_4 $m = 0.5/2.5 = 0.2,\ n = 1.0/2.5 = 0.40,\quad (I_N)_4 = 0.0328$

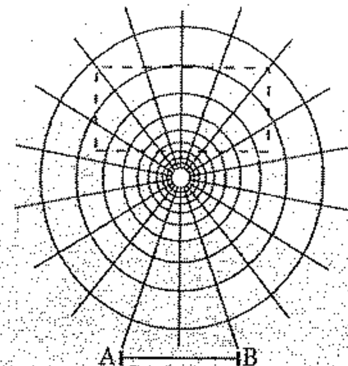
Therefore, $\sigma_z = 80[0.1914 - 0.059 - 0.1013 + 0.0328]$

$= \mathbf{5.12\ kN/m^2}$

NEWMARK'S INFLUENCE CHART

- Newmark developed influence chart to compute vertical stress, horizontal and shear stress also due to the uniformly loaded area of any shape, irregular or geometric, below any point, inside or out side the loaded area.

- This chart is based on Boussinesq's Equation.



Influence value per unit pressure = 0.005

- The chart as shown in the above figure essentially consists of n no. of radial lines and m no. of concentric circles.
- To find out vertical stress at any point below or outside the loaded area. Plan of the loaded area is drawn such that depth Z at which stress is being computed equals the length AB as shown on chart.
- Further, the plan is placed over influence chart such that the point below which stress is required coincides with the center of chart. Then count the number of influence area (N) covered by the plan area.

  The vertical stress is given by.

$$\boxed{\sigma_z = \frac{1}{m \times n} \times q \times N}$$

where,

m = no. of concentric circle
n = no. of radial lines.
q = Intensity of load
N = Equivalent no. of areas.

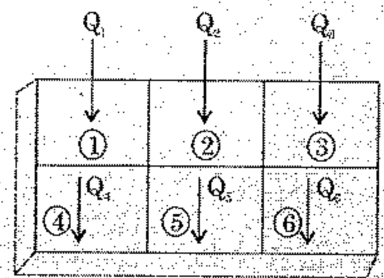
- It should be noted that all the area unit will have equal influence at the center and wether the area unit is inside or outside the loaded area even though it will have same influence at the center of chart have same influence at the center of chart where vertical stress is to computed.

Note: *Similarly like Newmarks influence chart Fenske's chart are developed which are based on Westergaard's equation.*

APPROXIMATE STRESS DISTRIBUTION METHODS FOR LOADED AREAS.

1. Equivalent Point Load Method

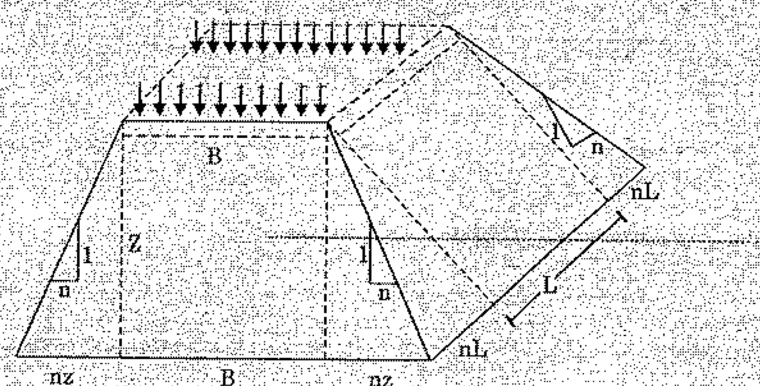
- In this method the loaded area is divided into smaller areas and then by assuming an equivalent point load acting at the centeriod of these units. Then total vertical stress σ_z is calculated either by Boussinique's or wester gaards theory, as the sum of all the vertical stresses calculated for different area units.



$$\sigma_z = K_{B_1}\frac{Q_1}{Z^2} + K_{B_2}\frac{Q_2}{Z^2} + K_{B_3}\frac{Q_3}{Z^2} + K_{B_4}\frac{Q_4}{Z^2} + K_{B_5}\frac{Q_5}{Z^2} + K_{B_6}\frac{Q_6}{Z^2}$$

2. Load distribution Moethod

- According to this method if a loaded area of dimension L × B has to be distributed over an area to depth Z below the loaded area. If will get distributed in a shape of trapazoid with the slanting sides having a slope of 1 : n (1v : nH).

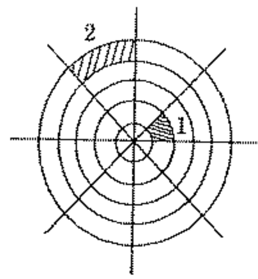


σ_z at the depth z below the loaded area will be given as.

$$\boxed{\sigma_z = \frac{q\times(B\times L)}{(B+2nz)-(L+2nz)}}$$

OBJECTIVE TYPE QUESTIONS

1. Standard Newmark's influence chart is shown in the given figure. If loaded equally the areas marked 1 and 2 will yield pressures at the centre such that

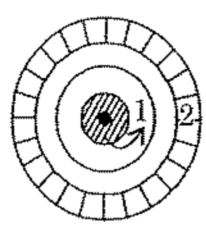


(a) 1 yield more than 2
(b) 2 yield more than 1
(c) 1 and 2 yield the same
(d) 1 yield exactly half of that of 2

2. A concentrated load of 50 t acts vertically at a point on the soil surface. If Boussinesq's equation is applied for computation of stress, then the ratio of vertical stresses at depths of 3 m and 5 m respectively vertically below the point of application of load will be
(a) 0.36
(b) 0.60
(c) 1.66
(d) 2.77

3. For a vertical concentrated load acting on the surface of a semi-infinite elastic soil mass, the vertical normal stress at depth 'z' is proportional to
(a) z
(b) 1/z
(c) z^2
(d) $1/z^2$

4. A part of the Newmark's influence chart with four concentric circles is shown in the figure below. If the hatched areas 1 and 2 are loaded separately with the same intensity of loading, then the intensity of pressure yielded



(a) by 1 will be more than that yielded by 2
(b) by 2 will be more than that yielded by 1
(c) by 1 and 2 will be equal
(d) at the centre will be inversely proportional to the radii of the two circles

5. Influence factor for immediate settlement of footing depends on its
(a) size and shape
(b) rigidity alone
(c) location and size
(d) size, shape, rigidity and location

6. In the case of stratified soil layers, the best equation that can be adopted for computing the pressure distribution is
(a) Prandtl's
(b) Skempton's
(c) Westergaard's
(d) Boussinesq's

7. In a Newmark's influence chart for stress distribution, there are 10 concentric circles and 50 radial lines. The influence factor of the chart is

(a) 0.0002
(b) 0.002
(c) 0.02
(d) 0.2

8. A footing of 3 m × 3 m size transmits a load of 1800 kN. The angle of load dispersion in soil a = $\tan^{-1}$ 0.5. What is the stress created by the footing load at a depth of 5 m?

(a) 26.12 kN/m^2
(b) 27.12 kN/m^2
(c) 28.12 kN/m^2
(d) 29.12 kN/m^2

9. Westergaard's formula for vertical stress gives greater value of stress than that by the Boussinesq's formula, when r/z exceeds

(a) 1.5
(b) 2.5
(c) 3.5
(d) 4.0

10 A point load of 650 kN is applied on the surface of a thick layer of clay. Using Boussinesq's elastic analysis, what is approximate value of the estimated vertical stress at a depth 2 m and a radial distance of 1.0 m from the point of application of load?

(a) 55 kN/m^2
(b) 44 kN/m^2
(c) 41 kN/m^2
(d) 37 kN/m^2

11. A line load of infinite length has an intensity q per unit length. What is the vertical stress s_z at a depth z below the earth at the centre of the load?

(a) $\sigma_z = \dfrac{2qz}{\pi}$
(b) $\sigma_z = \dfrac{2q}{\pi z}$
(c) $\sigma_z = \dfrac{2qz^2}{\pi}$
(d) $\sigma_z = \dfrac{2q}{\pi z^2}$

12. Consider the following statements:

1. Westergaard's theory of stress distribution is more appropriate for soil deposits which exhibit large lateral strain.
2. Newmark's influence chart can be used for the determination of vertical stress under any slope of a loaded area.

Which of these statements is/are correct?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

13. Westergaard's analysis for stress distribution beneath loaded areas is applicable to

(a) sandy soils
(b) clayey soils
(c) stratified soils
(d) silty soils

14. Consider the following characteristics of soil layer:

1. Poisson's ratio
2. Young's modulus
3. Finite nature of soil layer
4. Effect of water table
5. Rigidity of footing

Westergaard's analysis for pressure distribution in soils utilizes

(a) 1, 2, 4 and 5
(b) 2, 3, 4 and 5
(c) 3, 4 and 5
(d) 1 and 2

15. Soil pressure distribution below a rigid footing on the surface of a cohesive soil is
    (a) maximum at the centre and minimum at edges
    (b) minimum at the centre and maximum at edges
    (c) uniform throughout
    (d) maximum at one end and minimum at the other end
16. According to Boussinesq's theory, the vertical stress at a point in a semi-infinite soil mass depends upon
    (a) point load, co-ordinates of the point and modulus of elasticity of soil
    (b) point load, co-ordinates of the point, modulus of elasticity of soil and its Poisson's ratio
    (c) point load and co-ordinates of the point
    (d) point load, co-ordinates of the point, modulus of elasticity of soil and its density
17. A circular area of radius R on the surface of a semi-infinite soil mass is uniformly loaded with a loading intensity of 'q'. The vertical stress s_z directly below its centre at a depth 'z' is given by

(a) $\frac{q}{z}\times\frac{2}{\pi}\left[\frac{1}{1+\left(\frac{R^2}{z}\right)}\right]^2$

(b) $q\left[1-\left(\frac{1}{1+\left(\frac{R^2}{z}\right)}\right)^{3/2}\right]$

(c) $\frac{3q}{2\pi z^2}\left[\frac{1}{1+\left(\frac{R}{z}\right)^2}\right]^{5/2}$

(d) $\frac{q}{2\pi z}\left[\frac{1}{1+\left(\frac{R}{z}\right)^2}\right]^{3/2}$

18. The stress distribution at a depth beneath a loaded area is determined using Newmark's influence chart which indicates an influence value of 0.005. The number of segments covered by the loaded area in the chart is 20 and the intensity of loading on the area is 10 t/m^2. The intensity of stress distribution at that depth is
    (a) 1 t/m^2 (b) 2 t/m^2 (c) 5 t/m^2 (d) 10 t/m^2
19. Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

	List-I		List-II
A.	Stress distribution due to point load in homogeneous isotropic medium	1.	Stein Brenner
B.	Stress distribution due to point load in an anisotropic soil medium	2.	Newmark
C.	Influence chart for stress distribution in a rectangular area	3.	Boussinesq
D.	Influence chart for stress distribution in irregularly shaped areas	4.	Westergaard

Codes:

	A	B	C	D
(a)	4	3	2	1
(b)	3	4	2	1
(c)	3	4	1	2

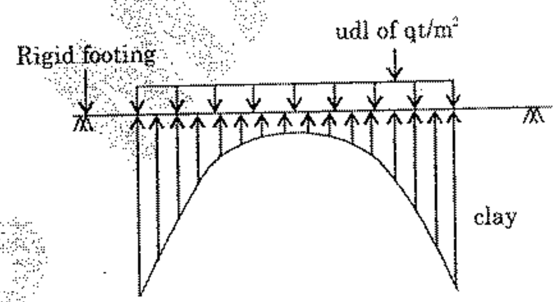
20. What is Newmark's chart used for?
   (a) To know the safe bearing capacity of a footing
   (b) To know the settlement of a foundation
   (c) To know the stress intensity at any depth due to a loaded foundation
   (d) To know the allowable bearing pressure on the foundation

21. There are two footings resting on the ground surface. One footing is square of dimension 'B'. The other is strip footing of width 'B'. Both of them are subjected to a loading intensity of q. The pressure intensity at any depth below the base of the footing along the centre line would be
   (a) equal in both footings
   (b) large for square footing and small for strip footing
   (c) large for strip footing and small for square footing
   (d) more for strip footing at shallow depth (£ B) and more for square footing at large depth (> B)

22. The vertical stress at some depth below the corner of a 3 m × 3 m rectangular footing due to a certain load intensity is 100 kN/m^2. What will be the vertical stress in kN/m^2 below the centre of a 6 m × 6 m rectangular footing at the same depth and same load intensity?
   (a) 25
   (b) 100
   (c) 200
   (d) 400

23. A footing 2 m × 1 m exerts a uniform pressure of 150 kN/m^2 on the soil. Assuming a load dispersion of 2 vertical to 1 horizontal, the average vertical stress (kN/m^2) at 1.0 m below the footing is
   (a) 50 (b) 75 (c) 80 (d) 100

24. The given figure shows the contact pressure distribution in pure clayey soil subjected to a uniformly distributed load (udl) through rigid footing (placed on the surface).

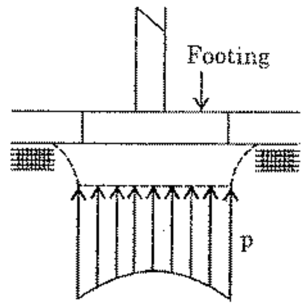


Which of the following would cause the contact pressure distribution maximum at the centre and decrease towards the outer edges leading to parabolic shape?
1. When udl is transmitted through rigid footing placed on the surface of a cohesionless soil.
2. When udl is transmitted through flexible footing placed on the surface of a cohesive soil.
3. When udl is transmitted through flexible footing placed on the surface of a pure clay.

Select the correct answer using the codes given below:
   (a) 1, 2 and 3
   (b) 1 and 2
   (c) 2 and 3
   (d) 1 alone

25. The contact pressure distribution below a rigid footing on the surface of a clay soil is
   (a) uniform for the full width
   (b) maximum at the edges and minimum at the centre
   (c) maximum at the centre and minimum at the edges
   (d) none of the above

26. The figure given below represents the contact pressure distribution underneath a



(a) rigid footing on saturated clay
(b) rigid footing on sand
(c) flexible footing on saturated clay
(d) flexible footing on sand

ANSWERS

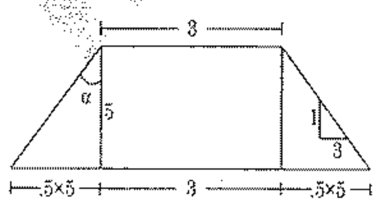
1. (c)	2. (d)	3. (d)	4. (c)	5. (d)	6. (c)	7. (b)	8. (c)	9. (a)	10. (b)
11. (b)	12. (d)	13. (c)	14. (a)	15. (b)	16. (c)	17. (b)	18. (a)	19. (c)	20. (c)
21. (c)	22. (d)	23. (a)	24. (d)	25. (b)	26. (a)				

Hints

2. $s_z \propto \frac{1}{z^2}$

$\frac{\sigma_{z@3m}}{\sigma_{z@5m}} = \frac{25}{9} = \mathbf{2.77}$

8. $\tan\alpha = 0.5 = \frac{P}{B}$



$$\sigma_z = \frac{1800}{(3+2\times0.5\times5)^2}$$

$$\sigma_z = \frac{1800}{8^2}$$

$$\sigma_z = 28.125 \text{ kN/m}^2$$

10. $\sigma_z = \frac{3}{2\pi}\frac{q}{z^2}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$

$$\sigma_z = \frac{3}{2\pi}\times\frac{650}{2^2}\left[\frac{1}{1+\left(\frac{1}{2}\right)^2}\right]^{5/2}$$

$= 44.41 \text{ kN/m}^2$

18. $\sigma_z = I_2 zN$

$= 0.005 \times 10 \times 20$

$= 1 \text{ t/m}^2$

23. $\sigma_z = \frac{150\times2\times1}{(2+2\times.5\times1)\times(1+2\times.5\times1)}$

$= \frac{150\times2\times1}{3\times2} = 50$

CHAPTER

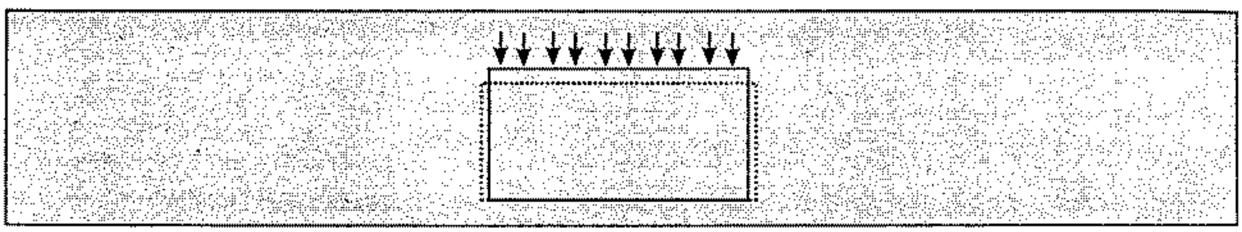
8

Compressibility and Consolidation

INTRODUCTION

- When a soil mass is subjected to a compressive force, like all other materials, its volume decreases.
- The property of the soil due to which a decrease in volume occurs under compressive forces is known as the compressibility of soil.
- The decrease in volume of soil, under stress is because of

  1. Compression and Expulsion of Pore Air.
  2. Compression and Expulsion of Pore Water.
  3. Gradual readjustment of clay particles into more stable configuration.



- Reduction in volume of soil mass results in change of lateral and vertical dimensions of soil mass.
- As soil being infinitely large in the lateral direction, hence the change in dimension in this direction is considered to be negligible, but there is significant change in vertical direction which is termed as settlement of soil.
- In other words settlement of soil is the gradual sinking of the structure due to compression of the soil below.
- Total settement of soil is expressed as three components.

$$S_t = S_{immidiate} + S_{1^\circ\text{-consolidation}} + S_{2^\circ\text{-consolidation}}$$

IMMEDIATE SETTLEMENT

- If the soil is initially partially saturated, expulsion of air as well as compression of pore air may take place with the application of external loads which is called Initial Compression. It is a immidiate phenomenon.
- After the initial compression, soil reaches into fully saturated state, further reduction in volume occurs due to expulsion of pore water i.e. water present in the soils.
- Immidiate settlement can also occur if significant lateral strain takes place. This is due to deformation of soil under undrained condition. This immidiate settlement can be calculated from elastic theory.

PRIMARY CONSOLIDATION

- Primary-consolidation occurs due to expulsion of excess pore water pressure generated due to increase in total stress. It is a time dependent phenomenon.

  For Example:

- When a structure is built over a layer of saturated clay.
- When water table is lowered permanently in a structure overlaying a clay layer.

  Magnitude of settlement due to 1°-consolidation depends on:

  1. Compresibility of soil
  2. Magnitude of stress increase
  3. Thickness of soil layer
  4. Permeability of soil

2°-CONSOLIDATION

- Experimentally], it has been shown that compression of soil layer do not cease when excess pore water pressure has been completely dissipated to zero. It *continues at a gradually decreasing rate under constant effective stress.*
- 2°-Consolidation is thought to be due to *gradual readjustment of clay particles into a more stable configuration* following the structural disturbance caused by the decrease in void ratio.
- Rate of 2°-consolidation is thought to be controlled by highly viscous film of absorbed water, surrounding the clay mineral particles in soil.
- *A very slow viscous flow of adsorbed water takes place from the zones of film contact, allowing the particles to move closer together.*
- *Viscosity of film increases as particles move closer, resulting in a decrease in the rate of compression of the soil.*
- It is more important For peat for organic soil.
- It is unimportant for preconsolidated clay and stiff clay,

  Thus,

- For Stiff clay (unconfined compressive stress > 200 KN/m²)

$$S_t = S_{\text{immidiate}} + S_{1^\circ\text{-consolidation}}$$

- For Granular Soils

$$S_t = S_{(\text{immidiate})}$$

Note: In coarse grained soil, any volume change resulting from change in loading occurs immediately after the loading increases and water pressure dissipates rapidly due to high permeability. This is called drained loading. In fine soils with low permeability, the soil is undrained as load is applied, slow see page occurs and excess pore water disspates leading to consolidation. Hence in coasrse grained soil 1° consolidation = 0.

SILENT POINTS ABOUT CONSOLIDATION

- In the analysis of consolidation theory we consider only *one-dimensional consolidation*. One-dimensional consolidation means that deformation occurs in only vertical direction.

$$\therefore \qquad \boxed{\frac{\Delta H}{H} = \frac{\Delta V}{V}}$$

Where, ΔV = Change in Volume

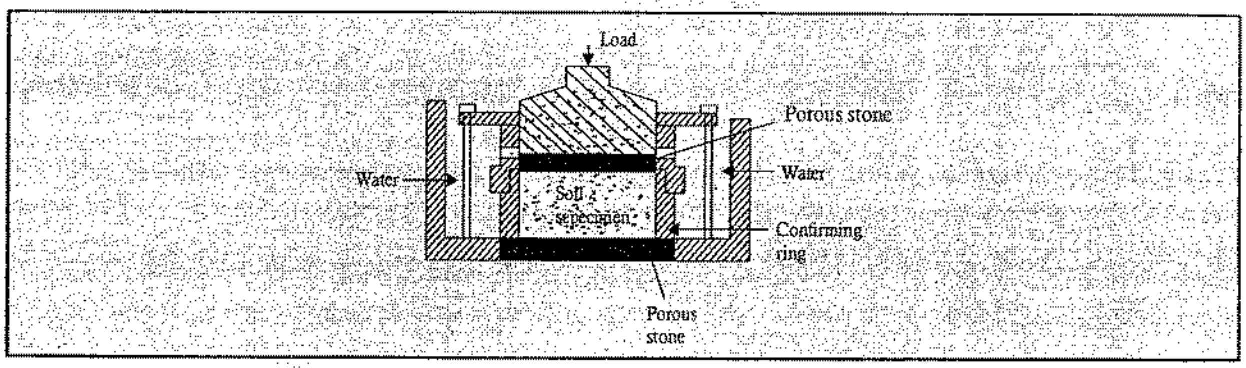
V = Original volume

ΔH = Change in depth

H = Original depth.

- Due to one-dimensional consolidation assumption, *lateral strain will be zero.*
- One-dimensional consolidation is realised when
  - (a) loaded area is large
  - (b) there is lateral confinement such that lateral strain = 0.

ODEOMETER TEST



- The characteristics of soil during one-dimensional consolidation or swelling can be determined by mean of *oedometer test/Consolidation test.*
- Apparatus is known as odeometer or Consolidometer.
- Soil is loaded in increments of vertical stress.
- Under each increment of loading, soil is allowed to consolidate till there is no or little further compression (usually load is kept for 24 hrs).
- Compression reading is noted at suitable intervals during this period (This gives the rate of consolidation).
- After the end of the increment period, when excess pore water has dissipated completely, the applied pressure becomes equal to the effective stress in the specimen.
- The 24 hr. reading gives the final compression under each stress increment.
- After one increment has been kept for around 24 hr. next increment is applied and same procedure of recording the compression at various time interval, is repeated.
- After the consolidation under the last stress increment is over, the specimen is unloaded in two or three stages and the soil is allowed to swell.

- Only the final swell readings are taken at each unloading stage.
- After complete unloading, soil specimen is taken out, dried in the oven to determine.
  (a) *weight* of solids (W_s)
  (b) final water content (w_f)
- The results of Consolidation test are presented by plotting a graph between void ratio at the end of each increment period v/s the corresponding effective stress.
- The void ratio at the end of each increment period is also known as increment void ratio.

EFFECTIVE STRESS AND VOID RATIO RELATIONSHIP

- Equillibrium void ratio at each stress level is found out by two different methods.

1. Height of Solid Method

$$\boxed{\text{Hs} = \frac{W_s}{G_s \gamma_w . A}}$$

$$\boxed{\text{e} = \frac{\text{H} - H_s}{H_s}}$$

$$\left[e = \frac{A \times H - A \times H_S}{A \times H_S} = \frac{V - V_S}{V_S}\right]$$

Where, W_s = dry wt of soil
G_s = Specific gravity of soil (must be known)
A = Cross section area of soil specimen
H = height of soil speciment at the end of a particular stress increment
$H = H_1 \pm \Delta H$
H_1 = Height of specimen at the begining of stress increment
ΔH = Change in thickness under the stress increment
H_S = Height of solid

2. Change in void ratio method

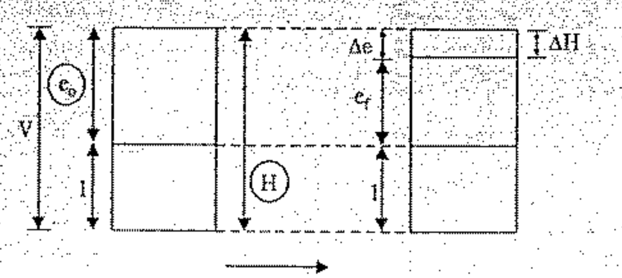
Soil at the end of test is assumed to be saturated (*i.e.* s = 1)

Hence from $\boxed{S.e = w.G}$

$$\boxed{e_{f0} = w_{f0} \cdot G_s}$$

Where, e_{f0} = Void ratio at the end of all stress increment *i.e.* at the end of the test.
w_{f_0} = Water content at the end of test (Noted after oven drying)

Due to effective stress measurment $\Delta\bar{\sigma}$



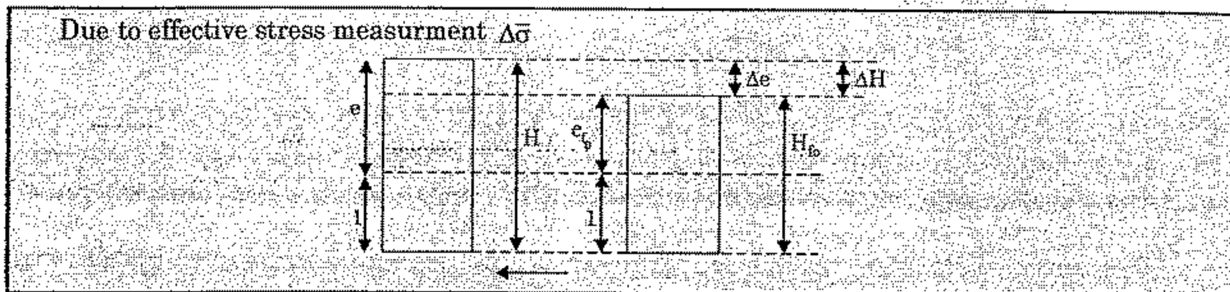
For one dimensional consolidation

$$\boxed{\frac{\Delta V}{V}=\frac{\Delta H}{H}=\frac{\Delta e}{1+e_0}}$$

- Knowing H, e_0 and ΔH, Δe can be calculated and from the above expression. Further the equilibrium void ratio at the end of every effective stress increment can be calculated as $\boxed{e = e_0 - \Delta e}$

Another methods:

- By working backwards from the known value of e_f & H_f at the end of each effective stress increment, we can calculate equilibrium void ratio.



$$\boxed{\frac{\Delta H}{H_{f0}}=\frac{\Delta e}{1+e_{f0}}}$$

- Knowing ΔH, H_{fo} & $\boxed{e_{fo} = w_{fo} G_s}$, Δe can be calculated. Then working backwards, $\boxed{e = e_{fo} + \Delta e}$ can be calculated.
- Thus void ratio at the end of various stress increments can be calculated.

Example 1.

Applied stress (kN/m^2)	Sample thickness (mm)
0	19.7
50	19.3
100	19.15
200	18.85
400	18.56
800	18.36
0	18.90

Water content of the sample at the end of test 28% and $G_S = 2.72$. Find the void ratio at each applied stress.

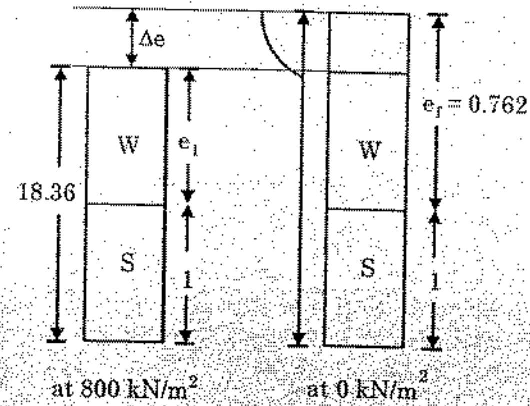
Sol. Let the final void ratio of the sample be e_f.

$\therefore$ From water-soil relationship

We have $\qquad e_f \cdot S = wG$

$\Rightarrow \qquad e_f \times 1 = (0.28 \times 2.72)$

$$e_f = 0.762 \qquad \dots (i)$$



$\therefore$ We have $$\frac{\Delta H}{H_0} = \frac{\Delta e}{(1+e_0)}$$

$$\Rightarrow \quad \frac{(18.36-18.90)}{18.90} = \frac{\Delta e}{(1+0.762)}$$

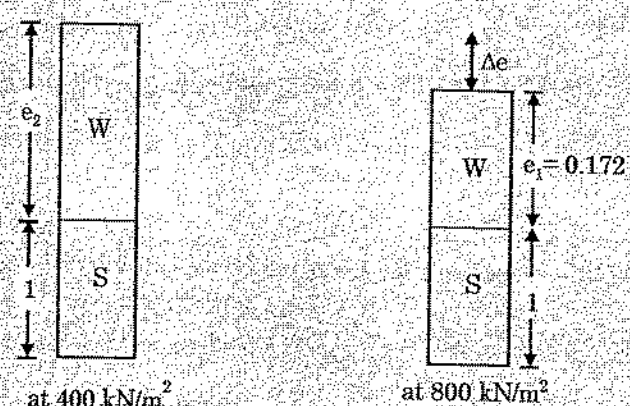
$$\Rightarrow \quad \Delta e = -0.050$$

Thefore void ratio at 800 kN/m² $= (e_f - \Delta_e)$

$= (0.762 - 0.050)$

$= 0.712$

For 400 – 800 kN/m² Stress range



$\therefore$ We have $$\frac{\Delta e}{1+e} = \frac{\Delta H}{(H_0)}$$

$$\Rightarrow \quad \frac{\Delta e}{(1+0.712)} = \frac{(18.56-18.36)}{18.36}$$

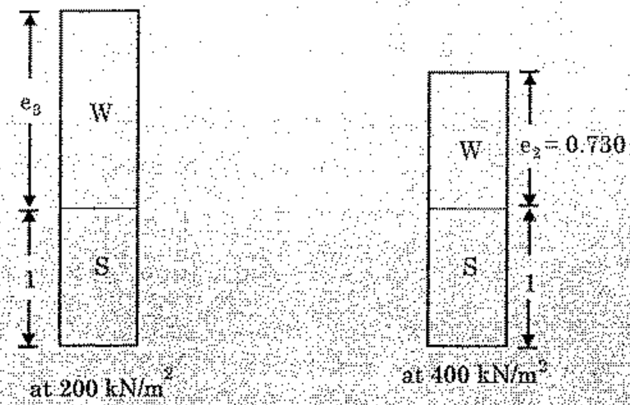
$$\therefore \quad \Delta e = 0.018$$

$\therefore$ Void Ratio at 400 kN/m² $= e_2 = (e_f + \Delta_e)$

$= (0.712 + 0.018)$

$= 0.730$

For 400 – 200 kN/m² Stress range



$\Rightarrow$ We have $\qquad \dfrac{\Delta H}{H_0} = \dfrac{\Delta e}{(1+e_0)}$

$\Rightarrow$ $\qquad \Delta e = \dfrac{(18.85-18.56)}{(18.56)} \times (1+0.730)$

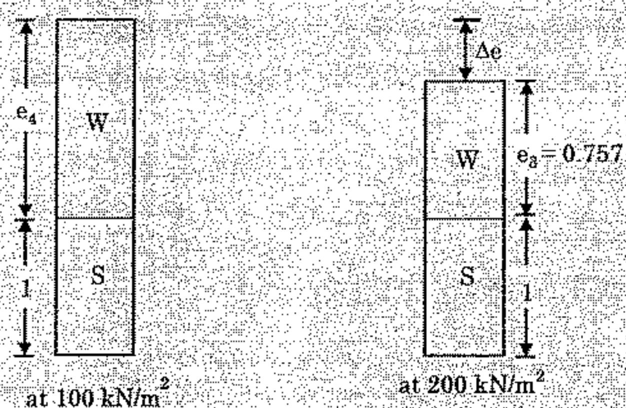
$= 0.0270$

$\Rightarrow$ Void ratio at 200 kN/m² $\qquad e_3 = (e_2 + \Delta e)$

$= (0.730 + 0.0270)$

$= 0.757$

For 200 – 100 kN/m² stress range



We have $\qquad \dfrac{\Delta H}{H_0} = \dfrac{\Delta e}{(1+e_0)}$

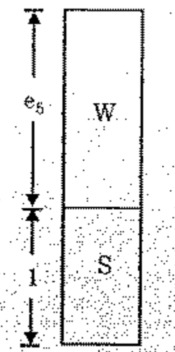
$\therefore$ $\qquad \Delta e = \dfrac{(19.15-18.15)}{(18.85)} \times (1+0.757)$

$= 0.02796$

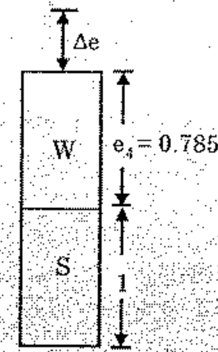
Void ratio at 100 kN/m² $= (0.757 + 0.02796)$

$e_4 = 0.785$

For 100 – 50 kN/m² stress range



at 50 kN/m²



at 100 kN/m²

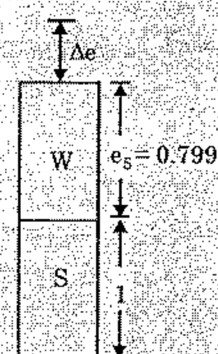
We have
$$\frac{\Delta H}{H_0} = \frac{\Delta e}{(1+e_0)}$$

$$\therefore \quad \Delta e = \frac{(19.3-19.15)}{19.15}\times(1+0.785)$$
$$= 0.01398$$

Void ratio at 50 kN/m²,
$$e_5 = (e_4 + \Delta e)$$
$$= (0.785 + 0.01398)$$
$$= 0.799$$

For 50 – 0 kN/m² stress range

at 0 kN/m²



at 50 kN/m²

We
$$\frac{\Delta H}{H_0} = \frac{\Delta e}{(1+e_0)}$$

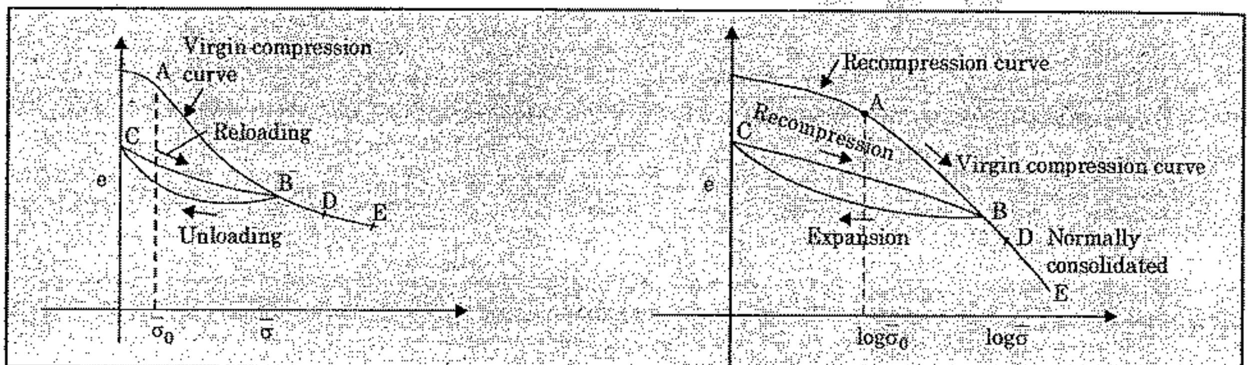
$$\Rightarrow \quad \Delta e = \frac{(19.7-19.3)}{19.3}\times(1+0.799)$$
$$= 0.03728$$

Threfore void ratio at 0 kN/m2,
$$e_6 = (0.799 + 0.03728) = 0.836$$

Applied stress (kN/m²)	Sample thickness (mm)	void ratio (e)
0	19.7	0.836
50	19.3	0.799
100	19.15	0.785
200	18.85	0.757
400	18.56	0.730
800	18.36	0.712
0	18.90	0.762

COMPRESSIBILITY CHARACTERISTIC

- Shape of the curve are related to the stress history of clays.



- Soil tend to retain the effect of stress changes that have taken place in their geological history, in the form of their structure.
- A soil is said to be *Normally-consolidated* when the existing effective stress is the stress it has ever experienced in its stress history.
- The maximum value of stress that the soil has ever experienced is called *preconsolidation stress* $\bar{\sigma}_o$.
- *If existing effective stress* $\bar{\sigma} > \bar{\sigma}_0$, *soil is said to be* Normally-consolidated.
- If $\bar{\sigma} < \bar{\sigma}_0$ soil is said to be over consolidated soil/Preconsolidated soil.

$$\boxed{\frac{\bar{\sigma}_o}{\bar{\sigma}} = \text{Over Consolidation Ratio (OCR)}}$$

- For Normally Consolidated soil, OCR ≤ 1
- For Over Consolidated soil, OCR > 1
- Over consolidation is due to several factors such as
  - (a) Erosion of overburden soil
  - (b) Permanent rise in water table.
  - (c) Melting of Ice sheets after glaciation etc.
- Normally Consolidated soil is much more compressible than *over consolidated-soil.*
- Amount of deformation per unit increase in stress is called compressibility.
- e-log $\bar{\sigma}$ for a normally consolidated soil is always a straight line.
- e-log $\bar{\sigma}$ for an over-consolidated soil has convex curvature upwards.
- State of over consolidated soil is represented by a point on, expansion or recompression parts of e-log $\bar{\sigma}$ plot.

- Recompression curve ultimately join the virgin compression line. The further compression then occurs along the virgin line.
- During compression, changes in soil structure continuously take place and the clay does not revert back to original structure during expansion.

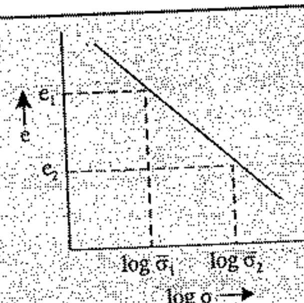
COMPRESSIBILITY OF CLAY CAN BE REPRESENTED BY ONE OF THE FOLLOWING COEFFICIENT.

1. Compresion Index (C_c):

- It is the slope of linear portion of e-log $\bar{\sigma}$ plot and is a dimensionless number.

$$\boxed{C_c = \frac{e_1 - e_2}{\log_{10}\bar{\sigma}_2 - \log\bar{\sigma}_1} = \frac{\Delta e}{\log_{10}(\bar{\sigma}_2/\bar{\sigma}_1)}}$$

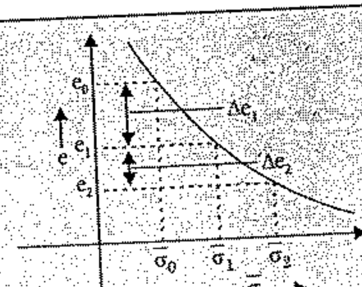
- Note that C_c has significance only for **Normally Consolidated soil.**



- C_c has a constant value for a given type of soil.
- It is not a function of effective stress.
- Higher the compression index, higher is the resulting vertical deformation of clay. *i.e.* $\Delta H \propto C_c$
- The expansion part of e-log $\bar{\sigma}$ plot can be aproximated to a straight line. The slope of which is called expansion index or recompression index.
- C_c is a plot between Airthmatic scale v/s log scale.

2. Coefficient of compressibility (a_v)

$$\boxed{a_v = \frac{\Delta e}{\Delta\bar{\sigma}}}$$



- Coefficient of compressibility decreases with increase in effective stress.
- With each increment of effective stress soil becomes more densified, hence resistance to further compression with same effective stress increment increases.
- a_v will be negative (–ve) as the void ratio decreases with increase in stress.
- It is a plot between airthmatic scale v/s airthmatic scale.

3. Coefficient of volume compressibility (m_v)

$$m_v = \frac{\text{Volume change per unit volume}}{\text{increase in effective stress}}$$

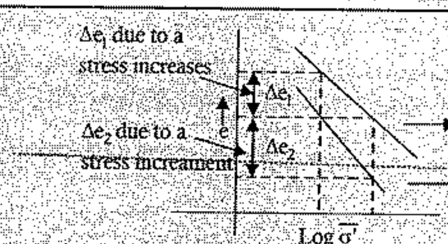
$$m_v = \frac{\Delta V / V}{\Delta\bar{\sigma}} = \frac{\Delta H / H_0}{\Delta\bar{\sigma}} = \frac{\Delta e / 1 + e_0}{\Delta\bar{\sigma}}$$

$$\boxed{m_v = \frac{\left(\dfrac{\Delta e}{1+e_o}\right)}{\Delta\bar{\sigma}} = \frac{a_v}{1+e_o}}$$

- The value of m_v for a particular soil is not constant but depends on stress range over which it is calculated.

  Oedometer test can also be used to calculate coefficient of consolidation (C_v). C_v indicates the rate of consolidation.

Note: Whenever possible, preconsolidation pressure for an over consolidated clay should not be exceeded in construction. Note that Compression will not be large if effective stress is below $\bar{\sigma}_0$ (i.e pre consolidation stress)



- Slope of line representing virgin compression of the in-situ soil will be slightly greater than the slope of virgin line obtained in laboratory. This is to disturbance of the soil used in laboratory.

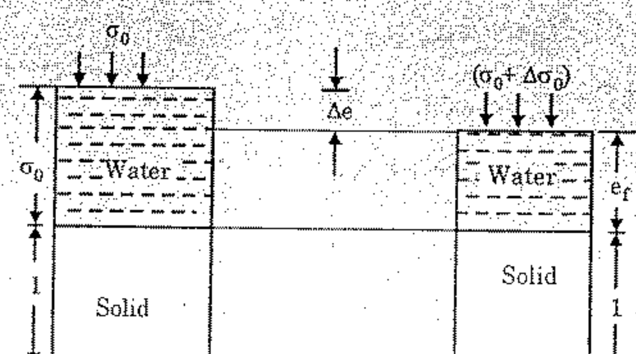
Example 2.

In a laboratory consolidation test, the void ratio of the samples reduced from 0.85 to 0.73 as the pressure was increased from 1 to 2 kg/cm^2. If the coefficient of permeability of the soil be 3.3×10^{-4}. Determine

(i) coefficient of volume change

(ii) coefficient of consolidation

Sol.



As per Question

(i)

$$e_0 = 0.85$$
$$e_f = 0.73$$
$$\Delta\sigma_0 = 2 - 1 \text{ kg/cm}^2$$
$$\Delta e = (0.85 - 0.73) = 0.12$$
$$K = 3.3 \times 10^{-4} \text{ cm/sec.}$$

$\therefore$ Coefficient of volume change $m_v = \dfrac{\Delta e}{(1+e_0)} \times \dfrac{1}{(\Delta\sigma_0)}$

$$= \frac{0.12}{(1+0.85)} \times \frac{1}{2}$$

$$[m_v = 0.065 \text{ cm}^2\text{/kg}]$$
$$= 0.065 \times 10^{-3} \text{ cm}^2\text{/gm}$$

(ii) Coefficient of consolidation $C_v = \dfrac{k}{(m_v \gamma_w)}$

$$\Rightarrow \quad C_v = \frac{3.3 \times 10^{-4}}{(0.065 \times 10^{-3} \times 1)} = 5.077 \text{ (cm}^2\text{/sec)}$$

COMPUTATION OF SETTLEMENT

- We will consider settlement due to 1° and 2° consolidation in this chapter. Settlement due to immediate settlement will be discussed in shallow foundation.
- Below are few relationship to calculate the Primary settlement.

1. $\boxed{\dfrac{\Delta H}{H_o} = \dfrac{\Delta e}{1+e_o}}$,

   Find $\Delta H = S_{1^\circ \text{ consolidation}}$

2. $\boxed{\Delta H = \dfrac{\Delta e}{1+e_o} . H_o = \dfrac{a_v \Delta\bar{\sigma}}{1+e_o} H_o = m_v . \Delta\sigma . H_o = S_{1^\circ \text{ consolidation}}}$

3. When layer thickness is more m_V & $\Delta\bar{\sigma}$ changes with depth. In such a case, the layer is divided into sublayers and m_V and $\Delta\bar{\sigma}$ are calculated at the centre of each sub layer.

$$\boxed{S_{1^\circ - \text{consolidation}} = \Sigma m_{vi} \Delta\sigma_i H_i}$$

4. In the case of normally consolidated soil

$$\frac{\Delta H}{H_0} = \frac{\Delta e}{1+e_0}$$

$$\Delta H = C_c \times \log_{10}\left(\frac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_0}\right)\frac{1}{1+e_0} \times H_0$$

$$C_c = \left(\frac{\Delta e}{\log_{10} \dfrac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_0}}\right)$$

$$\boxed{\Delta H = \frac{C_c H_o}{1+e_o}\log_{10}\left(\frac{\bar{\sigma}_o + \Delta\bar{\sigma}}{\bar{\sigma}_o}\right)}$$

In this case also if C_c is different in different layers of soil, than ΔH is calculated by calculating settlement in different layers and summing up the settlements to get total settlement.

5. If the soil is over consolidated, then check if $\bar{\sigma}_o + \Delta\bar{\sigma}$ is greater than $\bar{\sigma}_c$ or not.

   If not, use recompression index (C_r) in place of C_c.

   Hence $\boxed{\Delta H = \frac{C_r H_o}{1+e_o}\log\frac{\bar{\sigma}_o + \Delta\bar{\sigma}}{\bar{\sigma}}}$ over consolidated

   if $\bar{\sigma}_o + \Delta\bar{\sigma} > \bar{\sigma}_c$

   then, $\boxed{\Delta H = \frac{C_r H_o}{1+e_o}\log\frac{\bar{\sigma}_c}{\bar{\sigma}_o} + \frac{C_c H_o}{1+e_o}\log\frac{\bar{\sigma}_o + \Delta\bar{\sigma}}{\bar{\sigma}_c}}$

   Where,

   e_0 are H_0 the void ratio and height at $\bar{\sigma}_c$.

   However use of e_o and H_o in place of e_o' & H_o' will not introduce much error.

   Thus, $\boxed{\Delta H = \frac{C_r H_o}{1+e_o}\log\frac{\bar{\sigma}_c}{\bar{\sigma}_o} + \frac{C_c H_o}{1+e_o}\log\frac{\bar{\sigma}_o + \Delta\bar{\sigma}}{\bar{\sigma}_c}}$

Example 3.

In normally consolidated clay stratum of 3 m thickness has two permeable layers at its top and bottom. The liquid limit and initial void ratio of the clay are 36% and 0.82 respectively while the initial overburden pressure at the middle of the clay layer is 2 kg/cm². Due to the construction of new building this pressure increases by 1.5 kg/cm² compute the probable consolidation settlement of the building.

Sol. Data given

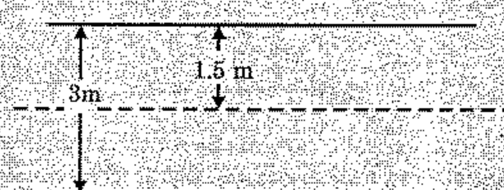
$\sigma_0 = 2\text{kg/cm}^2$

$\Delta\sigma = 1.5 \text{ kg/cm}^2$

$w_L = 36\%$

$e_0 = 0.82$

$H_0 = 3\text{m} = 300 \text{ cm}$



We know that, consolidation settlement of the building

$$\Delta H = \frac{H_0 C_c}{1+e_0}\log_{10}\left(\frac{\sigma_0 + \Delta\sigma}{\sigma_0}\right) \quad \text{... (i)}$$

$$\therefore \quad C_c = 0.009\,(w_L - 10)$$
$$= 0.009\,(36 - 10)$$
$$= 0.234$$

$$\therefore \quad \Delta H = \left(\frac{H_0 C_c}{1+e_0}\right)\log_{10}\left(\frac{\sigma_0+\Delta\sigma}{\sigma_0}\right)$$

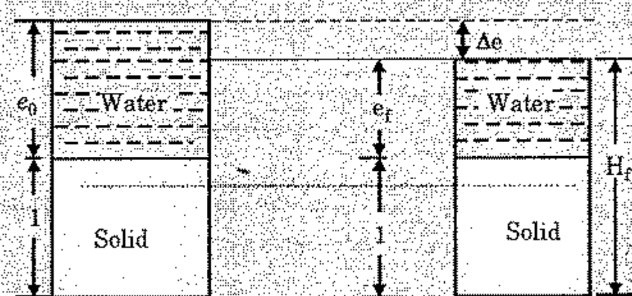
$$= \frac{300\times 0.234}{(1+0.82)}\log_{10}\left(\frac{2+1.5}{2}\right)$$

$$= 9.37 \text{ cm.}$$

Example 4.

Due to construction of a new structure the average vertical pressure at the centre of a 2.5 m thick clay increases from 1 kg/cm^2 to 2 kg/cm^2. A laboratory consolidation test was performed on 92 cm thick undisturbed sample of the clay. Under applied stress of 1kg/cm^2 and 2 kg/cm^2 the equilibrium thickness of the sample were found to be 1.76 cm and 1.63 cm respectively. on removing the stress completely the thickness increased to 1.88 cm. The final moisture content and the specific gravity of same were found to be 29% and 2.71 respectively. Compute the probable consolidation settlement of the structure.

Sol.



Set e_f and H_f be the final void ratio and thickness of the sample

$$\Rightarrow \quad e_f s = w\,G$$

$$\Rightarrow \quad e_f \times 1 = (0.29 \times 2.71)$$

$$\therefore \quad e_f = 0.786 \qquad \text{... }(i)$$

Hence $e_f = 0.786$ and $H_f = 1.88$ cm

From above figure, we have

$$\frac{\Delta H}{H_f} = \frac{\Delta e}{(1+e_f)}$$

$$\Rightarrow \quad \Delta e = \frac{\Delta H}{H_f}(1+e_f)$$

$$\Rightarrow \quad \Delta e = \frac{(1+0.786)}{1.88}\times \Delta H$$

$$\Delta e = 0.95\ \Delta H \qquad \text{... }(ii)$$

When $\sigma = 2\text{kg/cm}^2$

$$\Delta H = (1.88 - 1.63)$$

$$= 0.25$$

$$\therefore \quad \Delta e = 0.95\ \Delta H$$

$$= (0.95 \times 0.25)$$
$$= 0.238$$

Hence void ratio at $\sigma = 2.0$ kg/cm²

$$e_1 = (e_f - \Delta e)$$
$$= (0.786 - 0.238)$$
$$= 0.548$$

Again when $\sigma = 1.0$ kg/cm²

$$\Delta H = (1.88 - 1.76)$$
$$= 0.12 \text{ cm.}$$
$$\Delta e = (0.95 \times 0.12) = 0.114$$

Hence void ratio at $\sigma = 1.0$ kg/cm²

$$e_2 = (e_f - \Delta e)$$
$$= 0.786 - 0.114$$
$$= 0.672$$

Let m_v be the average, value of the co-efficient of volume change

In the pressure range of 1.0 kg/cm² to 2.0 kg/cm²

∴ We have

$$m_v = \left(\frac{\Delta e}{1+e_0}\right) \times \frac{1}{\Delta \sigma}$$

$$= \frac{(0.672 - 0.548)}{(1 + \cancel{0.548}\ 0.672)} \times \frac{1}{(2.0 - 1.0)}$$

$$= \cancel{0.08 \text{ cm}^2/\text{kg}} = 0.074 \text{ cm}^2/\text{kg} \quad \text{... (iii)}$$

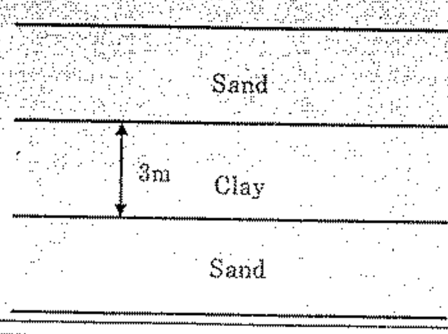
∴ Required consolidation settlement of the clay layer in the field

$$\Delta H = m_v H_0 \Delta \sigma$$
$$= 0.0\cancel{8}74 \times 250 \times (2.0 - 1.0)$$
$$= \cancel{20 \text{ cm}}\ 18.54 \text{ cm}$$

Example 5.

A 3m thick layer of silty clay is sandwitched between two layer of Dense sand. The effective overburden pressure at the centre of the silty clay layer is 2 kg/cm². However due to construction of a raft foundation test was performed on a 2.5 cm thick sample of the silty clay. Under applied stresses of 2 kg/cm² and 4kg/cm² the compression of the sample were found to be 0.26 cm and 0.38 cm respectively. Compute the probable consolidation settlement of the raft.

Sol. We know that



$\Delta H = m_v H_0 \bar{\sigma}\Delta$ ... (i)

Where H_0 = Initial thickness

= 2.5 cm for the soil sample and 300 cm for the the soil in-situ.

m_v = coefficient of volume change for the pressure range of 2kg/cm² to 4.0 kg/cm²

For laboratory test

$\therefore$ Initial thickness of the sample = 2.5 cm

Thickness under a pressure of 2 kg/cm² = (2.5 – 0.26) = 2.24 m

Thickness under a pressure of 4 kg/cm² = (2.5 – 0.38)

= 2.12 cm

$\therefore$ $\Delta H = (2.24 - 2.12)$

= **0.12 cm**

$\Delta\sigma = 4 - 2 = 2\text{kg/cm}^2$

$\Rightarrow$ We have $\Delta H = (m_v \, \Delta\bar{\sigma} \, H_0)$

$\Rightarrow$ $0.12 = m_v \times 2 \times \cancel{2.5}$ 2.24

$m_v = \cancel{0.024}\ \text{cm}^2/\text{kg}$ 0.02678 ... (ii)

$\therefore$ The consolidation settlement of the silty clay layer

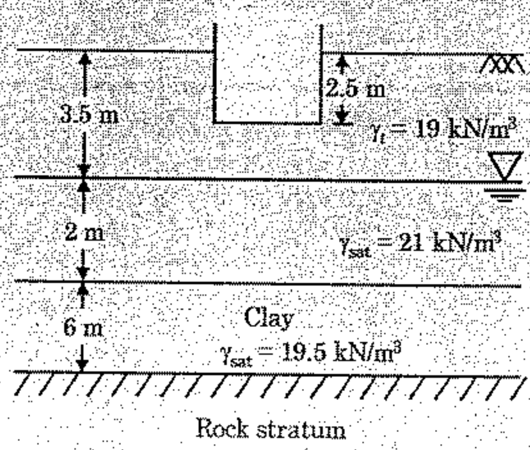
$\Delta H = m_v H_0 \Delta\sigma$

$= (\cancel{0.024} \times 300 \times 2)$ 0.02678

$[\Delta H = \cancel{14.4\ \text{cm}}]$ 16.07 cm

Example 6.

It is estimated that additional stress on the clay layer due to weight of building is 150 kN/m² at the top of the clay and 70 kN/m² at the bottom of the clay. Compute the settlement of the clay layer due to construction of building



Data of consolidation test

Applied stress (kN/m²)	Void Ratio
25	0.89
50	0.882
100	0.865
200	0.830
400	0.798
800	0.765

Sol. Effective stress at the top of the clay layer

$$\bar{\sigma}_1 = (\gamma_1\, h_1 + \gamma_{sub}\, h_2)$$
$$= (19 \times 3.5) + (21 - 9.8) \times 2$$
$$= 88.9 \text{ kN/m}^2$$

Effective stress at the bottom of the clay layer

$$\bar{\sigma}_2 = \gamma_1 h_1 + \gamma_{sub}\, h_2 + (\gamma_{sub})_{clay} \times h_3$$
$$= 19 \times 3.5 + (21 - 9.8) \times 2 + (19.5 - 9.8) \times 6$$
$$= 147.1 \text{ kN/m}^2$$

$\therefore$ Average initial effective stress at the centre of clay layers

$$\sigma_0 = \frac{(88.88 + 147.1)}{2}$$
$$\sigma_0 = 117.95 \text{ kN/m}^2 \quad \ldots (i)$$

Average increment of effective stress in clay layer

$$\Delta\sigma_0 = \left(\frac{150+70}{2}\right) = 110 \text{ kN/m}^2 \quad \ldots (ii)$$

$\therefore$
$$(\sigma_0 + \Delta\sigma_0) = 117.95 + 110$$
$$= 227.95 \text{ kN/m}^2 \quad \ldots (iii)$$

Void ratio corresponding to 227.95 kN/m²

$$e_2 = 0.830 + \frac{(0.798 - 0.830)}{(400 - 200)} \times (227.95 - 200)$$
$$e_1 = 0.8255$$

For
$$\sigma_0 = 117.95 \text{ kN/m}^2$$

void ratio
$$e_2 = 0.865 + \frac{(0.830 - 0.865)}{(200 - 100)}(117.95 - 100)$$
$$= \mathbf{0.859}$$

$\therefore$
$$\frac{\Delta e}{1+e_0} = \frac{0.859 - 0.8255}{(1 + 0.859)}$$
$$= 1.718 \times 10^{-2}$$

The settlement of clay layer due to construction of building will be given by

$$\frac{\Delta H}{H_0} = \left(\frac{\Delta e}{1+e_0}\right)$$

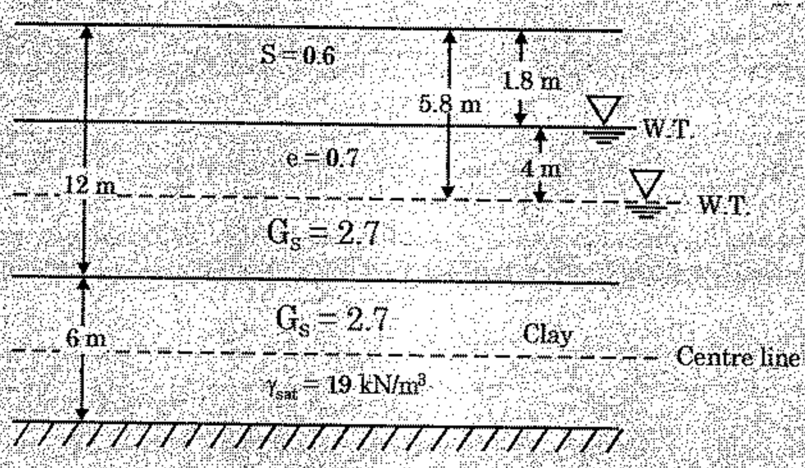
$\therefore$
$$\Delta H = (1.718 \times 10^{-2}) \times 6000 \text{ mm}$$
$$= 107.13 \text{ mm}$$

Example 7.

A school building stands over a 12 m thick stratum of sand beneath the sand stratum, there is a clay layer of 6m thickness and this is underclain by an impermeable rock stratum, the sand has a void ratio of 0.7 and W.T lies at a depth of 1.8 m below ground surface, above W.T. The avg. degree of saturation is 0.6, G_S for sand and clay both is 2.7, γ_{sat} for clay 19 kN/m^3, water is pumped from the sand strata to supply water to school and as a result W.T falls down by 4m assuming school building to apply insignificant loading. Calculate probable settlement of school as a result of pumping. The consolidation test data of clay gave the following result

$\bar{\sigma}$(kN/m^2)	e
160	0.798
192.8	0.754

Sol.



Before lowering

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{(1+e)}$$

$$= \frac{(2.7+0.7)}{(1+0.7)}\times 9.81 = 19.62 \text{ kN/m}^3$$

$$\gamma_t = \frac{(G+eS)\gamma_w}{(1+e)} = \frac{(2.7+0.7\times 0.6)}{(1+0.7)}\times 9.81 = 18 \text{ kN/m}^3$$

$\therefore$ Effective stress at the centre of clay layer

$$\bar{\sigma}_1 = 1.8\times 18 + 10.2\times(19.62-9.81) + (19-9.81)\times 3$$

$$= 160.03 \text{ kN/m}^2 \quad \ldots (i)$$

After lowering of water table effective stress at the centre of clay will increase.

The sand being a free draining soil will not remain saturated after lowering of W.T, we shall assume it to have degree of saturation 0.6.

$\therefore$ Effective stress at centre of clay layer after lowering

$$= 18\times 5.8 + 6.2\times(19.62-9.81) + 3\times(19-9.81)$$

$$= 192.79 \text{ kN/m}^2 \quad \ldots (ii)$$

and at $\sigma_0 = 160.03 \text{ kN/m}^2$

Void ratio $e_1 = 0.798$

at $(\sigma_0 + \Delta\sigma_0)$, void ratio $e_2 = 0.754$

$$\therefore \quad \Delta e = (0.798 - 0.754)$$

$$= 0.044$$

$$H_0 = 6\text{m} = 6000 \text{ mm}$$

$$e_0 = 0.798$$

$\therefore$ Probable settlement of school as a result of pumping will be given by

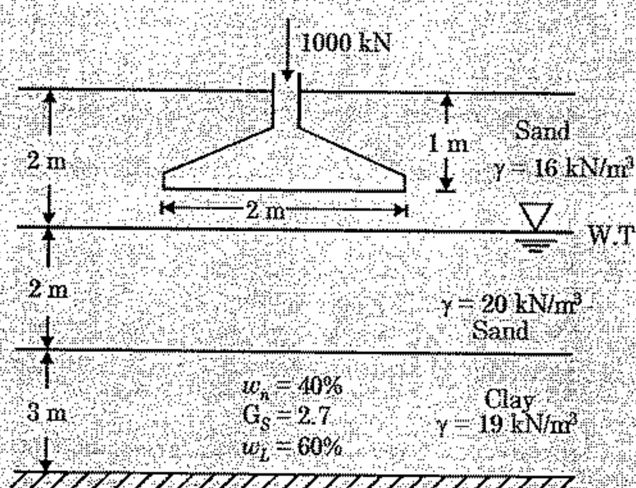
$$\frac{\Delta H}{H_0} = \frac{\Delta e}{(1+e_0)}$$

$$\therefore \quad \Delta H = \frac{0.044 \times 6000}{(1+0.798)}$$

$$\boxed{\therefore \Delta H = 146.829 \text{ mm}}$$

Example 8.

The subsoil profile of a proposed site of construction is shown below, a square footing of 2m × 2m carries a load of 1000 kN and is laid with its base at 1m depth below ground surface assuming that post construction settlement in sand is negligble determine the consolidation settlement of clay layer on account of construction. There is geological evidence, clay is *NC*, use 2 vertical and 1 horizontal for load dispression to estimate the stress increase in clay layer.



Rock Soil Profile

Sol. For clay layer

Data given: $w_n = 40\%, \quad w_L = 60\%$

$$\therefore \quad C_c = 0.009\,(w_L - 10)$$

$$= 0.009\,(60 - 10)$$

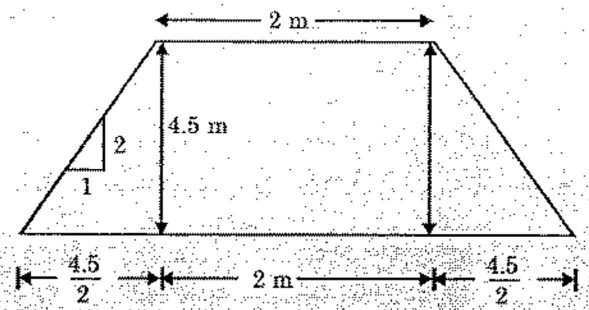
$$C_c = 0.45$$

We have $eS = wG$

$$\Rightarrow \quad e \times 1 = 0.40 \times 2.7$$

$$\therefore \quad e_0 = 1.08$$

Effective Stress at the centre of clay layer before the construction



$$\bar{\sigma}_0 = \gamma_1 h_1 + \gamma_{sub} h_2 + (\gamma_{sub})_{clay} h_3$$

$$= 16 \times 2 + (20 - 9.81) \times 2 + (19 - 9.81) \times 1.5$$

$$\bar{\sigma}_0 = 66.165 \text{ kN/m2} \quad \text{... } (i)$$

$\therefore$ Change in effective stress due to 1000 kN

$$\Delta\sigma_0 = \frac{1000}{(B+Z)(L+Z)}$$

$$\Delta\sigma_0 = \frac{1000}{(2+4.5)(2+4.5)}$$

$$= 23.67 \text{ kN/m}^2 \quad \text{... } (ii)$$

$\therefore$ Consolidation settlement of the clay layer

$$\Delta H = \frac{C_c H_0}{(1+e_0)} \log_{10}\left(\frac{\sigma_0 + \Delta\sigma_0}{\sigma_0}\right)$$

$$= \frac{0.45 \times 3000}{(1+1.08)} \log_{10}\left(\frac{66.165+23.67}{66.165}\right)$$

$$\boxed{\Delta H = 86.20 \text{ mm}}$$

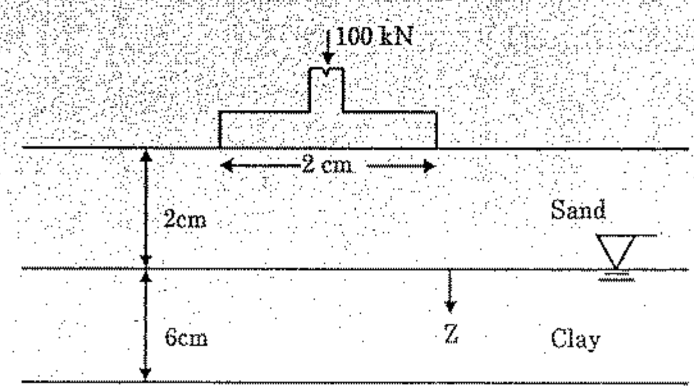
Example 9.

The load from the strip foundation as shown below is assumed to he distributed over the soil stratum at 2 vertical : 1 horizontal, the value of m_v (coifficient of volume compressihility) varies over the depth of clay stratum as

$$m_v = (22 + 1.2z^2) \times 10^{-5} \text{ m}^2 / \text{kN}$$

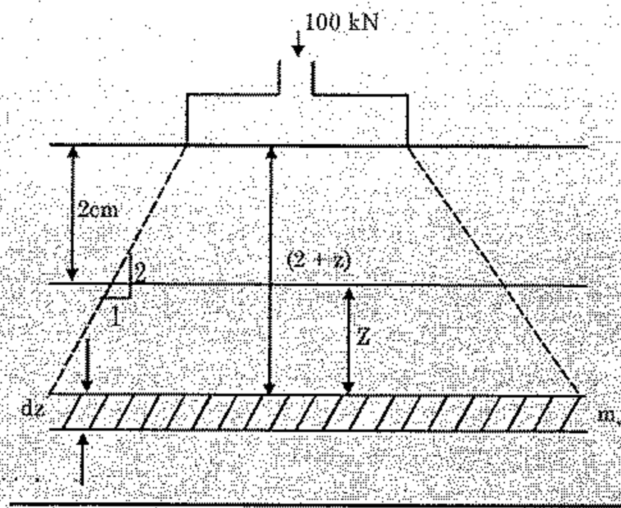
Where z is the distance from the top of clay layer.

Calculate the consolidation of the clay stratum (assuming one dimensional consolidation)



Sol. Data given $m_v = (22 \times 10^{-5} + 1.2 \times 10^{-5} z^2)\ m^2/kN$

Let us assume a thickness dz, for which m_v remains constant



Using 2 : 1 dispersion

$$\Delta\sigma' = \frac{100}{[2+(2+z)]\times 1} = \frac{100}{(4+z)}\ kN/m^2$$

We have,

$$\Delta H = H_0\, m_v\, \Delta\sigma'$$

$$\therefore \quad \Delta H = \int_0^6 \left(22+1.2Z^2\right)\times 10^{-5}\times\frac{100}{(4+z)}\times dz$$

$$= 10^{-3}\int_0^6 \frac{\left(22+1.2z^2\right)}{(4+z)}dz$$

Put

$$4+z = \phi$$

$$d\phi = dz$$

$$\Delta H = 10^{-3}\int_4^{10}\frac{\left[\left(22+1.2(\phi-4)^2\right]\right.}{\phi}d\phi$$

$$= 10^{-3}\int_4^{10}\frac{\left(41.2-9.6\phi+1.2\phi^2\right)}{\phi}d\phi$$

$$= 10^{-3}[41.2\{\log\phi\}_4^{10} - 9.6(10-4)+\frac{1.2}{2}(10^2-4^2)]$$

$$= 30.55\times 10^{-3}\ m$$

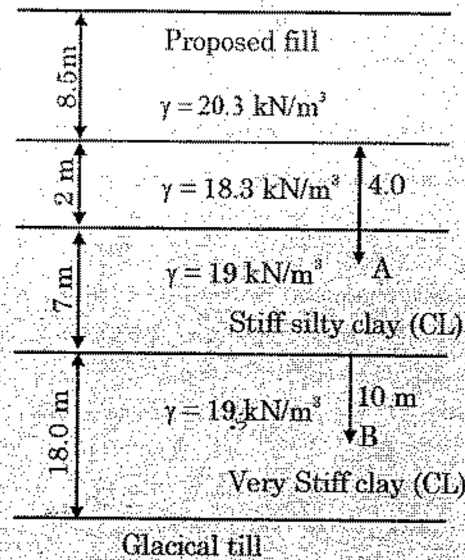
$$= 30.55\ mm$$

Example 10.

A 8.5m deep compacted Fill is to be placed over the soil profile shown in figure below. Consolidation tests on samples from points A and B producted the Following results :

Sample A	Sample B
C_c — 0.25	0.20
C_r — 0.08	0.06
e_0 — 0.66	0.45
σ_c' — ~~1.01 k Pa~~ 101 kPa	510 kPa

Preconsolidation stress at other points in each layers can be calculated using formula



$$\sigma'_m = \sigma'_c - \sigma_{z_o}$$

Where, σ'_m = Over consolidation margin

Which is assumed constant (effective)

σ'_c = preconsolidation stress ~~18.0~~ m (Effective)

σ'_{z_o} = Effective stress at the level under consideration compute the ultimate consolidation due to the weight of thin fill (for each layer compute settlement by calculating $\Delta\sigma'$ at the centre of layer)

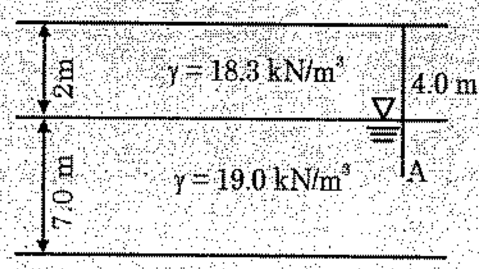
Sol. Data given :

$$\sigma'_m = \sigma'_c - \sigma'_{z_o}$$

$$\therefore \quad \sigma'_c = (\sigma'_m + \sigma'_{z_o})$$

Where, σ'_{z_o} = Effective stress at the level under consideration,

For sample A



$$\sigma'_{z_o} = (2 \times 18.3) + (19 - 10) \times 2$$

$$= 54.6 \text{ kN/m}^2$$

$$\therefore \quad \sigma'_m = (\sigma'_c - \sigma'_{z_o})$$

$$= (101 - 54.6)$$
$$= 46.4 \text{ kN/m}^2 \quad \text{... (i)}$$

$\therefore$ At the centre of first layer effective stress before application of proposed fill

$$\sigma_o' = 2 \times 18.3 + 2.5 \times (19 - 10)$$
$$= 59.1 \text{ kN/m}^2$$

$\therefore$ $\Delta\sigma_o$ = After placing the proposed fill

$$= (8.5 \times 20.3)$$
$$= 172.55 \text{ kN/m}^2$$

$\therefore$ Final stress at centre $= (\sigma_o' + \Delta\sigma_o')$

$$= (58.1 + 172.55)$$
$$= 231.65 \text{ kN/m}^2 \quad \text{... (ii)}$$

Pre consolidation stress at the point

$$= (\sigma_m' + \sigma_o')$$
$$= 59.1 + 46.4)$$
$$= 105.5 \text{ kN/m}^2 \quad \text{... (iii)}$$

$\therefore$ Settlement of first layer

$$\Delta H_1 = \frac{C_r H_o}{(1+e_o)} \log_{10}\left(\frac{\sigma_{Co}}{\sigma_o'}\right) + \left(\frac{C_c H_o}{1+e_o}\right) \log_{10}\left(\frac{\sigma_o' + \Delta\sigma_o}{\sigma_{c_o}}\right)$$

$$= \frac{0.08 \times 9}{1.66} \log_{10}\left(\frac{105.5}{59.1}\right) + \frac{0.25 \times 9}{1.66} \log_{10}\left(\frac{231.65}{105.5}\right)$$

$$= (0.1092 + 0.463)$$
$$= 0.572 \text{ m.} \quad \text{... (iv)}$$

$$\therefore \quad \sigma_m' = (\sigma_c' - \sigma_{z_o}')$$
$$= 510 - (2 \times 18.3 + (19 - 10) \times 7 + 10 \times (19.5 - 10))$$
$$= 315.4 \text{ kN/m}^2$$

$$\sigma_c' = (\sigma_m' + \sigma_o')$$
$$= (315.4 + 2 \times 18.3 + 9 \times 7 + 9.5 \times 9)$$
$$= 500.5 \text{ kN/m}^2 \quad \text{... (v)}$$

$\therefore$ Final stress at the centre of clay of sample B

$$= (\sigma_o' + \Delta\sigma_o')$$
$$= (185.1 + 8.5 \times 203)$$
$$= 357.65 \text{ kN/m}^2 \quad \text{... (vi)}$$

$\therefore$ From (v) and (vi). If is Evident that, preconsolidation stress is more than final stress at this level. Therefore only preconsolidation settlement will occur.

$$\therefore \quad \Delta H_2 = \frac{C_r H_o}{(1+e_o)} \log_{10}\left(\frac{\sigma_{co}}{\sigma_o}\right)$$

$$= \frac{0.06 \times 18}{1.45} \log_{10}\left(\frac{257.65}{185.1}\right)$$

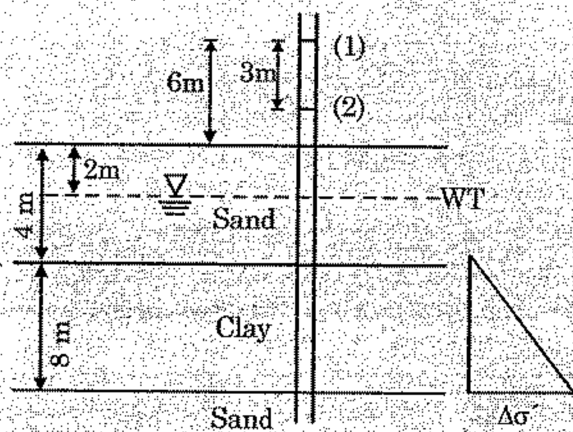
$$= 0.21 \text{ m} \quad \text{... (vii)}$$

$\therefore$ Total settlment

$$\Delta H = (\Delta H_1 + \Delta H_2)$$
$$= (0.572 + 0.21)$$
$$= 0.78 \text{ m}$$
$$= 780 \text{ mm.}$$

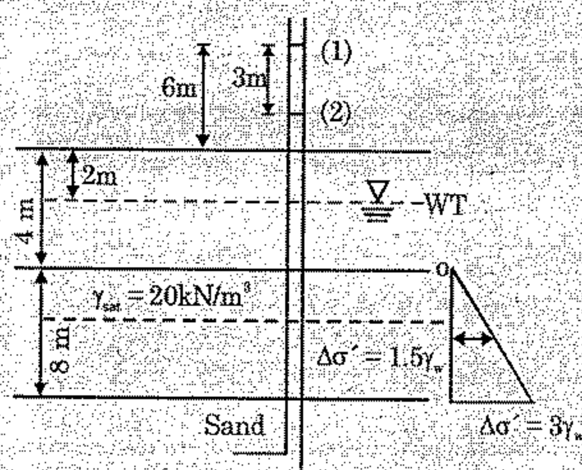
Example 11.

A layer of clay 8m thick lies between two layer of sand. The upper sand layer extends from ground level to a depth of 4m. The water table being at a depth of 2m. The lower sand layer is under artesion pressure, the piezometric level being 6m above ground level. The saturated unit wight of clay and sand respectively are 20 kN/m³. For the clay $m_v = 9.4 \times 10^{-4}$ m²/kN and $C_v = 4.5 \times 10^{-8}$ m²/sec. Due to pumping from the artesion layer the piezonetric level falls by 3m over a period of 2 years. Find final consolidation settlement.



Sol. Data given :

$$m_v = 9.4 \times 10^{-4} \text{ m}^2/\text{kN}$$
$$C_v = 4.5 \times 10^{-8} \text{ m}^2/\text{sec.}$$



The vertical effective stress remains unchanged at the top of the clay layer but will be increased by $3\gamma_w$ at the bottom of the layers due to decrease in the porce water pressure in the adjascent artesion layer

∴ Hence At the centre of the clay layer

$$\Delta\sigma' = 1.5\,\gamma_w \qquad \text{... (i)}$$

∴ Final consolidation at the centre of the clay layers

$$\Delta H = m_v\, \Delta\sigma' H_0$$
$$= (9.4 \times 10^{-4} \times 1.5\, r_w \times 8) = (9.4 \times 10^{-4} \times 1.5 \times 10 \times 8)$$
$$= 0.1128 \text{ m}$$

$$\boxed{\therefore \quad \Delta H = 112\text{mm.}}$$

SECONDARY CONSOLIDATION SETTLEMENT

- Secondary consolidation is significant only for highly plastic soil.
- Secondary compression index is defined as

$$C_\alpha = \frac{\Delta e}{\Delta \log t},$$

where, Δe = decrease in void-ratio corresponding to time Δt.

$$\frac{\Delta H}{H_o} = \frac{\Delta e}{1+e_o}$$

$$\boxed{\Delta H = \frac{C_\alpha . H_o}{1+e_o} \log_{10} \frac{t_2}{t_1}}$$

Where, e_o = void ratio at the end of 1° consolidation.
H_o = height at the end of 1°-consolidation.

For NC soil, $\boxed{C_\alpha \text{ is } 4\% - 6\% \text{ of the value of } \frac{C_c}{1+e_0}}$

Note:

1. 2° consolidation occurs at constant effective stress.
2. 2°-consolidations not related to desipation of pore woter pressure.
3. 2°-consolidotions occurs due to readjustment of soil skeleton after disturonce during 10°-consolidotion
4. Rate of 2°-consolidations is controlled by viscous absorbed loyer surrounding clay particles in soil.

DETERMINATION OF C_c BY EMPIRICAL RELATIONSHIP

1. For undisturbed clay with medium sensitivity.

$$\left(\text{Sensitivity} = \frac{q_{u \text{ undisturbed}}}{q_{u \text{ remoulded}}} < 4\right)$$

$$C_c = 0.009\,(w_L - 10)$$

   Where, w_L is in percentage (*i.e.* liquid limit)

2. For remoulded clay $C_c = 0.007\,(w_L - 7)$
3. $C_c = 1.15\,(e_o - 0.35)$, where, e_o initial void ratio before consolidation.
4. $C_c = 1.15 \times 10^{-2} \times w_n$ for organic soil

   Where, w_n = natural water content in percent
5. $C_c = 0.37\,(e_0 + 0.003\,w_L + 0.0004\,w_n - 0.34)$
6. $C_c = 0.3\,(e_0 - 0.27)$—inorganic cohesive soil, clayey silt, silty clay.
7. For most Normally Consolidated soil of medium sensitivity $C_c = 0.2 - 0.5$
   For organic clay $C_c > 4$
   For Peat $C_c \to 10–15$

   These results are used for preliminary estimate of settlement.

TIME RATE OF CONSOLIDATION

- In consolidation analysis, problem is two fold:
  (i) Computation of settlement
  (ii) Rate of settlement due to applied load.
- Rate of settlement is directly related to the rate of dissipation of pore water pressure. Therefore to predict the time rate of consolidation, a mathematical theory was for proposed by Terzaghi for one-dimensional consolidation
- This theory predicts the excess pore water pressure at any elapsed time and at any location.
- Basic Assumptions of Terzagi one-dimensional consolidation:
  (1) Soil is homogeneous
  (2) Soil is fully saturated
  (3) Soil particles and water are incompresible.
  (4) Compression and pore water flow are one-dimensional (vertical).
  (5) Strains are small.
  (6) Coefficient of permeability (k), Coefficient of volume compresibility (m_v), coefficient of compressibility (a_v) are constant throughout the soil.
  (7) Darcy's law is valid at all hydraullic gradient.
  (8) There is a unique relationship, independent of time, between void ratio and effective stress.
  i.e. $\boxed{\Delta e = \frac{-a_v}{\Delta\sigma}}$ *i.e.* a_v = constant over the stress increment
  (9) Secondary compression is neglected because it exist only at constant effective stress.
  The differential equation proposed by Terzaghi for one dimensional consolidation is

$$\boxed{\frac{\partial u}{\partial t} = C_V \frac{\partial^2 u}{\partial z^2}}$$

u = excess pore water pressure at any time and at any location z

$$\boxed{C_V = \frac{k_z(1+e_o)}{a_v \gamma_w}}$$

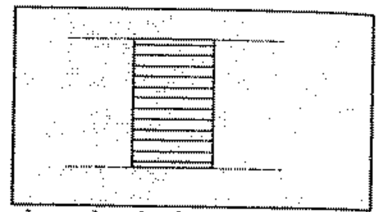
C_V = Coefficient of consolidation

- C_V is not really a constant parameter but it is a function of the stress increments.

Note: *Coefficient of consolidation is a soil parameter governing the time-rate of consolidation. C_V can also be calculated by odeometer test.*

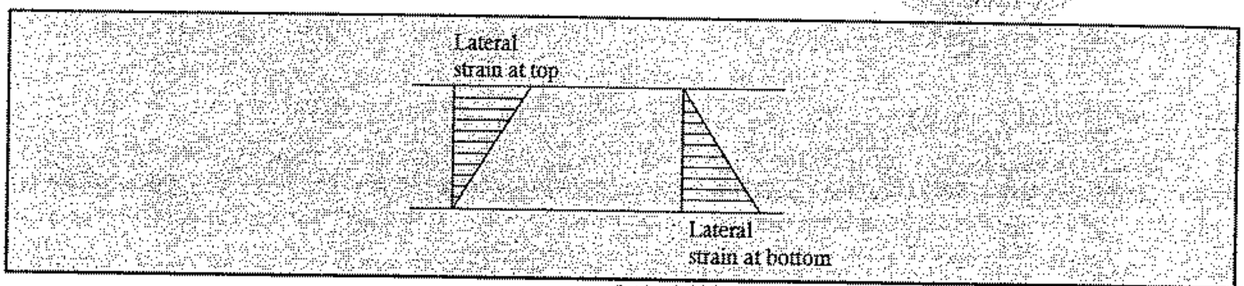
SOLUTION OF CONSOLIDATION EQUATION

- For solution of Terzaghi one-dimensional consolidation equation, rigorous mathematical solution of differential equation in terms of a fourier series expansion is required.
- The progress of consolidation can be shown by plotting a series of curves of u_e (excess pore water pressure) against z for different values of 't'.



- Such curves are called isochrones.
- The form of the curves depend on
  - (a) initial distribution of excess pore water pressure.
  - (b) Drainage condition at the boundaries of clay layer.
- The distribution of initial excess pore water pressure will be uniform along the thickness only when loaded area is large, relative to the thickness of consolidating layer i.e. when no lateral strain occurs. Initial excess pore water pressure is constant throughout the depth, when lateral strain at all depth = 0.

Note:



These type of Initial pore water pressure distribution will occur if there is lateral strain at top or at bottom.

- In the solution of Terzaghi's one dimensional consolidation equation following, non dimensional parameters are defined.
- Drainage path ratio $\boxed{Z = \frac{z}{H}}$
- Time factor $\boxed{T_v = \frac{C_v t}{H^2}}$
- Degree of consolidation $\boxed{U_z = \frac{u_i - u_z}{u_i}}$

  Where,

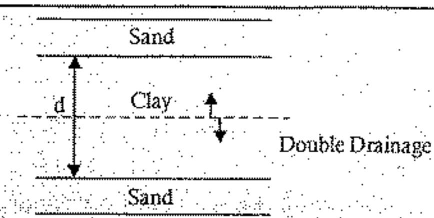
  z = depth of any point form top of the layer.

  H = Max. distance that water has to travel to reach a drainage face. *i.e. the length of longest drainage path.*

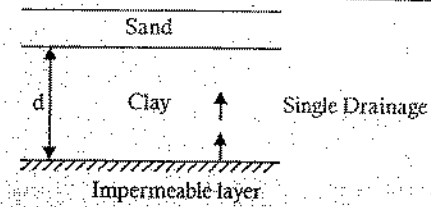
  u_i = initial excess pore water pressure ($u_i = \Delta\sigma$ for all location (*i.e.* for all z))

  u_z = excess pore water pressure at any depth z.

  U_z = represents the stage of consolidation at a certain location in the consolidating layer.
- If the soil has both upper and lower boundaries as free draining, the soil layer is called open-layer or soil under double drainage condition.
- If however, only one layer is free draining, the soil layer is called half-closed layer or soil under single-drainage condition.



In this case $H = d/2$



In this case $H = d$

BOUNDARY CONDITIONS AND INITIAL CONDITION FOR ONE-DIMENSIONAL CONSOLIDATION

Double Drainage System

1. At $t = 0$, $u = u_i$ at all locations (*i.e.* at all z)

   $u_i = \Delta\sigma$

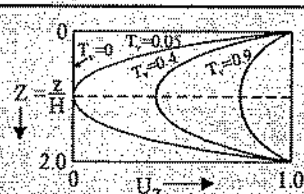
2. as $t \to \infty$, $u = 0$ for all z-location
3. at $t > 0$, $z = 0$, $u = 0$
4. at $t > 0$, $z = \mathrm{d}$, $u = 0$

Note: At boundaries pore water pressure immediately disspates to zero

Single Drainage System

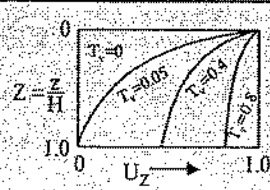
1. at $t = 0$, $u = u_i$ for all z-location
2. at $t \to \infty$, $u = 0$ for all z-location
3. at $t > 0$, $z = 0$, $u = 0$

- Solution of Terzaghi, equation is plotted in the form of **isochrones.**



(Degree of Consolidation)

Isochrone for double drianage condition



Degree of Consolidation Isochrone
(For single drainage)

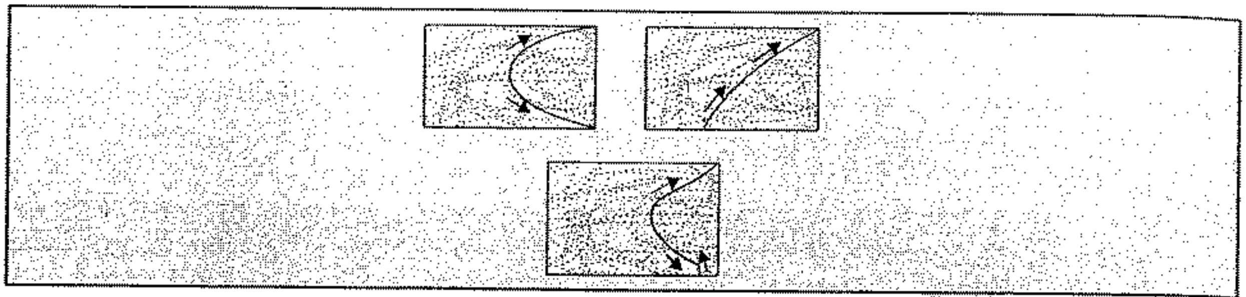
- Slope of Isochrone at any depth gives the hydraulic gradient

$$\boxed{i = \frac{\partial h}{\partial z} = \frac{\partial u}{\gamma_w \partial z}}$$

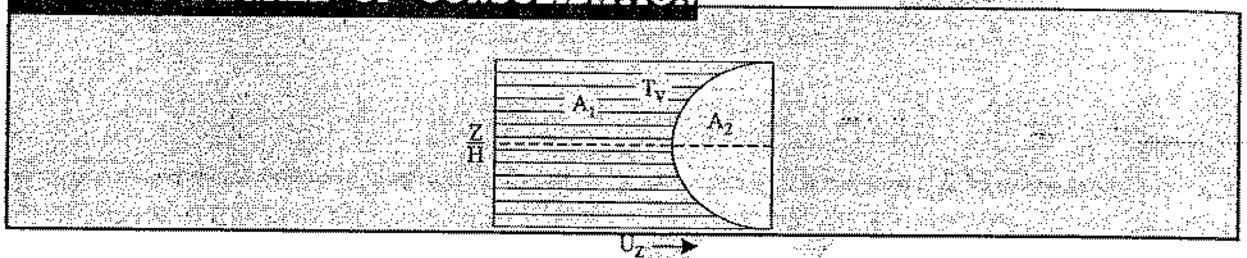
Where, h = head

u = excess pore water pressure

- The slope also indicates the direction of flow



AVERAGE DEGREE OF CONSOLIDATION



- In practical problems it is the average degree of consolidation' (U) over the depth of the layer as a whole that is of interest.
- Consolidation settlement at any time 't' = $U \times$ final settlement

  A_1 = Shaded area *(excess pore pressure dissipated)*

  A_2 = un shaded area *(excess pore pressure remaining)*

$$\text{Average degree of consolidation } (U) = \frac{A_1}{A_1 + A_2}$$

- It has been observed that, $U = f(T_v)$
- For $U \le 60\%$, $\boxed{T_v = \frac{\pi}{4}U^2}$
- For $U > 60\%$, $T_v = 1.781 - 0.933\log(100 - U)$
- Few Important values of T_v for different U are an tabulated below

$U(\%)$	T_V
0	0
50	0.197
60	0.287
90	0.848

Note that

for U > 60%, $T_v > 0.287$

- We can also find degree of Consolidation from few other relations.

(i) $U = \dfrac{\Delta h}{\Delta H} \times 100$ (ii) $U = \dfrac{e_0 - e}{e_0 - e_f}$ (iii) $U = \dfrac{u_i - u}{u_i - u_f}$

Where, Δh = Settlement of any stage

ΔH = settlement at the end of consolidation.

e_0 = initial void ratio

e = void ratio at any stage in between

e_f = final void ratio at the end of a stress increments.

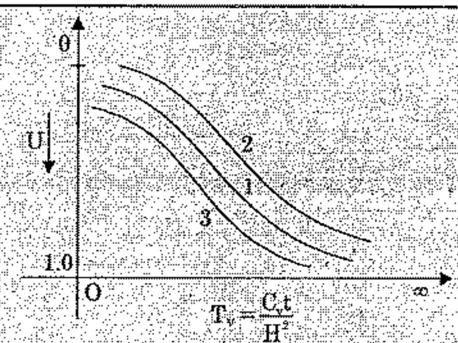
u_i = Initial Pore Water Pressure.

u = Pore Water Pressure at any stage in between.

u_f = Pore Water Pressure at the end of stress increment.

GRAPHICAL SOLUTION OF PORE WATER PRESSURE

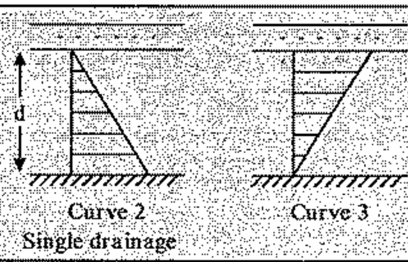
Note that as U increases T_v increases.



Relationship between u & T_v

Curve 1 represents condition corresponding to initial pore water pressure variations with depth as shown below

Clause 2 & 3-represents conditions coresponding to



Note: That time required to reach a particular degree of consolidation is proportional to square of length of drainage path and inversly proportional to C_V.

$$t = \frac{T_u H^2}{C_v}$$

i.e.

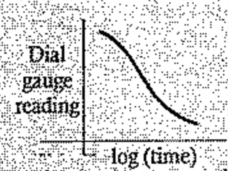
$$\boxed{t \propto \frac{H^2}{C_V}}$$

DETERMINATION OF COEFFICIENT OF CONSOLIDATION (C_v)

- The curve of deformation versus time obtained from oedometer test is similar to U versus T_V curve. This property is used to determine C_v from oedometer test by curve fitting technique.
- Two methods are used.
  - (a) Casagrandes logarithm of time fitting method.
  - (b) Taylor's square root of time fitting method.

(a) Casagrande's Method/Logarithm of Time Fitting method

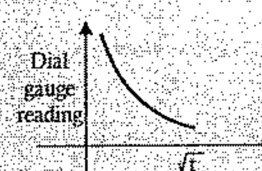
- Time for 50% consolidation is noted.



$$\boxed{C_V = \frac{T_{50} H^2}{t_{50}}} \quad T_{50} = 0.196$$

(b) In Taylors metbod/Square root of time fitting method.

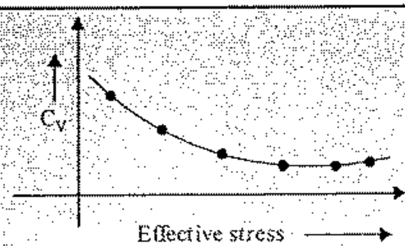
- Time for 90% consolidation is noted.



$$\boxed{C_V = \frac{T_{90} H^2}{t_{90}}} \quad T_{90} = 0.848$$

Note: Taylor curve is suitable because curve gets plotted as time progresses, and as soon as 90% settlement point is obtained, next increment of loading is applied. Thus time to test reduces.

Note: Coefficient of consolidation can be obtained for each one of the stress increments.



C_V value obtained from any of the above curve fitting techniques is plotted against effective stress. From the above curve appropriate value of C_V for any value of effective stress can be calculated.

Note: Coefficient of consolidation can be used to calculate coefficient of permeability for fine grained soil. Value of C_V decreases as liquid limit of soil increases.

$$C_v = \frac{K(1+e_o)}{a_v \gamma_w} = \frac{K}{m_v \gamma_w}$$

$$\boxed{k = C_v . m_v \gamma_w}$$

COMPRESSION RATIOS

Relative magnitude of initial compression, primary consolidation and secondary consolidation are expressed by following ratios.

(a) Initial compression ratio

$$r_0 = \frac{R_i - R_o}{R_i - R_f} = r_o$$

(b) 1°-Consolidation ratio (r_P)

$$r_P = \frac{R_o - R_{100}}{R_i - R_f}$$

(c) Secondary compression ratio (r_s)

$$r_S = 1 - (r_o + r_p)$$

Where, R_i = initial dial gauge reading

R_o = dial gauge reading corresponding to beginning of 1° consolidation.

R_{100} = dial gauge reading corresponding to completion of 1°-consolidation.

R_f = final dial gauge reading

R_F & R_i = Can be known from experiment.

$R_o - R_i$ = Initial Compression

R_0, R_{90}, R_{100} are calculated from curve fitting techniques of Taylors method.

Initial compresion is due to

(a) Compression of small quality of air in soil

(b) Imperfect saturation

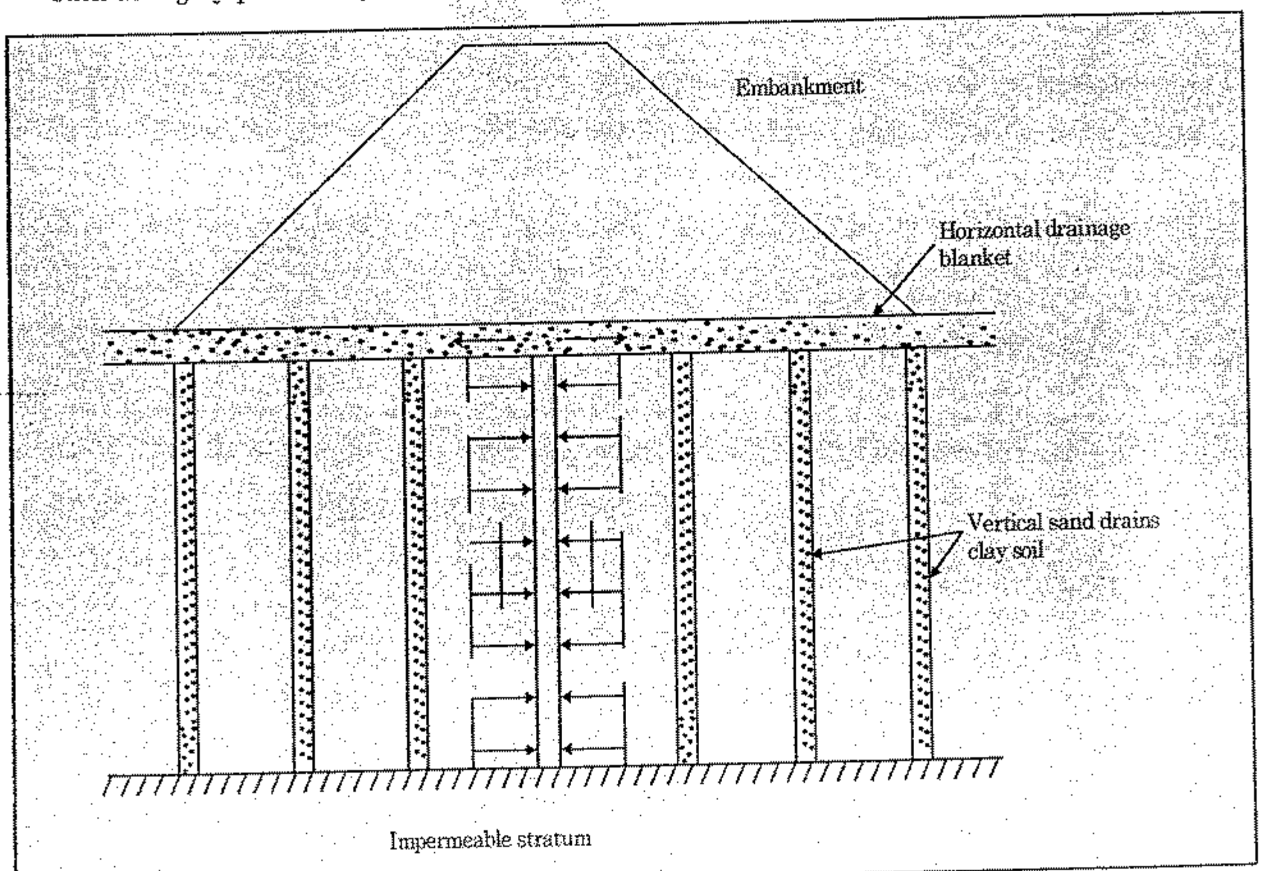
(c) Vertical elastic compression

(d) Lateral expansion of the soil specimen when imperfectly mounted.

VERTICAL SAND DRAINS

- The principle of the functioning of a sand drain is based on the consolidation theory of radially and vertically drained clay systems.
- The slow rate of consolidation of saturated clays of low permeability may be accelerated by means of vertical sand drains which provide for radial drainage, resulting in shortening of the drainage path.
- Consolidation is then due mainly to horizontal radial drainage which results in a faster dissipation of excess pore water pressure. The customary vertical drainage becomes less important.

- Magnitude of consolidation settlement, however, is unaffected; only the rate of settlement is increased.
- Sand drains are mostly used when embankments are to be constructed over a highly compressible caly layer.
- Sand drains installed in the clay accelerate consolidation and would enable the embankment to be brought into service much sooner. The post-construction settlement of a settlement-sensitive pavement or a railway line over an embankment would be greatly reduced.
- There would be an increase in the speed at which shear strength is generated in the clay layer, which would permit a faster rate of construction of the embankment.
- Sand drains are installed by driving a casing through the clay layer and making vertical boreholes.
- The casing is withdrawn after the boreholes are backfilled with a suitably graded sand having a permeability of at lest 1,000 times more than of the consolidating clay.
- Sand drains vary in diameter from about 450 to 600 mm, spacing depends on the type and permeability of the soil, and in practice varies from 1.8 to 4.5 m centre to centre. The spacings must obviously be less than the thickness of the clay layer.
- A successful design of the sand drain system depends basically on the correct estimation of the soil parameters, especially the coefficients of consolidation in both the horizontal and vertical directions, i.e., c_h and c_v.c_h is usually greater than c_v, because in any natural clay stratum, k_H is greater than k_v. The higher the ratio c_h/c_v, the more beneficial a sand drain system will be, since the radial drainage will be governed by c_h.
- During the installation of sand drains, the clay around the drains may get remoulded, thus reducing the value of coefficient of consolidation. This is known as the smear effect.
- Another shortcoming of the sand drains is their inability to control secondary compression because this reason, sand drains are not successful in soils having a high secondsary compression coefficient, such as highly plastic clays and peat.

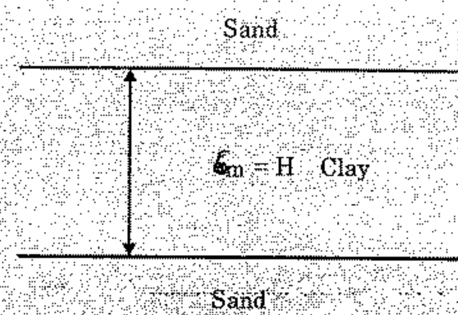


Example 12.

A 6m. thick clay layer is drained at both top and bottom the coefficient of consolidation of the soil is 5×10^{-4} cm²/sec. Determine the time required for 50% consolidation of the layer due to an external load.

Sol. Data Given:

$$C_v = 5 \times 10^{-4} \text{ cm/sec}$$
$$U = 50\% = 0.50$$



We know that, for 50% consolidation

Time factor $$Tv_{50} = \frac{\pi}{4}(0.50)^2 = 0.197$$

For Double drainage condition $$h = \left(\frac{H}{2}\right) = \frac{6}{2} = 3\text{m}$$

$\therefore$ Time required for 50% consolidation $$t_{50} = \left(\frac{Tv_{50} \times h^2}{C_v}\right)$$

$$\therefore \quad t_{50} = \left(\frac{0.197 \times (300)^2}{5 \times 10^{-4}}\right) = 35460000 \text{ sec}$$

$$\therefore \quad t_{50} = \frac{35460000}{(86400)} = 410.4 \text{ Days.}$$

Example 13.

A raft footing in to be constructed on a 7.5 m thick clay layer which lies between two sand layers Which lies between two sand layers. In order to predict the time rate of settlement of the bulding a 25 cm thick undisturbed sample of the soil was tested in the laboratory under double drainage condition. The sample was found to have undergone 50% consolidation in 12.5 minute. Determine the time req for 50% settlement of tbe building.

Sol. We know that, for 50% consolidation

$$T_v = \frac{C_v \times t}{h^2}$$

$$\Rightarrow \quad C_v = \left(\frac{T_v \times h^2}{t}\right) \quad \ldots(i)$$

In the Laboratory test

$$Tv_{50} = \frac{\pi}{4}(0.50)^2 = 0.197$$
$$t = 12.5 \text{ min}$$
$$h = \frac{H}{2} = \frac{2.5}{2} = 0.125 \text{ cm}$$

$$\therefore \quad C_v = \frac{Tv_{50} \times (1.25)^2}{12.5}$$

$$= \frac{0.197 \times (1.25)^2}{12.5} = 0.0246 \text{ cm}^2/\text{min} \quad \text{... } (ii)$$

In case of actual building

$$h = \frac{7.5 \times 100}{2} = 375 \text{ cm}$$

$$t = \frac{Tv_{50} \times h^2}{C_v} = \frac{(0.197 \times (375)^2)}{0.0246} \text{min}$$

$$= \frac{11261}{(60 \times 24)} = 782 \text{ days}$$

Example 14.

In a laboratory consolidation Test, a 2.5 cm, thick sample of clay reached 60% consolidation in 17 minutes under double drainge condition. Determine the time req for 60% consolidation of a layer of this soil in the field under the following conditions.

(i) When a 3 m thick layer of the given soil in sandwitched between two sand layers.

(ii) When a 5 m thick layer of the soil is overlain by a sand layer and underlain by a deep layer of intact shale.

Sol. Data given:

$$U = 60\%$$

$$t = 17 \text{ minuite.}$$

For double drainage condition $h = \frac{2.5}{2} = 0.125$ cm

Time factor for 60% consolidation

$$Tv_{60} = 1.781 - 0.933 \log_{10} (100 - U)$$

$$Tv_{60} = 1.781 - 0.933 \log_{10} (100 - 60)$$

$$Tv_{60} = 0.286 \quad \text{... } (i)$$

Therefore $\quad C_v = \frac{Tv_{60} \times h^2}{t}$

$$= \frac{0.286 \times (0.125)^2}{17} = 0.0263 \text{ cm}^2/\text{min}$$

(i) The soil layer is drained at both top and bottom

$$H = \frac{3 \times 100}{2} = 150 \text{ cm}$$

$$\therefore \quad t = \frac{Tv_{60} \times h^2}{C_v} = \frac{0.0263 \times (150)^2}{(0.0263)} \text{min}$$

$$= \frac{244800}{(24 \times 60)} = 170 \text{ days}$$

(*ii*) In this case the soil layer is drained at top only.

$$\therefore \quad H = 5 \text{ m} = 500 \text{ cm}$$

$$t = \frac{Tv_{60} \times H^2}{C_v}$$

$$= \frac{(0.286 \times (500)^2}{0.0263}$$

$$= 2718631 \text{ min}$$

$$= \frac{2718631}{(24 \times 60)} = 1888 \text{ days}$$

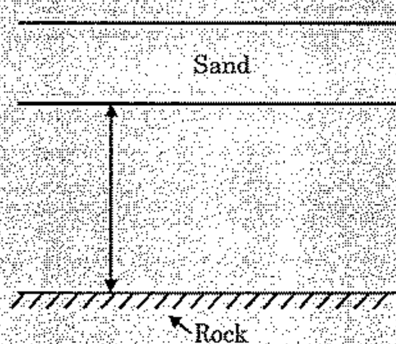
Example 15.

The consolidation settlement of a new structure founded on a 5 m thick layer is estimated as 6.5 cm. The structure was founded to have settled by 1.6 cm. In 6 months after the completion of a construction. If the clay layer is underlain by a rock and overlain by a layer of coarse sand, Determine.

(*i*) The time required for 50% consolidation to occur.

(*ii*) The amount of settlement which will take place in next six months.

Sol. **Data Given**



$$H = 6.5 \text{ cm}$$
$$\Delta H = 1.6 \text{ cm}$$

$\therefore$ Degree of consolidation occured in the first six month $= \left(\frac{1.6}{6.5} \times 100\right)$

$$U = 24.62\%$$

$\therefore$ Time factor for 24.62% consolidation

$$Tv_{24.62} = \frac{\pi}{4}(0.2462)^2 = 0.048 \quad \text{... (i)}$$

As single Drainage condition is prevailing

$$h = 5\text{m}, \; t = 30 \times 6 = 180 \text{ days}$$

$$\therefore \quad C_v = \frac{T_v \times h^2}{t}$$

$$\therefore \quad C_v = \frac{(0.048 \times 5^2)}{30 \times 6} = 6.67 \times 10^{-3} \text{m}^2/\text{day}$$

(i) For 50% consolidation, time factor $Tv = 0.197$

$$\therefore \quad t = \frac{Tv_{50} \times h^2}{C_v}$$

$$= \frac{(0.197 \times (5)^2)}{(6.67 \times 10^{-3})}$$

$$= 738.4 \text{ days.} = 2 \text{ years and } 8.4 \text{ days}$$

(ii) Let U be the degree of consolidation that will take place in the next six months i.e At the end of one years

- Since the completion of constrution.
- We have already found that the time required for 50% consolidation is 2 year and 8.4 days. Thus degree of consolidation occured in one year must be less than 50%.

Therefore corresponding time factor $= \frac{\pi}{4}\left(\frac{U}{100}\right)^2$ ... (i)

$$T_v = \frac{C_v \times t}{h^2} = \frac{(6.67 \times 10^{-3} \times 365)}{(5)^2}$$

$$= 0.0974 \quad \text{... (ii)}$$

from (i) and (ii) $\quad \frac{\pi}{4}\left(\frac{U}{100}\right)^2 = 0.0974$

$$\therefore \quad \left(\frac{U}{100}\right)^2 = \left(\frac{0.0974 \times 4}{\pi}\right)$$

$$\therefore \quad U = 100\sqrt{\frac{0.0974 \times 4}{\pi}}$$

$$U = 35.22\% \quad \text{... (iii)}$$

If x be the amount of settlement

$$\Rightarrow \quad U = \frac{x}{6.5} \times 100$$

$$\Rightarrow \quad x = \frac{6.5U}{100}$$

$$\Rightarrow \quad x = \frac{6.5 \times 35.22}{100}$$

$$\boxed{\therefore x = 2.29 \text{ cm}}$$

Example 16.

Undisturbed samples were collected from a 3m thick clay stratum which lies between two sand strata. A laboratory consolidation test was performed on a 2.5 cm. thick sample of the clay. During the test water was allowed to drain out only through the top of the sample. The time required for 50% consolidation was found to be 35 minutes. Determine the time required for 60% and 90% consolidation In the field.

Sol. For the laboratory test

Data given: The sample was tested under single drainage conditions

Therefore

$$h = H = 2.5 \text{ cm.}$$
$$U = 50\%$$
$$t = 35 \text{ minute}$$

$$\therefore \quad Tv_{50} = \frac{\pi}{4}\left(\frac{U}{100}\right)^2 = \frac{\pi}{4}\times(0.50)^2$$
$$= 0.197$$

$$\therefore \quad t = \frac{Tv_{50}\times h^2}{C_v}$$

$$\therefore \quad C_v = \frac{0.197\times(2.5)^2}{35}$$

$$C_v = 0.035 \text{ cm}^2/\text{min} \quad \ldots(i)$$

Now For 60% Consolidation

$$Tv_{60} = 1.781 - 0.933\log_{10}(100 - U)$$
$$= 1.781 - 0.933\log_{10}(100 - 60)$$
$$= 0.286$$

For double drainage condition

$$h = \left(\frac{H}{2}\right) = \frac{3}{2} = 1.50 \text{ m}$$

$$t_{60} = \left(\frac{Tv_{60}\times h^2}{C_v}\right)$$

$$= \frac{(0.286\times(150)^2)}{(0.035)}$$
$$= 183857 \text{ min}$$
$$= 127.7 \text{ days}$$
$$= 128 \text{ days.}$$

For 90% consolidation

$$Tv_{90} = 1.781 - 0.933\log_{10}(100 - 90)$$
$$= 0.848$$

$$\therefore \quad t_{90} = \frac{Tv_{90}\times h^2}{C_v}$$

$$= \frac{0.848\times(150)^2}{(0.035)}$$
$$= 545143 \text{ min}$$
$$= 378 \text{ days.}$$

Example 17.

June

The construction of a multistoryed building started in Jannuary 1989 and was completed in 1990. The total consolidation settlement of the building was estimated to be 8 cm. The average setlement of the building was measured in december 1991 and was found to be 2.2 cm. Compute the probable settlement of the building in jan. 2001

Sol. Let C_v be the coefficient of consolidation of the soil in the appropriate pressure Range.

∴ Time Elapsed from June 1990 to December 1991 = 1.5 years

Degre of consolidation occured in 1.5 years

$$= \frac{2.2\times 100}{8} = 27.5\% \quad \dots (i)$$

∴

$$T_v = \frac{\pi}{4}U^2$$

$$T_v = 0.059$$

But

$$T_v = \frac{C_v \times t}{H^2}$$

$$\Rightarrow \quad \frac{C_v}{H^2} = \left(\frac{T_v}{t}\right)$$

$$= \frac{0.059}{1.5}$$

$$= 0.039$$

Again, time elapsed from June 1990 to Jan. 2001 = 10.5 years

Let U be the corresponding degree of consolidation

Assuming

$$U > 53\%$$

$$Tv = 1.781 - 0.933 \log_{10}(100 - U) \quad \dots (ii)$$

But

$$T_v = \left(\frac{C_v \times t}{H^2}\right) = 0.039 \times 10.5 = 0.4095 \quad \dots (iii)$$

From (ii) and (iii)

$$1.781 - 0.933 \log_{10}(100 - U) = 0.4095$$

$$\Rightarrow \quad 1.3715 = 0.933 \log_{10}(100 - U)$$

$$\Rightarrow \quad \log(100 - U) = 1.47$$

$$\Rightarrow \quad U = 70.49\% \quad \dots (iv)$$

∴ Amount of consolidation settlement in Jan. 2001

$$U = \frac{\Delta H}{H} \times 100$$

∴

$$70.49 = \frac{\Delta H}{8} \times 100$$

∴

$$\Delta H = \left(\frac{70.49 \times 8}{100}\right)$$

$$[\Delta H = 5.64 \text{ cm}]$$

Example 18.

A 2 m thick layer of saturated clay lies in between two permeable layers. The clay has the following properties liquid limit = 45%

Coefficient of permeablity $K = 2.8 \times 10^{-7}$ cm/sec

Initial void Ratio = 1.25

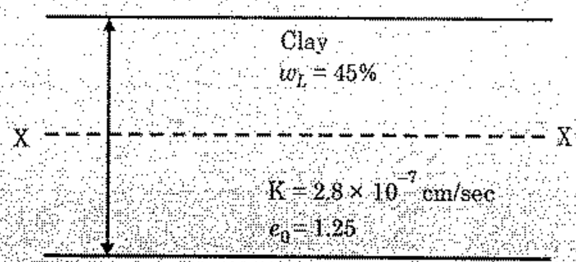
The initial effective over burden pressure at the middle of the clay layer is 2 kg/cm^2 and is likely to increase to 4kg/cm^2 due to

The construction of a new building Determine

(i) Final void ratio of the clay

(ii) Settlement of the proposed building

(iii) Time req. for 50% consolidation



Sol. Data given

$$\sigma_0 = 2 \text{ kg/cm2}$$

$$\Delta\sigma = 4 - 2 = 2 \text{ kg/cm}^2$$

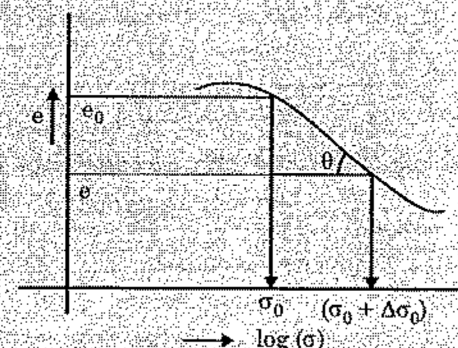
$\therefore$ We have $C_c = 0.009\,(w_L - 10)$

$$= 0.009\,(45 - 10)$$

$$C_c = 0.315 \qquad \ldots (i)$$

But by definiation

$$C_c = \frac{\Delta e}{\log_{10}\left(\frac{\sigma_0 + \Delta\sigma}{\sigma_0}\right)}$$



$$\Rightarrow \qquad \Delta e = C_c \log_{10}\left(\frac{2+2}{2}\right)$$

$$= 0.315 \log_{10}\left(\frac{4}{2}\right)$$

$$= 0.095$$

$\therefore$ Final void ratio $e_f = e_0 - \Delta e$

$$= (1.25 - 0.095)$$

$$e_f = 1.155 \qquad \ldots (ii)$$

(ii) Let ΔH be the consolidation settlement of the clay layer.

$$\frac{\Delta H}{H} = \frac{\Delta e}{(1+e_0)}$$

$$\Rightarrow \qquad \Delta H = \frac{2 \times 0.095 \times 100}{(1+1.25)}$$

$$= 8.44 \text{ cm}$$

(iii) In the pressure range of 2 to 4 kg/cm^2

$$m_v = \frac{\Delta e}{(1+e_0) \times \Delta\sigma}$$

$$= \frac{0.095}{(1+1.25)} \times \frac{1}{2}$$

$$= 0.021 \text{ cm}^2\text{/kg}$$

$\because$ $K = 2.8 \times 10^{-7}$ cm/sec

$$m_v = 0.021 \text{ cm}^2\text{/kg}$$

$$\gamma_w = 1\text{gm/cc} = 10^{-3} \text{ kg/cc}$$

$$\therefore \qquad C_v = \frac{K}{(m_v\,\gamma_w)}$$

$$= \frac{2.8\times10^{-7}}{(0.021\times10^{-3})}$$

$$= 0.0133\ \text{cm}^2/\text{sec}$$

For 50% consolidation

$$Tv_{50} = \frac{\pi}{4}(0.5)^2 = 0.197$$

$$\therefore\quad t = \frac{T_v h^2}{C_v} = \frac{0.197\times\left(\frac{200}{2}\right)^2}{0.0133}\ \text{sec}$$

$$t = 148120.3008\ \text{sec}$$

$$t = 1.71\ \text{days}$$

Example 19.

A 6 m thick clay stratum is overlain]by 8 m thick stratum of coarse sand and is underlain by an impermeable sale. A raft footing, supported the column of a building is to be founded at a depth of 1.2 m below G.L. The size of raft is 8.5 m × 13.6 m, and it is loaded uniformly with a stress intensity of 9.2 t/m². The water table is located at 2 m below the G.L the unit weight of sand above and below Water table are 1.90 and 2.10 t/m³. the properties of clay are as follows:

Initial void rat]io = 0.72

Specific gravity of solids = 2.71

liquid limit = 42%

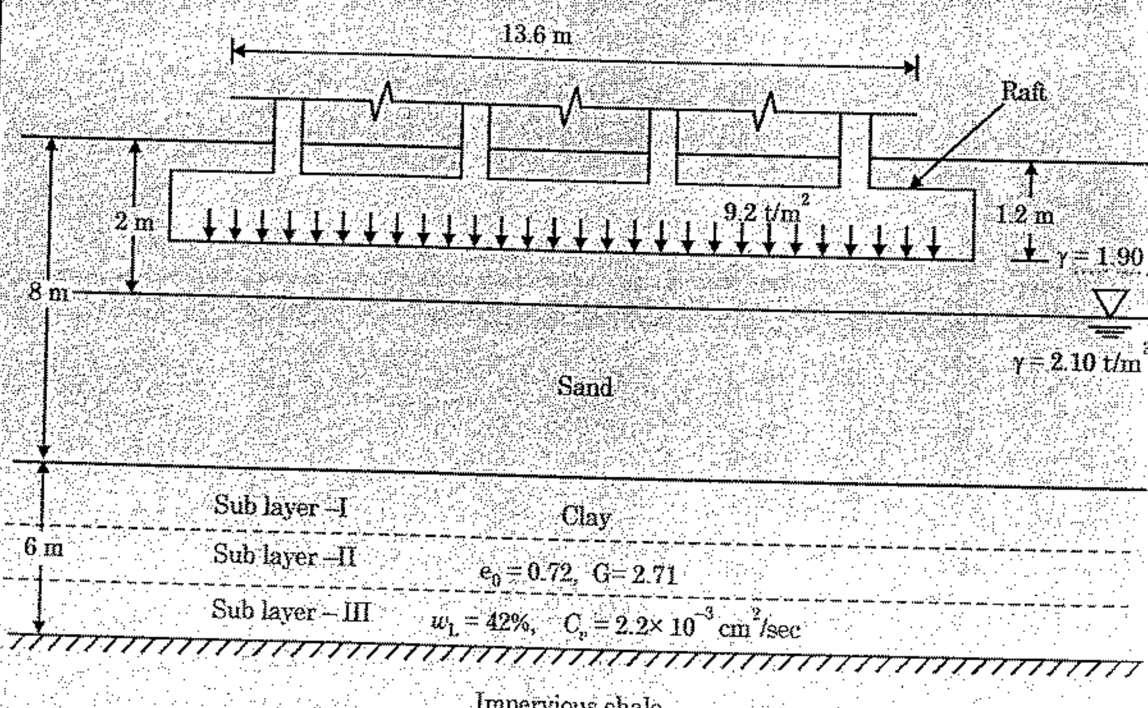
Coeffcient of consolidation = 2.2×10^{-3} cm²/sec

Determine

(i) Probable settlement of the raft

(ii) The time required to undergo a settlement of 5 cm

Sol.



The clay layer is divided into three sub layers of thickness 2m each.

For the settlement of each layer

We have $$\Delta H = \frac{H_0\, C_c}{(1+e_0)} \log_{10}\left(\frac{\sigma_0 + \Delta\sigma}{\sigma_0}\right) \quad \text{... }(i)$$

The computation of settlement for the first sub layer

$$C_c = 0.009\,(w_L - 10)$$

$$\therefore \quad C_c = 0.009\,(42 - 10)$$

$$= 0.288$$

$$e_0 = 0.72$$

$$H_0 = 2\text{m} = 200 \text{ cm}$$

$\therefore$ Depth of middle of the sub-layer below GL $= 8 + \frac{2}{2} = 9\text{m}$

$\therefore$ σ_0 = Initial effective overburden stress at a depth of 9 m below G.L

$$= (\gamma_1\, h_1 + \gamma_{sub}\, h_2 + \gamma_{clay}\, h_3) \quad \text{... }(ii)$$

$$\gamma_{sat} = 2.10 \text{ t/m}^3;\ \gamma_w = 1.0 \text{ t/m}^3$$

$$\therefore \quad \gamma_{sub} = (2.10 - 1.0) = 1.10 \text{ t/m}^3$$

$$\gamma_{clay} = \frac{(G+e)\gamma_w}{(1+e)} = \frac{(2.71+0.72)\times 1}{(1+0.72)}$$

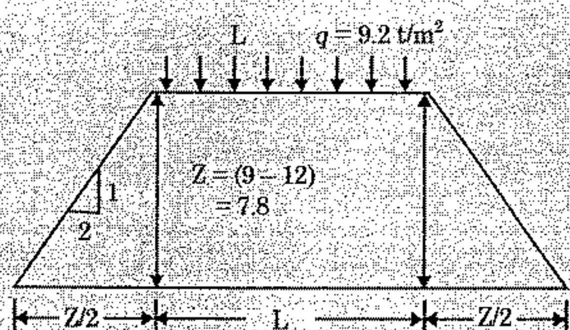
$$= 2.0 \text{ t/m}^3$$

$$\therefore \quad \sigma_0 = 1.9 \times 2 + 1.10 \times 6 + (2-1) \times 1$$

$$= 11.4 \text{ t/m}^2$$

$$= 1.14 \text{ kg/cm}^2$$

Using 2:1 Despersion Method



$$\Delta\sigma = \frac{q\,BL}{(B+Z)(L+Z)}$$

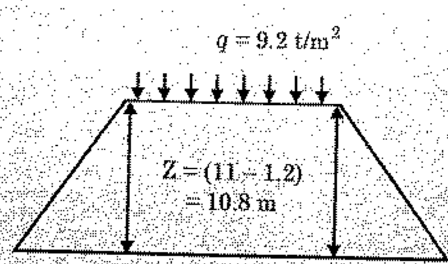
$$= \frac{9.2 \times 8.5 \times 13.6}{(8.5+7.8)\times(13.6+7.8)}$$

$$= 3.05 \text{ t/m}^2$$

$$= 0.305 \text{ kg/cm}^2$$

$$\therefore \quad DH_1 = \frac{0.288\times 200}{(1+0.72)}\log_{10}\left(\frac{1.14+0.305}{1.14}\right) = 3.45 \text{ cm} \quad \ldots (iii)$$

Similarily for second layer



$$\sigma_0 = (\gamma_1\ h_1 + \gamma_{sub}\ h_2 + \gamma_{clay}\ h_3)$$
$$= (1.9 \times 2 + 1.10 \times 6 + 1 \times 3)$$
$$= 13.4 \text{ t/m}^2$$
$$= 1.34 \text{ kg/cm}^2$$

$$\therefore \quad \Delta\sigma = \frac{qB_L}{(B+Z)(L+Z)}$$
$$= \frac{9.2\times 13.6\times 8.5}{(13.6+9.8)(8.5+9.8)}$$
$$= 2.484 \text{ t/m}^2$$
$$= 0.2484 \text{ kg/cm}^2$$

$$\therefore \quad \Delta H_2 = \frac{0.288\times 200}{(1+0.72)}\log_{10}\left(\frac{1.34+0.2484}{1.34}\right)$$
$$= 2.47 \text{ cm} \quad \ldots (iv)$$

Similarly for Third sub layer

$$\sigma_0 = (\gamma_1 h_1 + \gamma_{sub}\ h_2 + \gamma_{clay}\ h_3)$$
$$= (1.9 \times 2 + 1.1 \times 6 + 1\times 5)$$
$$= 15.4 \text{ t/m}^2$$
$$= 1.54 \text{ kg/cm}^2$$

$$\Delta\sigma = \frac{qBL}{(B+Z)(L+Z)}$$
$$= \frac{9.2\times 13.6\times 8.5}{(13.6+11.8)+(8.5+11.8)}$$
$$= 2.06 \text{ t/m}^2 = 0.206 \text{ kg/cm}^2$$

$$\Delta H_3 = \frac{0.288\times 200}{(1+0.72)}\log_{10}\left(\frac{1.54+0.206}{1.54}\right) = 1.83 \text{ cm}$$

$\therefore$ Probable settlement of the clay layer

$$\Delta H = (\Delta H_1 + \Delta H_2 + \Delta H_3)$$
$$= (3.45 + 2.47 + 1.83)$$
$$= 7.75 \text{ cm}$$

(ii) Degree of consolidation coresssponding to a settlement of 5 cm

$$U = \frac{5}{7.75}\times 100 = 64.52\%$$

The corresponding time factor

$$T_v = 1.781 - 0.933 \ \log_{10}(100 - 64.52)$$
$$T_v = 0.335$$

As single drainage condition prevails at site

$$\therefore \quad t = \left(\frac{T_v H^2}{C_v}\right)$$
$$= \frac{0.335 \times (600)^2}{(2.2 \times 10^{-3})}$$
$$= 634 \text{ days}$$

Example 20.

A 2m wide strip footing in to be placed in a sand layer 2m thick at a depth of 1m below the ground surface. The sand is underlain by a layer of saturated clay which is 1 m thick. The clay overlies a bed of Dense sand. The ground water table is at the level of the top of the clay layer. The bulk unit weight of sand above the clay layer is 2t/m^3 and submerged unit weight of clay in 0.8 t/m^3 the footing is designed to carry a load of 22t/m^2. Compute the probable settlement of the footing below its centre. Also determine the elapsed time in which 10 percent and 90 percent of the ultimate setlement will occur. It is known from a graph between pressure and void ratio for clay that the void ratio corresponding to vertical pressure of 4.4 t/m^2 and 18 t/m^2 are respectively 1.16 and 1.02. The coefficient of consolidation for the soil is 4×10^{-4} cm^2/sec.

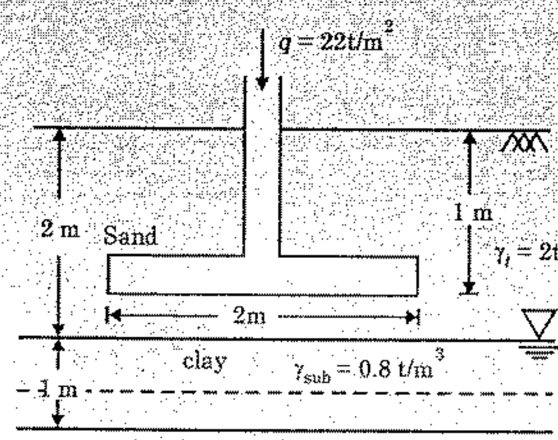
The vertical pressure (α_2) below the centerline of uniformly loaded strip footing (intensity q) of width B may be taken an

Depth	α_2
0.1 B	0.997 q
0.5 B	0.817 q
1.0 B	0.550 q
2.0 B	0.306 q

The time factor (T_v) corresponding to the degree of consolidation (U%) is as follows

U(%)	10	90
T_v	0.008	0.848

Sol.



Initial Effective stress at 2.5 m below the G.L before the construction

$$= \frac{0.55q + (0.306 - 0.550)q}{(2-1)B}(2.5-2)$$

$$= 0.550\ q + (-\ 0.061\ q)$$

$$\sigma_1 = 0.489\ q = 10.758\ \text{t/m}^2 \quad \dots (i)$$

Effective stress at 1.5 m below the footing after the construction $= \dfrac{(0.817 + 0.550)q}{2}$

$$\sigma_2 = 0.6835\ q = 15.037\ \text{tan/m}^2 \quad \dots (ii)$$

$\therefore$ Change in effective stress at the centre of clay

$$\Delta\bar{\sigma} = \sigma_2 - \sigma_1$$

$$= (15.037 - 10.758)$$

$$\Delta\bar{\sigma} = 4.279\ \text{t/m}^2 \quad \dots (iii)$$

$$\therefore \quad m_v = \frac{\Delta e}{(1+e)} \times \frac{1}{(\Delta\bar{\sigma})}$$

$$= \frac{(1.16 - 1.02)}{(1.1.16)} \times \frac{1}{(18 - 4.4)}$$

$$= 4.766 \times 10^{-3}\ \text{m}^2\text{/tonn} \quad \dots (iv)$$

$\therefore$ Probable settlement of the footing $\Delta H = (H_0\ m_v\ \Delta\bar{\sigma})$

$$= (1 \times 4.279 \times 4.766 \times 10^{-3})$$

$$= 0.02039\ \text{m}$$

$$= 20.4\ \text{mm}$$

Coeffecient of consolidation $C_v = 4 \times 10^{-4}\ \text{cm}^2\text{/sec}$

$$\therefore \quad Tv_{10\%} = 0.008$$

$\therefore$ Since there is double dranage condition

$$t = \frac{Tv_{10} \times \left(\frac{100}{2}\right)^2}{C_v}$$

$$= \frac{0.008 \times (50)^2}{4 \times 10^{-4}}\ \text{sec}$$

$$= 50{,}000\ \text{sec} = 0.579\ \text{day}$$

Time factor for 90% consolidation

$$Tv_{90} = 0.848$$

$$\therefore \quad t = \frac{Tv_{90}\left(\frac{h}{2}\right)^2}{C_v}$$

$$= \frac{0.848 \times (50)^2}{(4 \times 10^{-4})}\ \text{sec}$$

$$= 53{,}00{,}000\ \text{sec}$$

$$= 61.34\ \text{days}$$

Example 21.

A flexible footing of 2m × 2m size carries a total load of 490 kN, inclussive of its self weight. The footing rests on a sand layer having a modulus of elasticity of 40000 kN/m^2 and a poission's ratio of 0.38. Estimate the probable settlement below the centre and below any corner of the footing

Given I_f (corner) = 1.56, I_f (centre) = 1.12

Sol. W]e have

$$S_i = qB\frac{(1-\mu^2)}{E}I_f \quad \ldots (i)$$

where q = intensity of loading

$$= \frac{490}{(2\times2)} = 122.5 \text{ kN/m}^2$$

B = 2 m

μ = 0.38

E = 40000 kN/m^2

I_f (corner) = 0.56

∴ Immediate settlement below the corner

$$S_{i\,(\text{corner})} = \frac{qB(1-\mu^2)}{E}I_f$$

$$= \frac{22.5\times2[1-(0.38)^2]\times0.56}{40,000}$$

$$= 2.94 \text{ mm}$$

Immediate settlement below the centre

I_f (centre) = 1.12

$$\therefore S_i = \frac{122.5\times2\,[1-(0.38)^2]\times1.12}{40,000}$$

$$= 5.89 \text{ mm}$$

OBJECTIVE QUESTIONS

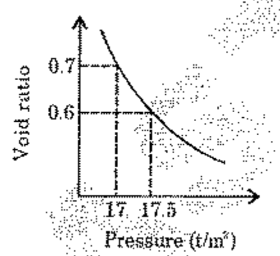
1. The natural void ratio of a saturated clay strata, 3 m thick is 0.90. The final void ratio of the clay at the end of consolidation is expected to be 0.71. The total consolidation settlement of the clay strata is

(a) 30 cm (b) 25 cm
(c) 20 cm (d) 15 cm

2. The identical clay samples of the same size, designated as A and B are subjected to consolidation test under identical loading conditions. Drainage takes place through one face in sample A and through both the faces in sample B. 50% consolidation of sample A occurs in 10 minutes. The time required for 50% consolidation to occur in sample B will be

(a) 40 minutes (b) 10 minutes
(c) 5 minutes (d) 2.5 minutes

3. The void ratio-pressure diagram is shown in the given figure. The coefficient of compressibility is



(a) 0.0050 m^2/t (b) 0.073 m^2/t
(c) 0.20 m^2/t (d) 0.25 m^2/t

4. Match **List-I** (Effect) with **List-II** (Reason) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Excessive settlement	1. Rise of water table
B. High expansivity	2. High compressibility
C. Reduction of bearing capacity	3. Montmorillonite
D. Acceleration of consolidation	4. Sand drains

Codes:

	A	B	C	D
(a)	4	1	2	3
(b)	2	3	4	1
(c)	4	1	3	2
(d)	2	3	1	4

5. The change that take place during the process of consolidation of a saturated clay would include

(a) an increase in pore water pressure and an increase in effective pressure
(b) an increase in pore water pressure and a decrease in effective pressure
(c) a decrease in pore water pressure and a decrease in effective pressure
(d) a decrease in pore water pressure and an increase in effective pressure

6. Match **List-I** (Property) with **List-II** (Slope of the curve) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Coefficient of compressibility	**1.** Stress-deformation
B. Compression index	**2.** Stress-void ratio
C. Coefficient of subgrade modulus	**3.** Volume-pressure
	4. Log stress-void ratio

Codes:

	A	B	C
(a)	4	2	1
(b)	4	3	2
(c)	2	4	1
(d)	3	4	1

7. The initial and final void ratios of a clay sample in a consolidation test are 1 and 0.5, respectively. If initial thickness of the sample is 2.4 cm, then its final thickness will be
(a) 1.3 cm
(b) 1.8 cm
(c) 1.9 cm
(d) 2.2 cm

8. Which one of the following soils has stress-strain response similar to that of dense sand? (OCR stands for over consolidation ratio)
(a) Over consolidated clays having high OCR
(b) Over consolidated clays having low OCR
(c) Normally consolidated clays
(d) Unconsolidated clays

9. Which one of the following pairs of parameters and expression is NOT correctly matched?

(a) Coefficient of consolidation ... $\frac{T_V H^2}{t}$

(b) Coefficient of volume compressibility ... $\frac{e_0 - e}{(1+e_0)(p-p_0)}$

(d) Over consolidation ratio $\sqrt{\frac{\text{Maximum previous effective pressure}}{\text{Existing effective pressure}}}$

(d) Modulus of volume change ... $\frac{a_v}{1+e_0}$

10. Consider the following :
1. Initial consolidation
2. Primary consolidation
3. Secondary consolidation

The three stages which would be relevant to consolidation of a soil deposit includes
(a) 1, 2 and 3
(b) 2, 3 and 4
(c) 1, 3 and 4
(d) 1, 2 and 4

11. Match **List-I** with **List-II** and select the correct answer using the codes given below the lists (notations have their usual meaning):

List-I		List-II	
A.	Coefficient of compressibility	1.	m_v
B.	Compression index	2.	$C_v t/H^2$
C.	Time factor	3.	a_v
D.	Coefficient of volume compressibility	4.	C_c

Codes:

	A	B	C	D
(a)	3	2	4	1
(b)	1	2	4	3
(c)	1	4	2	3
(d)	3	4	2	1

12. Reduction in volume of soil primarily due to squeezing out of water from the voids is called
(a) primary consolidation
(b) plastic flow
(c) creep
(d) secondary consolidation

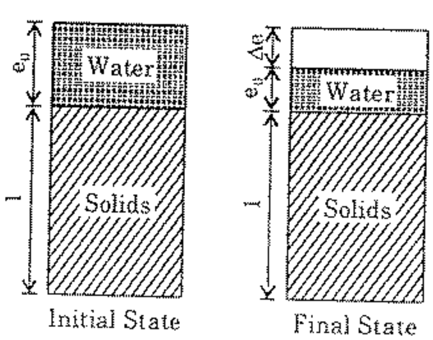
13. Excess pore pressure distributions within the thickness of a soil sample tested in oedometer sometime after loading is shown in the figure below labelled 1, 2, 3 and 4. Which one of these figures, refers to a situation where the operator forgot to put on the porous stones at the top and bottom of the sample before the test?
(a) 1
(b) 2
(c) 3
(d) 4

14. In soil consolidation process, the following events take place after loading:
1. Decrease in excess pore pressure.
2. Increase in total stress.
3. Development of excess pore pressure.
4. Increase in effective stress.

The correct sequence of these events is
(a) 3, 2, 1, 4
(b) 2, 3, 1, 4
(c) 2, 3, 4, 1
(d) 3, 2, 4, 1

15. In the phase diagrams given the change due to initial state changing into final state is shown due to consolidation. Depth of soil layer undergoing consolidation is H; e_0 is initial void ratio; e_f is final void ratio; De is change in void ratio.

Indicate which of the following expressions gives settlement of the layer.



(a) $H\log_{10}\left(\frac{\Delta e}{1+e_0}\right)$ (b) $\log_{10}\left(H\frac{\Delta e}{1+e_0}\right)$

(c) $\frac{\Delta e}{1+e_0}$ (d) $H\frac{\Delta e}{1+e_0}$

16. Which one of the following statements regarding coefficient of consolidation C_v is correct?

(a) $C_v \propto k$ (b) $C_v \propto \frac{1}{k}$

(c) $\frac{\Delta e}{1+e_0}$ (d) $H\frac{\Delta e}{1+e_0}$

17. Consider the following statements:

Theory of consolidation predicts settlement due to primary consolidation; it cannot include settlement due to initial compression nor due to secondary consolidation. This happens because of the following assumptions made by the theory:

1. Soil grains and water are incompressible.
2. Soil is fully saturated.
3. Compression takes place in vertical direction only.
4. Time lag in consolidation is entirely due to low permeability of soil.

Which of these statements are correct?

(a) 1, 2 and 3 (b) 2 and 3

(c) 3 and 4 (d) 1, 2, 3 and 4

18. The one dimensional theory of consolidation proposed by Terzaghi derives its name due to the fact that

(a) only vertical dimensions of the soil sample is used for consolidation test and lateral dimensions are neglected

(b) water in the soil sample in conventional consolidometer escapes in the lateral directions resulting into settlements only in vertical direction

(c) normal stress on the sample is applied in only one (vertical) direction

(d) lateral movements of soil grains are not permitted across any vertical boundary resulting into only vertical settlements to account for the decrease in volume due to escape of water from soil sample

19. For a certain loading condition, a saturated clay layer undergoes 40% consolidation in a period of 178 days. What would be the additional time required for further 20% consolidation to occur?

(a) 89 days (b) 222.5 days

(c) 329.5 days (d) 400.5 days

20. The installation of sand drains in clayey soil causes the soil adjacent to the sand drains to undergo which one of the following?

(a) Increase in porosity (b) Increase in compressibility

(c) Decrease in horizontal permeability (d) Decrease in shear strength

21. If Δp is increment of pressure on a normally consolidated saturated soil mass, as per Terzaghi's theory at the instant of application of pressure increment i.e., when time $t = 0$, what is the pore pressure developed in the soil mass?

(a) Zero
(b) Very much less than Dp
(c) Equal to Dp
(d) Greater than Dp

22. In a consolidation test the sample tested has height H; water content is w; specific gravity of solids G. After increasing the loading by an increment Δp, the height decrease in ΔH. Which one of the following expresses the corresponding change in void ratio De?

(a) $\Delta e = \dfrac{\Delta H}{H(1+wG)}$
(b) $\Delta e = \dfrac{\Delta H(1+wG)}{H}$
(c) $\Delta e = \dfrac{H(1+wG)}{\Delta H}$
(d) $\Delta e = \dfrac{H}{\Delta H(1+wG)}$

23. Consider the following statements:

1. Coefficient of consolidation normally increases with decreasing liquid limit of clay
2. The larger the value of coefficient of consolidation, the longer it takes for full consolidation to occur.

Which of these statements is/are correct?
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

24. A 1 m thick layer of saturated clay, drained at both faces, settles by 10 cm in one year. If a thin layer of pervious soil is introduced in the middle of this layer, then what will be the period during which the settlement of 10 cm will be completed?
(a) 4 years
(b) 0.5 year
(c) 0.25 year
(d) 2 years

25. Considerable loss of shear strength due to shock or disturbance is exhibited by
(a) under-consolidated clays
(b) normally consolidated clays
(c) over-consolidated clays
(d) organic soil

26. Two specimens of clay A and B are tested in a consolidation apparatus. If

$(m_v)_A = 3.6 \times 10^{-4}$ m²/kN

$(m_v)_B = 1.8 \times 10^{-4}$ m²/kN

$(C_v)_A = 3.8 \times 10^{-4}$ cm²/s

$(C_v)_B = 1.9 \times 10^{-4}$ cm²/s,

then the ratio k_A/k_B is equal to
((a) 0.0625
(b) 0.25
(c) 1.0
(d) 4.0

27. In consolidation testing, curve fitting method is used to determine
(a) compression index
(b) swelling index
(c) coefficient of consolidation
(d) time factor

28. Compression index of a soil helps to determine
(a) total time required for consolidation
(b) time required for 50 per cent consolidation
(c) total settlement of clay layer
(d) pre-consolidation pressure of clay

29. When the degree of consolidation is 50%, the time factor is about
(a) 0.2 (b) 0.5 (c) 1.0 (d) 2.0

30. Consider the following soil parameters and conditions:
1. Drinage conditions
2. Thickness of layer
3. Initial void ratio
4. Over-burden pressure

The magnitude of settlement is influenced by
(a) 1, 2 and 3 (b) 2, 3 and 4 (c) 1, 3 and 4 (d) 1, 2 and 4

31. In a saturated clay layer undergoing consolidation with single drinage at its top, the pore water pressure would be the maximum at its
(a) top
(b) middle
(c) bottom
(d) top as well as the bottom

32. A saturated clay stratum of thickness 10 m, bounded on top and bottom by medium coarse sand layers, has a coefficient of consolidation of 0.002 cm^2/s. If this stratum is subjected to loading, it is likely that it would undergo 50% of its primary consolidation in
(a) 1136 days (b) 227 days (c) 284 days (d) 568 days

33. Consider the following statements regarding settlement of foundations:
1. Differential settlement of foundation leads to structural damage of the superstructure.
2. In non-cohesive soils, the major component of settlement is due to consolidation.
3. Lowering of ground water table contributes to settlement of foundations.

Which of these statements are correct?
(a) 1 and 2
(b) 1 and 3
(c) 2 and 3
(d) 1, 2 and 3

34. The time 't' required for attaining a certain degree of consolidation of a clay layer is proportional to
(a) H^2 and C_v
(b) H^2 and $1/C_v$
(c) $1/H^2$ and C_v
(d) $1/H^2$ and $1/C_v$.

35. Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

List-I	List-II
A. Elastic settlement	1. Constant effective stress with change in volume of soil
B. Primary consolidation	2. Dissipation of excess pore water pressure
C. Secondary consolidation	3. Occurs within a short period
D. Creep	4. Compression and rearrangement of particles

Codes:

	A	B	C	D
(a)	3	2	1	4
(b)	4	3	1	2
(c)	3	2	4	1
(d)	4	3	2	1

36. Given that for an over-consolidated clay soil deposit, the pressure under which the deposit has been fully consolidated in the past is 125 kN/m^2, and the present overburden pressure is 75 kN/m^2, the over-consolidation ratio of the soil deposit is

(a) 75/125 (b) 50/75

(c) 125/75 (d) 200/75

37. When the primary consolidation process in a soil is complete, then

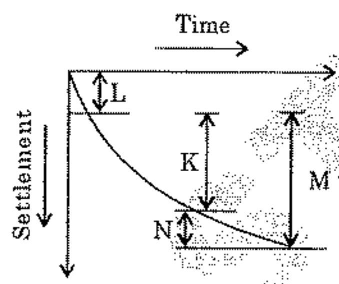
(a) the hydrostatic pressure will become zero

(b) the excess pore water pressure will become zero

(c) both the hydrostatic and excess pore water pressure will become zero

(d) the effective stress will become zero

38. Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:



	List-I	List-II
A.	Immediate settlement	1. K
B.	Primary consolidation	2. L
C.	Secondary compression	3. M
D.	Time-dependent settlement	4. N

Codes:

	A	B	C	D
(a)	1	2	4	3
(b)	2	1	4	3
(c)	2	1	3	4
(d)	1	2	3	4

39. If a saturated soil sample is consolidated, the degree of saturation will

(a) reduce (b) increase

(c) remain constant (d) become zero

40. Rate of settlement of a compressible layer is dependent upon

(a) applied load, thickness of layer and time factor

(b) applied load and time factor

(c) thickness of layer and time factor

(d) applied load and thickness of layer

41. Loading of a saturated fine-grained soil results in the following processes:

1. Shear failure
2. Immediate settlement
3. Pore pressure generation
4. Volumetric deformation

What is the correct sequence of the processes given above?

(a) 1-4-3-2
(b) 2-3-4-1
(c) 1-3-4-2
(d) 2-4-3-1

42. Consider the following statements:

1. In the laboratory consolidation test, initial compression is the result of displacement of soil particles.
2. Primary consolidation is due to dissipation of pore water pressure.
3. Secondary compression starts after complete dissipation of pore water pressure.
4. Primary consolidation and secondary compression occur simultaneously.

Which of these statements are correct?

(a) 1 and 4 only
(b) 2 and 3 only
(c) 2 and 4 only
(d) 1 and 3 only

43. Match **List-I** (Cause) with **List-II** (Effect) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Tamping	**1.** Shearing
B. Consolidation	**2.** Piping
C. Triaxial compression	**3.** Expulsion of air
D. Seepage	**4.** Reduction water

Codes:

	A	B	C	D
(a)	2	4	1	3
(b)	3	1	4	2
(c)	2	1	4	3
(d)	3	4	1	2

44. Which of the following are the reasons for pre-consolidation of a clay layer?

1. Desiccation of upper layers
2. R ising of water table
3. Removal of construction load
4. Withdrawal of a glacier

Select the correct answer using the codes given below:

(a) 1, 2 and 3
(b) 2, 3 and 4
(c) 1, 3 and 4
(d) 1, 2 and 4

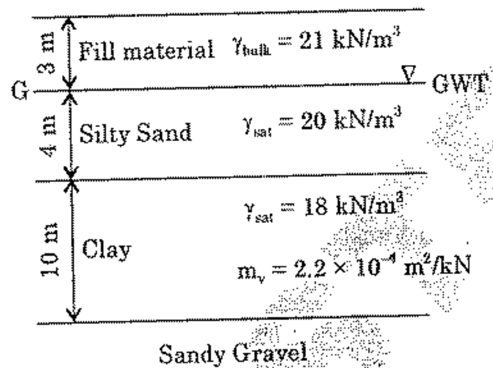
45. What is the condition until when consolidation continues?

(a) The pore water pressure becomes zero.
(b) The effective stress becomes zero.
(c) The total stress becomes zero.
(d) The excess pore water pressure becomes zero.

46. A double draining clay layer, 6 m thick, settles by 30 mm in three years under the influence of certain loads. Its final consolidation settlement has been estimated to be 120 mm. If a thin layer of sand having negligible thickness is introduced at a depth of 1.5 m below the top surface, the final consolidation settlement of clay layer will be

(a) 60 mm
(b) 120 mm
(c) 240 mm
(d) None of these

47. At a reclamation site for which the soil strata is shown in figure, a 3 m thick layer of a fill material is to be laid instantaneously on the top surface. If the coefficient of volume compressibility, m_v for clay is 2.2×10^{-4} m²/kN, the consolidation settlement of the clay layer due to placing of fill material will be



(a) 69.5 mm
(b) 139 mm
(c) 228 mm
(d) 278 mm

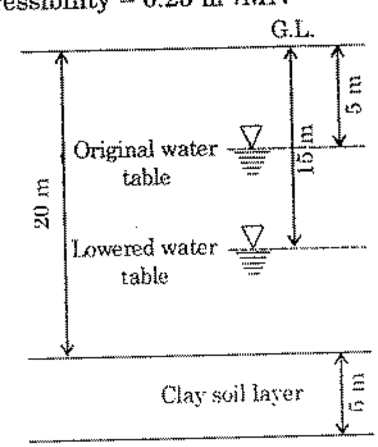
48. Root time method is used to determine

(a) T, time factor
(b) C_v, coefficient of consolidation
(c) a_v, coefficient of compressibility
(d) m_v, coefficient of volume compressibility

Statement for Linked Answer Questions 49 and 50 :

The ground conditions at a site are as shown in the figure. The water table at the site which was initially at a depth of 5 m below the ground level got permanently lowered to a depth of 15 m below the ground level due to pumping of water over a few years. Assume the following data:

(i) unit weight of water = 10 kN/m³
(ii) unit weight of sand above water table = 18 kN/m³
(iii) unit weight of sand and clay below the water table = 20 kN/m³
(iv) coefficient of volume compressibility = 0.25 m²/MN



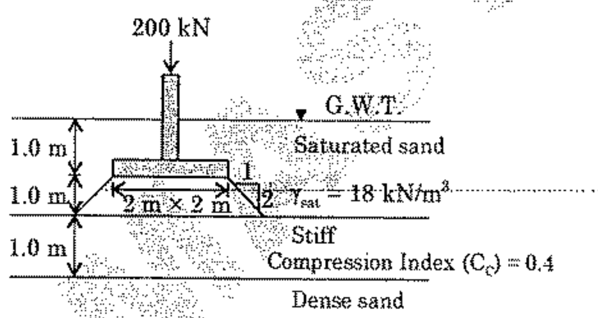
49. What is the change in the effective stress in kN/m^2 at mid-depth of the clay layer due to the lowering of the water table?
(a) 0
(b) 20
(c) 80
(d) 100

50. What is the compression of the clay layer in mm due to the lowering of the water table?
(a) 125
(b) 100
(c) 25
(d) 0

Statement for Linked Answer Qnestions 51 and 52 :

51. A saturated undisturbed sample from a clay strata has moisture content of 22.22% and specific weight of 2.7. Assuming γ_w = 10 kN/m^3, the void ratio and the saturated unit weight of the clay, respectively are
(a) 0.6 and 16.875 kN/m^3
(b) 0.3 and 20.625 kN/m^3
(c) 0.6 and 20.625 kN/m^3
(d) 0.3 and 16.975 kN/m^3

52. Using the properties of the clay layer derived from the above question, the consolidation settlement of the same clay layer under a square footing (neglecting its self weight) with additional data shown in the figure below (assume the stress distribution as 1 H : 2V from the edge of the footing and γ_w = 10 kN/m^3) is



(a) 32.78 mm
(b) 61.75 mm
(c) 79.5 mm
(d) 131.13 mm

Statement for Linked Answer Questions 53 and 54:
The average effective overburden pressure on 10 m thick homogeneous saturated clay layer is 150 kPa. Consolidation test on an undisturbed soil sample taken from the clay layer showed that the void ratio decreased from 0.6 to 0.5 by increasing the stress intensity from 100 kPa to 300 kPa (G = 2.65).

53. The initial void ratio of the clay layer is
(a) 0.209
(b) 0.563
(c) 0.746
(d) 1.000

54. The total consolidation settlement of the clay layer due to the construction of a structure imposing an additional stress intensity of 200 kPa is
(a) 0.10 m
(b) 0.25 m
(c) 0.35 m
(d) 0.50 m

Instructions :
The following items consists of two statements, one labelled as 'Assertion A' and the other labelled as 'Reason R'. You are to examine these two statements carefully and decide if the Assertion A and the Reason R are individually true and if so, whether the Reason is a correct explanation of the Assertion. Select your answers to the these items using the codes given below :

(a) both A and R are true and R is the correct explanation of A
(b) both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

55. **Assertion (A):** The rates of settlement of building structures on sandy clay soil are faster compared to those of buildings constructed on clayey soils.

Reason (R): The rate of consolidation is dependent on the permeability of soils and the permeability of sandy clay is more than that of clayey soil.

56. **Assertion (A):** A clay layer of thickness. H, with single drainage suffers an ultimate settlement of S under increase in pressure of Dp. When a thin sand layer is present at its mid depth, the layer, under otherwise identical conditions, will have no change in ultimate settlements.

Reason (R): The path of travel of water for the clay layer is H under single drainage. When a thin sand layer is present at mid depth of clay layer, the path of travel in lower half of the layer gets reduced to H/2 and that in upper half of the layer gets reduced to H/4. This considerably reduces the time required to realize ultimate settlement.

57. **Assertion (A):** The field consolidation curve is steeper than the laboratory curve.

Reason (R): Sample taken for laboratory test are subjected to disturbance.

58. **Assertion (A):** Rate of settlement of a consolidating layer depend upon its coefficient of consolidation which is directly proportional to permeability of consolidating layer and the drinage path available.

Reason (R): The excess hydrostatic pore pressure is relieved fast in soil of higher permeability and number of drinage layers available in the consolidating layer.

59. **Assertion (A):** Secondary consolidation takes place at a rate much slower than that of primary consolidation.

Reason (R): There is dissipation of excess pore water pressure during secondary consolidation.

60. **Assertion (A):** Terzaghi considers in his theory of consolidation only primary consolidation and neglects secondary consolidation.

Reason (R): Secondary consolidation begins after the end of primary consolidation.

ANSWERS

1. (a)	2. (d)	3. (c)	4. (d)	5. (d)	6. (c)	7. (b)	8. (a)	9. (d)	10. (a)
11. (d)	12. (a)	13. (c)	14. (b)	15. (d)	16. (a)	17. (d)	18. (d)	19. ~~(d)~~ (b)	20. (d)
21. (c)	22. (b)	23. (b)	24. ~~(b)~~ (c)	25. (c)	26. (d)	27. (c)	28. (c)	29. (a)	30. (b)
31. (c)	32. (c)	33. (b)	34. (b)	35. (c)	36. ~~(a)~~ (c)	37. ~~(c)~~ (b)	38. (b)	39. (c)	40. (c)
41. (b)	42. (b)	43. (d)	44. (b)	45. (d)	46. (b)	47. (b)	48. (b)	49. ~~(d)~~ (c)	50. ~~(a)~~ (b)
51. (c)	52. (b)	53. (b)	54. (d)	55. (a)	56. (b)	57. ~~(a)~~ (c)	58. (a)	59. (c)	60. (b)

Hints

1. $\dfrac{\Delta H}{H} = \dfrac{\Delta e}{1+e_0}$

$\Delta H = \dfrac{.9-.71}{1.9} = \dfrac{.19}{1.9} \times 3 = 0.3\text{m}$

2. $t \propto d^2$

3. $a_v = \dfrac{\Delta e}{\Delta \bar{\sigma}}$

$a_v = \dfrac{0.7-0.6}{17.5-17} = \dfrac{.1}{.5} = .2$

7. $\dfrac{\Delta H}{H} = \dfrac{\Delta e}{1+e_0}$

$\Delta H = \dfrac{0.5}{1+1} \times 2.4$

$\Delta H = .6$; $H = 2.4 - 0.6 = 1.8$

19. $T_v = \dfrac{C_v t}{n^2}$

$t \propto T_v$

$T_v \propto v^2 \therefore t \propto v^2$

$t_2 = t_1\left(\dfrac{.6}{.4}\right)^2 = 178 \times 2.25 = 400.5$ days

26. $k = C_v m_v \gamma_w$

$\dfrac{K_A}{K_B} = \dfrac{3.6\times10^{-4}\times3.8\times10^{-8}}{1.8\times10^{-4}\times1.9\times10^{-8}} = 4$

29. $T_v = \dfrac{\pi}{4}v^2 = \dfrac{\pi}{4}\times0.5^2 = 0.196 = 0.2$

32. $T_v = \dfrac{C_v \times t}{H^2}$

$\dfrac{0.196\times5\times100^2}{0.002} = 245\times10^5 \text{ sec} = 284 \text{days}$

47. $m_v = \dfrac{a_v}{1+e_0} = \dfrac{\Delta e}{\Delta\sigma(1+e_0)}$

$\dfrac{\Delta H}{H} = \dfrac{\Delta e}{1+e_0}$

$= 2.2\times10^{-4}\times21\times3 = 0.01386$

$\Delta H = \dfrac{\Delta e}{1+e_0}\times H = 0.01386\times10\times10^3 = 138.6$ mm ≈ 139 mm

49. $\Delta\sigma = (15-5)\times10 = 100\text{kN/m}^2$

50. $\Delta H = m_v \Delta\bar{\sigma} H_0$

$= \dfrac{.25}{10^3}\times100\times5$

$= 0.125$ m $= 125$mm

51. $S.e = w.G$

$e = .2222 \times 2.7$

$e = 0.6$

$\gamma_{sat} = \dfrac{G+e}{1+e}\gamma_w = \dfrac{2.7+0.6}{1+0.6}\times10 = 20.625$

52. $\sigma_0 = \gamma'_{sand}\times2 + \gamma'_{clay}\times0.5$

$= 8\times2 + 10.65\times0.5 = 21.31$

$\Delta\bar{\sigma} = \dfrac{200}{3.5\times3.5} = 16.33$

$\Delta H = \dfrac{H_0 C_c}{1+e_0}\log_{10}\left(\dfrac{\bar{\sigma}_0+\Delta\sigma}{\bar{\sigma}_0}\right)$

$\Delta H = \dfrac{1\times0.4}{1+0.6}\log_{10}\left(\dfrac{21.31+16.33}{21.31}\right)$

$= 0.0617\text{m} = 61.7\text{mm}$

53. $C_c = \dfrac{\Delta e}{\log_{10}\left(\dfrac{\bar{\sigma}_1}{\bar{\sigma}_0}\right)} = \dfrac{0.6-0.5}{\log_{10}\left(\dfrac{300}{100}\right)} = 0.21$

$C_c = \dfrac{e_0-.5}{\log_{10}\left(\dfrac{300}{150}\right)}$

$0.21 = \dfrac{e_0-.5}{\log_{10}^{2}}$

$e_0 = 0.563$

54. $\Delta H = \dfrac{C_c\times H}{1+e_0}\log_{10}\left(\dfrac{\bar{\sigma}_0+\Delta\bar{\sigma}}{\bar{\sigma}_0}\right)$

$= \dfrac{0.21}{1+.563}\times10\log_{10}\left(\dfrac{150+200}{150}\right) = 0.50$ m

CHAPTER

9

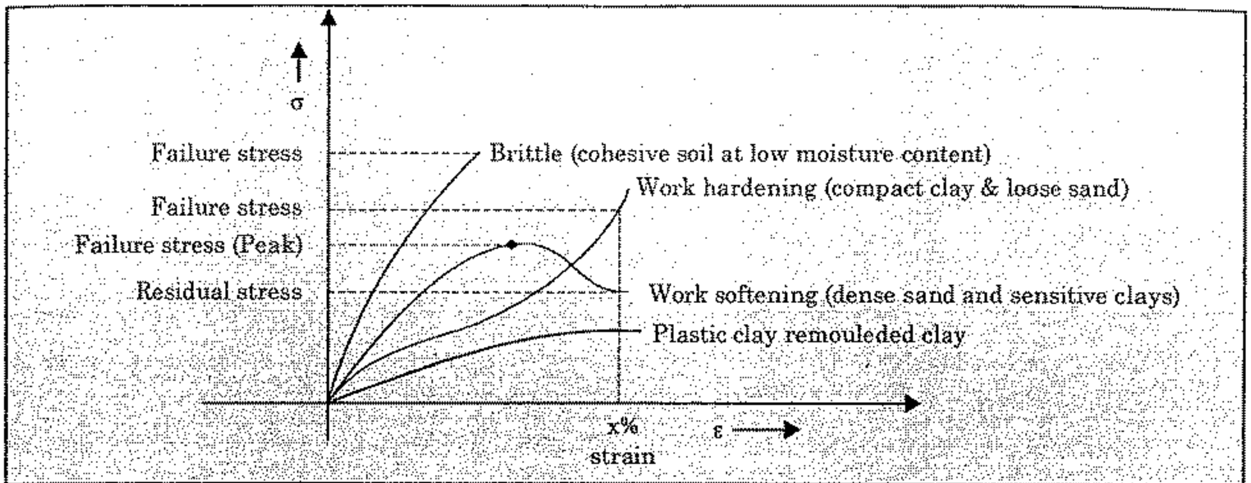
Shear Strength of Soil

INTRODUCTION

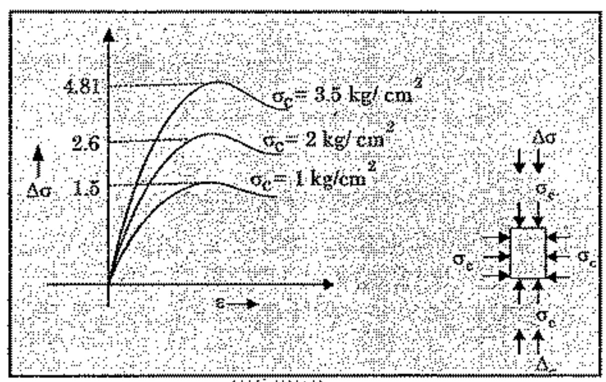
- Shear strength is the capacity to resist shear stress.
- If the value of shear stress on any plane or a surface at any point equals or exceeds the shear strength value, failure will occur in the soil mass because of the movement of a portion of soil mass along that particular plane or surface and soil is said to have failed in shear.
- Thus shear strength is a very important property of soil which keeps it in a stable equilibrium, under type of loading which produces shear stress.
- Shear strength of soil governs bearing capacity of soil, stability of slopes, Earth pressure retaining structure.

MECHANISM OF SHEAR RGESISTANCE (SHEAR STRENGTH)

- Shear strength is the resistance to shear deformation. It is categorised into two broad categories.
  (*a*) Frictional strength.
  (*b*) Cohesive strength.
- Frictional strength takes into account the particle to particle friction and also the inter-locking between particles.
- Cohesive strength takes into account
  (*a*) True cohesion between particles
  (*b*) Apparent cohesion between particles.
- To calculate the shear strength of soil and take preventive measures such that soil mass does not fail in shear, we have to define state of failure upon stressing to acertain:
  (*i*) **At what stress the failure will occur**: By knowing the stress at failure, we can design the system such that, the failure stress does not generate.
  (*ii*) **On what plane failure will occur**: By knowing the orientation of potential failure plane we can take suitable strengthening measure to prevent failure on that plane.
- To define the stress at which failure will occur, we use **stress-strain curve**. The following figure shows the stress-strain curve for various types of soils.

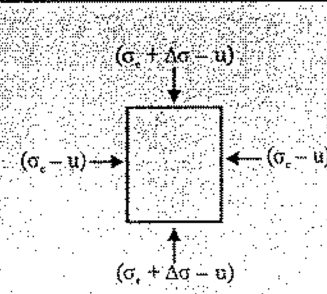


- For brittle soil, failure stress is taken corresponding to the Peak Point.
- For work softening material, failure is taken at peakpoint.
- For work hardening material or for plastic clay, failure stress shall be defined at **some % of strain**.
- Stress strain curve drawn could be Normal stress-Normal strain curve/or/Shear stress-Shear strain curve.
- In compression testing (*i.e.* in triaxial test) we plot Normal stress normal strain curve and in shear box test we plot shear-stress: shear strain curve
- Using the stress-strain curve we can derive shear stress on the potential rupture plane.
- We actually carryout 2-3 tests and from each of it we get stress-strain curve. The failure stress is noted from each stress-strain curve and is used as three different values for finding the shear strength parameter for the soil.
- The following example will illustrate how do we use the stress-strain curve to find out parameters which helps us in finding out the shear strength. let us take three samples and apply confining stress σ_c on the sample. Over this confined sample if additional axial stress $\Delta\sigma$ is applied and axial strain ε is noted, we get the curves as shown below:

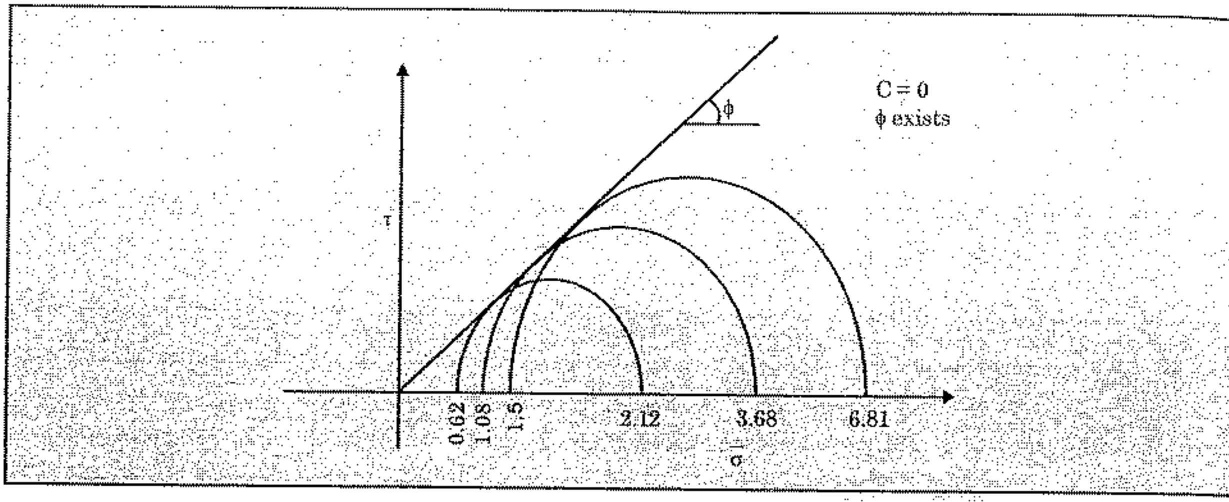


Effective stress condition at peak for each of the three tests are as shown below.

Test data no.	$\sigma_3 = \sigma_c$ minor principal stress	$\sigma_1 = \sigma_3 + \Delta\sigma = \sigma_c + \Delta\sigma$	Pore water pressure u
1	1 kg/cm^2	2.5 kg/cm^2	0.38
2	2 kg/cm^2	4.6 kg/cm^2	0.92
3	3.5 kg/cm^2	8.81 kg/cm^2	2.00



- Corresponding to these stress conditions in three sample, if mohr circles are plotted we get the mohr circles as shown below.



- If a common tangent is drawn to these three circles we get a failure envelope. The slope of failure envelope is called shear strength parameter ϕ and intercept will give the shear strength parameter 'C'.

  Shear strength is given by

$$\boxed{\tau = c + \bar{\sigma}\tan\phi}$$

  τ = effecting stress on the plane of failure.

- **Nearly all geotechnical strength analysis evaluate shear strength because whatever be the nature of loads, failure always occurs by shearing, it never occurs hy crusbing of particles.**
- Once we know the stress corresponding to failure, we need to know the plane on which failure will occur.
- To determine the failure plane and also to find out shear strength of soil, mohr-columb failure hypothesis is used.

MOHR'S HYPOTHESIS

- Shear stress on failure plane of failure reaches a value which is a unique function of normal stress on that plane.

  $\tau_{ff} = f(\sigma_{ff})$

  τ_{ff} = Shear stress on failure plane at failure. (It is also called shear strength)

  σ_{ff} = Normal stress on failure plane at failure.

COLUMB HYPOTHESIS

$$\boxed{\tau_f = C + \sigma\tan\phi}$$

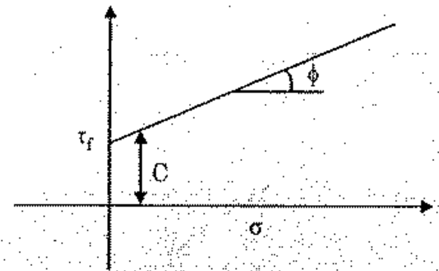
where,

τ_f = shear strength of soil

C = apparent cohesion of soil

σ = Normal stress on the plane of rupture

ϕ = angle of internal friction



Graphical representation of Coulomb equation

- C & ϕ are known as shear strength parameter of soil.
- **C & ϕ are not the inherent properties of soil. These are related to the type of test and the condition under which these are measured.**
- Normally for clayey soil $\phi = 0$, and for granular soil $C = 0$.
- Initially coulomb believed that σ = Total stress. Later on after the knowledge of effective stress it was realised that σ is actually the effective stress hence new defintion of shear strength came out.

$$\boxed{\tau_f = C' + \bar{\sigma}\tan\phi'}$$

$\bar{\sigma} = (\sigma - u)$ = effective stress

σ = total stress

u = pore water pressure

Where,

C' & ϕ' are effective stress shear strength parameter.

Hence,

$\tau_f = C + \sigma\tan\phi$ C & ϕ are total stress parameter
$\tau_f = C' + \bar{\sigma}\tan\phi'$ C' & ϕ' are effective stress parameter

Note: It is believed that frictional resistance is basically governed by normal stress between particles which in reality is governed by effective stress. Hence correct form of τ_f should be

$$\boxed{\tau_f = C' + \bar{\sigma}\tan\phi'}$$

Example 1

The subsoil at a site consists of a 10 m thick homogeneous layer of Dense Sand having the following properties

γ_d = 1.62 gm/cc G = 2.68, $\phi = 35°$

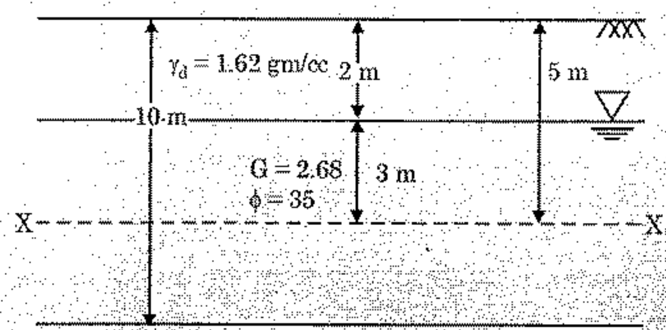
The natural ground water table lies at 2m below the ground surface

(i) Determine the shear strength of soil along a horizontal plane through the middle of sand layer.

(ii) If during mansoon the W.T rises to the ground level, how will the shear strength along the same plane change?

Assume that the soil is dry above W.T.

Sol.



We have $$\gamma_d = \left(\frac{G\gamma_w}{1+e}\right)$$

$$\Rightarrow \quad 1.62 = \frac{(2.68)\times 1}{1+e}$$

$$\Rightarrow \quad e = \left(\frac{2.68}{1.62}-1\right) = 0.654 \quad \text{... (i)}$$

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{(1+e)}$$

$$= \frac{(2.68+0.554)\times 1}{(1+0.654)}$$

$$= 2.02 \text{ gm/cc}$$

$$= 2.02 \text{ t/m}^3$$

$$\therefore \quad \gamma_{sub} = (\gamma_{sat} - \gamma_w) = (2.02 - 1) = 1.02 \text{ t/m}^3$$

The horizontal plane XX, under consideration is at a depth of 5m below the G.L

(i) The normal stress on the given plane

$$\sigma_{XX} = (\gamma_d Z_1 + \gamma_{sub} Z_2)$$
$$= (1.62 \times 2 + 1.02 \times 3)$$
$$= 6.3 \text{ t/m}^3$$

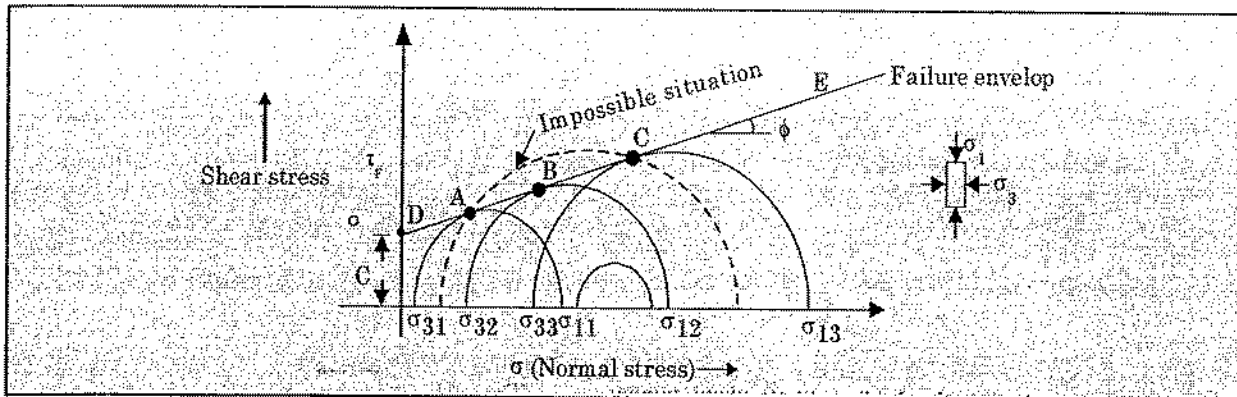
$\therefore$ Shear strength of the soil at this plane

$$\tau_{XX} = (c + \sigma \tan \phi)$$
$$= (0 + 6.3 \tan 35°)$$
$$= 4.41 \text{ t/m}^2$$

(ii) In the the IInd case the entire soil mass in submerged

$$\sigma = (\gamma_{sub} \cdot Z)$$
$$= (1.02 \times 5)$$
$$= 5.1 \text{ t/m}^2$$
$$\tau_{xx} = 0 + 5.1 \tan 35°$$
$$= 3.57 \text{ t/m}^2$$

- **Combining the Mohr & Coulomb criteria** it can be shown that it a series of Mohr's circles are drawn corresponding to different tests, carried out on different samples of a soil upto failure then a common tangent drawn on these circles represents a failure envolope. ie. failure will occure on a plane on which normal streses and shear stress is corresponding to the co-rdinates of the mohr-columb failure envelope.

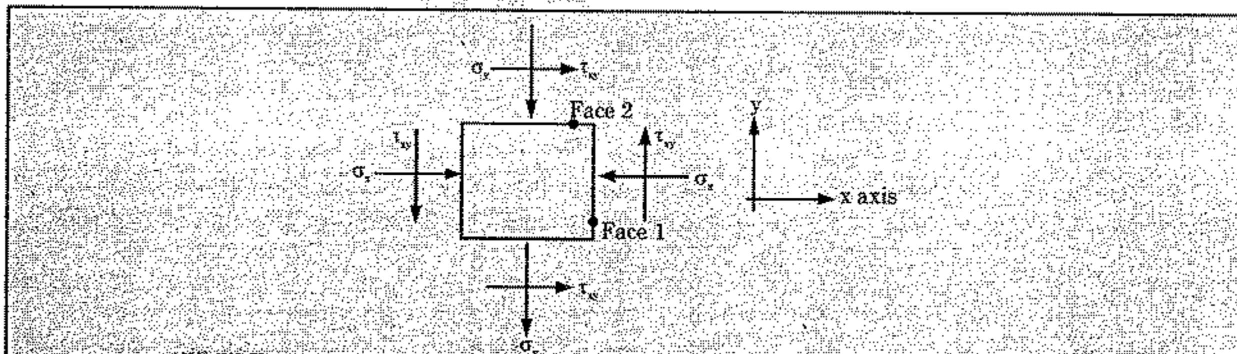


Note that there can not be a mohr circle which crosses the mohr failure envelope because there cannot be normal & shear stress inside the soil mass which gets plotted above failure envelope. This is because failure with occure in the soil mass before we reach a stress condition which gets plotted above failure envelope.

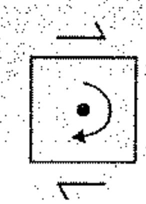
- Point A, B & C represents failure plane the orientation of these planes can be determined by usual Mohr-circle analysis. Thus failure plane also becomes known to us.

PROCEDURE FOR DRAWING MOHR CIRCLE

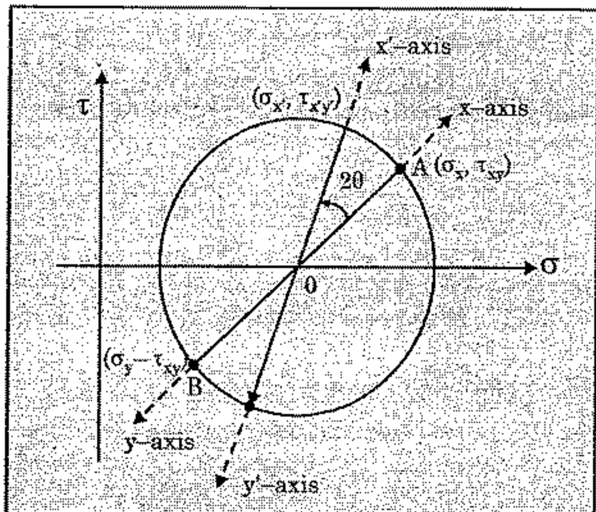
For the stress condition shown below the mohr circle can be drawn as follows :

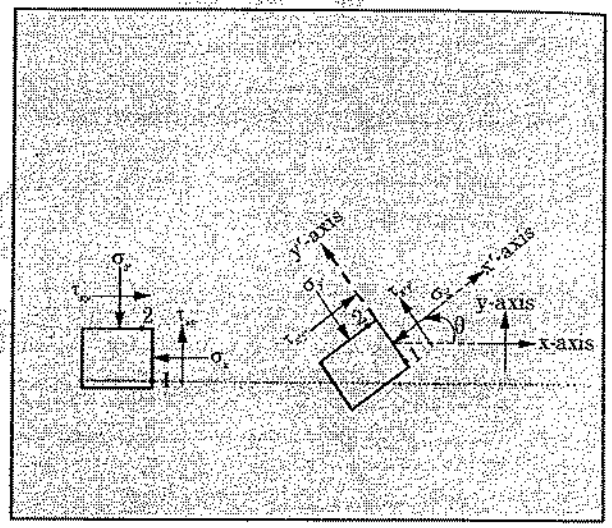


Sign convention

1. Compressive stress → +ve
2. Tensil stress → –ve
3.  Shear producing anti clockwise moment about centre of the element is taken as ⊕ve.
4.  Shear producing clockwise moment about centre of the element is taken as ⊖ve.

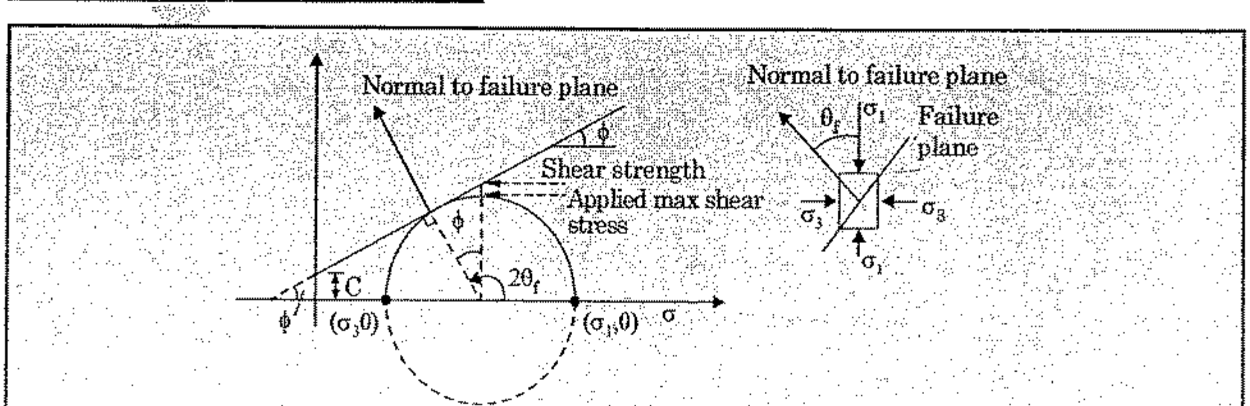
1. On a rectangular σ–τ axis, plot points having co-ordinates corresponding to **face 1** and **face 2** adoping the sign convention of mohr circle discussed above. In this case **face 1** has co-ordinates (σ_x, τ_{xy}) and face 2 has co-ordinate $(\sigma_x - \tau_{xy})$
2. Joint the points plotted by straight line. This line is the diameter of a circle whose centre is on the σ–axis (The line joining the two points will be a diameter only when the two points correspond to two faces which are perpendicular to each other)
3. As different planes are passed through the selected points in a stressed body, the normal & shearing stress components on these planes are represented by the co-ordinates of points whose position shifts around the circumference of mohr's circle.
4. The line joining centre of the circle to any point on its circumference represents the axis directed normal to the plane whose stress components are given by the co-ordinate of that point. Thus normal to the plane on which (σ_x, τ_{xy}) is acting i.e., normal to **face 1** is represented by line joining O & A in the following figure :





5. The angle between the radii to selected points on mohr circle is twice this angle between the normals to the actual planes represented by these points. The rotational sense of this angle corresponds to the rotational sense of actual angle between the normals to the planes. ie. if x'–axis is at an angle θ in anticlockwise direction from x-axis, then on mohr circle x'–radius is laid off at an anticlockwise angle 2θ from the x-radius.

RELATION BETWEEN A ANGLE OF FAILURE PLANE ANDE ANGLE OF SHEARING RESISTANCE (F)



σ_1 = major priciple stress

σ_3 = minor priciple stress

From the above diagram it is clear that

$2\theta_f = 90 + \phi$

$$\boxed{\theta_f = 45 + \frac{\phi}{2}}$$

- ***i.e.* the failure plane makes an angle of $\left(45 + \frac{\phi}{2}\right)$ degree with the major principal plane.**
- Thus knowing the ϕ value of the soil, potential rupture plane can be identified.
- It should be observed that failure does not occur on plane of max shear stress. Because available shear strength is greater than the max shear stress.

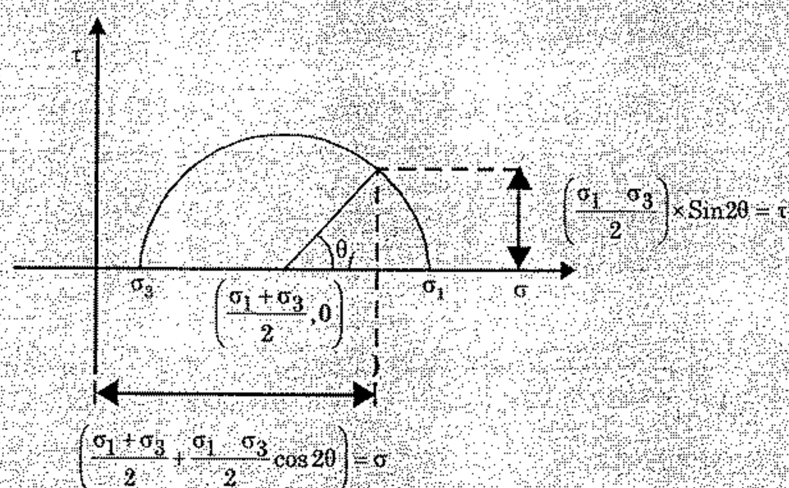
Example 2

Prove that failure plane is the one on which difference of shear stress and shear strength is min.

If η = (Shear stress – Shear strength)

$\eta = [\tau] - [C + \sigma \tan\phi]$

Sol.



$$\eta = \left(\frac{\sigma_1 - \sigma_3}{2}\sin 2\theta\right) - \left[C + \left(\frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos 2\theta\right)\tan\phi\right]$$

For η to be minimum,

$$\frac{d\eta}{d\theta} = 2\times\left(\frac{\sigma_1 - \sigma_3}{2}\right).\cos 2\theta - 2\times\left(\frac{\sigma_1 - \sigma_3}{2}\right)\times(-\sin 2\theta)\tan\phi = 0$$

$$\frac{d\eta}{d\theta} = (\sigma_1 - \sigma_3)[\cos 2\theta + \sin 2\theta \tan\phi] = 0$$

$\Rightarrow$ $\tan\phi = -\cot 2\theta$

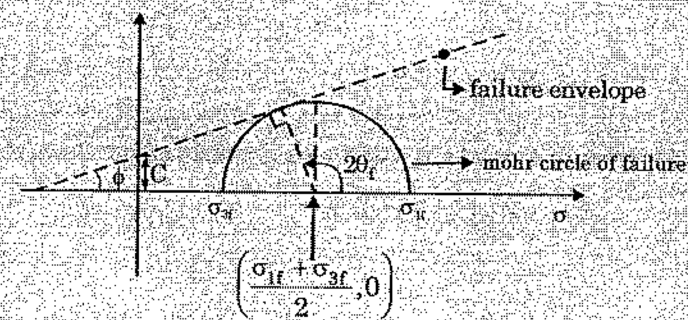
$\Rightarrow$ $2\theta = 90 + \phi$

$$\Rightarrow \qquad \boxed{\theta = 45 + \frac{\phi}{2}}$$

Thus the plane on which difference of shear stress & shear strength is minimum is given by $\theta = 45 + \dfrac{\phi}{2}$

But, $\theta = 45 + \dfrac{\phi}{2}$ is actually the θ for plane of failure as discussed above. Hence plane of failure is the one on which difference of shear stress and shear strength is minumum.

Relation between major & minor priciple stress at failure in a soil mass on the basis of mohr- columb criteria of failure.



- Radius $= \dfrac{\sigma_{1f} - \sigma_{3f}}{2}$

  Co-ordinate of center of Mohr circle is $\left(\dfrac{\sigma_{1f} + \sigma_{3f}}{2}, 0\right)$

  From the figure shown above, we have,

$$\sin\phi = \frac{\left(\frac{\sigma_{1f} - \sigma_{3f}}{2}\right)}{\left(\frac{\sigma_{1f} + \sigma_{3f}}{2}\right) + C\cot\phi}$$

$$\Rightarrow \qquad \boxed{\frac{\sigma_{1f} + \sigma_{3f}}{2}\sin\phi + C\cos\phi = \frac{\sigma_{1f} - \sigma_{3f}}{2}} \qquad \text{....(a)}$$

$$\Rightarrow \qquad \frac{\sigma_{1f}}{2}(1 - \sin\phi) = \frac{\sigma_{3f}}{2}(1 + \sin\phi) + C\cos\phi$$

$$\Rightarrow \qquad s_{1f} = \sigma_{3f}\left(\frac{1 + \sin\phi}{1 - \sin\phi}\right) + \frac{2C\cos\phi}{1 - \sin\phi}$$

$$\sigma_{1f} = \sigma_{3f}\left(\frac{1 + \sin\phi}{1 - \sin\phi}\right) + 2C\sqrt{\frac{1 - \sin^2\phi}{(1 - \sin\phi)^2}}$$

$$\boxed{\sigma_{1f} = \sigma_{3f}\left(\frac{1 + \sin\phi}{1 - \sin\phi}\right) + 2C\sqrt{\frac{1 + \sin\phi}{1 - \sin\phi}}} \qquad \text{....(A)}$$

Similarly,

$$\boxed{\sigma_{3f} = \sigma_{1f}\left(\frac{1-\sin\phi}{1+\sin\phi}\right) - 2C\sqrt{\frac{1-\sin\phi}{1+\sin\phi}}} \quad \text{....(B)}$$

As we know that

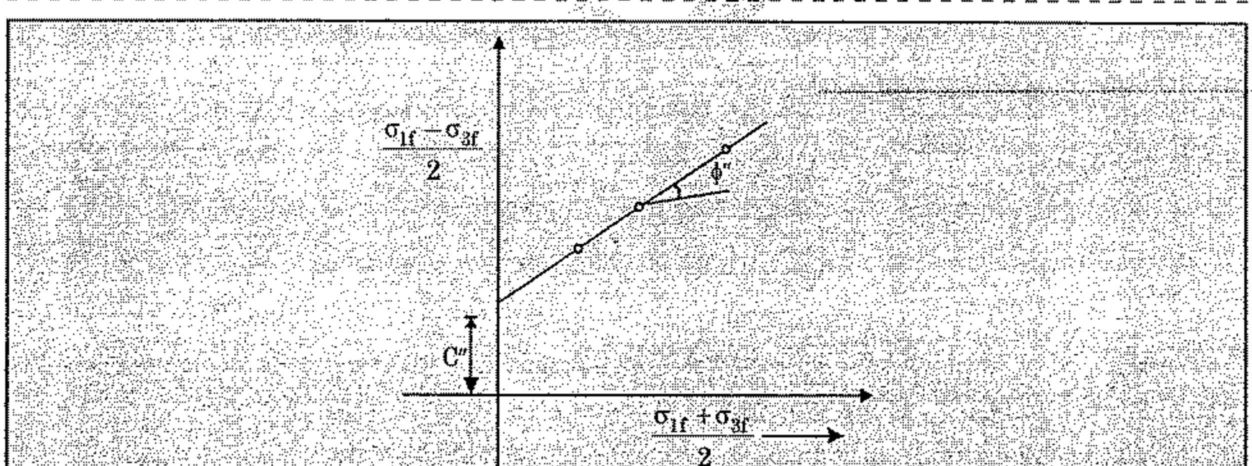
$$\frac{1-\sin\phi}{1+\sin\phi} = \tan^2\left(45-\frac{\phi}{2}\right)$$

$$\frac{1+\sin\phi}{1-\sin\phi} = \tan^2\left(45+\frac{\phi}{2}\right)$$

$$\Rightarrow \quad \boxed{\sigma_{1f} = \sigma_{3f}\tan^2\left(45+\frac{\phi}{2}\right) + 2C\tan\left(45+\frac{\phi}{2}\right)} \quad \text{....(C)}$$

$$\boxed{\sigma_{3f} = \sigma_{1f}\tan^2\left(45-\frac{\phi}{2}\right) - 2C\tan\left(45-\frac{\phi}{2}\right)} \quad \text{....(D)}$$

Note: From equation (a) it is clear that a straight line can be plotted by taking $\frac{\sigma_{1f}-\sigma_{3f}}{2}$ on y-axis and $\frac{\sigma_{1f}+\sigma_{3f}}{2}$ on x-asis as shown below :



where,

$C'' = C\cos\phi$

$\phi'' = \tan^{-1}(\sin\phi)$

- **This plot is called P–q plot.** This curve also estabilishes a relation between σ_{1f} & σ_{3f}.

MAXIMUM OBLIQUITY RELATIONSHIP

- Angled of obliquity (δ) is defined as the angle between resultant stress on a plane and normal stress on the plane.
- For the stress condition shown in the figure (i) below, angle of obliquity (δ) for plane AB is shown in the figure (ii).

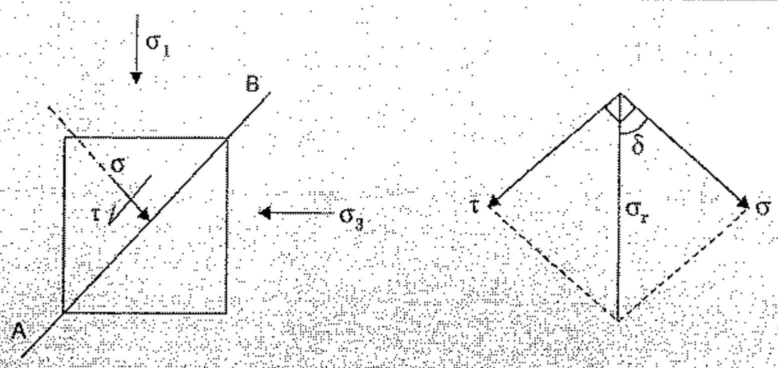


fig. (ii) fig. (ii)

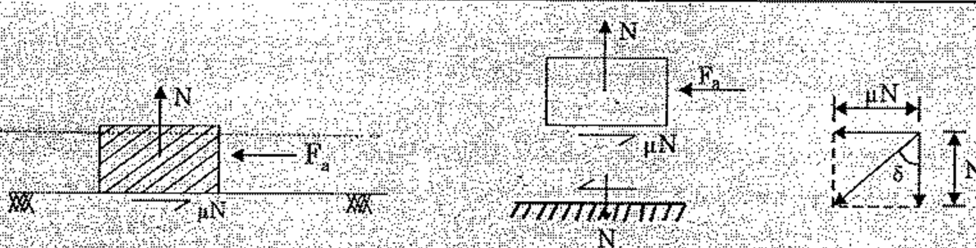
Note that

$\tan\delta = \dfrac{\tau}{\sigma}$

δ = angle of obliquity.

σ_r = resultant stress on the plane.

- Significance of angle of obloiquity has been described here by a mechanical model in which a soilid block has been placed over a surface.



$$\tan\delta = \frac{\mu N}{N} = \mu \text{ (co-efficient of friction)}$$

- As a force F_a is applied to the body trying to slide the body, a frictional force μN develops on the body trying to resist the tendency of sliding.
- As F_a is increased, μ will go on increasing. The sliding will start when μ becomes equal to coefficient of static friction. Under that situation, μ is max. (*i.e.* tan δ is max). when tan δ is maximum, δ is maximum. This maximum δ i.e., δ_{max} is called angle of friction ϕ. Note that δ is also the angle of obliquity.
- Comparing this with the stress situation in a soil mass having frictional nature of shear resistance, failure/sliding will occur **wben angle of obliquity is max.**

Example 3

Prove using mohr circle approach that angle of maximum obliquity is the criteria of failure for cohesion less soil.

Sol: We know that angle of obliquity is given by $\tan\delta = \dfrac{\tau}{\sigma}$

$$\frac{\tau}{\sigma} = \frac{\frac{\sigma_1-\sigma_3}{2}\sin 2\theta}{\frac{\sigma_1+\sigma_3}{2}+\frac{\sigma_1-\sigma_3}{2}\cos 2\theta}$$

$$= \alpha \text{ (say)}$$

For α to be max, $\frac{d\alpha}{d\theta}=0$

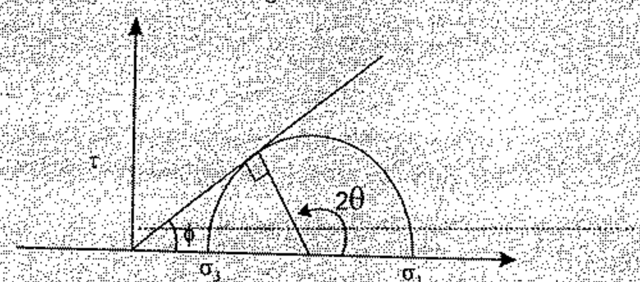
$$= \left(\frac{\sigma_1+\sigma_3}{2}+\frac{\sigma_1-\sigma_3}{2}\cos 2\theta\right)\times\left(\frac{\sigma_1-\sigma_3}{2}\right)2\cos 2\theta$$

$$-\left(\frac{\sigma_1-\sigma_3}{2}\sin 2\theta\right)\left(\frac{\sigma_1-\sigma_3}{2}\right)(-2\sin 2\theta)=0$$

$$\Rightarrow \frac{\sigma_1+\sigma_3}{2}\cos 2\theta+\frac{\sigma_1-\sigma_3}{2}=0$$

$$\frac{\sigma_1-\sigma_3}{2}=\frac{-(\sigma_1+\sigma_3)}{2}\cos 2\theta \quad \text{...(B)}$$

From Mohr-Columb criteria for soil having $C = 0$



$$\sin\phi = \frac{(\sigma_1-\sigma_3)/2}{\left(\frac{\sigma_1+\sigma_3}{2}\right)}=\frac{(\sigma_1-\sigma_3)}{(\sigma_1+\sigma_3)}$$

$$\Rightarrow (\sigma_1+\sigma_3)\sin\phi=(\sigma_1-\sigma_3)$$

$\Rightarrow$ $(\sigma_1+\sigma_3)\cos 2\theta+(\sigma_1+\sigma_3)\sin\phi=0$ (from equation (B))

$$\Rightarrow \cos 2\theta+\sin\phi = 0$$

$$\Rightarrow -\sin\phi = \cos 2\theta$$

$$\cos(90+\phi) = \cos 2\theta$$

$$2\theta = 90+\phi$$

$$\boxed{\theta=45+\frac{\phi}{2}}$$

Thus point of tangency on the Mohr circle from Mohr Columb criteria represents the plane on which angle of obliquity is max.

Max angle of obliquity governs the failure plane in a soil having $c=0$

$$\boxed{\frac{\sigma_{1f}}{\sigma_{3f}}=\frac{1+\sin\phi}{1-\sin\phi}=\tan^2\left(45+\frac{\phi}{2}\right)}$$

The above relation is obliquity relationship where s_1 & s_3 are principal stress under failure loading.

Example 4

A soil sample is subjected to a major principal stress of 2 kg/cm^2 and minor pricipal stress of 1.1 kg/cm^2. Determine the normal and shear stress acting on a plane at 30° to the major principal stress.

Sol. The normal stress σ, and the shear stess τ on any plane Inclined at θ to the major principal plan is given by

$$\sigma = \left(\frac{\sigma_1+\sigma_3}{2}\right)+\left(\frac{\sigma_1-\sigma_3}{2}\right)\cos 2\theta \quad \text{... (i)}$$

$$\tau = \frac{(\sigma_1-\sigma_3)}{2}\sin 2\theta \quad \text{... (ii)}$$

Data given

$$\sigma_1 = 2 \text{ kg/cm}^2$$

$$\sigma_3 = 1.1 \text{ kg/cm}^2$$

Note: *The given plane is inclined at 30° to the major principal stress. But the direction of major principal stress is perpendicular to the major principal plane.*

Therefore the angle of inclination between the given plane and the major principal plane is,

$$\theta = (90° - 30°) \quad \text{... (iii)}$$

$$\sigma = \left(\frac{\sigma_1+\sigma_3}{2}\right)+\left(\frac{\sigma_1-\sigma_3}{2}\right)\cos 2\theta$$

$$= \left(\frac{2+1.1}{2}\right)+\left(\frac{2-1.1}{2}\right)\cos 120°$$

$$= 1.55 + 0.45 \cos 120°$$

$$= 1.325 \text{ kg/cm}^2$$

$$\tau = \left(\frac{\sigma_1-\sigma_3}{2}\right)\sin 2\theta = \left(\frac{2-1.1}{2}\right)\sin 120°$$

$$= 0.45 \sin 120°$$

$$= 0.39 \text{ kg/cm}^2$$

DETERMINATION OF C & F WITH EFFECTIVE STRESS ANALYSIS & TOTAL STRESS ANALYSIS

- We know that C and ϕ are not the inherent properties of soil. These are related to the type of test & condition under which tests are performed.
- The tests performed to determine C and ϕ and hence shear strength of soil, must simulate the field conditions.
- **The field conditions could be**
  (a) Rapid or slow construction and hence rapid or slow application of load causing shear.
  (b) Drained and undrained conditions.
  (c) Types of soil, granular or clayey.

- We have already discussed in previous chapters that when load is applied to the soil, all of the load is not taken up by the soil grains immediately. Load is Ist of all borne by the pore water and thus *excess pore water pressure* develops. This excess pore water pressure slowly dissipates as water flows out of the soil and thereby the load gets transferred to the soil grains. This transfer of load to the soil grains causes effective stress to increase.
- If the rate of loading in a soil is such that water in the pores of soil gets sufficient time to drain out, the condition of loading is called **drained condition**.
- If however, the rate of loading is rapid such that the water in the pores doesnot get sufficient time to drainout, the condition of loading is called **undrained condition**.
- Drained and undrained conditions basically depends on the type of loading (Rapid or slow) and soil type. **These conditions are actually relative.**
- In granular soil if loading is slow/normal, drained conditions prevail. If loading is quite rapid, then even in granular soils undrained condition may prevail.
- In clayey soil, if loading is normal, undrained condition prevail. However, if loading is very slow drained condition may prevail.
- Under undrained condition excess pore water pressure will develop but under drained condition excess pore water pressure will not develop.
- Under drained condition, to determine the shear strength, we perform **effective stress analysis.**

$$\boxed{\tau_f = C' + \sigma' \times \tan\phi'}$$

- Effective stress used in field in this case will be given by (effective stress = Total stress– hydrostatic pore water pressure) because under drained condition excess pore water pressure doesnot develop hence pore water pressure used for analysis under drained condition is hydrostate pore water pressures.
- How ever if we use σ′ in lab for analysis, we use (effective stress = Total stress) because hydrostatic pressure in lab sample will be zsero.
- Thus, if loading condition in field is such that no excess pore water pressure develops, the test should be performed in the lab in such a way that no pore water pressure develops during testing. Thus we simulate the field condition.

TOTAL STRESS ANALYSIS

- Effective stress analysis can be performed when we can predict effective stress in the field. This is a simple case when pore water pressure present in field is hydrostatic. (*i.e.* drained condition). This is because hydrostate pore water pressure is easy to calculate. However, this analysis becomes complex when excess pore water presure is present. (*i.e.* undrained condition) because determination of excess pore water pressure in field is a difficult task.
- Excess pore water pressure develops due to both induced normal stress and induced shear stress.
- Pore water pressure at any time is given by

$$\boxed{u = u_h + (u_e)_{normal} + (u_e)_{shear}}$$

Where,

u_h = hydrosatic pore water pressure

$u_{e\ normal}$ = excess pore water pressure due to change in normal stress = U_e (Normal)

$u_{e\ shear}$ = excess pore water pressure due to shearing U_e (sear)

$u_{e\ normal} > 0$ if σ increases

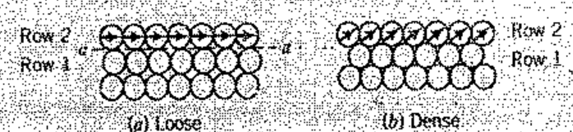
$U_{e\ normal} < 0$if σ decreases

$u_{e\ shear} > 0$, if soil tends to compress when sheared and drainage is not allowed.

$u_{e\ shear} < 0$, If soil tends to dilate when sheared and drainage is not allowed (*i.e.* water is not allowed to get in.) $u_{e\ normal}$ dominates over $u_{e\ shear}$. Hence if $\sigma > 0$, u_e is (+)ve and if $\sigma < 0$, u_e is (−)ve

- Loose sand and Normally Consolidated clays—Compresses when sheared, specially if stress is high.
- Dense sand & Over Consolidated clays—Dilates when sheared specially **when σ is less.**

Note: Following figure illustrates the compression & dilation on shearing



(a) Loose (b) Dense

- When loose state is sheared as shown in figure (a) by horizontal arrow, it tries to achieve the configuration as shown in fig. (b). Thus it tries to be denser. In this process compression occurs.
- When dense state is sheared as shown in figure (b), it tires to achieve the configuration in loose state as shown in figure (a). Thus, dense state when sheared tries to dilate.

- In the field it is difficult to calculate the excess pore water pressure. **Hence analysis in Lab is done based on total stress approach, instead of effective stress approacb.**
- Thus for undrained condition, total stress approach is used and shear strength parameter are obtained corresponding to total stress approach.

$$\boxed{\tau_f = C_T + \sigma \tan\phi_T}$$

where,

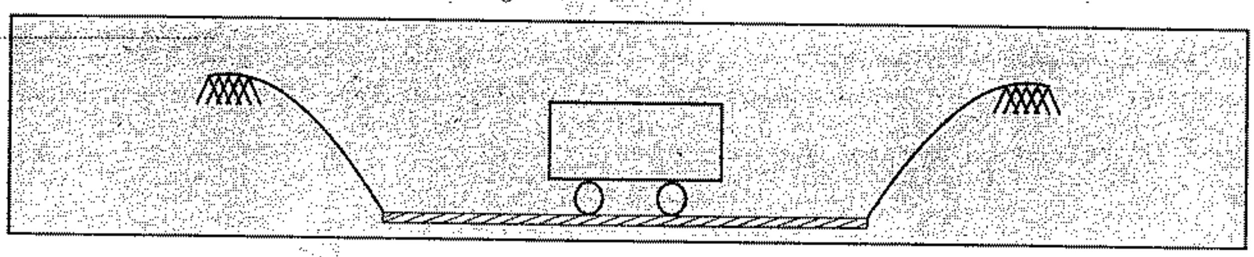
σ = total stress.

- C_T & ϕ_T are total stress shear strength parameters. C_T & ϕ_T in lab will be calculated and will be used in field conditions.
- Total stress analysis assumes that the excess pore water pressure developed in lab are the same as those in the field and its effect is implicity incorporated in C_T & ϕ_T. This assumption introduces some error in analysis, but there is no option. More accuracy will be achieved if lab test is performed in a way that simulates the field conditions as closely as possible.
- If somehow pore water pressure in the field can be measured during construction by installing peizometer, we can carryout effective stress analysis in the lab and C' & ϕ' found can be used in field.

ADDITIONNAL DISCUSSION ON C & ϕ VALUES

- Shear strength check in the soil must be carried out under most critical condition of failure. Hence shear strength parameters C & ϕ must be evaluated in the laboratory for that most critical conditions.

- For example, when loading condition and soil type is such that drained condition prevail, effective stress generated and applied load is such that the factor of safety against shear failure is least post constructrion
- *Hence for drainid condition, effective stress approach is used and post-construction stability is checked.*
- Thus, C & ϕ calculation in laboratory should be carried out in the effective stress range expected in post construction condition.
- Drained strength analysis should also be used to evaluate shear strength in soil in which excess pore water pressure has already dissipated. Thus for *longterm stability chek*, Drained strength analysis is done.
- In case of loading under undrained condition +ve pore water pressure develops. For example below earthern fills or below structureal foundation in clays, +ve pore water pressure develops under undraind condtion. *Under such situation, effective stress decreases due to increase in excess pore water pressure.* This excess pore water pressure slowly dissiptes and hence effective stress increases with time. This leads to gain in shear strength with time. On account of this most critical condition of shear failure occurs immidiately after construction.
- Thus, under **undrained condition with +ve pore water pressure**, stability should be checked immidiately after construction and Total stress analysis should be used.
- If under undrained condition, –ve pore water pressure develops, as in case of excavation done for road construction in which unloading leads to



generation of –ve pore water pressure, effective stress initially will be more. but as time passes this excess negative pore water pressure causes sucking of water and thereby it dessipates causing the effective stress to decrease with time. Under such situations, stability should be checked under long term condtion.

Thus for **undrained condtion with –ve pore water pressure** long **term stability should be checked and total stress analysis should be adoped to calculate requisite C & ϕ value.**

Conclusion

(1) Shear strength under drained condition:

- Drained analysis is used to evaluate long term stability.
- Effective stress apprach used.

(2) Shear strength under undrained condition, when +ve excess pore water pressure develops

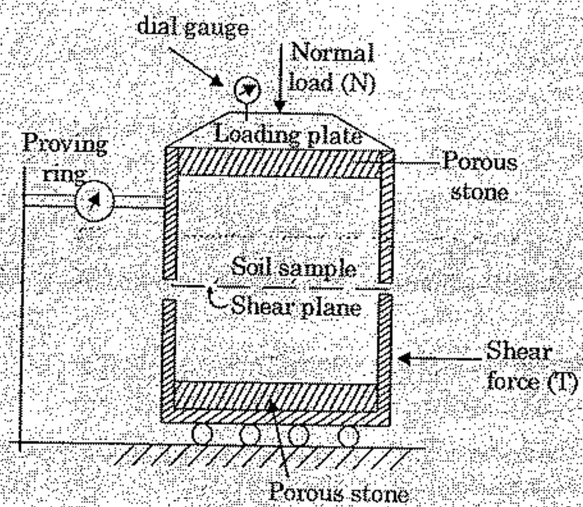
- We evaluate shear strength at the end of construction period.
- Total stress approach is used.

(3) Shear strength under undrained condition with (–)ve excess pore water pressure.

- Long term stability is analysed.
- As all pore water could have dispated in long term, effective stress approach is used with pore pressure as hydrostatic pore pressure.

VARIOUS LABORATORY TEST PERFORMED TO FIND OUT C & Φ

Direct shear test



* Volume expansion is measured using dial gauge

- Vertical load is applied through loading plate.
- Soil is sheared gradually by applying horizontal force.
- Shear is normally applied at **constant rate of strain.**
- Magnitude of shear load is measured by proving ring.
- **Shear deformation & Vertical deformation** are measured during test using dial gauge.
- Shear stress & Normal stress are found out by dividing the shear & normal load by **nominal area of specimen.**
- As drainage cannot be controlled in this test hence rate of loading should be such that pore water pressure does not develop i.e. it will be a drained condition testing.
- This test is good for free draining soil like sand & gravel.
- This test cannot be used for clays because drainage cannot be controlled.
- Effective stress & Total stress are same in this test.
- As the specimen fails along a predetermined plane, hence this test is useful for conditions where soil has predetermined faults/joints etc.

Advantages:

(1) quick, inexpensive & simple.

(2) easy to prepare sample.

Disadvantages:

(1) Drainage condition cannot be controlled & pore water pressure can not be measured.

(2) Failure plane is always horizontal and pre determined and which may not be the weakest plane.

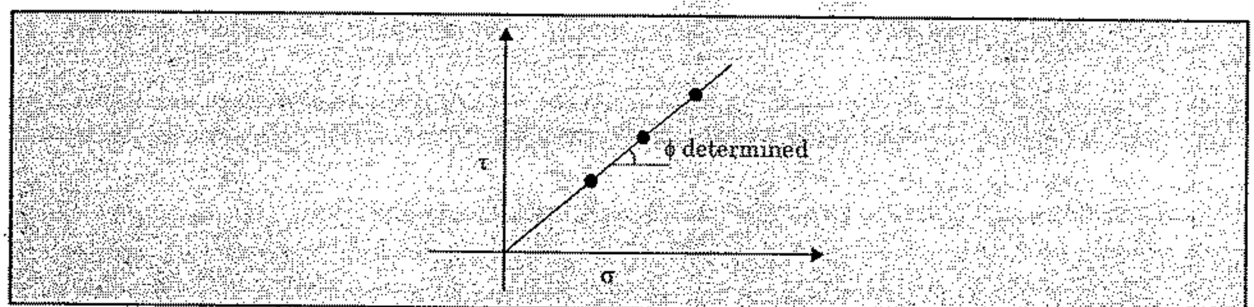
(3) Non uniform stress distribution on shear plane. Failure starts at edge & progresses towards centre.

(4) Area of specimen under Normal & shear does not remain constant during the test. Hence, calculation of Normal & shear stresses are done on the basis of Nominal area (original area) which is not correct.

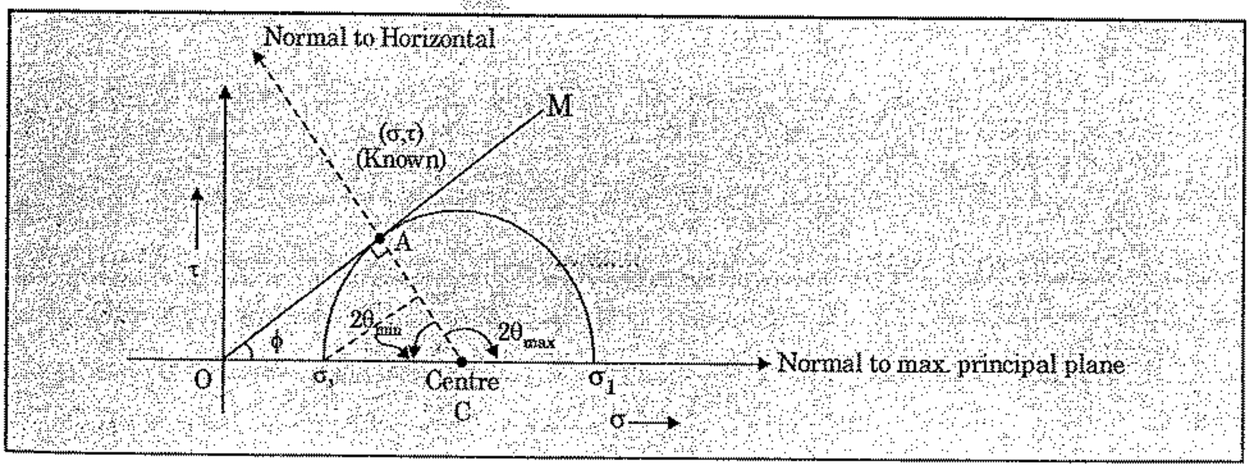
(5) Direction of principal planes are not known at every stage of the test. It is only when Mohr failure envelope is known that direction of principal stresses will be known. In fact there is a rotation of principal plane between the start of the test and failure of soil.

RESULTS OF THE TEST

- A number of specimens of soil are tested, each under a different vertical force, and value of shear stress at failure is plotted against normal stress for each test.
- Shear strength parameter are then obtained from best line fitting method from the plotted points.
- At least three tests are required to draw a line properly. Even if the line does not passes through the origin we forcibly pass it through origin because $C = 0$ (under drained condition cohesion does not get mobilized).



Mohr Circle for Direct Shear Test



Principal stresses are found out as follows:

- $A\ (\sigma,\ \tau)$ known $\Rightarrow$ OM line known.
- draw $\perp^r$ to line OM, it cuts σ-axis at C.
- from C as centre and AC as radius find σ_3 & σ_1.
- Major principal plane is at an angle $2\theta_{max}$ from horizontal and minor principal stress will be $\perp^r$ to major principal plane.

Example 4

A Direct shear box test on a remoulded sample of sand gave the following observation at the time of failure,

Normal load = 288 N

Shear load = 173 N

Cross sectional area of sample = 36 cm²

Determine

(*a*) Angle of internal friction

(*b*) Magnitude and direction of principal stress in the zone of failure

Sol. Data given:

Normal Load = 288 N

Shear Load = 173 N

C/S area of sample = 36 cm²

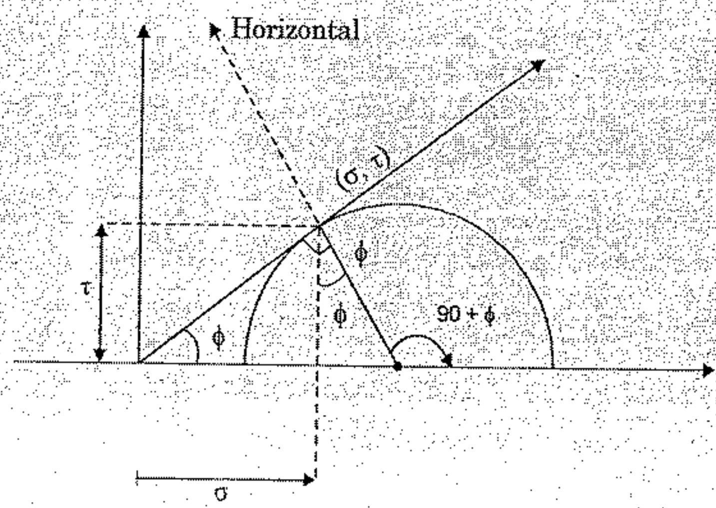
$$\tau_f = \frac{\text{Shear load}}{\text{C/S area of sample}}$$

$$= \frac{173 \times 10^{-3}}{(36 \times 10^{-4})} = 48.05 \text{ kN/m}^2$$

$$\sigma = \frac{\text{Normal load}}{\text{C/S area of sample}}$$

$$= \frac{288 \times 10^{-3}}{(36 \times 10^{-4})} = 80 \text{ kN/m}^2$$

From Mohr's circle diagram



$$\tan\phi = \left(\frac{\tau_f}{\sigma}\right)$$

$$\Rightarrow \qquad \phi = \tan^{-1}\left(\frac{48.05}{80}\right)$$

$$= (30.99)^\circ \qquad \ldots (i)$$

$$2\theta_f = \left(\frac{\pi}{2}+\phi\right)$$

$$\Rightarrow \qquad \theta_f = \left(\frac{\pi}{4}+\frac{\phi}{2}\right)$$

$$= \left(45+\frac{30.99}{2}\right) = (60.5)^\circ \qquad \ldots (ii)$$

We have

$$\sigma = \left(\frac{\sigma_1+\sigma_3}{2}\right)+\left(\frac{\sigma_1-\sigma_3}{2}\right)\cos(2\theta_f)$$

$$\Rightarrow \qquad 80 = \left(\frac{\sigma_1+\sigma_3}{2}\right)+\left(\frac{\sigma_1-\sigma_3}{2}\right)\cos(2\times 60.5)$$

$$\Rightarrow \qquad 160 = (\sigma_1+\sigma_3) - 0.515\,(\sigma_1-\sigma_3)$$

$$0.485\,\sigma_1 + 1.515\,\sigma_3 = 160 \qquad \ldots (iii)$$

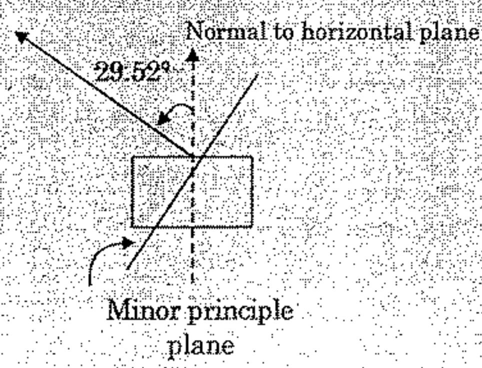
Again,

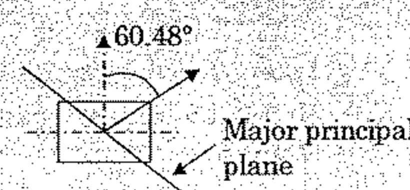
$$\tau_f = \left(\frac{\sigma_1-\sigma_3}{2}\right)\sin(2\times 60.5)$$

$$\Rightarrow \qquad 48.05\times 2 = 0.857\,\sigma_1 - 0.857\,\sigma_3$$

$$\Rightarrow \qquad 0.857\,\sigma_1 - 0.857\,\sigma_3 = 96.1 \qquad \ldots (iv)$$

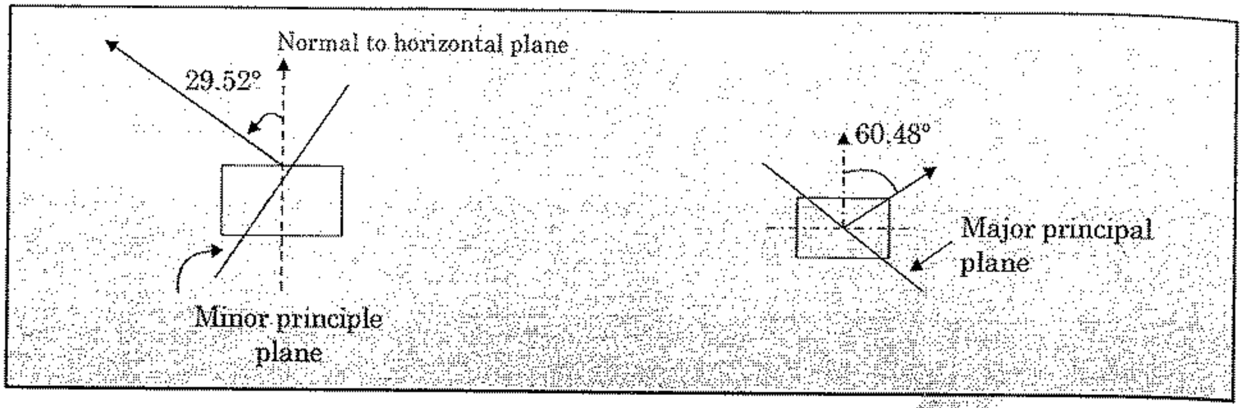
Solving (*iii*) and (*iv*) we have



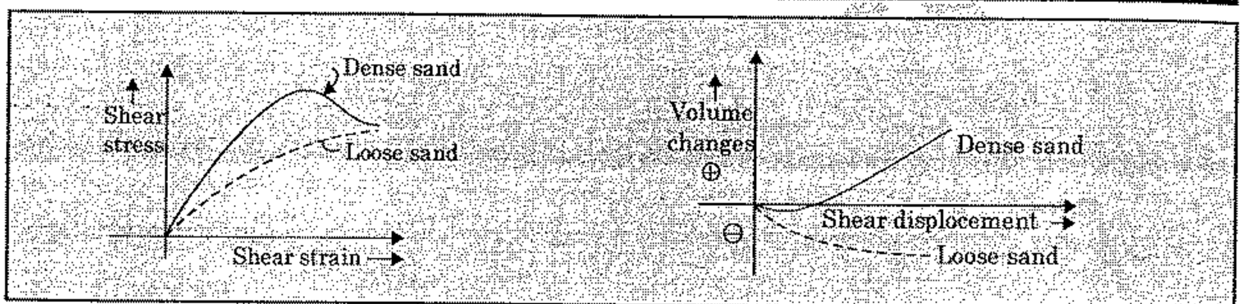


$$\Rightarrow \qquad \sigma_1 = 164.94 \text{ kN/m}^2$$

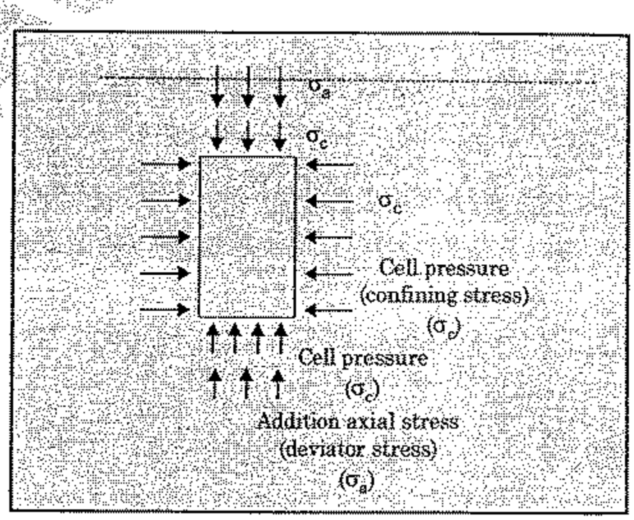
$$\sigma_2 = 52.81 \text{ kN/m}^2$$



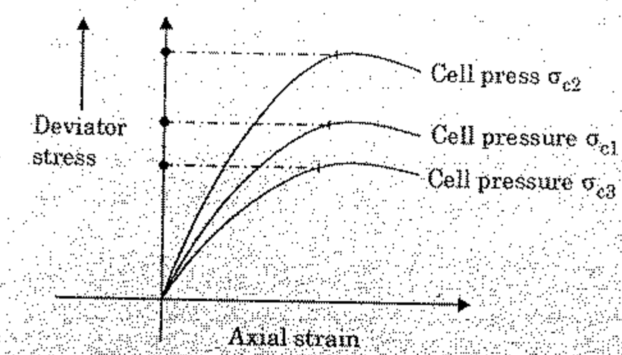
THE OTHER RESULTS THAT ARE OBTAINED IN DIRECT SHEAR TEST ARE:



Triaxial test



- This is the most widely used shear strength test and is suitable for all types of soil.
- Drainage can be controlled what ever be the type of soil i.e. sand can be tested under undrained conditions and clay can be tested under drained condition also. .
- Pore water pressure can be measured.
- Volume changes can also be measured.
- Failure plane is not pre determined.
- There is no rotation of principal stresses during test.
- Stress distribution on failure plane is fairly uniform.
- The trixial cell is filled with water and specimen is sealed inside a rubber membrane. Cell pressure is applied (called confining pressure) (σ_c).
- With cell pressure held, constant, additional axial stress is applied gradually untill sample fails at additional axial stress value of (σ_a).
- Application of additional axial stress, also called deviator stress, prodcues shear stresses with in soil mass on all planes except horizontal & vertial planes.
- Horizontal plane becomes major principal plane with major principal stress $\sigma_{1f} = \sigma_a + \sigma_c$.
- Vertical plane becomes minor principal plane with minor principal stress $\sigma_{3f} = \sigma_c$.
- In triaxial test **axial strain** and **deviator stress** is determined.



- By measuring change in height, axial strain is found out

$$\boxed{\varepsilon_a = \text{axial strain} = \frac{-dh}{h_o}}$$

- By measuring change in height and change in volume area of the specimen can be measured:

$$\boxed{\text{deviator stress } \sigma_a = (\sigma_1 - \sigma_3) = \frac{\text{Axial load}}{\text{corrected area}}}$$

Corrected area is calculated as follows:

- If specimen is assumed to remain cylindrical than

$$\boxed{A(h_o + dh) = V = V_o + dv}$$

$$A = \frac{V_0\left(1+\frac{dv}{v_0}\right)}{h_0\left(1+\frac{dh}{h_0}\right)}$$

$$A = \frac{A_0\left(1+\frac{dv}{v_0}\right)}{\left(1+\frac{dh}{h_0}\right)}$$

For axial compression and volume reduction

$$\boxed{A = \frac{A_0(1-\varepsilon_v)}{(1-\varepsilon_a)}}$$

For undrained test, $dv = 0$, as no pore water is dissipated hence no volume change occurs

$$\boxed{A = \frac{A_0}{1-\varepsilon_a}}$$

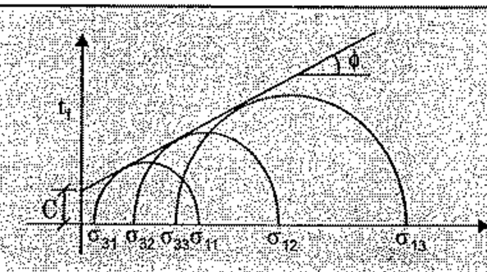
where,

$$\boxed{\frac{1}{1-\varepsilon_a} = \text{area correction}}$$

Ao = original area

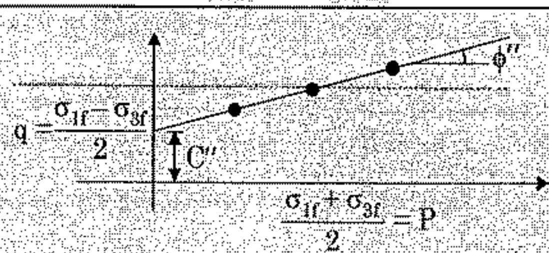
A = corrected area

- By performing tests on 2-3 specimen at different cell pressure and at different deviator stress, mohr circles are polotted and shear strength parameters C **& ϕ are found out from Common tangent or by using p-q plot.**



C – ϕ determination by mohr circle

$$\frac{\sigma_{1F}-\sigma_{3F}}{2} = \frac{\sigma_{1F}+\sigma_{3F}}{2}\sin\phi + c\cos\phi$$



C & ϕ determination using p – q plot

$$C'' = C\cos\phi$$

$$\tan\phi'' = \sin\phi$$

Note: At any time in a triaxial test either pore water line is open and drainage line is closed or vice versa. i.e. either pore water pressure measurement will be made under undrained condition or Volume change is measured under drained condition.

Example 5

A sample of dry sand tested in triaxial and direct shear test in the triaxial test the sample fails when major and minor principal stresses are 960 kN/m² and 260 kN/m² respectively what shear strength would be expected in direct shear test when normal stress is 230 kN/m²

Sol. Data given

Dry sand $\Rightarrow c = 0$

$$\sigma_1 = 960 \text{ kN/m}^2$$

$$\sigma_3 = 260 \text{ kN/m}^2$$

$\therefore$ We have $\sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi}$

$\Rightarrow \quad \sigma_1 = \sigma_3 N_\phi + 2\times 0\sqrt{N_\phi}$

$\Rightarrow \quad N_\phi = \left(\frac{\sigma_1}{\sigma_3}\right) = \left(\frac{960}{260}\right)$

$\Rightarrow \quad \tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right) = \left(\frac{960}{260}\right)$

$\Rightarrow \quad \phi = (35.01)^\circ \quad \text{... (i)}$

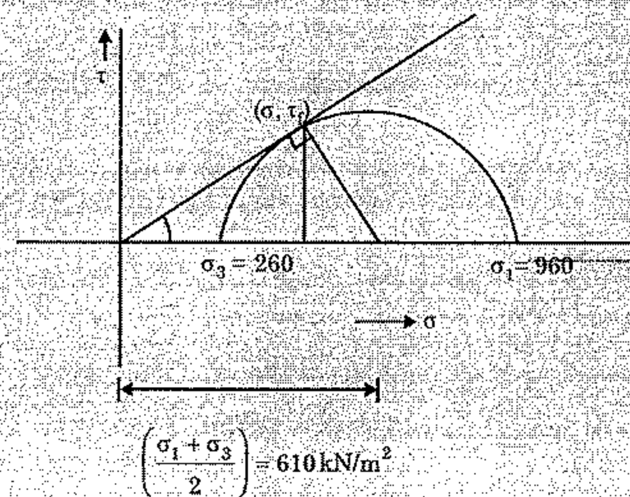
Assuming the same shear strength parameter in direct shear test.

Normal shear $\sigma = 230 \text{ kN/m}^2$

$\Rightarrow \quad \tau_f = (c + \sigma \tan \phi)$

$= 0 + 230 \tan (35.01)^\circ$

$= 161.11 \text{ kN/m}^2$



Example 6

The following are the result of a set of Drained triaxial tests, peformed on three indentical specimens of 38 mm diameter and 76 mm height.

Sample No.	Cell pressure (kN/m²)	Deveiatior load at failure (kN)	Change in volume (CC)	Axial deformation (mm)
1.	50	0.0711	–0.9	5.1
2.	100	0.0859	–13	7.0
3.	150	0.0956	–1.6	9.1

Determine the shear parameter of the soil.

Sol. Data Given:

Diameter of the specimen = 38 mm

= 3.8 cm

height of the sample h $= 76$ mm $= 7.6$ cm

$\therefore$ Corrected area $A_c = \dfrac{(V_1 \pm \Delta V)}{(L_1 - \Delta L)}$ ... (*i*)

$\therefore$ $V_1 =$ Initial Volume of the specimen

$= \dfrac{\pi}{4} d^2 h = \dfrac{\pi}{4} \times (3.8)^2 \times 7.6 = 86.19$ cc.

$L_1 = 7.6$ cm

For the first sample, $\Delta V = -0.9$ cc and $\Delta L = 5.1$ cm.

$$\therefore \quad A_c = \frac{(V_1 - \Delta V)}{(L_1 - \Delta L)} = \frac{(86.19 - 0.8)}{(7.6 - (5.1))}$$

$$= 12.03 \text{ cm}^2 = 12.03 \times 10^{-4} \text{ m}^2$$

$$\sigma_d = \frac{(0.0711)}{(12.03) \times 10^{-4}} = 59.10 \text{ kN/m}^2$$

$$\therefore \quad \sigma_1 = (\sigma_3 + \sigma_d) = (50 + 59.10) = 109.10 \text{ kN/m}^2$$

$\therefore$ The major principal stress for two other samples are computed in a similar manner are tabulated below.

Sample No.	σ_3 (kN/m²)	σ_3 (kN)	ΔV (cc)	ΔL (cm)	A_C (cm²)	σ_d (kN/m²)	σ_1 (kN/m²)
1.	50	0.0711	–0.9	5.1	12.03	59.10	109.10
2.	100	0.0859	–1.3	7.0	12.36	69.50	169.50
3.	150	0.0956	–1.6	9.1	12.65	75.61	225.61

We have $\sigma_1 = (\sigma_3 N_\phi + 2c\sqrt{N_\phi})$

$\Rightarrow \quad 109.10 = 50\, N_\phi + 2c\sqrt{N_\phi}$ ... (*i*)

$\Rightarrow \quad 169.50 = 100\, N_\phi + 2c\sqrt{N_\phi}$ ... (*ii*)

$225.61 = 150\, N_\phi + 2c\sqrt{N_\phi}$ ... (*iii*) From (*ii*) – (*i*) we have

$\Rightarrow \quad (169.50 - 109.10) = 50\, N_\phi$

$\Rightarrow \quad N_\phi = 1.208$

$\Rightarrow \quad \tan^2\left(\dfrac{\pi}{4} + \dfrac{\phi}{2}\right) = 1.208$

$\Rightarrow \quad \tan\left(\dfrac{\pi}{4} + \dfrac{\phi}{2}\right) = 1.208$

$\Rightarrow \quad \phi = (5.405)°$ ... (*iv*)

Put the value of N_ϕ in (*i*)

$\Rightarrow \quad 109.10 = (50 \times 1.208) + 2c\sqrt{1.208}$

$\Rightarrow \quad c = 22.15$ kN/m²

Example 7

The following results were obtained from a laboratory triaxial test with arrangements for pore pressure measurements:

Sampel No.	cell pressure (kg/cm²)	Deviator stress at failure (kg/cm²)	Pore pressure at failure (kg/cm²)
1.	1.0	2.02	0.41
2.	1.5	2.18	0.62
3.	2.0	2.37	0.70

Determine the shear parameters of the soil considering

(i) Total stresses (ii) Effective stress

Sol. Effective stress may be obained from the relation

$$\sigma' = (\sigma - u) \quad \text{... (i)}$$

The major and minor principal stress, considering the total stress analysis as well as effecive stress analysis are tabulated as below

Sample No.	σ_3 (kg/cm²)	σ_d (kg/cm²)	$\sigma_1 = (\sigma_3 + \sigma_d)$ (kg/cm²)	u (kg/cm²)	$\sigma_3' = (\sigma_3 - u)$ (kg/cm²)	$\sigma_1' = (\sigma_1 - u)$ (kg/cm²)
1.	1.0	2.02	3.02	0.41	~~0.41~~ 0.59	~~0.59~~ 2.61
2.	1.5	2.18	3.68	0.62	0.88	3.06
3.	2.0	2.37	4.37	0.70	1.30	3.67

Total Stress Analysis

We have,
$$\sigma_1 = (\sigma_3 N_\phi + 2c\sqrt{N_\phi}) \quad \text{... (i)}$$

$$\Rightarrow \quad 3.02 = 1 \times N_\phi + 2c\sqrt{N_\phi}$$

$$\Rightarrow \quad 3.02 = N_\phi + 2c\sqrt{N_\phi} \quad \text{... (ii)}$$

Similarly
$$3.68 = 1.5 N_\phi + 2c\sqrt{N_\phi} \quad \text{... (iii)}$$

$$4.37 = 2N_\phi + 2c\sqrt{N_\phi} \quad \text{... (iv)}$$

Form (iii) – (ii) we have

$$\Rightarrow \quad (3.68 - 3.02) = 0.5\, N_\phi$$

$$\Rightarrow \quad N_\phi = 1.32$$

$$\Rightarrow \quad \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) = 1.32$$

$$\Rightarrow \quad \tan\left(45 + \frac{\phi}{2}\right) = \sqrt{1.32}$$

$$\phi = (7.9)° \quad \text{... (v)}$$

put the the value of N_ϕ in (ii)

$$\Rightarrow \quad 3.02 = 1.32 + 2c\sqrt{1.32}$$

$$\Rightarrow \quad c = \frac{(3.02-1.32)}{(2\sqrt{1.32})} = 0.74 \text{ kg/cm}^2$$

From effective stress analysis: We have

$$\Rightarrow \quad \sigma'_1 = \sigma'_3 N_\phi + 2c'\sqrt{N_\phi}$$

From table ,

$$\Rightarrow \quad 2.59 = 0.59\ N_\phi + 2c'\sqrt{N_\phi} \quad \text{... (i)}$$

$$\Rightarrow \quad 3.06 = 0.88\ N_\phi + 2c'\sqrt{N_\phi} \quad \text{... (ii)}$$

$$\Rightarrow \quad 3.67 = 1.30\ N_\phi + 2c'\sqrt{N_\phi} \quad \text{... (iii)}$$

From (ii) – (i), we have

$$\Rightarrow \quad (3.06 - 2.59) = (0.88 - 0.59)$$

$$\Rightarrow \quad N_\phi = 1.62$$

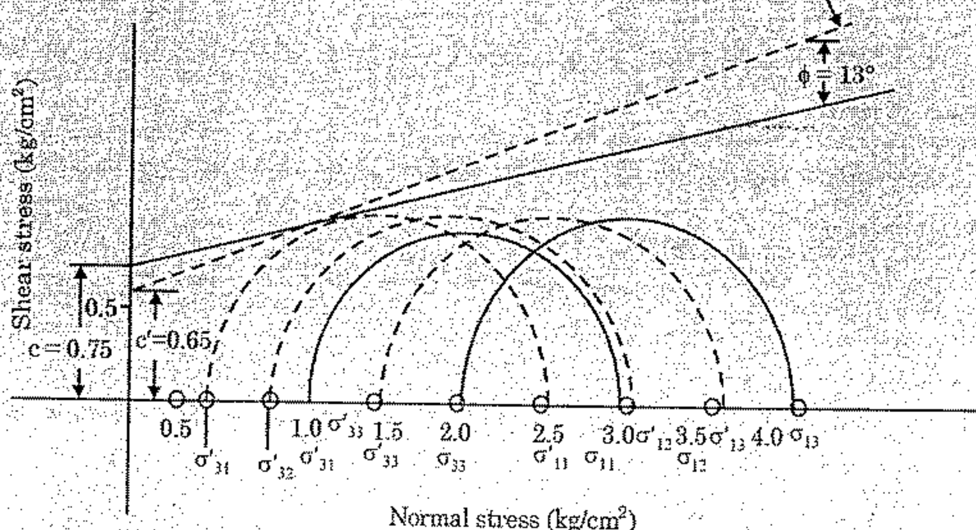
$$\Rightarrow \quad \tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right) = 1.62$$

$$\Rightarrow \quad \phi = (13.7)° \quad \text{... (iv)}$$

Put the value of N_ϕ, in (i)

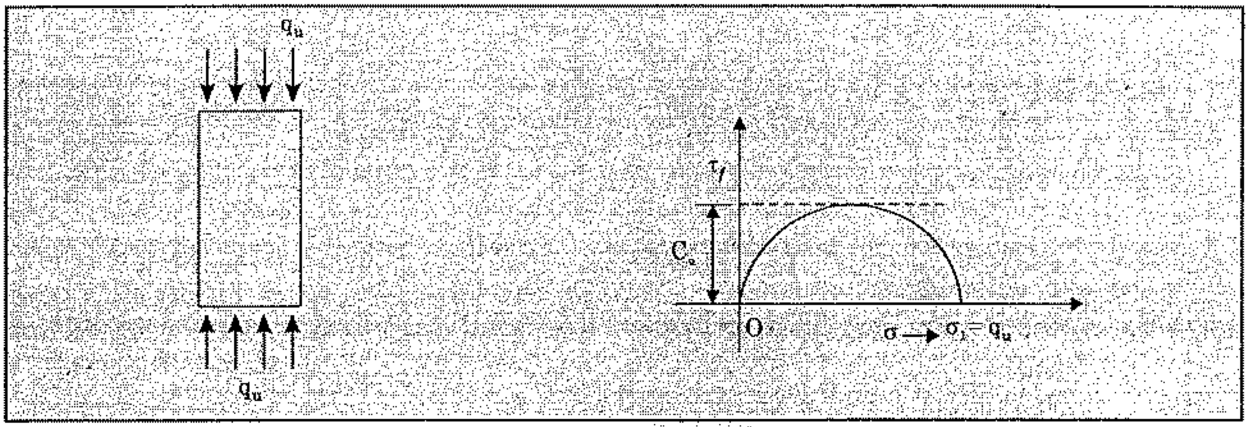
$$\Rightarrow \quad 2.59 = 0.59 \times 1.62 + 2c'\sqrt{1.62}$$

$$\Rightarrow \quad c' = \frac{(2.59-0.9558)}{(2\sqrt{1.62})} = 0.642 \text{ kg/cm}^2 \quad \text{... (v)}$$



Unconfined Compression Test

- No confining presure (i.e cell pressure) is applied.
- A special case of triaxial test with $\sigma_3 = 0$.
- Used to test cohesive soil.
- Load is rapidly applied. Hence it is an undrained test. Angle of internal friction is not mobilized. $\Rightarrow \phi_u = 0$
- As there is only one confining pressure $\sigma_3 = 0$ only one mohr circle is obtained.



$$\boxed{C_u = \frac{q_u}{2}}$$

σ_1 = axial stress at failure

q_u = unconfined compressive strength.

- As no lateral pressure (confining pressure) exist, sand/coarse grained soil can not stand in equipment. Hence *UCS* test is not done for these.

$$\boxed{q_u = \frac{P}{A_f} = \frac{P}{\left(\frac{A_0}{1-\varepsilon_a}\right)}}$$

where,

P = axial load applied

A_f = Area of x-sec at the time of failure.

A_o = Initial cross sectional area.

$$\varepsilon_a = \text{axial strain } = \frac{\Delta h}{h}.$$

- *UCS* test uses simple equipment. Can be Conducted in field easily.
- Used for rapid assessment of consistency of soil for classification clays are classified according to their consistency based on *UCS*.

Consistency	q_u (Kpa)
Very soft	0-24
Soft	24-48
Medium	48-46
Stiff	96-192
Very stiff	192-383
hard	>383

- Test is also used to find out sensitivity of clay soil. Other than fissured clay

$$\text{Sensitivity} = \frac{q_u\,(\text{undisturbed})}{q_u\ (\text{remoulded})}$$

Normally for clay sensitivity is $1 - 8$, for floaculated marine clay, sensitivity 10-80.

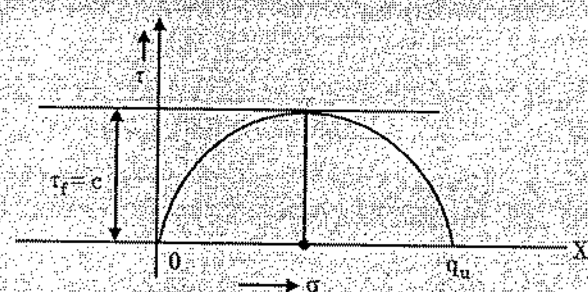
Example 8

An unconfined compression test was performed on an undisturbed sample of normally consolidated caly, having a diameter of 3.75 cm and 7.5 cm heigh failure occured under a vertical compressive load of 116.3 kg. The axial deformation recorded at failure was 0.9 cm. A Remoulded sample of the same soil failed under a compressive load of 68.2 kg and the corresponding axial deformation was 1.15 cm.

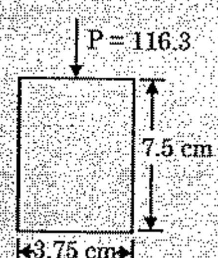
Determine the unconfined strength and cohesion of the soil in the undisturbed as well as Remoulded state.

Also determine the sensitivity of the soil and hence classify it accordingly.

Sol. (a) Undisturbed state



Initial Area of cross section of the sample $A_0 = \frac{\pi}{4}(3.75)^2$



$$= 11.04 \text{ cm}^2$$

Axial strain at failure $\quad \varepsilon_a = \left(\frac{\Delta L}{L}\right)$

$$\varepsilon_a = \left(\frac{0.9}{7.5}\right) = 0.12$$

$\therefore$ Corrected area $\quad A_c = \frac{A_0}{(1-\varepsilon)}$

$$= \frac{11.04}{(1-0.12)} = 12.55 \text{ cm}^2$$

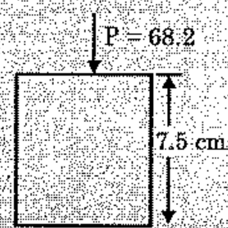
$\therefore$ Normal stress at failure $= \left(\dfrac{P}{A_c}\right) = \left(\dfrac{116.3}{12.55}\right) = 9.27$ kg/cm²

$\therefore$ Unconfined compressive strength $q_u = 9.27$ kg/cm²

and Cohesion $c = \left(\dfrac{q_u}{2}\right) = \left(\dfrac{9.27}{2}\right)$

$= 4.63$ kg/cm²

(b) In Remoulded state:



Axial Deformation = 1.15 cm

$$\varepsilon_a = \left(\frac{1.15}{7.5}\right) = 0.153$$

$$A_c = \frac{A_0}{(1-\varepsilon_a)} = \frac{11.04}{(1-0.153)}$$

$$= 13.04 \text{ cm}^2$$

$$\therefore \quad q_u = \left(\frac{P_u}{A_c}\right) = \left(\frac{68.2}{13.04}\right)$$

$$= 5.23 \text{ kg/cm}^2$$

$$\therefore \quad c = \left(\frac{q_u}{2}\right) = \left(\frac{5.23}{2}\right) = 2.62 \text{ kg/cm}^2$$

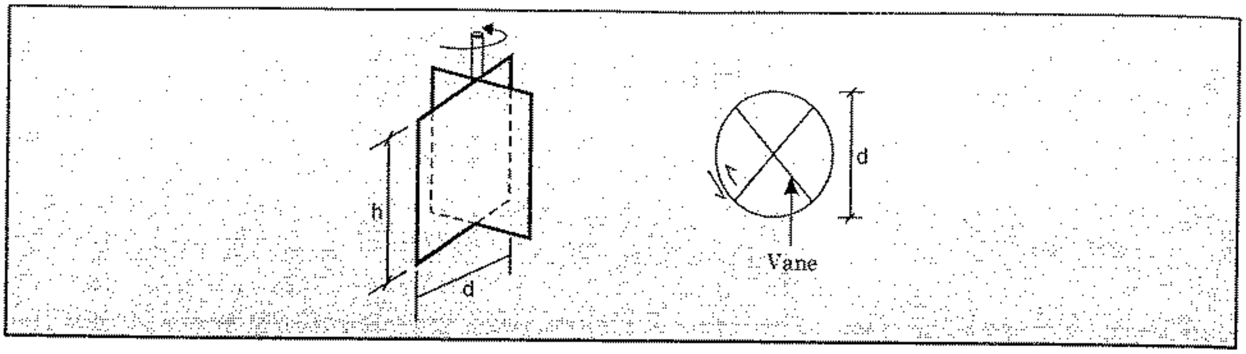
$$\therefore \quad \text{Sensitivity} = \frac{\text{Strength in the undisturbed state}}{\text{(Strength in the remoulded state)}}$$

$$= \frac{9.27}{5.23} = 1.7$$

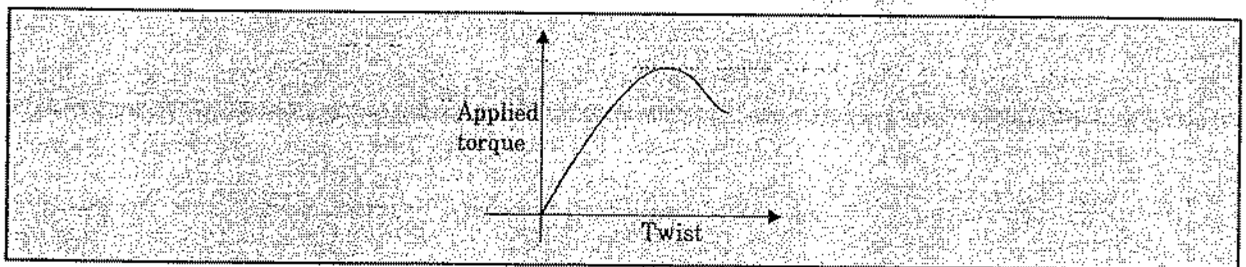
As the value of sensitivity lies between 1 and 2, the soil is classified an a low sensitive soil.

Vane shear test

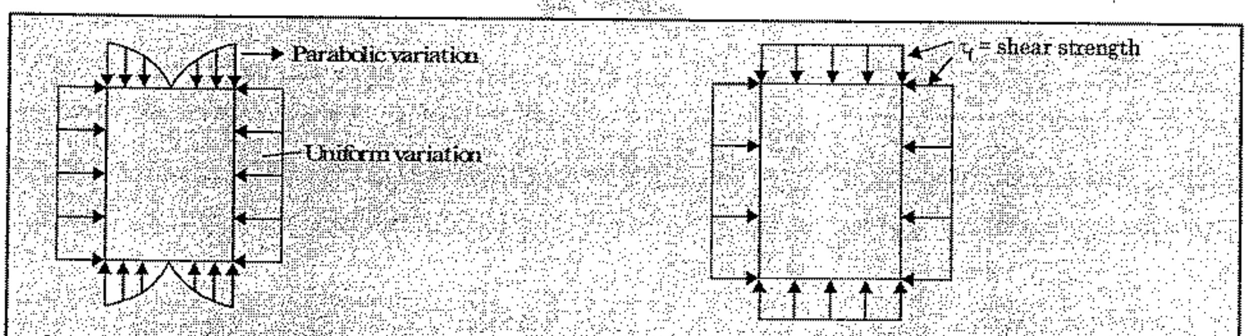
- **In plastic cohesive soil** which is very sensitive, obtaining undisturbed specimen is difficult. Shear strength of such soil may be significantly affected during sampling & handing.
- For such soil **Vane-shear test can be done in field.** Vane shear test is sometime also done in lab.
- Field equipment is large, where as lab equipment is smaller.



- Vane is push into soil gently and twisted untill soil fails.
- Max Torque applied is the total shear resistance.
- Shear failure occur over the surface and the ends of a cylinder having dia '*d*' equal to dia of vane.
- **Result of tbe test is as described below.**



- **Actual sbear stress variation** over the failure surface.



Before Failure of soil sample **After Failure of soil sample**

Applied Toruqe = T

$$T = \pi d h \tau_f \frac{d}{2} + 2\int_0^{d/2} 2\pi r dr.\tau_f r$$

$$T = \frac{\pi d^2 h \tau_f}{2} + \frac{4\pi\tau_f\left(\frac{d}{2}\right)^3}{3}$$

$$T = \frac{\pi d^2 h \tau_f}{2} + \frac{\pi d^3 \tau_f}{6}$$

$$T = \pi d^2 \tau_f \left[\frac{d}{6} + \frac{h}{2}\right]$$

$$\boxed{\tau_f = \frac{T}{\pi d^2\left(\frac{h}{2}+\frac{d}{6}\right)}}$$

When shearing is done such that the top end of the vane does not shear the soil.

$$\boxed{\tau_f = \frac{T}{\pi d^2\left(\frac{h}{2}+\frac{d}{12}\right)}}$$

Where,

$$\boxed{T_f = \text{shear strength} = C_u \text{ (undrained cohesion)}}$$

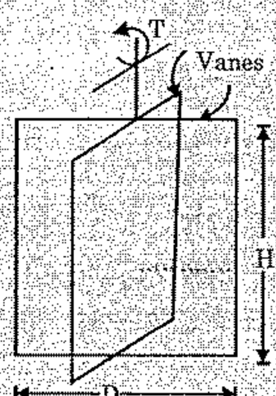
- If after the initial test, the vane is rotated rapidly several times, the soil becomes remoulded and the shear strength of remoulded clay can be calculated. Thus sensitivity of the clay soil is determined.

$$\boxed{\text{Sensitivity} = \frac{q_{u\,(\text{undisturbed})}}{q_{u\,(\text{remouded})}} = \frac{2c_u}{2c_{ur}} = \frac{c_u}{c_{ur}}}$$

Example 9

A Vane shear test was carried out in the field to determine the shearing strength of a deep-seated layer of soft clay. The Vane was 11.25 cm high and 7.5 cm across the baldes. The equivalent torque recorded at the torque head at failure was 417.5 kg. cm. The vane was then rotated very rapidly in order to completely remould the soil. It was found that the remoulded soil can be sheared by applying a torque of 283.2 kg.cm. Determine the shear strength of the soil in the undisturbed and remoulded states and its sensitivity.

Sol. Data Given H = 11.25 cm



D = 7.5 cm

In undisturbed state, T = 417.5 kg.cm

For a cohesive soil ϕ = 0

Therefore, $S = c = \dfrac{T}{\pi D^2\left(\frac{H}{2}+\frac{D}{6}\right)}$

$$\Rightarrow \qquad S = c = \frac{T}{\pi \times (7.5)^2 \left[\frac{11.25}{2} + \frac{7.5}{6}\right]}$$

$$S = c = \frac{417.5}{1113.67}$$

$$= 0.37 \text{ kg/cm}^2 \qquad \ldots (i)$$

In the Remoulded state, $T = 283.2$ kg.cm

$$S = \frac{T}{1113.67} = \left(\frac{283.2}{1113.67}\right)$$

$$= 0.250 \text{ kg/cm}^2 \qquad \ldots (ii)$$

$$\text{Sensitivity } (S_t) = \left(\frac{0.37}{0.250}\right) = 1.48$$

TYPES OF TRIAXIAL TEST

The triaxial test is performed in two stages.

1st stage	2nd stage
1. Drainage allowed (consolidated)	1. Volume change allowed (drained).
2. Drainage not allowed (unconsolidated)	2. No volume change allowed (undrained)

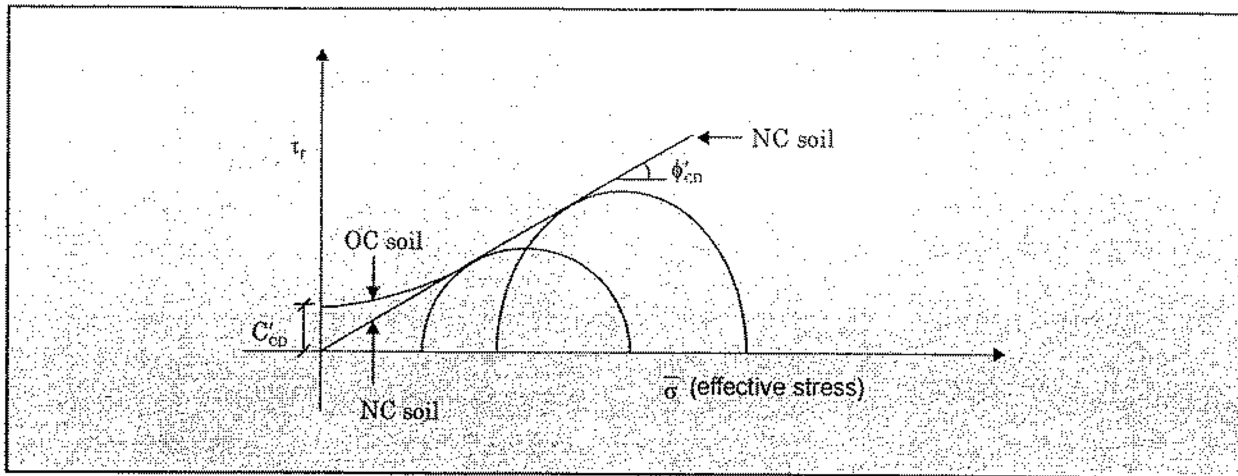
Accordingly, the type of tests are:

1. Consolidated drained test (CD Test)→Takes long time
2. Consolidated undrained test (CU Test).→Take 24 hr. in 1st stage & 2 hr. in 2nd stage.
3. Unconsolidated undrained test (UU Test)→Takes only 15 min.
4. Unconsolidated drained test→Not used (Not a realistic one as it does not occur in field).

Consolidated Drained Test (CD Test)

	Loading	Total stress	Pore water pressure	Effective stress
1st stage	Confining	$\sigma_1 = \sigma_c$	$u = 0$	$\bar{\sigma}_1 = \sigma_c$
	Pressure σ_c	$\sigma_3 = \sigma_c$	$u = 0$	$\bar{\sigma}_3 = \sigma_c$
2st stage	deviator stress	$\sigma_1 = \sigma_c + \Delta\sigma_a$	$u = 0$	$\bar{\sigma}_1 = \sigma_c + \Delta\sigma_a$
	$\Delta\sigma_a$	$\sigma_3 = \sigma_c$	$u = 0$	$\bar{\sigma}_3 = \sigma_c$

- Drainage is allowed in both stages.
- Loading rate is slow to allow water to expel out therefore Pore water pressure does not build up.
- We get effective stress parameter in this case.



$$\boxed{\tau_f = C_{CD} + \bar{\sigma}\tan\phi'_{CD}}$$

- If the soil is normally consolidated, Mohr failure envelope passes through origin.
- A normally consolidated soil is the one in which the current loading is the maximum loading to which the soil has even been subjected to in its stress history. Hence a NC soil at $\sigma_3 = 0$ will be just like a powder which has no shear strength or if it is saturated it will be like a slurly which has no shear strength. Thus at $\sigma = 0$ shear strength of NC soil will be zero and its failure envelope will pass through the origin.
- But an over consolidated soil will not have zero shear strength at $\sigma_3 = 0$. Hence it does not, pass through origin.
- The soil is over consolidated if the confining stress that we are applying is less than the force that the soil has already experienced.
- When applied load becomes more than over-consolidation pressure, the soil becomes normally consolidated, hence follows the curve corresponding to NC-condition.
- An over consolidated soil will show higher strength upto $\sigma_3 = \sigma_c$ (pre consolidation stress) and behaves like *NC* for $\sigma_3 > \sigma_c$.
- Thus initial part of the curve is nonlinear & later part is linear.
- Progressively bigger mohr circles are forming because initial void ratio will progressively decrease as more & more higher confining pressure, are applied. Hence progressively higher deviator stress is required for failure of the soil.

Use of CD Test

(1) Analysis of gradual loading condition.

(2) To check long term stability of embankment which has been in existence since long ago.

Shear strength calculation in field

$$\tau_f = C'_{CD} + \bar{\sigma}\tan\phi'_{CD}$$

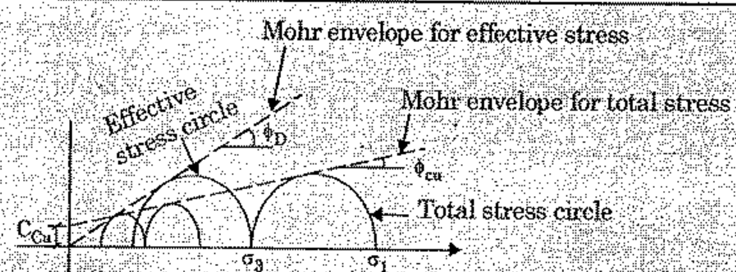
where,

$\bar{\sigma}$ is taken as the effective vertical stress in field.

CONSOLIDATED UNDRAINED TEST

Loading stress	Total stress	Pore water pressure	effective stress
Confining Pressure σ_c	$\sigma_1 = \sigma_c$ $\sigma_3 = \sigma_c$	$u = 0$ $u = 0$	$\bar{\sigma}_1 = \sigma_c$ $\bar{\sigma}_3 = \sigma_c$
Axial stress or deviator stress $\Delta\sigma_1$	$\sigma_1 = \sigma_c + \Delta\sigma_a$ $\sigma_3 = \sigma_c$	$u = u_0$ $u = u_0$	$\bar{\sigma}_1 = \sigma_c + \Delta\sigma_a - u_0$ $\bar{\sigma}_3 = \sigma_c - u_0$

- If during undrained stage, pore water pressure developed is exactly equal to $\Delta\sigma_a$, then $u_0 = \Delta\sigma_a$ should be taken.
- 1st stage—drainage permitted.
- 2nd stage—drainage not permitted, volume change not allowed.
- This test gives total stress parameter because unless drainage is permitted, stress does not become effective.
- Normally total deviator stress = Pore water pressure developed. However this may not be true under all conditions.
- Effective stress parameter can be calculated if pore water pressure is measured in the triaxial apparatus.
- Radius of Total stress circle & effective stress circle will be same if deviator stress approaches pore water pressure.



USE OF CU TEST

- To check stability under Sudden unloading such as dewatering or draw down condition.
- To check stability of an Embankment that has lived some of its life & is now being unloaded.

Note: • In *CU* test, undrained strength is determined after void ratio has been changed from initial value by the consolidation. The undrained strength is thus a function of this void ratio or of the corresponding all-round pressure σ_3' under which consoldiation took place.

- It should be realised that clay in insitu condition are consolidated under condition of zero lateral strains; the effective vertical & horizontal stress being unequal. *i.e.* clay has been consolidated anisotropically. A stress release then occurs on sampling. In the *CU* **test the specimen is then consolidated again under equal all-round pressures, normally equal to the value of effective vertical stress in at site.** *i.e.* the specimen is consolidated isotropically. Isotropic consoldiation in triaxial test under pressure equal to in situ effective vertial stress results in a void ratio lower than the in-situ value and therefore undrained strength higher than in-situ value.

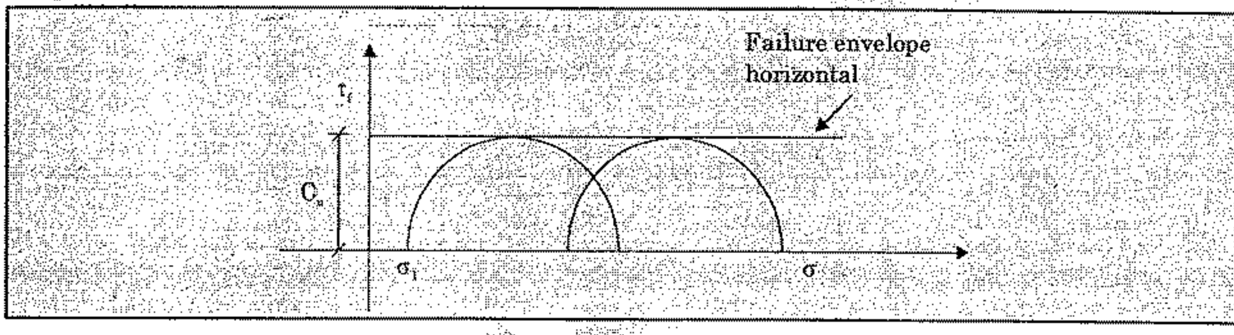
Shear strength equation for *CU* test are

$$\boxed{\tau_f = C_{cu} + \sigma \tan\phi_{cu}}$$

- For Normally consolidated soil, $C_{cu} = 0$ and For *OC*-soil C_{cu} exist.

UU TEST

- Drainage is not permitted during 1st & 2nd stage (sample is initially saturated).
- When specimen is placed under triaxial cell the initial pore water pressure is –ve due to capillary tension. Total stress being zero and effective stress (+) ve. (Amount not known in advance)
- After application of all round pressure, the effective stress in the specimen remains unchanged becuase under undrained condition, increase in all round pressure results in equal increase in pore water pressure.
- In deviator stress application also effective stress does not change because again undrained condition prevail.



- Thus **identical specimen** tested under various confining pressure will fail by equal increment of deviator stress.
- Due to undrained condition at both stages ϕ_u does not get moblised.
- **Total stress analysis** is performed as it is an undrained test.

Loading	Total stress	Neutral stress	Effective stress
at $t = 0$	$\sigma_1 = 0$	$u = -x$	$\bar{\sigma}_1 = x$
	$\sigma_3 = 0$	$u = -x$	$\bar{\sigma}_3 = x$
Confining	$\sigma_1 = \Delta\sigma_3$	$u = -x + \Delta\sigma_3$	$\bar{\sigma}_1 = x$
pressure $\Delta\sigma_3$	$\sigma_3 = \Delta\sigma_3$	$u = -x + \Delta\sigma_3$	$\bar{\sigma}_3 = x$
Axial	$\sigma_1 = \Delta\sigma_3 + \Delta\sigma_1$	$u = -x + \Delta\sigma_3 + \Delta\sigma_1$	$\bar{\sigma}_1 = x$
load $\Delta\sigma_1$	$\sigma_3 = \Delta\sigma_3$	$u = -x + \Delta\sigma_3 + \Delta\sigma_1$	$\bar{\sigma}_3 = x - \Delta\sigma_1$

- x is not known in advance, hence total stress analysis performed.
- As $\Delta\sigma_1$ for all test remain same, hence only one effective stress circle is obtained.
- Shear strength. $\boxed{\tau_f = C_u}$
- Test is suitable for soil of low permeability and/or when loading is very fast.
- It is a quick test—15 min time taken.

Use of UU Test

- Sudden loading such as rapid construction.
- Short term stability under construction pore water pressure *i.e.* during construction only.

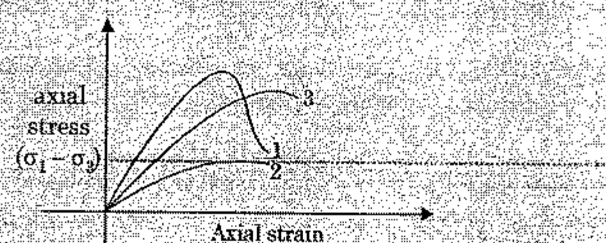
Note:

$$\boxed{\frac{Cu}{\bar{\sigma}_z} = 0.11 + 0.0037 I_p \%}$$

- For NC soil as plasticity index increases shear strength increase.
- Applicable for *NC* soil
- C_u is UU strength parameter
- $\bar{\sigma}_z$ = effective over burden at the depth at which sample has been obtained.

STRESS STRAIN & VOLUME CHANGE RELATIONSHIP FOR CLAYEY SOIL

- Direct shear test gives→**Shear stress v/s shear Strain curve.**
- Triaxial test gives **deviator stress v/s axial strain curve.**
- Unconfined compression test given→**axial stress v/s axial strain curve.**



Where,

curve 1, 2, 3 are obtaned for soil as shown below

(1) for Undisturbed sensitive clay

(2) for Remoulded sensitive clay

(3) for Insensitive day.

Properties of various curves

curve 1

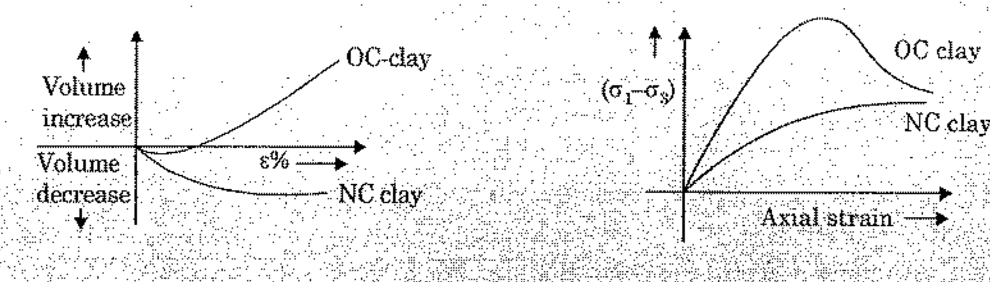
- Sharp peak at low strain.
- Specimen shears along a well defined plane.
- This is known as brittle failure.

curve 2

- Plastic or barrelled failure.
- Results in bulging effect of specimen.
- Precise value of ultimate deviator stress not obtained. Hence failure is defined on the basis of some arbitray value of strain (15-20%).

curve 3

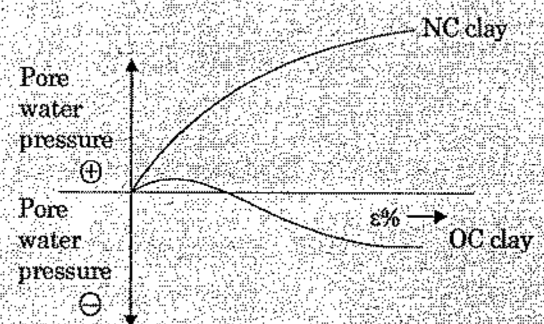
- Failure is taken corresponding to almost horizontal portion of curve.



Results of CD test

Note: • NC soil gets compressed as sheared.

- OC soil has dilation on shearing.
- OC soil has pronounced peak in stress-strain curve at low strain.

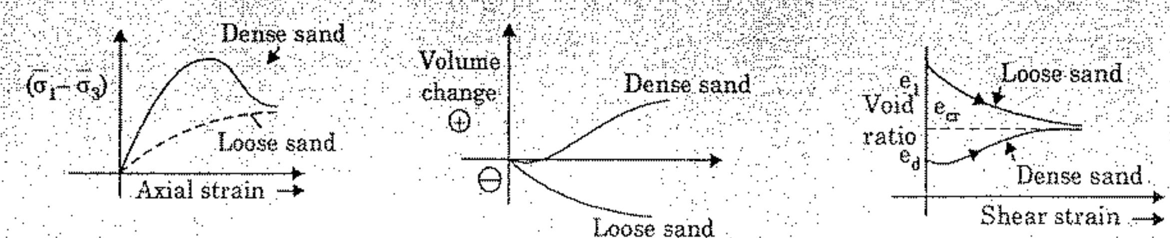


Result of CU Test

Note: • NC soil tries to get compressed when sheared but if drainage is not allowed than +ve pressure develops.

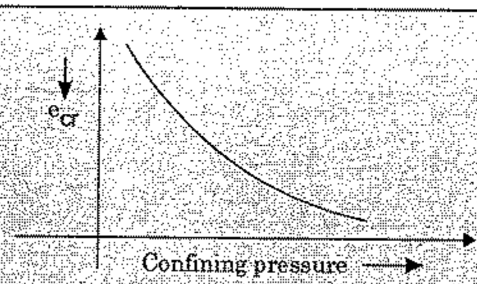
- OC soil tries to dilate when sheared due to reorientation of flaky clay particles. If drainage not allowed -ve pressure develops.
- Effect of strain rate: Undrained strength increases with increase in strain rate for clays.

Stress strain & volume cbange behaviour of sands



Results of CD Test

- Dense sand—Peak stress define shear strength.
- Loose sand—Some arbitrary value of strain is defined for shear strength.
- Note that if initial void ratio is more than e_{cr} volume decrease occurs and is initial void ratio is smaller than e_{cr} volume increase will occur.
- But e_{cr} depends on confining stress σ_3 as represented by the following curve.



- at $e = e_{cr}$ there is no tendency of volume change hence both drained & undrained strength will be same.
- at $e > e_{cr}$ there is a tendency of volume decrease. If undrained condition prevail, than +ve pore water pressure develops. Hence undrained strength will be smaller than drained strength.
- if $e < e_{cr}$, there is a tendency of volume increase. If undrained condition prevail, –ve pore water pressure develops. Thus undrained strength will be larger than drained strength.

Example 10

Three identical specimen of a partially saturated clay were subjected to an unconsolidated undrained Triaxial test and the following results were obtained:

Sample No.	Cell pressure (Kg/m2)	Deviator stress (Kg/cm²)
1	0.5	0.80
2	1.0	0.97
3	1.5	1.23

Determine the shear parametrs of the soil.

Sol. We know that

$$\sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi}$$

where $$N_\phi = \tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)$$

In case of the first sample:

$$\sigma_3 = 0.5,\ \sigma_1 = (\sigma_3 + \sigma_d)$$
$$= (0.5 + 0.8)$$
$$= 1.3 \text{ kg/cm}^2$$

$$\Rightarrow \quad \sigma_1 = \sigma_1 N_\phi + 2c\sqrt{N_\phi}$$

$$\Rightarrow \quad 1.3 = 0.5 N_\phi + 2c\sqrt{N_\phi} \quad \text{... (i)}$$

Similarly
$$1.97 = N_\phi + 2c\sqrt{N_\phi} \quad \text{... (ii)}$$

$$2.63 = 1.15 N_\phi + 2c\sqrt{N_\phi} \quad \text{... (iii)}$$

Subtracting (*i*) from (*ii*), we obtain

$$\Rightarrow \quad 0.5\, N_\phi = 0.67$$

$$\Rightarrow \quad N_\phi = 1.34$$

$$\Rightarrow \quad \tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right) = 1.34$$

$$\Rightarrow \quad \tan\left(\frac{\pi}{4}+\frac{\phi}{2}\right) = 1.157$$

$$\Rightarrow \quad \frac{\pi}{4}+\frac{\phi}{2} = (49.2)^\circ$$

$$\frac{\phi}{2} = (49.2)^\circ - 45^\circ$$

$$\Rightarrow \quad \phi = 8.4^\circ \quad \text{... (iv)}$$

Substituting for $\phi = 8.4^\circ$ in (*i*)

$$\Rightarrow \quad 0.5 \times 1.34 + 2c \times 1.157 = 1.3$$

$$\Rightarrow \quad \boxed{c = 0.27 \text{ kg/cm}^2}$$

Check: Substituting the value of c and ϕ in (*iii*)

$$\text{L.HS} = (1.5 \times 1.34) + 2 \times (0.27)\,(1.157)$$
$$= 2.63 = \text{R.H.S}$$

Example 11

A sample of dry coarse sand is tested in the laboratory triaxial apparatus in the undrained condition. Under a cell pressure of 2kg/cm^2 the sample failed when the deviator stress reached 4.38 kg/cm^2.

(*i*) Determine the shear parameter of the soil.

(*ii*) At what deviator stress will the soil fail if the cell pressure be 3 kg/cm^2

Sol. Data Given:

$$\sigma_3 = 2 \text{ kg/cm}^2$$
$$\sigma_d = 4.38 \text{ kg/cm}^2$$
$$\Rightarrow \quad \sigma_1 = (\sigma_3 + \sigma_d)$$
$$= (2 + 4.38)$$
$$= 6.38 \text{ kg/cm}^2 \quad \text{... (i)}$$

For dry coarse sand c = 0

$$\Rightarrow \quad \sigma_1 = \sigma_3 N\phi + 2c\sqrt{N_\phi}$$

$$\Rightarrow \quad 6.38 = 2\, N_{\phi}$$

$$\Rightarrow \quad N\phi = \left(\frac{6.38}{2}\right) = 3.19$$

$$\Rightarrow \quad \tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right) = 3.19$$

$$\Rightarrow \quad \tan\left(\frac{\pi}{4}+\frac{\phi}{2}\right) = \sqrt{3.19}$$

$$\Rightarrow \quad \frac{\pi}{4}+\frac{\phi}{2} = (61.39)°$$

$$\Rightarrow \quad \phi = (31.5)° \quad \text{... (ii)}$$

(ii)
$$\sigma_1 = (\sigma_3 N_{\phi} + 2\times 0\sqrt{N_{\phi}})$$

$$\sigma_1 = 3\times\tan^2\left(\frac{\pi}{4}+\frac{31.5}{2}\right)$$

$$= 9.56 \text{ kg/cm}^2$$

$$\therefore \quad \sigma_d = (\sigma_1 - \sigma_3)$$

$$= (9.56 - 3) = 6.56 \text{ kg/cm}^2 \quad \text{... (iii)}$$

Example 12

The shear strength parameter of a given soil are $c = 0.26$ kg/cm² and $\phi = 21°$. Undrained triaxial tests are to be carried out on specimens of the soil. Determine

(i) Deviator stress at which failure will occur if the cell pressure be 2.5 kg/cm²

(ii) The cell pressure during the test. If the sample fails when the deviator.

Stress reaches 1.68 kg/cm²

Sol. We have

$$\sigma_1 = (\sigma_3 N\phi + 2c\sqrt{N\phi}) \quad \text{... (i)}$$

Data given

$$c = 0.26 \text{ kg/cm}^2$$

$$\phi = 21°$$

$$\therefore \quad N_{\phi} = \tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right)$$

$$= \tan^2\left(\frac{\pi}{4}+\frac{21}{2}\right) = 2.117$$

$$\therefore \quad \sigma_1 = 2.117\,\sigma_3 + 2\times(0.26)\times 1.455$$

$$\sigma_1 = (2.117\,\sigma_3 + 0.757) \quad \text{... (ii)}$$

When

$$\sigma_3 = 2.5 \text{ kg/cm}^2$$

$$\therefore \quad \sigma_1 = (2.117\times 2.5 + 0.757)$$

$$= 6.05 \text{ kg/cm}^2$$

Therefore $\sigma_d = (\sigma_1 - \sigma_3)$

$$= (6.05 - 2.5) = 3.55 \text{ kg/cm}^2$$

Hence the required deviator stress at failure is 3.55 kg/cm²

(*ii*) Let the required cell pressure be x kg/cm²

$$\Rightarrow \quad \sigma_1 = (\sigma_d + \sigma_3)$$

$$= (\sigma_d + x)$$

$$= (1.68 + x) \quad \text{... (iii)}$$

Therefore from (*ii*) and (*iii*), we have

$$2.117\,x + 0.757 = (1.68 + x)$$

$$\Rightarrow \quad 1.117\,x = (1.68 - 0.757)$$

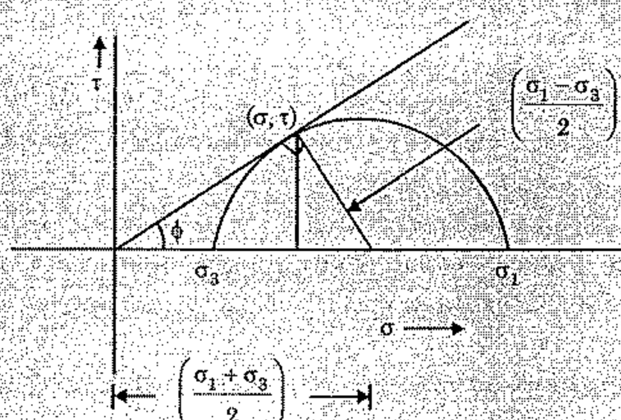
$$x = \frac{(1.68 - 0.757)}{(1.117)} = 0.83 \text{ kg/cm}^2$$

Hence the req. cell pressure is 0.83 kg/cm²

Example 13

A CU (Consolidated Undrained) test was conducted on a soil sample with cell pressure, $\sigma_c = 100$ kN/m². The deviator stress at failure, $(\sigma_1 - \sigma_3)$ was observed to be 60 kN/m² the soil is known to have a cohesion ($c' = 0$) and the angle of shearing resistance $\phi_u = 30°$ (Reffered to effective stress) and an undrained cohession $C_u = 0$ and angle of shearing resistance $\phi_u = 13.3°$ (Reffered to total stress). What was the pore water pressure at Failure?

Sol. Data given



Cell pressure $\sigma_3 = 100$ kN/m²

Deviator stress $\sigma_d = 60$ kN/m²

∴ Major principal stress at failure

$$\sigma_{1f} = (\sigma_c + \sigma_d)$$

$$= (100 + 60) = 160 \text{ kN/m}^2$$

Let pore pressure be u.

$\Rightarrow$ Then effective stresses

$$\sigma'_3 = (100 - u)$$

$$\sigma'_1 = (160 - u)$$

From Fig. (a),

$$\sin\phi = \frac{\left(\frac{\sigma_1 - \sigma_3}{2}\right)}{\left(\frac{\sigma_1 - \sigma_3}{2}\right)}$$

$$\Rightarrow \quad \sin\phi' = \frac{(\sigma'_1 - \sigma'_3)}{(\sigma'_1 - \sigma'_3)}$$

$$\Rightarrow \quad \sin 30^\circ = \frac{(160 - u - 100 + u)}{(160 - u + 100 - u)}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{60}{(260 - 2u)}$$

$$\Rightarrow \quad 260 - 2u = 120$$

$$\Rightarrow \quad 140 = 24\,u$$

$$u = \frac{140}{2} = 70\ \text{kN/m}^2$$

SOIL LIQUIFACTION

Loose sand has a tendency to get compressed when loaded. If rate of loading is larger and soil is **saturated,** +ve pore water will develop. This will reduces effective stress & hence strength. If effective stress reduces to zero, the soil will loose all its shear strength. This, phenomenon is known as **liquifaction.**

- Thus liquifaction occurs in **saturated loose sand.**
- It occurs during pile driving, vibration of machine, explosive blasting, earthquake shock.
- Once a complete loss of strength has occured in a limited mass of soil, the stress which were carried by the affected soil before its liquifaction, are transferred to the adjacent parts, throwing that part of the soil mass also into a state of liquifaction and process may continue.

PORE PRESSURE COEFFICIENTS

- Pore pressure coefficients are used to express the **response of pore water pressure to changes** in **total stress under undrained condition** and enable the initial value of excess pore water pressure to be determined.
- Values of coefficients maybe determined in the laboratory and can be used to predict the pore water pressure in field under similar stress condition.

$$\Delta u = B\left[\Delta\sigma_3 + A\left(\Delta\sigma_1 - \Delta\sigma_3\right)\right]$$

$$= B.\Delta\sigma_3 + AB\left(\Delta\sigma_1 - \Delta\sigma_3\right)$$

Let, $\Delta u = \Delta u_1 + \Delta u_2$

Where,

$$\Delta u_1 = B.\Delta\sigma_3$$

$\Rightarrow$ $$\boxed{B = \frac{\Delta u_1}{\Delta \sigma_3}}$$

Δu_1 = Change in pore water pressure due to an increase in cell pressure.

$\Delta u_2 = AB(\Delta\sigma_1 - \Delta\sigma_3)$

Δu_2 = Change in pore pressure due to increase in deviator stress ($\Delta\sigma_1 - \Delta\sigma_3$)

For a completely saturated soil,

$$B = 1$$

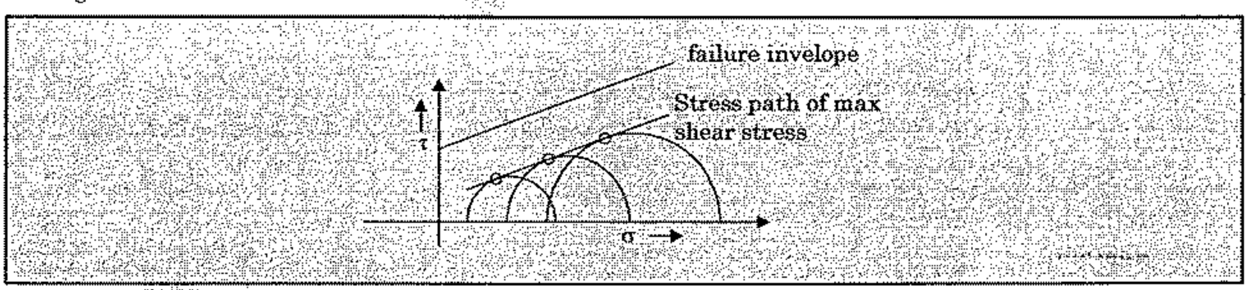
For a completely dry soil

$$B = 0$$

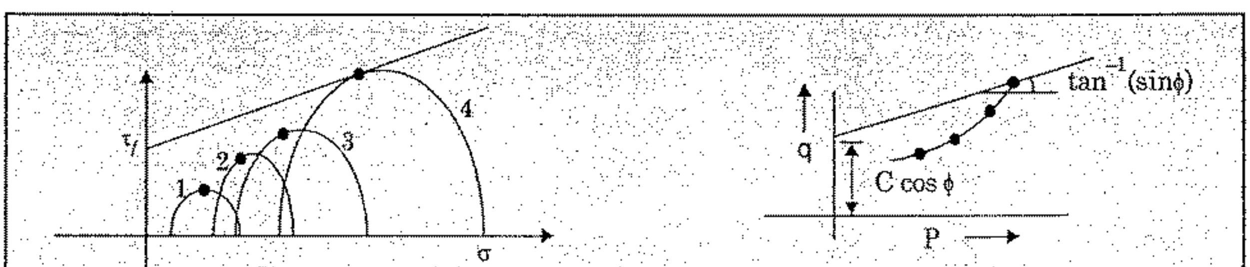
- **(B) varies with the stress range. Hence while calculating *A* from *AB*, Value of B corresponding to appropriate deviator stress range should be used.**
- B can be determined in *UU* test in 1st stage & *AB* can be measured in 2nd stage.
- A can also be found in (CD) test.
- Like *B*, *A* is not a constant & depends on type of soil, stress condition etc.
- For a given soil, A depends in **strain**, anisotropy, sample disturbance & *OCR*.
- Value of **A** may be as large as **2 to 3** for very loose saturated fine sand. It can be –0.5 to 0 for heavily over consolidated clays. For *NC* clays $A = 1$.
- For *OC* clays $A = f(OCR)$ for heavily over consolidated day, $A < 0$, typical value of A are – 0.2 to – 0.3.

STRESS PATH

Stress path is the locus of all points depicting a specific stress state as loading gradually progresses through various states of failure.



- By knowing the stress path, one knows what kind of stress history the soil has gone through and what has lead to failure. Knowledge of this throws insight into the behaviour of a soil on loading and how it generally tends towards failure. The following figure shows how the max stress in the soil has changed and what σ lead to failure.



Example 14

A saturated clay sample was obtained in field without allowing the water content and void ratio to change. The sample was subjected to the following stresses in field.

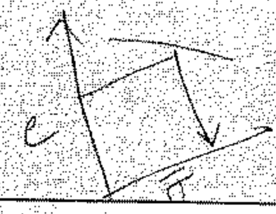
$\sigma = 240\ \text{kN/m}^2,\ u = 140\ \text{kN/m}^2,\ \bar{\sigma} = 100\ \text{kN/m}^2$

Sol. (1) On sampling what are the stresses acting on it.

On sampling total stress = 0.

But as void ratio does not change, $\bar{\sigma}$ is content

$$\sigma = 0$$
$$\bar{\sigma} = 100\ \text{kN/m}^2$$
$$u = -100\ \text{kN/m}^2$$



Example 15

Sample was placed in a triaxial cell & cell pressure applied = 100 kN/m² under undrained condition. What are the stress acting on it.

Sol.

$$\text{Total stress} = \sigma = 100\ \text{kN/m}^2$$
$$\text{Pore water pressure} = -100 + 100 = 0$$
$$\text{Effective stress} = 100\ \text{kN/m}^2$$

Example 16

Cell pressure was raised to 320 kN/m² with drainage valve closed. What are σ, u $\bar{\sigma}$

Sol.

$$\text{Total stress} = 320 = 320\ \text{kN/m}^2$$
$$\text{Pore water pressure} = 0 + 220 = 220\ \text{kN/m}^2$$
$$\text{Effective stress} = 100\ \text{kN/m}^2$$

Example 17

The drainage valve are opened and soil allowed to consolidate under the cell pressure what are σ, u $\bar{\sigma}$

Sol. Pore water pressure dissipates to zero hence. Total stress becomes effective

$$\sigma = 320\ \text{kN/m}^2$$
$$u = 0$$
$$\bar{\sigma} = 320\ \text{kN/m}^2$$

Example 18

Comment on nature of change in water content & degree of saturation after consolidation.

Sol. Water content will change degree of saturation remains constant.

Example 19

Drainage valve was closed and additional axial stress applied **with cell pressure held constant.** When additional stress was 100 kN/m², the pore water pressure was found to be 80 kN/m². What is the value of A at this stage.

(80-0)

$\Delta U_2 = AB(\Delta\sigma_1 + \Delta\sigma_3)$

→ 0 (cell p' kept constant)

$\boxed{AB = 0.8}$

Sol. We know that, $\dfrac{\Delta u}{(\Delta\sigma_1 - \Delta\sigma_3)} = AB$

for complete saturation, $B = 1$

$$A = \frac{\Delta u}{(\Delta\sigma_1 - \Delta\sigma_3)} = \frac{80}{100-0} = 0.8$$

Example 20

When an additional axial stress of 200 kN/m² had been applied, the sample failed and at that stage, the pure pressure coefficient $A = 1$.

inc in deviator stress

(Ambiguous statement)

(a) What is $\bar{\sigma}_{1f}$

(b) What was ϕ' value of the soil if it had $C' = 0$.

Sol.

$$\frac{\Delta u}{\Delta\sigma_1 - \Delta\sigma_3} = A$$

as confining pressure does not change $= \Delta\sigma_3 = 0$.

$$\Delta u = A\,\Delta\sigma_1 = 1 \times 200 \text{ kN/m}^2$$

Pore water pressure becomes 0 + 200 = 200 kN/m².

Total stress $\sigma_1 = 320 + 200 = 520$ kN/m².

$$\boxed{\bar{\sigma}_{1f} = 520 - 200 = 320 \text{ kN/m}^2}$$

Note at failure condition, effective stress $\bar{\sigma}_{1f}$ is same as before application of additional axial stress because drainage not allowed.

$$C' = 0$$

$$\frac{\bar{\sigma}_{1f}}{\bar{\sigma}_{3f}} = \frac{1+\sin\phi'}{1-\sin\phi'} = \tan^2\left(45 + \frac{\phi'}{2}\right)$$

$$\frac{320}{(320-200)} = \frac{1+\sin\phi'}{1-\sin\phi'} = \tan^2\left(45 + \frac{\phi}{2}\right)$$

$$\boxed{\phi' = 27.03°}$$

Example 21

Two identical specimens of a backfill soil, when tested in a triaxial compression test, gave the following stresses (in kg/cm^2) at failure.

Sample no.	All rou nd pressure	Axial stress	Pore Pressure
1	1.0	2.2	0.6
2	2.0	3.6	1.1

(i) Find axial stress at which, the third identical specimen of the same soil will fail if all round pressure is 2.8 kg/cm^2.

(ii) Find total & effective shear strength parameters of soil.

Sol.

$$\sigma_{1f} = \sigma_{3f} N_\phi + 2C\sqrt{N_\phi}$$

In terms of total stress

$$3.2 = 1N_\phi + 2C\sqrt{N_\phi} \quad \text{....(i)}$$

$$5.6 = 2N_\phi + 2C\sqrt{N_\phi} \quad \text{....(ii)}$$

$$2.4 = 1.0N_\phi$$

$$N_\phi = \frac{2.4}{1.0} = 2.400 \quad \text{....(iii)}$$

Put the value of (iii) in (i), we have

$$2C\sqrt{N_\phi} = \frac{3.2 - 2.4}{1.0} = 0.8$$

when

$$\sigma_3 = 2.8 \text{ kg/cm}^2$$

$$\sigma_{1f} = 2.8 \times 2.4 + 0.8$$

$$\sigma_{1f} = 7.52$$

$$\Delta\sigma_1 = \sigma_{1f} - \sigma_{3f}$$

$$\Delta\sigma_1 = \text{axial stress} = 7.52 - 2.8 = 4.72 \quad \text{....(iv)}$$

In terms of effective stress

$$2.6 = N_\phi \times 0.4 + 2C\sqrt{N_\phi}$$

$$4.5 = N_\phi \times 0.9 + 2C\sqrt{N_\phi}$$

$$1.9 = 0.5N_\phi$$

$$N_\phi = \frac{1.9}{0.5} = 3.8$$

$$2C\sqrt{N_\phi} = 2.6 - 3.8 \times 0.4$$
$$= 1.04$$

When cell pressure = 2.8

Note : No calculation can be done any further as pore water pressure is not known corresponding to allround pressure of 2.8 kg/cm.

Total stress approach

$$N_\phi = 2.400$$
$$2C\sqrt{N_\phi} = 0.800$$
$$\tan^2\left(45 + \frac{\phi}{2}\right) = 2.4$$
$$\boxed{\phi = 24.315°}$$
$$\boxed{C = 0.258 \text{ kg/cm}^2}$$

effective stress approach

$$N_\phi = 3.8$$
$$2C\sqrt{N_\phi} = 1.04$$
$$\boxed{\phi = 35.68°}$$
$$\boxed{C = 0.267 \text{ kg/cm}^2}$$

Example 22

A *CU* test was conduted on a soil sample with cell pressure $\sigma_C = 100 \text{kN/m}^2$
deviator stress $(\sigma_1 - \sigma_3) = 60 \text{ kN/m}^2$
The soil is known to have $C = 0$ and angle of internal friction $\phi = 30°$ (effective) and undrained cohession $C_u = 0$ and $\phi_u = 13.3°$ (total stress) what was the pore water pressure at failure.

Sol.
$$\tau_f = R \cos \phi$$
$$= \frac{160 - 100}{2} \cos 13.3$$
$$\tau_f = 30 \cos 13.3° = 29.195 \quad \text{....(i)}$$

From the coulomb's criterion, we know that

$$\tau_f = 0 + \sigma \tan \phi \quad \text{....(ii)}$$

From (i) and (ii), we have

$$\therefore \quad \sigma = \frac{30 \cos 13.3°}{\tan 13.30} = 123.51 \quad \text{....(iii)}$$

From effective stress approach

$$\tau_f = C + (\sigma - u)\tan 30° \quad ....(iv)$$

Put the value of τ_f and σ in (iv)

$$\therefore \quad \tau_f = (\sigma - u)\tan 30°$$

Therefore $\quad u = 72.94 \text{ kN/m}^2$

Example 23

In a triaxial test, a sample was consolidated under a cell pressure of 700 kN/m² and a back pressure of 350 kN/m². There after with drainage not allowed, the cell pressure was raised to 800 kN/m². Resulting is an increase in pore water pressure of 445 kN/m². The axial load was than increased to give a deviator stress of 575 kN/m² (while the cell pressure remianed at 800 kN/m²) and a pore water reading of 640 KN/m². Calculate pore pressure coefficient.

Sol. Back Pressure is applied to saturate the soil.

To ensure that application of back pressure does not result in an increase in effective stress in the sample, the cell pressure is also increased to the same level. Thus maintaining content effective stress.

During application of pressure, pore water pressure is equal to bulk pressure.

$$B = \frac{\Delta u_1}{\Delta \sigma_3} = \frac{445-350}{800-700} = 0.95$$

$$AB = \frac{\Delta u_2}{\Delta\sigma_1 - \Delta\sigma_3} = \frac{640-445}{575-0}$$

$$= 0.339$$

$$A = \frac{0.339}{0.95} = 0.357$$

Example 24

In a consolidated drained triaxial test, a specimen of clay fails at a cell pressure of 60 kN/m². The effective shear strength parameter are $C' = 15$ kN/m² and $\phi' = 20°$. Determine the compresive strength.

Sol. Data given, $C' = 15$ kN/m² and $\phi' = 20°$

$$\bar{\sigma}_{3f} = 60 \text{ kN/m}^2$$

$$\text{Compressive strength} = \sigma_{1f} - \sigma_{3f}$$

We know that,

$$\bar{\sigma}_{1f} = \sigma_{3f} + \tan^2\left(45 + \frac{\pi}{2}\right) + 2C\tan\left(45 + \frac{\phi}{2}\right)$$

$$= 60\tan^2(55°) + 2 \times 15\tan 55°$$

$$= 122.376 + 42.844$$

$$\bar{\sigma}_{1f} = 165.22 \text{ kN/m}^2$$

Therefore, Compressive strength $= \sigma_{1f} - \sigma_{3f}$

$$= 165.22 - 60 = 105.22 \text{ kN/m}^2$$

Example 25

In a CU triaxial test, a soil sample was consolidated at a cell pressure of 2 kg/cm^2 and a back pressure of 1 kg/cm^2 for 24 hours. On the next day, the cell pressure was increased to 3 kg/cm^2. this resulted in the development of a pore pressure of 0.08 kg/cm^2. The axial stress was the gradually increased to 4.5 kg/cm^2 which resulted in the failure of the soil. The pre pressure recorded at failure was 0.5 kg/cm^2 Determine skempton's pore pressure parameter A and B.

Sol. We have,

$$\Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$$

[Where A and B are skempton's pore pressure parameters]

In the first case,

$$\Delta\sigma_3 = 3 - 2 = 1 \text{ kg/cm}^2$$

$$\Delta\sigma_1 = 0$$

$$\Rightarrow \quad 0.08 = B[1 + A(0 - 1)]$$

$$\Rightarrow \quad B(1 - A) = 0.08 \qquad \ldots (i)$$

In the second case

$$\Delta\sigma_1 = (4.5 - 1)$$

$$= 3.5 \text{ kg/cm}^2$$

$$\Delta\sigma_3 = 0$$

$$\Rightarrow \quad (0.50 - 0.08) = B[0 + A(3.5 - 0)]$$

$$\Rightarrow \quad 0.42 = (AB \times 3.5)$$

$$\therefore \quad AB = \left(\frac{0.42}{3.5}\right) = \mathbf{0.12} \qquad \ldots (ii)$$

Put the value of $AB = 0.12$ From (ii) In (i)

$$\Rightarrow \quad B - AB = 0.08$$

$$\Rightarrow \quad B - 0.12 = 0.08$$

$$\Rightarrow \quad B = (0.08 + 0.08)$$

$$\boxed{\therefore\ B = 0.2} \qquad \ldots (iii)$$

Put the value of $B = 0.2$ In (ii)

$$A \times 0.2 = 0.12$$

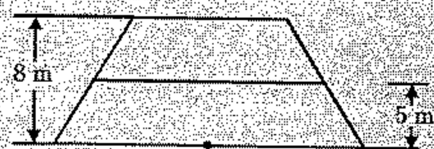
$$\Rightarrow \quad A = \left(\frac{0.12}{0.2}\right)$$

$$\boxed{A = 0.6}$$

Example 26

An Embankment is being built of a soil whose effective stress shear strength parameter c' and ϕ' are respectively $c' = 100$ kN/m^2 and $\phi' = 20°$. The unit weight of soil 17 kN/m^3. The pore pressure parameter are $A = 0.6$, $B = 0.8$. The height of Embankment has just been raised from 5 m to 8m. Determine the shear strength of the soil at the base of embankment. It can be assumed that lateral pressure at any point is one half the vertical pressure

Sol. Data given



$$c' = 100 \text{ kN/m}^2$$
$$\phi' = 20°$$
$$\gamma = 17 \text{ kN/m}^3$$
$$A = 0.6, \quad B = 0.8$$

From coulomb's theory, the shear strength at base footing, we have

$$\tau_f = (c' + \bar{\sigma}\tan\phi') \quad \text{... (i)}$$

where
$$\bar{\sigma} = (\sigma - u)$$
$$u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$$

$\Rightarrow$
$$\Delta\sigma_1 = (\Delta\sigma_3 N'_\phi + 2c'\sqrt{N\phi})$$

Assume initial pore water pressure to be zero.

$$\Delta u = u$$

$\Rightarrow$
$$u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$$

Total stress at the base of embankment is

$$= 17 \times 8 = 136 \text{ kN/m}^2 \quad \text{... (ii)}$$
$$\Delta\sigma_1 = 3 \times 17 = 51 \text{ kN/m}^2$$
$$\Delta\sigma_3 = \left(\frac{\Delta\sigma_1}{2}\right) = \frac{51}{2} = 25.5 \text{ kN/m}^2$$

$\therefore$
$$u = 0.8[25.5 + 0.6(51 - 25.5)]$$
$$= \mathbf{32.64} \quad \text{... (iii)}$$

$\therefore$
$$\bar{\sigma} = (\sigma - u)$$
$$= (136 - 32.64)$$
$$= 103.36 \text{ kN/m}^2$$

$\therefore$
$$\tau_f = c' + \bar{\sigma}\tan\phi'$$
$$= 100 + 103.36 \times \tan 20$$

$$\boxed{\tau_f = 137.62 \text{ kN/m}^2}$$

OBJECTIVE TYPE QUESTIONS

1. In a unconfined compression test on a saturated clay, the undrained shear strength was found to be 6 t/m^2. If a sample of the same soil is tested in an undrained condition in triaxial compression at a cell pressure of 20 t/m^2, then the major principal stress at failure will be

   (a) 48 t/m^2 (b) 32 t/m^2
   (c) 24 t/m^2 (d) 12 t/m^2

2. A laboratory vane shear test apparatus is used to determine the shear strength of a clay sample and only one end of the vane takes part in shearing the soil. If T = applied torque, H = height of vane and D = diameter of the vane, then shear strength of the clay is given by

   (a) $\dfrac{T}{\pi D^2\left(H+\dfrac{D}{6}\right)}$ (b) $\dfrac{T}{\pi D^2\left(\dfrac{H}{2}+\dfrac{D}{12}\right)}$

   (c) $\dfrac{T}{\pi D^2\left(H+\dfrac{D}{10}\right)}$ (d) $\dfrac{T}{\pi D^2\left(H+\dfrac{D}{12}\right)}$

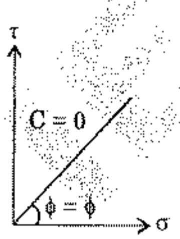
3. Which of the following laboratory triaxial test parameters should one specify to be carried out in connection with the initial stability of footing on saturated clay?

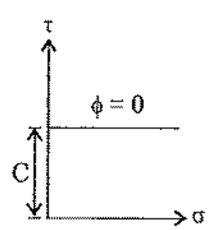
   1. c_{cu}, ϕ_{cu} - Consolidated undrained
   2. c_u, ϕ_u - Undrained
   3. c'_d, ϕ'_d - Drained

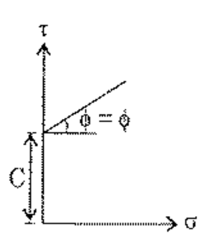
   Select the correct answer using the codes given below:

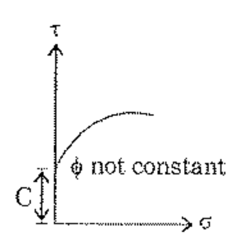
   (a) 1 alone (b) 2 alone
   (c) 1 and 3 (d) 1, 2 and 3

4. Which one of the following figure gives the failure envelope for a normally consolidated saturated clay sample tested in triaxial test under drained conditions?

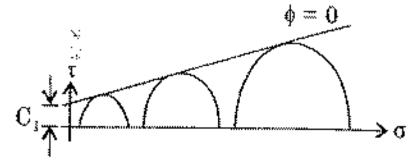
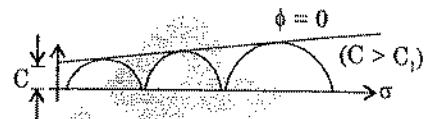
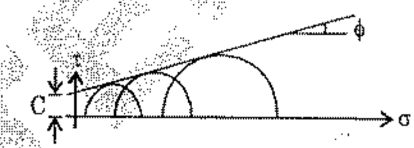
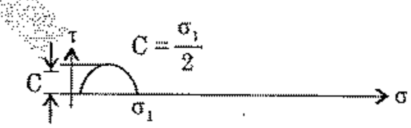
(a) 

(b) 

(c) 

(d) 

5. Match **List-I** (Type of shear tests) with **List-II** (Mohr circle and its envelope) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Undrained test on normally consolidated saturated clays	**1.** 
B. Consolidated undrained test on normally consolidated saturated clays	**2.** 
C. Drained tests on saturated cohesive soil	**3.** 
D. Unconfined test on clays	**4.** 

Codes:

	A	B	C	D
(a)	1	4	3	2
(b)	1	2	3	4
(c)	4	3	2	1
(d)	3	2	1	4

6. A soil fails under an axial vertical stress of 100 kN/m^2 in unconfined compression test. The failure plane makes an angle of 50° with the horizontal. The shear parameters 'c' and 'ϕ' respectively will be

   (a) 41.9 kN/m^2, 0° (b) 50.0 kN/m^2, 0°

   (c) 41.9 kN/m^2, 10° (d) 50.0 kN/m^2, 10°

7. A flownet constructed to determine the seepage through an earth dam which is homogeneous but anisotropic, gave four flow channels and sixteen equipotential drops. The coefficients of permeability in the horizontal and vertical directions are 4.0×10^{-7} m/s and 1×10^{-7} m/s respectively. If the storage head was 20 m, then the seepage per unit length of the dam (in m^3/s) would be

   (a) 5×10^{-7} (b) 10×10^{-7}

   (c) 20×10^{-7} (d) 40×10^{-7}

8. A clay soil specimen when tested in unconfined condition gave an unconfined compressive strength of 100 kN/m^2. A specimen of the same clay with the same initial condition is subjected to a UU triaxial test under a cell pressure of 100 kN/m^2. Axial stress (in kN/m^2) at failure would be

   (a) 150 (b) 200

   (c) 250 (d) 300

9. If 's' is the shear strength, 'c' and ϕ are shear strength parameters, and 'σ_n' is the normal stress at failure, then Coulomb's equation for shear strength of the soil can be represented by

(a) $c = s + \sigma_n \tan \phi$
(b) $c = s - \sigma_n \tan \phi$
(c) $s = \sigma_n + c \tan \phi$
(d) $s = c - \sigma_n \tan \phi$

10. A and B are Skempton's pore pressure coefficients. For saturated normally consolidated soils,

(a) $A > 1$ and $B > 1$
(b) $A > 1$ and $B < 1$
(c) $A < 1$ and $B > 1$
(d) $A < 1$ and $B = 1$

11. A dry sand specimen is put through a triaxial test. The cell pressure is 50 kPa and the deviator stress at failure is 100 kPa, the angle of internal friction for the sand specimen is

(a) 15°
(b) 30°
(c) 37°
(d) 45°

12. **Assertion (A):** In box shear test, the failure plane is predetermined and is horizontal.
Reason (R): The shear stress is applied in the vertical direction.

13. **Assertion (A):** In the case of unconfined compression test, Mohr's circle passes through the origin.
Reason (R): The major principal stress is zero.

14. Shear failure of soils takes place when

(a) the angle of obliquity is maximum
(b) maximum cohesion is reached in cohesive soils
(c) ϕ reaches its maximum value in cohsionless soils
(d) residual strength of the soil is exhausted

15. A triaxial test was conducted on a granular soil. At failure $\frac{\sigma_1}{\sigma_3} = 4$. The effective minor principal stress at failure was 100 kPa. The values of approximate ϕ and the principal stress difference at failure are, respectively

(a) 45° and 570 kPa
(b) 40° and 400 kPa
(c) 37° and 300 kPa
(d) 30° and 200 kPa

16. In the consolidated drained test on a saturated soil sample, pore water pressure is zero during

(a) consolidation stage only
(b) shearing stage only
(c) both consolidation and shearing stages
(d) loading stage

17. In a Mohr's diagram, a point above Mohr's envelope indicates

(a) imaginary condition
(b) safe condition
(c) imminent failure condition
(d) condition of maximum obliquity

18. Which one of the following is the reason for the likelihood of erroneous results of a Direct Shear Test on a saturated clay sample?

(a) The test amounts to undrained test
(b) Failure plane is predetermined
(c) Progressive failure might take place
(d) Drainage conditions are not controllable

19. Match **List-I** (Field problems) with **List-II** (Type of laboratory shear test) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Stability of a clay foundation of an embankment, whose rate of construction is such that some consolidation occurs	**1.** Undrained triaxial test
B. Initial stability of a footing on saturated clay	**2.** Drained triaxial test
C. Long-term stability of a slope in stiff, fissured clay	**3.** Consolidated undrained test
D. Foundation on soft marine clay deposits.	**4.** Quick vane shear test

Codes:

	A	B	C	D
(a)	1	3	4	2
(b)	1	3	2	4
(c)	3	1	2	4
(d)	3	1	4	2

20. In a direct shear test, the shear stress and normal stress on a dry sand sample at failure are 0.6 kg/cm^2 and 1 kg/cm^2 respectively. The angle of internal friction of the sand will be nearly

(a) 25°
(b) 31°
(c) 37°
(d) 43°

21. **Assertion (A):** Stress paths can be plotted for stress conditions during triaxial test.
Reason (R): It is not possible to control drainage in a triaxial test.

22. Match **List-I** (Test) with **List-Ii** (Property) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Proctor Test	**1.** Grain Size Analysis
B. Vane Test	**2.** Shear Strength
C. Penetration Test	**3.** Bearing Capacity
D. Hydrometer Test	**4.** Compaction

Codes:

	A	B	C	D
(a)	2	4	1	3
(b)	4	2	1	3
(c)	4	2	3	1
(d)	2	4	3	1

23. A soil sample tested in a triaxial compression apparatus failed when the total maximum and minimum principal stresses were 100 kPa and 40 kPa, respectively. The pore pressure measured at failure was 10 kPa. The effective principal stress ratio at failure is

(a) 2.5
(b) 3.0
(c) 2.75
(d) 2.0

24. Consider the following statements:

1. Quick condition and liquefaction of saturated sands are based on similar phenomenon.
2. Quick condition is associated with only earth dams.
3. Liquefaction is possible in dry sand.
4. Liquefaction is associated with increase in pore water pressure due to vibrations.

Which of these statements are correct?

(a) 2 and 4
(b) 1 and 4
(c) 1 and 2
(d) 1, 3 and 4

25. A wooden bridge in Assam failed and was observed to have arched up in the middle after the earthquake. The right abutment settled 40 cm and left abutment 30 cm. The bridge was supported on wooden piles, which floated up during the earthquake. The most probable cause of failure is

(a) shear failure of soil below abutments.
(b) excessive settlement below abutments due to increased forces.
(c) liquefaction of foundation soil below abutments and piles.
(d) failure of abutments due to dynamic earth pressure behind abutments.

26. In a triaxial test at failure, major principal stress was 180 kPa, minor principal stress was 100 kPa, and pore pressure was 20 kPa. The tangent of the angle of shearing resistance of the sandy soil tested is

(a) 1/3
(b) 2/7
(c) 1/2
(d) 1/6

27. Laboratory vane shear test can also be used to determine

(a) shear parameters of silty sand
(b) shear parameters of sandy clay
(c) liquid limit of silty clay
(d) plastic limit of clayey silt

28. **Assertion (A):** The ultimate strength of soil material is determined by the stresses in the potential failure plane.

Reason (R): The critical shear stress causing failure depends upon the properties of soil as well as on the normal stress in the failure plane.

29. A CD triaxial test was conducted on a granular soil. At failure $\frac{\sigma_1'}{\sigma_3'}$ was 3.0. The effective minor principal stress of failure was 75 kPa. The principal stress difference at failure will be

(a) 75 kPa
(b) 150 kPa
(c) 225 kPa
(d) 300 kPa

30. Consider the following statements:

1. All soils can experience liquefaction under vibrations.
2. Liquefaction is generally associated with sandy soils.
3. Liquefaction is not possible in normal clays.
4. Highly sensitive clays may undergo liquefaction under vibrations.

Which of these statements are correct?

(a) 1 and 3
(b) 2 and 4
(c) 2 and 3
(d) 2, 3 and 4

31. On which of the following factors does the behaviour of sand mass to cause liquefaction during an earthquake depend largely?

1. The number of stress cycles
2. The frequency and amplitude of vibrations of the earthquake shock
3. Angle of internal friction of sand
4. Relative density of sand

Select the correct answer using the codes given below:

(a) 1, 2, 3 and 4
(b) 2, 3 and 4
(c) 1 and 3
(d) 4 only

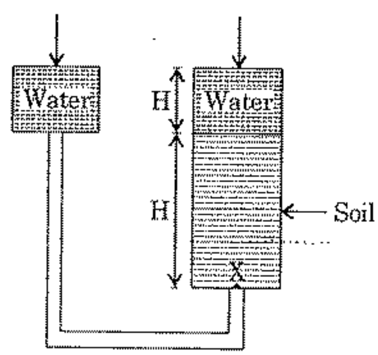
32. Match **List-I** (Situation) with **List-II** (Stress Path) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Embankment construction	**1.** Stress path moves in left side upward direction
B. Excavation of a pit	**2.** Stress path moves in left side downward direction
C. Hydrostatic loading	**3.** Stress path moves in right side upward direction
D. Lateral expansion of a backfill	**4.** Stress path moves along the horizontal axis

Codes:

	A	B	C	D
(a)	3	2	1	4
(b)	3	2	4	1
(c)	2	3	4	1
(d)	2	3	1	4

33. Which one of the following expressions represents the shear strength of soil at point X? Angle of shear resistance of soil is ϕ and symbols γ_w, γ_{sat}, γ_{sub}, γ_{dry} stand for unit weights of water, saturated soil, submerged soil and dry soil respectively.



(a) $\gamma_{dry}\ H\ \tan\ \phi$
(b) $[\gamma_{sat}\ H - (H + H_1)\gamma_w]\tan\ \phi$
(c) $\gamma_{sub}\ H\ \tan\ \phi$
(d) $(\gamma_{dry}\ H - H_1\ \gamma_w)\ \tan\ \phi$

34. A shear test was conducted on a soil sample. At failure the ratio of $\frac{\sigma_1 - \sigma_3}{2}$ to $\frac{\sigma_1 + \sigma_3}{2}$ is equal to unity. Which one of the following shear tests represents this condition?

(a) Drained triaxial compression test
(b) Undrained triaxial compression test
(c) Undrained triaxial compression test
(d) Unconfined compression test

35. The installation of sand drains in clayey soil causes the soil adjacent to the sand drains to undergo which one of the following?
(a) Increase in porosity
(b) Increase in compressibility
(c) Decrease in horizontal permeability
(d) Decrease in shear strength

36. **Assertion (A):** The drained tests using triaxial test apparatus are useful in the study of drainage of water through the soil sample and hence the permeability of the soil sample.
Reason (R): The permeability of soil is an important property useful in estimation of loss of imponded water through permeable soil layers below the earth dams.

37. Which one of the following is the appropriate triaxial test to assess the immediate stability of an unloading problem, such as an excavation of a clay slope?
(a) UU test
(b) CU test
(c) CD test
(d) Unconsolidated drained tests

38. Well-graded dense saturated sands have high shear strength because
(a) such sands have a better grade (superior type) of sand grains resulting in higher strength
(b) such sands have lower water content, which increases shear strength
(c) such sands have better interlocking of grains, higher inter-particle contacts and higher inter-particle frictional resistance resulting in higher strength
(d) presence of water in such sands induces capillary pressure generating higher inter granular stresses, which generate apparent cohesion and hence higher shear strength

39. A footing is resting on a fully saturated clayey strata. For checking the initial stability, shear parameters are used from which one of the following?
(a) Consolidated non-drained test
(b) Unconsolidated drained test
(c) Unconsolidated non-drained test
(d) Unconsolidated non-drained test with pore pressure measurement

40. In a shear test on cohesionless soils, if the initial void ratio is less than critical void ratio, the sample will
(a) increase in volume
(b) initially increase in volume and then remain constant
(c) decrease in volume
(d) initially decrease and then increase in volume

41. As the state of strain of an element of dense sand changes from plain strain to trivial strain condition, the effective angle of internal friction
(a) increases
(b) decreases
(c) remains constant
(d) first increases and then remains constant

42. Which one of the following factors are associated with the behaviour of sand mass during earthquake to cause liquefaction?
1. Number of stress cycles
2. The frequency and amplitude of vibration of waves generated by an earthquake.
3. Characteristics of sand
4. Relative density

Select the correct answer using the codes given below:

(a) 1, 2 and 3
(b) 1, 2, 3 and 4
(c) 2 and 4
(d) 3 and 4

43. Which one of the following different types of submerged soils is susceptible to liquiefaction under earthquake shocks?

(a) Dense sand
(b) Soft clay
(c) Loose silt
(d) Fissured clay

44. Consider the following statements:

1. A sand with its void ratio higher than its critical void ratio increases in volume when sheared.
2. A sand with its void ratio less than its critical void ratio increases in volume when sheared.
3. For a sand at critical void ratio, the volume change during shear is minimum.

Which of these statements are correct?

(a) 1, 2 and 3
(b) Only 1 and 2
(c) Only 2 and 3
(d) Only 1 and 3

45. Consider the following statements about the shearing resistance as a function of effective stress:

1. Effective stress on the failure plane governs the shearing resistance and not the total stress.
2. Two soils equally dense, consolidated to same effective stress will show different shear resistance with drainage and undrained condition.
3. The post peak drop off in shearing resistance is less pronounced in over consolidated clays and more in normally consolidated clays at same effective stress.

Which of these statements are correct?

(a) 1, 2 and 3
(b) Only 1 and 3
(c) Only 1 and 2
(d) Only 2 and 3

46. Consider the following statements:

1. Stress path is a locus of stress points developed by stress changes in the soil and can be obtained from Mohr's Stress Circle.
2. Stress path can be used to determine the intensity of stress at a point due to the application of uniformly applied circular loaded area.
3. Stress path has a value in giving insight into probable soil response-particularly if a part of the previous history stress path can be reproduced.

Which of these statements are correct?

(a) 1, 2 and 3
(b) Only 1 and 2
(c) Only 2 and 3
(d) Only 1 and 3

47. A point load of 650 kN is applied on the surface of a thick layer of clay. Using Boussinesq's elastic analysis, what is approximate value of the estimated vertical stress at a depth 2 m and a radial distance of 1.0 m from the point of application of load?

(a) 55 kN/m^2
(b) 44 kN/m^2
(c) 41 kN/m^2
(d) 37 kN/m^2

48. What does the confining pressure used in triaxial compression tests on an undisturbed soil sample represent?

(a) The in-situ total normal stress
(b) The in-situ total lateral stress
(c) The in-situ effective principal stress
(d) The in-situ shear stress

49. Consider the following statements:
Liquefaction is a phenomenon

1. observed in fine sands
2. associated with development of positive pore pressure

Which of these statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

50. Assertion (A): Shear strength parameters of sand can be estimated by conducting unconfined compression test.

Reason (R): The effective angle of shearing resistance of sand is nearly the same for dry and saturated sands, in drained condition.

51. Consider the following statements:

1. Pore pressure parameter A is a constant for a soil.
2. The shear strength of soil is a function of the effective stress in the soil and not of the total stress in the soil.

Which of these statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

52. Match **List-I** (Equipments) with **List-II** (Use) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Hydrometer	**1.** Determination of shear parameters
B. Plate load test set up	**2.** Determination of specific gravity
C. Pycnometer	**3.** Determination of bearing capacity of soils
D. Direct shear apparatus	**4.** Grain size distribution tests for clays

Codes:

	A	B	C	D
(a)	2	1	4	3
(b)	4	3	2	1
(c)	2	3	4	1
(d)	4	1	2	3

53. Which one of the following conditions is valid in case of unconfined compression test in comparison to triaxial test?

(a) Minor principal stress = 0
(b) Minor principal stress = 0.5 × major principal stress
(c) Minor principal stress = major principal stress
(d) Major principal stress = 3 × minor principal stress

54. In an unconfined compression test on stiff clay, if the failure plane made an angle of 52° to the horizontal, what would be the angle of shearing resistance?

(a) 16° (b) 14°
(c) 12° (d) 13°

55. Consider the following statements:
Apparent cohesion in sands is exhibited mainly due to

1. reduction in density.
2. increase in density.

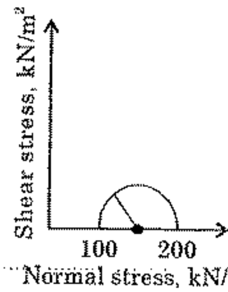
Which of these statements is/are correct?

(a) 1, 2 and 3
(b) 1 and 3 only
(c) 2 and 3 only
(d) 2 only

56. Liquefaction of foundation soil during an earthquake shall not be the reason for cracking of

(a) only floors in the building
(b) walls and roof in the building
(c) beams and columns in the building
(d) only balcony in the building

57. The results of a consolidated drained triaxial shear test on a normally consolidated clay are shown in the figure.



The angle of internal friction is

(a) $\sin^{-1}\left(\frac{1}{3}\right)$
(b) $\sin^{-1}\left(\frac{2}{3}\right)$
(c) $\sin^{-1}\left(\frac{1}{3}\right)$
(d) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

58. Which one of the following statements provides the best argument that direct shear tests are not suited for determining shear parameters of a clay soil?

(a) Failure plane is not the weakest plane.
(b) Pore pressures developed cannot be measured.
(c) Satisfactory strain levels cannot be maintained.
(d) Adequate consolidation cannot be ensured.

59. The upstream slope of an earth dam under steady seepage condition is

(a) equipotential line
(b) phreatic line
(c) flowline
(d) seepage line

60. Consider the following statements related to triaxial test:

1. Failure occurs along pre-determined plane.
2. Intermediate and minor principal stresses are equal.
3. Volume changes can be measured.
4. Field conditions can be simulated.

Which of these statements are correct?

(a) 1, 2 and 3
(b) 1, 2 and 4
(c) 1, 3 and 4
(d) 2, 3 and 4

61. A vane 20 cm long and 10 cm in diameter was pressed into a sub marine clay at the bottom of a bore hole. Torque was applied gradually and failure occurred at 1000 kg-cm. The cohesion of the clay in kg/cm^2 is

(a) $\frac{1}{\pi} \times \frac{6}{7}$
(b) $\frac{1}{\pi} \times \frac{5}{7}$
(c) $\frac{1}{\pi} \times \frac{4}{7}$
(d) $\frac{1}{\pi} \times \frac{3}{7}$

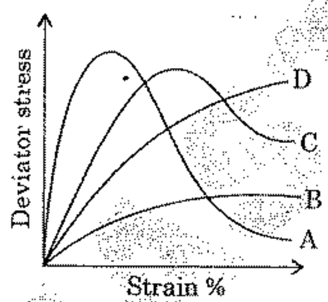
62. **Assertion (A):** Mohr's circle for unconfined compression test passes through the origin.
Reason (R): In an unconfine compression test, the axial stress is equal to confining stress.

63. Consider the following statements relating to triaxial tests:
1. Change in length of the specimen in a triaxial shear test is required to be accounted for in the calculation of stresses at failure.
2. In 'Drained Triaxial Shear Test', drainage is permitted under a specified all-around pressure until consolidation is complte. The principal stress difference is then applied with no drinage being permitted.
3. Unconfined compression test is a special case of triaxial shear test.

Which of these statements are correct?

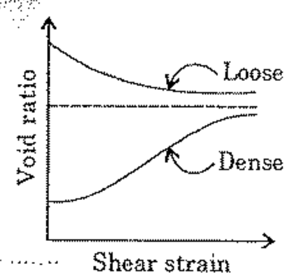
(a) 1 and 2
(b) 1 and 3
(c) 2 and 3
(d) 1, 2 and 3

64. A highly sensitive undisturbed clay sample was brought to the laboratory and UU test was performed. The curve of deviator stress versus strain % plot will be (among the four curves shown in the given figure) the one labelled



(a) A
(b) B
(c) C
(d) D

65. The given figure shows the relation between void ratio and shear strain for sand under two density conditions. The void ratio corresponding to the dshed line is called



(a) optimum void ratio
(b) critical void ratio
(c) residual void ratio
(d) undisturbed void ratio

66. A vane shear test on a soil sample gives a moment of total resistance M. The shear stress at failure, S being more or less uniform at top and bottom surface of cylinder of soil, is given by (where H = Height of vane, D = Diameter of vane)

(a) $S = \dfrac{2M}{\pi D^2 H}$
(b) $S = \dfrac{2M}{\pi D^2 (H + D)}$
(c) $S = \dfrac{2M}{\pi D^2 \left(H + \dfrac{D}{3}\right)}$
(d) $S = \dfrac{2M}{\pi D H}$

67. For a partially saturated soil, Δu, the increase in pore water pressure, when no drainage is permitted, is expressed as (where A and B are Skempton's pore pressure parameters and $\Delta\sigma_1$ and $\Delta\sigma_3$ are major and minor principal stress increaments)

(a) $\Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$
(b) $\Delta u = A[\Delta\sigma_3 + B(\Delta\sigma_1 - \Delta\sigma_3)]$
(c) $\Delta u = \Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)$
(d) $\Delta u = \Delta\sigma_3 + B(\Delta\sigma_1 - \Delta\sigma_3)$

68. Given that the effective angle of internal friction of a soil is 10°, the angle between the failure plane and the major principal plane will be

(a) 17.5°
(b) 27.5°
(c) 40°
(d) 50°

69. In which one of the following situations would one use shear parameters obtained from consolidated quick test with pore pressure measurements?

(a) Foundation on silty sands
(b) Excavation in saturated clays
(c) Pervious dams and slope stability
(d) Determination of earth pressures in saturated clays

70. Consider the following limitations:

1. Can be performed only on purely cohesionless soils
2. Plane of failure is predetermined
3. There is virtually no control on drainage
4. Non-uniform distribution of stresses
5. Principal stresses in the sample cannot be determined

The limitations inherent in direct shear test include

(a) 1, 2 and 3
(b) 2, 3 and 4
(c) 3, 4 and 5
(d) 1, 2 and 5

71. Match **List-I** (Field problems) with **List-II** (Type of laboratoryshear test to the carried out) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Stability of a clay foundation of an embankment whose rate of construction is such that some consolidation occurs during construction	1. Undrained triaxial test 2. Drained triaxial test
B. Initial stability of footing on saturated clay	
C. Long-term stability of a slope in stiff fissured clay	3. Consolidated undrained triaxial test
D. Foundation on soft marine clay deposit	4. Quick vane shear test

Codes:

	A	B	C	D
(a)	1	3	2	4
(b)	1	3	4	2
(c)	3	1	2	4
(d)	3	1	4	2

72. Which one of the following parameters can be used to estimate the angle of internal friction of a sandy soil?

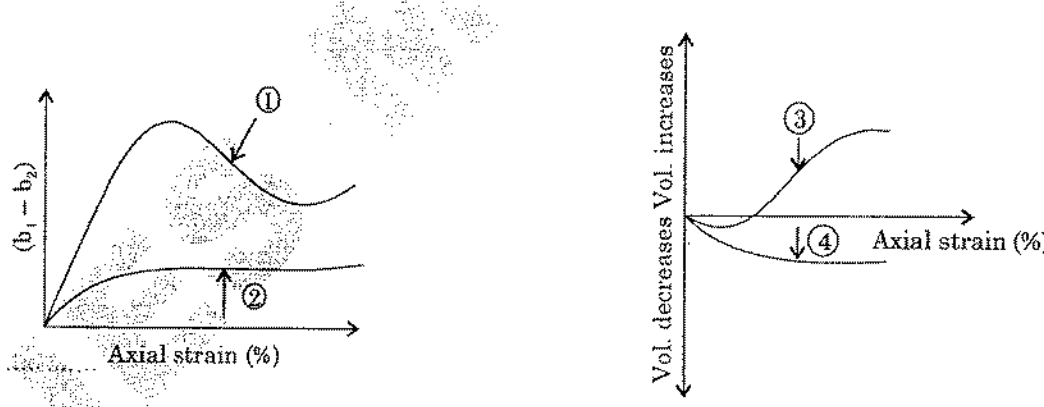
(a) Particle size
(b) Roughness of particle
(c) Particle size distribution
(d) Density index

73. Which one of the following diagrams correctly illustrates the Mohr's stress conditions of unconfined shear test on cohesive soil (x-axis: Normal stress; y-axis: Shear stress)?

74. A soil specimen having a cohesion c = 106 kN/m^2 and $\phi = 6°$ is tested in an unconfined compression test apparatus. The angle which the failure plane of the sample will make with the axis of the sample is

(a) 42°
(b) 45°
(c) 48°
(d) 51°

75. A CD triaxial test is performed on a clay soil. The given figures show two curves each for deviator stress vs axial strain % and volume change vs axial strain %. If the clay is over consolidated, then the results would be as in curves



(a) 1 and 3
(b) 1 and 4
(c) 2 and 3
(d) 2 and 4

76. A completely saturated normally consolidated clay is tested in a triaxial test under consoidated undrained condition. The value of pore pressure coefficient at failure, A_f is given by ($\Delta\sigma_3$ = change in cell pressure; $\Delta\sigma_1$ = change in axial stress, Δu = corresponding change in pore pressure)

(a) $\left(\dfrac{\Delta u - \Delta\sigma_1}{\Delta u - \Delta\sigma_3}\right)$

(b) $\left(\dfrac{\Delta u - \Delta\sigma_1}{\Delta\sigma_1 - \Delta\sigma_3}\right)$

(c) $\left(\dfrac{\Delta u - \Delta\sigma_3}{\Delta u - \Delta\sigma_1}\right)$

(d) $\left(\dfrac{\Delta u - \Delta\sigma_3}{\Delta\sigma_1 - \Delta\sigma_3}\right)$

77. Given that for a sample,

Critical void ratio = 0.50

Initial void ratio = 0.60

If the sand sample is subjected to continued shear, its volume will

(a) increase
(b) decrease
(c) not change
(d) initially increase and then decrease

78. A clear dry sand sample is tested in a direct shear test. The normal stress and the shear stress at failure are both equal to 120 kN/m^2. The angle of shearing resistance of the sand will be

(a) 25°
(b) 35°
(c) 45°
(d) 55°

79. An initial cross-sectional area of a clay sample was 15 cm^2. The failure strain was 25% in an unconfined compression test. The corrected area of the sample at failure would be

(a) 15 cm^3
(b) 20 cm^2
(c) 25 cm^2
(d) 30 cm^2

80. Consider the following statements:

The vane shear test is

1. a direct test to determine shear strength parameters of saturated clays
2. mostly useful in cohesionless soils
3. used for determining undrained shear strength of normally consolidated, sensitive clays

Which of these statements are correct?

(a) 1 and 4
(b) 2 and 4
(c) 1 and 3
(d) 2 and 3

81. A clayey sample tested in unconfined compression test is failed at a normal stress of 100 kN/m^2 and the failure plane made an angle of 45° with the horizontal. If the same sample is tested in triaxial test using lateral pressure of 30 kN/m^2, then deviator stress, shear stress on principal plane and cohesion, respectively, would be

(a) 70 kN/m^2, 70 kN/m^2, 100 kN/m^2
(b) 70 kN/m^2, zero, 50 kN/m^2
(c) 100 kN/m^2, 70 kN/m^2, zero
(d) 100 kN/m^2, zero, 50 kN/m^2

82. Consider the following statements:

Of these statements:

(a) both A and R are true and R is the correct explanation of A
(b) both A and R are true but R is not a correct explanation of A
(c) a is true but R is false
(d) a is false but R is true

Assertion (A): With box-shear aparatus, quicka dn consolidated quick tests can be made only on clay samples.

Reason (R): The soils other than clays are so permeable that drainage takes place even under a very rapid application of load on the sample.

83. **Assertion (A):** The critical shear stress causing failure of soil below foundation depends upon properties of soil as well as on normal stress on the failure plane.

Reason (R): The ultimate strength of material is determined by the stresses in the potential failure plane (or plane of slip)

84. Consider the following steps in conducting consolidated drained triaxial test:
    1. Opening of the drainage valve
    2. Application of the back pressure
    3. Application of the cell pressure
    4. Shearing

    Which one of the following is the correct sequence of the steps given above?

    (a) 1-3-2-4 (b) 2-4-1-3
    (c) 1-4-2-3 (d) 2-3-1-4

85. Match **List-I** (Nomenclature) with **List-II** (Associated with) and select the correct answer using the codes below the lists:

List-I	List-II
A. Isobar	**1.** Pore pressure
B. Isochrone	**2.** Seepage
C. Conjugate plane	**3.** External loading
D. Concentric parabola	**4.** Shear strength

Codes:

	A	B	C	D
(a)	4	1	3	2
(b)	3	2	4	1
(c)	4	2	3	1
(d)	3	1	4	2

86. During the first stage of triaxial test when the cell pressure is increased from 0.10 N/mm^2 to 0.26 N/mm^2, the pore water pressure increases from 0.07 N/mm^2 to 0.15 N/mm^2. What is the value of the Skempton's pore pressure parameter B?

    (a) 0.5 (b) –0.5
    (c) 2.0 (d) –2.0

87. Match **List-I** (Investigator) with **List-II** (Equation or Law) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Skempton	**1.** $v = ki$
B. Coulomb	**2.** $\bar{\sigma} = \sigma - u$
C. Terzaghi	**3.** $S = c + \sigma \tan \phi$
D. Darcy	**4.** $U = B[\sigma_3 + A(\sigma_1 - \sigma_3)]$

Codes:

	A	B	C	D
(a)	1	2	3	4
(b)	4	3	2	1
(c)	4	2	3	1
(d)	1	3	2	4

88. Field vane shear is the approximate field test for obtaining the shear strength of which one of the following?
    (a) Stiff clay
    (b) Weathered rock
    (c) Sand
    (d) Soft clay

89. Consider the following statements:
    1. The parameters c, ϕ obtained using Coulomb's theory are emprirical.
    2. Mohr's envelop is a straight line.
    3. The characteristics of soil are not used in the construction of Mohr's circle.
    4. The strength of a soil is a function of effective stress.

    Which of these statements are correct?
    (a) 1, 2 and 3 only
    (b) 1, 3 and 4 only
    (c) 1, 2 and 4 only
    (d) 2, 3 and 4 only

90. Undrained strength of a clay soil can be obtained by conducting which of the following tests?
    1. UC test
    2. Vane shear test
    3. Cyclic triaxial test
    4. Slow direct shear test

    Select the correct answer using the codes given below.
    (a) 1 and 2
    (b) 1 and 4
    (c) 2 only
    (d) 1, 2 and 3

91. Which one of the following shear test on a saturated clay soil gives a unique effective stress Mohr's cycle?
    (a) Drained triaxial test
    (b) Unconsolidated undrained triaxial test
    (c) Consolidated undrained triaxial test
    (d) Consolidated drained triaxial test

92. What is the appropriate field test to determine the in-situ undrained shear strength of a soft clay?
    (a) Static cone penetration test
    (b) Standard penetration test
    (c) Plate load test
    (d) Vane shear test

93. The confining pressure and the deviator stress on a triaxial sample are, respectively, 100 kN/m^2 and 300 kN/m^2. What is the normal stress acting on the plane of maximum shear stress?
    (a) 150 kN/m^2
    (b) 200 kN/m^2
    (c) 250 kN/m^2
    (d) 400 kN/m^2

94. A sand when sheared is expected to undergo an increase in volume at what value of its void ratio?
    (a) Equal to critical void ratio
    (b) Less than critical void ratio
    (c) Greater than critical void ratio
    (d) Euqal to the maximum void ratio

95. In an undrained triaxial test on a saturated clay, the Poisson's ratio is
    (a) $\dfrac{\sigma_3}{\sigma_1 + \sigma_3}$
    (b) $\dfrac{\sigma_3}{\sigma_1 - \sigma_3}$
    (c) $\dfrac{\sigma_1 - \sigma_3}{\sigma_3}$
    (d) $\dfrac{\sigma_1 + \sigma_3}{\sigma_3}$

96. The undrained cohesion of a remoulded clay soil is 10 kN/m^2. If the sensitivity of the clay is 20, the corresponding remoulded compressive strength is

(a) 5 kN/m^2 (b) 10 kN/m^2
(c) 20 kN/m^2 (d) 200 kN/m^2

97. For a triaxial shear test conducted on a sand specimen at a confining pressure of 100 kN/m^2 under drained conditions, resulted in a deviator stress $(\sigma_1 - \sigma_3)$ at failure of 100 kN/m^2. The angle of shearing resistance of the soil would be

(a) 18.43° (b) 19.47°
(c) 26.56° (d) 30°

98. A sample of saturated cohesionless soil tested in a drained triaxial compression test showed an angle of internal friction of 30°. The deviatoric stress at failure for the sample at a confining pressure of 200 kPa is equal to

(a) 200 kPa (b) 400 kPa
(c) 600 kPa (d) 800 kPa

99. A clay soil sample is tested in a triaxial apparatus in consolidated-drained conditions at a cell pressure of 100 kN/m^2. What will be the pore water pressure at a deviator stress of 40 kN/m^2?

(a) 0 (b) 20 kN/m^2
(c) 40 kN/m^2 (d) 60 kN/m^2

100. A direct shear test was conducted on a cohesionless soil (c = 0) specimen under a normal stress of 200 kN/m^2. The specimen failed at a shear stress of 100 kN/m^2. The angle of internal friction of the soil (degrees) is

(a) 26.6 (b) 29.5
(c) 30.0 (d) 32.6

ANSWERS

1. (b)	2. (b)	3. (b)	4. (a)	5. (b)	6. (c)	7. (b)	8. (b)	9. (b)	10. (d)
11. (b)	12. (c)	13. (c)	14. (a)	15. (c)	16. (c)	17. (a)	18. (b)	19. (c)	20. (b)
21. (c)	22. (c)	23. (b)	24. (b)	25. (c)	26. (a)	27. ~~(c)~~ (b)	28. (a)	29. (b)	30. (d)
31. ~~(d)~~ (a)	32. (b)	33. (c)	34. (d)	35. (c)	36. ~~(b)~~ (d)	37. ~~(a)~~ (b)	38. (c)	39. ~~(d)~~ (a)	40. (d)
41. ~~(a)~~ (b)	42. (b)	43. (c)	44. (c)	45. ~~(c)~~ (a)	46. (d)	47. (b)	48. ~~(b)~~ (a)	49. (c)	50. (d)
51. (b)	52. (b)	53. (a)	54. (b)	55. (b)	56. (d)	57. (a)	58. ~~(a)~~ (b)	59. (a)	60. (d)
61. (a)	62. (c)	63. (b)	64. ~~(c)~~ (a)	65. (b)	66. (c)	67. (a)	68. (d)	69. (b)	70. (b)
71. (c)	72. (d)	73. (d)	74. (c)	75. (a)	76. (d)	77. (b)	78. (c)	79. (b)	80. (c)
81. (d)	82. ~~(a)~~ (d)	83. (a)	84. (d)	85. (d)	86. (a)	87. (b)	88. (d)	89. (b)	90. (a)
91. (b)	92. (d)	93. (c)	94. (b)	95. (a)	96. (c)	97. (b)	98. (b)	99. (a)	100. (a)

CHAPTER

10

Stability of Slopes

INTRODUCTION

- A slope in a soil mass is ecnountered when the elevation of the ground surface gradually changes from a lower level to a higher one. Such a slope may be either natural (in hilly region) or man-made (in artificially constructed embankment or excavations).
- The soil mass bounded by a slope has a tendency to slide down. The principal factor causing such a sliding failure is the self-weight of the soil. However, the failure may be aggravated due to seepage of water or seismic forces. Every man-made slope has to be properly designed to ascertain the safety of the slope against sliding failure.
- Various methods are available for analysing the stability of slopes. Generally these methods are based on the following assumptions:

1. Any slope stability problem is a two-dimensional one.
2. The shear parameters of the soil are constant along any possible slip surface.
3. In problems involving seepage of water, the flownet can be constructed and the seepage forces can be determined.

STABILITY OF INFINITE SLOPES

- In Fig. (a), X-X represents an infinite slope which is inclined to the horizontal at an angle β. On any plane YY (YY||XX) at a depth z below the ground level the soil properties and the overburden pressure are constant, Hence, failure may occur a along a plane parallel to the slope at some depth. The conditions for such a failure may be analysed by considering the equilibrium of the soil prism ABCD of width b.

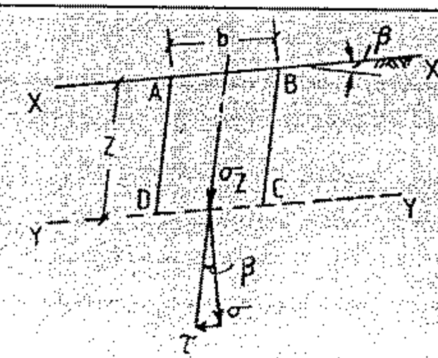


Fig (a)

Considering unit thickness, volume of the prism $V = z\ b \cos\beta$

and, weight of the prism, $W = \gamma z\ b \cos\beta$

Vertical stress on YY due to self-weight,

$$\boxed{\sigma_z = \frac{W}{b} = \gamma z \cos\beta} \qquad ...(i)$$

This vertical stress can be resolved into the following two components:

$$\boxed{\sigma = \sigma_z \cos\beta = \gamma z\, b \cos^2\beta} \qquad ...(ii)$$

and,

$$\boxed{\tau = \sigma_z \sin\beta = \gamma z \cos\beta \sin\beta} \qquad ...(iii)$$

Failure will occur if the shear stress τ exceeds the shear strength τ_f of the soil. The factor of safety against such failure is given by.

$$\boxed{F = \frac{\tau_f}{\tau}} \qquad ...(iv)$$

(i) **Cohesionless soils :** from Coulomb's equation, we have

$$\tau_f = c + \sigma \tan\phi$$

For a cohesionless soil, $c = 0$,

$$\tau_f = \sigma \tan\phi$$

Substituting in eqn. (iv)

$$F = \frac{\sigma \tan\phi}{\tau}$$

Again, substituting the expressions for σ and τ.

$$\boxed{F = \frac{\gamma z \cos^2\beta \tan\phi}{\gamma z \cos\beta \sin\beta} = \frac{\tan\phi}{\tan\beta}} \qquad ...(v)$$

When $\phi = \beta$, $F = 1$. Thus a slope in a cohesicnless soil is stable till $\beta \le \phi$, provided that no external force is present.

(ii) **c – ϕ soils:** In this case, the factor of safety against slope failure is given by,

$$\boxed{F = \frac{c + \sigma \tan\phi}{\tau}}$$

or

$$\boxed{F = \frac{c + \gamma z \cos^2\beta \tan\phi}{\gamma z \cos\beta \sin\beta}} \qquad ...(vi)$$

Let H_c be the critical height of the slope for which F = 1 (i.e. $\tau_f = \tau$)

$$\therefore \quad \gamma H_c \cos\beta \sin\beta = c + \gamma H_c \cos^2\beta \tan\phi$$

or,

$$H_c = \frac{c}{\gamma \cos\beta(\cos\beta \tan\phi - \tan\beta)}$$

or,

$$\boxed{H_c = \frac{c}{\gamma \cos^2 \beta(\tan\phi - \sin\beta)}} \quad \text{...(vii)}$$

Eqn. (vii) may also be written as:

$$\boxed{\frac{c}{\gamma H_c} = \cos^2 \beta(\tan\phi - \tan\beta)} \quad \text{...(viii)}$$

or,

$$\boxed{S_n = \cos^2\beta\ (\tan\beta - \tan\phi)} \quad \text{...(ix)}$$

where,

S_n, is a dimensionless quantity known as the stability number and is given by:

$$\boxed{S_n = \frac{c}{\gamma H_c}} \quad \text{...(x)}$$

If a factor of safety F_c is applied to the cohesion such that the mobilised cohesion at a depth H is,

$$\boxed{c_m = \frac{c}{F_c}} \quad \text{...(xi)}$$

Then,

$$\boxed{S_n = \frac{c_m}{\gamma H} = \frac{c}{F_c \gamma H}} \quad \text{...(xii)}$$

From eqns. (x) and (xii), we get,

$$\frac{c}{\gamma H_c} = \frac{c}{F_c \gamma H}$$

or,

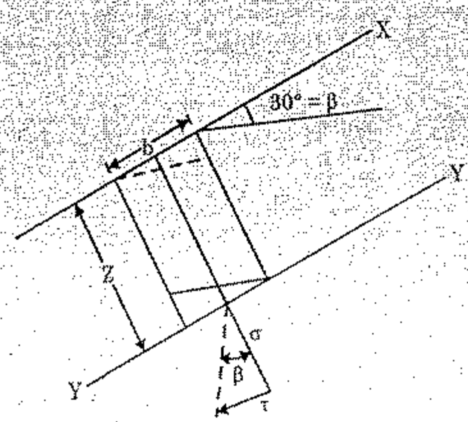
$$\boxed{F_c = \frac{H_c}{H} = F_H}$$

Hence, the factor of safety against cohesion, F_c, is the same as the factor of safety with respect to height, F_H.

Example 1

A slope of infinite extent is made in a dense sand layer at angle of 30° to the horizontal. Determine the factor of safety of the slope against shear failure if the angle of internal frcition of the soil be 36°.

Sol. With reference to figure (*a*) XX respresents the given slope and YY is a plane parallel to it at a depth *z*



Vertical stress on YY $\sigma = \sigma_z \cos\beta$

$$= \left(\frac{\gamma Z b \cos\beta}{b}\right)\cos\beta \qquad [\because \quad W = (\gamma Z b \cos\beta)]$$

$$= \gamma Z \cos^2\beta \qquad \text{... (i)}$$

Shear stress on YY $\tau = \sigma_Z \sin\beta$

$$= \left(\frac{\gamma Z b \cos\beta}{b}\right)\sin\beta$$

$$= (\gamma Z \sin\beta \cos\beta) \qquad \text{... (ii)}$$

Shear strength of the soil on the YY-plane

$$\tau_f = (c + \sigma \tan\phi) \qquad [\because \quad c = 0 \quad \text{for dense sand}]$$

$$= (0 + \gamma Z \cos^2\beta \tan\phi)$$

$$= \gamma Z \cos^2\beta \tan\phi \qquad \text{... (iii)}$$

Factor of safety against shear failure

$$F = \frac{\tau_f}{\tau} = \frac{\gamma Z \cos^2\beta \tan\phi}{(\gamma Z \sin\beta \cos\beta)}$$

$$F = \frac{\tan\phi}{\tan\beta} \qquad \text{... (iv)}$$

Data given

$\phi = 36°$

$\beta = 30°$

$$F = \left(\frac{\tan\phi}{\tan\beta}\right) = \left(\frac{\tan 36°}{\tan 30°}\right)$$

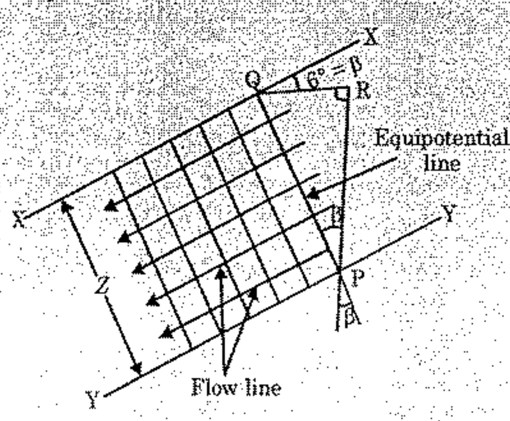
$$\boxed{F = 1.258}$$

Example 2

A slope inclined at 16° to the horizontal is to made in a cohesionless deposit having the followinmg properties $G = 2.70$, $e = 0.72$, $\phi = 35°$.

Determine the factor of safety of the slope agaist shear failure if water percolates in a direction parallel to the surface of the slope.

Sol. From Figure



$$PR = PQ\cos\beta \quad \text{... } (i)$$

$\because$ $$PQ = Z\cos\beta$$

$\therefore$ $$PR = (Z\cos\beta)\cos\beta$$

$$= Z\cos^2\beta$$

$\therefore$ Pizeometric height $= PR = Z\cos^2\beta$

Therefore Neutral stress at point $P = (\gamma_w Z\cos^2\beta)$

Total normal stress at point $P = \sigma_Z\cos^2\beta$

$$= (\gamma_{sat} Z\cos^2\beta)$$

$\therefore$ Effective stress at $P = \gamma_{sat} Z\cos^2\beta - \gamma_w Z\cos^2\beta$

$$= (\gamma_{sat} - \gamma_w) Z\cos^2\beta$$

$$= (\gamma_{sub} Z\cos^2\beta)$$

Shear stress at $P = \sigma_Z\cos\beta\sin\beta$

$$\tau = (\gamma_{sat} Z)\cos\beta\sin\beta \quad \text{... } (ii)$$

Shear strength of the soil on yy

$$\tau_f = (\bar{\sigma}_P\tan\phi + 0)$$

$$= (\gamma_{sat} Z\cos^2\beta\tan\phi) \quad \text{... } (iii)$$

$\therefore$ Factor of safety $$F = \left(\frac{\tau_f}{\tau}\right)$$

$$= \frac{\gamma_{sub} Z\cos^2\beta\tan\phi}{\gamma_{sat} Z\cos\beta\sin\beta}$$

$$\left[F = \frac{(\gamma_{sub}\tan\phi)}{(\gamma_{sat}\tan\beta)}\right]$$

Data Given:

$$\beta = 16°$$

$$G = 2.70$$

$$e = 0.72$$

$$\phi = 35°$$

$\therefore$ $$\gamma_{sub} = (\gamma_{sat} - \gamma_w)$$

$$= \frac{(G+e)\gamma_w}{(1+e)} - \gamma_w$$

$$= \frac{(G+e-1-e)\gamma_w}{(1+e)} = \frac{(G-1)\gamma_w}{(1+e)}$$

$\therefore$ $$\gamma_{sub} = \frac{(2.70-1)\times 1}{(1+0.72)} = 0.988 \text{ t/m}^3$$

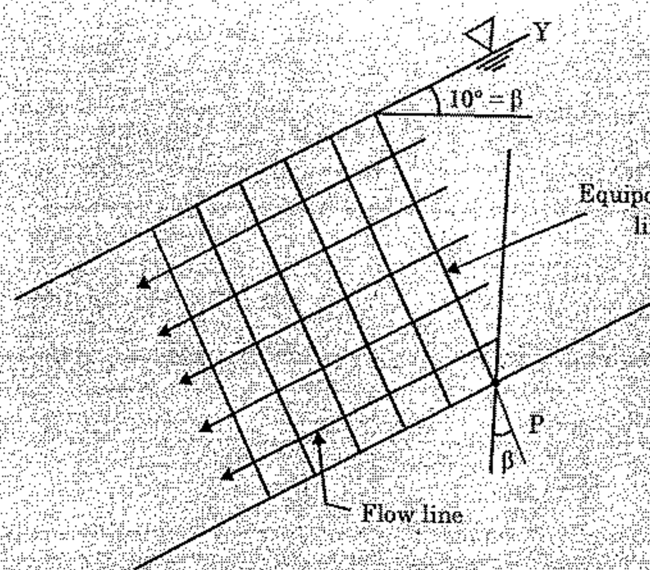
$$\gamma_{sat} = \frac{(G+e)\gamma_w}{(1+e)} = \frac{(2.70+0.72)\times 1}{(1+0.72)} = 1.99 \text{ t/m}^2$$

$\therefore$ $$F = \frac{(0.988\times\tan 35)}{(1.99\times\tan 16°)} = 1.21$$

Example 3

A long natural slope in an overconsolidated clay ($c' = 10\ kN/m^2$, $\phi' = 25°$, $\gamma_{sat} = 20\ kN/m^3$) is inclined at 10° to the horizontal. The water table is at the surface and seepage is parallel to the slope. If a plane slip had developled at a depth of 5 m below the surface, determine the factor of saftey take $\gamma_w = 10\ kN/m^3$.

Sol. Data given



$$c' = 10\ kN/m^2$$
$$\phi' = 25°$$
$$\gamma_{sat} = 20\ kN/m^3$$
$$\gamma_w = 10\ kN/m^3$$
$$\beta = 10°$$
$$\gamma_{sub} = (\gamma_{sat} - \gamma_w) = (20 - 10) = 10\ kN/m^3$$

We know the factor of safety of cohessive soil

$$F = \frac{c + \gamma_{sub} Z \cos^2\beta \tan\phi}{(\gamma_{sat} Z \sin\beta \cos\beta)}$$

$$= \frac{(10 + 10 \times 5 \cos^2 10 \tan 25)}{(20 \times 5 \sin 10 \cos 10)}$$

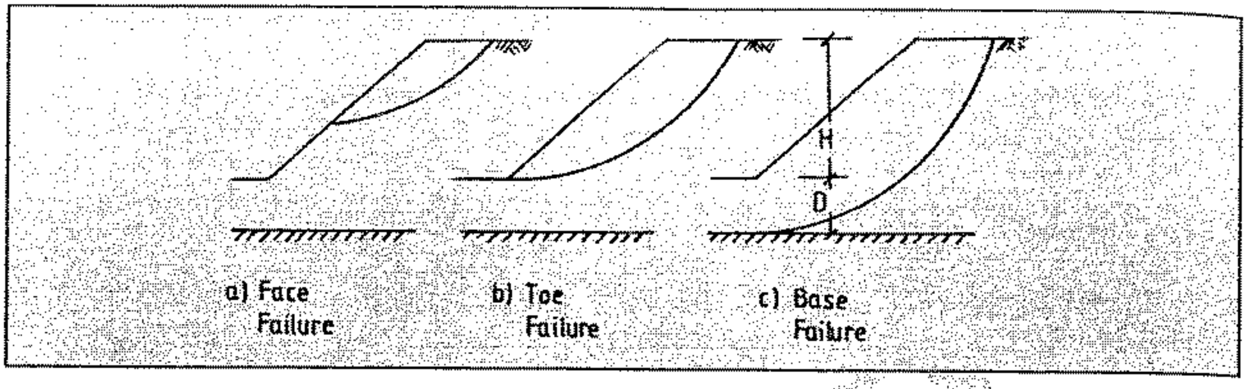
$$= 1.90$$

STABILITY OF FINITE SLOPES

- In case of slopes of limited extent, three types of failure may occur. These are : face failure, toe failure and base failure.
- Various methods of analysing the failure of finite slopes are discussed below.

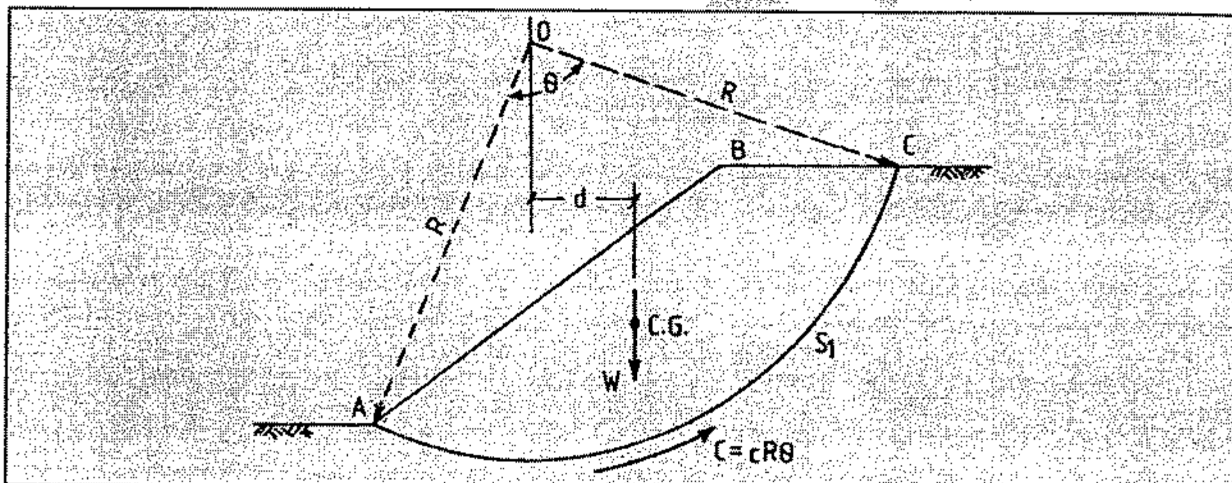
SWEDISH CIRCLE METHOD

In this method, the surface of sliding is assumed to be an arc of a circle.



(a) Purely Cohesive soils:

- Let Let AB represent the slope whose stability has to be investigated. A trail slip circle AS_1C is drawn with O as centre and OA = OC = R as radius.



Let W be weight of the soil mass AS_1CB acting vertically downwards through the centre of gravity and c be the unit cohesion of the soil. The self-weight tends to cause the sliding while the shear resistance along the plane AS_1C counteracts it.

Now, arc length $AS_1C = R \times \theta$

where,

$\theta = \angle AOC$ (expressed in radians)

$\therefore$ Total shear resistance along the plane $AS_1C = R\,\theta\,c$

Restoring moment = shear resistance × lever arm

or, $$\boxed{M_R = R\,\theta\,c \times R = R^2\,\theta\,c} \quad ...(xiii)$$

Considering unit thickness of the soil mass,

$$W = A \times 1 \times \gamma = A\,\gamma$$

where,

γ = unit weight of the soil

A = cross-sectional area of the sector AS_1CB.

- The area A can be determined either by using a planimeter or by drawing the figure to a proper scale on a graph paper and counting the number of divisions of the graph paper covered by the area.

  Now, disturbing moment, $M_d = W \times d$

  where,

  d = lever arm of W with respect to O.

- The distance d may be determined by dividing the area into an arbitrary number of segments of small width, and taking moments of all these segments about O.

  Thus, the factor of safety against slope failure,

  or, $$\boxed{F = \frac{M_R}{M_D} = \frac{cR^2\theta}{Wd}} \quad \text{(xiv)}$$

- A number of trial slip circles are chosen and the factor of safety with respect to each of them is computed. A curve is then plotted to show the variation of factor of safety with various slip circles. The slip circle corresponding to the minimum factor of safety is identified from this curve. This is the potential slip surface, and the corresponding factor of safety is the factor of safety against failure of the slope AB.

(b) Cohesive frictional soils:

With reference to Fig., (a) trial slip circle AS_1C is taken and the sector AS_1CB is divided into a number of vertical slices, preferably of equal width. The forces acting on each slice are:

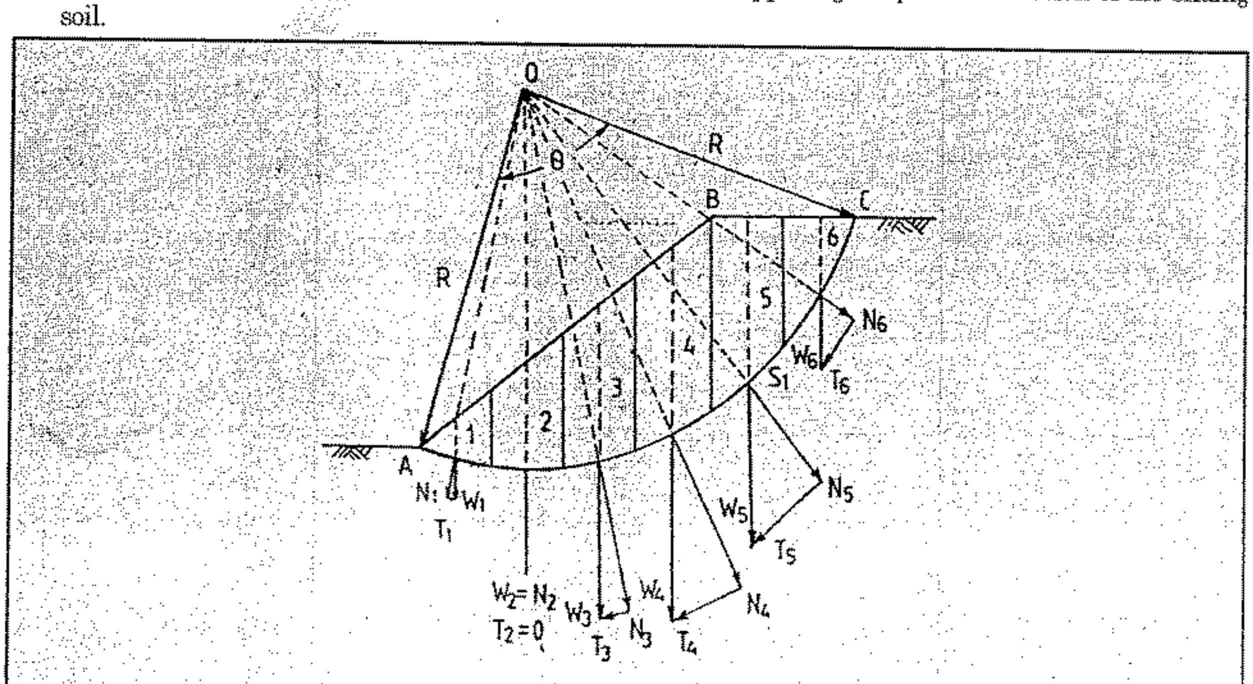
(i) Self-weight, W, of the slice, acting vertically downwards through the centre of gravity. Considering unit thickness of the slice,

$$\boxed{W = \gamma \times b_a \times l_a} \quad ...(xv)$$

where,

b_a and l_a represent the average height and length of the slice respectively.

(ii) The cohesive force, C, acting along the arc in a direction opposing the probable motion of the sliding soil.



$$\boxed{c = cl_a} \quad ...(xvi)$$

where,

c = unit cohesion,

l_a = average length of slice

(iii) Lateral thrust from adjacent slices, E_L and E_R. In simplified analysis it is assumed that, $E_L = E_R$. Hence the effects of these two forces are neglected.

(iv) Soil reaction R across the arc. According to the laws of friction, when the soil is about to slide, R will be inclined to the normal at an angle ϕ.

(v) The vertical stresses, V_L and V_R, which are equal and opposite to each other and hence need not be considered.

- The weight W is resolved into a normal component N and a tangential component T.
- For some of the slices T will enhance the failure, for the others it will resist the failure. The algebraic sum of the normal and tangential components are obtained from:

$$\boxed{\Sigma T = \Sigma (W \sin \alpha)} \quad ...(xvii)$$

and,

$$\boxed{\Sigma N = \Sigma (W \cos \alpha)} \quad ...(xviii)$$

Now, driving moment,

$$\boxed{M_D = R \Sigma T} \quad ...(xix)$$

and, Restoring moment, $M_R = R[c \Sigma \Delta l + \Sigma N \tan \phi]$

But, $\Sigma \Delta l$ = total length of arc $AS_1C = R\theta$

$\therefore$

$$\boxed{M_R = R[cR\theta + \Sigma N \tan \phi]} \quad ...(xx)$$

$\therefore$ Factor of safety,

$$\boxed{F = \frac{M_R}{M_D} = \frac{R[cR\theta + \Sigma N \tan\phi]}{R \Sigma T}} \quad ...(xxi)$$

or,

$$\boxed{F = \frac{cR\theta + \Sigma N \tan\phi}{\Sigma T}} \quad ...(xxi)$$

- A number of trial slip circles should be considered and the factor of safety for each should be determined.
- The one corresponding to the minimum factor of safety is the critical slip surface.

Example 4

An Embankment 10 m high is inclined at an angle of 36° to the horizontal. A stability analysis by the method of slices given the following forces per running meter

Σ Shearing forces = 450 kN

Σ Normal forces = 900 kN

Σ Neutral forces = 216 kN

The length of the failure arc in 27 m. Laboratory tests on the soil indicate the effective values c′ and ϕ′ as 20 kN/m² and 18° respectively

Determine the F.O.S of the slope with respect to

(a) Shearing strength and

(b) Cohesion.

Sol. Factor of safety w.r.t shearing strength

$$F_S = \frac{c'(\gamma\theta) + \Sigma(N-U)\tan\phi'}{\Sigma N}$$

$$= \frac{20 + 27 + (900-216)\tan 18^\circ}{45^\circ}$$

$$F_S = 1.70 \qquad \text{... } (i)$$

Factor of safety w.r.t cohesion

$$F_C = \frac{c'(\gamma\theta)}{\Sigma T}$$

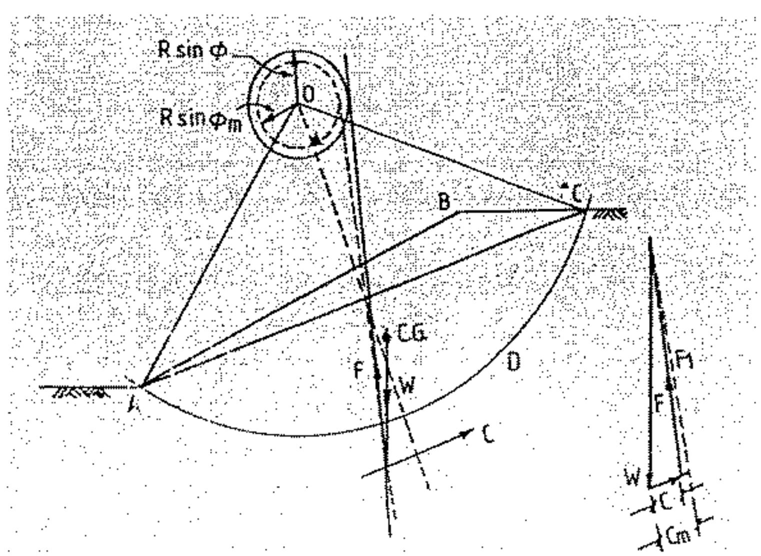
$$= \frac{20 \times 27}{450} = 1.20$$

FRICTION CIRCLE METHOD

- This method is based on the assumption that the resultant force R on the rupture surface is tangential to a circle of radius γ = R sin ϕ which is concentric with the trial slip circle. Various steps involved are given below:

1. Draw the given slope to a chosen scale.
2. Select a trial slip circle of radius R, the centre of which is located at O Fig. (a)
3. Compute r (= R sin ϕ) and draw another circle of radius r, with O as the centre.
4. Now consider the equilibrium of the sliding soil mass under the following forces:
   (i) Self-weight W of the sector ABCD.
   (ii) The cohesive force C along the plane ADC, the magnitude and direction of which can be computed as follows:

- Let c be the unit cohesion. The arc ADC is divided into a number of small elements.
- Let C_1, C_2, ..., C_n be the mobilised cohesive forces along them.
- The resultant C of these forces can be determined by drawing a force polygon.
- Now, the mobilised unit cohesion, c_m', is given by:



(a) (b)

$$\Sigma T = \Sigma (W \sin \alpha) \quad ...(xvii)$$

$$c'_m = \frac{c'}{F_c} \quad ...(xxii)$$

where,

F_c = factor of safety with respect to cohesion.

- The cohesive force is given by

$$C = c'_m L_c = \frac{c' L_c}{F_c} \quad ...(xxiii)$$

- But, summing up the moments of all forces about O and equating to zero, we get,

$$C \times L_a \times R = C \times L_c \times a \quad ...(xxiv)$$

where,

a = perpendicular distance of line of action of C from the centre of the slip circle.

$$\therefore \quad a = \frac{L_a}{L_c} \times R \quad ...(xxv)$$

(iii) The other force is the soil reaction F, which is assumed to be tangential to the friction circle.

5. Draw the triangle of forces in the following manner:
   (i) Draw a vertical line ab to represent W.
   (ii) From A draw AC, making it parallel to the line of action of F_R.
   (iii) From b drop a perpendicular bd on ac. The line bd now represents, in magnitude and direction, the cohesive force C_R required to maintain the equilibrium of the soil mass ABCD along the chosen slip circle.
6. Determine the unit cohesion c_r required for stability, from:

$$\therefore \quad c_r = \frac{c}{L_c} \quad ...(xxvi)$$

7. The factor of safety w.r.t. cohesion is now obtained from:

$$F_c = \frac{\text{actual cohesion}}{\text{required cohesion}} = \frac{c}{c_r} \quad ...(xxvii)$$

8. The factor of safety w.r.t. shear strength can be obtained as follows:
   (i) Assume a certain factor of safety with respect to the angle of internal friction. Let it be F_ϕ. The mobilised angle of internal friction is then given by:

$$\tan \phi_m = \frac{\tan \phi}{F_\phi} \quad ...(xxviii)$$

   (ii) Draw a new friction circle with O as centre and r′ as radius, where,

$$r' = R \sin \phi_m \quad ...(xxix)$$

(iii) The factor of safety w.r.t. cohesion F_c is then obtained by forming another triangle of forces. Compare F_C and F_ϕ. If they are different, go for another trial.

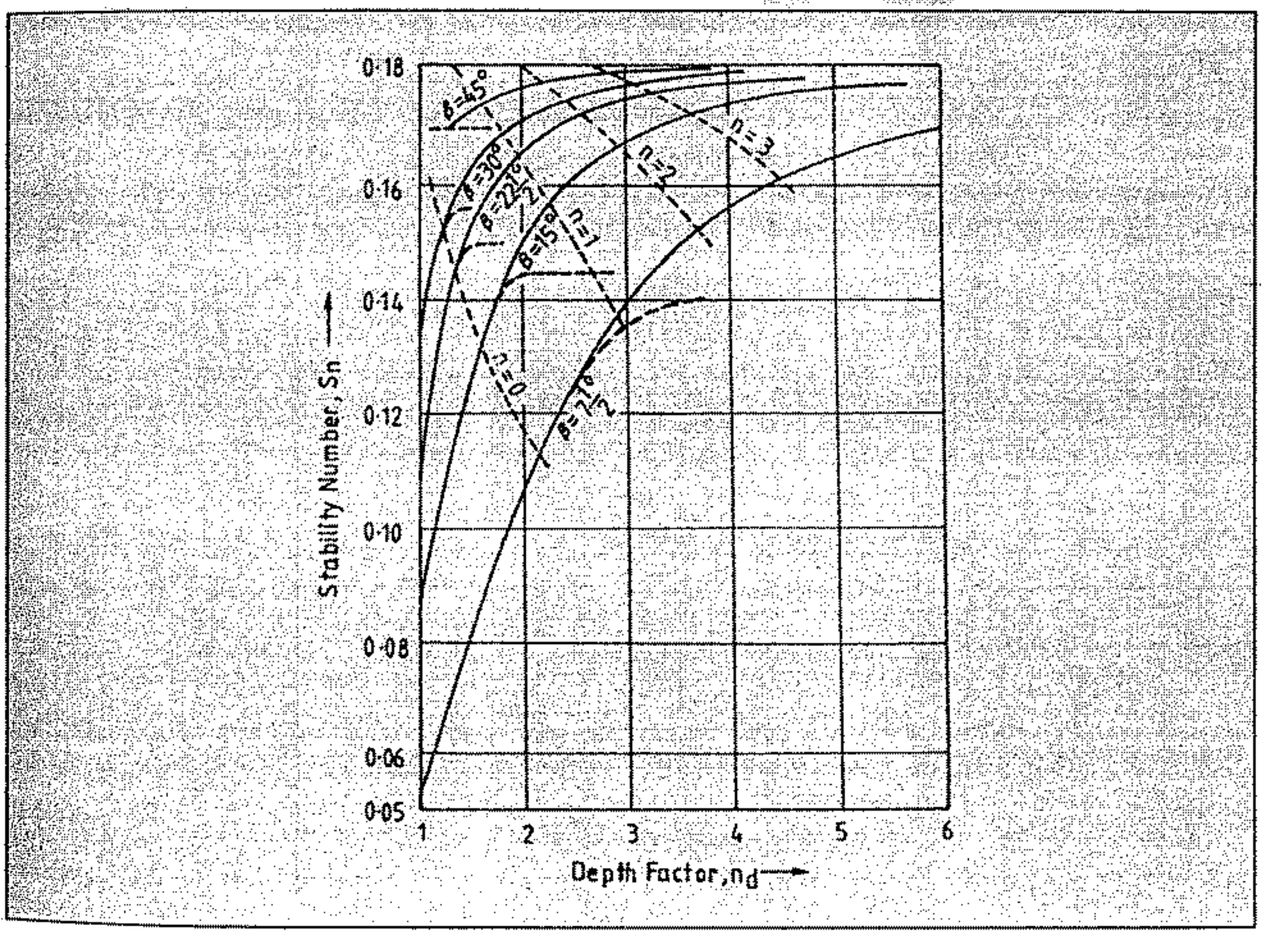
(iv) In this manner, adjust the radius of the circle until F_ϕ and F_c become equal to each other. This value is then accepted as the factor of safety for shear strength of the soil w.r.t. the given trial slip circle.

TAYLOR'S STABILITY NUMBER

- Taylor carried out stability analysis of a large number of slopes having various heights, slope angles and soil properties. On the basis of the results, he proposed a simple method by which the factor of safety of a given finite slope can be easilty determined with reasonable accuracy.
- Taylor introduced a dimensionless parameter, called Taylor's Stability Number, which is given by,

$$\boxed{S_n = \frac{c}{F_c \gamma H}} \quad \text{...(xxx)}$$

The value of S_n may be obtained from Figure below.



- The stability numbers are obtained for factor of safety w.r.t. cohesion, while the factor of safety w.r.t. friction, F_ϕ is initially taken as unity.
- The values of S_n obtained from Fig. are applicable for slip circles passing through the toe. However for slopes made in cohesive soils of limited depth and underlain by a hard stratum, the critical slip circle passes below the teo. In such cases, the value of S_n should be obtained from Fig. (b) In this figure, the depth factor plotted along the x-axis is defined as:

$$\boxed{n_d = \frac{D+H}{H}} \quad \text{...(xxxi)}$$

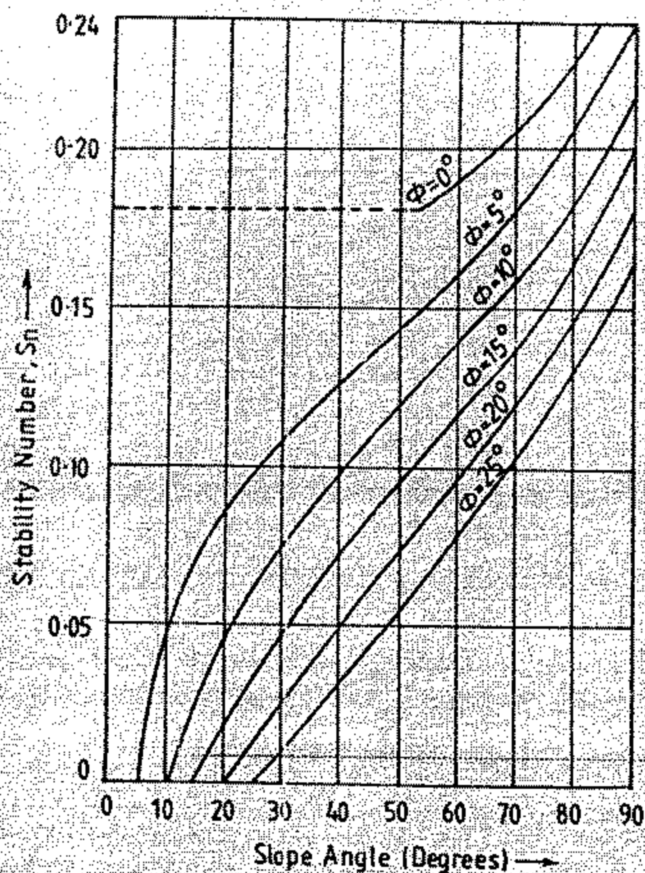


Fig. (b)

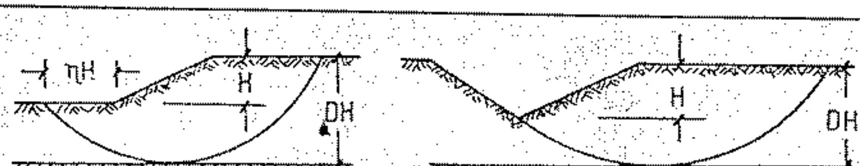
where,

D = Depth of hard stratum below toe

H = Height of slope above toe.

- Fig. (b) consists of a family of curves for various slope angles. Each curve consists of two parts. The portions drawn with firm lines are applicable to field conditions illustrated in Fig., while the portions drawn with broken lines are meant for the conditions shown in Fig. (d)
- The figure also consists of a third set of curves, shown with broken lines, for various values of n, where n represents the distances x of the rupture circle from the teo, as illustrated in Fig. (c) and is given by,

$$n = \frac{x}{H}$$



Example 5

A vertical cut is made in a clay deposit $c = 30$ kN/m^2, $\phi = 0$, $\gamma = 16$ kN/m^3. Find the maximum height of the cut which can be temporarily supported.

For $\phi = 0$ $S_n = 0.261$.

Sol. From Taylor stability No. we know that

$$S_n = \frac{c}{(F_C \gamma H)}$$

$$\Rightarrow \quad H = \frac{c}{(F_C \gamma S_n)} \qquad \text{[taking } F_c = 1.0\text{]}$$

$$= \frac{30}{(1.0 \times 16 \times 0.261)}$$

$$[H = 7.18]$$

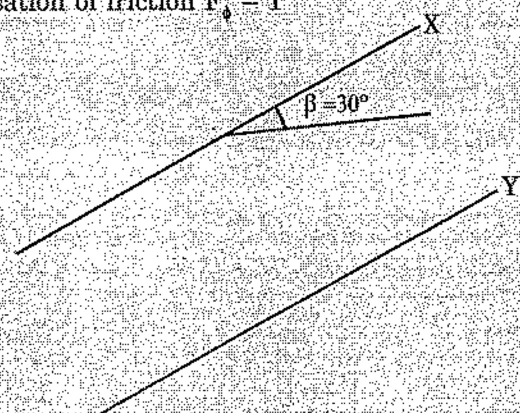
Example 6

A cutting is to made in a soil man having $\gamma = 1.8$ t/m^3 $C = 1.6$ t/m^2 and $\phi = 15°$ with side slope of 30° to the horizontal, upto a depth of 12 m below the ground level. Determine the factor of safety of the slope against shear failure. Assume that friction and cohesion are molrised to the same proportion of their ultimate values.

For $\phi = 15°$ and $\beta = 30°$

Stability no. 0.046

Sol. In case of full moblisation of friction $F_\phi = 1$



For $\quad \phi = 18°, \ \beta = 30°$

$$S_n = 0.046.$$

$$\Rightarrow \quad S_n = \left(\frac{c}{F_c \gamma H}\right)$$

$$\Rightarrow \quad F_C = \frac{c}{(S_n \gamma H)} = \frac{1.6}{(0.046 \times 1.8 \times 12)}$$

$$F_C = 1.61$$

However as friction will not be fully mobilised, the actual value of F_c will be less than thin and is to be found out by trials,

$$F_\phi = 1.25$$

$$\tan\phi = \frac{\tan 15°}{1.25} = 0.2143$$

$$\therefore \quad \phi = 12.1°$$

Referring to stability chart for $\beta = 30°$,

when $\phi = 10°, \quad S_n = 0.075$

when $\phi = 15°, \quad S_n = 0.046$

$\therefore$ When $\phi = 12.1°, \quad S_n = 0.46 + \frac{(0.075-0.046)(12.1-10)}{(15-10)} = 0.058$

$$F_c = \frac{1.6}{(0.058)(1.8)(12)} = 1.277 \approx 1.25$$

Hence, as F_c and F_ϕ are nearly equal, the factor of safety of the slope may be taken as 1.25.

Example 7

A cutting is to be made in clay for which the cohesion is 35 kN/m^2 and $\phi = 0$. The density of the soil is 20 kN/m^3. Find the maximum depth for cutting of side slope $1\frac{1}{2}$ to 1 if the F.O.S is to be 1.5.

Take stability no. For $1\frac{1}{2}$ to 1 slope and $\phi = 0$, $S_n = 0.17$

Sol. Data Given,

$$c = 35 \text{ kN/m}^2 \quad \phi = 0$$

$$\gamma = 20 \text{ kN/m}^3 \quad S_n = 0.17$$

$$F_c = 1.5$$

$$\therefore \quad C_m = \frac{c}{F_c} = \left(\frac{35}{1.5}\right) = \frac{70}{3} \text{ kN/m}^2$$

But $S_n = \frac{C_m}{\gamma H}$

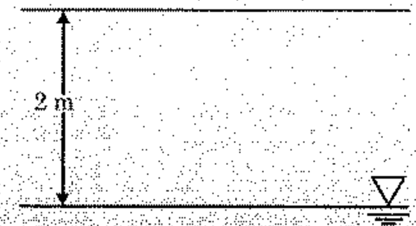
$$\Rightarrow \quad 0.17 = \frac{70}{3 \times 20 \times H}$$

$$\Rightarrow \quad H = \frac{70}{(3 \times 20 \times 0.17)} = 6.86 \text{ m}$$

Example 8

An embankment is inclined at an angle of 35° and its height is 15 m. The angle of shearing resistance is 15° and cohesion intercept is 200 kN/m^2. The unit weight of soil is 18.0 kN/m^3. If Taylor's stability no. is 0.06 find the F.O.S with respect to cohesion.

Sol. Data Given



$$\beta = 35°$$
$$H = 15 \text{ m}$$
$$\phi = 15° \quad c = 200 \text{ kN/m}^2$$
$$\gamma = 18 \text{ kN/m2}$$
$$S_n = 0.06$$

We know that $$S_n = \left(\frac{c_m}{\gamma H}\right)$$

$\Rightarrow$ $$0.06 = \frac{c_m}{(15 \times 15)}$$

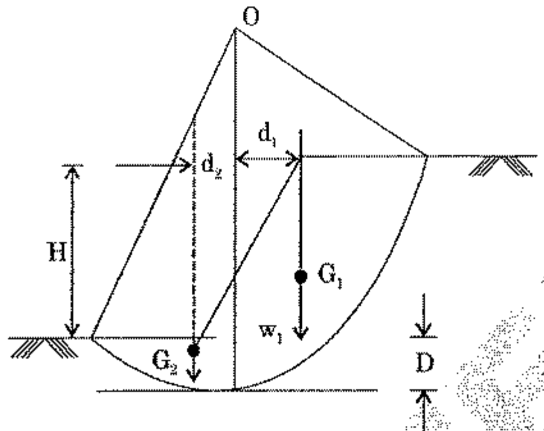
$$c_m = (0.06 \times 18 \times 15)$$
$$= 16.2 \text{ kN/m}^3$$

Factor of safety *w.r.t* cohesion

$$F_C = \left(\frac{c}{c_m}\right) = \left(\frac{200}{16.2}\right)$$
$$= 12.3$$

OBJECTIVE TYPE QUESTIONS

1. The depth factor D_f in slope failure in the situation shown in the figure will be



(a) greater than one
(b) less than one
(c) equal to one
(d) equal to zero

2. If an infinite slope of clay at a depth 5 m has cohesion of 1 t/m^2 and unit weight of 2 t/m^3, then the stabilitynumber will be
(a) 0.1
(b) 0.2
(c) 0.3
(d) 0.4

3. Consider the following statements:
Dewatering increases the slope stability of a cohesionless soil mainly because
1. it causes change in pH
2. it reduces pore water pressure
Which of the statements is/are correct?
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

4. Taylor's stability number curves are used for the analysis of stability of slopes. The angle of shearing resistance used in the chart is the
(a) effective angle
(b) apparent angle
(c) mobilized angle
(d) weighted angle

5. An excavation was made at a slope angle of 54° in homogeneous clay. When the depth of excavation reached 8 m, a slip occurred. The slip surface was likely to have passed through a point
(a) above the toe of the slope
(b) below the toe
(c) through the toe
(d) near the mid-point of the slope

6. A cohesionless soil having an angle of shearing resistance of ϕ, is standing at a slope angle of 'i'. The factor of safety of the slope is

(a) $\dfrac{\tan i}{\tan \phi}$
(b) $\tan i - \tan \phi$

(c) $\dfrac{\tan \phi}{\tan i}$
(d) $\tan \phi - \tan i$

7. For a fully saturated clay Skempton's pore pressure parameter B is
   (a) 0
   (b) between 0 and 1
   (c) 1
   (d) more than 1

8. For stability analysis of slopes of purely cohesive soils, the critical centre is taken to lie at the intersection of
   (a) the perpendicular bisector of the slope and the locus of the centre
   (b) the perpendicular drawn at one-third slope from the toe and the locus of the centre
   (c) the perpendicular drawn at two-third slope from the toe and the locus of the centre
   (d) directional angles

9. A slope is to be constructed at an angle of 30° to the horizontal from a soil having the properties, $c = 15\ kN/m^2$, $\phi = 22.5°$, $\gamma = 19\ kN/m^3$. Taylor's stability number is 0.046. If a factor of safety (with respect to cohesion) of 1.5 is required, then the safe height of the slope will be
   (a) 25.8 m
   (b) 19.1 m
   (c) 17.2 m
   (d) 11.5 m

10. Consider the following forces:
    1. Weight of the sliding wedge of slope
    2. Resultant reaction R of the slip
    3. Total cohesive resistance developed along the slip circle
    4. Critical height of slope

    Which of these are taken into consideration in the friction circle method for the equilibrium of sliding sector in the stability analysis of slope?
    (a) 1, 2 and 4
    (b) 1, 3 and 4
    (c) 1, 2 and 3
    (d) 2, 3 and 4

11. **Assertion (A):** The factor of safety obtained in the Fellenius method of slices is conservative.
    Reason (R): In the Fellenius method, the effect of horizontal forces acting on the sides of slices is neglected, but the effect of shearing forces acting on the sides of slices in included.

12. Which one of the following pairs is **not** correctly matched?
    (a) Swedish arc – Stability of slopes
    (b) Critical heigh – Stability number
    (c) Critical void ratio – Rapid draw down
    (d) Base failure – Soft clay

13. Cohesion is $15\ kN/m^2$, the unit weight of soil is $20\ kN/m^3$, the factor of safety is 1.5 and stability number is 0.05; the safe maximum height of the slope is
    (a) 5.0 m
    (b) 8.0 m
    (c) 10.0 m
    (d) 12.0 m

14. A granular soil possesses saturated density of $20\ kN/m^3$. Its effective angle of internal friction is 35 degrees. If the desired factor of safety is 1.5, the safe angle of slope for this soil, when seepage occurs at and parallel to the slope surface, will be
    (a) 25°
    (b) 23°
    (c) 20°
    (d) 13°

Common Data for Questions 15 and 16:

A canal having side slopes 1:1 is proposed to be constructed in a cohesive soil to a depth of 10 m below the ground surface. The soil properties are $\phi_u = 15°$, $c_u = 12$ kPa, $e = 1.0$, $G_s = 2.65$.

15. If Taylor's Stability Number, S_n is 0.08 and if the canal flows full, the factor of safety with respect to cohesion against failure of the canal bank slopes will be

(a) 3.7
(b) 1.85
(c) 1.0
(d) None of these

16. If there is a sudden drawdown of water in the canal and if Taylor's Stability Number for the reduced value of ϕ_w is 0.126, the factor of safety with respect to cohesion against the failure of bank slopes will be

(a) 1.85
(b) 1.18
(c) 0.84
(d) 0.53

17. An infinite soil slope with an inclination of 35° is subjected to seepage parallel to its surface. The soil has $c' = 100$ kN/m^2 and $\phi' = 30°$. Using the concept of mobilized cohesion and friction, at a factor of safety of 1.5 with respect to shear strength, the mobilized friction angle is

(a) 20.02°
(b) 21.05°
(c) 23.33°
(d) 30.00°

18. For two infinite slopes (one in dry condition and other in submerged condition) in a sand deposit having the angle of shearing resistance 30°, factor of safety was determined as 1.5 (for both slopes). The slope angles would have been

(a) 21.05° for dry slope and 21.05° for submerged slope
(b) 19.47° for dry slope and 18.40° for submerged slope
(c) 18.4° for dry slope and 21.05° for submerged slope
(d) 22.6° for dry slope and 19.47° for submerged slope

19. **List-I** below gives the possible types of failure for a finite soil slope and **List-II** gives the reasons for these different types of failure. Match the items in **List-I** with the items in **List-II** and select the correct answer from the codes given below the lists:

List-I	List-II
A. Base failure	**1.** Soils above and below the toe have same strength
B. Face failure	**2.** Soil above the toe is comparatively weaker
C. Toe failure	**3.** Soil above the toe is comparatively stronger

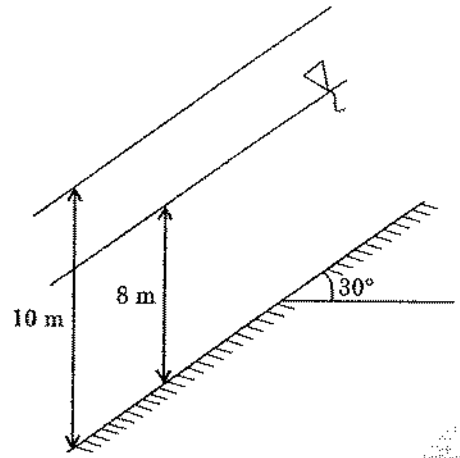
Codes:

	A	B	C
(a)	1	2	3
(b)	2	3	1
(c)	2	1	3
(d)	3	2	1

20. The water content of a saturated soil and the specific gravity of soil solids were found to be 30% and 2.70, respectively. Assuming the unit weight of water to be 10 kN/m^3, the saturated unit weight (kN/m^3) and the void ratio of the soil are

(a) 19.4, 0.81
(b) 18.5, 0.30
(c) 19.4, 0.45
(d) 18.5, 0.45

21. The factor of safety of an infinite soil slope shown in the figure having the properties $c = 0$, $\phi = 35°$, $\gamma_{dry} = 16$ kN/m³ and $\gamma_{sat} = 20$ kN/m³ is approximately equal to



(a) 0.70
(b) 0.80
(c) 1.00
(d) 1.20

ANSWERS

1. (a)	2. (a)	3. (b)	4. (a)	5. (c)	6. (c)	7. (c)	8. (d)	9. (d)	10. (c)
11. (a)	12. (c)	13. (c)	14. (d)	15. (b)	16. (d)	17. (b)	18. (a)	19. (d)	20. (a)
21. (a)									

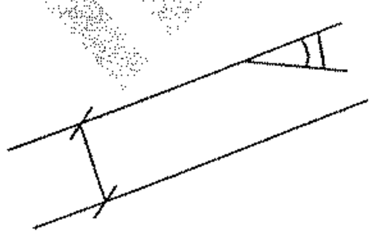
Hints

2. $S_n = \left(\frac{C_m}{\gamma H}\right)$

$= \left(\frac{1}{2\times5}\right) = 0.1$

$= 0.1$

6.



$\text{F.O.S0} = \left(\frac{\tan\phi}{\tan i}\right)$

For Cohesionless soil

7. For fully saturated clay skemption's pore pressure para meter B = 1.

9. $C_m = \frac{C}{1.5} = \frac{\cancel{15}}{\cancel{1.5}} = 10$

$S_n = 0.046 = \frac{10}{\gamma H}$

$H = \frac{10}{(19\times0.046)}$

$= 11.5$

13. $C = 15$ kNm² $\gamma_t = 20$ kNm²

$F = 1.5$

$S_n = 0.05$

$S_n = \left(\frac{C}{F\gamma H}\right)$

$\therefore \quad H = \frac{\cancel{15}}{(\cancel{1.5}\times\cancel{20}\times0.05)}$

$= 10$ m.

14. $\phi = 35°$ F = 1.5

$$\tan\beta = \left(\frac{\tan\phi}{1.5}\right)\frac{\gamma_{sub}}{\gamma_{sat}}$$
$$\beta = (13.14)°$$

For Dry $= \tan\beta = \left(\frac{\tan 30}{1.5}\right)$

$\beta = (21.05)°$ For Dry condition.

15. $$S_n = \left(\frac{C_m}{\gamma_{sub}H}\right),\ \gamma_{sat} = \frac{(G+e)\gamma_w}{(1+e)}$$
$$= \frac{(2.65+9)}{2}9.81$$
$$= 17.90\ \text{kNm}^3$$
$$C_m = (0.08 \times 8.09 \times 10)$$
$$= 6.472$$
$$\text{F.O.S.} = \left(\frac{12}{6.472}\right) = 1.85$$

16. $$S_n = \frac{C_m}{\gamma_{sat}H}$$
$$Cm = 0.126 \times 17.90 \times 10) = 22.554$$
$$\text{F.O.S.} = \left(\frac{12}{22.554}\right)$$

17. $$\tan\phi_m = \frac{\tan\phi}{1.58}$$
$$\phi_m = \tan^1\left[\frac{\tan 30}{1.5}\right]$$
$$= 21.05$$

20. $$W = 0.30\ G = 2.70$$
$$\gamma_w = 10\ \text{kNm}^3$$
$$\gamma_{sat} = \frac{(G+e)\gamma_w}{(1+e)}$$
$$= \frac{(2.70+e)\gamma_w}{(1+e)}$$
$$e \times 1 = (0.30 \times 2.70)$$
$$= 0.81$$
$$\gamma_{sat} = \frac{(2.70+0.81)\times 10}{(1+0.81)}$$
$$= 19.4\ \text{kNm}^3$$

21. $$\text{F.O.S.} = \left(\frac{\tau}{\tau_\gamma}\right)$$
$$\text{F.O.S} = \left(1-\frac{\gamma_w h}{\gamma_{sat}z}\right)\frac{\tan\phi}{\tan\beta}$$
$$= \left[1-\frac{9.8\times 2}{20\times 10}\right]\frac{\tan 35}{\tan 30}$$
$$= 1.094$$

IG

CHAPTER

11

Earth Pressure and Retaining Walls

INTRODUCTION

- Soil mass is stable when the slope of the surface of the soil mass is flatter than the safe slope. But at some places the space is limited, it is not possible to provide flat slope and the soil is to be retained at a slope steeper than the safe one.
- Therefore to retain this soil mass in a stable state a retaining structure is provided to provide the lateral support to the soil mass.
- In the design of these retaining structure it becomes imparative to know the magnitude and line of action of Earth pressure, where earth pressure is the lateral force exerted by the soil on any structure retaining that soil.
- The magnitude of the lateral earth pressure depends upon a number of factors, such as the mode of movement of wall, the flexibility of the wall, the properties of the soil, and drainage conditions.

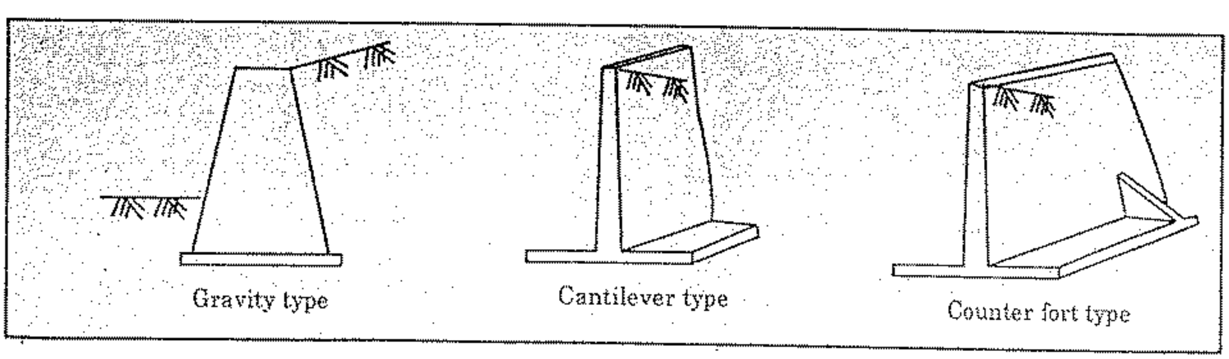
RETAINING STRUCTURES

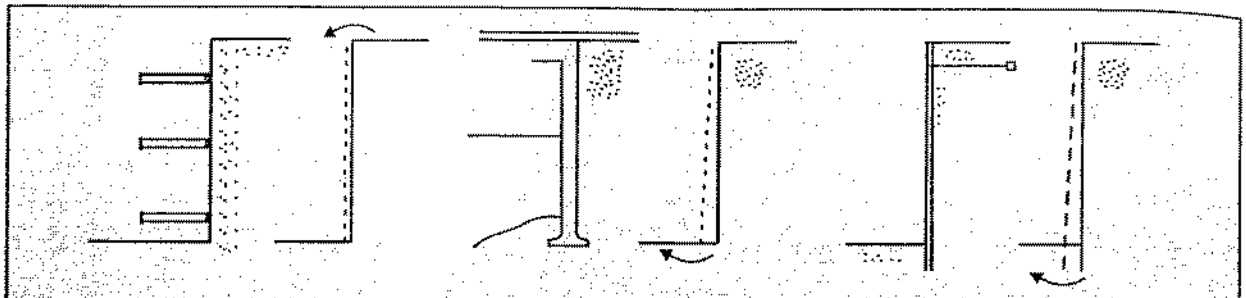
A structure which is used to hold back a soil mass is called a Retaining Structure

Types of Retaining structure

1. Retaining wall – Gravity type, Cantilever type, Counterfort type.
2. Bracings in cuts
3. Abutment of a Bridge
4. Sheet Pile/Anchored sheet Pile

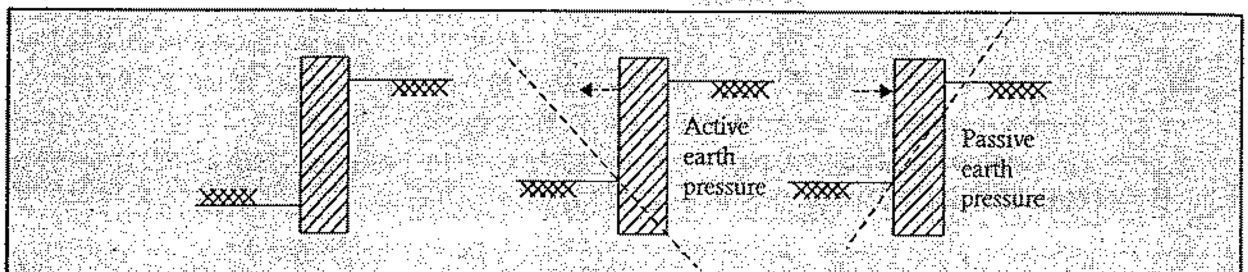
Some types of Retaining Walls.





TYPES OF LATERAL EARTH PRESSURE

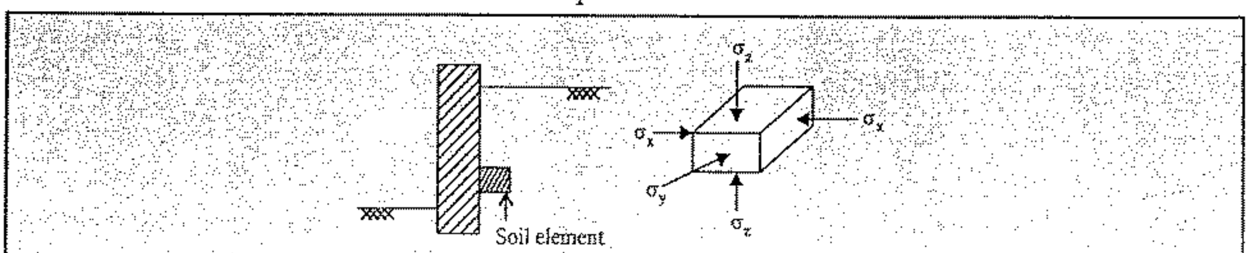
- Lateral earth pressure can be divided into 3 categories, depending upon the movement of retaining wall with respect to back fill soil.
  1. Earth Pressure **At Rest** – wall does not move at all
  2. **Active** Earth Pressure – wall moves away from the backfill soil.
  3. **Passive** Earth Pressure – wall moves towards the backfill soil.



Earth Pressure at Rest

- A soil element in its natural state at any depth z below the ground surface is not subjected to any strain, the element in this condition is known as at rest condition.
- It is possible to evaluate earth pressure at rest using the theory of elasticity.
- For analysis of earth pressure at rest, consider a soil mass element at depth z below the ground surface and following assumptions are made.

  Assumptions.
  1. Soil mass is homogenous, isotropic and semi infinite.
  2. Elastic modulus E & Poission's ratio μ is constant through out the depth.
- For Plane strain condition we can write. ε_x as.



$$\epsilon_x = \frac{1}{E}[\sigma_x - \mu(\sigma_z + \sigma_y)]$$

- We know that earth pressure at rest corresponds to zero lateral strain hence.

$\therefore$ $$\boxed{\frac{\sigma_x}{E}-\mu\frac{\sigma_y}{E}-\frac{\mu\sigma_z}{E}=0}$$

Note: Soil is not a perfectly elastic material. Hence, the above expression will not give the correct result.

$$-\frac{\sigma_x}{E}-\mu\left(\frac{-\sigma_y}{E}\right)-\mu\left(\frac{-\sigma_z}{E}\right) = 0$$

We know that, $\sigma_x = \sigma_x$

$\therefore$ $$\frac{\sigma_x}{E}(1-\mu) = \frac{\mu\sigma_z}{E}$$

$$\boxed{\sigma_x=\frac{\mu}{(1-\mu)}\sigma_z}$$

Thus,

$$\boxed{\sigma_x = K_o\sigma_z}$$

Where, k_0 = Earth pressure coefficient at Rest.

- For a perfectly cohension-less soil (c = 0)

$$\boxed{K_0=(1-\sin\phi)}$$

- If the soil is normally consolidated, (N.C. soil)

$$\boxed{K_0=0.19+0.233\log_{10}(I_P)}$$

Where, I_p = Plasticity index

- For over consolidated soil (OC soil)

$$\boxed{K_{o(OC)}=K_{0(NC)}\sqrt{O.C.R.}}$$

$$\boxed{O.C.R.=\left(\frac{\sigma}{\sigma_0}\right)}$$

Where,

σ_0 = Pre consolidation stress

Value of K_0 for various soils are as tabulated below

Soil Type	(K_O)
(1) For Dense sand	0.4 – 0.5
(2) Loose sand	0.45 – 0.5
(3) Mechanically compacted sand	0.8 – 1.0
(4) N.C. clays	0.5 – 0.6
(5) O.C. clays	1.0 – 4.0

Note: We know that $\frac{\sigma_x}{\sigma_z}=\frac{\mu}{1-\mu}$

on solving above equation we can write that $\frac{\sigma_x}{\sigma_z+\sigma_x}=\mu$

Example 1

A 5m high rigid retaining wall has to retain a backfill of dry cohesionless soil having the following properties.

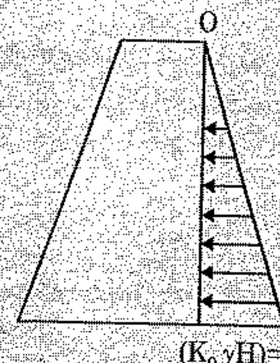
$G = 2.68$, $\mu = 0.36$, $e = 0.74$, $\phi = 30°$

(i) Plot the distribution of lateral Earth pressure on the walls

(ii) Determine the magnitude and point of application of the resultant thrust

(iii) Compute the % change in the lateral thrust if the water table rises from a great depth to the top of the backfill.

Sol. Bulk density of the dry backfill



$$\gamma_d = \left(\frac{G\gamma_w}{1+e}\right)$$

$$\gamma_d = \left(\frac{2.68\times1}{1+0.74}\right)$$

$$= 1.54 \text{ t/m}^3$$

As the wall is rigid, the lateral pressure exerted by the backfill is Earth pressure at rest.

Coefficient of Earth pressure at rest $K_0 = \left(\frac{\mu}{1-\mu}\right)$

$$\therefore \quad K_0 = \left(\frac{0.36}{1-0.36}\right) = 0.5625$$

$\therefore$ at the top of the wall ($Z = 0$), $p_a = 0$

At the base of the wall $Pa = (K_0\ \gamma)Z$

$$= (0.5625 \times 1.54 \times 5)$$

$$= 4.33 \text{ t/m}^2$$

(i) Resultant lateral thrust on the wall (considering unit width)

$$P_0 = \frac{1}{2}(K_0\ \gamma H)\times H = \frac{1}{2}\times(0.5625)\times1.54\times(5)^2 = 10.83 \text{ t/m}$$

The Resultant thrust applied at a height of $\left(\frac{5}{3}\right)$

$$= 1.67 \text{ m above the base of wall}$$

(ii) If the W.T rises to the top of the backfill, the soil will get fully submerged.

$$\gamma_{sub} = \frac{(G-1)\gamma_w}{(1+e)} = \frac{(2.68-1)\times1}{(1+0.74)}$$

$$= 0.965 \text{ t/m}^3$$

$$\therefore \quad \text{Resultant thrust} = \frac{1}{2}(K_0\gamma_{sub}H)H + \frac{1}{2}\gamma_w H^2$$

$$= \frac{1}{2}\times(0.5624)\times0.965\times(5)^2 + \frac{1}{2}\times1\times(5)^2$$

$$= 19.285 \text{ t/m}$$

(iii) % Increase in lateral thrust

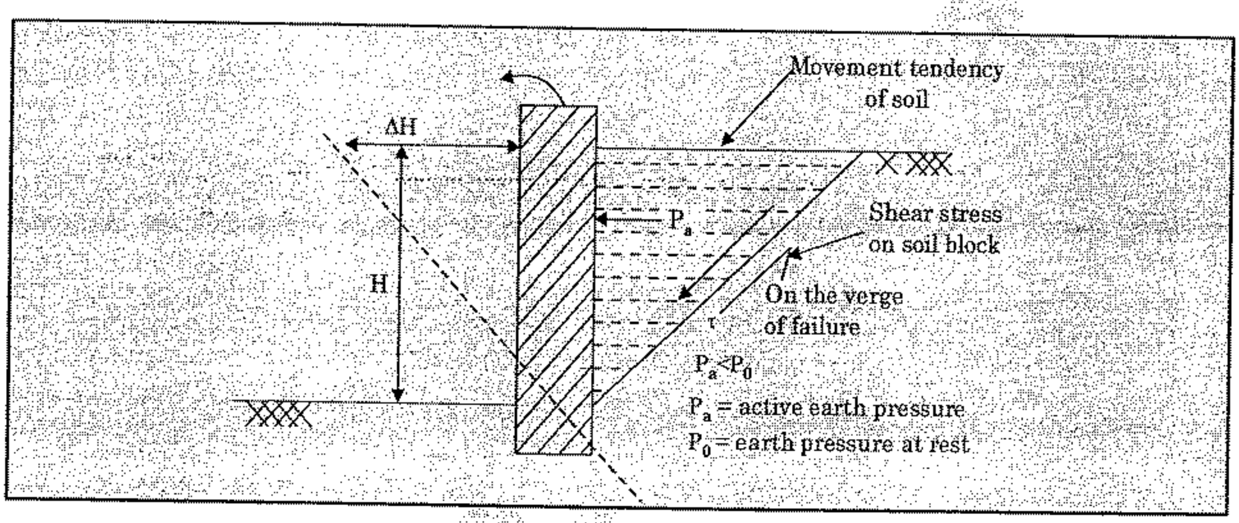
$$= \frac{(19.285 - 10.83)}{(10.83)} \times 100$$

$$= 78\%$$

ACTIVE AND PASSIVE EARTH PRESSURE

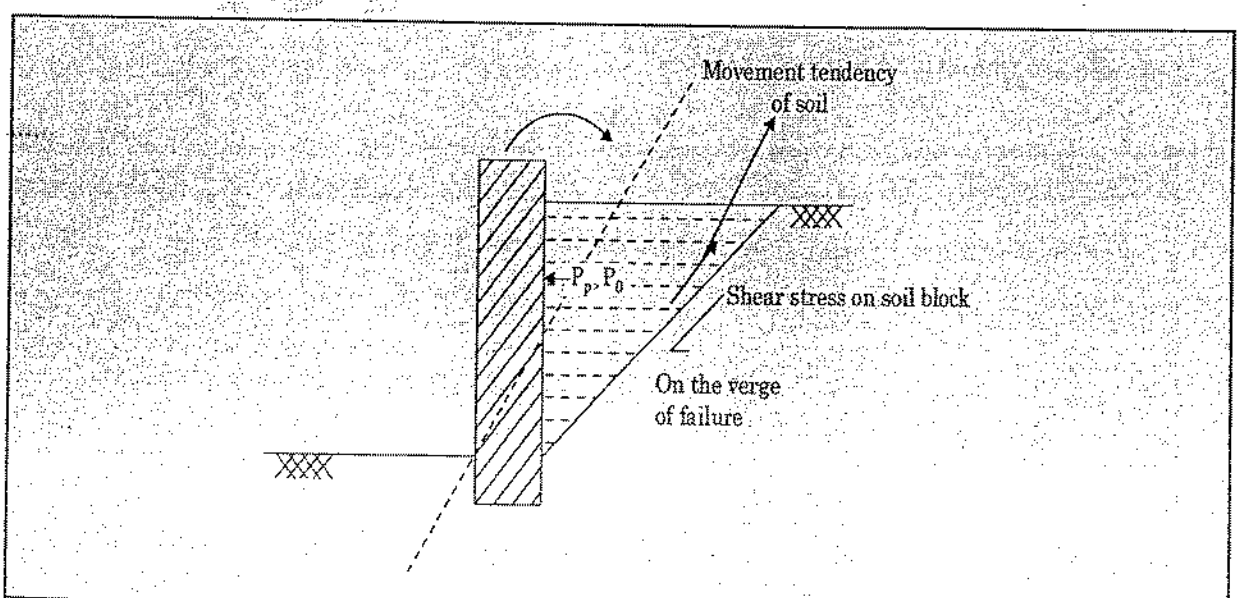
- Because of movement of wall, a block of soil mass adjacent to the retaining wall tends to break away from rest of the soil mass and due to this phenomenon, active/passive earth pressure developes.

Active Earth Pressure



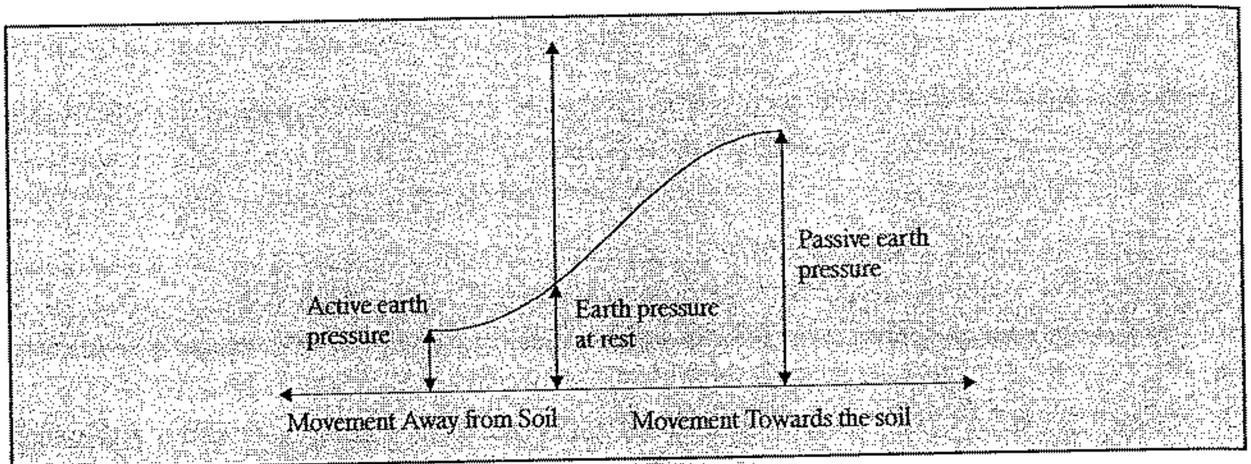
- When wall moves away from the soil, the block of soil has the tendency to move downwards and outwards. Hence, shear stress is mobilized on the soil.
- Because of the Nature of the shear stress developed in the direction opposite to movement of wall, active Earth pressure on wall will fall below. (the pressure at Rest condition) and this pressure developed on the wall when the soil is on the verge of failure is called Active Earth pressure.

Passive Earth Pressure



- Pressure developed an wall when wall moves towards the soil, a block of soil has the tendency to move up and inside. Hence, shear stress is mobilised on the soil.
- Because of the component of mobilised shear in the direction of wall, pressure on the wall increases beyond the pressure at rest condition and this is called Passive Earth Pressure.
- Full value of passive earth pressure is mobilized when the soil is on the verge of shear failure.

RELATIONSHIP BETWEEN WALL MOVEMENT & LATERAL PRESSURE



Note: *Movement of wall required for generation of Active and Passive earth pressure are as given below:*

1. $\left(\frac{\Delta H}{H}\times 100\right)$ = (0.2)% → For Dense sand } For Active Earth pressure
2. = (0.5)% → For Loose sand } For Active Earth pressure
3. $\left(\frac{\Delta H}{H}\times 100\right)$ = (2.0)% → For Dense sand for generating passive earth pressure } For Passive Earth pressure
4. = (5.0–10)% → For Loose sand } For Passive Earth pressure

EARTH PRESSURE THEORIES

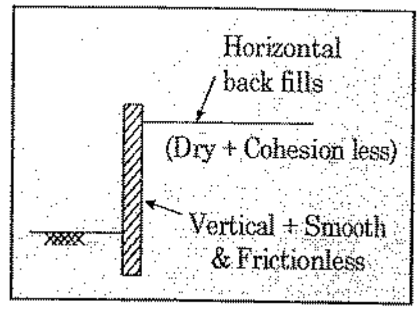
- There are two classical theories of Earth Pressure
  - (a) Rankine's theory (1857)
  - (b) Coulomb's theory (1776)
- Rankine theory came later and is considered to be simpler than Coulomb's theory.

Rankine Theory

- Rankine's theory considered stress in soil mass when it attains plastic equillibrium. Here, by plastic equillibrium we infer that every point in the soil mass experience shear failure, under the effect of shear stress developed.

Note: *In reality entire soil mass never experience plastic equillibrium. Hence development of earth pressure is a concequence of, a portion of soil mass coming under plastic equillibrium.*

Assumptions in Rankine's theory

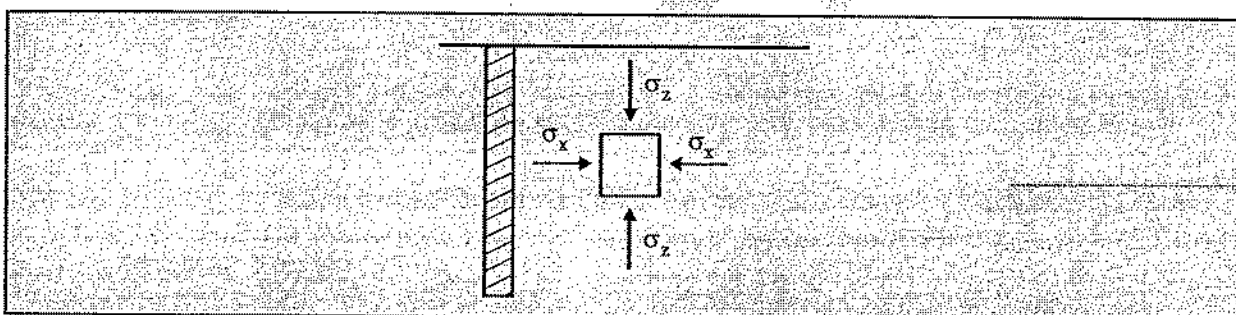


1. Soil is semi-infinite, homogeneous, isotropic, dry and cohesionless.
2. Soil is in a state of plastic condition at the time of active and passive pressure generation.
3. The backfill soil is horizontal.
4. Back of wall is vertical and smooth.
5. Rupture surface is a planar surface which is obtained by considering the plastic equilibrium of soil.

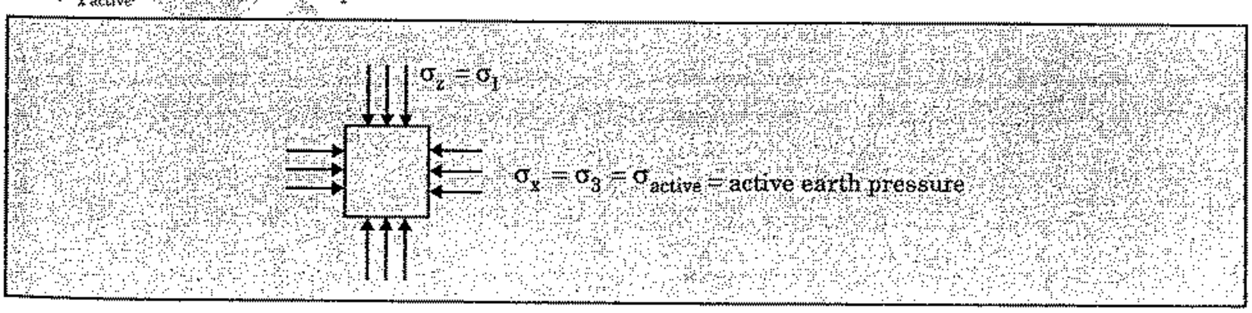
Note: 1st Assumption states that this theory was orignally deviced for cohession less soil but lather it was extended to cohesive as well as submerged soils also.

Active Earth pressure

- Consider a soil mass element at a depth Z below the ground surface, which is subjected to vertical stress; σ_z and Horizontal stress σ_x .
- As there is no shear stress acting on horizontal and vertical planes σ_x and σ_z are principal stresses.



- From the previous discussion we know that σ_z is major principal stress (σ_1) and σ_x is minor principal (σ_3) stress (from the theory of earth pressure at rest).
- As wall starts moving away from soil, (σ_z) remains constant but (σ_x) will go on reducing and on the verge of shear failure (i.e., when soil attains plastic equillibrium) its value will become ($\sigma_{x\,active}$) = Active Earth pressure.



- The value of active earth pressure p_a or σ_{active} or σ_3 can be determined in terms of σ_z or σ_1.

  We know that

$$\sigma_1 = \sigma_3\left(\frac{1+\text{Sin}\phi}{1-\text{Sin}\phi}\right)+2C\sqrt{\frac{1+\text{Sin}\phi}{1-\text{Sin}\phi}}$$

Here,
$$\sigma_1 = \sigma_z = \gamma z$$
$$\sigma_3 = p_a$$
$$\sigma_z = p_a\left(\frac{1+\text{Sin}\,\phi}{1-\text{Sin}\,\phi}\right) + 2C\sqrt{\frac{1+\text{Sin}\,\phi}{1-\text{Sin}\,\phi}}$$

or
$$p_a = \sigma_z\left(\frac{1-\text{Sin}\,\phi}{1+\text{Sin}\,\phi}\right) - 2C\sqrt{\frac{1-\text{Sin}\,\phi}{1+\text{Sin}\,\phi}}$$

Where,
$$K_a = \frac{1-\text{Sin}\phi}{1+\text{Sin}\phi}$$
K_a = Coefficient of active earth pressure

$$\boxed{p_a = K_a\sigma_z - 2C\sqrt{K_a}}$$

Note: *C & ϕ values used are effective stress parameter because drained condition is assumed to prevail in retaining walls due to provision of weep hooles. Hence σ_z used should be effective stress.*

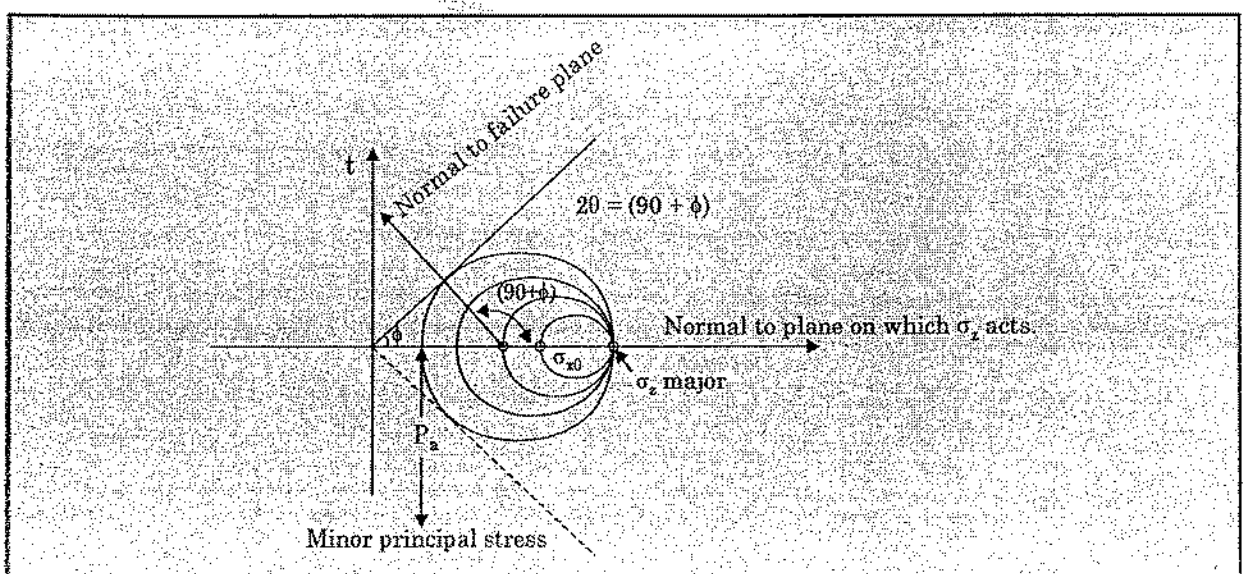
But for cohesionless soil, $\boxed{C = 0}$

$\therefore$ $\boxed{P_a = K_a\sigma_z}$

also, $\boxed{K_a = \dfrac{(1-\sin\phi)}{(1+\sin\phi)} = \tan^2\left(45 - \dfrac{\phi}{2}\right)}$

- After drawing the Mohr circle for two state of stress develope i.e., σ_1 & σ_3 we can show that the two plane of failures are inclined at

1. $45° + \dfrac{\phi}{2}$ with the horizontal (major principal plane)



Passive Earth pressure

- In case of movement of wall towards the soil, soil mass experiences a uniform compression in the horizontal direction. Hence the value of σ_z increases from its orignal value i.e. at rest condition.

- As the deformation of soil mass goes on increasing, a state comes at which $\sigma_x = \sigma_z$. But at the state of failure i.e., when the soil mass attains plastic equillibrium σ_x becomes greater than σ_z $(\sigma_x > \sigma_z)$.
- For this condition σ_z becomes minor principal stress (σ_3) and σ_x becomes major principal stress (σ_1).
- At the time of plastic failure when σ_x is maximum the soil is said to be in Passive Rankine state and σ_x (σ_1) is known as Passive Earth Pressure (p_p).

  We know that

$$\sigma_1 = \sigma_3\left(\frac{1+\operatorname{Sin}\phi}{1-\operatorname{Sin}\phi}\right) + 2C\sqrt{\left(\frac{1+\operatorname{Sin}\phi}{1-\operatorname{Sin}\phi}\right)}$$

$$\sigma_1 = p_p$$

$$\sigma_3 = \gamma z = \sigma_z$$

$$\therefore \quad p_p = \sigma_z\left(\frac{1+\operatorname{Sin}\phi}{1-\operatorname{Sin}\phi}\right) + 2C\sqrt{\frac{1+\operatorname{Sin}\phi}{1-\operatorname{Sin}\phi}}$$

where, $$K_p = \frac{1+\operatorname{Sin}\phi}{1-\operatorname{Sin}\phi}$$

K_p = Coefficient of Passive earth Pressure.

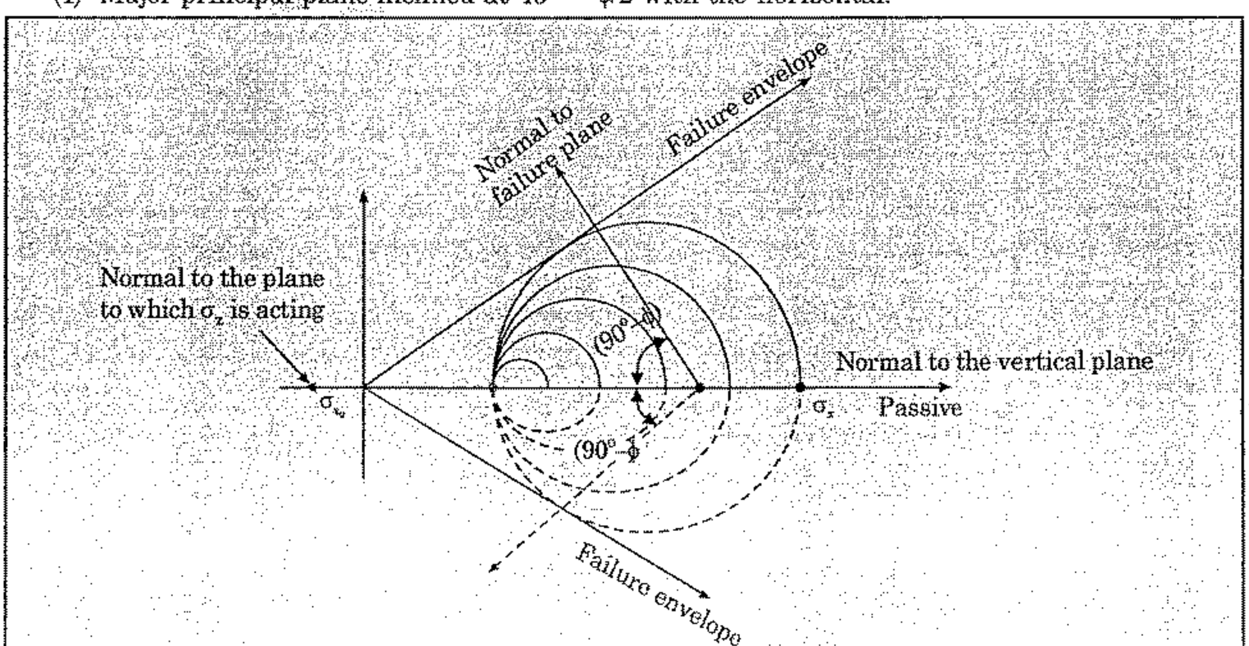
$$\therefore \quad \boxed{p_p = \sigma_z K_p + 2C\sqrt{K_p}}$$

- But for cohesionless soil, C = 0

$$\boxed{p_p = \sigma_z k_p}$$

- After drawing the Mohr circle for two state of stress we can observe that there are two failure planes (or principal planes).

  (1) Major principal plane inclined at 45° – ϕ/2 with the horizontal.



Note: $K_a = \dfrac{1-\text{Sin }\phi}{1+\text{Sin }\phi} = \tan^2(45-\phi/2)$

$K_p = \dfrac{1+\text{Sin }\phi}{1-\text{Sin }\phi} = \tan^2(45+\phi/2)$

$K_a = \dfrac{1}{K_p}$

Example 2

A counterfort wall of 10 m height retain non-cohesive backfill. The void ratio and angle of internal friction 0.7 and 30° in loose state and they are 0.4 and 40°. In dense state calculate and compare the active and passive earth pressure in both cases, take specific gravity $G_S = 2.7$. Give your comment on the results.

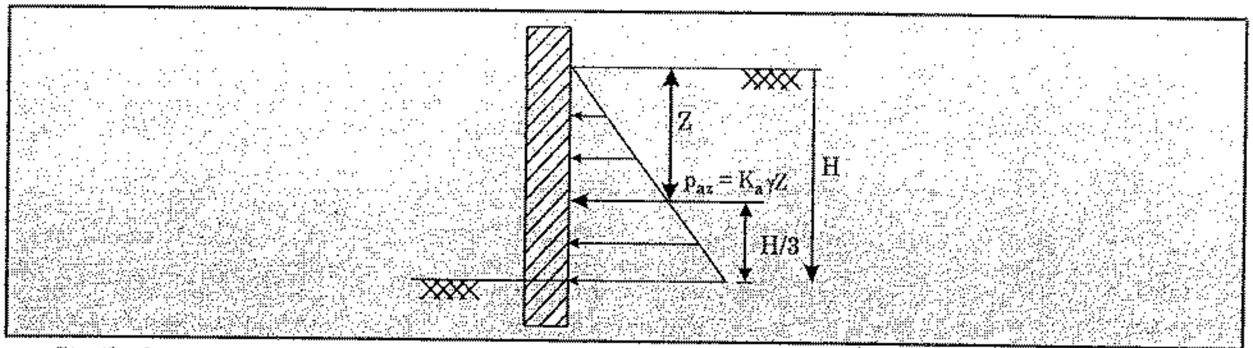
Sol.

Loose State	**Dense State**
Data Given $H = 10$ m	$H = 10$
$e = 0.70,\ \phi = 30°$	$e = 0.40\ \ \phi = 40°$
$G_S = 2.7$	$G_S = 2.7$
$\gamma_t = \left(\dfrac{G\gamma_w}{1+e}\right) = \dfrac{(2.70\times 9.81)}{1+0.70} = 15.58\text{ kN/m}^3$	
(i) $K_a = \left(\dfrac{1-\sin\phi}{1+\sin\phi}\right) = \left(\dfrac{1-\sin 30°}{1+\sin 30°}\right) = \left(\dfrac{1}{3}\right)$	(i) $K_a = \left(\dfrac{1-\sin 40°}{1+\sin 40°}\right) = 0.217$
(ii) $K_P = \dfrac{1}{K_a} = 3$	(ii) $K_p = \left(\dfrac{1}{K_a}\right) = \left(\dfrac{1}{0.217}\right) = 4.59$
(iii) $P_A = \dfrac{1}{2}(K_a\gamma H)H$	(iii) $P_A = \dfrac{1}{2}\times(K_a\gamma H)H$
$= \dfrac{1}{2}\times\left(\dfrac{1}{3}\right)\times 15.58\times(10)^2$	$= \dfrac{1}{2}\times(0.217\times 15.58)\times(10)^2$
$= 259.67$ kN/m	$= 169.38$ kNm
(iv) $P_P = \dfrac{1}{2}(K_p\gamma H)H$	(iv) $P_P = \dfrac{1}{2}(K_p\gamma H)H$
$= \dfrac{1}{2}\times 3\times 15.59\times(10)^2$	$= \dfrac{1}{2}4.59\times\times 15.58\times(10)^2$
$= 2337.05$ kN/m	$= 3575.61$ kNm

Comment: By compacting the soil active earth pressure decreaseS but passive earth pressure increases.

VARIOUS CASES OF EARTH PRESSURES

Case I: Cohesionless soil on a vertical smooth wall



- Similarly passive earth pressure, per unit length of wall

$$\boxed{p_{pz} = K_p.\gamma.z}$$

- Force due to Active Earth pressure per unit length of wall

$$F_a = \frac{1}{2}\times(K_a.\gamma.H)\times H\times 1$$

$$\boxed{F_a = \frac{K_A.\gamma H^2}{2}}$$

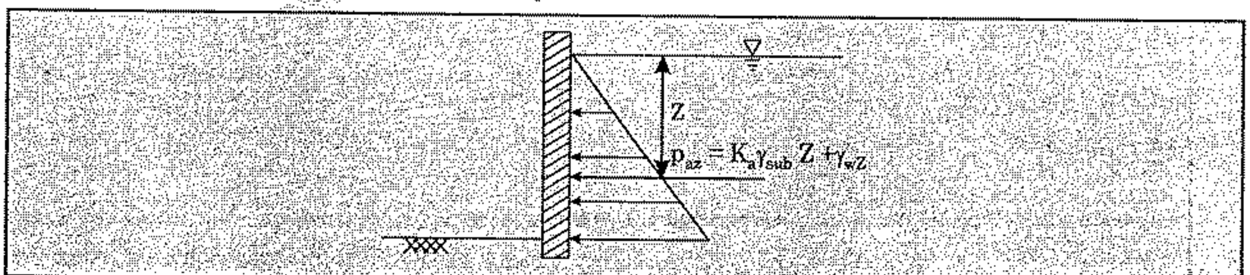
- Simillarly Force due to Passive Earth pressure per unit length of wall.

$$\boxed{F_p = \frac{K_p.\gamma.H^2}{2}}$$

- F_p & F_a will be acting as H/3 height from the base of wall.

Case (II): Submerged cohesionless soil on vertical somooth wall

- Here we deviate from Rankine's Assumption of Cohesionless soil.



$$\boxed{p_{az} = K_a.\gamma_{sub}.Z + \gamma_w.Z}$$

$$\boxed{p_{pz} = K_p.\gamma_{sub}.Z + \gamma_w.Z}$$

- Force on wall due to active Earth pressure per unit length of wall

$$F_a = \frac{1}{2}(K_a.\gamma_{sub}.H + \gamma_w.H)\times H\times 1$$

$$\boxed{F_a = \frac{K_A.\gamma_{sub}.H^2}{2} + \frac{\gamma_w.H^2}{2}}$$

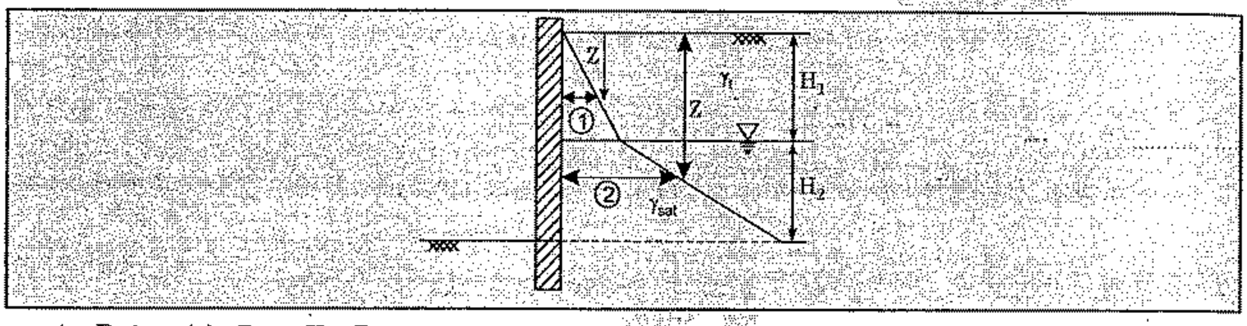
- Similarly $$\boxed{F_P = \left(\frac{k_p \cdot \gamma_{sub} H^2}{2} + \frac{\gamma_w H^2}{2}\right)}$$

Line of action of F_a & F_p will be at H/3 distance from base of wall.

Note: For the undrained condition in a fully saturated clay, the active and passive pressures are calculated using the parameter c_u (f_u being zero) and the total unit weight g_{sat} (*i.e* the water in the soil pores not considered separately).

(3) Case III—Partialy Submerged cohession less soil on vertical smooth wall.

- If a soil is partially submerged as in the case shown below, then active earth pressure is calculate as shown below

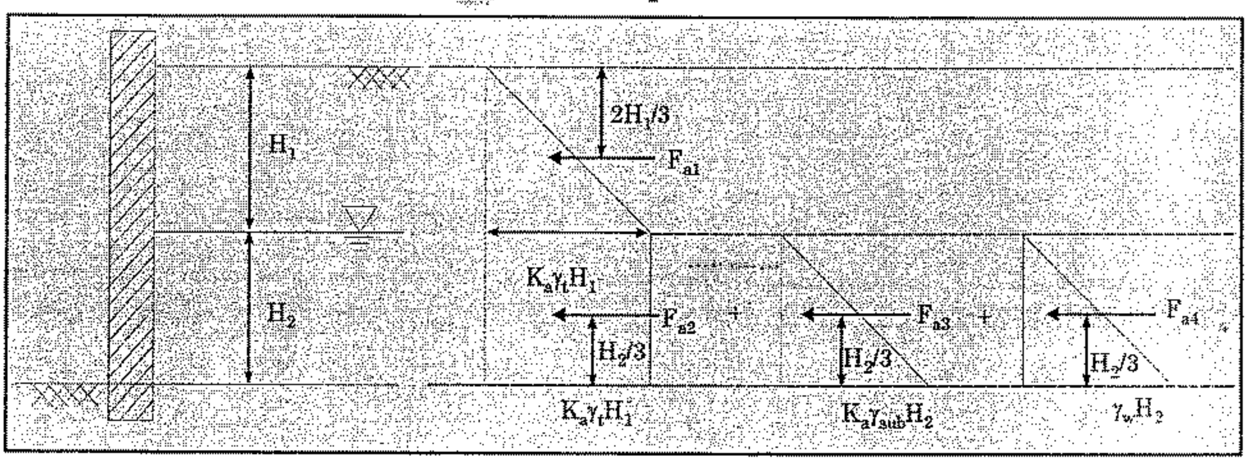


At Point (1), $P_a = K_a \gamma_t Z$

At Point (2), $P_a = \underbrace{K_a[\gamma_t H_1 + \gamma_{sub}(Z - H_1)]}_{\text{effective stress}} + \gamma_w(Z - H_1)$

Note: Water pressure due to hydrostatic condition is same in all direction. Hence, hydrostatic pressure is not multiplied by K_a .

Calculation of Force due to active earth pressure.



$F_a = \left(\frac{1}{2} \cdot K_a \cdot \gamma_t \cdot H_1^2\right)$ acting at $2H_1/3$ from top

$F_{a2} = (K_a \cdot \gamma_t \cdot H_1 \cdot H_2)$ acting at $H_2/2$ from base of wall

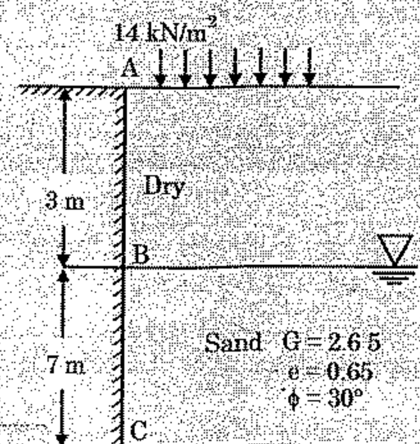
$F_{a3} = \left(\frac{K_a \cdot \gamma_{sub} \cdot H_2^2}{2}\right)$ acting at $H_2/3$ from base of wall

$$F_{a4} = \left(\frac{\gamma_w . H_2^2}{2}\right) \text{ acting at } H_2/3 \text{ from base of wall}$$

Point of application of Resultant force from the bottom of base $= \dfrac{Fa_1\left(H_1 + H_2 - \dfrac{2H_1}{3}\right) + Fa_2\left(\dfrac{H_2}{2}\right) + (Fa_3 + Fa_4)\left(\dfrac{H_2}{3}\right)}{Fa_1 + Fa_2 + Fa_3 + Fa_4}$

Example 3

For an earth retaining Str. Shown in figure below. Construct Earth pressure diagram for active state. Find the total thrust per unit length of wall.



Sol. Dry Density of the soil above W.T

$$\gamma_d = \frac{G\gamma_w}{(1+e)} = \frac{(2.65 \times 9.81)}{(1+0.65)}$$

$$= 15.76 \text{ kN/m}^3$$

Surcharge Intensity $q = 14$ kN/m²

Submerged density of the soil below W.T

$$\gamma_{sub} = \left(\frac{G-1}{1+e}\right)\gamma_w = \frac{(2.65-1)\times 9.81}{1+0.65} = 9.81 \text{ kN/m}^3$$

$$K_a = \frac{1-\sin 30°}{(1+\sin 30°)} = \left(\frac{1}{3}\right)$$

$\therefore$ At point A $Z = 0$ $\quad p_A = (K_a q + K_a \gamma \times 0)$

$$\therefore \quad p_A = K_a q = \left(\frac{1}{3} \times 14\right) = 4.67 \text{ kN/m}^2$$

At point B $Z = 3$ m $\quad p_A = (K_a q + K_a \gamma_d \times 3)$

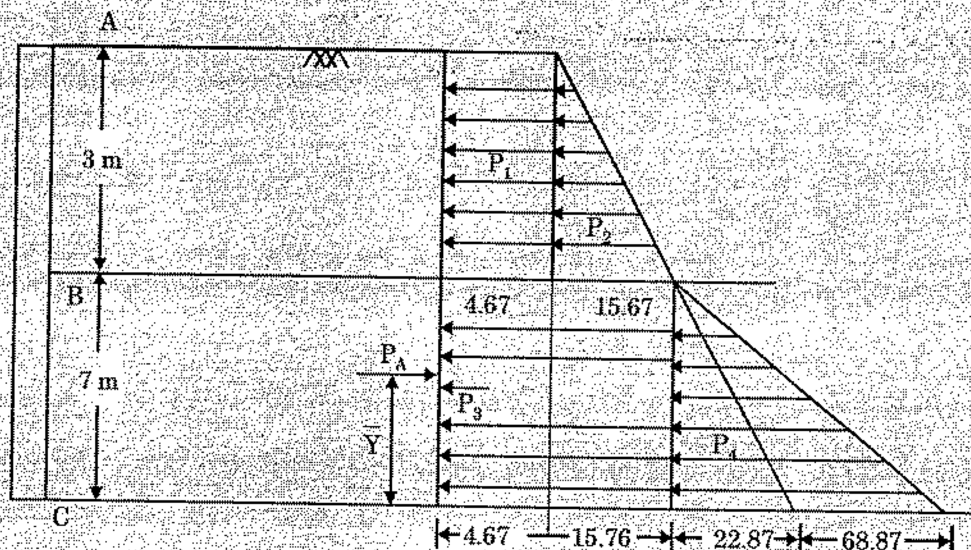
$$= \left(4.67 + \frac{1}{3} \times 15.76 \times 3\right)$$

$$= 20.43 \text{ kN/m}^2 \quad \ldots (i)$$

In Sand

$$\text{Equivalent surcharge} = (q + \gamma_d h_1)$$
$$= (14 + 15.76 \times 3)$$
$$q_1 = 61.28 \text{ kN/m}^2$$
$$\therefore \quad p_B = K_a q_1$$
$$= \frac{1}{3} \times 61.28 = 20.43 \text{ kN/m}^2$$

At Point C
$$p_c = (K_a q_1 + \gamma_w Z_2 + \gamma_{sub} h_2 K_a)$$
$$= \left(20.43 + 9.81 + 7 \times \frac{1}{3} \times 9.81 \times 7\right)$$
$$= (20.43 + 68.67 + 22.87) = 111.97 \text{ kN/m}^2$$



$$\therefore \quad P_1 = (3 \times 4.67) = 14.01 \text{ kN/m}, \qquad Y_1 = \left(7 + \frac{3}{2}\right) = 8.5 \text{ m}$$

$$P_2 = \left(\frac{1}{2} \times 15.7 \times 3\right) = 23.64 \text{ kN/m}, \qquad Y_2 = 7 + \frac{1}{3} \times 3 = 8\text{m}$$

$$P_3 = (4.67 + 15.76) \times 143.01 \text{ kN/m}, \quad Y_3 = \frac{7}{2} = 3.5\text{m}$$

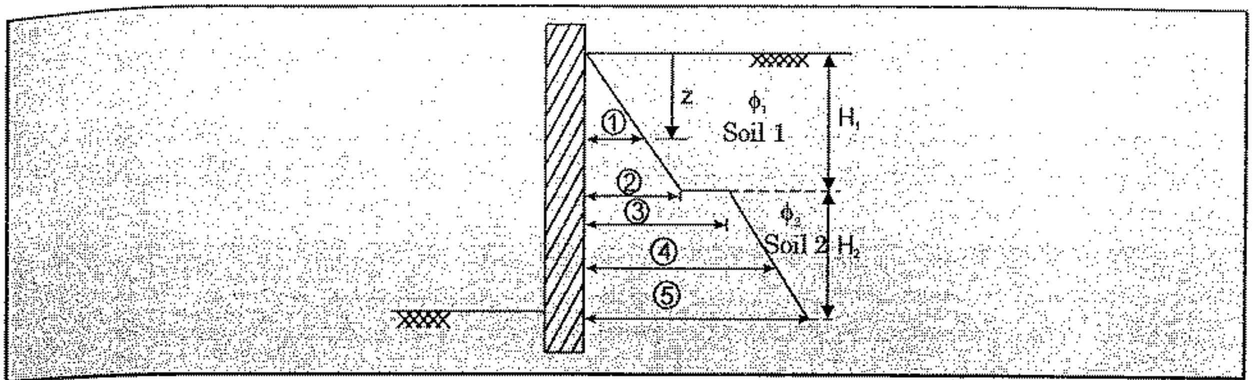
$$P_4 = \frac{1}{2}(22.87 + 68.67) \times 7 = 320.39 \qquad Y_4 = \frac{7}{3} = 2.33\text{m}$$

Total active Thrust on the wall

$$P_A = \sum_{i=1}^{n} p_i = (P_1 + P_2 + P_3 + P_4) = 501.05\text{kN.}$$

$$\text{Resultant of action thrust} = \frac{P_1 Y_1 + P_2 Y_2 + P_2 Y_2 + P_3 Y_3 + P_4 Y_4)}{P_A} = 3.105\text{m}$$

Case-IV: Backfill with two different Soil: [ϕ_1 & ϕ_2]



(1) $p_{a1} = K_{a1}.\gamma_1.Z$

(2) $p_{a2} = K_{a1}.\gamma_1.H_1$ (At depth H_1 in first soil)

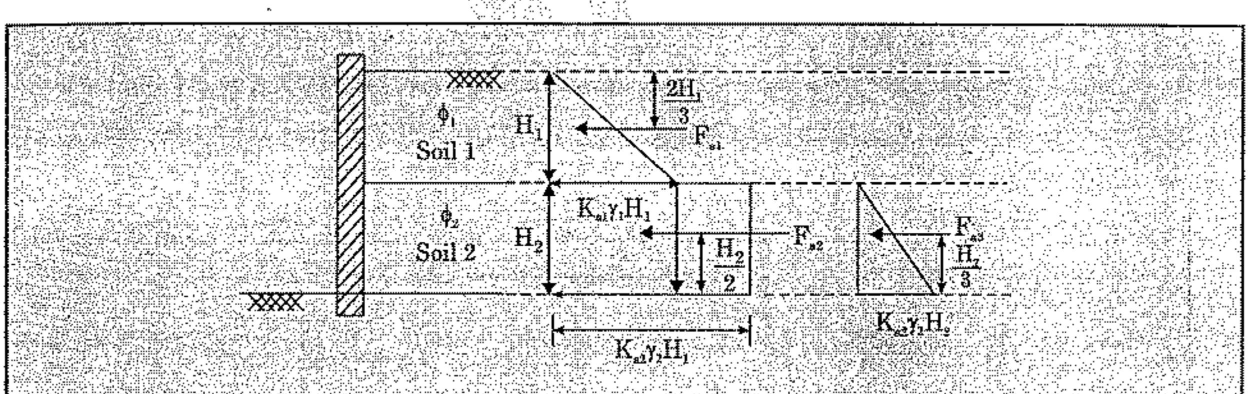
(3) $p_{a3} = K_{a2}(\gamma_1.H_1)$ (At depth H_1 in second soil)

(4) $p_{a4} = Ka_2\left[\gamma_1 H_1 + \gamma_2(Z - H_1)\right]$

(5) $Pa_5 = Ka_2\left[\gamma_1 H_1 + \gamma_2 H_2\right]$

Note: Similarly, passive earth pressure can be calculated

Calculation of forces



$$F_{a1} = \frac{1}{2}.Ka_1.\gamma_1.H_1^2$$

$$F_{a2} = Ka_2.\gamma_1 H_1.H_2$$

$$F_{a3} = \frac{1}{2}.Ka_2\gamma_2.H_2^2$$

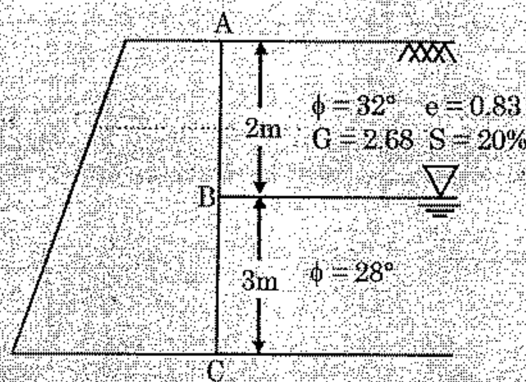
Resultant force location from bottom: $= \left[\dfrac{Fa_1\left(H_1 + H_2 - \dfrac{2H_1}{3}\right) + Fa_2\left(\dfrac{H_2}{2}\right) + Fa_3\left(\dfrac{H_2}{3}\right)}{\left(F_{a1} + F_{a2} + F_{a3}\right)}\right]$

Example 4

A Retaining wall a smooth vertical back has to retain a backfill of cohesionless soil a height of 5m above G.L. The soil has a void ratio of 0.83 and the specific gravity of soil solids is 2.68. The W.T is located at a depth of 2m below the top of the backfill. The soil above the water table is 20% saturated. The angle of internal eriction of the soil above and below W.T are found to be 32° and 28° respectively. Plot the distribution of active earth premure on the wall and determine the magnitude and point of application of resultant thrust.

Sol. Bulk Density of the soil above W.T

$$\gamma_t = \frac{(G+eS)\gamma_w}{1+e} = \frac{(2.68+0.83\times 0.20)\times 1}{(1+0.83)} = 1.55 \text{ t/m}^3$$



Submerged density of soil below W.T

$$\gamma_{sub} = \frac{(G-1)\gamma_w}{(1+e)} = \frac{(2.68-1)\times 1}{(1+0.83)}$$

$$= 0.92 \text{ t/m}^3$$

Active Earth pressure above W.T:

$$K_{a_1} = \left(\frac{1-\sin\phi}{1+\sin\phi}\right) = \frac{1-\sin 32°}{(1+\sin 32°)} = 0.307$$

At A $(Z = 0)$ $p_A = 0$

At B $(Z = 2\text{m})$ $p_B = (K_{a_1} Z_1 \gamma)$

$= (0.807 \times 2 \times 1.55)$

$= 0.95 \text{ t/m}^2$

Active Earth pressure below W.T: In thin case the upper layer (*i.e* the moist soil above W.T) should be treated an a uniform surcharge, For which the intensity q is equal to the self weight of the layer

$$q = \gamma Z_1 = (1.55 \times 2)$$

$$= 3.10 \text{ t/m}^2$$

Now $K_{a_2} = \left(\frac{1-\sin\phi}{1+\sin\phi}\right)$

$$= \left(\frac{1-\sin 28°}{1+\sin 28°}\right) = 0.361$$

At $B\ (Z' = 0)\ p_B = K_{a_2}\ q$

$= (0.361 \times 3.10)$

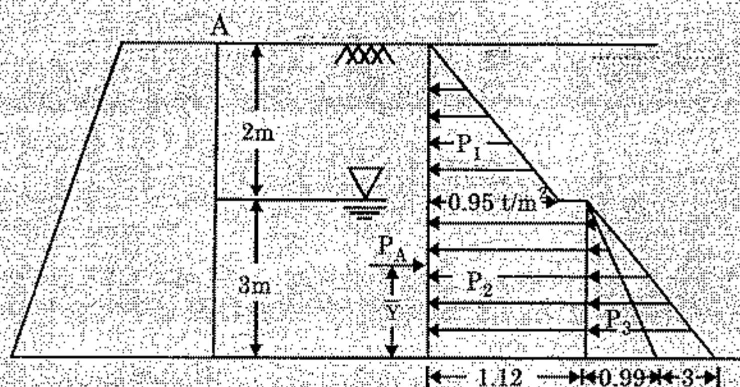
$= 1.12\ t/m^2$

At $C\ (Z' = 3m)\ p_c = (K_{a_2} q + K_{a_2} \gamma_{sub} Z' + \gamma_w Z')$

$= (1.12 + 0.361 \times 0.92 \times 3 + 1 \times 3)$

$= 1.12 + 0.99 + 3$

$= 5.11\ t/m^2$



$P_1 = \frac{1}{2} \times (0.95) \times 2 = 0.95$ t/m $\qquad Y_1 = 3 + \frac{2}{3} = 3.67$ m

$P_2 = 1.12 \times 3 = 3.36$ t/m $\qquad Y_2 = \frac{3}{2} = 1.5$ m

$P_3 = \frac{1}{2}(0.99 + 3) \times 3 = 5.98$ t/m $\qquad Y_3 = \frac{3}{3} = 1$ m

∴ Resultant thrust $P_A = (P_1 + P_2 + P_3)$

$= (0.95 + 3.36 + 5.98)$

$= 10.29$ t/m run.

$$\bar{Y} = \frac{(P_1Y_1 + P_2Y_2 + P_3 \times Y_3)}{P_A}$$

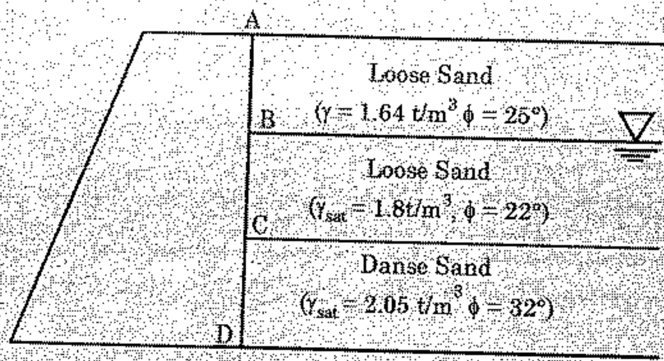
$$= \frac{(0.95 \times 3.67 + 3.36 \times 1.5 + 5.98 \times 1)}{10.29}$$

$= 1.41$ m.

∴ The Resultant thrust of 10.29 t/m run is applied at 1.41 m above the base of the wall.

Example 5

For the Retaining wall show in figure plot the distribution of active earth pressure and determine the magnitude and point of application of the resultant active thrust.



Sol. Active pressure exerted by various strata are as follows

Stratum –I

$$K_{a_1} = \frac{1-\sin 25°}{(1+\sin 25°)}$$

$$= 0.406$$

$$p_A = 0$$

$$p_B = K_{a_1}\ \gamma_1 H_1 = (0.406 \times 64 \times 1.0)$$

$$= 0.67 \text{ t/m}^2$$

Stratum –II: This stratum in fully submerged. while computing the Active earth pressure in this region, stratum I is to be treated as surcharge of Intensity q_1 where

$$q_1 = \gamma_1 H_1 = 1.64 \times 1.0 = 1.64 \text{ t/m}^2$$

$$K_{a_2} = \frac{(1-\sin 22°)}{(1+\sin 22°)} = 0.455$$

$$p_B = K_{a_2} q_1$$

$$= (1.64 \times 0.455)$$

$$= 0.75 \text{ t/m}^2$$

$$p_C = (K_{a_2}\ q_1 + K_{a_2}\ \gamma'\ H_2 + \gamma_w\ H_2)$$

$$= (0.455 \times 1.64 + 0.45 \times (1.80 - 1) \times 1.2 + 1 \times 1.2)$$

$$= 2.39 \text{ t/m}^2$$

Stratum –III: Equivalent surcharge

$$q_2 = (\gamma_1 H_2 + \gamma_2' H_2)$$

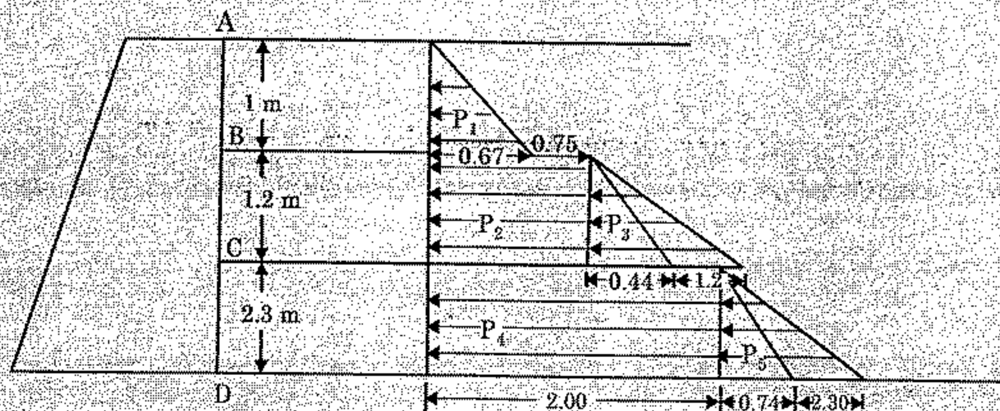
$$= 1.64 \times 1 + (1.80 - 1) \times 1.2$$

$$= 2.60 \text{ t/m}^2$$

$$K_{a_3} = \left(\frac{1-\sin 32°}{1+\sin 32°}\right) = 0.307$$

$$p_c = (K_{a_3} q_2 + \gamma_w \ H_2)$$
$$= 0.307 \times 2.60 + 1.0 \times 1.2$$
$$= 0.80 + 1.2 = 2.00 + \text{t/m}^2$$

$$p_D = (p_D + K_{a_3} \gamma' \ H_2 + \gamma'_w \ H_3)$$
$$= 2 + 0.307 \times (2.05 - 1) \times 2.3 + 1.0 \times 2.3$$
$$= 2 + 0.74 + 2.3 = \mathbf{5.04 \ t/m^2}$$



∴ $P_1 = \frac{1}{2} \times (0.67) \times 1 = 0.335$ t/m $\qquad Y_1 = 3.5 + \frac{1}{3} = 3.83$ m

$P_2 = (0.75 \times 1.2) = 0.90$ t/m $\qquad Y_2 = \left(2.3 + \frac{1.2}{2}\right) = 2.9$

$P_3 = \frac{1}{2}(0.44 + 1.2) \times 1.2 = 0.984$ t/m $\qquad Y_3 = \left(2.3 + \frac{1.2}{2}\right) = 2.7$m

$P_4 = 2.3 \times 2 = 4.6$ t/m $\qquad Y_4 = \frac{2.3}{2} = 1.15$m

$P_5 = \frac{1}{2}(0.74 + 2.30) \times 2.3$ $\qquad Y_5 = \frac{2.3}{3} = 0.77$m

$= 3.496$ t/m

∴ $P_A = \sum_{i=1}^{n} P_i$

$= (0.335 + 0.90 + 0.984 + 4.6 + 3.496)$

$= 10.315$ tonn.

$$\bar{Y} = \frac{(P_1Y_1 + P_2Y_2 + P_3Y_3 + P_4Y_3 + P_5Y_5)}{P_A}$$

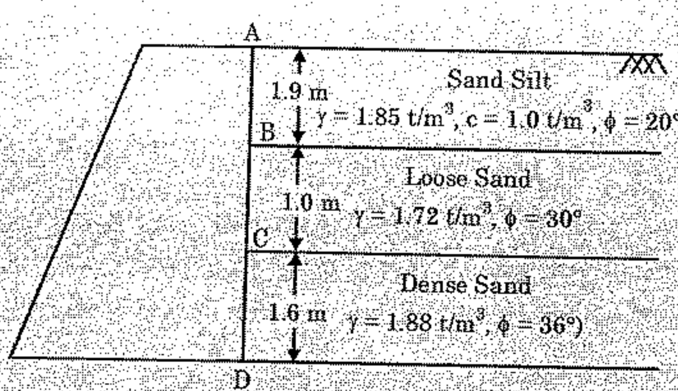
$$= \frac{(0.33 \times 3.83 + 0.90 \times 2.9 + 0.984 \times 2.7 + 4.6 \times 1.15 + 3.496 \times 0.77)}{10.315}$$

$\bar{Y} = 1.409$ m

Hence the resultant active thrust of 10.315 t/m run is applied at 1.409 m above the base of the wall.

Example 6

A retaining wall of 5 m height has to retain a stratified backfill as shown in figure. Find out the magnitude of total active thrust on the wall and locate the point of application.



Sol.

Sand silty layer

$$p_a = K_a \gamma Z - 2c\sqrt{K_a}$$

at $Z = 0,\ p_a = -2c\sqrt{K_a}$

$$\therefore \quad K_a = \left(\frac{1-\sin 20°}{1+\sin 20°}\right) = \mathbf{0.490}$$

$$\therefore \quad p_a = -2c\sqrt{K_a} = -2\times 1.0\times\sqrt{0.490}$$

$$= -1.400 \text{ t/m}^2$$

$$\therefore \quad p_b = (K_a\gamma Z - 2c\sqrt{K_a})$$

$$= (0.490\times 1.85\times 1.9 - 2\times 1.0\times\sqrt{0.490})$$

$$= 0.32 \text{ t/m}^2$$

$$\therefore \quad H_c = \frac{2c}{\sqrt{K_a}\gamma} = \frac{2\times 1.0}{(\sqrt{0.490}\times 1.85)} = 1.54\text{ m}$$

(ii) Loose Sand Layer

$$K_{a_2} = \frac{1-\sin 30°}{(1+\sin 30°)} = \left(\frac{1}{3}\right)$$

$\therefore$ Equivalent surcharge $q_1 = (\gamma_1 h_1)$

$$= (1.85\times 1.90) = 3.515 \text{ t/m}^2$$

$$\therefore \quad p_B = K_{a_2} q_1 = \frac{1}{3}\times 3.515 = 1.172 \text{ t/m}^2$$

$$p_C = (K_{a_2} q_1 + K_{a_2}\gamma_2 H_2)$$

the

$$= \left(1.172 + \frac{1}{3} \times 1.72 \times 1\right)$$

$$= (1.192 + 0.573) = 1.745 \text{ t/m}^2$$

For Dense sand layer

Equivalent surcharge

$$q_2 = (1.85 \times 1.9 + 1.72 \times 1)$$

$$= 5.235 \text{ t/m}^2$$

$$\therefore \quad p_C = (K_{a_3} q_2)$$

$$\therefore \quad K_{a_3} = \frac{(1 - \sin 36°)}{(1 + \sin 36°)} = 0.2596$$

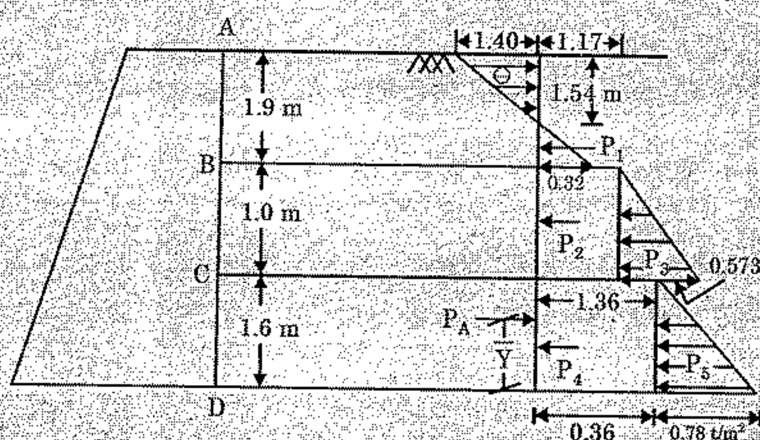
$$\therefore \quad p_C = (0.2596 \times 5.235)$$

$$= 1.36 \text{ t/m}^2$$

$$p_D = (K_{a_3} q_2 + K_{a_3} \gamma H_3)$$

$$= (1.36 + 0.2596 \times 1.88 \times 1.6)$$

$$= 2.14 \text{ t/m}^2$$



$$P_1 = \frac{1}{2}(1.9 - 1.54) \times 0.32 = 0.06 \text{ tonn} \qquad Y_1 = \left(2.6 + \frac{0.35}{3}\right) = 2.72 \text{ m}$$

$$P_2 = (1.17 \times 1.0) = 1.17 \text{ tonn} \qquad Y_2 = 1.6 + \frac{1}{2} = 2.1 \text{ m}$$

$$P_3 = \frac{1}{2} \times (0.573 \times 1) = 0.2685 \text{ tonn} \qquad Y_3 = \left(1.6 + \frac{1}{3}\right) = 1.93 \text{ m}$$

$$P_4 = (1.36 \times 1.6) = 2.18 \text{ tonn} \qquad Y_4 = \frac{1.6}{2} = 0.8 \text{ m}$$

$$P_5 = \frac{1}{2} \times 0.78 \times 1.6 = 0.62 \text{ tonn} \qquad Y_5 = \frac{1.6}{3} = 0.53 \text{ m}$$

$$\therefore \quad P_A = \sum_{i=1}^{n} p_i$$

$$= (P_1 + P_2 + P_3 + P_4 + P_5)$$

$$= 4.30 \text{ tonn}$$

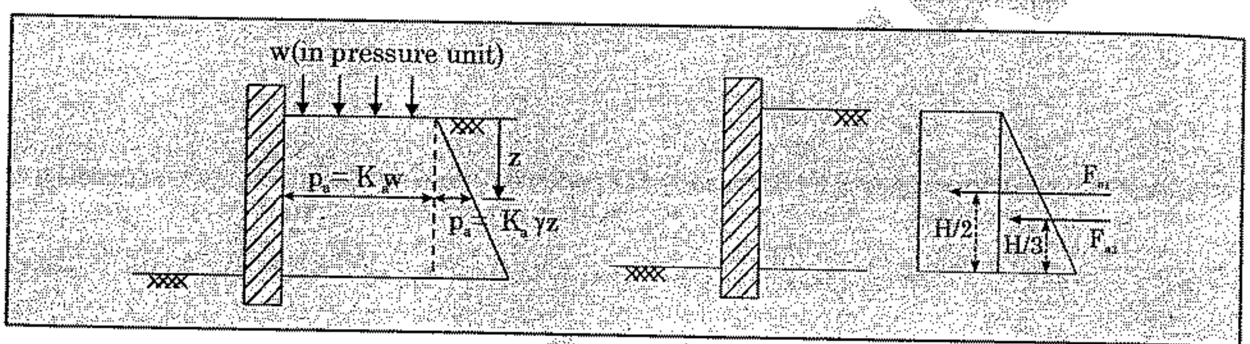
$$\therefore \quad \bar{Y} = \frac{(P_1Y_1 + P_2Y_2 + P_3Y_3 + P_4Y_4 + P_5Y_5)}{P_A}$$

$$= \frac{(0.06 \times 2.72 + 1.17 \times 2.1 + 0.2685 \times 1.93 + 2.18 \times 0.8 + 0.62 \times 0.53)}{4.30}$$

$$[\bar{Y} = 1.21 \text{ m}]$$

$\therefore$ The point of application of $P_A = 4.30$ tonn is located at 1.21 m. above the base of the wall.

Case V– Soil with surcharge load:



$$Fa_1 = Ka_1 wH$$

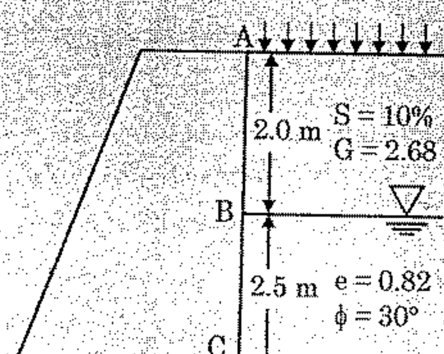
$$Fa_2 = Ka_1 \frac{\gamma H^2}{2}$$

$$\text{Resultant force location from bottom} = \frac{Fa_1 \times \frac{H}{2} + Fa_2 \times \frac{H}{3}}{Fa_1 + Fa_2}$$

Example 7

A Retaining wall with a smooth vertical backfill bas to retain a sand hackfill upto a beight of 4.5 m a uniform surcharge of 5t/m^2 is placede over tbe backfill. The water table is 2m below G.L. The specific gravity of solids and the void ratio of the backfill are 2.68 and 0.82 respectively. The soil above the water table has the degree of saturation of 10%. The angle of internal Friction of the soil, botb above and below W.T is 30°. Determine the magnitude and point of application of the resultant active thrust on the wall.

Sol.



Bulk density of the soil above W.T

$$\gamma_t = \frac{(G+eS)\gamma_w}{(1+e)} = \frac{(2.68+0.82\times0.10)\times1}{(1+0.82)} = 1.517 \text{ t/m}^2$$

Submerged density of the soil below W.T

$$\gamma_{sub} = \frac{(G-1)\gamma_w}{(1+e)} = \frac{(2.68-1)\times1}{(1+0.82)} = 0.923 \text{ t/m}^3$$

Coefficient of active Earth pressure

$$K_a = \frac{1-\sin30°}{(1+\sin30°)} = \frac{1}{2\times\frac{3}{2}} = \frac{1}{3}$$

Active pressure at B due to moist soil above W.T

$$= (K_a\,\gamma Z) = \left(\frac{1}{3}\times1.517\times2\right) = 1.01 \text{ t/m}^2$$

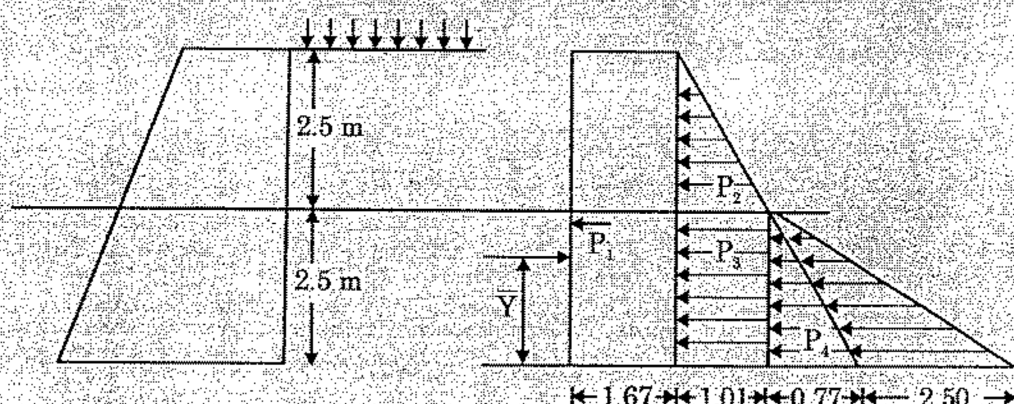
Active pressure due to surcharge $= (K_a q) = \frac{1}{3}\times5 = 1.67 \text{ t/m}^2$

Active pressure at c due to submerged soil $= (K_a\,\gamma_{sub}\cdot Z)$

$$= \left(\frac{1}{3}\times0.923\times2.5\right) = \mathbf{0.77\ t/m^2}$$

Lateral pressure exerted by water

$$= (\gamma_w Z) = 1\times2.5 = 2.5 \text{ t/m}^2$$



$$P_1 = 1.67\times4.5 = 7.51 \text{ tonn} \qquad Y_1 = \left(\frac{4.5}{2}\right) = 2.25 \text{ m}$$

$$P_2 = \frac{1}{2}\times(1.01)\times2 = 1.01 \text{ tonn} \qquad Y_2 = 2.5\frac{2}{3} = 3.17 \text{ m}$$

$$P_3 = (1.01\times2.5) = 2.525 \text{ tonn} \qquad Y_3 = \left(\frac{2.5}{2}\right) = 1.25 \text{ m}$$

$$P_4 = \frac{1}{2}(0.77+2.50)\times2.5 = 4.09 \text{ tonn} \qquad Y_4 = \frac{2.5}{2} = 0.83 \text{ m}$$

Resultant thrust $P_A = (P_1 + P_2 + P_3 + P_4)$

$= (7.51 + 1.01 + 2.252 + 4.09)$

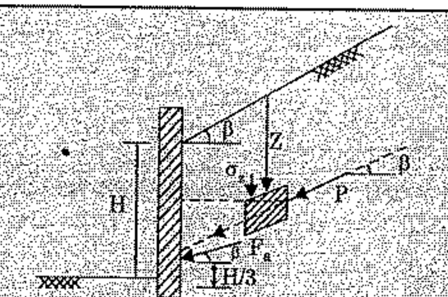
$= 15.13$ tonn per m. Run. ... (i)

The point of application of this thrust above the bass of the wall

$$\bar{Y} = \frac{(7.51 \times 2.25) + (1.01 \times 3.17) + (2.525 \times 1.25) + (4.09 \times 0.83)}{(15.13)}$$

$$\left[\bar{Y} = \frac{26.64}{(15.13)} = 1.76 \text{ m}\right]$$

(6) Case-VI: Soil with Inclined Backfill:



$$\sigma_z = \gamma z.\cos\beta$$

$$p_a = k_a \gamma z \cos\beta$$

$$K_a = \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}$$

$$K_p = \frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}$$

- Resultant force will be acting at a distance $\left(\frac{H}{3}\right)$ from bottom.

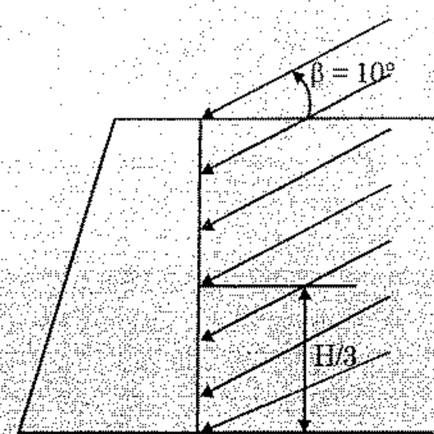
$$F_a = \frac{1}{2}.K_a.YH^2.\cos\beta$$

Note : If we take $\beta = 0°$ *i.e.* back fill is horizontal then $k_a = \frac{1-\sin\phi}{1+\sin\phi}$ & $k_p = \frac{1+\sin\phi}{1-\sin\phi}$

Example 8

A 5 m high masonary Retaining wall has to retain a backfill of sandy soil having a unit weight of 1.82 gm/cc and an angle of internal friction of 32°. The surace of the backfill is inclined at an angle of 10° to the horizontal determine the magnitude and point of application of the active thrust on the wall.

Sol.



Data given: $\phi = 32°$

$\beta = 10°$

$$K_a = \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{(\cos^2\beta - \cos^2\phi}}$$

$$K_a = \cos 10° \frac{(\cos 10° - \sqrt{\cos^2 10 - \cos^2 32°})}{(\cos 10° + \sqrt{\cos^2 10 - \cos^2 32°})}$$

$$K_a = 0.296$$

$$P_a = \frac{1}{2} K_a \gamma H^2$$

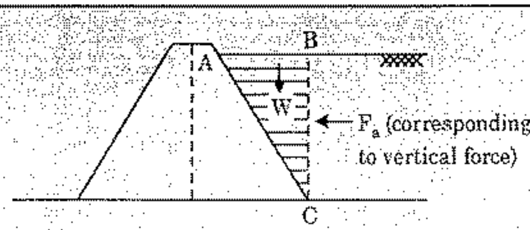
$$= \frac{1}{2} K_a \gamma H^2$$

$$= \frac{1}{2} \times (0.296) \times 1.82 \times (5)^2 = \mathbf{6.734\ t/m}$$

This thrust is inclined at 10° to the horizontal and is applied at a height of $\left(\frac{5}{3}\right) = 1.67$m above the base of wall.

(7) **Case VII: Horizontal Back fill with inclined wall**

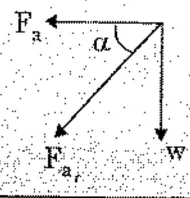
- In case of Inclined wall we draw a vertical line from the base of wall which cuts the back fill of wall at point B.
- We can calculate Earth pressure along vertical line BC as $F_a = \frac{1}{2} k_a \gamma H^2$.
- If W is the weight of soil in region ABC then



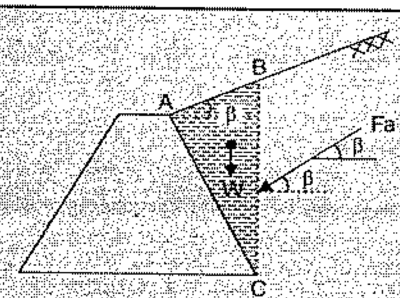
the Resultant force of (W) and (F_a) is the **Earth pressure** on the inclined back wall.

- Direction will be that of the **Resultant** of (W) and (F_a)

$$\boxed{\tan\alpha = \frac{W}{F_a}}$$



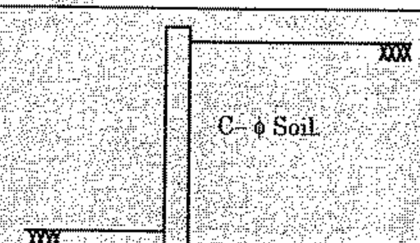
(8) Case-VIII: Inclined backfill with inclined wall



- In case of inclined backfill with inclined wall we draw a vertical line from the base of wall which cuts the back fill of point B.
- The lateral earth pressure is calculated along the line BC as $F_a = \frac{1}{2}\, k_a\, \gamma\, H^2 \cos\beta$ which will be acting at angle β from the horizontal. Weight of soil in ΔABC as W.
- Resultant of W & F_a will give resultant Earth Pressure F_{ar}.

Case IX— Active Earth Pressure acting on Cohesive Soil.

Orignally Rankine's theory was given for cohesionless soil but Bell extended this theory to C–φ soil. Consider a vertical smooth retaining wall, retaining a c–φ soil.



Further we know that.

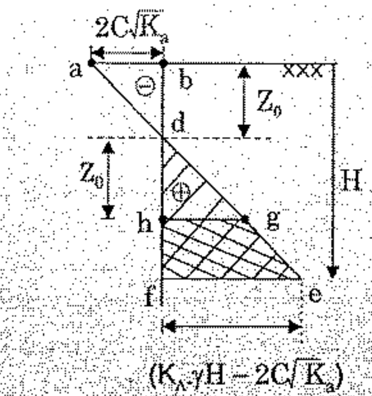
$$p_a = \sigma_z\left(\frac{1-\sin\phi}{1+\sin\phi}\right) - 2C\sqrt{\frac{(1-\sin\phi)}{(1+\sin\phi)}}$$

$$\therefore \qquad \boxed{P_a = K_A \sigma_z - 2C\sqrt{K_A}}$$

at $\qquad Z = 0 \;\; Pa = -2C\sqrt{Ka}$

and $\qquad Pa = 0 \;\;\text{at}\;\; Z = Z_0 = \dfrac{2C}{\gamma\sqrt{Ka}}$

Active earth Pressure distribution is shown below in the diagram.



Active earth pressure

- We can observe that by virtue of cohesive property, cohesive soil has tendency to develope tension (or negative pressure) in the soil in upper reaches up to depth Z_0 and also active earth pressure becomes zero at depth Z_0.
- Another most important observation can be made from the above pressure distribution diagram that net total active earth pressure is zero upto a depth $2Z_0$, which implies that we can make an unbraced cut in clayey soil upto a depth of $2Z_0$.
- Hence critical depth of vertical cut in a cohessive soil is given by.

$$\boxed{H_C = 2Z_0 = \frac{4C}{\gamma\sqrt{Ka}}}$$

- In case of no loss of contact has occurred between the wall and soil, active earth pressure on wall will correspond to area under (e f g h)
- However for design purpose, we will assume that the contact force has occured and hence active earth pressure on wall will correspond to (fde) as shown in the figure above.

Hence,

$$F_a = \frac{1}{2}\left(K_A.\gamma.Z - 2C\sqrt{K_a}\right).(Z - Z_0)$$

We know that

$$Z_0 = \frac{2C}{\gamma\sqrt{Ka}}$$

$$F_a = \frac{1}{2}\left(K_a.\gamma.Z - 2C\sqrt{K_a}\right).\left(Z - \frac{2C}{\gamma\sqrt{k_a}}\right)$$

$$F_a = \frac{1}{2}k_a.\gamma.Z^2 - C\sqrt{k_a}.Z - C\sqrt{k_a}.Z + \frac{2C^2}{\gamma}$$

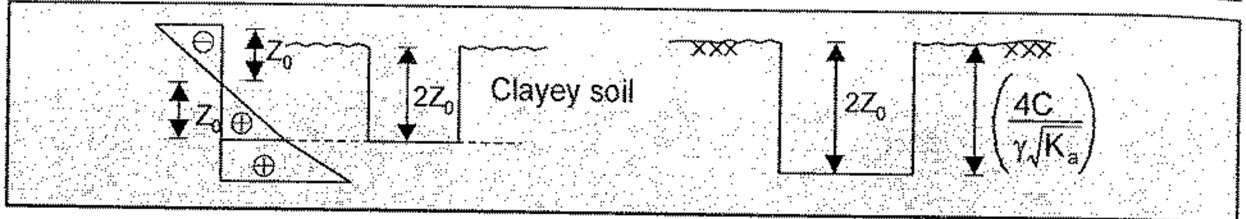
$$\boxed{F_a = \frac{k_a\gamma Z^2}{2} - 2C\sqrt{K_a}.Z + \frac{2C^2}{\gamma}}$$

However, in case of No loss of contact

$$F_a = \left[\frac{K_a.\gamma.Z^2}{2} - 2c\sqrt{K_a}\,Z + \frac{2c^2}{\gamma}\right] - \left[\frac{1}{2}\times\frac{2C}{\gamma\sqrt{K_a}}\times 2C\sqrt{K_A}\right] - \left[\frac{2C^2}{\gamma}\right]$$

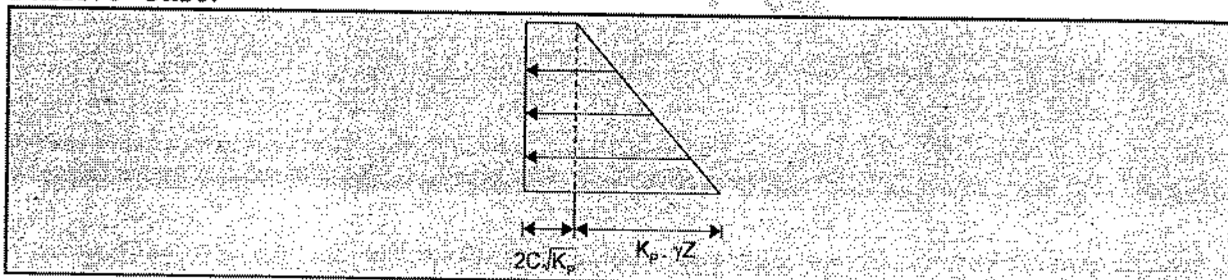
$$\therefore \quad \boxed{F_a = \left[\frac{K_a.\gamma.Z^2}{2} - 2C\sqrt{K_a}\,Z\right]}$$

Note: As there is no Net Earth pressure upto a depth of ($2Z_0$) we can make an **unbraced cut** in clayey soil upto a depth of ($2Z_0$).



- For purely cohesive soil, $H_C = 2Z_0 = \dfrac{4C}{\gamma}$, as $K_a = 1$ for purely cohesive soil ($\phi = 0$).
- In actual practice tension developed in the upper reaches of the soil can not be relied upon to act on the wall, since cracks are likely to develop with in the tension Zone causing loss of contact between wall and soil mass.
- Hence for design consideration we assume that loss of contact has occured between soil mass and retaining wall in the tension zone and in this case active earth pressure on the wall will correspond to area under (f d e).

Passive Case:



$$P_P = K_P \gamma Z + 2C\sqrt{K_P}$$

Example 9

A retaining wall with a smooth vertical backface has to retain a backfill of c–ϕ soil upto 5 m above G.L The surface of the backfill is horizontal and it has the following properties $\gamma = 1.8$ t/m^3, $c = 1.5$ t/m^2 $\phi = 12°$

(*i*) Plot the distribution of active earth presure on the wall

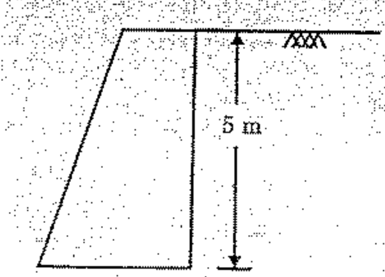
(*ii*) Determine the magnitude and point of application of active thrust.

(*iii*) Determine the depth of zone of tension cracks.

(*iv*) Determine the intensity of Fictions uniform surcharge which if placed over the backfill, can prevent the formations of tension cracks.

(*v*) Compute the resultant active thrust after placing the surcharge.

Sol.



For $c-\phi$ soil the intensity of action earth pressure at any depths Z is given by:

$$p_a = (K_a \gamma Z = 2c\sqrt{K_a}) \quad \text{... (i)}$$

At the top of the wall, $Z = 0$

$$p_a = -2c\sqrt{K_a}$$

$$\therefore \quad K_a = \frac{(1-\sin 12°)}{(1+\sin 12)} = 0.6557$$

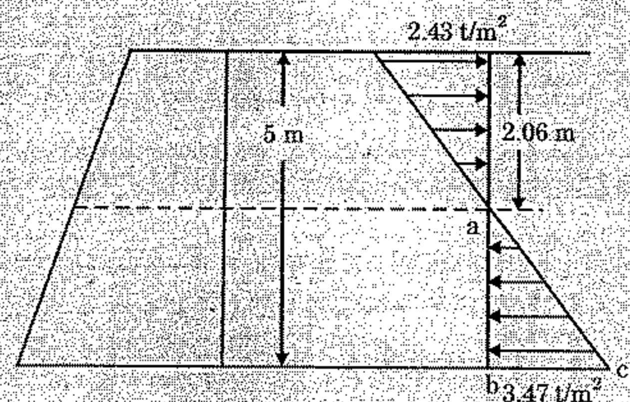
$$\therefore \quad p_a = -2\times 1.5\times\sqrt{0.6557} = -2.43 \text{ t/m}^2 \quad \text{... (ii)}$$

At the base of the wall, $Z = 5$m

$$\therefore \quad p_a = (K_a \gamma Z - 2c\sqrt{K_a})$$

$$= (0.6557 \times 1.8 \times S - 2.43) = 3.47 \text{ t/m}^2 \quad \text{... (iii)}$$

Hence the pressure distribution diagram.



The depth of tension crack

$$p_a = 0 = K_a\gamma H_c - 2c\sqrt{K_a}$$

$$\Rightarrow \quad \frac{2c\sqrt{K_a}}{K_a\gamma} = H_c$$

$$H_c = \left(\frac{2c}{\sqrt{K_a}\gamma}\right) = \frac{2\times 1.5}{(\sqrt{0.6557}\times 1.8)} = 2.06 \text{ m.}$$

(ii) The resultant active thrust is given by the part abc of the pressure distribution diagram.

$$\therefore \quad P_A = \frac{1}{2}\times(5-H_c)\times 3.47$$

$$= \frac{1}{2}(5-2.06)\times 3.47 = 5.10 \text{ t/m run} \quad \text{... (iv)}$$

The point of application of P_A to located at $\frac{2.94}{3} = 0.98$ m. above the base of the wall.

(iii) The maximum negative pressure intennity developed at the top of the wall $= -2.43$ t/m².

Evidently, the formation of tension cracks can be prevented by placing a surcharge q on the backfill which can neutralise this –ve pressure

Therefore

$$p_A = (q + \gamma Z) \times K_a - 2c\sqrt{K_a} \quad \text{... (v)}$$

$$\text{at } Z = 0 \quad P_A = qK_a - 2c\sqrt{K_a}$$

But the magnitude of q is such that, at $Z = 0$, $p_A = 0$

$$q K_a = 2c\sqrt{K_a}$$

$$\Rightarrow \quad q = \frac{2c\sqrt{K_a}}{K_a} = \left(\frac{2c}{\sqrt{K_a}}\right)$$

$$\therefore \quad q = \left(\frac{2\times1.5}{\sqrt{0.6557}}\right) = 3.70 \text{ t/m}^2$$

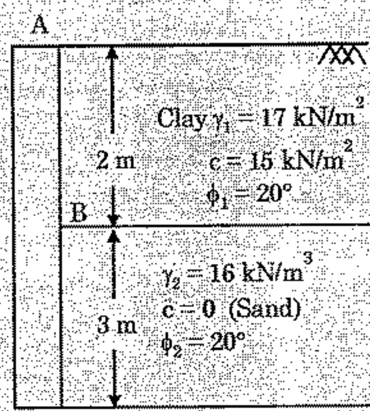
Again at $Z = H$

$$p_A = (q+\gamma H)K_a - 2c\sqrt{K_a}$$

$$\therefore \quad p_A = (3.70+1.8\times5)\times0.6557 - 2\times1.5\sqrt{0.6557}$$

Example 10

Find active thurst $P_A = ?$ when tension crack has not yet appeared



Sol.

For soil I

$$p_a = (K_a\gamma Z - 2c\sqrt{K_a}) \quad \text{... (i)}$$

$$K_{a_1} = \left(\frac{1-\sin\phi}{1+\sin\phi}\right) = \left(\frac{1-\sin 20°}{1+\sin 20°}\right) = 0.490$$

at $Z = 0$,

$$p_a = (K_a\gamma\times0 - 2c\sqrt{K_a})$$

$$= (-2c\sqrt{K_a})$$

$$= -2\times15\times\sqrt{0.490} = -21.006 \text{ kN/m}^2$$

at point B

$Z = 2\text{m}, \qquad p_B = (K_{a_1}\gamma \times 2 - 2c\sqrt{K_{a_1}})$

$$= (0.490 \times 17 \times 2 - 2 \times 15\sqrt{0.490}) = -4.34 \text{ kN/m}^2$$

At point B in soil (*ii*)

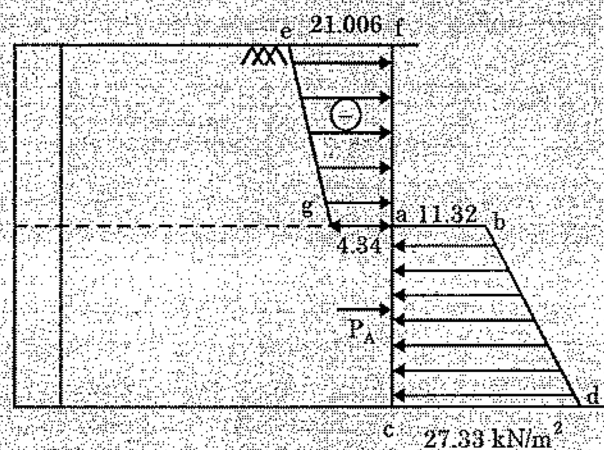
Equivalent surcharge $= (\gamma_1\ h_1) = 17 \times 2 = 34 \text{ kN/m}^2$

$$\therefore \quad K_{a_2} = \frac{(1-\sin 30)}{(1+\sin 30°)} = \left(\frac{1}{3}\right)$$

$$p_b = (K_{a_2} q) = \frac{1}{3} \times 34 = 11.34 \text{ kN/m}^2$$

$$p_c = K_{a_2}(\gamma_1 h_1 + \gamma_2 h_2) - 2 \times 0\sqrt{K_{a_2}} = \frac{1}{3}(17 \times 2 + 16 \times 3) = 27.33 \text{ kNm}^2$$

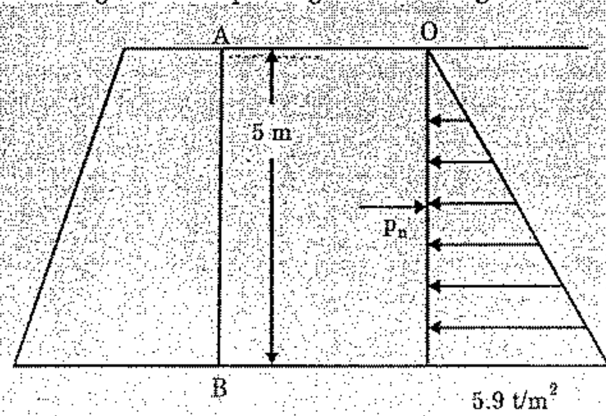
Pressure distribution diagram



When tension crack has not developed total force will be c/p to

$$\text{Area }(abcdefgh)\ P_A = \left(\frac{11.32 + 27.33}{2}\right) \times 3 \times 1 - \left(\frac{21 + 4.34}{2}\right) \times 2 \times 1 = 32.648 \text{ kN/m} = 5.9 \text{ t/m}^2$$

The pressure distribution diagram after placing the surcharge



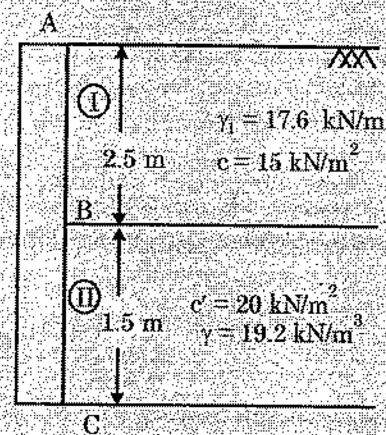
(*iv*) $P_A = \frac{1}{2} \times (5.9) \times 5 = 14.75 \text{ t/m}$ applied at a height of $\left(\frac{5}{3}\right) = 1.67$ m above the base of wall

Example 11

A 4m high vertical wall supports a saturated cohesive soil ($\phi = 0$) with horizontal backfill, the top 2.5 m backfill has a density of 17.6 kNm³ and apparent cohesion of 15 kN/m² the Bulk density and apparent cohesion of the bottom 1.5 m is 19.2 kN/m³ and 20 kN/m² Respectively. If tension crack developed then what would be the total active pressure on the wall. Also draw the pressure distribution diagram.

Sol.

For Soil I



$$p_a = K_a \gamma Z - 2c\sqrt{K_a}$$

$$K_a = \left(\frac{1-\sin 0}{1-\sin 0}\right) = 1$$

$$K_p = \frac{1}{K_a} = 1$$

at $Z = 0$,

$$p_a = -2c\sqrt{K_a}$$

$$= -2 \times 15 = -30 \text{ kN/m}^2$$

$$p_b = (K_a \gamma z - 2c\sqrt{K_a})$$

at $Z = 2.5$ m

$$p_b = (1 \times 17.6 \times 2.5 - 30)$$

$$= 14.0 \text{ kN/m}^2$$

$$H_0 = \frac{2c}{\gamma\sqrt{K_a}} = \frac{2\times 15}{17.6\times 1} = 1.705 \text{ m}$$

For soil II

$$K_{a_2} = \left(\frac{1-\sin 0}{1-\sin 0}\right) = 1.0$$

Equivalent surcharge

$$q = \gamma h$$

$$= (17.6 \times 2.5)$$

$$= 44 \text{ kN/m}^2$$

$$p_b = (K_{a_2} q) - 2c'\sqrt{K_a}$$

$$= (1\times 44 - 2 \times 20)$$

$$= 4 \text{ kN/m}^2$$

At point C

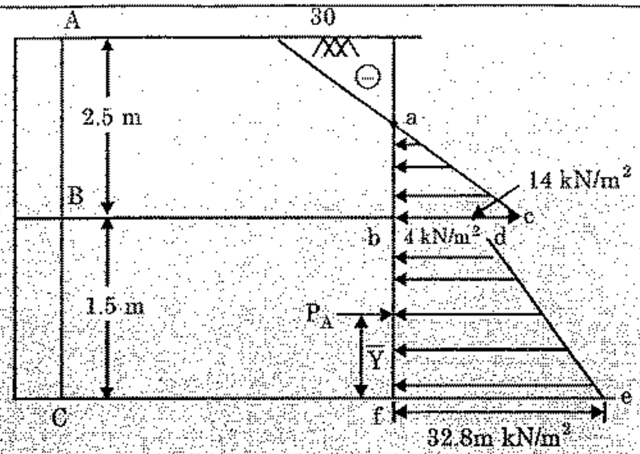
$$p_c = (K_{a_2}(\gamma_1 h_1 + \gamma_2 h_2) - 2c' - \sqrt{K_{a_2}})$$

$$= 1 \times (17.6 \times 2.5 + 19.2 \times 1.5) - 2 \times 20\sqrt{1}$$

$$= (44 + 28.8 - 40)$$

$$= 32.8 \text{ kN/m}^2$$

p
d
k
n



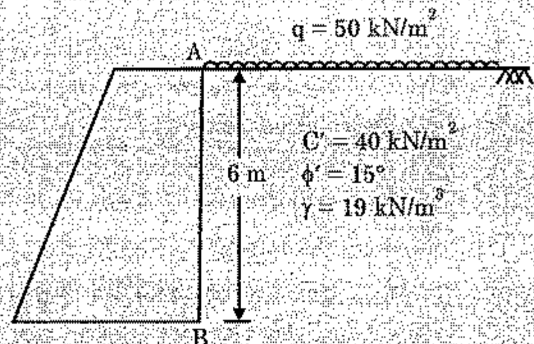
The total active thrust when crack has developed C/P to shaded region *abcdef*.

$$P_A = \frac{1}{2}\times 14(2.5-1.705)\times 1+\frac{(4+32.8)}{2}\times 1.5 = \mathbf{33.165\ kN/m\ run.}$$

Example 12

A Retaining wall 6m hight with a smooth vertical back is pushed against a soil mass having $c' = 40$ kN/m², $\phi' = 15°$ and $\gamma = 19$ kN/m³. Using Theory, compute the Total pressure and the point of application of the resultant thrust, If the horizontal soil surface carries a uniform surcharge load of **50 kN/m2**.

Sol.



Calculations should be done using passive earth pressure.

$$K_a = \frac{1-\sin 15°}{1+\sin 15°} = 0.588$$

$$\therefore \quad p_a = K_a(q+\gamma z)-2c\sqrt{K_a} \quad \text{... (i)}$$

At point A, $Z = 0$ $\quad p_a = (K_a q - 2c\sqrt{K_a})$

$$= (0.588\times 50 - 2\times 40\times\sqrt{0.588}) = -31.945 \text{ kN/m}^2 \quad \text{... (ii)}$$

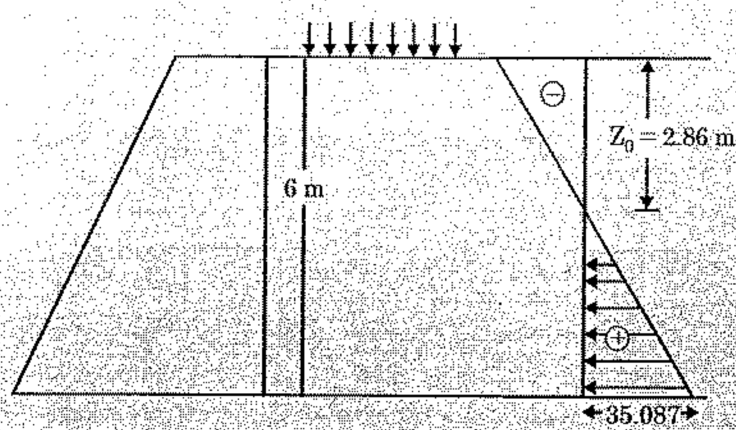
At point B $Z = 6$m $\quad p_b = K_a(q+\gamma\times H)-2c\sqrt{K_a}$

$$= 0.588(50+19\times 6)-2\times 40\sqrt{0.588} = 35.0.87 \text{ kN/m}^2 \quad \text{... (iii)}$$

$p_a = 0 \Rightarrow$ $\quad Z_0 = \left(\frac{2C}{\sqrt{K_a}}\cdot\frac{q}{\gamma}\right) = \frac{2\times 40}{(\sqrt{0.588}\times 19)} - \frac{50}{19}$

$$= 2.86 \text{ m} \quad \text{... (iv)}$$

Pressure distribution diagram



Assuming that, the soil is the zone of tension crack does not exerts any pressure on the wall.

Therefore Total Thrust $= \frac{1}{2} \times 35.087 \times (6-2.86) = 55.086$ kN/m sum

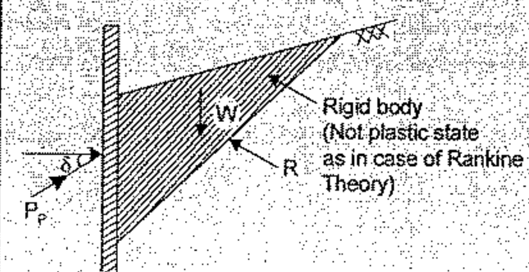
$\therefore$ Point of application the thrust form bottom $= \frac{(6-2.86)}{3} = \mathbf{1.05\ m}$

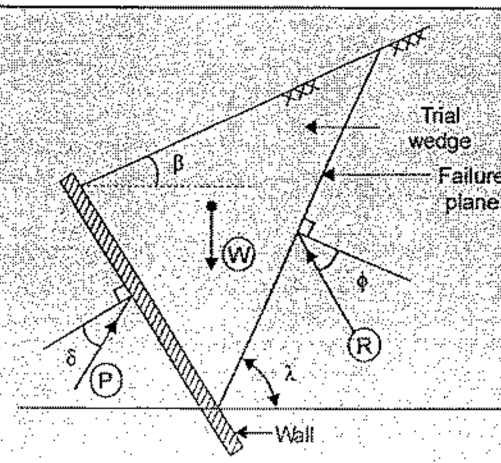
COULOMB'S THEORY OF EARTH PRESSURE

Assumptions:

1. The backfill is dry, cohesionles, Isotropic.
2. Back of wall can be inclined.
3. Backfill can be inclined.
4. There would be friction between the wall and the soil.
5. Failure plane in assumed to be a **plane surface** [actually curved].
6. Sliding wedge is assumed to be a rigid body.

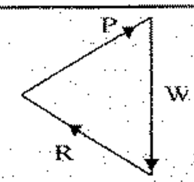
- The position and line of action of earth pressure will be known in advance.
- It will be at (1/3rd) depth from bottom and will be inclined at an angle of (δ) from the Normal to the wall.





- Trial wedge will have forces like (W), (R), and (P). The direction of all three will be known.
- Magnitude of (W) will be known.
- Trial wedge will be assumed at some angle (λ) from horizontal.

Force Triangle



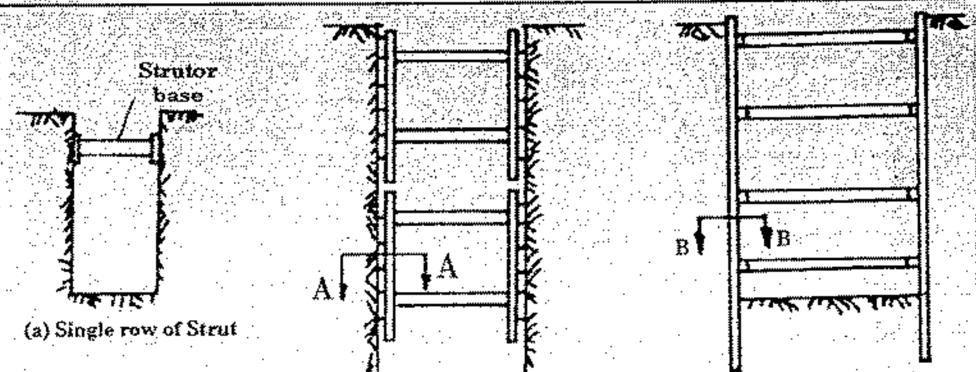
- In the force triangle, magnitude of W is known and all angles of the triangle are known. Hence, P can be calculated.
- By assuming various trial wedges at different trial angle (λ) the value of P will be calculated.
- **For Active Earth Pressure,** P is the **highest value** obtained for various trial wedges.
- **For Passive Earth Pressure,** P will be the **minimum value** that will cause the wedge to move.

Note:

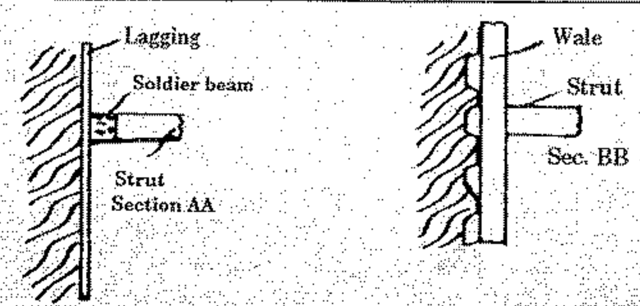
- Rankine's Earth pressure Theory.
  (1) **Overestimates** the **Active Earth** pressure.
  (2) **Underestimates** the **pasive Earth** pressure.
- Retaining walls are designed for **Active Earth pressure.**
- ***On compacting the soil***
  (1) Active Earth pressure (↓) decreases
  (2) Passive Earth pressure (↑) increases.

BRACED EXCAVATIONS

- Permanent earth cuts are usually made either with stable side slopes or supported by retaining walls.
- Temporary cuts, on the other hand, are made with as steep a slope as possible, depending on the soil conditions or made with vertical sides and are suitable braced.
- If the depth of the cut is less than 6m, it is usually termed a shallow cut.
- The design of a sheeting and bracing system for deep cuts (with depth more than 6m) require an estimate of subsoil characteristics, dimensions of the cut,
- If an supported cut with vertical sides is made in a cohesive soil, tension cracks are likely to appear shortly after excavation near the surface of the ground adjoining the cut.
- The presence of tension cracks considerably reduces the critical height, and the horizontal cross members are called struts or braces are provided at upper ends to eliminates tension crack. The struts are usually of timber (square in section) tightened by wedges or screws and support horizontal timber.



(a) Single row of Strut



(b) Lagging and Soldier beam

(c) Sheet pile and wale

- If the depth of a narrow cut exceeds $H_c/2$, solidier beams that bear against horizontal boards termed lagging the structs are wedged against horizontal timber section called wales that support vertical members known as sheeting.

STRUT LOADS

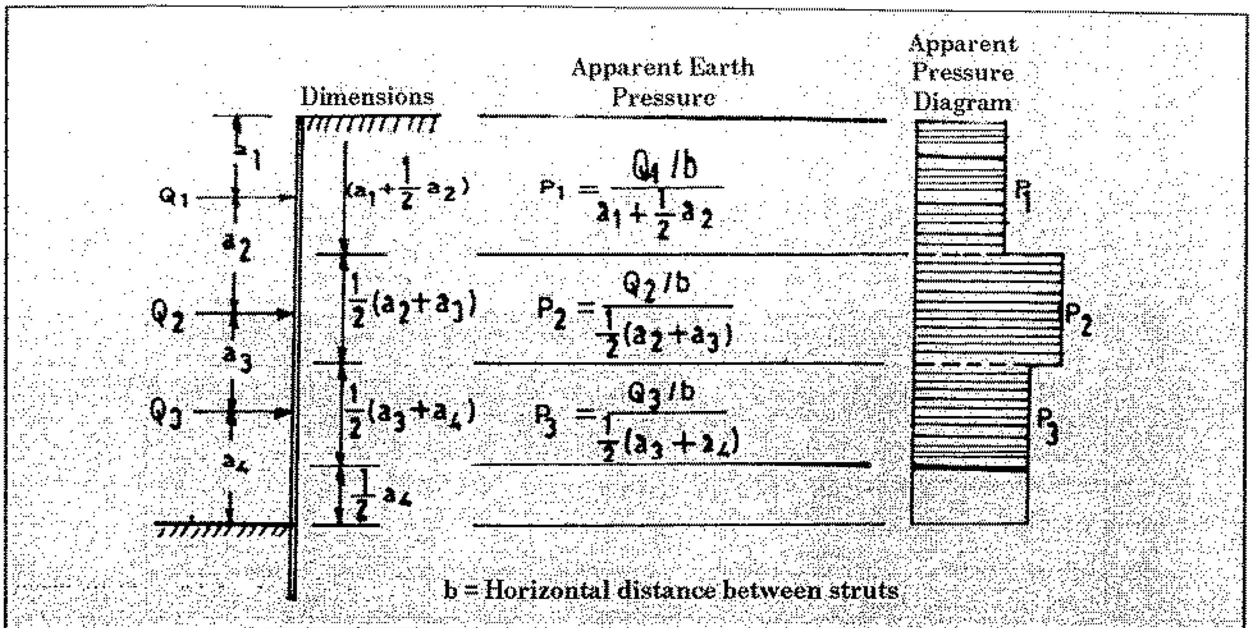
- The retaining wall tends to tilt about its base, while bracings tend to tilt about its top where they are restrained from movement. This explains the difference in the pattern of distribution of lateral pressure in the two cases.
- There is another major point of difference. The retaining wall is a single structural unit, and it will remain safe if it is designed to withstand the total lateral thrust acting against its full height.
- The bracings for a cut are composed of several units as compared to a rigid retaining wall which is a single unit. It is necessary that the stability of each unit/member is examined.
- The strut being the major unit, the estimation of strut loads is of large importance.
- If a strut fails on account of an understimation of the load coming on it, its share of load is transferred to adjoining struts, leading to a possible failure in them too.
- Because of progressive failure, the entire bracing system may fail.
- The load taken by a strut is a function of
  (a) The deformation condition.
  (b) The force with which the wedges are driven.
  (c) The time-elapse between the excavation of cut and installation of supports.

Procedure

(i) It is assumed that the load on each strut is equal to the total earth pressure acting on the sheeting over a rectangular area extending horizontally half the distance to the next vertical row of struts on either side and also vertically half the distance to the horizontal sets of struts immediately above or below.

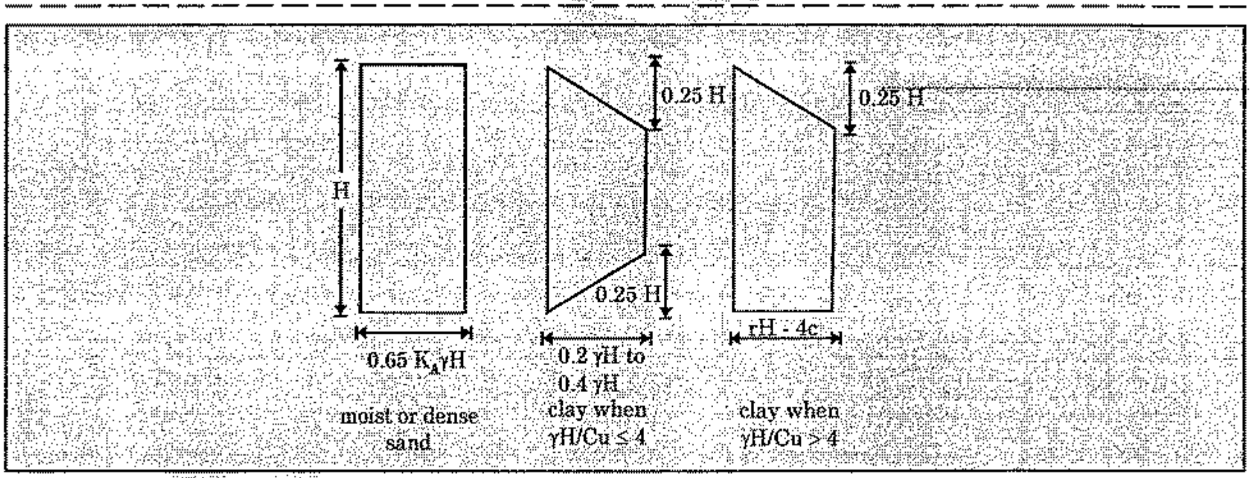
(ii) The earth pressure is assumed to be uniformly distributed over the rectangular area.

(iii) For purposes of calculations, the bottom of the cut is assumed to be a struts.



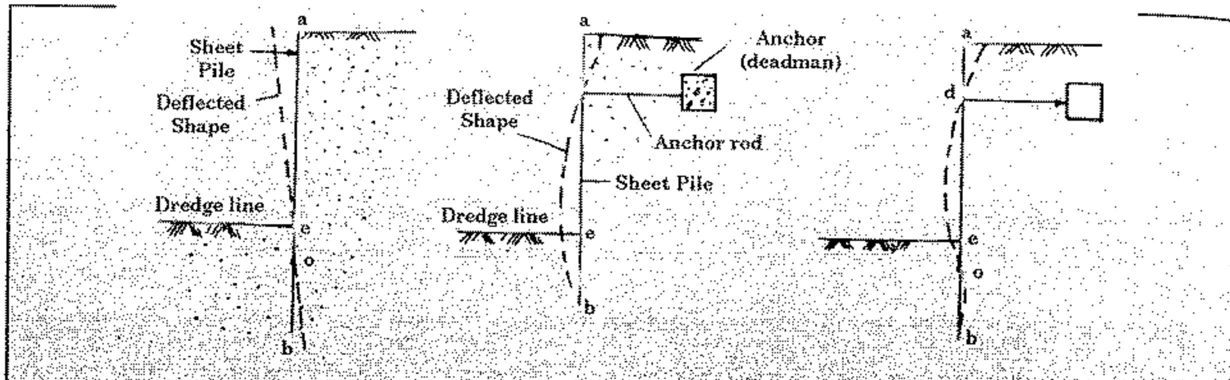
Note: (i) In cohesionless deposits, the earth pressure at the ground surface must be zero. The actual earth pressure distribution is likely to be parabolic, and under field conditions, it will vary widely from one vertical section to another along the length of a cut.

(ii) Apparent earth pressure diagrams for cuts



FLEXIBLE RETAINING STRUCTURES

- A sheet pile wall is an earth and water retaining structure which behaves as fixed vertical cantilevers in resisting lateral earth pressure.
- If the wall height is large, support against lateral pressure is provided by embedment in the ground as well as by the tie rods near the top. This type of earth retaining structure is called an anchored bulkhead.
- An anchored bulkhead may be a dredged type or backfilled type

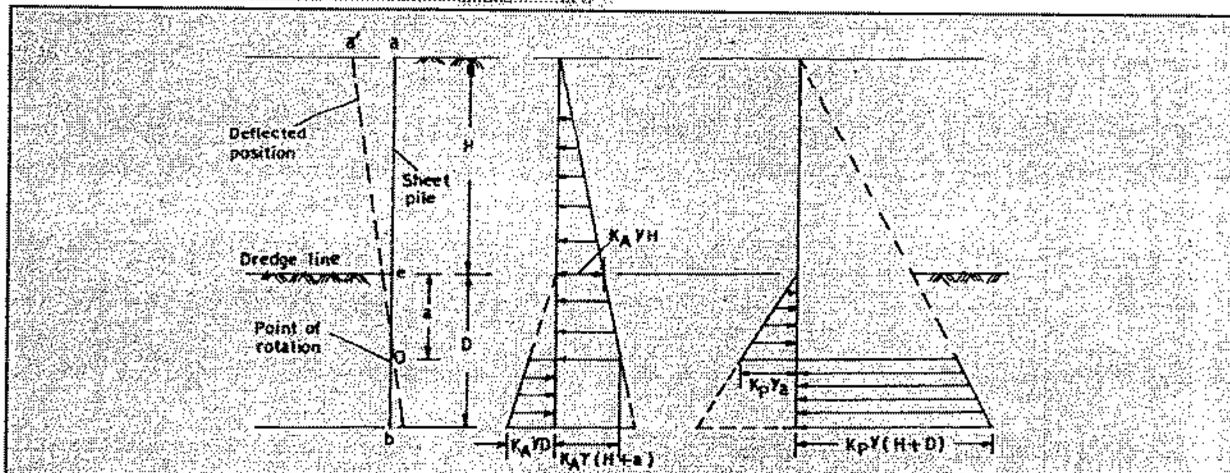


(a) Cantilever sheet pile (b) Anchored bulkhead to shallow depth (c) Anchored bulkhead driven to deeper depth

CANTILEVER SHEET PILE WALL

Cantilever Sheet Pile in Granular Soils

- The cantilever sheet pile of the wall.
- Between the dredge line (point e) and the point of rotation (point *o*), as the wall tends to move towards the soil in front of the wall, passive conditions are generated on this side.
- Below the point of rotation, the active and passive conditions generated on the two sides are reversed.
- Stability to the sheet pile wall is provided by the passive pressure over the embedded depth of sheet pile.



(a) Cantilever sheet pile (b) Active earth pressure diagram (c) Passive earth pressure diagram

Cantilever sheet piling

- The point of rotation, O located at a distance a below the dredge line has zero earth pressure. The magnitude of earth pressure at locations e, O and b are calculated as below

$$p_{Ae} = K_A\gamma H$$

$$p_{Ae} = (K_p - K_A)\gamma a$$

$$a = \frac{p_{Ae}}{(K_p - K_A)\gamma}$$

$$p_{po} = K_p \gamma (H + a) - K_A \gamma a$$
$$= (K_p - K_A) \gamma a + K_p \gamma H$$
$$p_{pb_1} = p_{p_o} + (K_p - K_A) \gamma Y$$
$$= (K_p - K_A) \gamma D + K_p \gamma H$$
$$p_{pb_2} = (K_p - K_A) \gamma Y$$

Let R_a be the resultant of all forces above point O, acting at a distance $\bar{y}$ above O. The distance Z is calculated from equilibrium equation $\sum F_H = 0$

$$R_a + (p_{pb_1} + p_{pb_2})\frac{Z}{2} - p_{pb_2}\frac{Y}{2} = 0$$

$$Z = \frac{p_{pb_2} y - 2R_a}{(p_{pb_1} + p_{pb_2})}$$

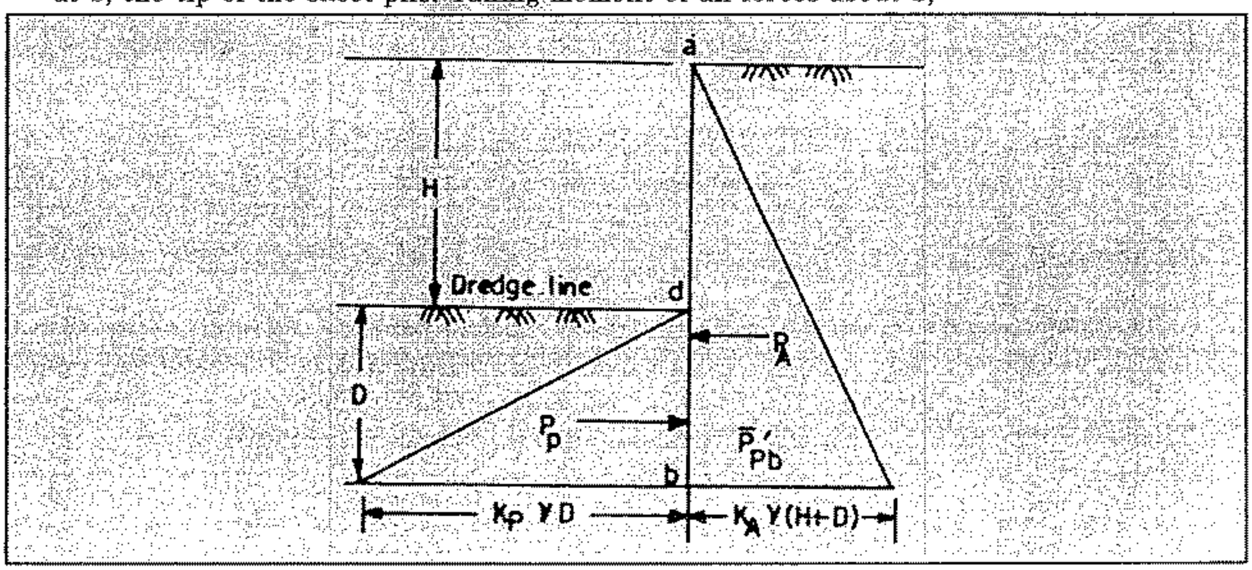
Taking moments about the base of pile, $\sum M = 0$

$$R_a (Y + \bar{y}) + \frac{Z}{3}(p_{pb_1} + p_{pb_2})\frac{Z}{2} - p_{pb_2}\left(\frac{Y}{2}\right)\left(\frac{Y}{3}\right) = 0$$

$$6 R_a (Y + \bar{y}) + Z^2(p_{pb_2} + p_{pb_1}) - p_{pb_2} Y^2 = 0$$

Approximate Analysis

- In the approximate analysis, the active earth pressure on the back of the wall and the passive earth pressure in front of the wall are assumed to extend upto the base of the wall.
- The passive pressure due to the length of the wall O b, near the bottom is assumed to act at b, the tip of the sheet pile. Taking moment of all forces about b,



$$\frac{1}{2}K_p \gamma D^2 \times \frac{D}{3} = \frac{1}{2}K_A \gamma (H + D)^2\left(\frac{H + D}{3}\right)$$

By solving the above equation value of D is obtained.

Cantilever sheet pile in Cohesive Soil $(\phi_u = 0)$

- The analysis of a cantilever sheet pile in cohesive soils is carried out in a similar way to that in granular soils.
- But certain phenomena such as consolidation of clay in passive pressure zones, formation by solving the above cubic equation of tension cracks in the active zone may need additional consideration.

We know that, in cohesive soils

$$p_A = \bar{q}\,K_A - 2C_u\sqrt{K_A}$$

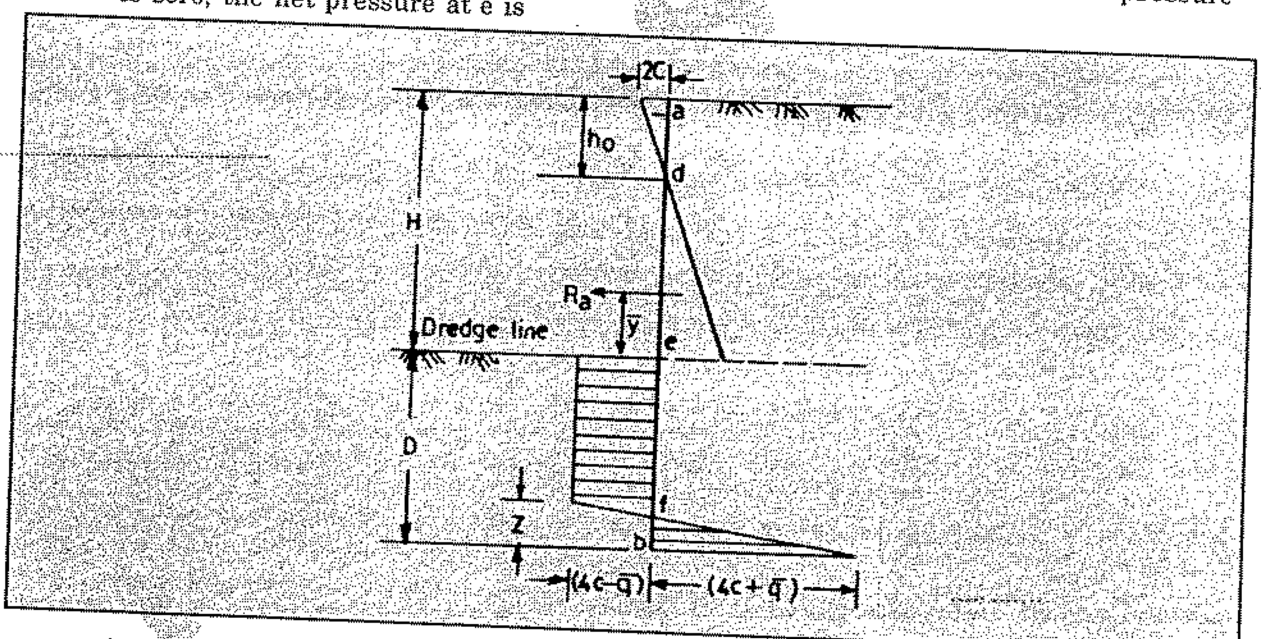
Where, $\bar{q}$ = effective vertical stress at any depth.

for $\phi_u = 0, \quad K_A = K_p = 1.$

$$p_A = \bar{q} - 2C_u$$

$$p_p = \bar{q} + 2C_u$$

- Active pressure at point a is equal to $-2C_u$ and at e is equal to $\bar{q} - 2C_u$, where $\bar{q}$ is effective vertical stress at e equal to γH.
- At the dredge line i.e., point e on the left of the sheet piling, the overburden pressure is zero, the net pressure at e is



$$(p_p - p_A)e = (0 + 2C_u) - (\bar{q} - 2C_u) = 4C_u - \bar{q} \text{ acting to the right.}$$

Similarly, at the base of the wall, i.e., point b, the net pressure is

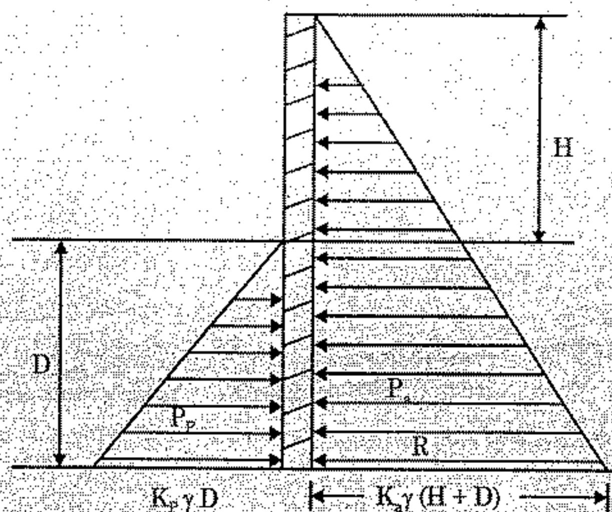
$$(p_p - p_A) = (\bar{q} + \gamma' D + 2C_u) - (\gamma' D - 2C_u)$$

$$= 4cu + \bar{q} \text{ acting to the left.}$$

Example 13

With suitable illustrations describe the simplified analysis method for designing the depth of embedment of a cantilever short pile for a 6 m deep excavation in a sandy soil layer for $\gamma = 18\ kN/m^3$ and $\phi = 35°$ for a factor of safety of 20°

Sol.



Let D be the depth of the embedment and H be the height of the cantilever sheet pile above the dredge level.

Assuming a concentrated force R acting at the foot of the pile. For equilbrium the moment of active pressure on the right and passive resistance on the left about the point of reaction R must be balanced.

$$\Sigma M = 0 \Rightarrow P_p \times \frac{D}{3} - P_a \times \frac{(H+D)}{3} = 0$$

We will provide F.O.S = 2, against passive force which is providing stability.

$$\therefore \quad \frac{P_P}{2}\frac{D}{3} - \frac{P_a(H+D)}{3} = 0 \quad \text{... (i)}$$

$$P_P = \frac{1}{2}K_p\gamma D^2$$

$$P_a = \frac{1}{2}K_a\gamma(H+D)^2$$

Put the value of P_P and Pa in (*i*)

$$\Rightarrow \frac{1}{6}\times\frac{1}{2}K_p\gamma D^2\times D - \left(\frac{H+D}{3}\right)\times\frac{1}{2}Ka\gamma(H+D)^2 = 0$$

$$\Rightarrow \frac{\gamma}{6}\left[\frac{K_p D^3}{2} - K_a(H+D)^3\right] = 0$$

$$\Rightarrow K_pD^3 - 2\,K_a\,(H+D)^3 = 0 \quad \text{... (ii)}$$

Data given

$$\phi = 35^\circ \quad H = 6\text{ m}$$

$$\gamma = 18\text{ kN/m}^3$$

$$\text{F.O.S} = 2.0$$

$$\therefore \quad K_a = \frac{1-\sin\phi}{(1+\sin\phi)} = \frac{1-\sin 35^\circ}{(1+\sin 35^\circ)} = 0.271$$

$$K_P = \left(\frac{1}{Ka}\right) = \mathbf{3.69}$$

$$\therefore \quad 3.69D^3 - 2 \times 271(6 + D)^3 = 0$$

$$\Rightarrow \quad 3.69D^3 - 0.542\,[(6)^3 + D^3 + 3 \times 6D\,(D + 6)] = 0$$

$$\Rightarrow \quad 3.69\,D^3 - 0.542\,[216 + D^3 + 108\,D] = 0$$

$$\Rightarrow \quad 3.148D^3 - 9.756\,D^2 - 58.536\,D - 117.072 = 0 \quad \text{... (iii)}$$

Solving above equation. for D.

$$\boxed{D = 6.71\text{ m}}$$

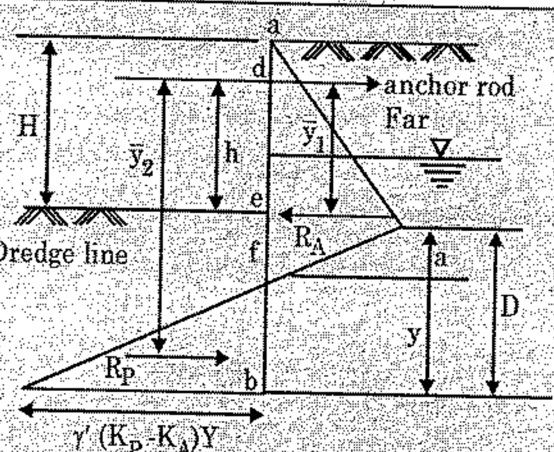
$\therefore$ Therefore depth of embedment for sheet pile $D = 6.71$ m

ANCHORED BULKHEAD

- The stability of an anchored sheet pile depends not only on the passive earth pressure but also on the anchor rod.
- Driving depth that is required in an anchored sheet pile is, thus, considerably smaller than in a cantilever sheet pile.

(a) Granular Soil :

- The earth pressure diagram for the anchord bulkhead in a granular soil.



(a) Granular Soil

F_{ar} = force in the anchor rod

K_A and K_p = coefficients of active and passive earth pressure for the backfill.

R_A = resultant active earth pressure acting at $\bar{y}_1$ below the anchor rod level

R_p = resultant passive earth pressure acting at $\bar{y}_2$ below the ancbor rod

For equilibrium,

$$F_{ar} + R_p - R_A = 0$$

- Tbe depth a to the point of zero pressure can be determined from

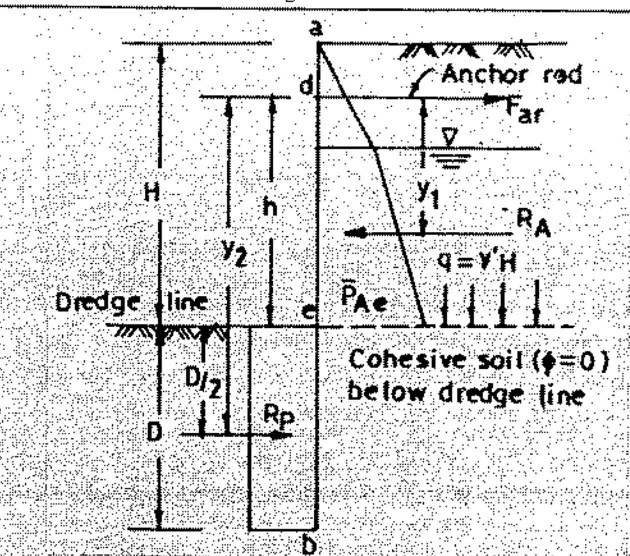
$$a = \frac{p_{Ae}}{\gamma'(K_p - K_a)}$$

- Taking moments about the anchor rod level and satisfying the condition $\Sigma M = 0$,

$$R_A\bar{y}_1 = R_p\bar{y}_2$$

(b) Cohesive Soil :

- In case the soil below the dredge line is cohesive ($\phi_u = 0$) and the backfill is granular the earth pressure diagram will be as shown in figure.



(b) Cohesive soil

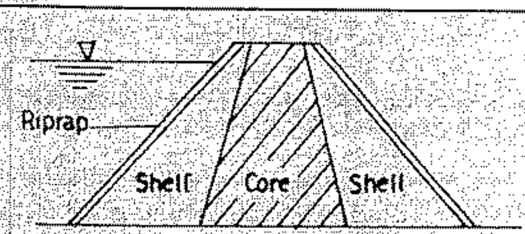
COFFER DAM

A Coffer Dam is a temporary structure constructed usually in a river, take, etc., to keep the working area dry for construction of other structures. After the construction of coffer dam is the area is dewatered by pumping.

Types of Coffer Dam

Earth Embankments

- This the simplest type of coffer dam. This type of coffer dam is made by placing the fill at a suitable location at a stable slope.



Earth Embankment

- To reduce the quantity of seepage, impervious material as core is used.
- These coffer dams have practically no height limition.

Cantilever Sheet Pile

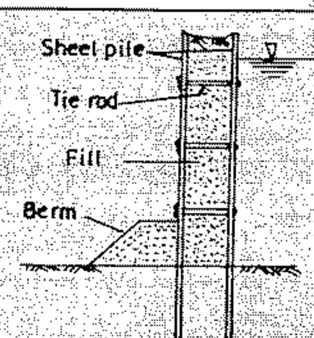
- Single sheet pile coffer dams are constructed by driving sheet piling of steel or timber.

Single Sheet Pile

- These coffer dams are considered superior to earth embankments from the point of view of cost of placement of fill and its subsequent removal and the econimy of space.
- They are useful for small heights only
- Single pile coffer dams are susceptible to large leakage and flood damage.

Double Wall Coffer Dams.

- These coffer dams consist of two lines of sheeting tied together, with the space between the sheeting filled with soil.

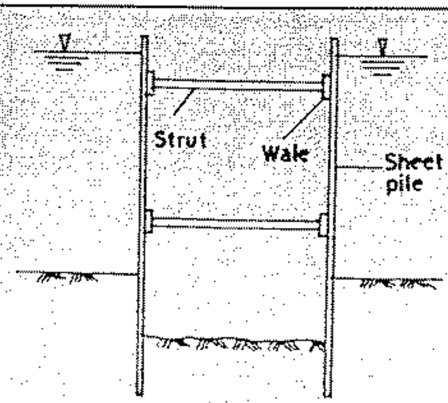


Double Wall Coffer Dam

- Two rows of sheeting may be connected to each other by a combination of wales and tie rods.
- Double-walled coffer dams are suitable for moderate heights.

Braced Coffer Dams

- A braced coffer dam is formed by driving vertical sheeting into the ground. The vertical sheeting is held in position through horizontal beams called wales.

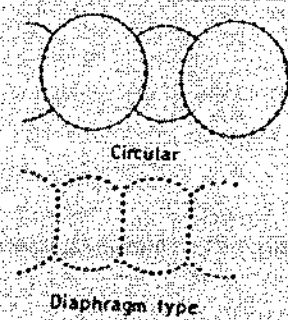


Braced Coffer Dam

- Braced coffer dams are more commonly used as land coffer dams for supporting soil during excavation.
- They are economical for small to moderate heights.

Cellular Coffer Dams

- A cellular coffer dam is made by driving sheet piles to form a series of cells which are later filled with a suitable soil.
- The cells are interconnected for water-tightness and are self stabilising against lateral pressure of water and soil.



Cellular Coffer Dam

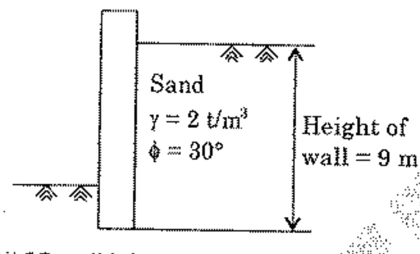
- The celluar coffer dams are more advantageous as compared to the other types of coffer dams. These are easy to construct and are economical as compared to the braced coffer dams, particularly for a large area or head of water.
- Cellular coffer dams are relatively more water-tight as compared to the braced coffer dams.
- Compared to the embankment type, the cellular coffer dam is smaller and occupies less area, impossing less restrictions on channel flow or navigation.

OBJECTIVE TYPE QUESTIONS

1. A retaining wall retains a sand strata with $\phi = 30°$ up to its top. If a uniform surcharge of $12\ t/m^2$ is subsequently put on the sand strata, then the increase in the lateral earth pressure intensity on the retaining wall will be

(a) $1\ t/m^2$ (b) $2\ t/m^2$ (c) $4\ t/m^2$ (d) $8\ t/m^2$

2. Active earth pressure per metre length on the retaining wall with a smooth vertical back as shown in the given figure will be



(a) 81 t (b) 27 t (c) 2 t (d) 1 t

3. A retaining wall with vertical back retains a cohesionless dry backfill at an inclination of β with the horizontal. The backfill has an angle of internal friction φ, unit weight γ and height of the wall is H. The passive earth pressure on the wall is given by (where P_p = Total passive earth pressure)

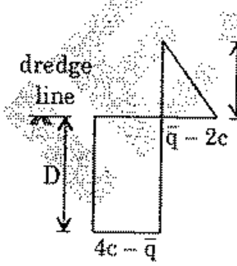
(a) $P_p = \frac{1}{2}\gamma H^2 \frac{\cos\beta - \sqrt{\cos^2 - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}$

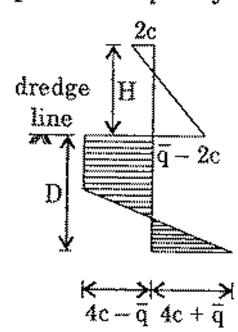
(b) $P_p = \frac{1}{2}\gamma H^2 \frac{\cos\beta - \sqrt{\cos^2\beta + \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta + \cos^2\phi}}$

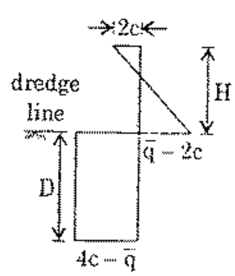
(c) $P_p = \frac{1}{2}\gamma H^2 \frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}$

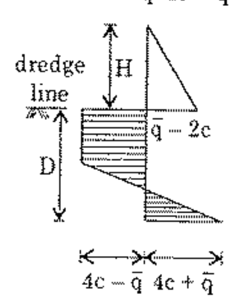
(d) $P_p = \frac{1}{2}\gamma H^2 \frac{\cos\beta + \sqrt{\cos^2\beta + \cos^2\phi}}{\cos\beta - \sqrt{\cos^2\beta + \cos^2\phi}}$

4. In the following figure, if
H = height of wall above dredge line,
$\bar{q}$ = effective vertical stress at any depth,
c = unit cohesion,
and passive pressure is shown hatched in the figures, then the earth pressure distribution diagram used for analysis of a cantilever sheet pile embedded to a depth D in a purely cohesive soil will be as in

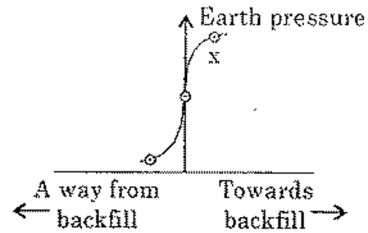
(a)


(b)


(c)


(d)


5. Earth pressure and resultant possibilities of wall movement are shown in the diagram below.



Lateral movement of retaining wall

The point marked X in the diagram denotes

(a) earth pressure at rest
(b) active earth pressure
(c) arching active pressure
(d) passsive earth pressure

6. Given that for a soil deposit,

K_o = earth pressure coefficient at rest
K_a = active earth pressure coefficient
K_p = passive earth pressure coefficient
μ = Poisson's ratio.

The value of $(1 - \mu)/\mu$ is given by

(a) K_a/K_p
(b) K_o/K_a
(c) K_p/K_a
(d) $1/K_o$

7. Consider the following statements:
Rankine's theory and Coulomb's theory give same values of coefficients of active and passive earth pressure when

1. the retaining wall has a vertical back
2. the backfill is cohesionless
3. angle of slope of backfill is equal to the angle of internal friction
4. angle of slope of backfill is 0°
5. angle of wall friction δ is 0°
6. angle of wall friction δ is equal to ϕ

Which of these statements is/are correct?

(a) 1, 2, 3 and 5
(b) 1, 2, 4 and 5
(c) 2, 3 and 6
(d) 1, 4 and 6

8. Consider the following statements:

1. Coulomb's earth pressure theory does not take the roughness of wall into consideration.
2. In case of non-cohesive soils, the coefficients of active earth pressure and earth pressure at rest are equal.
3. Any movement of retaining wall away from the fill corresponds to active earth pressure condition.

Which of these statements is/are correct?

(a) 1 alone
(b) 1 and 2
(c) 2 alone
(d) 3 alone

9. Match **List-I** (Type of structure) with **List-II** (Type of pressure exerted by sandy back fill) and select the correct answer using the codes given below the lists:

List-I	List-II
A. A mosonry retaining wall founded on compressible clay	1. Active pressure
B. Pressure on the back of a cantilever sheet pile wall near the embedded end	2. Earth pressure at rest
C. A masonry retaining wall founded on rock	3. Passive earth pressure

Codes:

	A	B	C
(a)	1	3	2
(b)	3	2	1
(c)	3	1	2
(d)	2	3	1

10. Given that $c = 2\ t/m^2$, $\phi = 0°$ and $\gamma = 2\ t/m^3$, the depth of tension crack developing in a cohesive soil backfill would be
(a) 1 m (b) 2 m (c) 3 m (d) 4 m

11. The correct sequence of the given parameters in descending order of earth pressure intensity is
(a) active, passive, at rest (b) passive, active, at rest
(c) passive, at rest, active (d) at rest, passive, active

12. The nature of earth pressure above dredge line behind a cantilever sheet pile wall is
(a) active (b) passive (c) at rest (d) active and passive

13. An earth-retaining structure may be subjected to the following lateral earth pressures:
1. Earth pressure at rest 2. Passive earth pressure
3. Active earth pressure
The correct sequence of the increasing order of the magnitudes of these pressures is
(a) 3, 2, 1 (b) 1, 3, 2 (c) 1, 2, 3 (d) 3, 1, 2

14. For a sand having an internal friction of 30°, the ratio of passive to active lateral earth pressure will be
(a) 1 (b) 3 (c) 6 (d) 9

15. When movement of a wall under the earth pressures from the backfill was prevented the coefficient of earth pressure was recorded as 0.5. The ratio of the coefficient of passive and active earth pressure of the backfill is
(a) 1/3 (b) 3 (c) 1/9 (d) 9

16. The wall friction of the retaining wall
(a) decreases active earth pressure but increases passive earth pressure
(d) decreases passive earth pressure but increases active earth pressure
(c) decrease both active and passive earth pressure
(d) increase both active and passive earth pressure

17. If the coefficient of active earth pressure is 1/3, then what is the value of coefficient of passive earth pressure?
(a) 1/9 (b) 1/3 (c) 3 (d) 1

18. Why are weep holes provided at the back of retaining walls?

(a) To reduce the active earth pressure on the walls.
(b) To reduce the build-up of hydrostatic pressure.
(c) To provide better compaction.
(d) To increase the passive earth pressure.

19. A vertical cut is to be made in saturated clay with c = 15 kN/m^2, ϕ = 0°, and γ = 20 kN/m^3. What is the theoretical depth to which the clay can be excavated without side collapse?

(a) 6 m (b) 2 m (c) 2.5 m (d) 3 m

20. What is the intensity of active earth pressure at a depth of 10.0 m in dry sand with an angle of shearing resistance of 30° and unit weight of 18 kN/m^3?

(a) 50 kN/m^2 (b) 60 kN/m^2 (c) 70 kN/m^2 (d) 80 kN/m^2

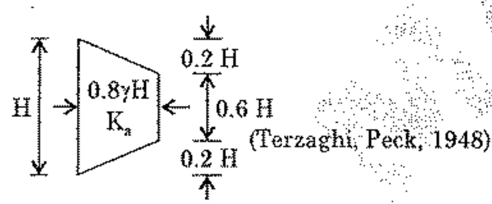
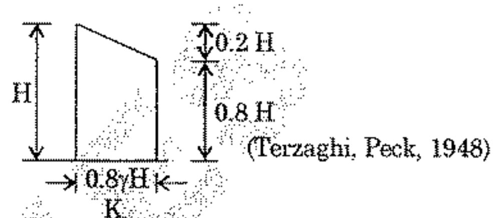
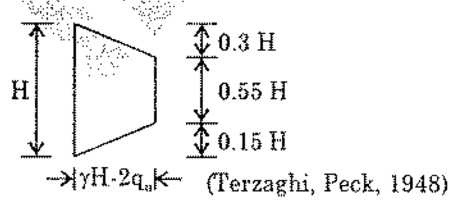
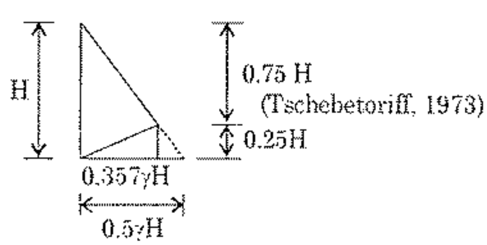
21. Consider the following statements:

1. The yield of a retaining wall required to reach plastic equilibrium in active case is more than that in passive case.
2. Culman's graphical method is simplified version of the mohr general trial wedge method.
3. For a masonry gravity retaining wall coulomb's theory of earth pressure is preferred for designing.

Which of these statements is/are correct?

(a) 1, 2 and 3 (b) 1 and 2 only (c) 2 and 3 only (d) 3 only

22. Match **List-I** (Pressure distribution for strutted excavation of foundation trench) with **List-II** (Soil type) and select the correct answer using the codes given below the lists:

List	List-II
A. 	1. Dense sand
B. 	2. Moderately stiff clay
C. 	3. Loose sand
D. 	4. Plastic clay

Codes:

	A	B	C	D
(a)	1	3	4	2
(b)	4	2	1	3
(c)	2	4	3	1
(d)	1	2	3	4

23. For no tension at the base of the gravity wall or at any horizontal section within the wall, the resultant of total active pressure and weight of wall must pass

(a) through the middle third of the base
(b) outside the middle third of the base
(c) through one of the ends of the base
(d) through the middle quarter of the base

24. Given that for a soil backfill, K_a = coefficient of active earth pressure, K_p = coefficient of passive earth pressure and K_o = coefficient of earth pressure at rest. Which one of the following represents the correct relationship between K_a, K_o and K_p?

(a) $K_o = K_p/2$
(b) $K_o = (K_a + K_p)/2$
(c) $K_o = (K_p - K_a)/2$
(d) None of these

25. A vertical retaining wall retains a c – ϕ backfill and carries a surcharge of uniform intensity 'q' per unit area. The depth Z_0 from the top of the wall where the active earth pressure is zero is given by ($\alpha = 45° + \phi/2$ and γ = unit weight of the soil)

(a) $\frac{q}{\gamma}$
(b) $\frac{2c}{\gamma}\tan\alpha - \frac{q}{\gamma}$
(c) $\frac{2c}{\gamma}\tan\alpha + \frac{q}{\gamma}$
(d) $\frac{2c}{\gamma}\tan\alpha$

26. A cantilever sheet pile derives its stability from

(a) lateral resistance of soil
(b) self weight
(c) the deadman
(d) the anchor rod

27. Deflection of a sheet pile in a braced cut

(a) increases from top to bottom
(b) decreases from top to bottom
(c) increases from top and then decreases
(d) decreases from top and then increases

28. Consider the following assumptions:

1. The backfill is dry and homogeneous.
2. The sliding wedge acts as a rigid body.
3. The back face of the wall is a plane.
4. The position and direction of the earth thrust are known.

Which of these assumptions are common to Rankine's and Coulomb's earth pressure theories?

(a) 1 and 3
(b) 2 and 4
(c) 1 and 4
(d) 2 and 3

29. Consider the following statements:

1. Culmann's graphical method of determining the earth pressure is based on Coulomb's wedge theory.
2. Rankine's theory of lateral earth pressure is more versatile than Coulomb's theory.
3. A gravity retaining wall together with the retained backfill and supporting soil is an indeterminate system.

Which of these statements are correct?

(a) 1 and 2 (b) 1 and 3 (c) 2 and 3 (d) 1, 2 and 3

30. Consider the following statements:

Active earth pressure will be developed in the backfill when the

1. horizontal strain is $\leq 0.5\%$
2. horizontal strain is $\geq 1\%$
3. mobilized shearing resistance along the failure plane is a minimum.
4. mobilized shearing resistance along the failure plane is a maximum.

Which of these statements are correct?

(a) 1 and 3 (b) 1 and 4
(c) 2 and 3 (d) 2 and 4

31. Saturated unit weight of a soil is 20 kN/m^3 and unit weight of water is 10 kN/m^3. If the ground water table is at the surface of soil and lateral earth pressure coefficient of soil is 0.4, effective lateral stress at 10 m depth will be

(a) –20 kPa (b) 40 kPa
(c) 80 kPa (d) 180 kPa

32. A 3 m high retaining wall is supporting a saturated sand (saturated due to capillary action) of bulk density 18 kN/m^3 and angle of shearing resistance 30°. The change in magnitude of active earth pressure at the base due to rise in ground water table from the base of the footing to the ground surface shall (γ_w = 10 kN/m^3)

(a) increase by 20 kN/m^2 (b) decrease by 20 kN/m^2
(c) increase by 30 kN/m^2 (d) decrease by 30 kN/m^2

Instructions :

The following items consists of two statements, one labelled as ***'Assertion A'*** *and the other labelled as* ***'Reason R'****. You are to examine these two statements carefully and decide if the Assertion A and the Reason R are individually true and if so, whether the Reason is a correct explanation of the Assertion. Select your answers to the these items using the codes given below :*

(a) both A and R are true and R is the correct explanation of A
(b) both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

33. Assertion (A): The safe height ($2z_0$) to which an unsupported vertical cut in clay can be made is $4c/\gamma$.

Reason (R): Active earth pressure of cohesive backfill shows that the negative pressure (tension) is developed at top level. This tension decrease to zero at depth z_0 and total net pressure up to a depth $2z_0$ is zero.

34. Assertion (A): Passive earth pressure is always greater than the earth pressure at rest and active earth pressure.

Reason (R): In passive state the structure becomes the actuating element and soil becomes the resisting element to maintain the stability.

35. Consider the following statements:
Assertion (A): The state of earth pressure at rest is the state of equilibrium with zero strain condition.
Reason (R): In rest condition neither the wall nor the soil moves.

36. **Assertion (A):** Rankine's earth pressure theory should not be used for concrete retaining walls and Coulomb's theory should not be used for estimating passive earth pressures.
Reason (R): Rankine assumed that the retaining wall has a vertical back and Coulomb assumed that the resultant reaction due to earth pressure acts at one-third the height of the wall.

37. **Assertion (A):** Rankine's earth pressure theory is a simplified form of Coulomb's earth pressure theory.
Reason (R): Coulomb's theory considers effect of pore pressures.

38. Consider the following statements:
Assertion (A): Clays are an excellent backfill material.
Reason (R): It is difficult to compact them.

ANSWERS

1. (c)	2. (b)	3. (c)	4. (b)	5. (d)	6. (d)	7. (b)	8. (d)	9. (a)	10. (b)
11. (c)	12. (a)	13. (d)	14. (d)	15. (d)	16. (a)	17. (c)	18. (b)	19. (d)	20. (b)
21. (c)	22. (a)	23. (a)	24. (d)	25. (b)	26. (a)	27. (a)	28. (a)	29. (b)	30. (b)
31. (b)	32. (a)	33. (a)	34. (a)	35. (a)	36. (b)	37. (b)	38. (d)		

Hints

1. $\Delta p_a = k_a F$

$= \frac{1}{3} \times 12 = 4 \text{ t/m}^2$

2. $\frac{1}{2} k_a \gamma H^2$

$\frac{1}{2} \tan^2\left(45 - \frac{30}{2}\right) \times 2 \times 9^2$

$= \frac{1}{\cancel{2}} \times \frac{1}{3} \times \cancel{2} \times 9^2 = 27\text{t}$

10. $2Z = \frac{4C}{\gamma}$

$Z = \frac{4 \times 2}{2 \times 2} = 2\text{m}$

14. $\frac{k_p}{K_a} = \tan^4 (45 + f/2) = 3^2 = 9$

15. $1 - \text{Sin } f = 1/2$

$\sin \phi = 1/2$

$$\frac{k_p}{K_a} = \frac{(1+\sin\phi)^2}{(1-\sin\phi)^2} = \frac{(1.5)^2}{(.5)^2} = 9$$

17. $K_p = \frac{1}{Ka}$

19. $H = \frac{4c}{\gamma}$

$= \frac{4 \times 5}{20} = 3 \text{ m}$

20. $p_a = k_a \gamma_d H$

$= \frac{1}{3} \times 18 \times 10 = 60 \text{ kN/m}^2$

31. $p_a = k_a \gamma_{sub} \times H$

$= 0.4 \times 10 \times 10 = 40 \text{ kP}_a$

CHAPTER

12

Shallow Foundation

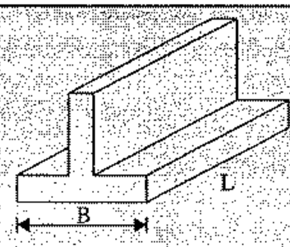
INTRODUCTION

- Footings are generally the lowermost supporting part of the structure known as sub-structure and are the last structural elements through which load is transferred to foundation comprising soil/rock.
- Structural elements transfer the applied loads from one part of the building to the other. These are in turn transmitted to the foundation which transfers it to the underlying soil/rock.

TYPE OF FOOTINGS

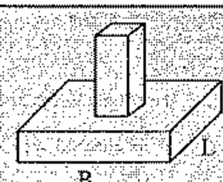
1. Strip footing:

These are also known as wall footing to support wall. [If L >> B] → strip footing.



2. Isolated footing:

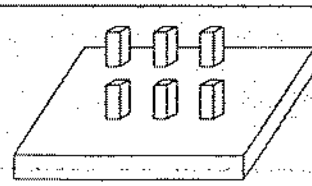
These are also known as spread footing. Isolated footing is **used below the column.**



Isolated footing

3. Raft/Mat foundation:

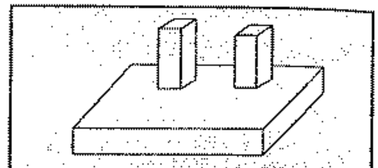
- These type of foundations are large continous footing which support all collumns and walls of a structure and are constructed when soil is weak.



Raft foundation

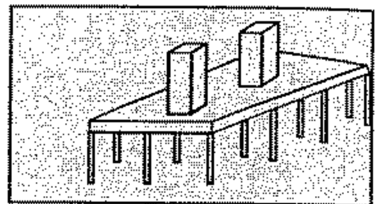
4. Combined Footings :

These footings are usually constructed due to space limitations and support two or more columns. They may be either rectangular or trapazoidal in shape.


Combined footing

5. Pile foundation :

These are used to transmit heavy column loads to a group of piles joined at top by a pile cap. The piles transmit the structural loads to the underlying soil through friction and bearing.



Such type of foundation system is usually adopted when the material below footing is too weak to support the structure and it becomes essential to transfer loads to better strata underlying weaker strata. These foundations are very expensive.

- The choice of a particular type of foundation depends on
  (a) Magnitude of loads. (b) Nature of subsoil strata. (c) Nature of Superstructure.

General requirements for foundation:

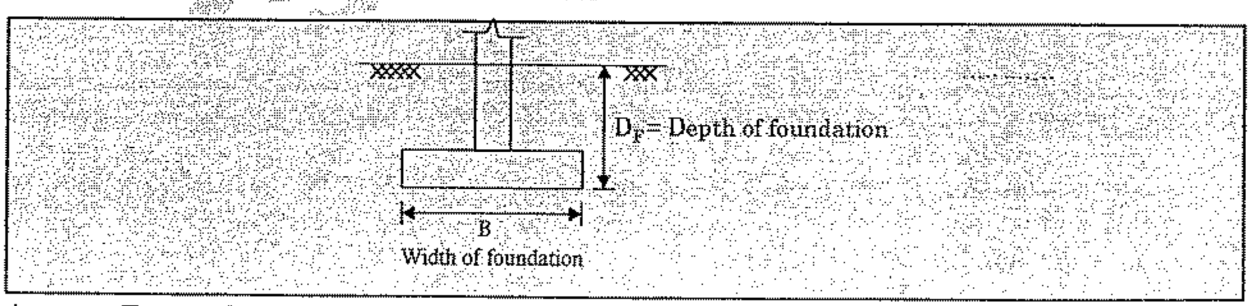
For satisfactory performance, a foundation must satisfy the following three basic criteria.

1. Shear failure criteria or Bearing capacity — i.e Foundation must be safe against shear failure
2. Determent criteria — i.e settlement of foundation esp. differential settlement must be within the perminible limit.
3. Location & depth criteria — Foundation must be located at such a depth that is performance is not affected by Seasonal volume changes of soil due to swelling & shrenkage and also by the presence of adjoining structure.

- Generally, for sandy soil settlement is certical (except for narrow footing and in loose sand)
- For clayey soil, the shear strength is critical.

Note: Generally for sandy soil: Settlement criteria is more critical and for clayey soil, shear failure criteria is critical as they have low shear strength

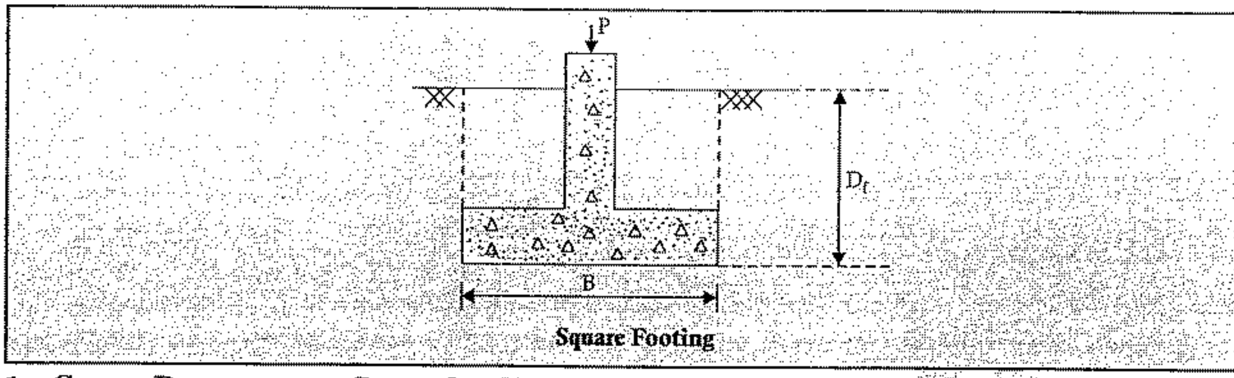
CLASSIFICATION OF FOUNDATION



As per Terzaghi,

- If $\left(\frac{D_f}{B}\right) \leq 1 \rightarrow$ The foundation is called Shallow foundation.
- Shallow foundation transfers the load at smaller depth. e.g.: Combined footing, Raft Foundation, Isolated footing
- If $\left(\frac{D_f}{B}\right) \geq 15 \rightarrow$ The foundation is called Deep foundation eg Pile Foundation.

BASIC DEFINITIONS



1. Gross Pressure or Gross loading intensity(q_g)

$(q_g) = \frac{P}{B^2} + \frac{\gamma D_f \times B^2}{B^2}$ (Assuming that unit weight of soil = unit weight of water)

$\Rightarrow$ $$\boxed{q_g = \frac{P}{B^2} + \gamma D_f}$$

It is the total pressure intensity at the base of footing.

2. Net pressure Intensity (q_n)

- It is generally the loading intensity at the base of footing in excess of the load intensity that the soil was originally subjected to that causes deformation in soil.

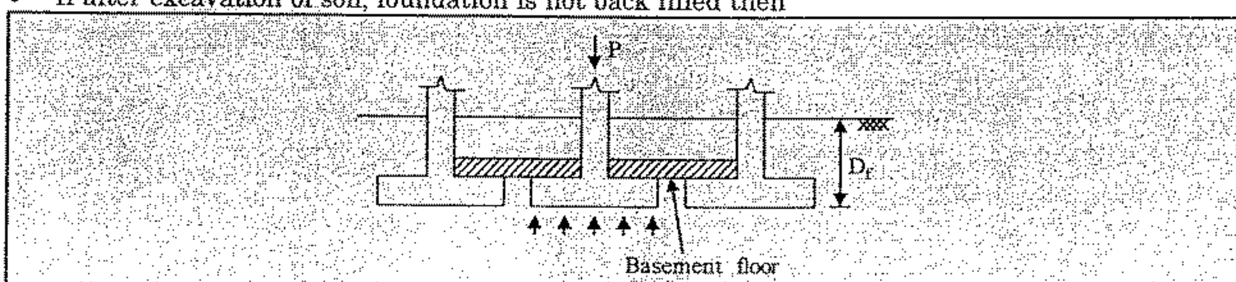
  Hence, net pressure intensity = $\frac{P}{B^2}$

$$\boxed{q_n = q_g - \gamma D_f}$$

- For safe design, $q_n \leq$ Net allowable bearing pressure.

  i.e $$\boxed{\frac{P}{B^2} \leq q_{netalloable}}$$

- From the above formula calculation of B i.e proportioning of foundation is done.
- If after excavation of soil, foundation is not back filled then



$\Rightarrow$ $$\boxed{q_n = \left(\frac{P}{B^2} - \gamma D_f\right)}$$

$$\boxed{\left(\frac{P}{B^2} - \gamma D_f \leq q_{net\,allowable}\right)}$$

- Note that when excavated soil is not back filled, load carrying capacity of the soil is significantly increased.
- When loads are very heavy and clayey soil is very soft the above concept is used to advantage. A raft foundation is constructed and soil is not back filled. If raft is constructed to such a depth that $\frac{P}{B^2} - \gamma.D_f = 0$, than soil is not called upon to resist any load. Such a raft is called fully compensated or floating raft.

3. Ultimate bearing capacity (q_u)

- The maximum gross intensity of loading that the soil can support before it fails in shear is called **ultimate bearing capacity (q_u).**

4. Net ultimate Bearing capacity (q_{nu})

- The maximum net intensity of loading at the base of the foundation that the soil can support before failing in shear.

$$\boxed{q_{nu} = q_u - \gamma D_f = \frac{P_u}{B^2}}$$

5. Net safe bearing capacity (q_{ns})

$$\boxed{q_{ns} = \frac{q_{nu}}{F.O.S} = \frac{P_u / B^2}{F.O.S}}$$

F.O.S. of 2 – 3 is adopted

6. Gross safe bearing capacity (q_s)

$$\boxed{q_s = \frac{q_{nu}}{F.O.S} + \gamma D_f}$$

- Max gross intensity of loading that the soil can safely carry with adequate safety against failure in shear is called (q_s).

7. Safe bearing pressure (q_{ps})

- Maximum net intensity of loading that can be allowed on soil without the settlement exceeding the permissible value.
- No factor of safety is used when dealing with settlement.

8. Allowable bearing pressure ($q_{a\ net}$)

- Maximum net intensity of loading that can be imposed on the soil with no possibility of shear failure or the possibility of excessive settlement.
- It is the smaller of **net bearing capicity (q_{ns})** and **safe bearing pressure (q_{ps}).**

Note: According to IS code Allowable bearing pressure ($q_{a\text{-}net}$) is Allowable bearing capacity.

BEARING CAPACITY OF SHALLOW FOUNDATION

- As discussed earlier bearing capacity of the soil is the ability of soil to carry loads without any failure.

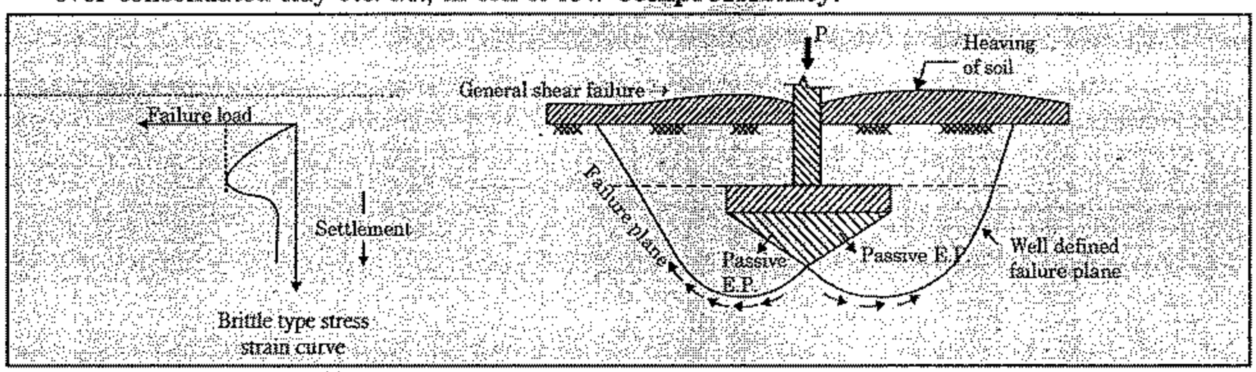
- To ensure no failure condition we shall know the bearing capacity of soil.
- Bearing capacity of soil is determined by
  (a) Analytical Method
  (b) Building codes
  (c) Field Test Method.
    (1) Standard Penetration test (SPT test)
    (2) Plate load test
    (3) Static Cone Penetration test

ANALYTICAL METHOD

- These methods are based on shear failure criteria:
- There are 3 modes of shear failure based on pattern of shear failure:
  (a) General shear failure. (b) Punching shear failure. (c) Local shear failure.

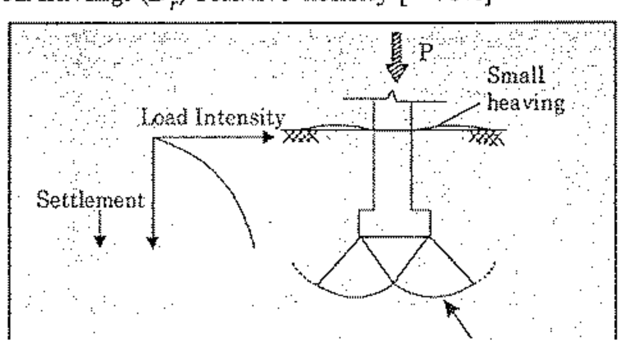
(a) General shear failure:

- General shear failure occurs in soil possssing **Brittle type shear stress curve.** Dense sand, silt, over consolidated clay etc. i.e., in soil of **low compressibility.**



- In this case **failure pattern is well defined and** a sudden shear failure is experienced with the Buldging (Heaving) of ground surface adjacent to foundation at both sides.
- However, final slip moment will occur only on one side accompanied **by tilling of foundation.**
- As the pressure increases towards the ultimate value, the state of plastic equilibrium is reached initially in the soil around the **edge of footing** and gradually spreads **down wards** and **outwards.**
- Ultimately, the state of plastic equilibrium in fully developed throughout the soil and the failure surface is as shown in the above diagram.
- Generally the General shear failure occure in soil having. (D_r) relative density [> 70%]

(b) Local shear failure:

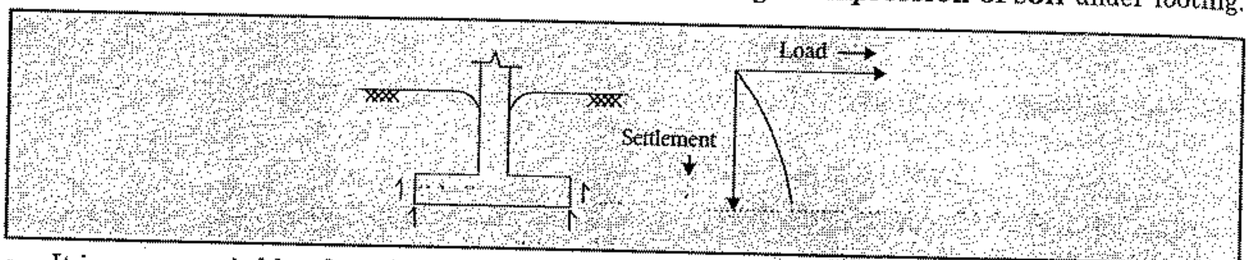


- In local shear failure there is considerable compression of soil under the footing and there is only partial development of state of plastic equilibrium.
- Failure surface, does not reach the ground surface and **only slight heaving of soil** adjacent to foundation occurs.

- Tilting of foundation **does not occur.**
- Ultimate bearing capacity is not well defined on load settlement curve.
- **In this case** failure is **not Sudden.**
- Generally occurs in soil having somewhat plastic stress-strain curve. E.g. Loose sand with Relative density (D_r) (30–70%)

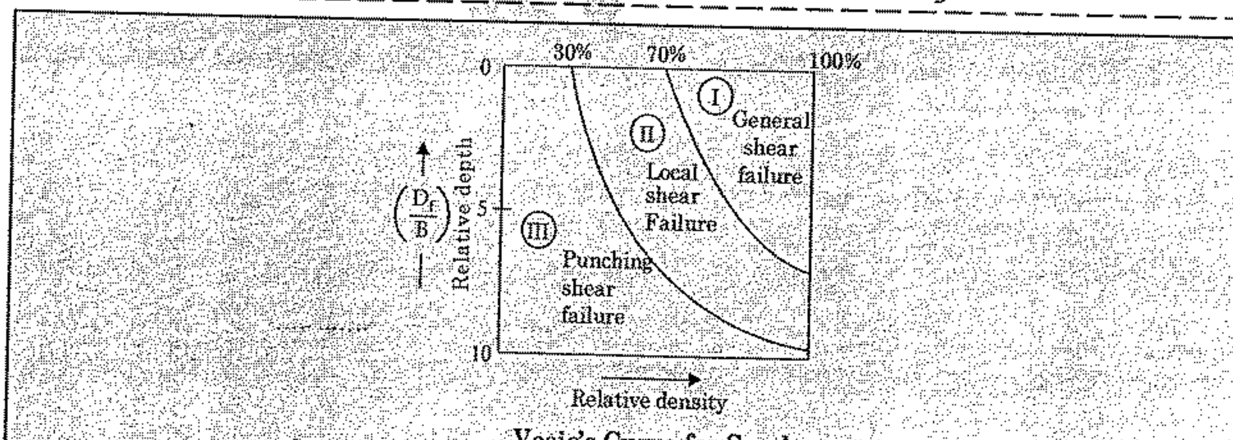
(c) Punching shear failure:

- Punching shear failure occurs when there is **relative high compression of soil** under footing.



- It is accompanied by **shearing** in **vertical direction around the edge of footing.**
- **No Heaving** of soil adjacent to footing occurs in this case
- No tilting of foundation occurs
- **Ultimate hearing** capacity is not well defined.
- Generally occurs in **very loose sand** with relative density (<30)%
- Shallow foundation in loose sand and **Deep foundations** have generally **punching shear failure.**
- Relatively large settlement occurs in this case.

***Note:** The graph below indicates the type of failure that can be expected for footings in sand.*



Vesic's Curve for Sand

Conclusion: (From the graph above)

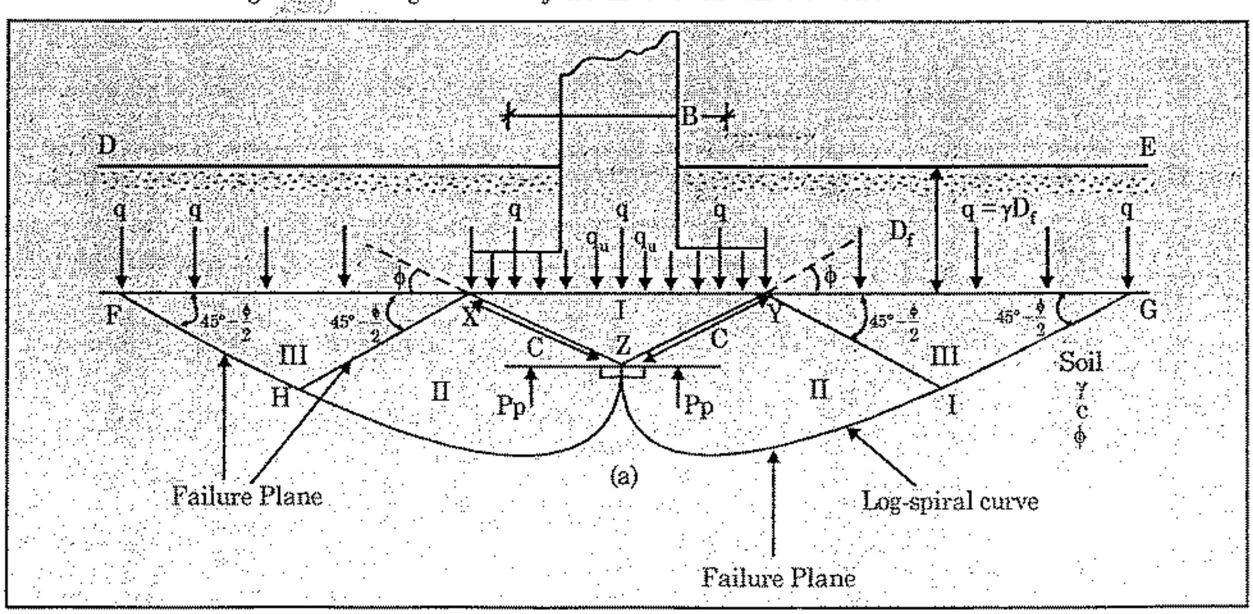
- As we go deeper, the **relative density** at which **the general shear failure** will occur **goes on increasing.**
- **Deep foundation** will always have **punching shear failure.**
- Generally for Granular Soil for $\phi > 36°$ → General shear failure occurs.
- For relative density (D_r) > [70%] → General shear failure occurs, for $30 < D_r < 70$ Local shear failure & for $D_r < 30$ punching shear failure
- For ($\phi \le 29°$) → Local shear failure is assumed

TERZAGHI'S BEARING CAPACITY THEORY

- The ultimate bearing capacity of shallow foundations are usually determined by using a bearing capacity theory in which a failure mechanism is analyzed and the load intensity at failure is expressed in terms of the shearing resistance mobilized and the geometry of the failure.
- The theory is based on limiting equilibrium approach where as the forces acting on the soil wedge immediately beneath the foundations are examined for static equilibrium condition and the ultimate bearing capacity is determined.
- Soil above the base of footing is not considered to be contributing to the strength of foundation i.e is shear failure is not considered in soil above base of footing. Thus the footing is considered as surface footing.
- Terzaghi developed bearing capacity equation for general shear failure for a **uniformly loaded strip footing of width B and infinite length carrying a uniform pressure q on the surface of a mass of homogeneous, isotropic soil.** Shear strength parameters for the soil are ϕ and c.

Assumptions :

1. Footing is a strip footing (L >> B)
2. Soil is Homogenous
3. 2-D plane strain condition prevails.
4. Base of footing is rough
5. The base of footing is laid down at shallow depth $\left(\frac{D_f}{B}\right) \leq 1$ i.e., shallow foundation.
6. Loading is vertical and symmetric i.e. (moment = 0)
7. General shear failure occurs.
8. Ground is Horizontal
9. Shearing resistance of soil between the **ground surface and base of footing** is **neglected.** Thus, footing considered as a surface footing with uniform surcharge = $(\gamma.D_f)$ at the base of footing.
10. Shear strength of soil is goverend by **mohr's coulmh** criteria.



Failure zone in the soil mass is divided into three zones

1. Zone (I) → zone of elastic equillibrium
2. Zone (II) → Radial shear zone
3. Zone (III) → Rankine's passive zone

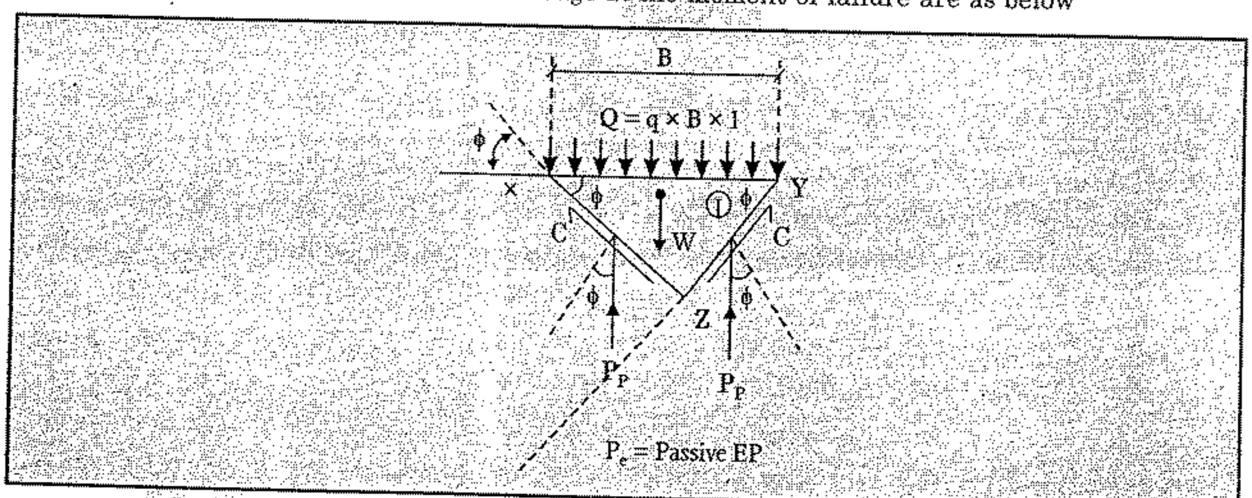
- Zone (1) Soil is prevented from undergoing any lateral movement by friction and adhesion between the base of footing and the soil. It remains in elastic equillibrium and is considered as a part of footing only.
- At failure, Zone I and Footing presses zone II & III and transforms them into a state of plasticity.

Zone II

- Wedges XZH and YZI are zones of radial shear. The curved lower boundary of these zones has the shape of a logarithmic spiral. The soil of this zone pushes the soil in Zone III.

Zone III

- The two wedges XHF and YIG are in a state of passive plastic equilibrium (passive Rankine zones). The boundaries of these zones make angles (45°– ϕ/2) with horizontal.
- When the soil mass below the footing is in the state of plastic equilibrium, the analysis of forces acting on the wedge XYZ gives the ultimate bearing capacity.
- The forces acting on the faces of this wedge at the moment of failure are as below



$$q_u \times B = 2P_p + 2c\sin\phi - \underbrace{\frac{1}{4}\gamma B^2 \tan\phi}_{\text{weight of wedge}}$$

- The above formula is obtained from the equillibrium of forces.
- Moment equillibrium is not satisfied.

$$q_u B = 2P_p + 2c_u \frac{B/2}{\cos\phi}\sin\phi - \frac{1}{4}\gamma B^2 \tan\phi$$

⇒
$$q_u B = 2P_p + Bc_u \tan\phi - \frac{1}{2}\gamma B^2 \tan\phi$$

Total passive earth pressure is made of three component

$$\boxed{P_p = P_{p\gamma} + P_{pc} + P_{pq}}$$

Where,

$P_{p\gamma}$ = Passive pressure due to weight of soil in shear zone.

P_{pc} = Passive earth pressure due to cohesion.

P_{pq} = Passive earth pressure due to surcharge.

$$q_u B = 2(P_{p\gamma} + P_{pc} + P_{pq}) + Bc_u \tan\phi - \frac{1}{2}\gamma B^2 \tan\phi$$

$$2P_{p\gamma} - \frac{1}{2}\gamma B^2 \tan\phi = \left(\frac{1}{2}B_\gamma N_\gamma\right)B \rightarrow \textit{Effect of soil weight in shear zone}$$

$$2P_{pc} + Bc_u \tan\phi = B(cN_c) \rightarrow \textit{Effect of cohesion}$$

$$2P_{pq} = B\times(qN_q) \rightarrow \textit{Effect of surcharge}$$

Thus, $$\boxed{q_u = cN_c + qN_q + \frac{1}{2}B\gamma N_\gamma}$$

Terzaghi's bearing capacity for strip footing.

- Thus ultimate bearing capacity for shallow **strip** footing as per Terzaghi is

$$\boxed{q_u = c.N_c + q.N_q + \frac{1}{2}.B\gamma N_\gamma}$$

- Net ultimate bearing capacity is given as below

$$\boxed{q_{nu} = c.N_c + q(N_q - 1) + \frac{1}{2}B.\gamma.N_\gamma}$$

Where,

$c.N_c$ = Effect of cohesion

$q.N_q$ = Effect of over burden

$B\gamma N_\gamma$ = Effect of soil in the shearing zone.

(N_C), (N_q) and (N_γ) are the bearing capacity factors

Terzaghi gave the following equatin for bearing capacity factors.

1. $\boxed{N_C = (N_q - 1).\cot\phi}$

2. $\boxed{N_q = \dfrac{a^2}{2.\cos^2(45+\phi/2)}}$

3. $\boxed{N_\gamma = \dfrac{1}{2}.\tan\phi\left[\dfrac{K_{p\gamma}}{\cos^2\phi} - 1\right]}$

$K_{p\gamma}$ = Passive earth pressure coefficient

where,

$$a = e^{(3\pi/4 - \phi/2).\tan\phi}$$

Hence, N_C, N_q and N_γ are function of (ϕ) only. As Kp is also a function of ϕ. The value of bearing capacity factors increases as the value of ϕ increases.

- As the failure plane is not assumed to extend above the horizontal plane at the label of base, this means that shear strength of soil above the base of footing is neglected. It is because of this assumption that this theory is valid only for shallow foundation. For layer depth of foundation this will introduce serious error.

In Clayey Soil

When $\phi = 0$,

$$\left.\begin{aligned} N_C &= 5.7 \\ N_q &= 1 \\ N_\gamma &= 0 \end{aligned}\right\} \text{ i.e., in clayey condition}$$

- N_c & N_γ has been obtained for $\phi \rightarrow \infty$.
- For end of construction stability of footing on saturated clay for when $\phi_u = 0$, we have

$$\boxed{q_u = (5.7c + q)}$$

and

$$\boxed{q_{nu} = 5.7c}$$

- This formula will be used for end of construction stability check for footing.
- Thus, (c_u) will be undrain shear strength parameters.

MODIFICATION OF TERZAGHI'S BEARING CAPACITY EQUATIONS FOR VARIOUS TYPES OF FOOTING

1. Square footing,

$$\boxed{q_{nu} = [1.3cN_C + q(N_q - 1)] + 0.4B\,\gamma.N_\gamma}$$

2. For circular footing,

$$\boxed{q_{nu} = [1.3cN_C + q(N_q - 1)] + 0.3B.\gamma.N_\gamma}$$

Where,

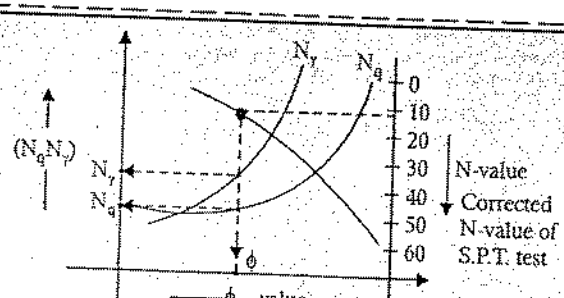
B = Dia of footing,

3. For rectangular, footing

$$\boxed{q_{nu} = \left(1 + 0.3\frac{B}{L}\right).cN_C + q(N_q - 1) + \left(1 - \frac{0.2B}{L}\right)\left(\frac{1}{2}BN_\gamma\right)}$$

Where, B < L

Note: For sands/Granular soil. We sometimes use correlation curve based on field datas to find out the values of (N_q) and (N_γ). One such correlation is correlation with S.P.T. data (standard penetration Test data) is as shown below.



Peck-Hensen Curve For Granular Soil only

- S.P.T. test is carried out in 4 or 5 Bore holes at the site of structure construction.
- The test data used in the above curve is **corresponding to the minimum average and value** obtained in various bore holes.
- The average value of (N) in any bore hole is the average of N-values for a depth from [(D_f) to (D_f + (1.5 B to 2B))].
- While using this curve we don't have to bother whethere the failure is a "local shear failure" or a "General shear failure".
- However, while using Terzaghi's equation, we will have to establish whether general or local shear failure occurs.
- In the peck-Hensen curve, the value of (N_q) and (N_γ) incorporate the effect due to local/general shear failure.
- While using Terzaghi's equation if local shear failure effect is to be considered, then:

(a) In place of 'c' we use $c_m = \frac{2}{3}c$

and in place of ϕ we use $\boxed{(\tan\phi)_m = \frac{2}{3}\tan\phi}$

Thus using c_m & ϕ_m, we calculate N_c', N_q' & N_γ' and use them to find bearing capacity.

Equation $\boxed{q_{nu} = \frac{2}{3}cN_c' + q(N_q' - 1) + \frac{1}{2}B\gamma N\gamma'}$

***Note:** We already know that for f > 36° general shear failure is assumed and for f < 29° local shear failure is assumed. For in between value of f, Nc, Nq & Ng can be obtained by interpolating between that corresponding to general and local shear failure.*

- However in the exam we do not consider local or general shear failure, rather than N_c, N_q & N_γ values given for any ϕ, should be adopted as such under the assumption that they have been recommended for the given value of ϕ.

Example 1

A 2m wide strip footing is formed at a depth of 1.5 m below the ground level in a homogenous bed of Dense Sand, having the following properties

$\phi = 36°$ $\gamma = 1.85$ tm^3, Determine the ultimate, net ultimate

Net safe bearing capacity of the footing. Given For

$\phi = 36°$; $N_c = 60°$ $N_q = 42$, $N_\gamma = 47$. Assume a F.O.S of 3

Sol. As $\phi = 36°$, general shear failure likely to occur.

For Dense Sand $c = 0$, $N_c = 60$, $N_q = 42$, $N_\gamma = 47$

(*i*) Ultimate Bearing Capacity

$$q_u = (cN_c + \gamma D\, N_q + 0.5\ B\gamma\ N_\gamma)$$

$$= \left(0\times60 + 1.85\times1.5\times42 + \frac{1}{2}\times1\times2\times1.85\times47\right)$$

$$= (116.55 + 86.95) = 203.5\ \text{t/m}^2$$

(ii) Net ultimate Bearing Capacity

$$q_{nu} = (q_u - \gamma D)$$
$$= (203.5 - 1.85 \times 1.5)$$
$$= 200.725 \text{ t/m}^2$$

(iii) Net safe Bearing Capacity

$$q_{ns} = \left(\frac{q_{nu}}{\text{F.O.S}}\right) = \left(\frac{200.725}{3}\right) = 66.908 \text{ t/m}^2$$

(iv) Safe Bearing Capacity

$$q_s = (q_{ns} + \gamma D)$$
$$= (66.908 + 1.85 \times 1.5)$$
$$= 69.68 \text{ t/m}^2$$

Example 2

Determine the safe load that can be carried by a square footing of 2.2m × 2.2 m size placed at a depth of 1.6 m below GL. The foundation soil has the following properties

$$\gamma = 1.65 \text{ t/m}^3\ c = 1.1 \text{ t/m}^2,\ \phi = 20°$$

Assume a F.O.S of 2.5. Given For $\phi = 20°$

$$N_c = 17.7,\ N_q = 7.4\ \ N_\gamma = 5.0$$
$$N_c' = 11.8\ \ N_q' = 3.8\ N_\gamma' = 1.3$$

Sol. The low value of unit weight suggests that the soil is in the loose state. Moreover $Q = 20° < 29°$

Hence a local shear failure is likely to occur.

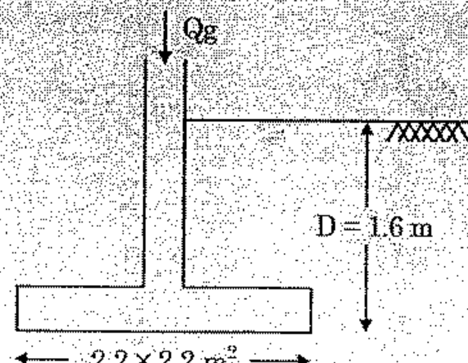
$$q_{nu} = 1.3c'\, N_c' + \gamma D \times (N_q' - 1) + 0.4\ B\gamma\ N_\gamma'$$

$$c' = \frac{2}{3}c = \frac{2}{3} \times 1.1$$

$$= \frac{2}{3} \times 1.1 = 0.73 \text{ tm}^2$$

$$\therefore\ q_{nu} = (1.3 \times 0.73 \times 11.8) + 1.65 \times 1.6\ (3.8 - 1) + (0.4 \times 1.65 \times 2.2 \times 1.3)$$
$$= 20.52 \text{ tm}^2$$

The safe bearing capacity of the footing



$$q_s = \left(\frac{q_{nu}}{\text{F.O.S}}\right) + \gamma D$$

$$= \left(\frac{20.52}{2.5}\right) + 1.65 \times 1.6$$

$$= 10.85 \text{ t/m}^2$$

$\therefore$ Gross safe load to be carried by the footing

$$= q_s \times (\text{Area of footing})$$
$$= 10.85 \times (2.2)^2$$
$$= 52.52 \text{ tonn}$$

Example 3

A concrete strip footing rectangular in cross section is located at ground level and extends 1.2m below the ground level. It carries UDL of 15000 kg/m. The soil profile consists of homogeneous clay 6m thick over laying rock. The clay properties are as under.

$$\text{Saturated unit bulk weight} = 1750 \text{ kg/m}^3$$
$$\text{Shear strength (undrained)} = 8500 \text{ kg/m}^2$$
$$\text{Compressibility} = 1 \times 10^{-4} \text{ m}^2/100 \text{ kg}$$

Determine,

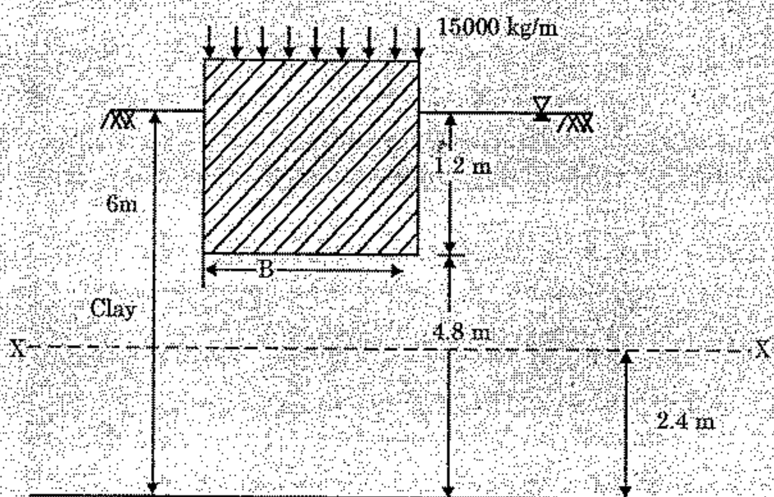
(i) Width of footing for F.O.S. of, F = 2

(ii) Ultimate consolidation settlement for F = 2

Assume bulk unit weight of concrete = 2500 kg/m³.

Neglect the spread of load beneath the footing and any side cohesion on the foundation.

Sol. As per Terzaghi's bearing capacity equation



For strip footing on clay soil $\phi = 0$, $N_q = 1$, $N_c = 5.7$, $N_\gamma = 0$

$$q_u = (cN_c + \gamma D_f \times N_q)$$
$$q_u = (5.7c + \gamma D_f)$$
$$\therefore \quad q_{nu} = (q_u - \gamma D_f)$$
$$= 5.7\,c \qquad \text{... (i)}$$

Data given, $\quad$ F.O.S. = 2, $C_u = \tau_f = 8500$ kg/m²

$$\text{Unit weight of concrete} = 2500 \text{ kg/m}^3$$
$$\text{Depth of footing} = 1.2 \text{ m}$$
$$\text{Udl} = 15000 \text{ kg/m}$$
$$\therefore \quad \text{Self wt. of footing} = 1.2B \times 1 \times 25000$$
$$= 3000B \text{ kg/m}$$
$$\therefore \quad \text{Total load} = (15000 + 3000\,B) \qquad \text{... (ii)}$$

$\Rightarrow$ Total load carried by the footing should be equal to the safe bearing capacity of soil

$$\Rightarrow \quad \frac{(1500+3000B)}{(B\times 1)} = \left(\frac{q_{nu}}{F.O.S.}\right)+\gamma D_f$$

$$\Rightarrow \quad 15000+3000B = \left[\frac{(5.7\times 8500)}{2}+1750\times 1.2\right]B$$

$$\Rightarrow \quad 15000+3000\,B = 26325\,B$$

$$\therefore \quad B = \frac{15000}{(26325-3000)} = 0.64\text{m}$$

$$\boxed{B = 0.64\text{m}}$$

(ii) Initial overburden pressure at the level of X – X

$$\bar{\sigma}_1 = (3.6\times \gamma_{sat}) = 3.6\times 1750 = 6300 \text{ kg/m}^2$$

Final effective overburden pressure at the level of X–X

$$\bar{\sigma}_2 = 1.2\times 2500+(2.4\times 1750)+\frac{15000}{0.64} = 30637.5 \text{ kg/m}^2$$

$$\therefore \quad \Delta\bar{\sigma} = \text{Change in effective overburden pressure}$$

$$= (\bar{\sigma}_2-\bar{\sigma}_1)$$

$$= (30637.5-6300) = 24337.5$$

Ultimate consolidation settlement

$$\Delta H = (m_v H_0 \Delta\bar{\sigma})$$

$$= \left(\frac{1\times 10^{-4}}{100}\times 4.8\times 24337.5\right)$$

$$= 0.11682 \text{ m.} = 116.8 \text{ mm}$$

Example 4

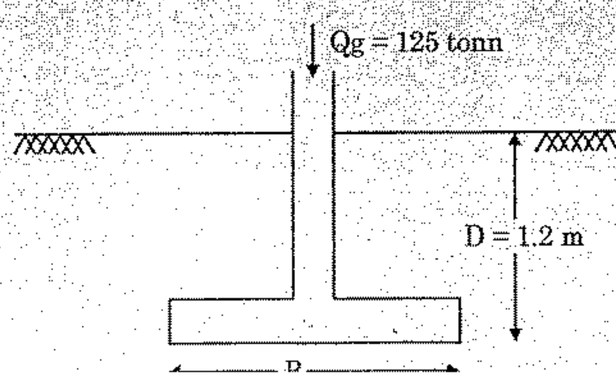
A column of a building, carrying a net vertical load of 125 tonn has to be supported by a square footing, the footing is to be placed at 1.2 m below G.L in a bomogeneous bed of soil having the following properties.

$$\gamma = 1.82 \text{ gm/cc}, \ \phi = 30°$$

Determine the minimum size of the footing required to have a factor of safety of 2.5 against shear failure.

Use Terzaghi's formula. For $\phi = 30°$, $N_c = 37.2$, $N_q = 22.5$ $N_\gamma = 19.7$, $c = 0$

Sol.



Data given: Net load on column from supper structure = 125 tonn

$$\therefore \quad q_{nu} = (1.3\ c\ N_c + \gamma D\ (N_q - 1) + 0.4\ B\gamma\ N_\gamma)$$

$$= 1.3\ N_c \times 0 + (1.82 \times 1.2 \times (22.5 - 1) + 0.4 \times 1.82 \times B \times 19.7$$

$$= (46.956 + 14.3416\ \text{B})$$

$$\Rightarrow \quad q_{ns} = \frac{q_{nu}}{F.O.S} = \left(\frac{46.956 + 14.3416\ \text{B}}{2.5}\right)$$

$$q_{ns} = 18.782 + 5.74\ B \quad \ldots (i)$$

$$\because \quad q_{ns} = \frac{P}{B^2} \quad \ldots (ii)$$

From (*i*) and (*ii*), we have

$$\frac{P}{B^2} = \left(\frac{46.956 + 14.3416\ \text{B}}{2.5}\right)$$

$$\frac{125}{B^2} = \left(\frac{46.956 + 14.3416\ \text{B}}{2.5}\right)$$

$$\Rightarrow \quad 312.5 = (46.956\text{B}^2 + 14.342\text{B}^3)$$

$$\Rightarrow \quad 14.342\text{B}^3 + 46.956\text{B}^2 - 312.5 = 0$$

By solving equation, we obtain

$$B = 2.03\ \text{m} \simeq \mathbf{2.10\ m.}$$

Example 5

If the size of the footing in above problem has to be restricted to 1.75 m × 1.75 m, at what depth the footing should be placed?

Sol.

Note: The bearing capacity of a footing placed in a cohensionless soil increases with depth. In above problem the depth of footing was specified as 1.2 m.

- The corresponding size for supporting a gross load of 125 tonn was found to be 2.1m × 2.1m.
- However if the size of footing has to be restricated to 1.75 × 1.75 m (such restriction are sometimes necessary for avoiding encroachement of adjacent land and If the column still has to with stand the same net load, its depth has to be increased).

Let d be the required depth.

$$q_{ns} = \frac{1}{\text{F}}[\gamma D(N_q - 1) + 0.4 B\ \gamma N_\gamma]$$

$$= \frac{1.82 \times d \times (22.5 - 1) + 0.4 \times 1.82 \times 1.75 \times 19.7}{2.5}$$

$$\therefore \quad q_{ns} = (15.652\ d + 10.039) \quad \ldots (i)$$

Again Actual contact pressure, $\frac{P}{B^2} = \frac{125}{\ldots} = 40.82\ \text{t/m}^2 \quad \ldots (ii)$

From (*i*) and (*ii*), we have

$$\Rightarrow \quad 15.652d + 10.039 = 40.82$$

$$\Rightarrow \quad d = \frac{(40.82 - 10.039)}{15.652}$$

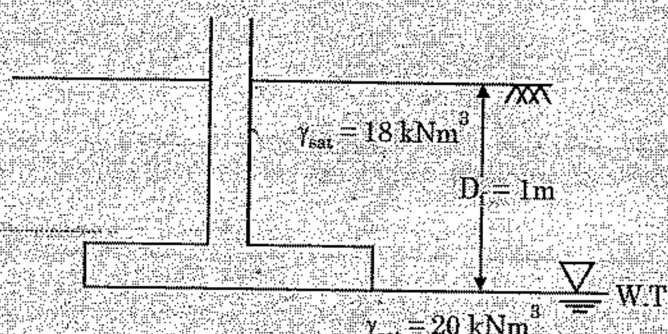
$$d = 1.97 \text{ m} \simeq 2 \text{ m}$$

The footing has to be founded at a depth of 2.00 m below G.L.

Example 6

A strip footing is to be designed to carry a gross load of 900 kN/m at a depth of 1m in a gravelly sand. The appropriate shear strength parameter are c = 0 and 38°. Determine the width of the footing if a F.O.S of 3. against shear failure is to be assured. Water table is found to be at the foundation level. Unit weight of soil above water table is 18 kn/m³ and that below water table is 20 kN/m³. For $\phi = 38°$, the bearing capacity factors are $N_q = 49$ and $N_\gamma = 67$ $\gamma_\omega = 9.8$ kN/m³.

Sol.



Data given:

$$Q = 900 \text{ kN/m}$$

$$D_f = 1\text{m}, \; c = 0, \; \phi = 38°$$

$$\text{F.O.S} = 3$$

The safe bearing capacity of strip footing on cohesionless soil is given by

$$q_s = \frac{1}{F}[cN_c + \gamma_t D_f (N_q - 1) + 0.5 B\gamma_{sub} N_\gamma] + \gamma_t D$$

$$= \frac{[(0 \times N_c + 18 \times 1 \times (49-1) + 0.5B \times (20-9.8) \times 67]}{3} + 18 \times 1$$

$$= \frac{(864 + 341.7B)}{3} + 18$$

$$= (306 + 113.9 \text{ B}) \quad \text{... } (i)$$

Now total load on footing including soil load of depth

$$D_f = \text{gross load on footing} = 900 \text{ kN/m}$$

$$\therefore \quad q_a = \frac{900}{(B \times 1)} \text{kN/m}^2$$

From (*i*) and (*ii*)

$$\Rightarrow \quad 306 + 113.9 \text{ B} = \frac{900}{\ldots}$$

$\Rightarrow \quad 113.9\,B^2 + 306\,B - 900 = 0$

$$\Rightarrow \quad B = \frac{-306 \pm \sqrt{(306)^2 + 4\times 900\times 113.9}}{(2\times 113.9)}$$

$$B = 1.772 \text{ m}$$

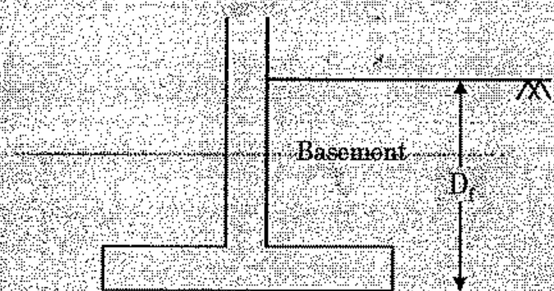
$$\Rightarrow \quad B = 1.8 \text{ m}$$

Example 7

A building has to be supported on a RC Raft foundation of dimension 14m × 21 m. The sub soil is clay, which has an average unconfined comprsive strength of 15 kN/m². The pressure on the soil due to the weight of the building and the loads that it will carry will be 140 kN/m² at the base of the raft. The building has provision for basement floors. At what depth should the bottom of the raft be placed to provide a F.O.S of 3 against shear failure?

$\gamma_{clay} = 19$ kN/m³. Use skempton's approach for bearing capacity calculation.

Sol. Data Given: Average unconfined strength $q_u = 15$ kN/m²



$$\Rightarrow \quad c = \left(\frac{q_u}{2}\right) = \frac{15}{2} = 7.5 \text{ kN/m}^2$$

Safe bearing pressure on the soil = 140 kN/m²

F.O.S against shear Failure = 3

Set the depth of raft foundation be D_f

We have for clay, $\phi = 0$ and for sucb a condition, net ultimate bearing capacity as per skempton

$$q_{nu} = cN_c \qquad \text{... (i)}$$

As per skempton the value of N_c is given by

$$N_c = 5.0\left[1 + \frac{0.2D_f}{B}\right]\left[1 + 0.2\frac{B}{L}\right]$$

$$= 5.0\left[1 + \frac{0.2\,D_f}{14}\right]\left[1 + 0.2\frac{14}{21}\right]$$

$$= 5.67\,(1 + 0.0143\,D_f) \qquad \text{... (ii)}$$

The building has a provision for basement, therefore

$$\therefore \quad \frac{P}{B^2} = \left(\frac{q_{nu}}{\text{F.O.S}} + \gamma D_f\right) \qquad \text{... (iii)}$$

Thus,

$$\Rightarrow \quad \frac{q_{nu}}{\text{F.O.S}} + \gamma D_f = 140$$

$$\Rightarrow \quad \frac{cN_c}{\text{F.O.S}} + \gamma D_f = 140$$

$$\Rightarrow \quad \frac{7.5}{3}\left[5.67\,(1 + 0.0143\,D_f)\right] + 19\,D_f = 140$$

$$\Rightarrow \quad D_f = 6.55 \text{ m} \qquad \ldots(iv)$$

Thus the bottom of the raft should be placed at a depth of 6.55 from the ground surface.

Example 8

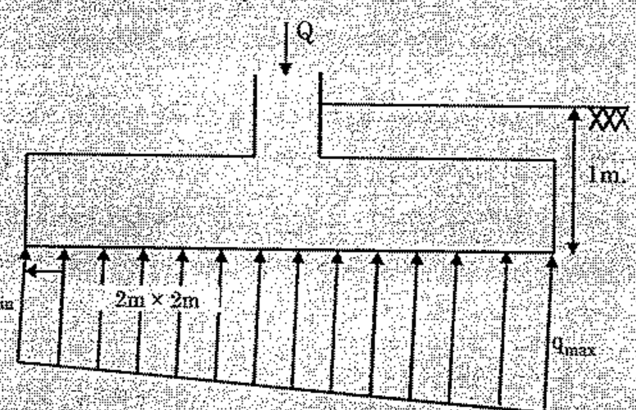
A footing 2m × 2m in plan is founded at 1m depth below GL in sand having angle of shearing resistance of 36°. Compute the allowable load at the base of the footing.

If (i) factor of safety against shear is 3. and (ii) the max. settlement of footing is not to exceed 50 mm.

The W.T is 1m below G.L. The allowable soil pressure : For 50 mm settlement with water table at very great depth is 50 t/m² saturated and dry unit weights of sand are 2 t/m³ and 1.6 t/m³. For $\phi = 36°$, $N_c = 50$, $N_q = 42$ and $N_\gamma = 46$. If the load on the footing is ecentric with $e_x = e_y = 0.25$m, what will be the allowable load?

Take soil above water table is dry.

Sol. Data given :



$$D_f = 1\text{m}$$
$$\phi = 36°$$
$$\text{F.O.S.} = 3.$$
$$N_c = 50,\ N_q = 42 \text{ and } N_\gamma = 46$$
$$\gamma_{sat} = 2 \text{ t/m}^3\ \gamma_{dry} = 1.6 \text{ t/m}^3$$

The W.T is 1ᵐ below the G.L.

The Net ultimate bearing capacity for a square footing is given by

$$q_{nu} = 1.3cN_c + \gamma_d D_f (N_q - 1) + (0.4B\gamma_{sub} N\gamma)$$

∵ Since the soil is dry sand, $c = 0$

$$\therefore \quad q_{nu} = 1.6 \times 1\,(42 - 1) + 0.4 \times 2 \times (2\text{-}1) \times 46 = 102.4 \text{ t/m}^2$$

∴ Safe bearing capacity $\quad q_s = \left(\dfrac{q_{nu}}{\text{F.O.S}} + \gamma D_f\right)$

$$q_s = \left(\frac{102.4}{3} + 1.6 \times 1\right)$$

$$= 35.73 \text{ t/m}^2 \quad \text{...(i)}$$

$\therefore$ Allowable load $= (q_s \times B^2)$

$= [35.73 \times (2)^2]$

$= 142.92$ tonn

(ii) The allowable soil pressure for 50 mm settlement

$q_a = 50 \text{ t/m}^2$

$\therefore$ Allowable load $= (q_a \times B^2)$

$= [50 \times (2)^2]$

$= 200$ tonn.

If the load on the footing is eccentric with $e_x = e_y = 0.25$ m.

$$\therefore \quad q_{max} = \frac{Q}{A}\left(1 + \frac{6e_x}{L} + \frac{6e_y}{B}\right)$$

$$q_{min} = \frac{Q}{A}\left(1 - \frac{6e_x}{L} - \frac{6e_y}{B}\right)$$

$\therefore$ $q_{max} = q_s = 35.73 \text{ t/m}^2$, $L = B = 2$m

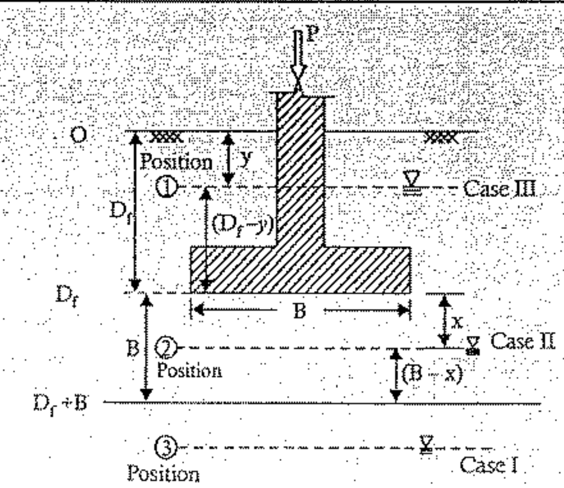
$$\Rightarrow \quad 35.73 = \frac{Q}{B^2}\left[1 + \frac{\cancel{6} \times 0.25}{\cancel{2}} + \frac{\cancel{6} \times 0.25}{\cancel{2}}\right]$$

$$\Rightarrow \quad Q = \frac{35.73 \times (2)^2}{(2.5)} = 57.17 \text{ tonn}$$

MODIFICATION OF TERZAGHI'S EQUATION FOR THE LOCATION OF WATER TABLE

We know that Terzaghi's equation for strip footing is

$$\boxed{q_{nu} = cN_C + q(N_q - 1) + \frac{1}{2}B\gamma N_\gamma}$$



- Soil in the zone from depth (D_f) to ($D_f + B$) is affected by the shear failure.
- For drained condition testing effective stress approach is used.

(1) Case (I): When water table is below depth $D = (D_f + B)$

$$\boxed{q_{nu} = c.N_c + \gamma_t.D_f(N_q - 1) + \frac{1}{2}B.\gamma_t.N_\gamma}$$

where,

γ_t = Bulk unit wt.

(2) Case (II): When water table is between depth $D = [D_f$ to $(D_f + B)]$.

$$\boxed{q_{nu} = c.N_c + \gamma_t.D_f(N_q - 1) + \frac{1}{2}[x\gamma_t + (B - x)\gamma_{sub}].N_\gamma}$$

Where,

$(x\gamma_t + (B - x)\gamma_{sub})$ is effective wt. of soil in the region from (D_f) to ($D_f + B$) at depth (D_f + B)

3. Case (III): When water table is in zone from depth (0 to D_F):

$$\boxed{q_{nu} = c.N_C + [\gamma_t.y + (D_f - y)\gamma_{sub}].(N_q - 1) + \frac{1}{2}B.\gamma_{sub}.N_\gamma}$$

Where $[\gamma_t y + (D_f - y)\gamma_{sub}]$ is the effective over burden *wt.* at the bottom of the foundation.

***Note:** For a soil having*

$C = 0$, $\gamma_{sat} = 20$ kN/m^2 and $\gamma_w = 10$ kN/m^2

Rise of water table from depth greater than ($D_f + B$) to ground level, reduces the net bearing capacity of the soil. To half of the value corresponding to water table at great depth.

ANOTHER WAY OF ACCOUNTING FOR THE EFFECT OF WATER TABLE:

$$\boxed{q_{nu} = c.N_c + D_f.\gamma_t.(N_q - 1).(R_w) + \frac{1}{2}B.\gamma_t.N_\gamma(R'_w)}$$

Where Rw & R'w are the correction factors for the terms involving Nq & Nγ respectively for the effect of water table.

$$R_w = 0.5\left(1 + \frac{Dw}{D_F}\right) \text{ when } 0 < \frac{Dw}{B} \le 1$$

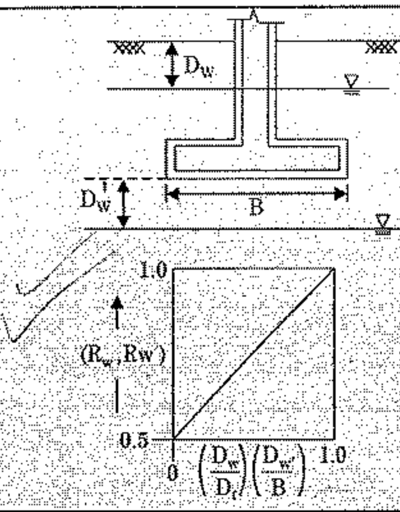
Where Dw = Depth of water table below the ground level limited to a depth of D_F

And

$$Rw' = 0.5\left(1 + \frac{Dw'}{B}\right) \text{ when } 0 < \frac{Dw'}{B} < 1$$

where Dw' = Depth of water table measured from the base of footing with a limiting value equal to width of footing.

The variation of Rw' with the ratio of Dw'/B & Rw with the ratio of Dw/D_F is show in figure below.



- When water table is at ground surface, in case of C & ϕ soil effective parameter shall be used in the 1st term and correction factor shall be applied in the II & III term of bearing capacity equation.
- But we can observed that in case of granular soil when water table is at ground surface bearing capacity becomes half.

Example 9

An R.C.C column footing of 1.8 m × 1.8 m size is founded at 1.5 m below G.L. The subsoil consists of a loose deposit of silty sand having the following properties

$\gamma_t = 1.75$ t/m³ $\phi = 20°$, $c = 1.1$ t/m², $\gamma_{sat} = 2$t/m³

Determine the ultimate bearing capacity of the footing when the ground water table in located at

(i) Ground level

(ii) 0.6 m below ground level

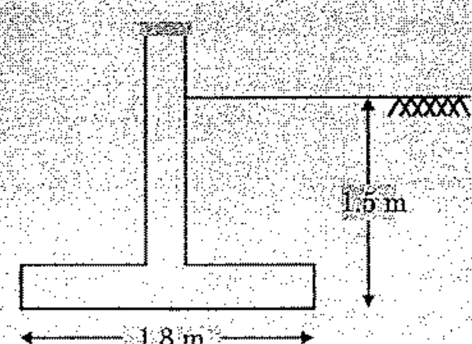
(iii) 2m below the base of footing

(iv) 4m below the base of footing

Given for $\phi = 20°$, $N_c' = 11.8$, $N_q' = 3.8$, $N_\gamma' = 1.3$

Sol. The soil is loose deposit of silty sand, and moreover $\phi = 20 < 28°$

Therefore assuming a local shear failure, the ultimate bearing capacity of a square footing is given by



$$q_u = (1.3\ c'\ N_c + \gamma D\ N_q + 0.4\ B\gamma\ N_\gamma) \quad \text{... (i)}$$

$$c' = \left(\frac{2}{3}c\right) = \frac{2}{3} \times 1.1$$

$= 0.73 \text{ t/m}^2$

$\therefore \quad q_u = (1.3 \times 0.73 \times 11.8) + q\,(3.8) + 0.4 \times 1.3 \times (B\gamma)$

$q_u = 11.198 + 3.8\,q + 0.52\,(B\gamma)$ ... (*ii*)

(*i*) When the W.T is located at ground level

$q = (\gamma_{sub} D)$

$q_u = 11.198 + 3.8 \times 1.5 \times (2.00 - 1) + 0.52 \times 1.8 \times (2.00 - 1)$

$q_u = 17.82 \text{ t/m}^2$

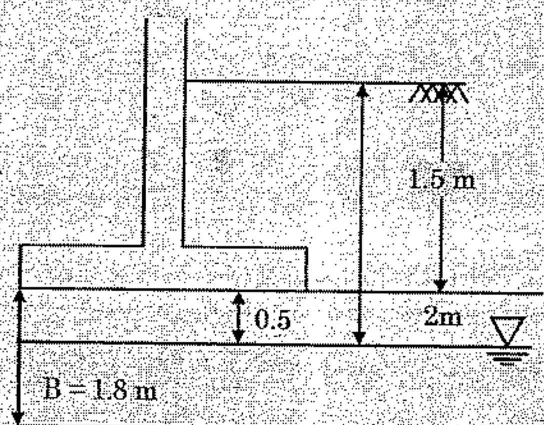
(*ii*) When the W.T is 0.6 m below ground level

$q_u = 11.198 + 3.8\,(0.6\gamma_t + 0.9\,\gamma_{sub}) + 0.52\ \ B\gamma_{sub}$

$= 11.198 + 3.8\,(0.6 \times 1.75 + 0.9 \times 1) + 0.52 \times 1.8 \times 1$

$= 19.54 \text{ t/m}^2$

(*iii*) When the water table is 2.0 m below the base of footing



$q_u = 11.198 + 3.8 \times \gamma_t D_f + 0.52\,[0.5\,\gamma_t + (1.8 - 0.5)\,\gamma_{sub}]$

$= 11.198 + (3.8 \times 1.75 \times 1.5) + 0.52\,[0.5 \times 1.75 + 1.3 \times 1]$

$= 22.304 \text{ t/m}^2$

(iv) When the ground water table is at 4m below the base of footing, no correction due to ground water table is necessary. In other words the ulitmate bearing capacity is not affected by the ground water table.

$q_u = 11.198 + 3.8 \times \gamma_t D_f + 0.52\ B\gamma_t$

$= (11.198 + 3.8 \times 1.75 \times 1.5 + 0.52 \times 1.8 \times 1.75)$

$= 22.81 \text{ t/m}^2$

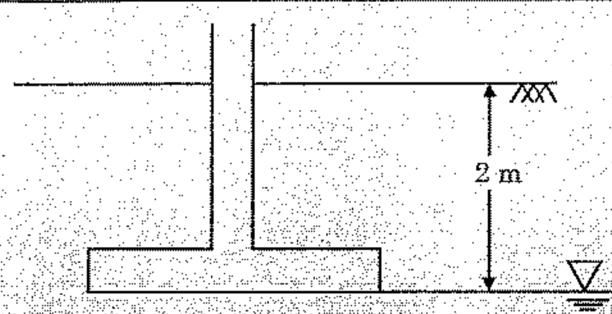
Note: It is evident from the above result, that, the bearing capacity of a footing increases with increasing depth of the ground water table.

Example 10

Using Terzaghi's Theory determine the ultimate bearing capacity of a strip footing 1.5 m wide resting on a saturated clay (C_u = 30 kNm², $\phi_u = 0$ and γ_{sat} = 20 kN/m³) at a depth of 2m from the ground level. If the W.T rises by 1m. Calculate the % Reduction in the ultimate bearing capacity.

Sol. For saturated clay

$\phi = 0;\ N_c = 5.7;\ \ N_q = 1;\ \ N_\gamma = 0$



∴ When W.T is 2m below the ground level

$$\Rightarrow \quad q_u = cN_c + \gamma D_f N_q + 0.5B\,\gamma N_\gamma$$
$$= (5.7\,c + \gamma_t\,D_f \times 1)$$
$$= (5.7 \times 30 + 20 \times 2)$$
$$= 211 \text{ kNm}^2 \quad \ldots (i)$$

When the water table rises by 1 m

$$q_u' = cN_c + (\gamma_t \times 1 + \gamma_{sub} \times 1) \times 1$$
$$= 5.7\,c + (\gamma_t + \gamma_{sub}) \times 1$$
$$= 5.7 \times 30 + [20 + (20 - 9.8)] \times 1$$

$$q_u' = 171 + 30.2 = 201.2 \text{ kN/m}^2 \quad \ldots (ii)$$

% Reduction in q_u

$$= \frac{(q_u - q_u')}{q_u} \times 100$$

$$= \frac{(211 - 201.2)}{211} \times 100 = 4.64\%$$

SKEMPTON'S BEARING CAPACITY OF SOIL FOR CLAYEY SOIL.

Skempton's analysis is applicable for saturated clay for which $\phi_u = 0$

- As per this theory, $q_{nu} = c_u . N_c$
- c_u is found out using the following tests
  (i) U–U Test
  (ii) Unconfined compression strength test. $c_u = \frac{q_u}{2}$
  (iii) Vane shear tests.
- The bearing capacity factor Nc is found out by laboratory test and field observations.
- Here (N_c) is a function of shape of footing and $\left(\frac{D_f}{B}\right)$. This theory does not neglect the shear strength of soil above foundation level. Hence, it can be applied even to deeper footings i.e., $\left[\left(\frac{D_f}{B}\right) > 1\right]$ also.

SKEMPTON PROPOSED FOLLOWING VALUES OF NC ACCORDING TO VARIOUS TYPES AND SHAPES OF FOOTINGS.

1. $N_c = 5\left[1+0.2\left(\frac{D_f}{B}\right)\right]$, $(N_c < 7.5) \rightarrow$ For strip footing

   If $D_f/B = 0$, $N_c = 5$ & If $D_f/B \geq 2.5$ $N_c = 7.5$

2. $N_c = 6\left[1+0.2\left(\frac{D_f}{B}\right)\right]$, $(N_c < 9.0) \rightarrow$ For square/circular footing.

   If $D_f/B = 0$, $N_c = 6$ & If $D_f/B \geq 2.5$, $N_c = 9$

3. $N_c = 5\left[1+0.2\left(\frac{D_f}{B}\right)\right]\left[1+\frac{0.2B}{L}\right]$ for $\left(\frac{D_f}{B} \leq 2.5\right)$ For rectangular footing

   If $\frac{D_f}{B} > 2.5$, $N_c = 7.5\left[1+\frac{0.2B}{L}\right]$

Here

Bigger dimension = Length.

Smaller dimension = Width

5. $N_{c_{Rectangular}} = N_{c\,square}\left[0.84+0.16\times\frac{B}{L}\right]$

Example 11

A square footing of (2.5m × 2.5m) size has been founded at 1.2 m below the ground level in a cobesion soil having a bulk density of 1.8 t/m³ and an unconfined compressive strength of 5.5 t/m². Determine the ultimate and safe bearing capacity of the footing for a F.O.S of 2.54 by

(i) Terzaaghi's Theory

(ii) Skempton's Theory

Sol. Data given: $D = 1.2$ m, $\gamma_t = 1.8$ tm³

$$q_u = 5.5 \text{ tm}^2$$

$$c = \left(\frac{q_u}{2}\right) = \left(\frac{5.5}{2}\right) = 2.75 \text{ tm}^2$$

$$q_u = (1.3\ cN_c + \gamma DN_q + 0.4B\gamma\ N\gamma)$$

For cohesive soil, $\phi = 0$, $N_c = 5.7$, $N_q = 1.0$, $N_\gamma = 0$

$$\therefore\ q_u = (1.3 \times 2.75 \times 5.7) + 1.8 \times 1.2 \times 1 + 0$$

$$= 22.54 \text{ t/m}^2$$

$$\therefore\ q_{nu} = (q_u - \gamma D)$$

$$= (22.54 - 1.8\ 5\ 1.2) = 20.378 \text{ t/m}^2$$

$$\therefore\ q_s = \left(\frac{q_{nu}}{\text{F.O.S}}\right) + \gamma D$$

$$= \left(\frac{20.378}{2.5} + (1.8 \times 1.2)\right) = \mathbf{10.21\ t/m^2}$$

(*ii*) By Skempton's Theory

$$\left(\frac{D}{B}\right) = \left(\frac{1.2}{2.5}\right) = 0.48 < 2.5$$

$$\therefore \quad Nc = \left(1 + \frac{0.2B}{B}\right)(N_c)_{surface}$$

$$= \left(1 + \frac{0.2 \times 12}{2.5}\right) \times 6.20 \qquad [\because (Nc) = 6.20 \text{ For square surface and circular}$$

$$= 6.79$$

$$\therefore \quad q_{nu} = cN_c$$

$$= (2.75 \times 6.79) = 18.67\ t/m^2$$

$$\therefore \quad q_u = (q_{nu} + \gamma D) = 20.84\ t/m^2$$

$$\therefore \quad q_s = \left(\frac{q_{nu}}{F.O.S}\right) + \gamma D$$

$$= \left(\frac{18.67}{2.5}\right) + 1.8 \times 1.2 = \mathbf{9.63\ t/m^2}$$

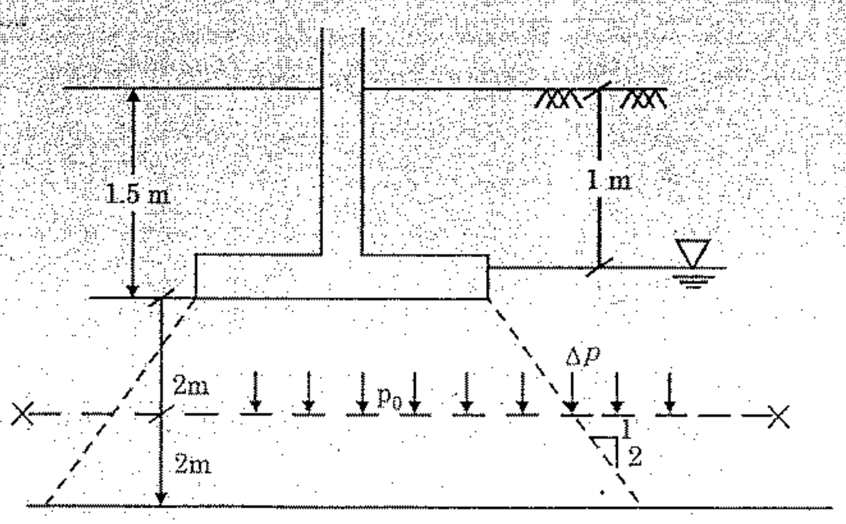
Example 12

Determine the allowable bearing capacity of a 2m × 2m square footing founded at a depth of 1.5 m below the G.L in a deep stratum of silty clay having the following avg. properties

$\gamma = 1.8\ t/m^3$, $c = 3t/m^2$ $\phi = 0°$ $C_c = 0.259$, $e_0 = 0.85$

The maximum permissible settlement of the footing is 7.5 cm. the highest position of the W.T at the site is at a depth of 1.0 m below G.L.

Sol.



(*i*) **Computation of bearing capacity**

We know from skempton's equation

$$q_{nu} = cN_c$$

$$\therefore \frac{D}{B} = \frac{1.5}{2} = 0.75 < 2.5$$

$$\therefore N_c = 5\left(1+\frac{1+0.2D}{L}\right)\left(1+\frac{0.2\,B}{L}\right)$$

$$= 5\left(1+\frac{0.2\times1.5}{2}\right)\left(\frac{1+0.2\times2}{2}\right)$$

$$= 5 \times 1.15 \times 1.2 = \mathbf{6.9}$$

$$\therefore q_m = cN_c$$

$$= 6.9\,c = 6.9 \times 3 = 20.7 \text{ t/m}^2 \qquad \text{... }(i)$$

$\therefore$ For a F.O.S of 2.5, the net safe bearing capacity is given by

$$q_{ns} = \frac{q_{ns}}{(\text{F.O.S})} = \left(\frac{20.7}{2.5}\right) = 8.28 \text{ t/m}^2$$

Computation of Settlement

As the underlying soil is saturated silty clay, only consolidation settlement will take place.

- The zone of influence below the base of footing is extended to max depth of twice the width of footing, i.e 4 m below the base.
- X–X is a horizontal plane through the middle of thin consolidation layer.

$$(\sigma_0)_{XX} = (\gamma_t\, z_1 + \gamma_{sub}\, z_2)$$

$$= (1.8 \times 1.0) + (1.8 - 1) \times (0.5 + 2.0)$$

$$= 3.8 \text{t/m}^2 = 0.38 \text{ kg/cm}^2$$

Using 2:1 dispersion method, stress increment at X–X

$$(\Delta\sigma)_{XX} = \frac{(8.28\times2.0\times2.0)}{(2.0+2.0)^2} \quad \text{(Assuming the footing to be loaded with 8.28 t/m}^2\text{)}$$

$$= 2.07 \text{ t/m}^2 = 0.207 \text{ kg/cm}^2$$

$$\therefore \Delta H = \frac{H.C_c}{(1+e_0)}\log_{10}\left(\frac{\sigma_0+\Delta\sigma}{\sigma_0}\right)$$

$$= \frac{400\times0.259}{(1+0.85)}\log_{10}\left(\frac{0.38+0.207}{0.38}\right) = \mathbf{10.58 \text{ cm}} \qquad \text{... }(ii)$$

As the Estimated Settlement is greater than the max. permissible limit of 7.5 cm. The allowable

$$\therefore \quad \frac{HC_c}{(1+e_0)}\log_{10}\left(\frac{(\sigma_0)_{XX}+\Delta\sigma}{\sigma_0}\right) = 7.5$$

$$\Rightarrow \quad \frac{400\times 0.259}{(1+0.85)}\log_{10}\left(\frac{0.387+\Delta\sigma}{0.38}\right) = 7.5$$

$$\Rightarrow \quad \frac{0.38+\Delta\sigma}{0.38} = 1.3612$$

$$\Rightarrow \quad \Delta\sigma = 0.1372 \text{ kg/cm}^2 = 1.372 \text{ t/m}^2$$

$$\Rightarrow \quad \Delta\sigma = \frac{qBL}{(B+Z)(L+Z)}$$

$$= \frac{qBL}{(B+Z)^2} = \frac{q\times(2)^2}{(2+2)^2}$$

$$\Rightarrow \quad 1.372 = \frac{q\times 4}{16}$$

$$\Rightarrow \quad q = (1.372\times 4)$$

$$q = \mathbf{5.49\ t/m^2 \simeq 5.5\ t/m^2}$$

Hence a loading intensity of 5.5 t/m^2 will result in a consolidation settlement of 7.5 cm. Therefore, the required allowable bearing capacity of the footing = **5.5 t/m^2**

MEYERHOFF'S APPROACH

Meyerhoff gave a simple general bearing capacity equation for the bearing capacity of the shallow foundation. The most important thing in this equation is that it can be applied for any depth of footing.

- Failure envelope is same as per Terzaghis method. But failure surface was assumed to go above foundation level. Hence, shearing strength of soil above the base of footing is taken into account.

$$\boxed{q_u = c.N_c[i_c S_c d_c] + q.N_q[i_q S_q d_q] + \frac{1}{2}B.\gamma.N_\gamma[i_\gamma.S_\gamma.d_\gamma]}$$

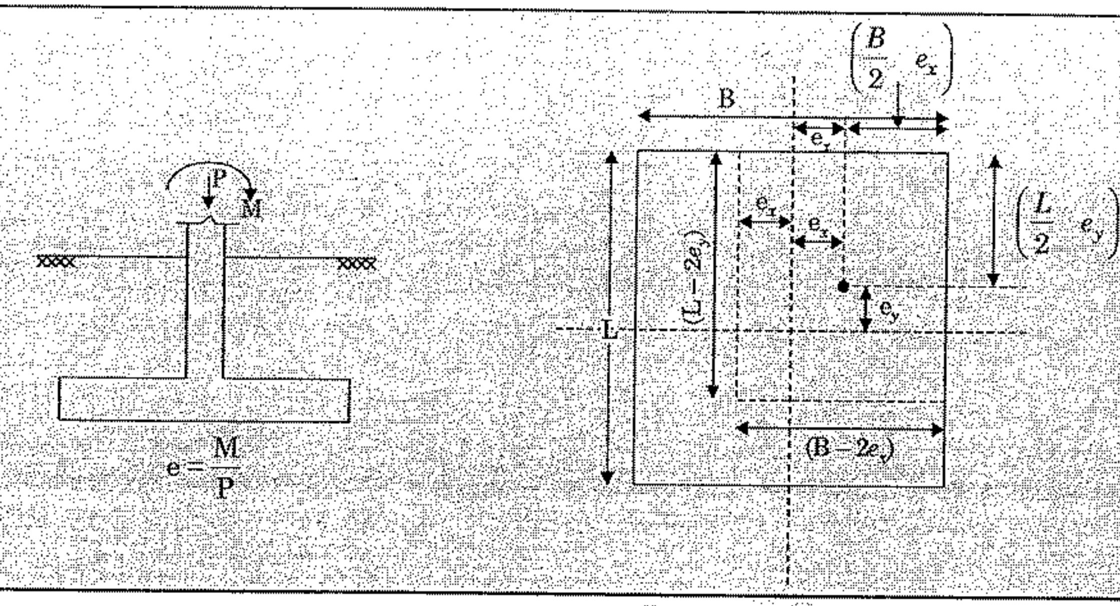
- Where, (*S*, *d*, *i*) are emperical correlation factor for shape, depth and inclination of loading respectively.
- Further he also gave emperical relations for Bearing capacity factors.

$$N_C = (N_q - 1)\cot\phi$$
$$N_q = \left[e^{\pi\tan\phi}\right]\left[\tan^2\left(45+\frac{\phi}{2}\right)\right]$$
$$N_\gamma = [N_q - 1].\tan(1.4\phi)$$

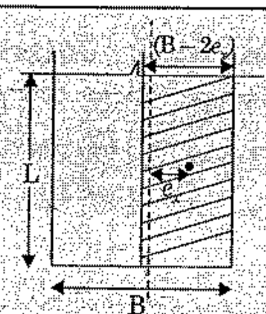
- For, Pure clayey soil ($\phi = 0$)

$$N_C = 5.14$$
$$N_q = 1$$
$$N_\gamma = 0$$

- To account for eccentricity of loading, the footing dimensions was proposed to be modified. Such that the load becomes concetnric with the reduced dimension of footing.



$$\therefore \quad \text{Load carrying capacity} = \left[\left(\frac{q_{nu}}{F.O.S.}\right)\times(B-2e_x)(L-2e_y)\right]$$



$$\text{Load carrying capacity/unit length} = \left[\left(\frac{q_{nu}}{F.O.S.}\right)\times(B-2e_x)\times 1\right]$$

HENSEN'S RECCOMENDATIONS

- Hensen proposed a bearing capacity equation as below:

$$q_u = \left[C.N_C.i_c S_c d_c + q.N_q.i_q s_q d_q + \frac{1}{2}B.\gamma.N_\gamma.i_\gamma s_\gamma d_\gamma\right]$$

- The above equation is applicable for $[\phi > 0]$ i.e. for granular soil.
- For $(\phi = 0)$, i.e. for purely cohesive soil, Hansen gave another equation as below.

$$q_u = C.N_C.(1+S_C+d_C-i_C)+\gamma.D_f$$

or

$$q_{nu} = C.N_C.(1+S_C+d_C-i_C)$$

where,

$$N_C = (N_q - 1)\cot\phi$$

$$\boxed{Nq = e^{(\pi \tan\phi)}.\tan^2\left(45+\phi/2\right)}$$

Sc, dc, ic are Hansen's correction factor for shape, depth and inclination

$$\boxed{N_\gamma = 1.5(N_q - 1).\tan\phi}$$

- N_C, N_q is same as Meyerhoff's.
- Eccentric loading treatment will be same as Meyerhoff's.

VESCIS BEARING CAPACITY

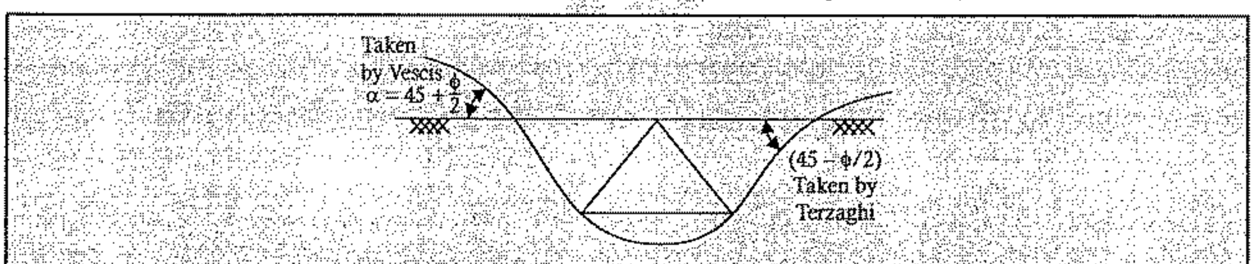
- Vescis proposed a bearing capacity equation as below.

$$\boxed{q_u = C.N_C[i_C.S_c d_c] + q.N_q[i_q.S_q.d_q] + \frac{1}{2}B\gamma.N_\gamma[i_\gamma.s_\gamma.d_\gamma]}$$

- Where, Sc dc, ic are vescis correction factor for shape, depth and inclination.

1. $N_\gamma = 2(N_q + 1)\tan\phi$
2. $Nc = (Nq - 1)\cot\phi$,
3. $Nq = e^{\pi\tan\phi}.\tan^2(45 + \phi/2)$

- Failure surface assumed by vescrics is simillar to Terzaghi's. But the angle of inclinalion of failure surface with the horizontal is 45 + ϕ/2 where as in case of Terzaghi it was (45– ϕ/2) as shown below



IS CODE METHOD

- IS –6403 –1981 recommends the equation for calculation of ultimate bearing capacity of shallow foundation as below :

$$\boxed{q_{nu} = cN_c(i_c S_c d_c) + q(N_q - 1)(i_q S_q d_q)\frac{1}{2}B\gamma N\gamma(i_\gamma S_\gamma d_\gamma) \times W'}$$

- Where N_c, N_q, N_γ are based of Vesic's formula and (W') is a factor that takes into account the water table correction.

1. If depth of water below ground > (D_f + B) then $\boxed{W' = 1}$
2. If water table at depth = (D_f) $\boxed{W' = 0.5}$
3. For in between case → Linear interpolation will be done.

- For cohesive soil net ultimate bearing capacity immidiately after construction i.e. undrained condition is given as below

$$q_{nu} = CN_c(i_c S_c d_c)$$

Where,

$N_C = 5.14$

$\text{Lim } [(N_q - 1) \cot\phi] = 0$

- C must be calculated using undrained conditions
- Bearing capacity in clay is used for end of construction stability check.

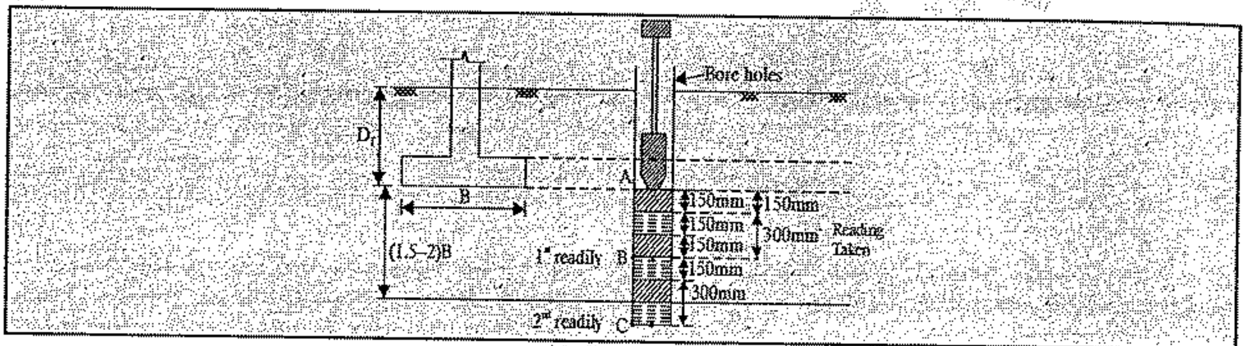
BEARING CAPACITY BASED ON FIELD TEST:

(A) S.P.T. Test [standard Penetration test]

(B) Plate Load test

(C) Static cone penetration test / Static Cone Resistance test.

(A) S.P.T. Test:



- Bearing capacity of Granular soil can be calculated by using corrected S.P.T.N-values.
- The values of N- is determined at No. of Selected bore holes and average value of corrected (N) is calculated for the depth from $[D_f + (1.5 - 2)B]$.
- While calculating the average value of (N), any value greater than 50% of average value is discarded.

e.g., if $\left[\dfrac{N_1 + N_2 + N_3 + N_4}{4}\right] = 30$

and, if $N_3 = 40$

Then (N_3) is accepted as $[30 \times 1.5 = 45]$ and $(40 < 45)$

- If $N_2 = 48$ then (N_2) is discarded as 48 is > 45

$\therefore$ New average is taken as $= \left[\dfrac{N_1 + N_3 + N_4}{3}\right]$

- Minimum of all the avg. values of various bore holes are used in design of foundations.
- Emperical correlations are available for N-value of granular soil:

N	ϕ	Relative Density D_r
< 4	(25 – 30)	0
(4 – 10)	(27 – 32)	15

- Bearing capacity of the granular soil can be calculated using Terzaghi's equation where, value of N_q & N_γ can be found out from Peck henson's curve or I.S code methode.
- Teng also gave emperical equation for calculation of bearing capacity of Granular soil as below.

TENG'S FORMULA

For Net ultimate Bearing capacity for granualr soil.

1. **For continuous or strip footing:**

$$q_{nu} = \frac{1}{6}\left[3N^2.B.R_w' + 5(100+N^2).D_f R_w\right]$$

2. **For square or circular footing:**

$$q_{nu} = \frac{1}{3}\left[N^2.B.R_w' + 3(100+N^2).D_f.R_w\right]$$

where,

q_{nu}= net ultimate bearing capacity in kN/m^2

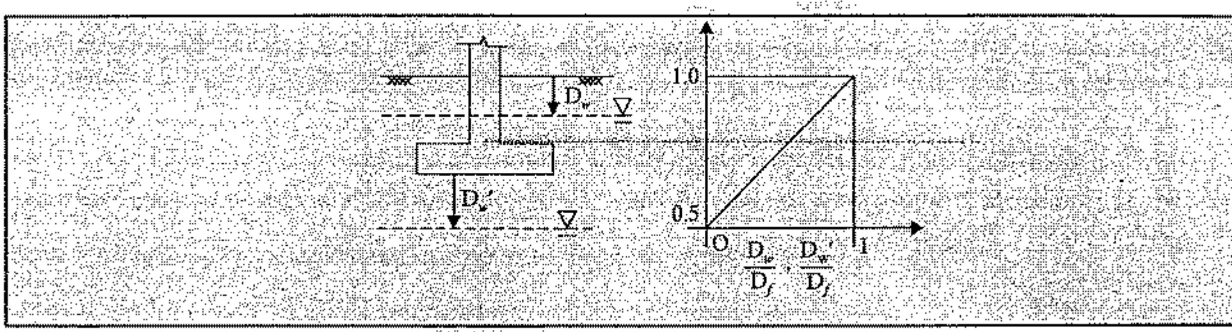
N = average corrected N value.

B = Width of footing in. meter (m)

But if $D_f > B$, use $D_f = B$

D_f = Depth of footing in meter.

R_w & R_w' = Water table correction factor.



S.P.T. Test Procedure:

- Suitable for **Granular soil.**
- Split spoon samples is used in the bore hole.
- Bore hole is advanced to a depth at which N-value is to be calculated.
- The split-spoon sampler is allowed to penetrate into the soil by applying impact load of 65 Kg having a free fall of 75 cm.
- The sampler is allowed to penetrate for 150 mm depth, but reading is not noted i.e.(No. of blows required for 150 mm penetration is not Noted).
- Then the sampler is allowed to penetrate, further for 300 mm and No. of blows required to penetrate the sampler to this 300 mm is the SPT N-value
- The next test is carried out at level 750 mm below the previous test reference level.
- If bore hole depth is large, then interval of next test is taken at a depth of (2.5 to 2) m or at the change of strata.

The following corrections are required for N-values obtained above.

1. Over burden correction:

- It is necessary because the N-value will have effect on it due to confinement of soil at various depth.
- Two granular soils possessing the same relative density but having different confining pressures are

- Since the confining pressure increases with depth, the N values at shallow depths are underestimated and the N values at larger depths are overestimated.
- Therefore, if no correction is applied to recorded N values, the relative densities at shallow depths will be under estimated and at higher depths, they will be overestimated.
- If N_0 = observed S.P.T. value Then,

$$\boxed{N_1 = N_0 \times \frac{350}{(\bar{\sigma}+70)}}$$

Where, $\bar{\sigma}$ = Effective stress at level of test (KN/m^2)

N_1 = Corrected N-value of overburden.

- Overburden correction will not be applied if $\bar{\sigma} > 280\ \text{KN/m}^2$

2. Dilatancy Correction:

- It is applied to the already corrected N-values for overburden pressure. Dilatancy correction is required only if $[N_1 > 15]$ in saturated fine sand and silt (i.e., when water table is above test level).
- $(N_1 > 15)$ basically represents the Dense sand which will have the tendency to dilate under rapid loading (undrained condition) and (–ve) pore water pressure will develope. Hence, observed (N) value will be more because shear resistance will increase.

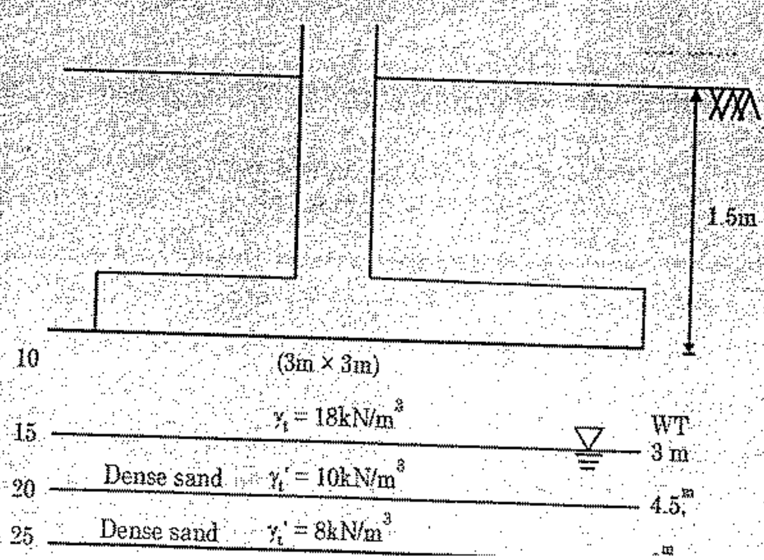
Corrected N value after Dilatancy Correction = $N_2 = 15 + \frac{1}{2}(N_1 - 15)$

- This correction becomes more significant for fine dense sand.

Example 13

Using peck hension equation determine net allowable bearing pressure for a permissible settlement of 40 mm. The SPT is conducted at 1.5 m interval as shown in fig. The observed SPT values are 10,15,20 and 25 at depths 1.5m, 3m, 4.5m and 6m.

Sol.



$$q_{net} = 0.44\, C_w\, N S_a$$

$$N_1 = N_0 \times \frac{350}{(\bar{\sigma}+70)} = \frac{10\times 350}{(18\times 1.5+70)} = 36.0824$$

$$N_2 = 15\times \frac{350}{70+(18\times 3)} = 42.338$$

$$N_3 = 20\times \frac{350}{(18\times 3+10\times 1.5)+70} = 50.36$$

$$N_4 = 25\times \frac{350}{70+(18\times 3+10\times 1.5\times 10+1.5\times 8)} = 57.944$$

Dilatancy correction : Correction will not be done for 1.5^m Depth become W.T is below that level.

$$N'_1 = 36.0824$$
$$N'_2 = 15 + 1/2\,(42.338 - 15) = 28.67$$
$$N'_3 = 15 + 1/2\,(50.36 - 20) = 32.68$$
$$N'_4 = 15 + 1/2\,(57.944 - 25) = 36.47$$
$$N_{avg} = 33.47$$
$$S_a = 40 \text{ mm}$$

$$C_w = 0.5\left[1+\frac{3}{3+1.5}\right] = 0.83$$

$$(q_a)_{net} = (0.44\times 0.83\times 33.47\times 40) = 489.011 \text{ kN/m}^2.$$

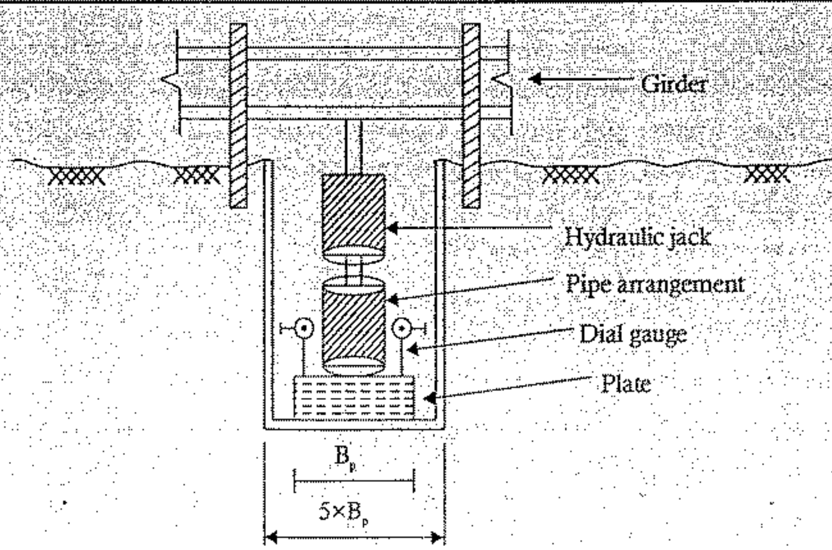
(B) Plate Load Test:

It is used to calculate:

(a) Ultimate Bearing capacity of soil.

(b) Allowable bearing pressure corresponding to a particular permissible settlement of foundation.

(c) Settlement of foundation itself.

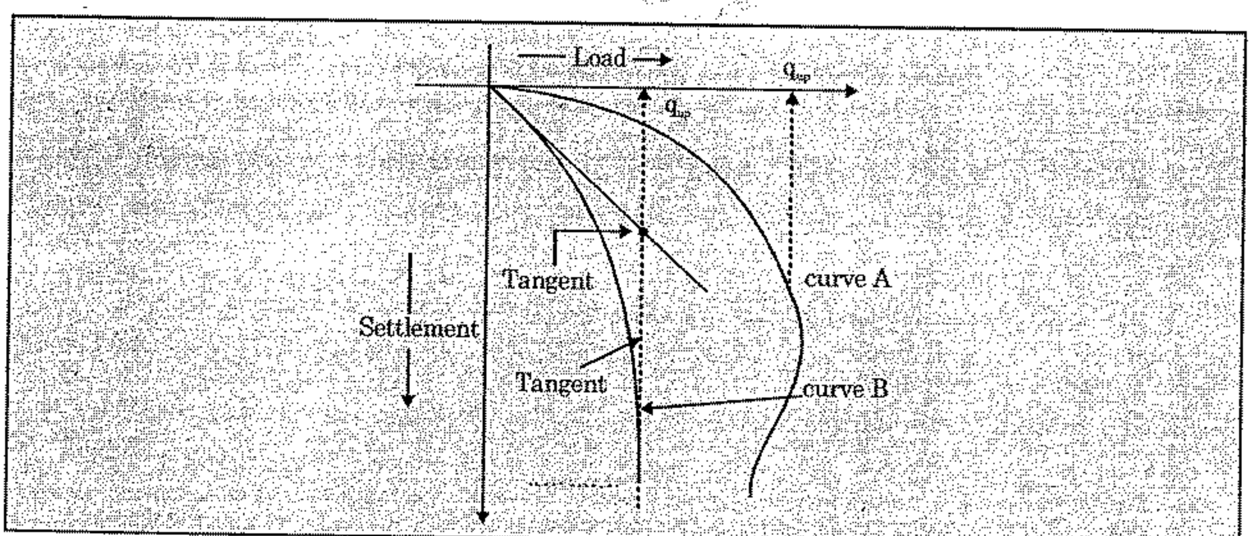


- Plate load test has been done according to IS : 1888-1992
- Plate load test is significant only for cohession less soils.

Procedure

1. Circular or square bearing plates of mild steels are used, of thickness not less than 25 mm and varying in size from 300 mm to 750 mm (300, 450, 600, 750) are used. Smaller size plates are used in dense or stiff soils where as larger plates are being used in loose or soft soils.
2. A pit of dimension 5 times the width of plate is excavated at the proposed depth of foundation i.e., D_f and test plate is placed in the center.

- This test is carried out at the level of proposed foundation and if water table is above the level at which test is to be carried out then water is pumped out before the placing of test plate.
- A load of $70 g/cm^2$ is first applied and released after some time.
- Loads are applied on test plate in increments of one-fifth the estimated safe load upto failure or at least until a settlement of 25 mm, whichever is earlier. At each load, settlement is recorded at time intervals of 1, 2.25, 4, 6.25, 9, 16 and 25 minutes and thereafter at intervals of one hour.
- The load settlement curve can be plotted from the data obtained.
- It is as short duration test. Hence, settlement is only the immediate settlement. It cannot measure consolidated settlement that is why,not relevent for clays.
- For clays settlement criteria is very much important in the determination of allowable bearing pressure of the foundation.



- In above load v/s settlement plot, Curve A shows a distinct peak or failure and ultimate bearing capacity can be easily found out corresponding to this peak intensity.
- Whereas when the failure is not clearly visible as in curve B, then the ultimate bearing capacity is found out by double tangent method.

BEARING CAPACITY CALCULATION

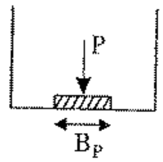
- As shown above the bearing capacity of the plate can be found out from load settlement curve.
- Further we can also find out bearing capacity of foundation from modified terzaghi's bearing capacity equation for square footing.

$$q_{up} = 0 + 0 + 0.4\, B_p \gamma N_\gamma$$

$$q_{up} = 0.4 B_p \gamma N_\gamma$$

- Note that in plate load test the over burden pressure = 0

Hence, $$\frac{P}{B^2} - \gamma D_f = q_{nu}$$



$$\Rightarrow \quad \frac{P}{B^2} = q_{nu} + \gamma D_f = q_{up}$$

q_{up} = Ultimate bearing capicity for plate

$$\Rightarrow \quad \frac{P}{B^2} = q_{up} = 0.4 B_p \gamma N_\gamma$$

- From the above equation we can calculte N_γ after knowing the q_{up} from the load settlement graph.
- With the help of N_γ, ϕ & N_c are calculated. Then either with the help of Terzaghi's equation or Peck Henson's curve Bearing capacity of the foundation is calculated.

Note: Approximately $\frac{q_{uf}}{q_{up}} = \frac{B_f}{B_p}$

where, B_f = width of foundations

B_p = width of plate

q_{uf} = ultimate bearing capacity of foundation

q_{up} = Ultimate bearing capacity of plate

II. Settlement computation

Case I : The plate load test carried out at the level of foundation

- The intensity of pressure applied at the base of plate will be taken as the Intensity of pressure at the base of foundation.
- Hence, following formula will be used for calculating the settlment of foundation from the settlement of plate.

$$\boxed{\frac{S_f}{S_p} = \left(\frac{B_f}{B_p} \times \left[\frac{B_p + 0.3}{B_f + 0.3}\right]\right)^2}$$

where,

(S_f) = Settlement of foundation of width B_f (meter).

(S_p) = Settlement of plate of width B_p (meter).

Case II : If the foundation is located at the depth deeper than the level of plate load test.

- Correction will be applied to the (S_F) value obtained from (S_p) value.

$$\boxed{S_{f(\text{correction})} = S_f \times \left[\frac{1}{\left(1 + \frac{D_2}{B_f}\right)}\right]^{0.5}}$$

Where,

D_2 = Depth of foundation from the l

III. Allowable bearing pressure calculation from settlement criteria:

- Plate load test is being carried out at the level of foundation
- From the permissible settlement of foundation, settlement of plate will be calculated. From this settlment of plate, load intensity at the base of plate will be calculated from load settlement curve of plate.
- Thus load intensity will be **allowable bearing pressure.**

Here,

$$\frac{S_f}{S_P} = \left[\frac{B_f}{B_P}\left(\frac{B_P + 0.3}{B_f + 0.3}\right)\right]^2$$

S_f = Permissible settlement of foundation.

S_P = Settlement of plate.

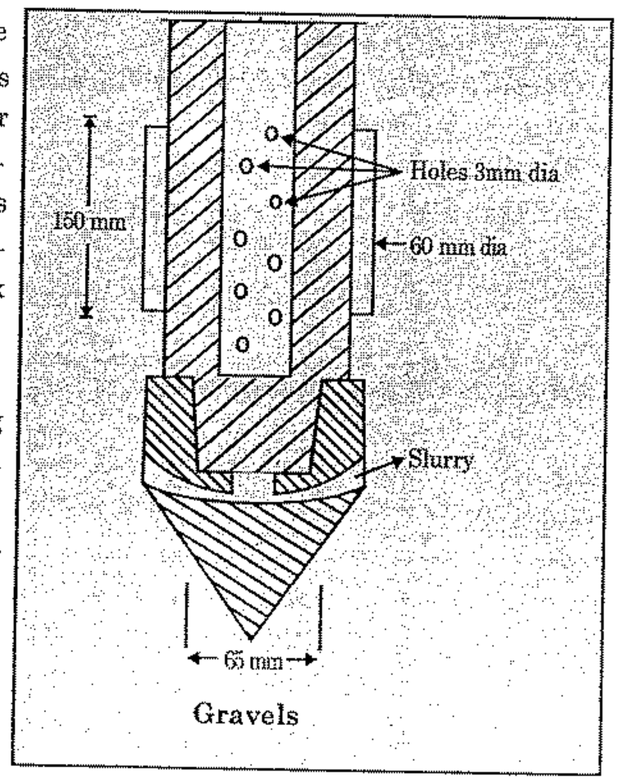
- If there is level difference between the **plate load test and base of foundation:**

Then,
$$S_{f\,(\text{corrected})} = S_f \times \left[\frac{1}{\left(1 + \frac{D_2}{B_f}\right)}\right]^{0.5}$$

- $S_{f\,(\text{corrected})}$ will be taken as permissible settlement and corresponding S_p will be calcualted
- From S_p, load intensity will be calculated from load settlement curve. This will be the allowable bearing pressure.

STATIC CONE PENETRATION TEST (CPT)

- The static cone penetration test, simply called the cone penetration test (CPT), is a simple test that is now widely used in place of SPT, particularly for soft clays, silts and fine to medium sand deposits.
- This test is performed to obtain a continuous record of the soil resistance by penetrating a cone.
- The cone will be 10 cm² in area with an apex angle of 60°.
- The cone is penetrated slowly and steadily.
- The cone is fixed on the lower end of a steel driving rod which passes through a steel tube, with external diameter equal to the base of the cone.
- The cone and the sleve are pushed into the soil at a rate of 20 mm/sec upto a depth of 100 mm. The resistance of soil offered to the penetration is recorded as cone penetration resistance



Note: *SCPT is not suitable for dense sands.*

BEARING CAPACITY OF FOOTING ON LAYERED SOIL

In case of stratified soils present below the base of foundation having different values of C & ϕ then weighted average values of C & ϕ are used to calculate the bearing capacity factors.

FACTORS AFFECTING BEARING CAPACITY

1. Cohession less Soils.

Utimate bearing capacity on cohessionless soil is given by.

$$q_u = qN_q + \frac{1}{2} B\gamma N\gamma$$

The bearing capacity of cohession less soil (C = 0) depends on following factors

(i) Relative density or friction angle ϕ (ii) Width of footing

(iii) Depth of footing (iv) Unit weight of soil

(v) Position of Ground water table.

- Higher is the Relative density, higher will be the ϕ value. Hence higher will be the Nγ & Nq, which results in increase in bearing capacity.

  But in loose soil deposit the increase in bearing capacity is very less (almost negligible) in comparision to medium and high relative density sands with the same amount of increase in friction angle value. This is because in loose soil deposit Nγ and Nq value is very less in comparision to dense soils.
- The ultimate bearing capacity increases with the width of footing in cohession less soil.
- As depth of foundation increases, the surcharge (q) at the base of foundation increases which leads in increase in bearing capacity of foundation.
- We can observe from the above equation that as the unit weight of soil increases the bearing capacity of cohession less soil also increases.
- From the previous discussion in this chapter we have come to know that as the water table rises the effective unit weight as soil decreases, which leads to decrease in the bearing capacity of soil.

Note: If water table rises from depth equal to B (width) below the base of foundation to the ground level the bearing capacity of granular soil become 1/2 of the bearing capacity of soil when water table is B meter below the base of foundation.

2. Purely cohesive soil (f = 0).

The ultimate bearing capacity of purely cohessive soil is given as below :

$$q_u = cNc + q$$

- The bearing capacity of a footing on purely cohessive soil is mainly dependent on the Cohession (C) of the soil.
- Increase in depth of foundation contributes to an increase in bearing capacity but only to the extent of increased surcharge (q).
- But net utimate bearing capacity $q_{nu} = cN_c$ remains same.
- As we can observe from the above quation that bearing capacity of footing on purely cohessive soil is independent of size of footing.

Note: Total load bearing capacity however gets increased as the width of footing increased but bearing capacity

ALLOWABLE BEARING CAPACITY: [$Q_{A\ NET}$]

- Based on permissible settlement of foundation, allowable bearing capacity is found out.
- Allowable bearing capacity is the minimum of
  (i) Bearing strength from shear failure criteria
  (ii) Bearing strength from settlement criteria
- To determine Allowable bearing capacity few emprical relations have be suggested as discussed below.

1. Peck Henson's Formula:

$$\boxed{q_{a(net)} = 0.44\, N.S.C_w}$$

Where,

$q_{a(net)}$ = Net allowable bearing capacity (KN/m²)

N = Corrected N-value of S.P.T for depth [1.5 – 2]B.

S = Permissible settlement in (mm)

C_w = Water table correction factor.

$$\boxed{C_w = 0.5\left[1 + \frac{D_w}{(D_f + B)}\right]}$$

Where, D_F = depth of foundation,

B = width of foundation

D_w = Depth of water table below the ground surface.

2. Teng's Formula:

- I.S. code adopted Teng's formula with some correction

$$\boxed{q_{a(net)} = 1.4(N-3)\left(\frac{B+0.3}{2B}\right)^2 S.C_W.C_D}$$

Where, $q_{a(net)}$ = Net allowable bearing capacity kN/m²

C_W = Water Table Correction Factor $\boxed{Cw = 0.5\left[1 + \frac{D_w}{B}\right]}$

Where, D_W = Depth of water table below the base of footing.

B = Width of footing.

C_D = Depth correction factor

$$\boxed{C_D = \left(1 + \frac{D_f}{B}\right) \le 2}$$

D_F = depth of foundation

B = width of foundation

S = Permessible settlement in (mm).

N = Corrected (N) value of S.P.T.

3. I.S. Code Method for Raft:

$$\boxed{q_{net(safe)} = 0.88\,N.S_a.C_W \ \text{kN/m}^2}$$

$$C_W = 0.5\left[1+\left(\frac{D_w}{D_f+B}\right)\right] \text{ (Peck hensen formula)}$$

S_a = Permissible settlement in (mm)

SETTLEMENT OF FOUNDATION

$$\boxed{S = S_i + S_{1^\circ\,comp} + S_{2^\circ-comp}}$$

- Immediate settlement is computed using the theory of elasticity.
- Net elastic settlement for a **flexible surface foundation based on theory of elasticity is:**

$$\boxed{S_i = \frac{q_n B (1-\mu^2)}{E_S} I_f}$$

Where,

S_i = Immediate elastic settlement [Both for sandy soil and Clayey soil]

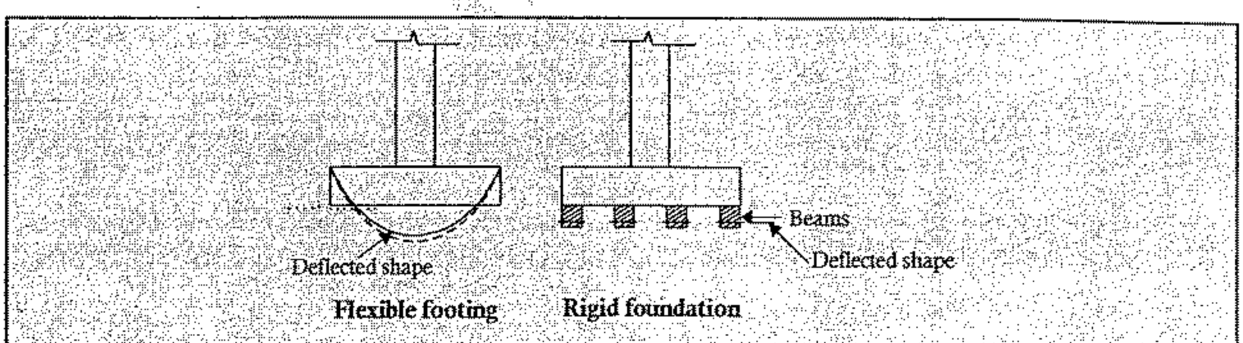
q_n = Net foundation pressure.

B = Width of foundation

μ = Poission's ratio

E_S = Modulus of elasticity

I_f = Influence factor which depends on the shape and rigidity of structure.



(E_S) Can be calculated using:

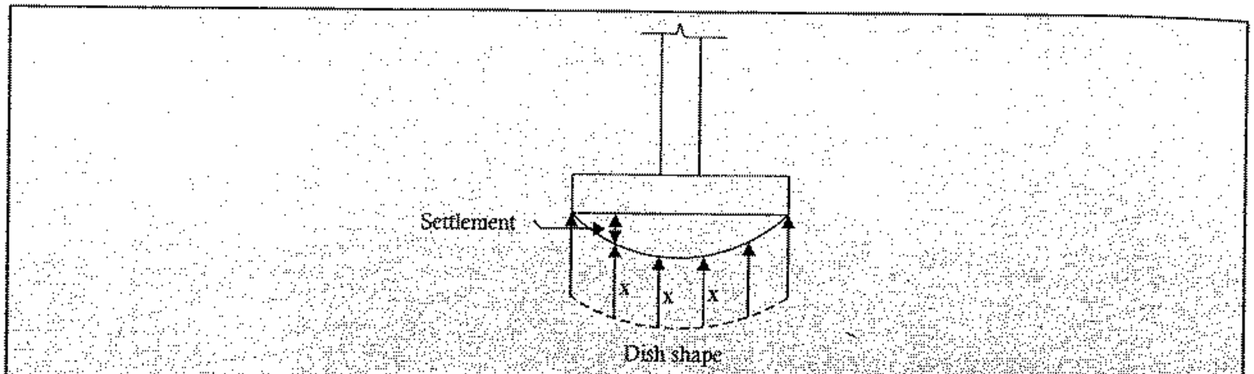
(i) Triaxial test

(ii) Field Test

$$\boxed{\underset{\text{(Immediate)}}{S_{rigid}} = \underset{\text{(Immediate)}}{S_{flexible}} \times 0.8}$$

1. Flexible footing over clayey soil

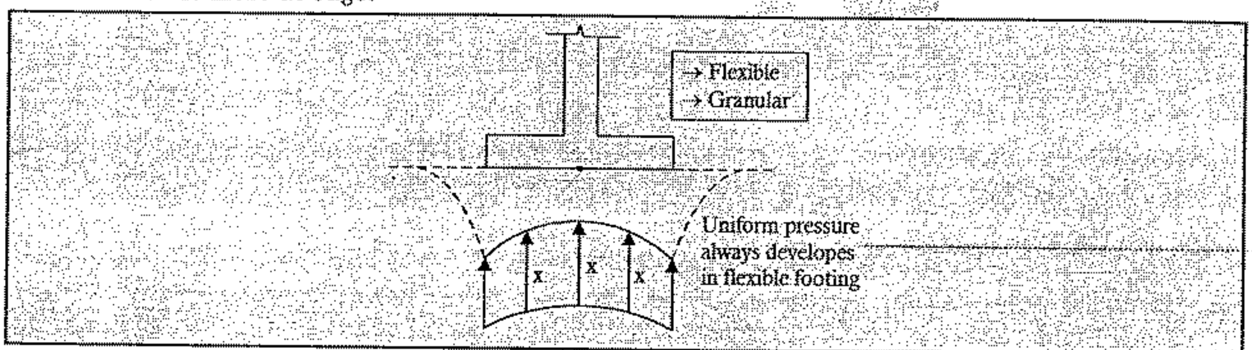
- In flexible footing, the contact pressure at the interface between footing and soil is uniformly distributed producing dish-shape pattern in clayey soil.



Flexible footing over clayey soil

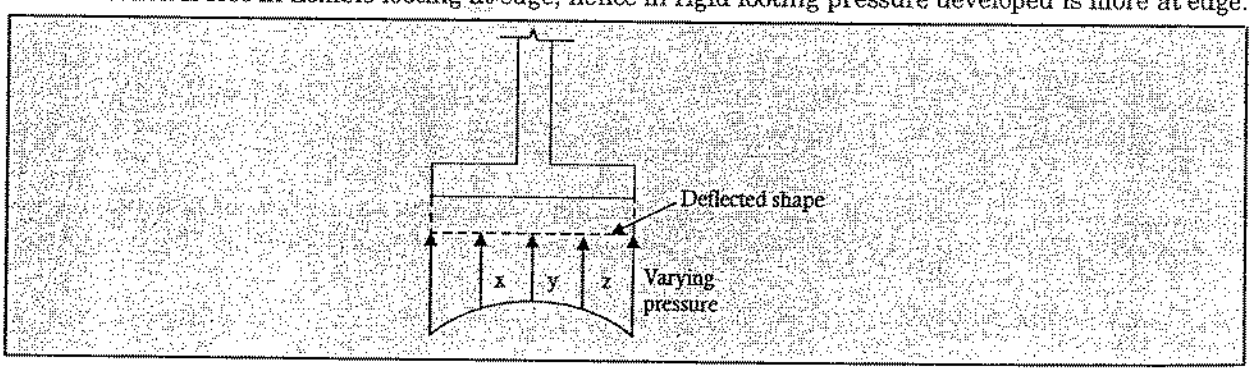
2. ~~Rigid~~ Flexible footing over Gannular soil

- In granular soil, modulus of elasticty (E_S) varies across the width being maximum at the centre and minimum at edge. As E is maximum at centre, defflection is less at centre. As E is less at edge deflection is more at edge.

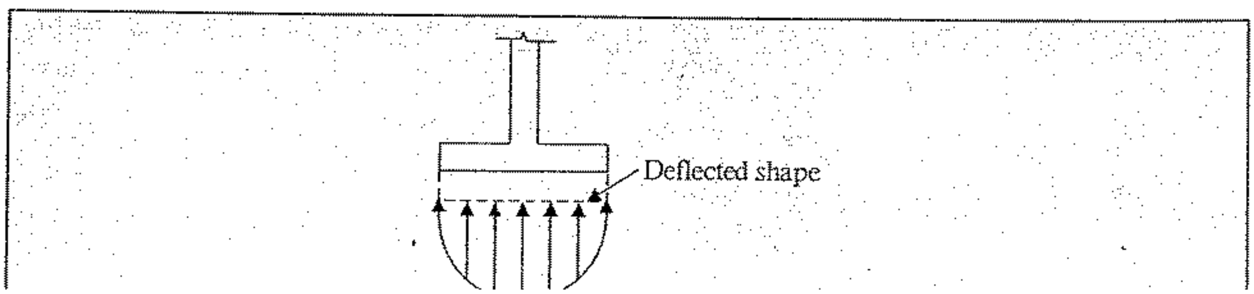


3. Rigid footing on Clayey soil

- In case of flexible footing, deflection is more at centre. Hence pressure developed at centre is less. Deflection is less in flexible footing at edge, hence in rigid footing pressure developed is more at edge.



4. Rigid footing on Granular soil



PERMISSIBLE SETTLEMENTS IN SHALLOW FOUNDATION

	Total Settlement (mm)	Differential Settlement (mm)	Angular Distortion (mm)
Isolated footing on clay	65	0.0015L	1/666
Isolated footing on sand	40	0.0015L	1/666
Raft on clay	65-100	0.0021L	1/500
Raft on sand	40-65	0.0021L	1/500

Example 14

Find the settlement of a footing of width 1.2 m with the following plate load test results for

(i) Sand soil

(ii) Clay soil

Settlement of the plate 15mm, width of the plate 400 mm.

Sol. Data given

Width of footing , $B_f = 1.2$ m

Width of plate $B_p = 400$ mm $= 0.4$ m

Settlement of plate, $S_p = 15$ mm

Settlement of footing, $S_f = ?$

(i) For sand soil,

$$S_f = S_p\left[\frac{B_f(B_p+0.3)}{B_p(B_f+0.3)}\right]^2$$

$$S_f = 15\times\left[\frac{1.2\times(0.4+0.3)}{0.4\times(1.2+0.3)}\right]^2$$

$$= 29.4 \text{ mm} \quad \text{... (i)}$$

(ii) For clayey soil,

$$S_f = S_p\times\left(\frac{B_f}{B_p}\right)$$

$$= 15\times\frac{1.2}{0.4} = 45 \text{ mm} \quad \text{... (ii)}$$

Example 15

The following data was obtained from plate load test carried out on a 60 cm square test plate at a depth of 2 m below ground surface on a sandy soil which extends upto a large depth. Determine the settlement of a foundation 3.0 m × 3.0 m carrying a load of 110 tonn and located at a depth of 2 m below ground surface.

Load Intensity (t/m^2)	0	5	10	15	20	25	30	35	40
Settlement (mm)	0	2.0	4.0	7.5	11	16.3	23.5	34.0	45.0

Water table is located at a large depth from the ground surface.

Sol. Data given:

$$B_p = 60 \text{ cm}$$
$$B_f = 3.0 \text{ m} = 300 \text{ cm}$$

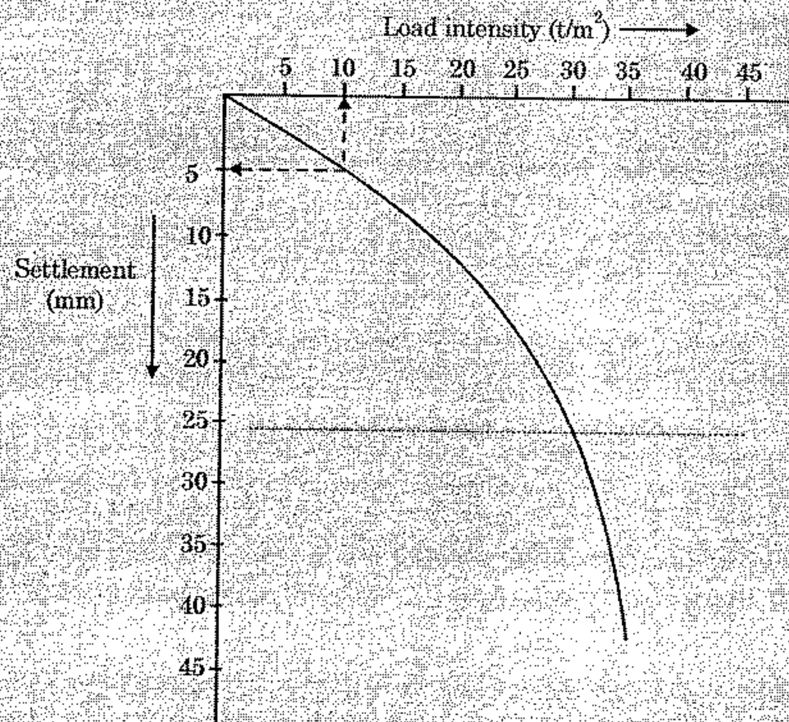
Load carried by foundatioon = 3.0 m = 300 cm

∴ Load carried by foundation = 110 tonn

∴ Size foundation 3.0×3.0 m²

∴ Load intensity at the base of foundation $= \left(\frac{110}{3 \times 3}\right)$

$= 12.2$ tm/²



From the load settlement given, for a load intensity of 12.2 t/m²,

The settlement of test plate = 5 mm

$$\text{Settlment of foundation } s_f = S_p \times \left[\frac{B_f (B_p + 300)}{B_p (B_f + 300)}\right]^2$$

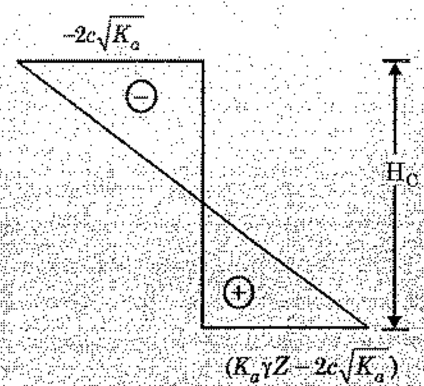
$$S_f = 5 \times \left[\frac{300 \times (60+30)}{60 \times (300+30)}\right]^2$$

$$= 9.3 \text{ mm}$$

Example 16

When a vertical face excavation was made in deposit of clay, it failed at a depth of 2.8 m of excavation. Find the shear strengths parameters of the soil if its bulk density is 17 kN/m³ in the deposit, at same other location, a plate load test was conducted with 30 cm square plate, placed at a depth of 1 m below the G.L. The ultimate load was 13.5 kN, water table was at a 4 m below the ground G.L. Calculate the net safe bearing capacity for a 1.5 m wide strip footing to be founded at a depth of 1.5

Sol. Data given:



For Clay $\phi = 0°$

Bulk density $\gamma = 17$ kN/m³

Critical height $H_c = 2.8$ m

We have $p_a = K_a\gamma Z - 2c\sqrt{K_a}$

$$\Rightarrow \quad 0 = K_a\gamma Z - 2c\sqrt{K_a}$$

$$\Rightarrow \quad \frac{2c\sqrt{K_a}}{K_a\gamma} = Z_0$$

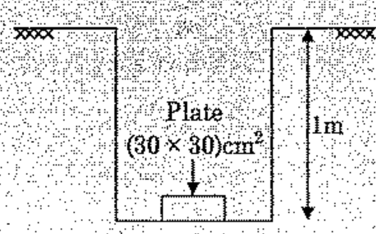
$$\therefore \quad \text{Critical height } H_c = 2Z_0 = \left(\frac{4c}{\sqrt{K_a}\times\gamma}\right) \quad \text{... (i)}$$

$$\Rightarrow \quad 2.8 = \frac{4c}{\sqrt{K_a}\,\gamma} \qquad \left[\because K_a = \left(\frac{1-\sin 0}{1+\sin 0}\right) = 1\right]$$

$$= \frac{4c}{1\times 17}$$

$$\therefore \quad c = \left(\frac{2.8\times 17}{4}\right) = 11.9 \text{ kN/m}^2 \quad \text{... (ii)}$$

For plate load test on square plate



Size of plate $= 30 \times 30$ cm²

Load on plate $= 13.5$ kN

$\therefore$ Ulimate load capacity of soil under plate

$$= \frac{13.5}{(0.3\times 0.3)} = 150 \text{ kN/m}^2$$

$$q_u = 1.3c\ N_c + \gamma D_f\ N_q + 0.4\ B\gamma N_\gamma$$

For cohesive soil, $N_c = 5.7\ N_q = 1,\ N_\gamma = 0$

$\Rightarrow$ $150 = (1.3c \times 5.7) + 17 \times 1.0 + 0$

$\Rightarrow$ $\dfrac{(150-17\times1.0)}{(1.3\times5.7)} = c$

$\Rightarrow$ $c = 17.95\ \text{kN/m}^2$ ... (*iii*)

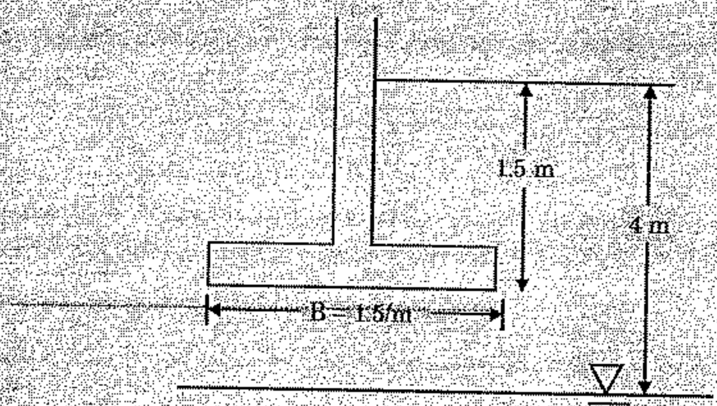
Hence From (*ii*) and (*iii*), lesser of two will be the value of cohesion

$$c = 11.9\ \text{kN/m}^2$$

For Strip Footing

$\therefore$ Depth of water table below the test level = 4 – 1.5 = 2.5m

$\because$ Depth is much more than the width of footing, therefore there will be no influence of W.T



$$q_u = (cN_c + \gamma D_f\ N_q + 0.5\ B\gamma N\gamma)$$

For $\phi = 0,\ \ N_c = 5.7\ N_q = 1,\ N_\gamma = 0$

$\Rightarrow$ $q_u = (5.7\ c + \gamma D_f)$

$\therefore$ $q_{nu} = 5.7\ c$

$= (5.7 \times 11.9)$

$= 67.83\ \text{kN/m}^2$

$\therefore$ Net safe bearing capacity $q_s = \left(\dfrac{q_{nu}}{\text{F.O.S}}\right)$

$= \left(\dfrac{67.83}{3}\right)$

$= 22.61\ \text{kN/m}^2$

Example 17

A square footing is restricted to carry a net load of 1200 kN. Determine the size of the footing if the depth of foundation is 2m. and tolerable settlement is 40 mm.
The soil is sandy with N = 12 and F.O.S = 3.
water table is very deep. Use Teng's equation.

Sol. Data given :

Net load $Q = 1200$ kN

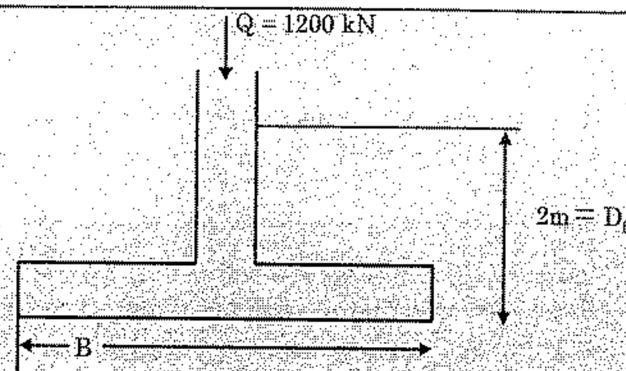
$D_f = 2$m

$S = 40$ mm

$C = 0$ (Sandy soil)

$N = 12$ F.D.S. $= 3$

$B = ?$



We know that from Teng's equation
The net safe bearing capacity

$$q_{ns} = 1.4\,(N-3)\left[\frac{B+0.3}{2B}\right]^2 S\,C_w\,C_D \qquad \ldots (i)$$

Where, N = Corrected standard penetration

B = Width of footing in meters

S = Tolerable settlement in mm.

C_W = Water table correction factor

$$= \frac{1}{2}\left(1+\frac{D_w}{B}\right)$$

D_w = Depthe of W.T. below the base of foundation

$\because$ Water table is very deep.

Therefore,

$$C_w = \frac{1}{2}\times\left(1+\frac{B}{B}\right)=1$$

$$C_D = \text{Depth factor} = \left(1+\frac{D_f}{B}\right)\le 2$$

$$= \left(1+\frac{2}{B}\right)$$

$$\therefore \quad q_{ns} = 1.4\,(12-3)\left[\frac{B+0.3}{2B}\right]^2\times 40\times 1\times\left(1+\frac{2}{B}\right)$$

$$\therefore \quad \frac{\text{Net load}}{B^2} = \left(\frac{q_{ns}}{F.O.S}\right)$$

$$\Rightarrow \quad \frac{1200\times F.O.S}{B^2} = 1.4\times 9\left[\frac{(B+0.3)^2}{4B^2}\right]\times 40\left(1+\frac{2}{B}\right)$$

$$\Rightarrow \quad \frac{1200\times 3}{B^2} = \frac{126}{B^2}\,(B+0.3)^2\left(\frac{B+2}{B}\right)$$

$$\Rightarrow \quad 3600\,B = 126\,[B^2+0.09+0.6B]\,(B+2)$$

$$\Rightarrow \quad B^3+0.09B^2+0.6B^2+2B^2+0.18+1.2B = 28.57B$$

$$\Rightarrow \quad B^3+2.6B^2-7.28B+0.18 = 0 \qquad \ldots (ii)$$

$\Rightarrow$ By solving above equation, $B = 4.08$ m

Therefore the size of the footing is 4.08×4.08 m^2.

OBJECTIVE TYPE QUESTIONS

1. Bearing capacity of a soil strata supporting a footing of size 3 m × 3 m will not be affected by the presence of ground water table located at a depth which is
   (a) 1.0 m below the base of the footing
   (b) 1.5 m below the base of the footing
   (c) 2.5 m below the base of the footing
   (d) 3 m below the base of the footing
2. A rectangular footing 1 m × 2 m is placed at a depth of 2 m in a saturated clay having an unconfined compressive strength of 100 kN/m^2. According to Skempton, the net ultimate bearing capacity is
   (a) 420 kN/m^2
   (b) 412.5 kN/m^2
   (c) 385 kN/m^2
   (d) 350 kN/m^2
3. A raft foundation is to be constructed on a sandy soil. The maximum differential settlement and limiting maximum settlement as recommended by Indian Standard code are:

Max. differential settlement	Limiting max. settlement
(a) 40 mm	65 mm to 100 mm
(b) 40 mm	40 mm to 65 mm
(c) 25 mm	65 mm to 100 mm
(d) 25 mm	40 mm to 65 mm

4. Terzaghi's consolidation theory is applicable to one-dimensionalconsolidation test
   (a) for small load increment ratios
   (b) for large load increment ratios
   (c) for a load increment ratio of nearly one
   (d) in situations where there is no excess pore pressure
5. Consider the following statements associated with local shear failure of soils:
   1. Failure is sudden with well-defined ultimate load.
   2. This failure occurs in highly compressible soils.
   3. Failure is preceded by large settlement.

   Which of these statements are correct?
   (a) 1, 2 and 3
   (b) 1 and 2
   (c) 2 and 3
   (d) 1 and 3
6. Rafts resting on sands can be allowed double of the allowable soil pressure when
   (a) permissible settlement is doubled
   (b) length is doubled
   (c) depth factor is increased
   (d) water table is lowered
7. Match List-I (Investigator) with List-II (Equation) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Skempton	1. $v = ki$
B. Coulomb	2. $\sigma' = \sigma - u$
C. Stokes	3. $v = \dfrac{D^2(\gamma_s - \gamma_w)}{18\eta}$
D. Terzaghi	4. $s = c + \sigma \tan \phi$
	5. [illegible]

Codes:

	A	B	C	D
(a)	4	5	3	2
(b)	5	4	3	2
(c)	4	5	1	3
(d)	5	4	2	3

8. As per IS:1904-1986 the permissible angular distortion with respect to sheet structures in isolated foundations resting on plastic clay is

(a) $\frac{2}{3000}$ (b) $\frac{1}{3000}$

(c) $\frac{1}{300}$ (d) $\frac{2}{300}$

9. As per Terzaghi's equation, the bearing capacity of strip footing resting on cohesive soil ($c = 10\ kN/m^2$) for unit depth and unit width (assume N_c as 5.7) is

(a) $47\ kN/m^2$ (b) $57\ kN/m^2$

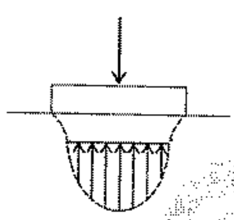
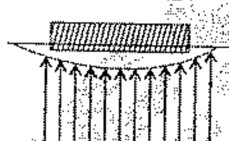
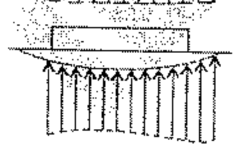
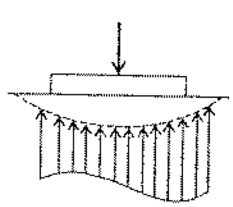
(c) $67\ kN/m^2$ (d) $77\ kN/m^2$

10. A raft of 6 m × 9 m is founded at a depth of 3 m in a cohesive soil having $c = 120\ kN/m^2$. The ultimate net bearing capacity of the soil using the Terzaghi's theory will benearly

(a) $820\ kN/m^2$ (b) $920\ kN/m^2$

(c) $1036\ kN/m^2$ (d) $1067\ kN/m^2$

11. Match List-I (Contact pressure distribution diagrams) with List-II (Description of footings) and select the correct answer using the codes given below the lists:

List-I	List-II
A. 	1. Rigid footing on cohesive soil
B. 	2. Flexible footing on cohesive soil
C. 	3. Rigid footing on cohesion-less soil at ground level
D. 	4. Flexible footing on cohesionless soil at ground level

Codes:

	A	B	C	D
(a)	3	1	4	2
(b)	4	2	3	1
(c)	3	2	4	1
(d)	4	1	3	2

12. Two circular footings of diameters D_1 and D_2 are resting on the surface of a purely cohesive soil. The ratio $D_1/D_2 = 2$. If the ultimate load carrying capacity of the footing of diameter D_1 is 200 kN/m^2, then the ultimate bearing capacity (in kN/m^2) of the footing of diameter D_2 will be

(a) 100 (b) 200
(c) 314 (d) 571

13. The minimum bearing capacity of a soil under a given footing occurs when the groundwater table at the location is at

(a) the base of the footing
(b) the ground level
(c) a depth equal to one-half the width of footing
(d) a depth equal to the width of footing

14. A rigid strip footing subjected to vertical central load fails under general shear failure. The rupture surface developed is

(a) circular one sided rotational failure
(b) plane surface of failure originating at the edge of footing and extending downwards with orientation of Rankine passive state equilibrium
(c) symmetrical failure surface consisting of segment of spiral and Rankine's passive state failure surface
(d) symmetrical failure surface consisting of segments of log spiral and plane failure surface as per Rankine active state of equilibrium

15. In a plate load test on sandy soil, the test plate of 60 cm × 60 cm undergoes a settlement of 5 mm at a pressure of 12×10^4 N/m^2. What will be the expected settlement of 3 m × 3m footing under same pressure?

(a) 25 mm (b) 20 mm
(c) 15 mm (d) 9 mm

16. The net ultimate bearing capacity of a square footing on surface of a saturated clay having unconfined compression strength of 50 kN/m^2 (using Skempton's equation) is

(a) 250 kN/m^2 (b) 180 kN/m^2
(c) ~~150 kN/m^2~~ 175 kN/m^2 (d) 125 kN/m^2

17. If two foundations, one narrow and another wide, are resting on a bed of sand carrying the same intensity of load per unit area, then which one is likely to fail early?

(a) Narrow foundation
(b) Wider foundation
(c) Both will fail simultaneously
(d) Difficult to judge since other conditions are unknown

18. The contact pressure distribution under a rigid footing on a cohesionless soil would be

(a) uniform throughout
(b) zero at centre and maximum at edges
(c) zero at edges and maximum at centre
(d) maximum at edges and minimum at centre

19. When a load test was conducted by putting a 60 cm square plate on top of a sandy deposit, the ultimate bearing capacity was observed as 60 kN/m^2. What is the ultimate bearing capacity for a strip footing of 1.2 m width to be placed on the surface of the same soil?
(a) 75 kN/m^2
(b) 120 kN/m^2
(c) 150 kN/m^2
(d) 160 kN/m^2

20. A plate load test is conducted on a cohesionless soil with a test plate having width B_p(cm) and settlement of this plate S_p(cm) is obtained at the same load intensity as a foundation. A footing having a width B_f(cm) is to be constructed as foundation. What is the settlement S_f(cm) experienced by this footing?
(a) $S_f = S_p\,\{[B_f(B_p + 30)]/[B_p(B_f + 30)]\}^2$
(b) $S_f = S_p\{[B_p(B_f + 30)]/[B_f(B_p + 30)]\}^2$
(c) $S_f = S_p[B_f/B_p]$
(d) $S_f = S_p[B_p/B_f]$

21. In a plate load test, how is the ultimate load estimated from the load settlement curve on a log-log graph?
(a) Directly
(b) By drawing tangents to the curve at the initial and final points
(c) By the secant method
(d) At 0.2 per cent of the maximum settlement

22. In which one of the following zones in a logarithmic spiral shape of failure surface assumed in the case of bearing capacity analysis of c-ϕ soils?
(a) Active zone
(b) Passive zone
(c) Radial shear zone
(d) Surcharge zone

23. A test plate 30 cm square, settles by 12 mm under a load of 4.5 kN in a sandy soil. By how much will a footing 2 m × 2 m subjected to a load of 200 kN settle?
(a) 36.3 mm
(b) 20.87 mm
(c) 75.75 mm
(d) 18.15 mm

24. Consider the following statements:
1. The bearing capacity of a footing on clay does not significantly get altered by the presence of water table.
2. The bearing capacity of a footing on saturated clay ($\phi = 0$) is a function of its size.

Which of these statements is/are correct?
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

25. Which factors influence the bearing capacity of a purely cohesionless soil?
1. Relative density of soil
2. Width and depth of footing
3. Unit weight of soil

Select the correct answer using the codes given below:
(a) 1 and 2 only
(b) 2 and 3 only
(c) 1 and 3 only
(d) 1, 2 and 3

26. For a proposed building, raft foundation, isolated footings and combined footings are being considered. These foundations are to be listed in the decreasing order of preference in terms of performance. Which one of the following is the correct order of listing?
(a) Raft foundation – Combined footings – Isolated footings
(b) Isolated footings – Raft foundation – Combined footings
(c) Combined footings – Raft foundation – Isolated footings
(d) Combined footings – Isolated footings – Raft foundation

27. The net ultimate bearing capacity of a purely cohesive soil
   (a) depends on the width of the footing and is independent of the depth of the footing
   (b) depends on the width as well as the depth of the footing
   (c) depends on the depth, but is independent of the width, of the footing
   (d) is independent of both the width and the depth of the footing

28. A soil has a low allowable bearing capacity. The soil deposit contains compressible loess. A foundation is to be provided for a structure carrying a heavy load. Which one of the following foundation types is to be adopted?
   (a) Strap footing
   (b) Continuous footing
   (c) Raft foundation
   (d) Combined spread foundation

29. Consider the following statements:
   Increasing width of footing results in
   1. increase in settlement of a consolidating clay layer
   2. increase in bearing capacity of sandy soils
   3. decrease in bearing capacity of clays

   Which of these statements is/are correct?
   (a) 1 only
   (b) 1 and 2 only
   (c) 2 and 3 only
   (d) 1, 2 and 3

30. A plate load test is carried out on a 300 mm × 300 mm plate placed at 2 m below the ground level to determine the bearing capacity of a 2 m × 2 m footing placed at same depth of 2 m on a homogeneous sand deposit extending 10 m below ground. The ground water table is 3 m below the ground level. Which of the following factors does not require a correction to the bearing capacity determined based on the load test?
   (a) Absence of the overburden pressure during the test
   (b) Size of the plate is much smaller than the footing size
   (c) Influence of the ground water table
   (d) Settlement is recorded only over a limited period of one or two days

31. Terzaghi's equation of ultimate bearing capacity for a footing may be used for square footing resting on pure clay soil with the correction factor
   (a) 0.4
   (b) 0.6
   (c) 1.2
   (d) 1.3

32. Match List-I (Bearing capacity terms) with List-II (Definition) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Ultimate bearing capacity	1. Net loading intensity at which neither soil fails in shear nor is there any excessive settlement
B. Net safe bearing capacity	2. The maximum pressure which soil can carry safely without risk of shear failure
C. Safe bearing capacity	3. Net ultimate bearing capacity divided by factor of safety
D. Allowable bearing pressure	4. Minimum gross pressure intensity at the base of foundation at which soil fails in shear

Codes:

	A	B	C	D
(a)	4	3	2	1
(b)	2	1	4	3
(c)	4	2	3	1
(d)	2	1	3	4

33. The determination of ultimate bearing capacity of an eccentrically loaded square footing depends upon the concept of useful

(a) square (b) width (c) triangle (d) circle

34. All theoretical approaches indicate that at greater depths the bearing capacity of pile base in sands should be practically independent of the size and be proportional to

(a) shape of pile (b) overburden pressure (σ)
(c) frictional angle (ϕ) (d) shape of pile and friction angle (ϕ)

35. The allowable bearing capacity at 25 mm allowable settlement for a footing in a sandy soil is 15 t/m^2. The allowable bearing capacity for the same footing permitting a settlement of 40 mm is

(a) 24 t/m^2 (b) 30 t/m^2 (c) 35 t/m^2 (d) 40 t/m^2

36. According to Terzaghi's theory, the ultimate bearing capacity of a strip footing with smooth base at ground level on a pure cohesive soil having cohesion 'c' will be

(a) 2.57c (b) 4.50c (c) 5.14c (d) 9.14c

37. A square footing is to be proposed on a cohesionless soil with an average N value of 40. The allowable bearing pressure of this footing will be governed by

(a) general shear failure (b) local shear failure
(c) progressive failure (d) settlement criteria

38. According to Skempton's formula for a surface footing of square shape, the ultimate bearing capacity on a purely cohesive soil of cohesion 'c' is

(a) 1.4c (b) 6.0c 0.6 c
(c) 7.4c (d) 9.0c

39. A 1 m × 1 m surface footing in a saturated clay soil with $\phi_u = 0°$ has the ultimate bearing capacity of 4q. The ultimate bearing capacity of a surface footing of dimensions 3 m × 3 m on the same soil will be

(a) $\frac{4q}{3}$ (b) 4q
(c) 4q $\cos^{-1}(1/3)$ (d) 4q $\sin^{-1}(1/3)$

40. A fully compensated raft foundation for a building is

(a) designed as very rigid raft
(b) designed as a completely flexible raft
(c) such that the weight of the excavated soil is equal to the load due to the building
(d) supported by piles of short length

41. Which one of the following equations represents Terzaghi's ultimate bearing capacity equation (Notations have their usual meaning)?

(a) $q_u = \gamma D_f \tan^4\left(45° + \frac{\phi}{2}\right)$

(b) $q_u = CN_c + \gamma D_f N_q + 0.5\gamma BN_\gamma$

(c) $q_u = CN_c S_c d_c i_c + D_f N_q S_q d_q i_q + 0.5\gamma\ BN_\gamma S_\gamma d_\gamma i_\gamma$

(d) $q_u = 5.14C + D$

42. Consider the following statements:
The safe bearing pressure for a raft on sand will be higher than that for an individual footing because

1. density of sand increases with depth
2. the permissible total settlement is twice that of a footing
3. differential settlement is less for a raft
4. raft is thicker than individual footings

Which of these statements are correct?
(a) 1 and 2 (b) 1 and 3
(c) 2 and 3 (d) 2 and 4

43. In Terzaghi's bearing capacity analysis, the soil wedge immeditely below the footing remains in a state of
(a) plastic equilibrium (b) radial shear
(c) elastic equilibrium (d) linear shear

44. Consider the following factors:
1. Geometrics of the footings
2. Settlement and its rate for foundation soil
3. Permeability of the soil
4. Shear parameters of the soil
5. Taylor's stability number

Which of these factors are to be taken into reckoning for assessing the allowable bearing capacity of a foundation soil?
(a) 1, 2 and 3 (b) 1, 2 and 4
(c) 2, 3 and 4 (d) 3, 4 and 5

45. Match List-I (Method) with List-II (Used for) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Meyerhof's analysis	1. Earth pressure
B. Newmark's chart	2. Stability analysis
C. Bishop's analysis	3. Seepage analysis
D. Rebhann's construction	4. Bearing capacity
	5. Vertical stress

Codes:

	A	B	C	D
(a)	4	1	2	5
(b)	2	5	3	1
(c)	4	5	2	1
(d)	2	1	3	5

46. Plate load test is useful to estimate
(a) bearing capacity of foundation (b) settlement of foundation
(c) both bearing capacity and settlement (d) depth of foundation

47. A c-ϕ soil has failed in local shear. Which one of the following pairs of shear parameters must be used to evaluate the bearing pressure of the soil?

(a) $c_m = c$ and $\phi_m = \frac{2}{3}\phi$
(b) $c_m = \frac{2}{3}c$ and $\phi_m = \frac{2}{3}\phi$
(c) $c_m = \frac{2}{3}c$ and $\phi_m = \phi$
(d) $c_m = \frac{2}{3}c$ and $\phi_m = \tan^{-1}(\frac{2}{3}\tan\phi)$

48. What will be the immediate settlement of a column footing 1.5 m diameter that is constructed upon an unsaturated clay layer? Given that total load carried by the column is 150 kN; modulus of elasticity of the soil = 7000 kPa; Poisson's ratio = 0.25; Assume the footing to be rigid and influence factor I_f as 0.79

(a) 13.5 mm
(b) 27 mm
(c) 54 mm
(d) 135 mm

49. Consider the following statements:
1. Hansen's equation accounts for influence of depth, shape and inclination of load.
2. Local shear failure takes place in dense and/or very dense sand and also in stiff and hard clays.
3. The region of failure of soil below foundations consists of three different zones.

Which of these statements are correct?

(a) 1, 2 and 3
(b) 1 and 2
(c) 2 and 3
(d) 1 and 3

50. On which of the following do the numerical values of Terzaghi's bearing capacity factors depend?
(a) Angle of internal friction of soil and depth of foundation
(b) Angle of internal friction of soil only
(c) Coefficient of curvature of soil and bulk density of soil
(d) Uniformity coefficient of soil and dry density of soil

51. A raft foundation with a basement floor is placed at a depth of 4 m below ground level. The superstructure imposes a load of 150 kN/m^2 on the raft. The unit weight of the soil is 20 kN/m^3. What are the values of the gross and net loading pressures on the soil, respectively?

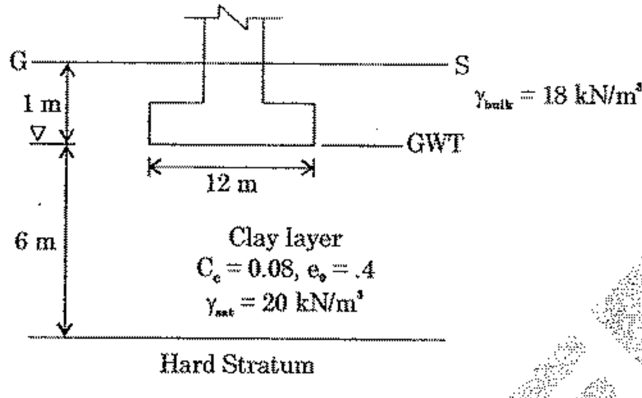
(a) 230 kN/m^2, 150 kN/m^2
(b) 150 kN/m^2, 230 kN/m^2
(c) 150 kN/m^2, 70 kn/m^2
(d) 80 kN/m^2, 150 kN/m^2

52. The rise of water table below a foundation influences the bearing capacity of the foundation soil by decreasing its
(a) cohesion
(b) effective unit weight of soil
(c) effective angle of shearing resistance and effective unit weight of soil
(d) effective angle of shearing resistance

53. Permissible settlement is relatively higher for which of the following?
(a) Isolated footings on clays
(b) Isolated footings on sands
(c) Rafts on clays
(d) Rafts on sands

Common Data for Questions 54 and 58 :

Figure shows the geometry of a strip footing supporting the load bearing walls of a three storied building and the properties of clay layer.



54. If the pressure acting on the footing is 40 kPa, the consolidation settlement of the footing will be
(a) 0.89 mm (b) 8.9 mm (c) 89.0 mm (d) None of these

55. If the elastic modulus and the Poisson's ratio of the clay layer are respectively 50×10^3 kPa and 0.4 and if the influence factor for the strip footing is 1.75, the elastic settlement of the footing will be
(a) 0.41 mm (b) 1.41 mm (c) 14.1 mm (d) None of these

56. Two circular footings of diameters D_1 and D_2 are resting on the surface of the same purely cohesive soil. The ratio of their gross ultimate bearing capacities is..........

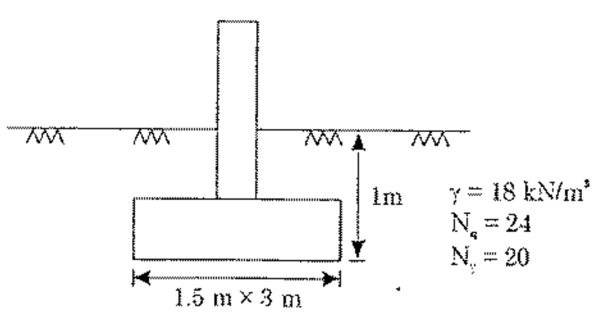
(a) $\frac{D_1}{D_2}$ (b) 1.0 (c) $\left(\frac{D_1}{D_2}\right)^2$ (d) $\frac{D_2}{D_1}$

57. A strip footing (8 m wide) is designed for a total settlement of 40 mm. The safe bearing capacity (shear) was 150 kN/m² and safe allowable soil pressure was 100 kN/m². Due to importance of the structure, now the footing is to be redesigned for total settlement of 25 mm. The new width of the footing will be
(a) 5 m (b) 8 m (c) 12 m (d) 12.8 m

58. A test plate 30 cm × 30 cm resting on a sand deposit settles by 10 mm under a certain loading intensity. A footing 150 cm × 200 cm resting on the same sand deposit and loaded to the same load intensity settles by
(a) 2.0 mm (b) 27.8 mm (c) 30.2 mm (d) 50.0 mm

Statement for Linked Answer Questions 59 and 61 :

A column is supported on a footing as shown in the figure below. The water table is at a depth of 10 m below the base of the footing.



59. The net ultimate bearing capacity (kN/m^2) of the footing based on Terzaghi's bearing capacity equation is
(a) 216 (b) 432 (c) 630 (d) 846

60. The safe load (kN) that the footing can carry with a factor of safety 3 is
(a) 282 (b) 648 (c) 945 (d) 1269

Instructions :

The following items consists of two statements, one labelled as ***'Assertion A'*** *and the other labelled as* ***'Reason R'****. You are to examine these two statements carefully and decide if the Assertion A and the Reason R are individually true and if so, whether the Reason is a correct explanation of the Assertion. Select your answers to the these items using the codes given below :*

(a) both A and R are true and R is the correct explanation of A
(b) both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

61. Assertion (A): Terzaghi's bearing capacity theory overestimates the bearing capacity of footings.
Reason (R): Terzaghi's theory neglects the shearing resistance of rupture surface in the soil above the foundation level.

62. Assertion (A): Raft foundations are the best type of shallow foundations to support heavy structures.
Reason (R): Raft foundations has the ability to redistribute load coming on weak pockets of soil below raft to the adjacent soils where the stresses are less severe for the soil at that place. This reduces the chances of differential settlements and increases the bearing capacity.

63. Assertion (A): Plate load test carried out at the site gives field test data which is useful in evaluation of bearing capacity and settlements. It is normally conducted at the level of the proposed foundation.
Reason (R): Plate load test is reliable because it reflects the true behaviour of foundation stratum below the proposed level of foundation and extending up to large depth below.

64. Assertion (A): The bearing capacity of a footing always gets affected by the location of ground water table.
Reason (R): Water in soil affects the shear strength parameters as well as the unit weight.

65. Assertion (A): In case of sands, the position of the water table is of great practical importance in the determination of bearing capacity of a footing on sand.
Reason (R): The unit weights of most sands, whether dry, moist or saturated, lie within a narrow range of values.

66. Assertion (A): The ultimate bearing capacity of a non-cohesive soil increases considerably with depth below ground level.
Reason (R): In cohesive soil, the ultimate bearing capacity is independent of foundation width.

67. Consider the following statements:
Assertion (A): Presence of water table in the vicinity of a foundation causes reduction in its load carrying capacity.
Reason (R): Presence of water table results in reduction of the effective unit weight of soil.

ANSWERS

1. (d)	2. (c)	3. (d)	4. (a)	5. (c)	6. (a)	7. (b)	8. (c)	9. (b)	10. (a)
11. (c)	12. (b)	13. (b)	14. (c)	15. (d)	16. (c)	17. (a)	18. (d)	19. (b)	20. (a)
21. (b)	22. (c)	23. (a)	24. (d)	25. (d)	26. (a)	27. (d)	28. (c)	29. (b)	30. (c)
31. (d)	2. (a)	33. (b)	34. (c)	35. (a)	36. (c)	37. (a)	38. (b)	39. (b)	40. (c)
41. (b)	42. (b)	43. (c)	44. (b)	45. (c)	46. (c)	47. (d)	48. (a)	49. (d)	50. (b)
51. (d)	52. (c)	53. (c)	54. (c)	55. (b)	56. (b)	57. (a)	58. (b)	59. (c)	60. (c)
61. (a)	62. (a)	63. (c)	64. (d)	65. (b)	66. (b)	67. (a)			

Hints

2.

$$q_{nu} = cNc$$

$$C = \frac{q_u}{2} = 50\text{kN/m}^2$$

$$N_c = 5\left[1+0.2\frac{DF}{B}\right]\left[1+0.22\frac{B}{L}\right]$$

$$= 5\left[1+0.2\times\frac{2}{1}\right]\left[1+0.2\frac{1}{2}\right]$$

$$= 5[1.4][1.1] = 7.7$$

$$q_{nu} = 50\times 7.7 = 385\ \text{kN/m}^2$$

9.

$$q_{nu} = cN_c = 10\times 5.7 = 57\ \text{kN/m}^2$$

10.

$$q_{nu} = \left(1+0.3\frac{B}{L}\right)cN_c + q\,(N_q-1)$$

For clay $N_c = 5.7$; $N_q = 1$

$$\therefore\ q_{nu} = \left(1+0.3\frac{B}{L}\right)cN_c$$

$$= \left(1+0.3\times\frac{6}{9}\right)\times 120\times 5.7$$

$$= 1.2\times 120\times 5.7 = 820.8$$

15.

$$\frac{S_f}{Sp} = \left(\frac{B_f}{Bp}\times\frac{Bp+0.3}{B_f+0.3}\right)^2$$

$$S_f = 0.005\times\left[\frac{3}{.6}\times\frac{.6+0.3}{3+.3}\right]^2$$

$$S_f = 0.005\left[\frac{3}{.6}\times\frac{.9}{3.3}\right]^2$$

$$S_f = 0.009\ \text{m} = 9\ \text{mm}$$

16.

$$q_{nu} = 5.7\,C = 5.7\times 25 = 142.5$$

19.

$$\frac{q_{uf}}{q_{up}} = \frac{B_f}{Bp}$$

$$q_{uf} = 60\times\frac{1.2}{0.6} = 120$$

23.

$$\frac{S_f}{S_p} = \left[\frac{B_f}{B_p}\left(\frac{B_p+0.3}{B_f+0.3}\right)\right]^2$$

$$S_f = 12\times\left[\frac{2}{.3}\left(\frac{0.6}{2.3}\right)\right]^2$$

$$S_f = 36.3\ \text{mm}$$

48.

$$S_i = \frac{qB\left(1-\mu^2\right)}{E}If$$

$$q = \frac{Q}{\left(\frac{\tau}{\gamma}d^2\right)} = 150$$

$$= 84.88$$

$$S_i = \frac{84.88\times 1.5\left(1-25^2\right)}{7000} = 13.5\text{mm}$$

51. $q_g = 150 + 20 \times 4 = 230$

$q_{net} = 150$

New width of footing $= \left(\frac{8 \times 62.5}{100}\right) = 5\,\text{m}$

54. $\Delta H = \frac{C_c H_0}{1 + e_0} \log_{10}\left(\frac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_0}\right)$

$\bar{\sigma}_0 = (18 \times 1) + (20 - 10) \times 3 = 48\ \text{kNm}^3$

$\Delta\bar{\sigma} = 40$

$$\Delta H = \frac{0.08 \times 6}{(1 + .4)} \log_{10}\left(\frac{88}{48}\right) = 0.090\,\text{m} = 90\ \text{mm}$$

55. $S_i = \frac{qB(1 - \mu^2)}{E} I_f$

$$= \frac{40 \times 1.2\left(1 - (0.4)^2\right) \times 1}{50 \times 10^3} = 1.41\ \text{mm}$$

57. $q_{ap} = \frac{25}{40} \times 100 = 62.5\ \text{kN/m}^2$

58. $\frac{s_p}{s_f} = \left[\frac{B_p(B_f + 0.3)}{B_f(B_p + 0.3)}\right]^2$

$$\therefore 10 \times \left[\frac{1.5(0.3 + 0.3)}{0.3(1.5 + 0.3)}\right]^2 = 27.8\ \text{m}$$

59. For c = 0 and $\phi > 0$

$q_u = \left(0.4B\gamma N_\gamma + \gamma D N_q\right)$

$= (0.4 \times 18 \times 1.5 \times 20 + 18 \times 1 \times 24)$

$= 648\ \text{kN/m}^2$

$q_{net} = \left(q_u - \gamma D_f\right) = (648 - 18 \times 1) = 630\,\text{kN/m}^2$

60. Safe load $= \frac{630}{3}(1.5 \times 3) = 945\,\text{kN}$

CHAPTER

13

Deep Foundation

INTRODUCTION

- In situations where soil at shallow depth is poor, inorder to transmit load safely, the depth of foundation has to be increased till the suitable soil strata is met. Inview of increased depth, such foundations are called Deep foundation, Well foundation, Pile Foundation and Pier Foundation are Deep foundations.
- Pile is a small dia shaft which can be driven or installed into ground, Where as Piers and well Foundation are large dia shafts, constructed by excavation and sunk to the required depth.

CLASSIFICATION OF PILE

Piles are classified in a number of ways based on different criteria

1. Material and composition
2. Mode of transfer of load
3. Method of installation
4. Function or Action
5. Displacement of Soil

(1) Classification Based on Material & Composition.

(a) Timber piles

(b) Steel piles

(c) Concrete piles

(d) **Composite piles** : These may be either of concrete and timber or concrete and steel. Composit piles are rarely used in practice as it is difficult to provide joint between two different materials.

(2) Classification based on mode of transfer of load.

(a) End-bearing piles

- Used to transfer load through the pile tip to a suitable bearing stratum, passing soft soil or water.

(b) Friction piles

- Used to transfer loads to a depth in a frictional material by means of skin friction along the surface area of the pile.
- Friction piles are also called as Floating piles, as they do not reach the hard stratum.

(c) Combined End bearing and Friction pile.

- Used to transfer load through the combine action of end bearing and friction along the surface area of pile.

(3) Classification based on Metbod of Installation.

(a) Driven piles

- Timber, steel, or precast concrete piles may be driven into position either vertically or at an inclination.
- If they are inclined then they are called as "Batter" or "Raking" piles.
- Pile hammers and pile-driving equipment are used for driving piles.

(b) Bored and Cast-in-situ piles

- Only concrete piles can be cast-in-situ. As the Holes are drilled and filled with concrete.
- These are straight-bored piles or 'under-reamed' with one or more bulbs at intervals. Reinforcements is used according to the requirements.

(c) Driven and cast-in-situ piles.

- A closed end casing or a shell is driven into the ground. Later casing is filled with concrete. eg. Franki piles.

(d) Jack piles

- These are piles diven into soil by means of a hydraullic jack.

(e) Screw piles

- These piles are screwed into the soil by means of hydraullic jack

4. Classification Based on Function or Action

(a) Load Bearing Piles

- Used to transfer the load of the structure to a suitable stratum by end bearing, by friction or by both.

(b) Tension or uplift piles

- Used to anchor structures subjected to uplift due to hydrostatic pressure or overturning moment due to horizontal forces. [Friction gets mobiled in downward direction in Tension.]

(c) Compaction piles

- Used to compact loose granular soils in order to increase the bearing capacity.
- As they are not required to carry any load, the material required may be strong; in fact, sand may be used to form the pile.
- Pile tube, driven to compact the soil, is gradually taken out and sand is filled in its place and forming a required pile called as a 'sand pile'.

(d) Anchor piles

- Used to provide anchorage against horizontal pull from water or sheet piling.

(e) Fender piles

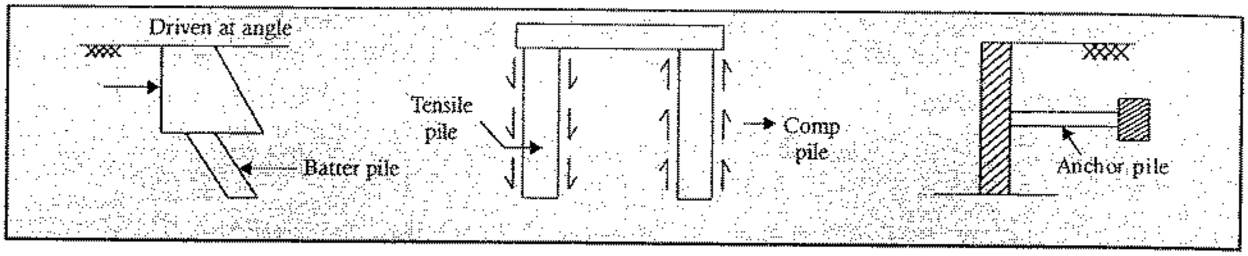
- Used to protect water-front structures against impact from ships and other floating objects.

(f) Sheet piles

- Used as bulkheads, or cut-offs to reduce seepage and uplift in hydraulic structures.

(g) Batter piles

- Used to resist horizontal and inclined forces, specially in water front structures.

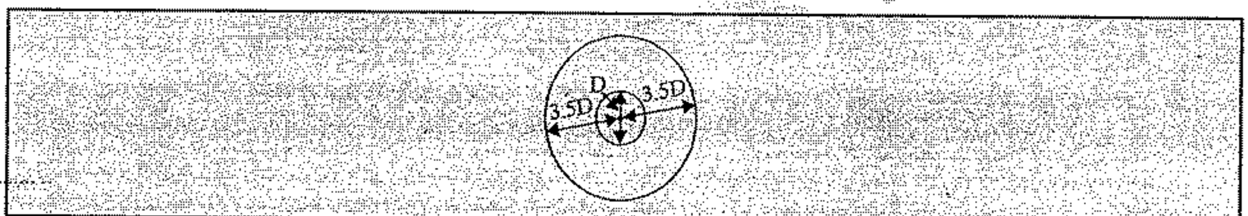


5. Classification Based on Displacement of soil.

- It has been observed that the best way of classification is on the basis of effect of installation on the soil. Accordingly we have:

(a) Displacement pile

- If during installation a large volume of soil is displaced laterally or upward is called displacement pile.
- In loose sand such a pile densifies the sand upto a distance of 3.5 times the diameter of pile measured from the centre of pile. This compaction leads to increase in the shearing resistance within the zone of influence.



- In case of clays, large displacement piles remoulds the soil to a distance of (2 × diameter of pile).
- During pile driving, High pore water pressures are setup around the pile. Soil regains its strength only after a period of time when excess pore water pressure has dissipated.
- Driven piles are preferred in loose to medium sand and are less prefered in case of clays & Dense sand. Example of large displacement piles are :

  (i) Driven cast-in-situ piles. (ii) Driven prestressed piles.

  (iii) Steel piles. (iv) Timber piles etc.
- Rolled steel section piles, screw piles and open ended Hollow sections piles are examples of **Small displacement piles.** Small displacement piles are used when ground displacement and ground disturbances are to be minimised;

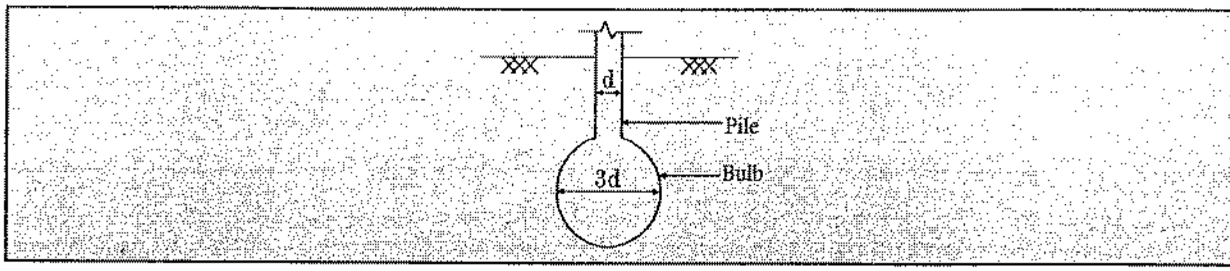
(b) Non-displacement pile

- No displacement of soil during installation.
- In such piles voids are formed in the soil by boring or excavation, and then these voids are filled with concrete.
- Sides of the void can be supported either permanently by casing or temporarily by using Bentonite slurry.
- Non displacement piles can be further classified as:

  (i) Bored cast-in-situ.

  (ii) Bored precast.
- There is no heaving of ground, no noise and no vibration in this case.
- Length of these piles can be easily varied at site and very long and large dia pile can be installed.

- Enlarged end upto 3-times of pile diameter can be made in case of clays.



- Construction process enables inspection of the excavated soil and its comparasion with soil exploration data.
- During the installation of pile, soil on the sides of bore hole softens due to contact with water used during boring **or concreting.** It results in loss of shear strength (Obviously temporarily).
- There is difficulty in concreting under water.
- Pile should be casted or installed immediately after boring.

Note: **Driven concrete piles** are generally of diameter upto **500 mm.** But **Bored piles** may even be 2-3 m.

PILE LOAD CAPACITY IN COMPRESSION

- General requirement for satisfactory performance of pile foundation are:
  (a) Safety against shear failure.
  (b) Safety against settlement.
- Load capacity of pile can be estimated by several methods:
  (1) Static pile load formula.
  (2) Pile load test.
  (3) Pile driving formula.
  (4) Correlation with penetration data.

(A) Single pile load capacity

(1) Static pile load capacity

- When a compersive load (P) is applied at the top of pile, the pile will tend to move vertically downward relative to soil. Due to this shear or friction developes between soil and surface of shaft. As a result applied load is distributed as frictional load along certain length of pile.
- As load is increased full frictional resistance is mobilised over complete length of pile but by that time point bearing resistance will be very less.
- When full point bearing resistance is mobilised, the frictional stress will drop from its maximum value.
- The maximum frictional resistance mobilised is called (Q_f) when the load exceeds (Q_f), the point bearing starts mobilising. This load is known as point load (at base). This point load goes on increasing till failure occurs by punching shear.
- Load in bearing at this stage is called **ultimate point load** (Q_{pu}).
- Hence, for calculation purpose, we take Ultimate load Q_u $\boxed{Q_u = (Q_{pu} + Q_f)}$, although this is not correct because when maximum point bearing is developed, friction reduced from its maximum value.

  (i) If $[Q_{P_u} >>> Q_F]$ Pile is called **Point Bearing pile.**

  (ii) If $[Q_{P_u} <<< Q_F]$ Friction pile

- For full mobilisation of **friction, Relative moment** required is [0.5 – 1]% of Dia of pile.
- For full point Bearing mobilisation, relative moment required is = (10 – 20)% of diameter of pile.
- Also it has been observed that, when ultimate skin frictional resistance is mobilised, only a fraction of ultimate point bearing will get mobilised and when ultimate point bearing is mobilised, only a fraction of ultimate friction is mobilised.
- General equation for Point Bearing Resistance, q_{pu} for a C-ϕ soil may be written as below.

$$\boxed{q_{pu} = CN_c + \bar{\sigma} N_q + 0.5 B\gamma N_\gamma}$$

Where,

B = width or diameter of the pile

$\bar{\sigma}$ = effective overburden presure at the tip of the pile, equal to γL

N_c, N_q and N_γ = bearing capacity factors.

C_c = unit cohesion

B = Diameter of pile

γ = effective unit weight of soil

- In deep foundation, it has been observed that value of $0.5\,\gamma BN_\gamma$ is very less in comparission to the value of $\bar{\sigma} N_q$. Therefore we can neglect this term. and the equation for ultimate point load for C– ϕ soil can be written as below.

$$\boxed{q_{pu} = cN_c + \bar{\sigma}\, N_q}$$

For Granular soil, C = 0

$$\therefore \quad \boxed{q_{pu} = \bar{\sigma}\, N_q}$$

For pure clay soil $\phi = 0$

$$\boxed{q_{pu} = CNc} \qquad (\text{As } N_q = 0 \text{ for pure clay})$$

- The ultimate point load can be expressed as

$$\ldots\ldots\ldots \quad \boxed{Q_{pu} = q_{pu} \cdot A_b}$$

where, q_{pu} = Unit point bearing resistance

A_b = Cross sectional area of pile as its base.

- The ultimate skin friction resistance, Q_F can be expressed as.

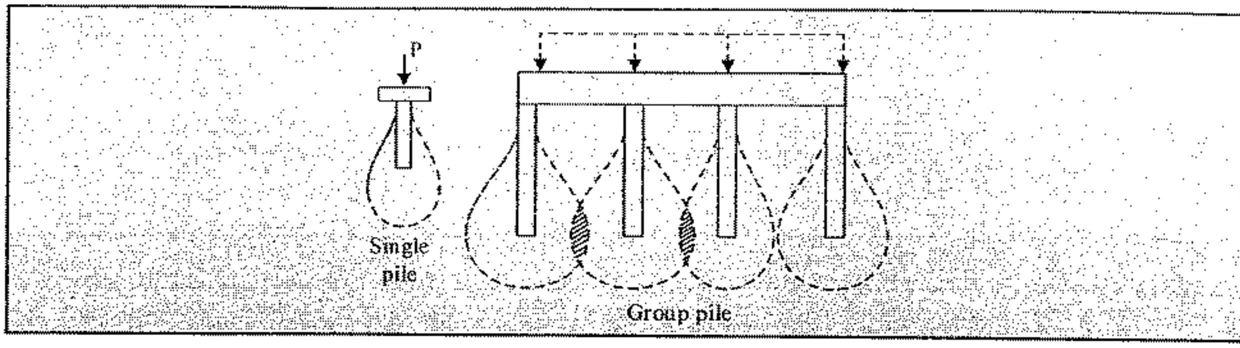
$$\boxed{Q_F = F_S \cdot A_S}$$

where, F_S = Unit skin friction resistance.

A_S = Surface area of pile in contact with soil.

- Therefore the ultimate load capacity Qu, can be expressed sum of both ultimate point load and ultimate skin friction resistance

$$Q_u = q_{pu} A_b + F_S A_S$$



PILES IN GRANULAR SOILS (SAND & GRAND)

- As we have observed above that ultimate load capacity for a single pile in granular soil is given as

$$Q_u = q_{up} \cdot A_b + F_S \cdot A_S$$

Driven Pile

1. Point bearing resistance.

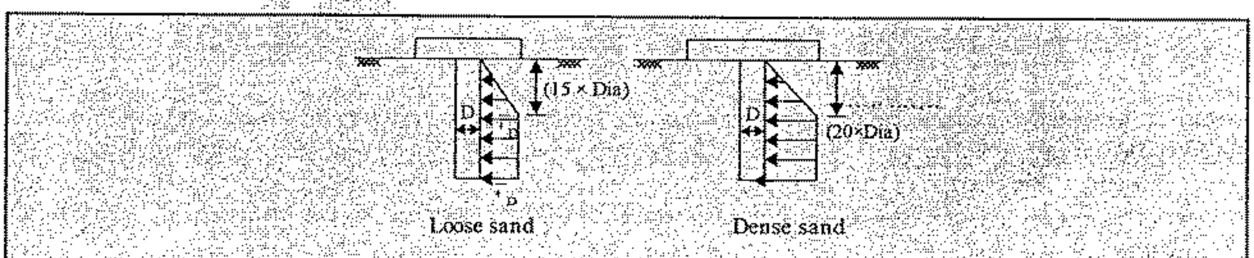
   We know that

$$\boxed{q_{pu} = \bar{\sigma} \cdot N_q}$$

where, $\bar{\sigma}$ = effective stress at the base of the pile.

N_q = Given by I.S. code as per (ϕ)-values.

- It has been observed that the value of effective stress increases up to a certain depth of sand, after which the increase in value is insignificant i.e. after certain depth it is assumed to be constant.
- Hence in case of sand the value of $\bar{\sigma}$ is taken corresponding to depth.
- It should be noted that this phenomenon is only accounted for sand.
  (i) (15 × Diameter pile)—for **Loose to Medium sand**
  (ii) (20 × Dia)—for **Dense sand.**



Note : *As per IS code q_{pu} is given as below*

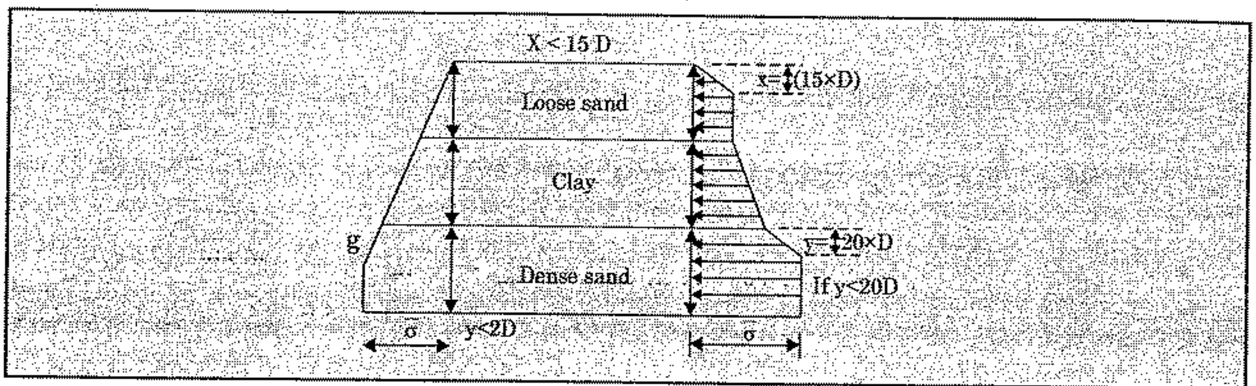
$$\boxed{q_{pu} = \bar{\sigma}\, Nq + \frac{1}{2} B\gamma N_\gamma}$$

where N_γ is taken corresponding to general shear failure.

- However for calculation purpose in design of foundation we neglect the value of $\frac{1}{2}$ BγN_γ as its value is very less in comparission to $\bar{\sigma} N_q$.

$$\boxed{Q_{pu} = (q_{pu} \times A_{base})}$$

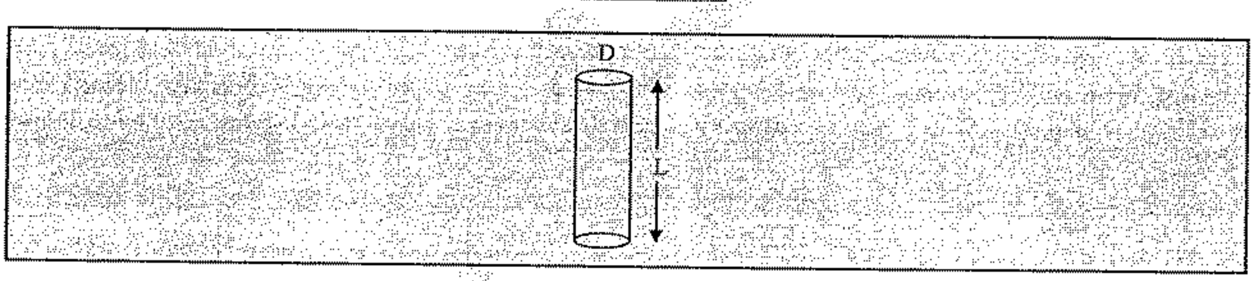
$$\text{F.O.S.} = 2.5 \text{ for } Q_f$$



Skin Friction (For Granular Soil).

- As discussed above the ultimate frictional resistance Q_f is given as below.

$$\boxed{Q_f = f_s \cdot A_s}$$



where, A_s = Surface Area of pile shaft = πDL

f_s = Frictional Resistance = μ.N.

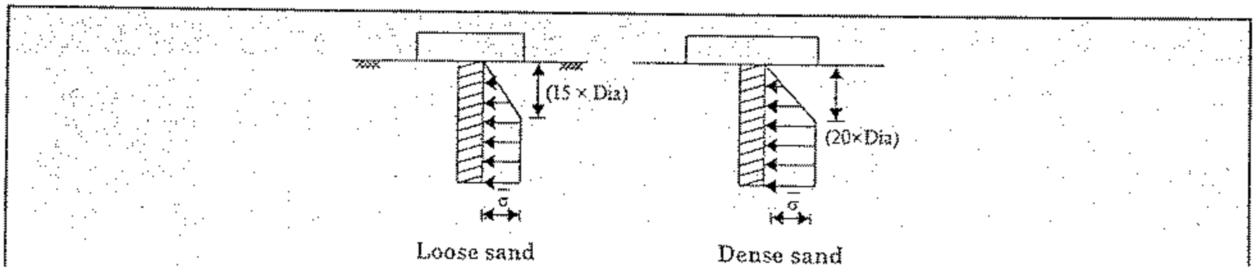
N = Lateral earth pressure at any time

N = K$\bar{\sigma}$

μ = Coefficient of friction between soil and pile material.

μ = tan δ

- δ is friction angle between soil & pile. If it would have been between soil & soil, then we would have taken $\delta = \phi$.
- Thus, f_s = K$\bar{\sigma}$ tan δ.
- But we know that $\bar{\sigma}$ is not constant throughout the depth of embeded pile length.



- Now utimate skin frictional resistance will be given as below :

$$\boxed{Q_F = (K.\bar{\sigma}_{av}\tan\delta).A_s}$$

- According to IS code $\delta = \phi$ and $K = 1 - 3$ for driven piles in loose to medium sands.
- Few values of K & δ are also given by few scientist as per the material of pile driven in loose and dense sand.

		Values of K	
Pilematerial	δ	Loose	Dense
(1) Steel	20°	0.5	1.0
(2) Concrete	$\frac{3}{4}\phi$	1	2
(3) Timber	$\frac{2}{3}\phi$	1.5	4.0

Note : δ can also be calculated as δ average over the depth of pile which will be taken as weighted average.

- In that case. Total frictional resistance will be calculated as:

$$Q_f = K.\tan(\delta)\left[\sigma_{avg1}.A_1 + \sigma_{avg2}.A_2 + ...\right]$$

Bored cast In-Situ pile

- Due to boring condition, the value of (K) is very small. Hence, Load carrying capacity is very small as compared to driven pile.
- (K) is usually taken as (0.5) for calculation. Although the range is [0.3 – 0.75) and $\delta = \phi$.
- Rest of calculations will be done as driven piles.
- It has been observed that
- Point Bearing Resistance of Board cast insitu piles. $= \left[\frac{1}{2}\times \text{ Point bearing resistance of driven pile}\right]$
- Factor of Saftey taken is = 2.5

Note: (1) Maximum upper limit of **frictional stress** (f_s) is = 110 kN/m^2.

(2) Max. upper limit of point **bearing stress:**

(*a*) **11000 kN/m^2** → for **Silica sand**

(*b*) **5000 kN/m^2** → for **calcarious** sand

(3) Any value more than this can be neglected in design.

PILES IN CLAYS

- Piles in clays generally carry most of the load by virtue of **skin friction.**

We know that

- $$\boxed{Q_u = \underbrace{q_{up}.A_b}_{\text{Point load}} + \underbrace{f_s.A_s}_{\text{Friction}}}$$

- In case of clays.

$$q_{up} = C_{ub} \cdot Nc \ \& \ f_s = \alpha C_u$$

$$\therefore \quad \boxed{Q_u = \left(C_{u\,base} \times N_c\right).A_b + \left(\alpha.C_u\right).A_s}$$

Where, α = Adhesion factor

C_u = Undrained cohesion in the embeded length of pile.

C_{ub} = Undrained cohesion at the base of the pile.

N_c = 9 [As per skempton for deep foundation]

α = depends on the adhesion between soil and pile.

α = adhesion factor, α depends upon the adhesion between soil and pile and undrained shear strength of pile.

- Smaller the undrained strength, softer is the consistency of the soil. Hence, greater is the tendency to adhere to the pile.

$$\alpha = 1 \rightarrow \text{for very loose clays}$$
$$\alpha = 0.3 \rightarrow \text{for very stiff clays.}$$

- When pile is driven in clay stratum, the soil gets remoulded and looses its strength. However, if left for sometimes it regains its strength.
- At least 30 days should be elapsed after the driving of pile before it is loaded so that the pile developes the full frictional resistance.
- Static load formula may be used only as the guide for Q_{up} estimate. F.O.S = 2.5 should be used to calculate Q_{up} for piles in clay.
- More reliance is based on pile load test.

Example 1.

A 12 m long, 300 mm diameter concrete pile is driven in a uniform deposit of **dense sand.** Water Table is at great depth and is not likely to rise. The avg. dry unit wt. of sand is **18 kN/m³.** Use N_q = 137, Calculate the safe load capacity of a single pile with a F.O.S. of 2.5, $\phi = 40°$.

Sol. (1) Driven/Bored Driven pile

(2) Sand/clay → Dense sand

So, (1) Driven pile

(2) Pile is in dense sand.

Here, in case of pile we only work for $P_{ultimate}$ and not for $P_{net\ ultimate}$.

$$\boxed{q_{pu} = 0 + \bar{\sigma}.N_q + 0.5\,B.\gamma.N_\gamma}$$

Here, 3rd term is negligible. Hence, it is neglected.

$$\therefore \quad \boxed{q_{pu} = 0 + \bar{\sigma}.N_q}$$

Calculation for $\bar{\sigma}$

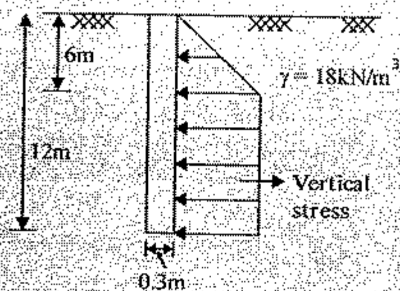
we know that, in case of dense sands $\bar{\sigma}$ is calculated at a depth of 20 × D.

$$\because \quad 20 \times D = 20 \times 0.3 = 6 \text{ cm}$$

$$\therefore \quad \bar{\sigma} = \gamma \times 20 \times D$$
$$= 18 \times 20 \times 0.3 = 108 \text{ kN/m}^2$$

$$Q_{pu} = q_{pu} \cdot A_b$$

$$\therefore \quad Q_{pu} = [0 + 108 \times 137] \times \frac{\pi}{4}(0.3)^2$$
$$= 1045.86 \text{ kN}$$



Also, $Q_f = f_s \cdot A_s$

where, $f_s = k.\bar{\sigma}_{avg} \tan(\delta)$

Here $Q_f = f_{s1}.A_1 + f_{s2}.A_2$

$k = 2$

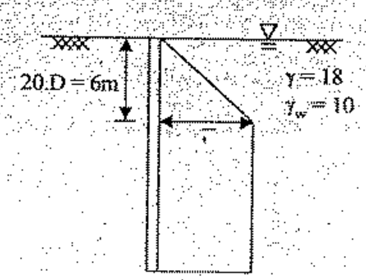
$$\delta = \left(\frac{3}{4}\phi\right) = \frac{3}{4} \times 40 = \boxed{30°}$$

$$Q_f = 2.\tan 30° \left[\left(\frac{108+0}{2} \times \pi \times 0.3 \times 6\right) + (108 \times \pi \times 0.3 \times 6)\right]$$
$$= 1057.807 \text{ kN}$$
$$Q_{up} = 1057.807 + 1045.86 \text{ kN}$$
$$= 2103.667 \text{ kN}$$
$$Q_{safe} = \left[\frac{Q_{up}}{F.O.S.}\right] = 841.452 \text{ kN}$$

Exampl 2.

For **50 mm** side square section concrete pile **15 m** long is driven in a deep deposite of uniform clay. The laboratory unconfined compression test on undisturbed sample indicates an avg. value of (Q_u) unconfined compresive Strength = 75 kN/m². Calculate the ultimate load capacity of pile. Take $\alpha = 0.8$.

Sol.



$$\bar{\sigma} = (18 \times 6 - 10 \times 6)$$
$$= 48 \text{ kN/m}^2$$

$$Q_u = \underbrace{C_{ub}}_{37.5\ Kn/m^2} \cdot \underbrace{N_C}_{9} \cdot \underbrace{A_b}_{0.45\times0.45} + \underbrace{\alpha}_{0.8} \cdot \underbrace{C_{uavg}}_{\frac{q_u}{2}} \cdot \underbrace{A_S}_{0.45\times4\times15}$$

$$C_{uavg} = \frac{75}{2}$$

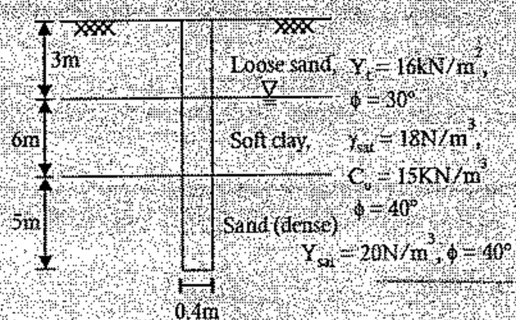
$$= 37.5 = (\text{Perimeter} \times \text{Length})$$

Here, (C_{ub}) at the base of pile is not given. Hence, avg. values will be taken as (C_{ub}).

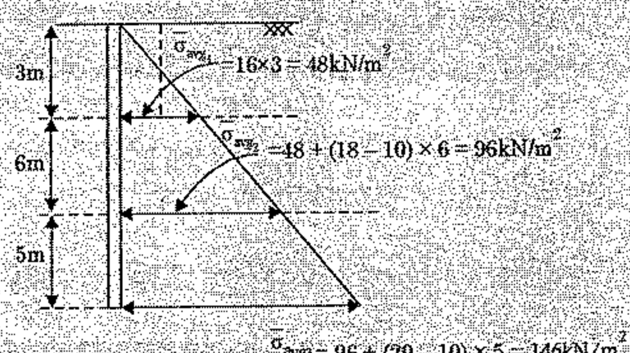
$$Q_u = 878.34 \text{ kN}$$

Example 3.

Determine the allowable pile load capacity of 40 cm dia driven concrete pile as shown below. Take $N_2 = 160$.



Sol.



- Critical depth for loose sand $= 15D = (15\times0.4) = 6$m
- As depth of loose sand < 6 m, full depth of loose sand will be effective force $(\bar{\sigma})$

Check:

- For dense sand, critical depth = (20D) $= (20\times0.4) = 8m$
- But depth of dense sand layer < 8 m. ∴ Full depth of dense sand will contribute to $(\bar{\sigma})$.

$$\bar{\sigma}_{av_1} = \left(\frac{48+0}{2}\right) = 24 \text{ kN/m}^2$$

$$\sigma_{av_3} = \frac{96+146}{2} = 121 \text{ kN/m}^2$$

$$Q = (Q \; + Q)$$

$$Q_u = Q_{pu} + Q_f$$
$$= \bar{\sigma} Nq.\frac{\pi}{4}(0.4)^2 + [(K_1.\bar{\sigma}_{avi}.\tan\delta_1 \times \pi \times 0.4 \times 3)$$
$$(\alpha C_u.\pi(0.4)\times 6) + K_3\bar{\sigma}_{avg1}.\tan(\delta_3).\pi \times 0.4 \times 3]$$
$$Q_u = 3963.97 \text{ kN}$$

We know that, $\quad F.O.S. = 2.5$

$\therefore \quad Q_{allowable} = \left(\dfrac{3963.97}{2.5}\right) = 1585.58 \text{ kN}$

Example 4.

Precast concrete pile of size 50 cm × 50 cm is to be driven into clay strata. The unconfined compressive strength of which is **110 kN/m²**. Compute the length of pile required to carry safe working load of **450 kN** with F.O.S. = 2.5, $\alpha = 0.6$.

Sol. Soil is clay

$\therefore$ Point bearing

$$Q_u = C_{ub}.N_c.A_b + \sigma.C_{uav}.A_s$$
$$Q_u = 450 \times F.O.S. = (450 \times 2.5)$$
$$(450 \times 2.5) = \left(\frac{110}{2}\right)\times 9 \times (0.5)^2 + 0.6\left(\frac{110}{2}\right)\times 4 \times 0.5 \times L$$

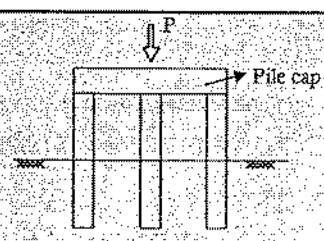
On solving the above equation we get,

$$L = 15.2 \text{ m}$$

GROUP ACTION OF PILES

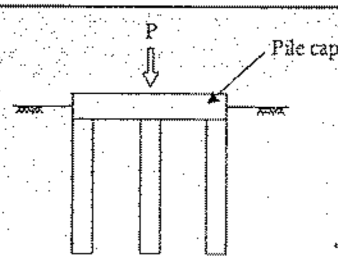
- Piles are always used in group.
- When single pile is used as a driven pile, there is uncertainty regarding vertical installation of piles. Hence, in case of driven piles, minimum number of piles used are three (3).
- Whereas in case of bored piles verticality can be ensured. Hence, single piles can also be used. However we always provide group of piles.

Free standing pile group



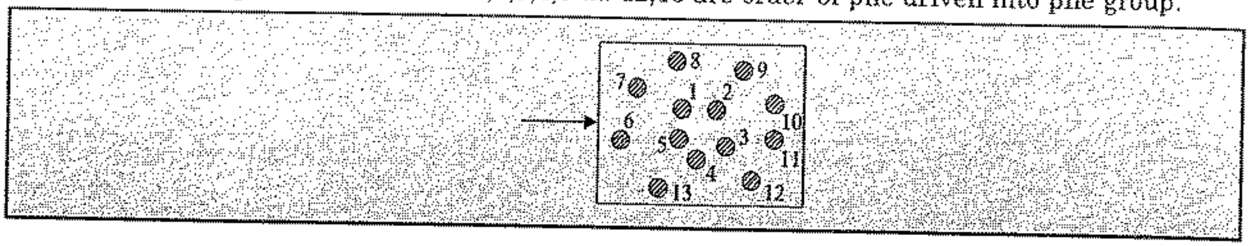
Pile cap above the ground

Pile foundation



Pile cap Below/on the ground

- In case of expanding soil, at top layer, **Free standing pile group** is used.
- To avoid ground tightening, pile in sand should **begin at centre** and **moved outward.**
- In the diagram shown below 1,2,3,4,5 .... 12,13 are order of pile driven into pile group.



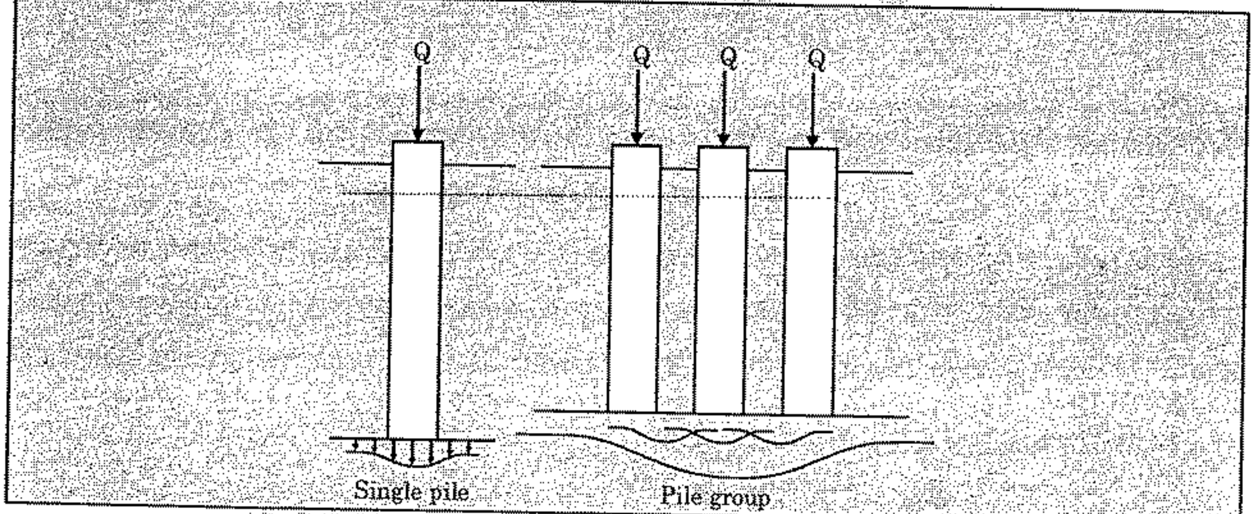
ULTIMATE LOAD CAPACITY OF PILE GROUP :

Group efficiency = $\boxed{\eta = \dfrac{Q_{ug}}{n\,Q_u}}$

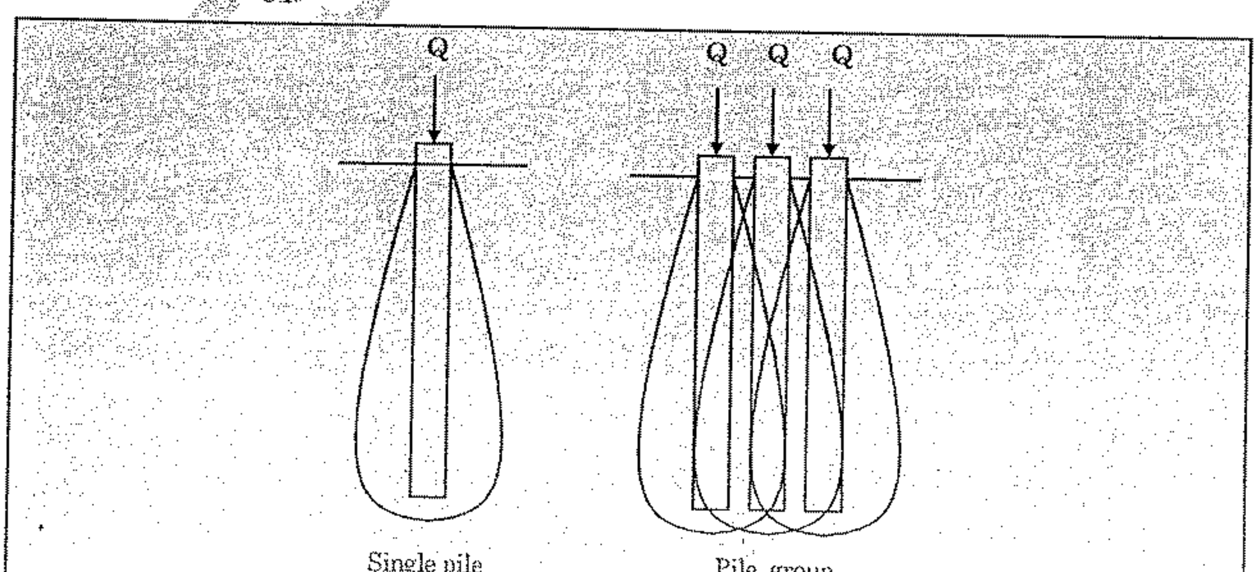
Where, n = No. of piles

Q_{ug} = ultimate load capacity of pile group.

Q_u = ultimate load capacity of single pile.



(a) Point bearing piles



(b) Friction pile

- Disturbances of soil during installationof piles and overlap of stress zones of various piles may cause group efficiency to be less than 1 (<1).
- If spacing between the piles in a pile group is increased, efficiency may approach unity (1).

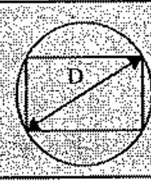
Min. spacing between piles According to [I.S. code]

(1) 2.5 × Dia → for point bearing piles [Centre to centre] Imp

(2) 3 × Dia → for friction piles.

(3) 2 × Dia → for loose sand or fill deposite.

- In case of non-circular piles, diameter of circumscribed circle will be taken as the diameter



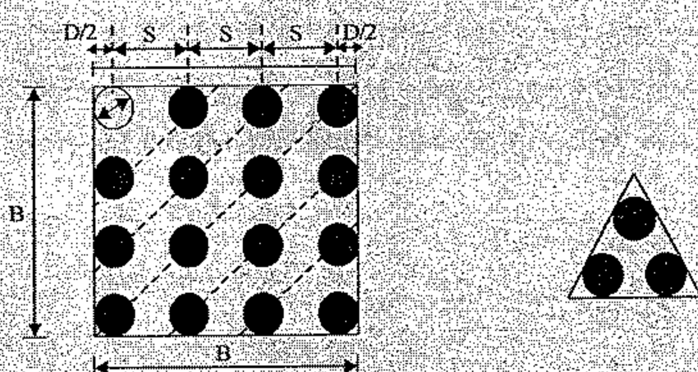
Ultimate Bearing capacity of pile group

(1) Clay:

The group of pile may fail as:

(a) Block failure

(b) Individual pile failure



- In case of clay, block failure generally occurs when → S < (2 to 3) D
- In case of clay, individual failure generally occurs

  when $\boxed{S > 8D}$

- In block failure, soil is bound by perimeter of pile group and embeded length of pile is taken as one unit or block.
- Utimate load capacity of the pile group for block failure is given by,
- $Q_{ug} = C_{ub}.N_c.$ (Area of Base of Block) + $\alpha.C_u.$ (Perimeter of block × length of pile)

Where, C_{ub} = Undrained strength of clay at the base of the pile group.

N_c = Brearing capacity factor, 9 for deep foundation.

α = average adhesion factor over the length of block.

C_u = average undrained strength of clay over the length of block.

- Value of $\alpha = 1$ in case of pile group block because there is interaction between soil particles in pile block with the soil particles adjacent to the block.
- Value of $\alpha < 1$ in case of single pile
- Q_{ug} for individual pile failure is given as below

$$Q_{ug} = nQ_u.$$

- Whereas safe load capacity of pile group is given as below

$$Q_s = \left[\frac{\text{Minimum of } (Q_{ug}, n.Q_u)}{F.O.S.}\right]$$

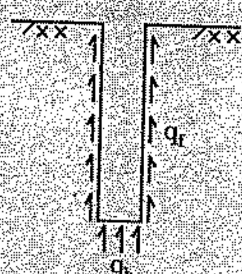
2. Pile Group in sands

- It has been observed that group efficiency of driven piles in loose or medium dense sand is $\eta > 1$. This is because soil around and between the piles get compacted due to the vibration caused during the driving operations.
- Whereas in dense sand above phenomenon is not true.
- For design purpose we never take group efficiency greater than 1. Hence an efficiency factor of 1 is commonly assumed in design.

Example 5.

An RCC pile of 18 m, overall length is driven into a deep stratum of soft clay having an unconfined compresive strength of 3.5 t/m². The diameter of the pile is 30 cm. Determine the safe load that can be carried by the pile with a factor of safety of 3.0.

Sol. Data Given,



$$q_u = 3.5 \text{ t/m}^2$$

$$c = \left(\frac{q_u}{2}\right) = \frac{3.5}{2} = 1.75 \text{ t/m}^2$$

$$d = 30 \text{ cm} = 0.30 \text{ m}, \ L = 18.0 \text{ m}$$

$$\text{F.O.S} = 3.0$$

$$c = \left(\frac{q_u}{2}\right)$$

By the static Formula, we know that

$$Q_u = (Q_f + Q_b)$$

$$= (q_f \times A_f + q_b \times A_b)$$

$$\therefore \quad N_c = 9.0$$

$$q_b = CN_c = 9.0\ C$$

$$q_f = \alpha C = 0.95\ C \qquad [\because \text{ For soft clay } \alpha = 0.95]$$

$$\therefore \quad A_b = \frac{\pi}{4}d^2 = \frac{\pi}{4}\times(0.3)^2 = 0.07068\ \text{m}^2$$

$$A_f = \pi dL = 3.14 \times 0.3 \times 18$$
$$= 16.956\ \text{m}^2$$

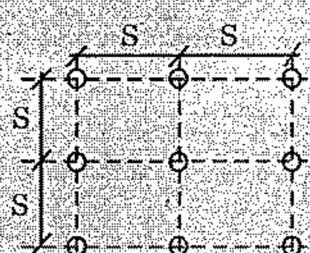
$$\therefore \quad Q_u = (q_f \times A_f + q_b \times A_b)$$
$$= (0.95c \times 16.95b + 9c \times \frac{\pi}{4} \times (0.3)^2$$
$$= (0.95 \times 1.75 \times 16.956 + 9 \times 1.75 \times 0.07068)$$
$$= (28.189) + 1.1025)$$
$$= 29.3\ \text{tonn.}$$

Safe load $Q_s = \left(\frac{Q_u}{F_s}\right) = \left(\frac{29.3}{3.0}\right) = 9.76$ tonn.

Example 6.

200 mm diameter, 8m long piles are used as foundation for a column in a uniform deposit of medium clay (Unconfined compressive strength = 100 kN/m² and adhesion Factor = 0.9) There are nine piles arrqanged In a square pattern of 3×3. For a group efficiency of = 1.0 Find the spacing between the piles (neglect bearing)

Sol. Data Given $q_u = 100$ kNm²



$$\therefore \quad c = \left(\frac{q_u}{2}\right) = 50\ \text{kN/m}^2$$

$$\alpha = 0.9$$

Neglecting the Bearing, Individual pile capacity

$$Q_u = (\alpha\ Cu)\ A_f$$
$$= (0.9 \times 50 \times \pi \times d \times L) = (0.9 \times 50 \times \pi \times 0.2 \times 8)$$
$$= 226.19\ \text{kN} \qquad \text{... (i)}$$

For Nine pile $n = 9$

$$Q_{ug} = nQ_u$$
$$= (9 \times 226.19)$$
$$= 2035.71\ \text{kN} \qquad \text{(ii)}$$

(ii) Considering As a Block

Neglecting skin Friction

$$Q_{ug} = \alpha\, C_u \times 4B \times L$$

$$= 0.90 \times 50 \times 4 \times (2S + d) \times 8 \quad \text{... (iii)}$$

From (ii) and (iii)

$$\eta = 1$$

$$\Rightarrow \quad (Q_{ug})_{\text{Individual}} = (Q_{ug})_{\text{Block}}$$

$$\Rightarrow \quad 2035.71 = 0.90 \times 50 \times 4 \times 8\ (2S + 0.2)$$

$$\Rightarrow \quad S = 0.356 \text{ m}$$

Proved $\quad S = 0.6$ m

$$\therefore \quad B = (2S + d)$$

$$= (2 \times 0.6 + 0.2)$$

$$= 1.4 \text{ m} < 3 \text{ m}$$

Example 7.

Design a pile group to carry 400 kN in clay with a unconfined stregnth of 60 kNm². Adhersion Factor may be taken as 0.6. The piles are of size 300 mm and 6m. long. Assuming F.O.S = 2.5

Sol. Data given: $\quad q_u = 60 \text{ kN/m}^2$

$$c = \left(\frac{q_u}{2}\right) = \frac{60}{2} = 30 \text{ kN/m}^2$$

$$\text{Load to be carried} = 400 \times 2.5$$

$$= 1000 \text{ kN} \quad \text{... (i)}$$

$$\therefore \quad \text{No. of piles} = \frac{1000}{(\text{Single pile load capacity})}$$

$$\text{Single pile load Capacity } Q_u = (C_u N_c) A_b + (\alpha\, C_u) A_f$$

$$= 30 \times 9 \times \frac{\pi}{4}(0.3)^2 + (0.6 \times 30 \times \pi \times 0.3 \times 6)$$

$$= 120.87 \text{ kN}$$

$$\therefore \quad \text{No. of pile} = \frac{1000}{(120.87)} = 8.27$$

$$= 9 \text{ pile.}$$

Note: If Instead of 8.27 no of piles, we are c... ...ng Nine, then for group efficiency equal to one Load carrying capacity will be more than 1000 kN. We have to Ensure that group capacity should be equal to 1000 kN, hence block size can be calculated corresponding to 1000 kN. Hence bigger than this size will be adopted.

$$\Rightarrow \quad Q_{ug} = (C_u N_c) B^2 + (\alpha\, C_u) \times 4B \times 6$$

$$1000 = 30 \times 9B^2 + (0.6 \times 30 \times 4 \times 6B) \quad \text{... (ii)}$$

$\Rightarrow$ Solving above equation $\Rightarrow$ B = 1.008 m

$\Rightarrow$ **Spacing S**

$$\Rightarrow \quad 2S + D = (1.008)$$

$$\Rightarrow \quad S = \frac{(1.008 - 0.300)}{2} = 0.354\text{m}$$

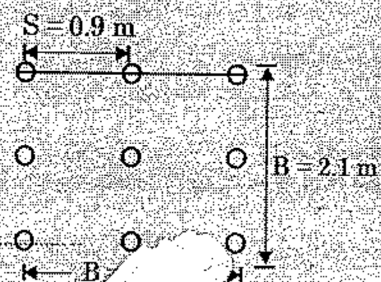
$$\text{Providing } S_{min} = 3D$$

$$= 3 \times 0.3 = 0.9 \text{ m}$$

Example 8.

Nine piles opf 300 mm dia and 8 m length are arranged In a square pattern for the foundation for a column in a uniform deposit of medium stiff clay (q_u = 100 kN/m²) If the centre to centre spacing of piles is 900 mm and adhesion factor = 0.9, calculate the capacity of the pile group assuming a F.S of 2.5.

Sol. Data given



$$\text{Die of pile } D = 300 \text{ mm}$$

$$= 0.3 \text{ m}$$

$$\text{Length of pile } L = 8 \text{ m}$$

Adhesion Factor $\alpha = 0.9$

$$C = \frac{q_u}{2} = \frac{100}{2} = 50\text{kN/m}^2$$

Ultimate load of pile group (considering individual pile load capacity)

$$Q_g = nQ_u$$

$$= 9\,[CN_C A_b + \alpha_c A_f]$$

$$= 9\left[50 \times 9 \frac{\pi}{4} \times (0.3)^2 + (0.9 \times 50)(\pi \times 0.3 \times 8)\right]$$

$$= 3339.9 \text{ kN} \quad \ldots (i)$$

Ulitmate load of pile group (considering as a block)

$$Q_g = (q_u A_b + q_f A_f)$$

$$= CN_c B^2 + 0.9 \times 50\ (4\ BL)$$

$$= 50 \times 9 \times (2.1)2 + 0.9 \times 50 \times 4 \times (2.1) \times 8$$

$$= 5008.5 \text{ kN} \quad \ldots (ii)$$

From (*i*) and (*ii*), lower value will be the ulimate load

Therefore, $Q_u = 3339.9$

$$\therefore \quad \text{Safer load } Q_a = \left(\frac{3339.9}{2.5}\right)$$

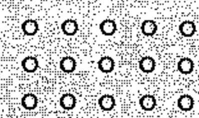
$$= 1335.9 \text{ kN}$$

Example 9.

A Raft foundation is supported by pile group consisting of 15 piles arranged in 3 rows. The diameter and length of each pile are 300 mm and 15 m. respectively. The spacing between the piles is 1.2 m. The foundation soil consists of a soft clay layer having $c = 3.2 t/m^2$ and $\gamma = 1.9 t/m^3$. Determine the capacity of the pile group.

Sol. (*i*) Data given : $c = 3.2 t/m^2$

$\gamma = 1.9\ t/m^3$



considering Individual action of piles:

$$q_f = \alpha c$$
$$= 0.90 \times 3.2 \quad (\text{assuming } \alpha = 0.9)$$
$$= 2.88\ t/m^2$$
$$A_f = (\pi\, d\, l) = (\pi \times 0.300 \times 15)$$
$$= 14.14\ m^2$$
$$q_b = cNc$$
$$= 9c$$
$$= 9 \times 3.2 = 28.8 t/m^2$$
$$A_b = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.300)^2$$
$$= 0.071\ m^2$$

$\therefore$ Individual capacity of each pile

$$Q_u = (Q_f + Q_b)$$
$$= (q_f A_f + q_b A_b)$$
$$= (2.88 \times 14.14 + 28.8 \times 0.071)$$
$$= 40.7232 + 2.035$$
$$= 42.75 \text{ tonn.}$$

$\therefore$ Group capacity $Q_{ug} = n\, Q_u$

$$= (15 \times 42.75) = 641.37 \text{ tonn.}$$

(*ii*) Considering group action of piles :

$$L = (1.2 \times 4 + 0.300)$$
$$= 5.1\ m$$
$$B = (1.2 \times 5 + 0.30)$$
$$= 2.7\ m$$
$$Q_g = q_b BL + (2B + 2L) \times D_f \times \alpha c - \gamma\ D_f \times (BL)$$
$$= 9c\ BL + (2B + 2L)\ D_f \times (\alpha_f) - \gamma\ D_f\ BL$$
$$= (9 \times 3.2 \times 2.7 \times 5.1) + (2 \times 2.7 + 2 \times 5.1) \times 15 \times 0.9 \times 3.2) - 1.9 \times 15 \times 2.7 \times 5.1$$
$$= 678.05 \text{ tonn} > 641.37 \text{ tonn.}$$

Therefore, the ultimate bearing capacity of the pile group is 641.37 tonn.

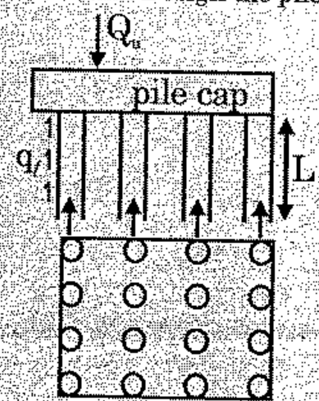
$\therefore$ Safe bearing capacity w.r. to F.O.S of 2.5

$$= \left(\frac{641.37}{2.5}\right) = 256 \text{ tonn.}$$

Example 10

A Raft foundation has to be supported by a group ... te piles. The gross load to be carried by the pile group is 250 tonn inclusive of the heig... e pile cap. The subsoil consists of a 25m thick stratum of normally consolidated clay hav... unconfined comprensive strangth of 4.8 t/m^2 and on effective unit weight of 0.9t/m^3. Design the pile group with a F.O.S of 3 against shown failure.

Sol.



Let use 16 nos. of 400 mm ϕ RCC piles in a square formation.

Let the spacing s be equal to 3d.

$$s = 3d = 3 \times 0.400 = 1.2\text{m}$$

Let L be the length of each pile

$$c = \left(\frac{q_u}{2}\right) = \frac{4.8}{2} = 2.4\text{t/m}^2$$

Assuming $\alpha = 0.90$

$$q_f = c\alpha$$
$$= 0.9 \times 2.4 = 2.16 \text{ t/m}^2$$
$$q_b = cN_c = 9c$$
$$= 9 \times 2.4 = 21.6 \text{ t/m2}$$
$$A_f = \pi BL = 0.40 \times \pi \times L = 1.257 \text{ L m}^2$$
$$A_b = \frac{\pi}{4}d^2 = \frac{\pi}{4} \times (0.4)^2 = 0.126 \text{ m}^2$$

$\therefore$ Capacity of each pile

$$Q_u = (q_f A_f + q_b A_b)$$
$$= (2.16 \times 1.257 \text{ L}) + (21.6 \times 0.126)$$
$$Q_u = (2.715 \text{ L} + 2.722) \quad \text{... } (i)$$

$\therefore$ Safe bearing capacity of each pile

$$Q_S = \left(\frac{Q_u}{3}\right) = (0.905 \text{ L} + 0.907) \quad \text{... } (ii)$$

Actual load carried by each pile $= \frac{250}{16} = 15.625$t. ... (*iii*)

From (*ii*) and (*iii*)

$\Rightarrow$ 0.905 L + 0.907 = (15.625)

$\therefore \quad L = \dfrac{(15.625 - 0.907)}{0.905}$

$= 16.50$ m

L = 16.50 m ... (iv)

Check for group action :

Considering the shear failure of a block of dimension $B \times L \times D$

$\Rightarrow \quad B = L = (3S + d)$

$= (3 \times 1.2 + 0.4) = 4$m.

$D = 16.5$m

$\therefore$ capacity of the pile group,

$$Q_g = (B \times L)\, q_b + (2B + qL)D \times \alpha c - \gamma\ D_f \times (B \times L)$$

$$= 21.6 \times 4^2 + (2 \times 472\ 1\ 4) \times 16.5 \times 2.16 - (0.9 \times 16.5) \times 4^2$$

$Q_g = 894.24$ tonn

$\therefore$ Safe bearing capacity of the pile group,

$$Q_{sg} = \left(\frac{894.24}{3}\right) = 298t > 250t$$

Hence the designed group of piles is safe from the consideration of block failure.

Example 11.

A bored concrete pile of 400 mm diameter and having an overlength of 12.5 m is embedded in a saturated stratum of $c - \phi$ soil having the following properties

$c = 15$ kN/m², $\phi = 20$ $\gamma_{sat} = 18$ kN/m³.

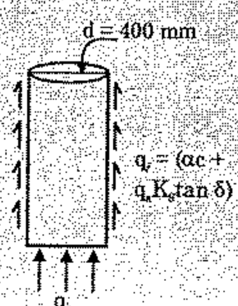
Determine the safe bearing capacity of the pile. Given $\phi = 20°$

The bearing capacity factors are

$N_c = 26$, $N_q = 10$, $N_\gamma = 4$

Assume reasonable values for all other factors.

Sol. For Pile Embedded In $c - \phi$ Soil



$$q_b = cN_c + \gamma D\,(N_q - 1) + \frac{1}{2}(B\gamma' N_\gamma)$$

$$= 15 \times 26 + (18 - 10) \times 12.5\ (10 - 1) + \frac{1}{2} \times 0.40 \times 4(18 - 10)$$

$= 1296.4$ kN/m²

$q_f = (\alpha c + \bar{q}_o K_s \tan\delta)$

Assume $\alpha = 0.5,\ K_s = 1,\ \frac{\delta}{\phi} = 0.80$

$$\delta = (0.80 \times 20) = 16°$$

$$\Rightarrow \quad q_f = (0.5 \times 15) + \left(\frac{\gamma H}{2}\right) \times K_s \tan\delta$$

$$= 75 + \frac{(18-10)\times 12.5}{2} \times 1 \times \tan 16$$

$$= 21.84 \text{ kNm}^2$$

Again $A_f = (\pi\, d\, l) = (\pi \times 0.40 \times 12.5)$

$$= 15.71 \text{ m}^2$$

$$A_b = \frac{\pi}{4}d^2 = \frac{\pi}{4}\times(0.4)^2 = 0.126 \text{ m}^2$$

$$Q_u = (21.84 \times 15.71) + (1296.4) \times 0.126$$

$$= 343.1 + 163.3$$

$$= 506.4 \text{ kN}$$

$$Q_s = \frac{Q_u}{(F.O.S)} = \frac{506.4}{3} = 168.8 \text{ kN}$$

Example 12.

A 450 mm × 450mm concrete pile 20.0 m long is driven into sand depoints with $\gamma = 17$ kN/m³ and $\phi' = 30°$. Find the ultimate load i.e. point load Q_p by Meyer hoff's method and jambu's method. Meyer hoff's $N_q = 55$, Atmospheric pressure = 100 kN/m², Jambu's.

$$N_q = 18.4$$

Sol. (i) As per Mayer hoff's method point load capacity of a pile In cohesionless soil is given by

$$Q_P = \gamma D_f N_q A_P$$

Data given : $\gamma = 17$ kN/m³, $D_f = 20.0$

$$N_q = 55,\ P_A = 100 \text{ kN/m}^2$$

$$A_P = 0.45 \times 0.45$$

$$= 0.2025 \text{ m}^2$$

$$\therefore \quad Q_P = (17 \times 20 \times 55 \times 0.2025)$$

$$= 3786.75 \text{ kN} \quad \text{... (i)}$$

but the value of Q_p should not exceed the limiting load carrying capacity at pile point

Therefore, $Q_p \leq (0.5\, P_a\, N_q \tan\phi' A_p)$

$$\leq 0.5 \times 100 \times 55 \tan 30° \times 0.2025$$

$$\leq 321.51 \quad \text{... (ii)}$$

Therefore the ultimate load carrying capacity of pile as per mayer hoff's method is 321.51 kN.

(ii) As per Jambu's method point load capacity of pile In cohesionless soils is given by

$$Q_P = \gamma D_f\ N_q'\ A_P$$

$$= 17 \times 20 \times 18.4 \times 0.2025$$

EFFICIENCY OF PILE GROUPS

- According to Converse - Labarre, the efficiency of a pile group or group efficiency of piles is the ratio of the actual group capacity to the sum of the individual pile capacities.
- The group efficiency is expressed as—

$$\eta_g = 1 - \theta\frac{(n-1)m + (m-1)\,n}{90 \times m \times n}$$

Where,

m = number of rows

n = number of piles in a row

$\theta = \tan^{-1}(d/s)$

d = diameter of pile

s = spacing of pile centre to centre

Example 13.

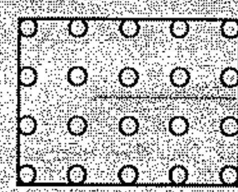
A group of 20 piles, Each having a diameter of 400 mm and 10 m long, are arranged. In 4 rows at a spacing of 1.0m c/c. The capacity of each pile is 380 kN. Determine the group capacity of the piles.

Sol. Data given

n = 20 piles

d = 400 mm

L = 10 m long, Q_u = 380 kN.



We know that the capacity of the pile group

$$Q_g = n\,Q_u\,\eta_g \quad \text{... (i)}$$

η_g calculation by converse labarre formula :

$$\eta_g = 1 - \frac{\theta}{90}\left(\frac{m(n-1)+n(m-1)}{mn}\right) \quad \text{.. (ii)}$$

Here

$m = 4$

$n = 5$

$$\theta = \tan^{-1}\left(\frac{d}{s}\right) = \tan^{-1}\left(\frac{400}{1000}\right)$$

$$= \tan^{-1}(0.4) = 21.8^\circ$$

$$\therefore \eta_g = 1 - \frac{21.8}{90}\left(\frac{4(5-1)+5(4-1)}{4\times 5}\right)$$

$$= \left(1 - \frac{21.8\times 31}{20\times 90}\right) = 0.624 = 62.4\%$$

Therefore, the capacity of the pile group

$$= (n\,Q_u\,\eta_g)$$

$$= (20 \times 380 \times 0.624)$$

$$= 4742.4 \text{ kN.}$$

***Settlement of pile groups**

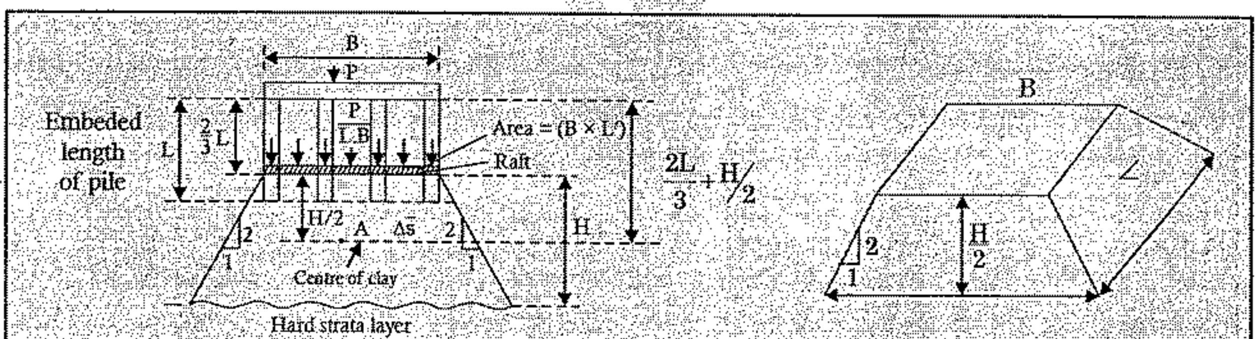
- Settlement of **pile group** is generally greater than the settlement of **individual pile** to **same loading.** [i.e. same loading per pile].
- This is because, the zone of influence of pile group is generally more than the individual pile

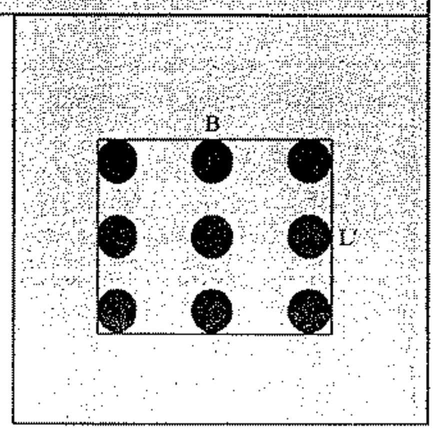
Settlement of Pile Groups in clay

- The widely used approach used for calculating settlement of a pile group in a clay is equivalent to raft method. As settlement of a pile group in clay can not be estimated from the data of load test on a single pile because of the time effect (i.e. consolidation and the effect of remoulding of soil due to pile driving which is also a kind of time effect.
- In this method for friction pile or displacement piles, the pile group is assumed to act as a single raft at a depth of 2/3rd of the pile length, in a homogenous stratum of clay.
- The load Q_g is assumed to be transffered from this level to the clay layer below at a distribution of 2 V: 1H in order to work out increase in $\Delta\bar{\sigma}$ at the mid depth of clay layer.
- In case of Bored or End bearing piles on firm stratum, the equivalent raft is assumed at the base of piles. A load spread of 2V : IH is assumed from this depth.
- In case of piles diven into firm or strong stratum of clay underlying a soft stratum of clay. A fictitious raft is assumed to act at a depth of 2/3rd of embeded length of pile in strong stratum from the top surface of strong layer and spreading out at 2 : 1 slope. The settlement of this fictitious raft is taken as settlement of pile group.

Settlement calculations are done same as discussed in consolidation chapter.

Case 1: When pile is emheded in uniform clay deposit.





$$\text{Area} = \left(B+\frac{H}{2}\right)\times\left(L+\frac{H}{2}\right)$$

$$= \left(B+\frac{H}{2}\right)\times\left(L+\frac{H}{2}\right)$$

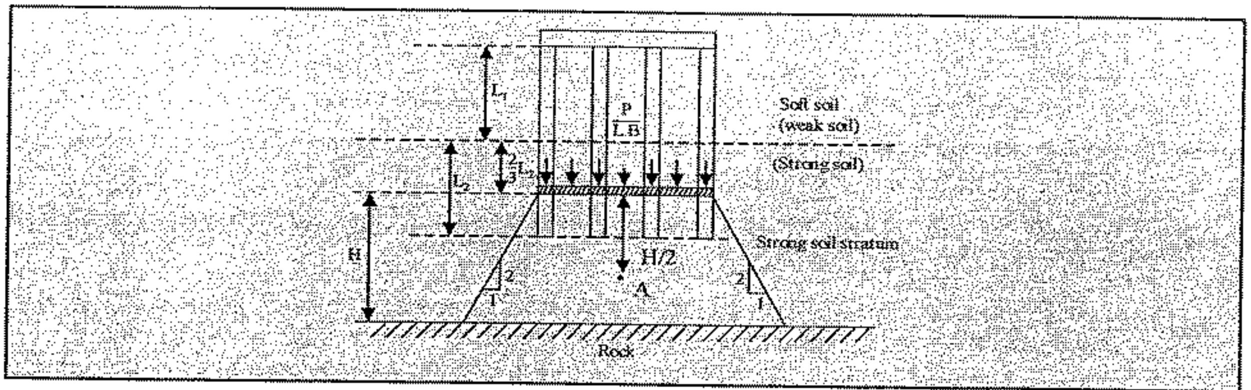
$$\bar{\sigma}_0 = \left[\frac{2}{3}L+\frac{H}{2}\right]\times\gamma_t$$

$$\Delta\bar{\sigma} = \frac{P}{\left(B+\frac{H}{2}\right)\left(L+\frac{H}{2}\right)}$$

$$\Delta H = \frac{C_c}{(1+e_o)}.H.\log\left(\frac{\bar{\sigma}_o+\Delta\bar{\sigma}}{\bar{\sigma}_o}\right)$$

Case-II

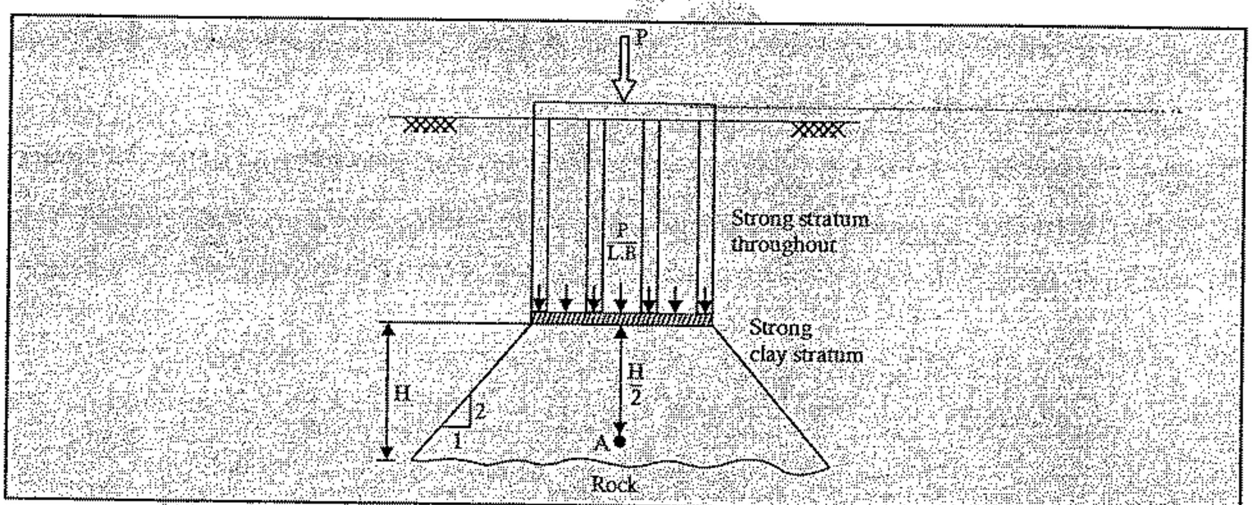
When piles are driven into **Strong Stratum** through an **overlying weak stratum.**



$$\Delta H = \frac{C_c}{(1+e_o)}.H.\log\left(\frac{\bar{\sigma}_o + \Delta\bar{\sigma}}{\bar{\sigma}_o}\right)$$

(3) Case III—In case of **bored piles** or **End bearing piles** resting on **firm stratum.**

Then, Equivalent Raft will be assumed at Tip of pile and Load dispersion is assumed from there 1H: 2V.



$$\boxed{\Delta H = \frac{C_c}{(1+e_o)}.H.\log_{10}\left(\frac{\bar{\sigma}_o + \Delta\bar{\sigma}}{\bar{\sigma}_o}\right)}$$

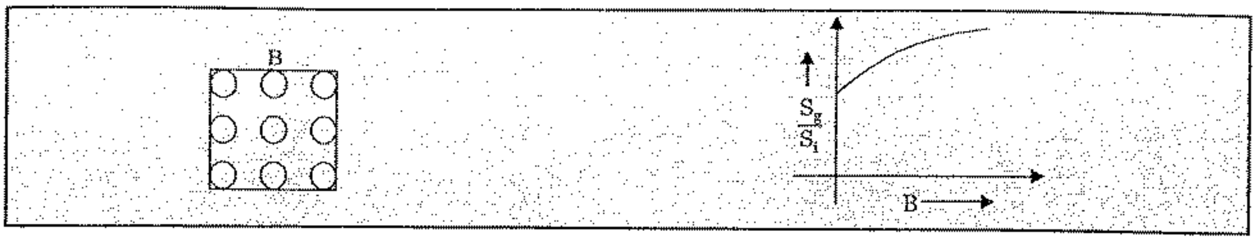
$\bar{\sigma}_o$ = Effective stress at (A)

*Pile group settlement for Sand

- Pile group settlement in sand is calculated from settlement of individual pile in a load test. Relation between them is expressed as settlement ratio.

$$\text{Group settlement Ratio} = \frac{s_g}{s_i} = \left(\frac{4B+2.7}{B+3.6}\right)^2$$

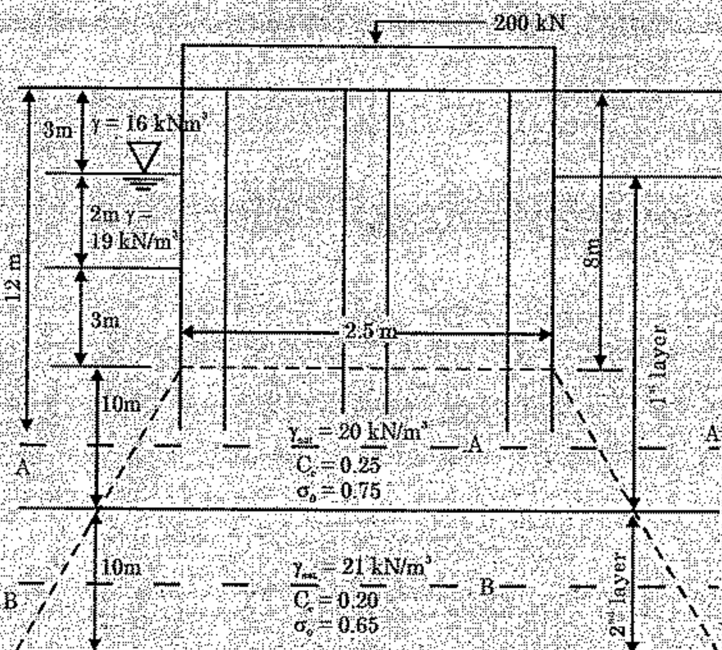
where, S_g = Group settlement at the same load of pile group.
B in (meter) = 'B' is the size of pile group in (meter)
S_i = settlement of individual pile calculated from the pile load test.



Note : Load carrying capacity of a pile group can be calculated either from shear **Strength criteria** and **Settlement criteria** and **minimum** value will be adopted.

Example 14.

A group of friction piles of 30 cm. diameter is subjected to a net load of 2000 kN, as shown in figure. Estimate the consolidation settlement.



Sol. σ_0 at point A, middle of Ist layer

$$= (3 \times 16 + 2 \times (19 - 10) + 8 \times 10$$

$$= 146 \text{ kN/m}^2$$

σ_0 at point B, middle of IInd layer

$$= (3 \times 16 + 2 \times (19 - 10) + 13 \times 10 + 5 \times (21 - 10)$$

$$= 251 \text{ kN/m}^2$$

cross sectional area at A = $(2.5 + 2 \times 5 \times 1/2)$

$$= 7.5 \text{ m}^2$$

$$\therefore \quad \Delta\sigma = \frac{2000}{(7.5 \times 7.5)} = 35.56 \text{ kN/m}^2$$

Cross sectional area at B = $(2.5 + 15 \times 2 \times 1/2)$

$$= 17.5 \text{m}^2$$

$$\therefore \quad \Delta\sigma = \frac{2000}{(17.5 \times 17.5)} = 6.53 \text{ kN/m}^2$$

$$\therefore \text{ Settlement of I}^{st} \text{ layer} = \frac{C_c H}{(1+e_0)} \log_{10}\left(\frac{\sigma_0 + \Delta\sigma}{\sigma_0}\right)$$

$$= \frac{0.25 \times 10}{(1+0.75)} \log_{10}\left(\frac{146+35.56}{146}\right)$$

$$= 0.135 \text{ m}$$

$$\text{Settlement of II}^{nd} \text{ layer} = \frac{C_c H}{(1+e_0)} \log_{10}\left(\frac{\sigma_0 + \Delta\sigma}{\sigma_0}\right)$$

$$= \frac{0.20 \times 10}{(1+0.65)} \log_{10}\left(\frac{251+6.53}{251}\right)$$

$$= 0.014$$

$$\text{Total settlement} = (0.135 + 0.014)$$

$$= 0.149 \text{ m.}$$

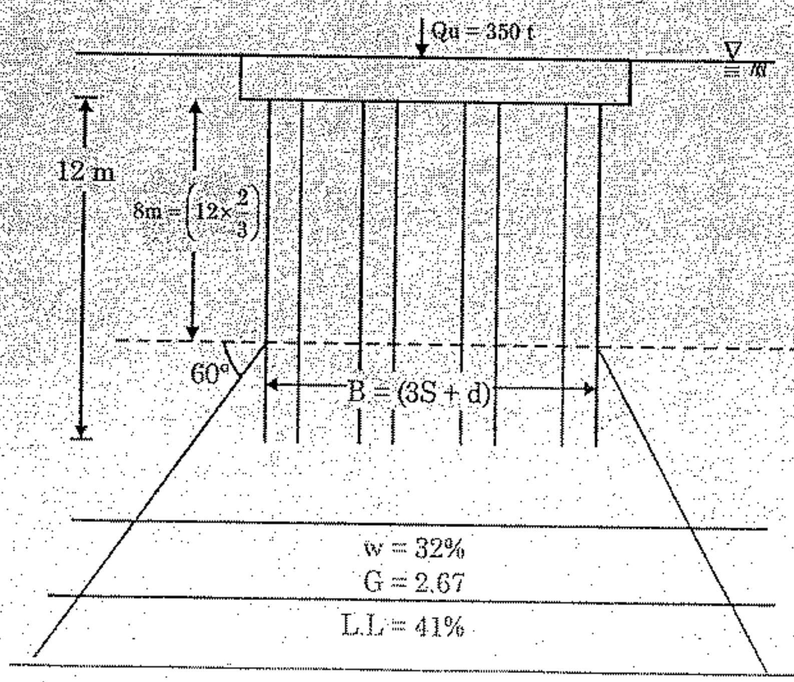
Example 15.

A Raft footing founded at a depth of 1.5 m below G.L. in a 19.5 thick stratum of normally consolidated clay underlain by a dense sand layer is to be. Supported by a group of 16 piles of length 12 m and the diameter 400 mm arranged in a square formation. The gross load to be carried by the pile group (including the self weight of the pile cap) is 350 t. The piles are spaced at 1.2m c/c. The W.T is located at the ground level. The properties of the foundation soil are :

w = 32% G = 2.67, L.L = 41%

Estimate the probable consolidation settlement of the pile group.

Sol.



The load from the pile group is assumed to be transmitted to the foundation soil at the lower one third point.

i.e. at a depth of $\frac{2}{3}\times 12 = 8$ m below the pile cap and $8 + 1.5 = 9.5$ below the ground level.

Therefore, thickness of the clay layer undergoing consolidation settlement = 10 m. Let us divide this zone into three sub layers of thickness 3m, 3m, and 4m respectively.

Data given
$$w = 32\% = 0.32$$
$$G = 2.67 \quad \text{L.L} = 41\%$$
$$eS = wG$$
$$\Rightarrow \quad e_0 = \frac{(0.32\times 2.67)}{1} = 0.854 \qquad \text{... }(i)$$
$$C_c = 0.009\,(w_L - 10)$$
$$= 0.009\,(41 - 10) = 0.279$$

Again
$$\gamma_{sat} = \frac{(G+e)\gamma_w}{(1+e_0)}$$
$$= \frac{(2.67+0.854)\times 1}{(1+0.854)}$$
$$= 1.90 \text{ t/m}^3$$
$$\gamma_{sub} = (\gamma_{sat} - \gamma_w) = 1.90 - 1 = 0.90 \text{ t/m}^3$$

Settlement of the Ist sub layer
$$\sigma_0 = (\gamma' z) = (1.5 + 8 + 3/2) \times 0.90 = 9.9 \text{ t/m}^2$$
$$\text{L} = \text{B} = (3\text{S} + \text{d}) = (3 \times 1.2 + 0.4) = 4\text{m}.$$

Assuming the load to be displaced along straight line inclined to the horizontal at 60°.

The area over which the gross load is distributed at the middle of the Ist layer
$$\text{A}_1 = (\text{L} + 2\text{H}_1/2 \ \tan 30°)\,(\text{B} + 2\text{H}_1/2 \ \tan 30°)$$
$$= (\text{B} + \text{H}_1 \tan 30°)^2$$
$$= (4 + 3\tan 30°)^2 = 32.86 \text{t/m}^2$$
$$\Delta\sigma = \left(\frac{\text{Q}}{\text{A}_1}\right) = \frac{350}{(32.86)} = 10.65 \text{ t/m}^2$$
$$\Delta\text{H}_1 = \frac{300\times 0.279}{(1+0.854)}\log_{10}\left(\frac{9.9+10.65}{9.9}\right)$$
$$= 14.32 \text{ cm} \qquad \text{... }(ii)$$

settlement of the IInd sub layer :
$$\sigma_0 = 0.90\,(1.5 + 8 + 3.0 + 3/2) = 12.6 \text{t/m}^2$$
$$\text{A}_2 = (4 + 2 \times 4.5 \tan 30)^2$$
$$= 84.57 \text{ m}^2$$
$$\Delta\sigma = \left(\frac{\text{Q}}{\text{A}_2}\right) = \frac{350}{(84.57)} = 4.14 \text{ t/m}^2$$

$$\Delta H_2 = \frac{300 \times 0.279}{(1+0.854)} \log_{10}\left(\frac{12.6+4.14}{12.6}\right)$$

$$= 5.57 \text{ cm} \qquad \ldots (iii)$$

settlement of the IIIrd sub layer

$$\sigma_0 = 0.90\ (1.5 + 8 + 3 + 3 + 4/2) = 15.75 \text{ t/m}^2$$

$$A_3 = (4 + 2 \times 8 \times \tan 30)^2$$

$$= 175.23 \text{ m2}$$

$$\Delta\sigma = \left(\frac{350}{175.23}\right) = 1.997 \text{ t/m}^3$$

$$\Delta H_3 = \frac{400 \times 0.279}{(1+0.854)} \log_{10}\left(\frac{15.75+1.997}{15.75}\right)$$

$$= 3.12 \text{ cm} \qquad \ldots (iv)$$

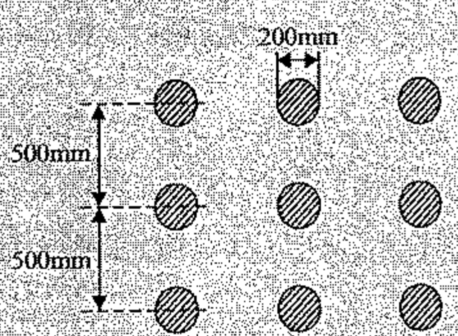
$$\text{Total settlement} = (\Delta H_1 + \Delta H_2 + \Delta H_3)$$

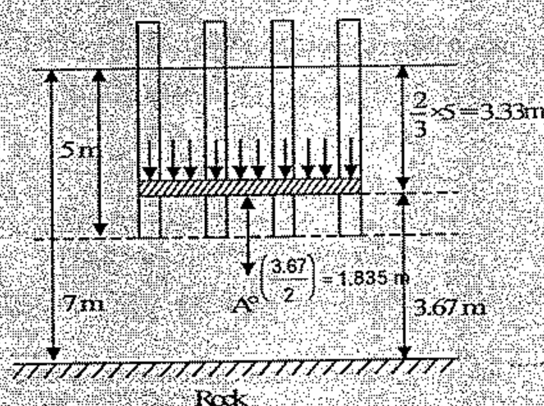
$$= (14.32 + 5.57 + 3.12)$$

$$= 23 \text{ cm.}$$

Example 16.

Calculate settlement of a pile group for the condition shown below:





$$\gamma_{sat} = 20 \text{ KN/m}^2$$

$$w_L = 40\%$$

$$w_P = 25.5\%$$

$$e = 1.05$$

- Assume soil to be fully submerged.
- N.C. clay.

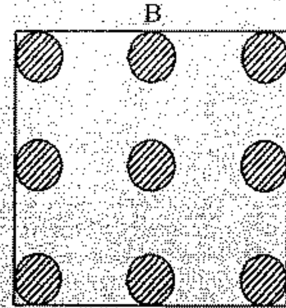
Sol. Find out $B \times B$ 1st ly.

$$B = 1.2 \text{ m}$$

$$[\ B = 2 \times 500 + 200\ ]$$

$$= 1200 \text{ mm}$$

$$= \boxed{1.2 \text{ m}}$$



$$\boxed{\Delta H = \frac{C_c . H}{(1+e_o)} . \log_{10}\left(\frac{\bar{\sigma}_o + \Delta\bar{\sigma}}{\bar{\sigma}_o}\right)}$$

where,

$$C_c = 0.009\,(w_L - 10)$$
$$= 0.009\,(40 - 10) = 0.27$$
$$H = \boxed{3.75\text{ m}}$$
$$e_o = \boxed{1.05}$$
$$\bar{\sigma}_o = \gamma_{sub} \times (3.33 + 1.835)$$
$$= (20 - 10)\,(3.33 + 1.835)$$
$$= \boxed{51.65\text{ kN/m}^2}$$
$$\Delta\bar{\sigma} = \left[\frac{500\text{ kN}}{(1.2+1.835)^2\text{ m}^2}\right]$$
$$= \boxed{54.33\text{ kN/m}^2}$$

$\therefore$
$$\Delta H = \frac{0.27}{(1+1.05)} \log_{10}\left(\frac{51.65+54.33}{51.65}\right) = 150.787\text{ mm}$$
$$= \boxed{150.787\text{ mm}}$$

Pile Capacity using Dynamic pile formula:

- These formulae are based on the penetration resistance imparted to pile driving.
- The basic of this formula is, Energy imparted = work done in pile driving.

$\therefore$
$$\boxed{\begin{array}{c}\text{Work done} = \text{Energy Imparted}\\ Q_u.S = W.H.\end{array}}$$

i.e.
$$\boxed{W \times H = Q_u \times S}$$

(1) Engineering News Formula

$$\boxed{Q_{allowable} = \frac{W.H.}{F.O.S(S+C)}}$$

Where, F.O.S. = 6

W = Load (kg)

H = Height of fall (cm)

S = Settlement/blow (cm)

= (S) is taken corresponding to last 5-blows of drop hammer also called as real set per blow

or = Last 20 blows of steam hammer.

C = Emperical factor

= 2.5 cm → for drop hamer

0.25 cm → for single acting steam Hammer.

Another form of Engineering News formula:

$$\boxed{Q_{KN} = \frac{166.64 \times E}{(S+2.54)}} \; Q = KN$$

C = E

Where, E = Energy per blow in (kilo Joule)

S = Settlement/Avg. penetration for Last 100 mm of driving per blow. min permissible value of S = 1.25 mm.

Example 17.

A timber pile is being driven with a drop hammer weighing 20 kN and having a free fall of 1m. The total penetration of the pile is the last five blows is 30 mm. Determine the load carrying capacity of the pile using the Engineering News Formula.

Sol. Data Given :

W = 20 kN

H = 1m = 100 cm.

S = avg. penetration of the pile in last n - blows

$= \frac{30}{5} = 6\text{mm} = 0.6 \text{ cm}.$

C = 2.5 cm For Drop hammer

From Engineering News Formula

We know that,
$$Q = \frac{WH}{6(S+C)}$$
$$= \frac{20 \times 100}{6(0.6+2.5)} = 107.5 \text{ kN}$$

Example 18.

What will be the penetration of square R.C. pile per blow which must be obtained in driving the pile with a 5 tonns drop hammer falling through 1.2 meter. Allowable load is 30 tonn.

Sol. Data Given:

W = 5 tonns (drop hammer)

H = 1.2 m = 120 cm

Q_a = 30 tonns, S = ?

∴ According to Engineering News Formula

We know that,

$$Q_a = \frac{WH}{6(S+C)}$$

S = avg. penetration in last n-blows

C = 2.5 – For Drop hammers

$$Q_a = \frac{WH}{6(S+2.5)}$$

$$\Rightarrow \quad 30 = \frac{5\times120}{6(S+2.5)}$$

$$\Rightarrow \quad \boxed{S = 0.83\text{ cm}}$$

(3) Modified Hilly Formula:

$$\boxed{\text{Ultimate Driving Resistance(R)} = \frac{W.h.\eta}{\left(S+\frac{C}{2}\right)}}$$

where, W = wt. of Hammer (Tonnes)

h = Ht. of fall (cm)

S = Final set per blow (Last one) [c.m.]

C = Total elastic compression per Blow.

C = Elastic compression of pile + Elastic compression of soil + Elastic compression of Dolly

η = Efficiency of blow. Depends on the coeff. of restitution.

Example 19.

Determine the Safe load that can be carried by a pile having a gross weight of 1.5 t, using the modified Hiley's Formula. Given,

Weight of hammer = 2.0 tonn

Height of free fall = 91 cm

Hammer Efficiency = 75%

Avg. penetration under the last 5 blows = 10 mm

Length of Pile = 22 mm

Dia of pile = 300 mm

Coefficient of Restitution = 0.55%

Sol. From the Modified Hiley's Formula

We know that, the Ultimate load on pile

$$Q_u = \frac{(\eta_h WH\eta_b)}{(S+C/2)} \quad \text{... (i)}$$

Where h_h = Efficiency of hammer = 75% = 0.75

W = 2.0 tonn

H = 91 cm.

S = Avg. penetration under the last 5 blows

$$= 10 \text{ mm} = 1 \text{ cm.}$$

$$eP = 0.55 \times 1.5 = 0.825 \text{ tonn.}$$

$$\because \quad W > eP.$$

$$\Rightarrow \quad \eta_b = \frac{W + e^2P}{(W+P)} = \frac{2+(0.55)^2 \times 1.5}{(2+1.5)} = 0.701.$$

In order to find out the value of Q_u, assume as a first approximation,

$$C = 2.5 \text{ cm}$$

$$\therefore \quad Q_u = \frac{\eta_h W H \eta_b}{(S + C/2)}$$

$$= \left(\frac{0.75 \times 2 \times 91 \times 0.701}{1 + 2.5/2}\right)$$

$$= 42.52 \text{ tonn}$$

Now using

$$C_1 = 1.77\left(\frac{Q_u}{A_p}\right) = 1.77 \times \frac{42.52}{\frac{\pi}{4}(30)^2} = 0.106 \text{ cm}$$

$$C_2 = 0.657\left(\frac{Q_u L}{A_P}\right) = 0.657 \times \frac{(42.52 \times 22)}{(\pi/4)(30)^2} = 0.869$$

$$C_3 = 3.55 \times \left(\frac{Q_u}{A_p}\right) = 3.55 \times \frac{42.52}{(\pi/4)(30)^2} = 0.213 \text{ cm}$$

$$\therefore \quad C = (C_1 + C_2 + C_3)$$

$$= 1.1885 < 2.5 \text{ cm}$$

Let

$$Q_u = 50 \text{ tonn}$$

$$\therefore \quad C = \frac{1.188 \times 50}{(42.52)} = 1.397$$

$$\therefore \quad Q_u = \frac{(0.75 \times 2 \times 91 \times 0.701)}{(1 + 1.397/2)} = 56.33 \text{ tonn.}$$

Let

$$Q_u = 55 \text{ tonn}$$

$$\therefore \quad C = \frac{1.188 \times 55}{(42.52)} = 1.537.$$

$$\therefore \quad Q_u = \frac{0.75 \times 2 \times 91 \times 0.7}{(1.0 + 1.537/2)}$$

$$= 54.03 \text{ tonn.}$$

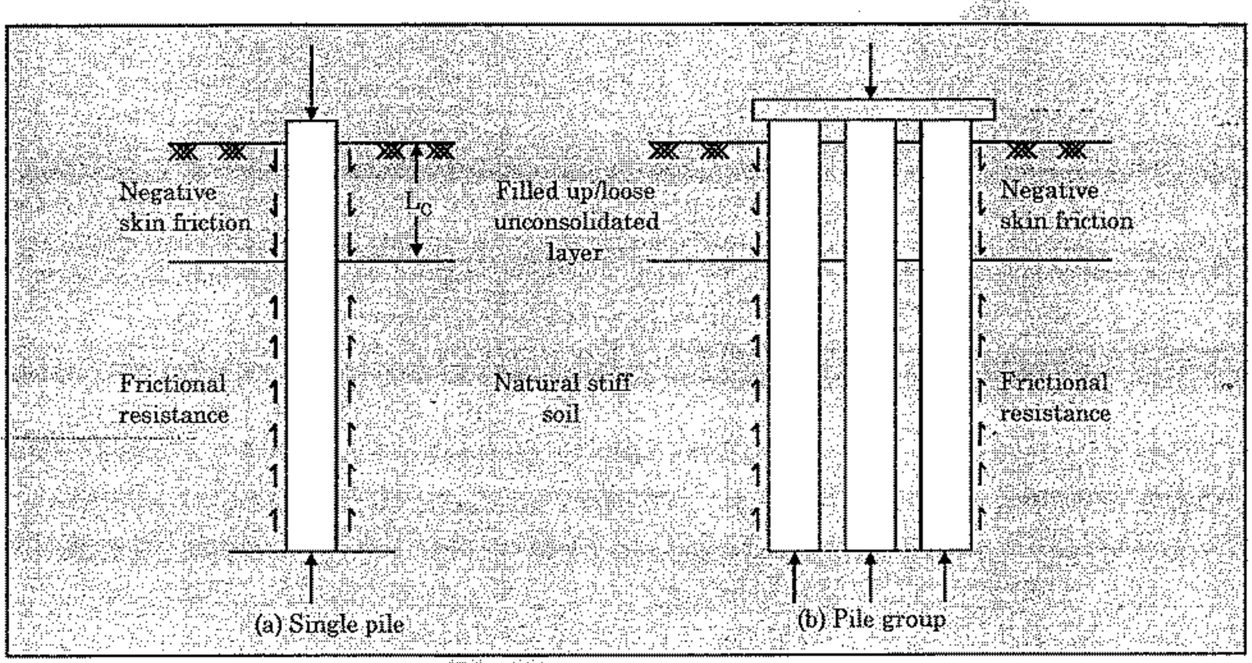
In the second iteration, the assumed and computed values of Q_u are quite close. Hence the ultimate load bearing capacity of the pile is 54 tonn.

$\therefore$ Therefore, the safe bearing capacity

$$Q_s = \left(\frac{Q_u}{F}\right) = \left(\frac{54}{2.5}\right) = 21.6 \text{ tonn.}$$

Negative Skin Friction

- Negative skin friction or 'down drag' is a phenomenon, which occurs when a portion soil layer surrounding a pile settles more than the pile.
- This condition can develope when a soft soil stratum loacated above the pile tip is subjected to a compressive loading, the soil may settle more than the pile, also by lowering of ground water table which includes consolidation of the soft soil.
- Normally friction between pile and soil helps in carrying the axial load. Where negative skin friction (or down ward drag) developed, increases the load acting on the pile because the weight of consolidating layer is transfered to pile by friction, thus imposing extra load on the pile.



- A 10mm relative movement of soil and pile may be sufficient for the full negative friction to mobilise.
- Negative skin friction can be reduded either by providing a casing around the pile or by providing a bitumen coating around the precast pile.

Negative Skin Friction in Single Piles

- In case of single pile negative skin friction is equal to the frictional resistance of the pile embeded in the settling soil.

1. Cohessive soils: $F_n = P \times L_c \times \alpha \times C_u$
2. Cohession less soil $F_n = \frac{1}{2} \times P \times L_c^2 \, \gamma k \tan \delta$

where, Fn = negative skin friction on single pile

P = perimeter of pile

Lc = Length of pile in settling zone.

α = adhesion Factor

Cu = Undrained cohession of the compresible layer

K = Lateral earth presure coefficient

δ = angle of friction between pile and soil range is ($1/2\ \phi - 2/3\ \phi$)

Negative skin friction in pile groups.

- In case of pile groups the magnitude of negative skin friction can be taken as

F_{ng} = Frictional force on the block + weight of soil enclosed in the block.

or F_{ng} = Negative skin friction due to single pile × Number of files in the pile group.

The maximum of value is taken.

$$F_{ng} = nFn$$

$$F_{ng} = C_u L_c P_g + \gamma L_c A_g$$

where, F_{ng} = Negative skin friction of pile group

n = No of piles in a pile group

C_u = Undrained cohession of the compressible layer

Lc = Hength of pile in compressible layer

P_g = Perimeter of pile group ....

γ = Unit weight of soil within the pile group uptill length Lc.

A_g = Area of pile group

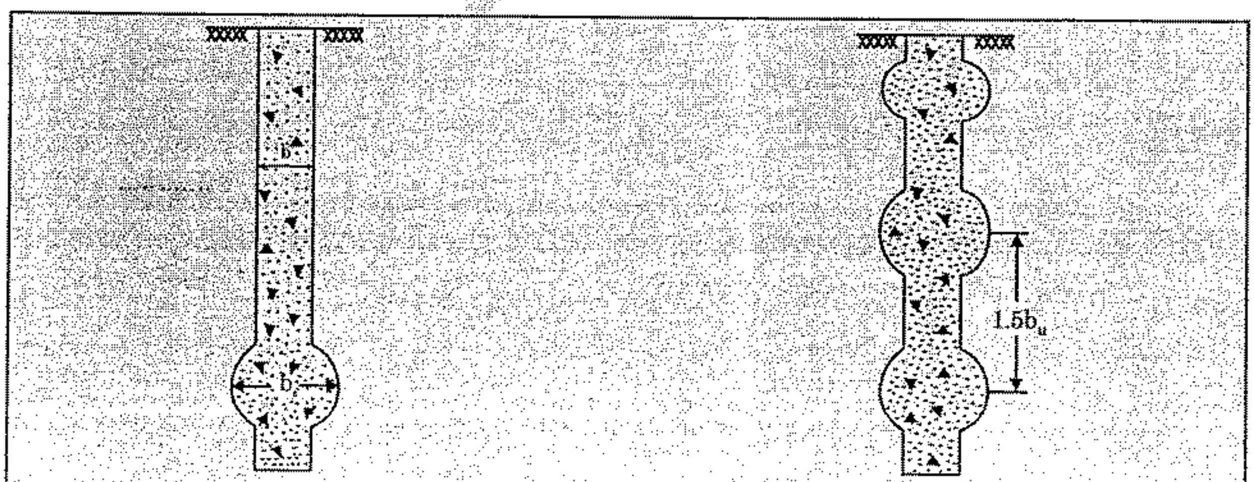
Effect of Negative skin friction on Factor of saftey.

- As it is necessary to subtract negative skin friction force from the total load that the pile can support. In such a case factor of saftey will be as below:

$$\text{F.O.S} = \frac{\text{Ultimate load capacity of single pile or a group of pile}}{\text{Working load + Negative Skin friction force}}$$

Under Reamed Piles

- An under reamed pile is a special type of bored pile with one or more bulb in the pile shaft as shown in the figure below.
- This bulb is called underream with diameter 2-3 times the dia of pile shaft.
- In case of more than one bulb provided the minimum spacing is 1.5 times the diameter of bulb.



(a) Single under-reamed pile (b) Multi-under-reamed pile

- The bearing capacity of pile increases because of the increase in base area and when the number of bulbs is increased from one to two the load carying capacity of the pile increases by 50%.
- Under reamed piles are very useful in case of expansive soils, where due to shrinkage and swelling of soil use of shallow spread footing is not suggestable.

Selection of Pile type

The selection of type of pile to be used in a project depends upon several factors.

(1) Type of structure and load on the pile→If magnitude of structural load is light then timber piles can be considered where as for heavy loads steel or reinforced concrete piles are used.

(2) Location of site→ in case of very busy and crowded area small or non diplacement piles are being used in place of large displacement piles.

(3) Soil conditions→ In case of loose and medium sand conditions displacement piles are suggested as they compact the soil around them. But in case of clayey soil, use of displacement piles cause heaving of soils, therefore Non displacement piles are used in such conditions.

(4) **Position of water table→** Driven piles are unaffected by the position of ground water table whereas in bored piles a close monitoring is required for the concreting below water table.

(5) Pile length. There are certain limitation to the length of pile.

Pile	Max. length
Timber	20 m
Driven concrete	30 m
Bored concrete pile	20 m
Steel pipe pile	upto any length

(6) Durability Enviromental conditions and pile material affects the durability of pile.

- Timber piles are liable to rot when subjected to alternate wetting and drying.
- Concrete piles are likely to detriorate in strength if sulphates are present in soil.
- Steel piles are likely to get corroded by the salts.

Pile load test

- Pile load test is only direct method for determining the allowable loads on the piles, and is considered to be most reliable due to the fact that, it is an insitu test.
- pile load test are basicly divided into two categories.

Pile load test
↓
Initial test | Routine test

- Initial test are carried out on test pile to asses the alowable load or to check the settlement at working load.
- Where as routine tests are carried out on working piles for the assesment of settlement under working load.

Note : *Test pile is a pile which is especially bored for the pourpose of conducting test and will not be part of foundation in the future.*

Working pile is a pile which is a part of foundation and is being used for the pourpose of testing at present. After the completion of test pile will be used as a foundation member.

- According to IS : 2911 Part IV for more than 200 piles there should be minimum of two intial test where as routine test is done on 0.5% to 2% of total number of piles.

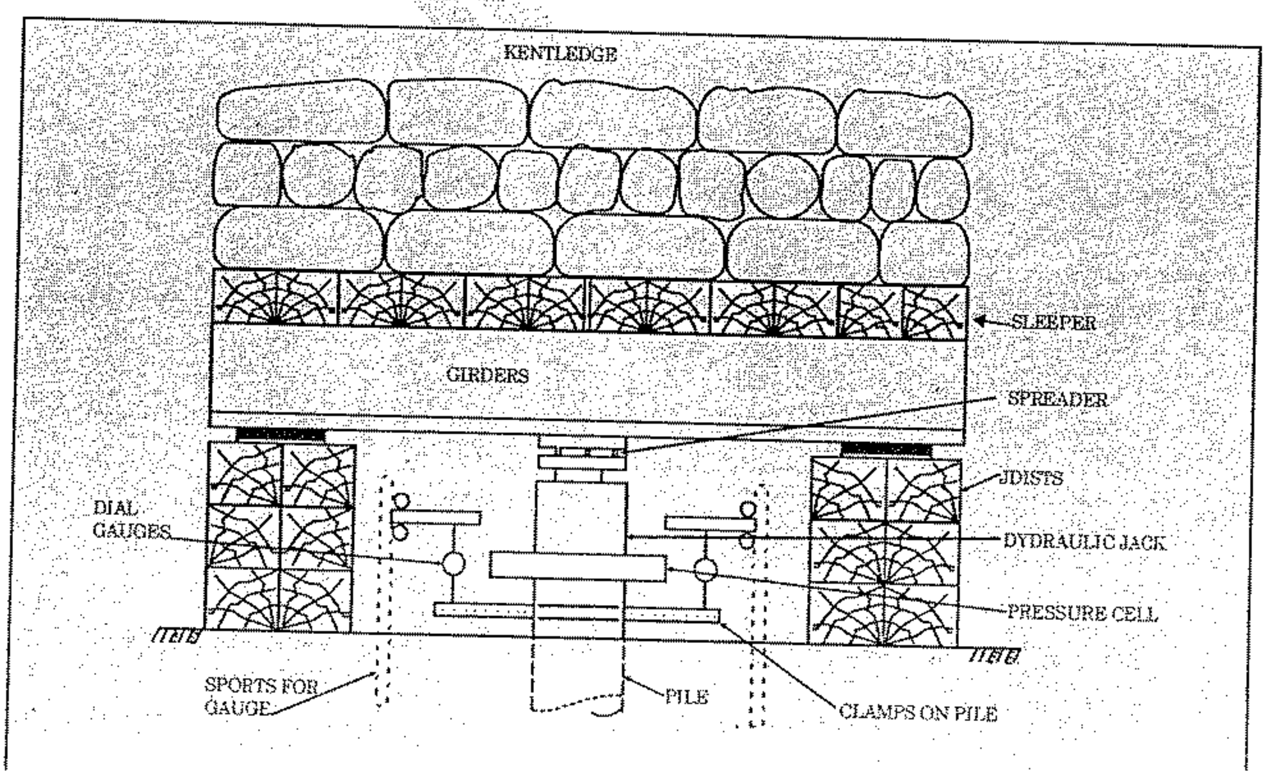
Note : Pile load test provide very useful results for design for cohessionless soil but in case of clays. The results obtained shall be used with great care as those are affected by the disturbance caused by pile driving, increase in pore water pressure and sufficient time is not given for consolidation.

Types of pile load test

1. **Vertical load test/compression load test** vertical load test is carried out to establish load settlement relationship under compression and to determine allowable load on pile.
2. **Lateral load test.** Lateral load test is carried out to determine safe lateral load on pile.
3. **Pullout test.** Pull out test is carried out to determine the safe tension on pile.
4. **Constant rate of penetration test.** Constant rate of penetration test is carried out to determine the ultimate load capacity of pile. In this test the load on the test is continously increased to maintain a constant rate of penetration (0.25 mm – 5.0mm per min). The ultimate load is determined from the load settlement curve drawn.
5. **Cyclic load test.** Cyclic load test is carried out when it is required to dermine, skin friction and end bearing seperately for a pile load on a singe pile.
   - **It is generally a intial test.** In this test load increments are applied in the increments 20%. of estimated safe load.
   - Loading and unloading is carried out alternatively at each stage.
   - The elastic rebound is measured at each stage with the help of dial gauges.

Note : Out of all of the above test generally vertical load test is carried out as an intial and routine test.

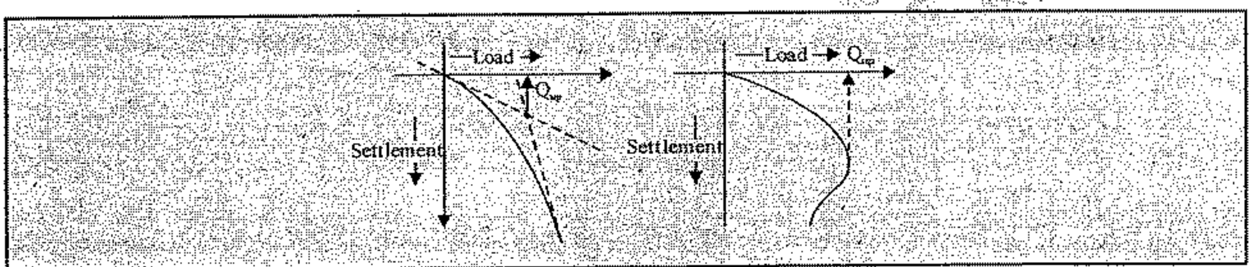
- The pile load test is performed by driving three piles in a row. The centre to centre spacing will vary from 1.8 m to 3.6m, and two outer piles serve as anchor piles.
- A strong beam is fixed to the anchor pile.



- The centre pile is the test pile and is jacked into the ground by a hydraulic jack placed between the top of the test pile and the beam.
- The final load which is 1.5 to 2.0 times the design load is reached is about 5 to 8 equal increments.
- Load increments and pile penetrations are recorded and plotted.
- Measurement of settlement is by dial gauges.
- Each increment of load is maintained till the rate of settlement is 0.25 mm per hour, and final load is maintained for 24 hours.
- Care should be taken to see that measurements are made with reference to a fixed mark outside the zone that could be affected by pile movements.

Usually two types of load settlement curves are obtained as shown below.

***Ultimate load will be calculated from loade settlement curve.**



Allowable load on single shall be minimum of the following cases : As per IS-2911 part IV.

(i) 50% of the ultimate load at which total settlement is equal to the1/10 of the diameter of pile.

(ii) 2/3rd of the load at which total settlement is 12 mm.

(iii) 2/3rd of the load at which net settlement is 6m (i.e. total settlement - elastic settlement).

Pile capacity on tbe basis of penetration resistance :

(1) S.P.T. Test data corelation

- If (N) = field S.P.T. Test value (un corrected) at a depth equal to depth of pile.

and, $(\bar{N})$ = Avg. field S.P.T. value over the depth of pile.
then

B = size of pile
D_f = depth of pile

$$q_b = \left(40.N.\frac{D_f}{B}\right) \rightarrow \text{unit} = (kN/m^2)$$

$$\not> 40(N)\,kN/m^2$$

$$q_s = (2\bar{N}) \rightarrow \text{unit} = (kN/m^2)$$

$$\not> \boxed{100}\,kN/m^2$$

Allowable value — $\boxed{Q_{allowable} = \frac{Q_{up}}{F.O.S.}}$

OBJECTIVE TYPE QUESTIONS

1. Consider the following statements regarding negative skin friction in piles:
   1. It is developed when the pile is driven through a recently deposited clay layer.
   2. It is developed when the pile is driven through a layer of dense sand.
   3. It is developed due to a sudden drawdown of the water table.

   Which of these statements is/are correct?

   (a) 1 alone (b) 2 alone
   (c) 2 and 3 (d) 1 and 3

2. A precast concrete pile is driven with a 50 kN hammer ralling through a height of 1.0 m with an efficiency of 0.6. The set value observed is 4 mm per blow and the combined temporary compression of the pile, cushion and the ground is 6 mm. As per Modified Hiley Formula, the ultimate resistance of the pile is

   (a) 3000 kN (b) 4285.7 kN
   (c) 8333 kN (d) 11905 kN

3. Match **List–I** (Soil property measured) with **List–II** (In-situ test) and select the correct answer using the codes given below the lists:

   List–I

   A. Modulusof subgrade reaction
   B. Relative density and strength
   C. Skin friction and point bearing
   D. Elastic constants

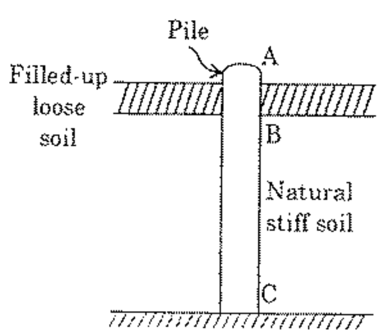
   Codes :

	A	B	C	D
(a)	1	3	2	4
(b)	1	2	4	3
(c)	2	4	1	3
(d)	3	4	1	2

4. A 30 cm diameter friction pile is embedded 10 m into a homogeneous consolidated deposit. Unit adhesion developed between clay and pile shaft is 4 t/m^2 and adhesion factor is 0.7. The safe load for factor of safety 2.5 will be

   (a) 21.50 t (b) 11.57 t
   (c) 10.55 t (d) 6.35 t

5. Which of the following statements are true for the pile shown in the figure?



1. Frictional resistance acts upwards throughout the length of the pile.
2. Negative skin friction acts over the length AB.
3. Frictional resistance acts upwards over the length BC.
4. There is point resistance at level C.

Select the correct answer using the codes given below:

(a) 1, 3 and 4 (b) 2, 3 and 4
(c) 1 and 2 (d) 2 and 3

6. Consider the following statements regarding under reamed piles:

1. They are used in expansive soils.
2. They are of precast reinforced concrete.
3. The ratio of bulb to shaft diameters is usually 2 to 3.
4. Minimum spacing between the piles should not be less than 1.5 times the bulb diameter.

Which of these statements are correct?

(a) 1, 2 and 3 (b) 1, 3 and 4
(c) 2, 3 and 4 (d) 1, 2 and 4

7. Minimum centre to centre spacing of friction piles of diameter (D) as per BIS codes is

(a) 1.5D (b) 2D
(c) 2.5D (d) 3D

8. For four free standing pile group having arc tan value of 18.3, the efficiency as per Converse Labarre formula, would be

(a) 76% (b) 78%
(c) 80% (d) 82%

9. Ratio of bearing capacity of double Under Reamed (UR) pile to that of single reamed pile is nearly

(a) 2 (b) 1.5
(c) 1.2 (d) 1.7

10. Match **List-I** (Suitable condition) with **List-II** (Foundations) and select the correct answer using the codes given below the lists:

List-I	List-II
A. When structural load is uniform and soil is soft clay, made up of marshy land.	1. Footings
B. When structural load is heavy and/or soil having low bearing capacity for considerable depth.	2. Piles
C. When soil is having good bearing capacity at shallow depth and structural load is within permissible limit.	3. Rafts
D. When structural load of bridge is to be transferred through sandy soil to bed rock.	4. Wells or piers

Codes:

	A	B	C	D
(a)	3	1	2	4
(b)	4	1	2	3
(c)	4	2	1	3
(d)	3	2	1	4

11. Depth of foundation depends upon
(a) scour depth, minimum grip length and Rankine depth
(b) scour depth, minimum grip length and depth of bearing stratum
(c) scour depth, Rankine depth and depth of bearing stratum
(d) minimum grip length, Rankine depth and depth of bearing stratum

12. In the Engineering News Record formula for determining the safe load carrying capacity of a pile, the factor of safety used is
(a) 2.5
(b) 3
(c) 4
(d) 6

13. In the case of a pile foundation, negative skin friction may occur at a load which is
(a) lower than the designed load
(b) higher than the designed load
(c) equal to the designed load
(d) of any magnitude

14. In under-reamed pile construction, the ratio of shaft diameter to bulb diameter is
(a) 1/1.5
(b) 1/2
(c) 1/2.5
(d) 1/4

15. Match **List-I** (Field test) with **List-II** (Property) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Pumping test	**1.** Bearing capacity
B. Plate load test	**2.** Load carrying capacity
C. Pile load test	**3.** Permeability

Codes:

	A	B	C
(a)	1	2	3
(b)	3	1	2
(c)	2	3	1
(d)	3	2	1

16. A single pile, 50 cm in diameter and 15 m long is driven in clay having an average un-confined compressive strength of 100 kN/m^2. The ultimate bearing capacity of the pile, neglecting end bearing, if any, and assuming shear mobilization factor of 0.8 around the pile is
(a) 942 kN
(b) 1884 kN
(c) 1177.5 kN
(d) 1334.5 kN

17. Match **List-I** (Method of estimating pile capacity) with **List-II** (Parameter to be estimated) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Dynamic formulae	**1.** Bearing capacity of cast-in-situ concrete pile
B. Static formulae	**2.** Separating end-bearing and friction-bearing powers of a pile
C. Pile load test	**3.** Bearing capacity of a timber pile
D. Cyclic pile load test	**4.** Settlement of a friction bearing pile

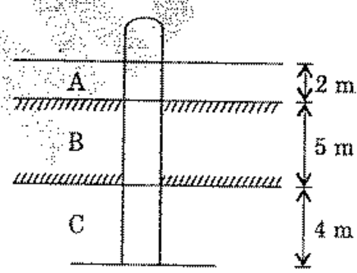
Codes:

	A	B	C	D
(a)	3	1	4	2
(b)	4	2	3	1
(c)	3	2	4	1
(d)	4	1	3	2

18. A pile of 0.50 m diameter and of length 10 m is embedded in a deposit of clay. The undrained strength parameters of the clay are cohesion = 60 kN/m^2 and the angle of internal friction = 0. The skin friction capacity (kN) of the pile for an adhesion factor of 0.6, is

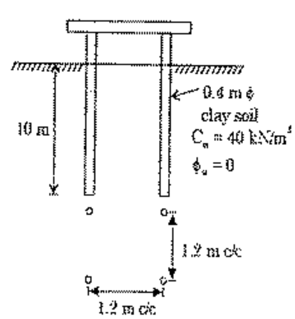
(a) 671
(b) 565
(c) 283
(d) 106

19. Skin frictional capacities of a 40 cm diameter driven concrete pile for the portions A, B and C are 17 kN, 63 kN and 503 kN respectively, and point load capacity is 11000 kN/m^2. Total pile load capacity will be



(a) 3743 kN
(b) 2864 kN
(c) 1965 kN
(d) 1529 kN

20. What is the ultimate capacity in kN of the pile group shown in the figure assuming the group to fail as a single block?



(a) 921.6
(b) 1177.6
(c) 2438.6
(d)

21. Match **List-I** (Type of Foundation) with **List-II** (Type of Soil) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Floating piles	**1.** Closed spaced columns resting on compressible soil
B. Micro piles	**2.** Expansive soils
C. Combined footing	**3.** Deep soft clays
D. Under-reamed piles	**4.** Loose sands

Codes:

	A	B	C	D
(a)	2	1	4	3
(b)	2	4	1	3
(c)	3	1	4	2
(d)	3	4	1	2

22. A square pile of section 30 cm × 30 cm and length 10 m penetrates a deposit of clay having c = 5 kN/m^2 and the mobilizing factor m = 0.8. What is the load carried by the pile by skin friction only?

(a) 192 kN
(b) 75 kN
(c) 60 kN
(d) 48 kN

23. Consider the following statements:

1. Pile foundations are usually provided when loads coming on the foundation are quite large. Such piles may often extend up to a large depth below ground level.
2. Precast piles inserted into the holes bored at the site do not get damaged while they are driven into the ground.

Which of these statements is/are correct?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

24. Match **List-I** (Type of foundation) with **List-II** (Use of the foundation) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Point bearing piles	**1.** To retain soil filling
B. Sheet piles	**2.** To transfer heavy loads to strong stratum below a weak stratum
C. Compaction piles	**3.** To resist lateral loads
D. Batter piles	**4.** To densify loose soils

Codes:

	A	B	C	D
(a)	3	1	4	2
(b)	3	4	1	2
(c)	2	1	4	3
(d)	2	4	1	3

25. Match **List-I** (Type of soil) with **List-II** (Suitable foundation) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Strong soil in surface layer	1. Raft foundation
B. Weak surface layer followed by rock at shallow depth below ground	2. Isolated footing
C. Swelling soil in surface layer extending up to a few meters below ground level	3. End bearing pile
D. Weak heterogeneous surface soil layer	4. Under-reamed piles

Codes:

	A	B	C	D
(a)	1	4	3	2
(b)	2	3	4	1
(c)	1	3	4	2
(d)	2	4	3	1

26. Match **List-I** (Type of pile) with **List-II** (Situation for use) and select the correct answer using the codes given below the lists:

List-I	List-II
A. End bearing pile	1. When weak foundation soil is to be compacted
B. Pedestal pile	2. When foundation soil is granular
C. Friction pile	3. When foundation soil is relatively weak
D. Sand piles	4. When hard formation or rock is at a shallow depth

Codes:

	A	B	C	D
(a)	4	2	3	1
(b)	1	3	2	4
(c)	4	3	2	1
(d)	1	2	3	4

27. While driving a large number of piles in loose sand
   (a) it is advantageous to follow a sequence of pile driving such that the inner piles are driven first and then proceed outwards
   (b) it is advantageous to follow a sequence of pile driving such that the piles near the periphery are driven first and inner piles are driven later
   (c) it is advantageous to follow a sequence of pile driving such that alternately inner and outer piles are driven
   (d) driving of piles can be done in any random order

28. Match **List-I** (Type of foundations) with **List-II** (Suitability) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Spread footings	**1.** Soft clay for 20 m followed by hard rock stratum
B. Under reamed piles	**2.** Up to 3 m black cotton soil followed bymedium dense sand
C. Raft foundation	**3.** Compact sand deposit extending to great depth
D. Deep foundation	**4.** Loose sand extending to great depth

Codes:

	A	B	C	D
(a)	4	1	3	2
(b)	3	2	4	1
(c)	4	2	3	1
(d)	3	1	4	2

29. Consider the following statements:
   1. Initial load tests and routine tests are carried out on test piles and working piles, respectively.
   2. A cyclic load test is performed to determine a pile's skin resistance and base resistance separately.
   3. In a pile load test, the safe load is taken as half the final load at which the settlement equal 10% of pile-diameter.

   Which of these statements are correct?
   (a) 1, 2 and 3
   (b) Only 1 and 2
   (c) Only 2 and 3
   (d) Only 1 and 3

30. Which of the following types of piles is likely to have the highest load capacity in compression?
   (a) Driven pre-cast concrete pile
   (b) Pre-cast pile placed in a pre-drilled bore
   (c) Driven steel pile pile
   (d) Steel pipe pile placed in a pre-drilled bore

31. Consider the following statements about the under-reamed pile in swelling soils:
   1. Its bulb provides anchor against movement due to volume changes of soil.
   2. It is a driven pile.
   3. Its bulb diameter is 2.5 times its shaft diameter.

   Which of these statements are correct?
   (a) 1, 2 and 3
   (b) 1 and 2 only
   (c) 2 and 3 only
   (d) 1 and 3 only

32. Consider the following statements:
   1. Underreamed piles are precast piles with one or more underreams in each pile.
   2. The ratio of pile shaft size to bulb size in an underreamed pile may be 0.33 to 0.50.
   3. In a multibulb underreamed pile, the load-carrying capacity is a function of the area of cross-section of the lowest bulb.

   Which of these statements is/are correct?
   (a) 1 only
   (b) 1 and 2 only
   (c) 2 and 3 only
   (d) 1, 2 and 3

33. Consider the following statements:
   1. Underreamed piles are designed as bearing piles.
   2. In multiple-bulb underreamed piles, the bulbs are spaced at 1.5 to 2.0 times the diameter of the underream, the centre of the first underream being at a minimum depth of 1.75 m.
   3. The length of traditional underreamed pile ranges from 3 ...

Which of these statements is/are correct?

(a) 2 and 3
(b) 1 and 2
(c) 2 only
(d) 1 and 3

34. All theoretical approaches indicate that at greater depths the bearing capacity of pile base in sands should be practically independent of the size and be proportional to

(a) shape of pile
(b) overburden pressure (σ)
(c) frictional angle (ϕ)
(d) shape of pile and friction angle (ϕ)

35. If the settlement of a single pile in sand is denoted by S and that of a group of N identical piles (each pile carrying the same load) by S_g, then the ratio S_g/S will

(a) be equal to 1 irrespective of width of the group
(b) be equal to N irrespective of width of the group
(c) decrease as the width of the group increases
(d) increase as the width of the group increases

36. Friction piles are most effective in

(a) soft clays
(b) dense sands
(c) organic soils
(d) filled up soils

37. Dolphin is a type of which one of the following?

(a) Pile foundation
(b) Isolated footing
(c) Raft foundation
(d) Caisson

38. Which of the following is **not** related to design of pile foundations?

(a) Pull out test
(b) Cyclic loading
(c) Plate load test
(d) Integrity test

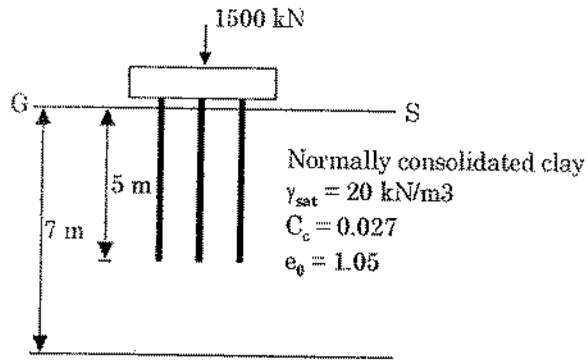
39. Two piles, one a bored cast-in-situ pile and another a precast driven pile, both of same length and diameter, are constructed in a loose sand deposit. If the bearing capacity of the bored pile is Q_1, and that of the precast driven pile is Q_2, then which one of the following is correct?

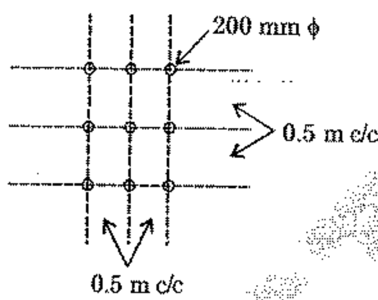
(a) $Q_1 > Q_2$
(b) $Q_1 = Q_2$
(c) $Q_1 < Q_2$
(d) No specific comparison may hold good

40. Which one of the following is responsible for development of negative skin friction on a pile?

(a) Downward movement of surrounding soil, irrespective of pile movement
(b) Downward movement of pile, irrespective of settlement of soil
(c) Settlement of pile more than that of surrounding soil
(d) Settlement of pile less than that of surrounding soil

41. For the (3 × 3) pile group shown in the figure, the settlement of pile group, in a normally consolidated clay stratum having properties as shown in the figure, will be

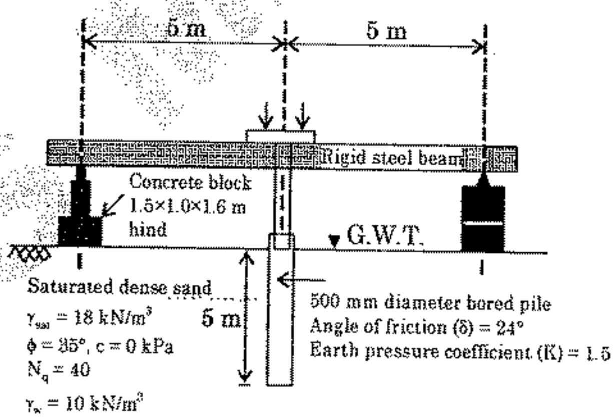




(a) 13.2 mm
(b) 12.775 mm
(c) 7.345 mm
(d) none of these

Common Data for Questions 42 and 43 :

Examine the test arrangement and the soil properties given below:



42. The maximum pressure that can be applied with a factor of safety of 3 through the concrete block, ensuring no bearing capacity failure in soil using Terzaghi's bearing capacity equation without considering the shape factor, depth factor and inclination factor is

(a) 26.67 kPa
(b) 60 kPa
(c) 90 kPa
(d) 120 kPa

43. The maximum resistance offered by the soil through skin friction while pulling out the pile from the ground is

(a) 104.9 kN
(b) 209.8 kN
(c) 236 kN
(d) 472 kN

Common Data for Questions 44 and 45:

A group of 16 piles of 10 m length and 0.5 m diameter is installed in a 10 m thick stiff clay layer underlain by rock. The pile-soil adhesion factor is 0.4; average shear strength of soil on the sides is 100 kPa; undrained shear strength of the soil at the base is also 100 kPa.

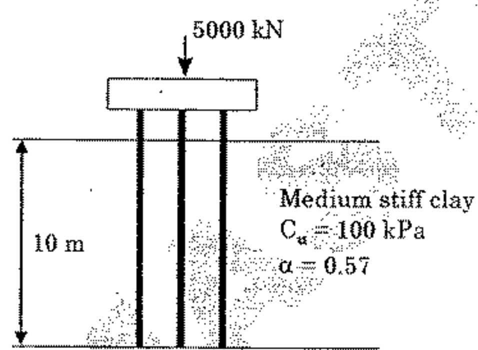
44. The base resistance of a single pile is

(a) 40.00 kN (b) 88.35 kN
(c) 100.00 kN (d) 176.71 kN

45. Assuming 100% efficiency, the group side resistance is

(a) 5026.5 kN (b) 10000.0 kN
(c) 10053.1 kN (d) 20106.0 kN

46. For the soil profile shown in figure below, the minimum number of precast concrete piles of 300 mm diameter required to safety carry the load for a given factor of safety of 2.5 (assuming 100% efficiency for the pile group) is equal to



(a) 10 (b) 15
(c) 20 (d) 25

Instructions :

*The following items consists of two statements, one labelled as **'Assertion A'** and the other labelled as **'Reason R'**. You are to examine these two statements carefully and decide if the Assertion A and the Reason R are individually true and if so, whether the Reason is a correct explanation of the Assertion. Select your answers to the these items using the codes given below :*

(a) both A and R are true and R is the correct explanation of A
(b) both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) *A is false but R is true*

47. Assertion (A): Pile driving formulae have very limited use in the case of cohesive soils.
Reason (R): Pile driving in cohesive soils results in liquefaction of clays.

48. Assertion (A): Generally driven piles are adopted in granular soils and not in clays.
Reason (R): Vibratory loading helps in densification of sands but it has adverse effects in clays.

49. Assertion (A): Under-reamed piles are suitable for loose filled up sites and black cotton soils.
Reason (R): Black cotton soils have expansive montmorillonite clay minerals.

50. Assertion (A): Negative skin friction will act on the piles of a group in filled up reclaimed soils or peat soils, which should be considered in design of pile groups.

Reason (R): The filled up or peat soils are not fully consolidated but start consolidating under their

51. **Assertion (A):** Dynamic formulae are not recommended for computing allowable loads of piles driven into cohesive soils.

Reason (R): In cohesive soils, the resistance to pile driving increases due to any sudden increase in pressure in the pore water.

52. **Assertion (A):** The load-carrying capacity of bored cast in situ pile in a sand soil is much less than that of driven pile of similar dimensions.

Reason (R): A driven pile generates much more point bearing resistance than a bored pile.

53. **Assertion (A):** Bearing capacity of an under-reamed pile is less than that of a straight bored pile of the same diameter.

Reason (R): Under-reamed piles have enlarged bulbs.

54. **Assertion (A):** Franki pile has an enlarged base of mushroom shape which gives the effect of spread footing.

Reason (R): The Franki pile is best suited for granular soil.

ANSWERS

1. (d)	2. (b)	3. (d)	4. (c)	5. (b)	6. (b)	7. (d)	8. (c)	9. (b)	10. (d)
11. (b)	12. (d)	13. (d)	14. (c)	15. (b)	16. (a)	17. (a)	18. (b)	19. (c)	20. (d)
21. (d)	22. (d)	23. (a)	24. (c)	25. (b)	26. (c)	27. (a)	28. (b)	29. (a)	30. (a)
31. (d)	32. (c)	33. (d)	34. (c)	35. (d)	36. (b)	37. (a)	38. (c)	39. (c)	40. (c)
41. (d)	42. (a)	43. (a)	44. (d)	45. (c)	46. (c)	47. (c)	48. (a)	49. (a)	50. (a)
51. (a)	52. (a)	53. (d)	54. (b)						

Hints

4. $q_s = \frac{q_u}{fos} = \frac{\propto C_u A_s}{fos}$

$$= \frac{0.7 \times 4 \times \pi \times 0.3 \times 10}{2.5} = 10.55\,t$$

8. $\eta_g = 1 - \theta\frac{(n-1)m + (m-1)n}{90 \times m \times n}$

$$= 1 - 18.3 \times \frac{2 \times 1 + 2 \times 1}{90 \times 2 \times 2} = 79.66\%$$

16. $q_u = \alpha C \pi D L$

$$= 0.8 \times 50 \times \pi \times .5 \times 15 = 942\,kN$$

19.

$$q_{nu} = 503 + 63 + 17 + 11000 \times \frac{\pi \times 0.4^2}{4} = 1965\,kN$$

22. $q_u = \alpha \times c \times P \times L$

$= 0.8 \times 5 \times 4 \times .3 \times 10$

$= 48$ kN

41. $\Delta H = \frac{c_c H_0}{(1 + e_0)} \log_{10}\left(\frac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_0}\right)$

Area of c/s at the centre of clay=(5.23 × 5.23)

$$\bar{\sigma}_0 = 20 \times \left(\frac{10}{3} + \frac{11}{6}\right) = 103.33\,kN/m^2$$

$$\Delta\bar{\sigma} = \frac{1500}{(5.23 \times 5.23)} = 54.84\,kN/m^2$$

$$\Delta H = \frac{\frac{11}{3} \times 0.027}{(1 + 1.05)} \log_{10}\left(\frac{103.33 + 54.84}{103.33}\right)$$

$\therefore\ \Delta H = 8.93\,mm$

43. $\bar{\sigma}_0 = \frac{\gamma H}{2} = \left(\frac{18-10}{2}\right)\times 5 = 20$

$Q_f = k\bar{\sigma}_0 A_s \times \tan\delta$

$= 1.5\times 20\times\pi\times 0.5\times 5\times\tan 24 = 104.90\,kN$

44. $A_b = \frac{\pi}{4}(0.5)^2 = 0.196$

$A_f = \pi d l = 15.71$; $\alpha = 0.4$

$c = 100\,kN/m^2$

$Q_b = (CN_c)\frac{\pi}{4}d^2$

$Q_b = (100\times 9\times 0.196) = 176.4\,kN$

45. $Q_{ug} = n\alpha C\pi DL$

$= 16\times 0.4\times 100\times\pi\times 0.5\times 10$

$= 10053.1$ kN

46. Capacity of single pile $Q_u = CN_C\frac{\pi}{4}d^2 + \alpha c\pi DL$

$= 100\times 9\times\frac{\pi}{4}\times 0.3^2 + 0.57\times 100\times\pi\times 0.3\times 10$

$= 618.28$ kN

$Q_{ns} = \frac{618.25}{2.5} = 247.312\,kN/m^2$

No. of Piles $= \frac{5000}{247.312} = 20.21 = 20\,Nos$

CHAPTER

14

Soil Exploration

INTRODUCTION

- The field and laboratory studies carried out for obtaining the necessary information about the soil characterstics including the position of ground water table is termed as Soil Exploration.
- The elements of soil exploration depend mostly on the importance and magnitude of the project, but generally should provide the following,
  (1) Information to determine the type of foundation required such as a shallow or deep foundation.
  (2) Necessary information with regards to the strength and compressibility characterstics of the subsoil to allow the design consultant to make recomendation on the safe bearing presure or pile load capacity.
- Soil exploration involves broadly the following.
  (1) Planning of a program for soil exploration.
  (2) Collection of disturbed and undisturbed soil or rock samples from the holes dilled in the field. The number and depths of holes depend upon the project.
  (3) Conducting all necessary in-situ tests for obtaining the strength and compressibility characteristic the soil or rock directly or indirectly.
  (4) Study of ground water conditions and collection of water samples for chemical analysis.
  (5) Geophysical exploration, if required.
  (6) Conducting all the necessary test on the samples of soil/rock and water collected.
  (7) Preperation of drawing, charts etc.
  (8) Analysis of the data collected.
  (9) Report preperation

METHOD OF EXPLORATION

- The sub soil explorations are generally carried out in 2 stages.
  (1) Preliminary stage → In this stage we do geological study of site and the site reconnoissance .
  (2) Detailed stage → Detailed site investigation is followed by preliminary stage.
- In this stage nature, sequence and thickness of various subsoil layers, their lateral variations physical properties, position of ground water and many more are done.
- Boring and detailed sampeling are done to acquire above information.

BORING OF HOLES

- Making and advancing of Bore holes is known as boring.
- It is a first step for the collection of sample.

METHODS OF BORING

(1) Auger Boring

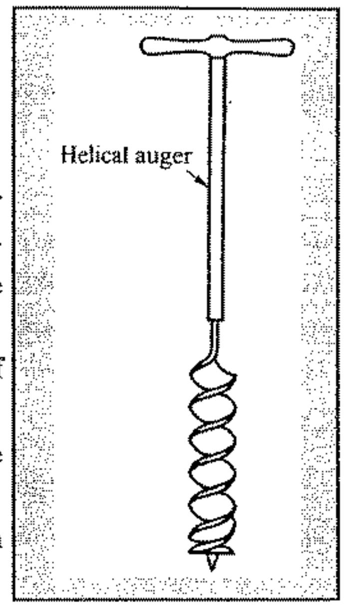
(2) Wash Boring

(3) Rotary Boring

(4) Percussion Boring

- The sutability of any method of boring depends on the nature of soil and the position of Ground water table.
- The ease and accuracy with which results are to be obtained also play a major role in deciding the method of boring .

1. Auger Boring



- In this type of boring is carried by an equipment called Auger,
- Auger is held vertically and pressed down while rotating it.
- Soil around the auger gets sheared and fills the annular space.
- As the annular space is filled, the auger is withdrawn and cleaned.
- The process is repeated by inserting the cleaned auger in the hole.
- Sampel obtained from Auger boring are highly disturbed and are used for identification purpose only.
- Hand operated augers are used for boring holes upto a depth of 6m.
- Power driven augers are used for greater boring depths or where hard or stiff soil strata are encountered.
- Auger broing is done in partially saturated sands, silts and medium to stiff clays.
- Auger boring is usually prefered from small depth of exploration of example shallow foundation, Highways and borrow pits.

Note: *A combination of shell and Auger is widely used these days. Shell is a heavy duty pipe with cutting edge at one end it is also called as sand bailer. Boring is always started with the auger and when when augering becomes difficult the shell is used. The shell is raised and let fall in the hole, which causes soil to enter into the shell which is emptied when full.*

(2) Wash Boring

- In this method a casing pipe is driven through a heavy drop hammer supported by a tripod and pully.
- Water is forced under pressure through the hollow drill rod and it is rotated up and down inside the casing pipe.
- The lower end of the drill rod is fitted with a sharp cutting edge or chopping bit, which cuts

- Soil water mixture floats up through the annular space between the casing pipe and the drill rod.
- Slurry or soil-water mixture flowing out provides an indication of the soil type. Whereas change in soil strata can be determined from the rate of progress and slurry flowing out.
- As the sample of soil is obtained in the form of soil water mixture, has no value because it is highly disturbed.

(3) Percussion Boring

- In this method a heavy drilling bit is alternately dropped and raised in such a manner that it grinds the underlying hard material to the consistency of a sand or silt.
- The bore hole is kept dry and only a small amount of water is added to form. A slurry with the material cut from the bit.
- The soil samples are obtained with the means of bailer, and the changes in character of the soil stratum can be determined by the rate of progress.
- In bouldery and gravelly stratum Percussion Boring is only suitable method.

(4) Rotatory Boring

- Rotatory Boring is of two types

(a) **Mud rotatory Boring** → A hole is drilled with the help of a rotating bit. Bentonite soil with some admixture is used as a drilling mud. Drill mud comes upwards through the anular space between the drill rod and the side of the hole.

(b) **Core drilling** → In this method core barrels with diamond bits are used in place of drilling bit, A solid core of rock formed inside the cylinder is taken out without jerk. Hence in this method samples obtained are with least disturbance.

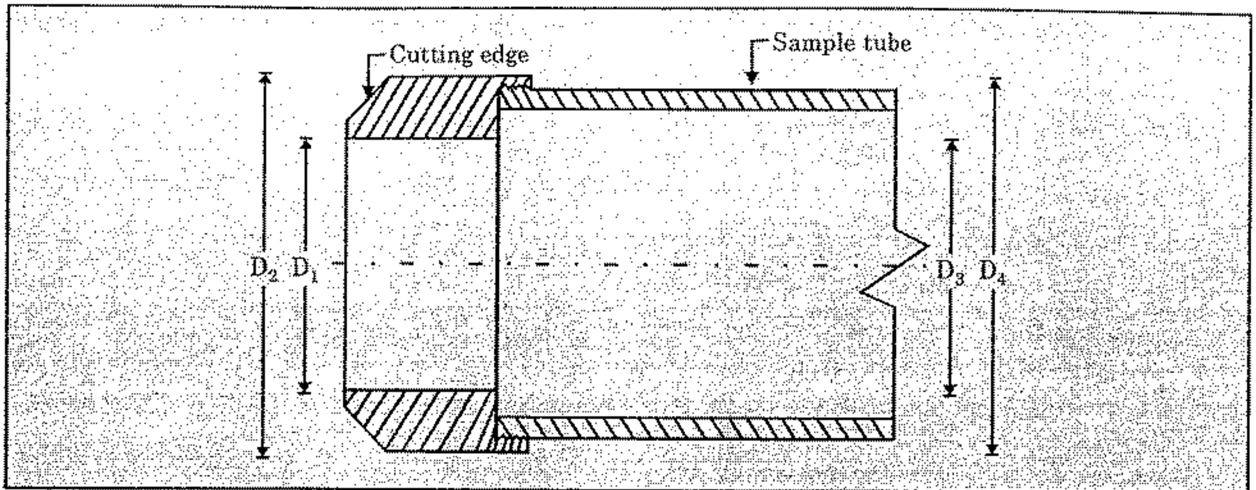
- Rotatory boring is useful in the soil which are highly resistant to auger and wash boring. eg. dense sand and dense clay.

SOIL SAMPLES

- Soil samples can be basicly classified into two samples disturbed samples & Undisturbed samples.
- Disturbed sample are those in which natural soil structure gets modified or destroyed during the sampling operation.
- But with suitable precautions, we can preserve the natural moisture content and the proportion of mineral constituents which is called as Representative samples, even though they are disturbed samples.
- Representative samples are being used for identification purpose where as Non representative samples are of no use.
- Undisturbed samples are those in which orignal soil structure is preserved as well as mineral properties have not undergone any change. Although its is impossible to obtain such samples in field but they play a major role in identification of soil properties such as soil structure, water content, shear strength, consolidation characterstics and etc.
- The extent of disturbance of sample due to the sampler depends upon.

  (a) Cutting edge

  (b) Inside wall friction



(a) Inside clearance, $C_i = \dfrac{D_3 - D_1}{D_1} \times 100$ per cent (1– 3%)

(b) Outside clearance, $C_0 = \dfrac{D_2 - D_4}{D_4} \times 100$ per cent (0– 2%) ($C_0 > C_i$)

(c) Area ratio, $A_r = \dfrac{D_2^2 - D_1^2}{D_1^2} \times 100$ per cent < 20 for stiff clay

< 10 for sensitive clay.

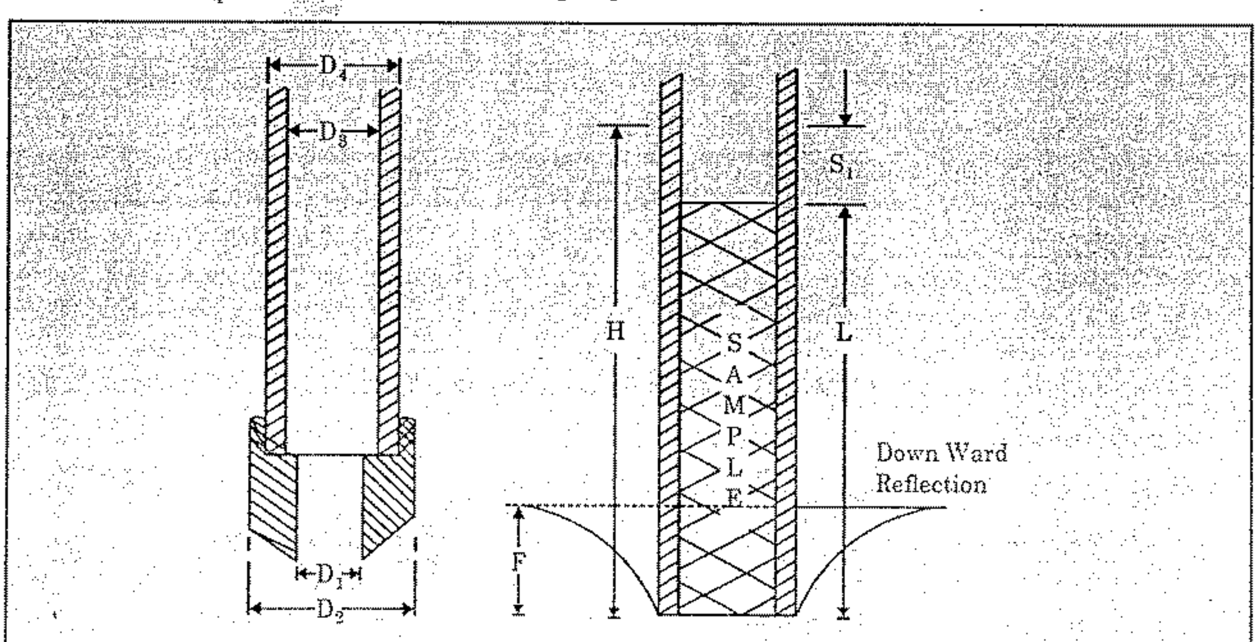
where,

D_1 = inside diameter of cutting edge/drive shoe

D_2 = outside diameter of cutting edge/drive shoe

D_3 = inside diameter of sample tube

D_4 = outside diameter of sampling tube



- In order to express the results of the sampeling operations and to have the satisfactory design requirements, following design criteria shall be used for samplers.

$$\text{Recovery Ratio} = \frac{L}{H}$$

where,

L = Length of sample before with drawl
H = Penetration of the sampler in the soil mass.

$$\text{Gross Recovery Ratio} = \frac{L_1}{H}$$

where,

L_1 = Gross length of the sample (equal to the distance from the top of the sample to the cutting edge irrespective of whether the lower part of the sample is lost or otherwise).

- From the above figure we can observe that H & L are not equal because of straining of soil layer and down ward deflection of soil adjacent to the sampler. Hence orignal thickness of the stratum is not H, it is H–f.

$$\therefore \text{ True recovery ratio} = \frac{L_1}{H-F}$$

Note: *Area ratio should be kept as lower possible. Which purely depends upon the efficiency of cutting edge.*

If Recovery ratio = 1 Good recovery
< 1 = Sample is compressed
> 1 = Sample is swelled

Type of test	Type of sample required
Natural water content	Undisturbed or SPT sample
Density	Undisturbed
Specific gravity	Representative or undisturbed
Grain size distribution	Rrepresentative or undisturbed
Atterberg limits	Representative or undisturbed
Coefficient of permeability	Undisturbed
Consolidation parameters	Undisturbed
Shear strength parameters	Undisturbed

SAMPLERS

The commonly used sampers are
(1) Open drive sampler
(2) Piston sampler
(3) Rotary sampler.

Open Drive Samplers

- Open drive samplers consists of a seamless open-end steel tube with a cutting edge. The tube is connected through a head to the drill rod.
- The sample head is provided with vents for the escape of water and air during sampling and alswo a ball check valve to retain the sample during the withdrawal of sampler.

- The sampling tubes are of two types.
  - (a) Thin Wall Samplers, and
  - (b) Thick Wall Samplers.

Thin Wall Samplers (IS: 2132-1972)

- Such samplers are made by cold-drawn seamless tube of brass, aluminium or any other suitable material.
- These sampling tubes are pushed into the soil in a continuous rapid motion without twisting.

Thick Wall Samplers

- Thick walled samplers have an area ratio of 10 to 25%. These are in the form of a solid tube or split tube with or without liner (IS 4640-1980).
- The split tube samplers are used in SPT determination.
- The soil sample is collected by thickwall sampler by repreated blows of a falling height.
- Thick walled samples are being used for obtaining disturbed but representative samples. Where as thin walled sampler is used for obtaining undisturbed samples.
- Open drive thin walled sampler can be used in all type of soil possesing some cohession.
- Such sampler can not penetrate in gravelly soil where as in too soft or two wet soil, cut soil sample can not be retained.

Piston sampler

- Piston sampler consists of two parts

(a) Sample cylinder

(b) Piston system

- A piston sampler is a drive sampler in which the lower end of the sampeling tube is closed with a piston. Piston fits thightly in the sampler cylinder and can be released or withdrawn at the time of sampeling.
- Piston sampler are useful in sampeling the saturated sands and soft and wet soils which can not be sampled by open drive sampler.

3. Rotatory Sampler

- A rotary samples are double walled tube samper with an inner removeable liner.
- The outer tube (rotating barrel) is having a cutting bit. The bit cuts an annular ring when the barrel is rotatted, while this inner tube will be stationary. This slides over the cut cylindrical sample by the outer rotating barrel, and the sample is collected in the inner liner.
- Rotary samplers are useful for sampling in firm to hard cohesive soils and rocks.
- The rock quality can be estimated by using the core recovery ratio termed as Rock quality designation, RQO.

DEPTH OF EXPLORORATION

- As a general rule, unless a bed rock is encounted, boring should be carried out to such a depth that the net increase in stress in the soil mass due to the super imposed external load is less than 10%.
- In case of square loaded areas isobar of 10% of intensity of loading is extended to a depth twice the width of foundation.

- I.S 1892 : 1979 recomeds following depth of Exploration.

Sl. No.	Type of foundation	Depth of exploration (D)
(i)	Isolated spread footing or raft	One and a half times the width B
(ii)	Adjacent footings with clear spacing less than twice the width	One and a half times the length (L) of the footing
(iii)	Adjacent rows of footings	
(iv)	Pile and well foundation	To a depth of one and a half times the width of structure from the bearing level (toe of pile or bottom of well)
(v)	(a) Road cuts	Equal to the bottom width of the cut
	(b) Fill	Two meters below ground level or equal to the height of the fill whichever is greater.

FIELD TESTS

- The field tests commonly used in subsurface investigation are :

1. Geophysical Methods
2. Penetration test
5. Pressuremeter test
3. Vane shear test
4. Plate load test

1. Geophysical Methods

The two commonly used geophysical methods are

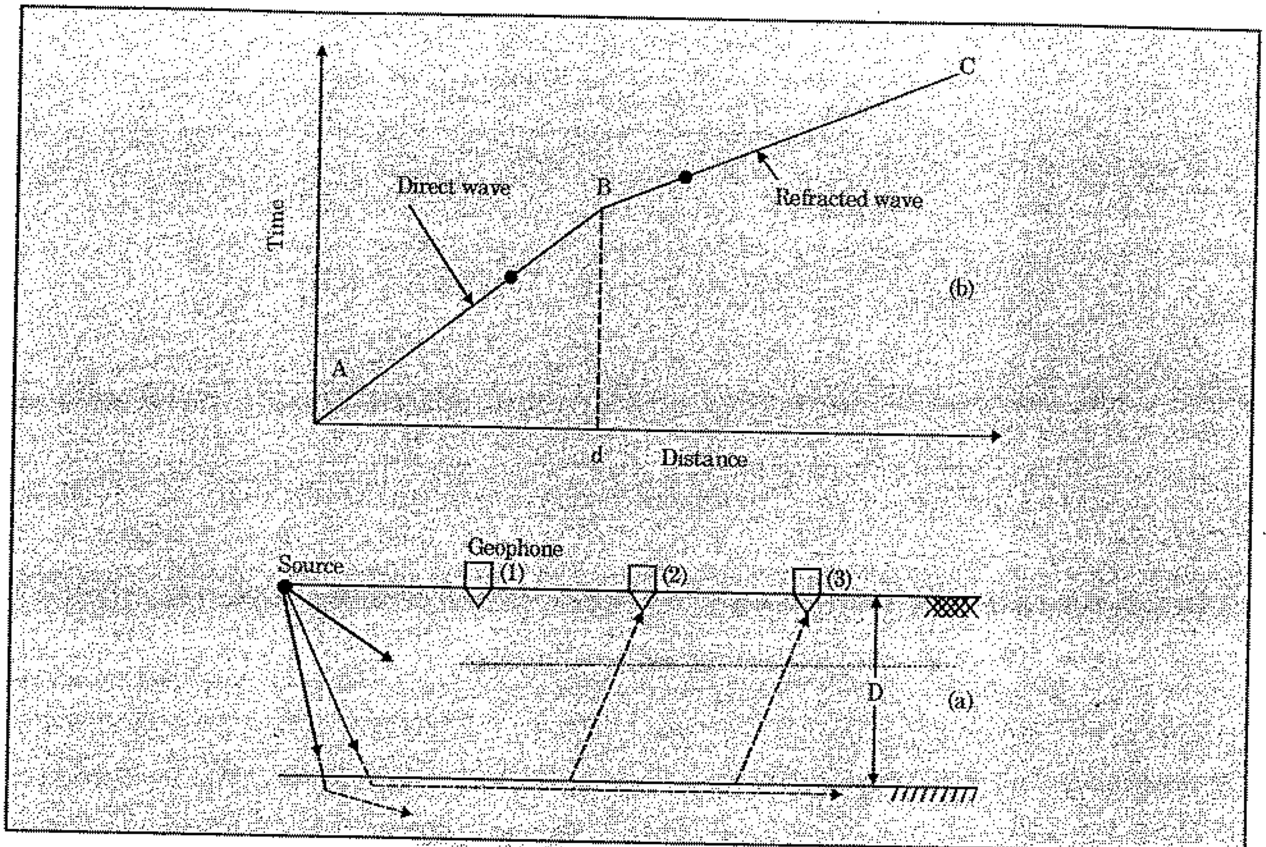
(a) Seismic Refraction method

(b) Electrical resistivity method

(a) Seismic Refraction Method

- Basic principal envolved in this method is that seismic waves have different velocities in different types of soil or rock, and waves get refracted when they cross the boundary between different types of soil.
- This method enables the determination of the general soil types and approximate depth of boundaries of stata or the bed rock.
- In this method a shock wave is generated either by striking a plate on the soil with a hammer or by exploding a charge at or near the ground surface.
- These radiating shock waves are recorded by a device called geophone, which records the time of travel of the waves.
- These geophones are installed at suitable known distances on the ground in a line from the source of shock.
- Some of the waves, termed direct or primary waves travel directly from the shock-source along the ground surface in the direction of the geophones. While other waves travel in a downward direction at various angles to the horizontal and will be refracted if they pass into a stratum of different seismic velocity.

- These reflected waves travel much faster if the underlying layer is denser.
- As the distance between the shock source and the geophone increases, refracted waves reach geophone earlier than the direct waves.



- The results are to be plotted. The break in distance time graph represents the point of simultaneous arrival of primary and refracted waves. This distance is known as "critical distance" which is a function of the thickness and velocity ratio of the deposit.
- If the spacing between source and geophone is less than d, the direct waves reaches the geophone earlier than the refracted wave, this time distance relationship is represented by a straight line AB through origin.
- If the spacing between source and geophone is greater than d, the refracted waves reaches the geophone earlier than the direct wave and is represend by a straight line BC on the time distance relationship plot.
- The slopes of lines AB and BC are the seismic velocities v_1 and v_2 of the upper and lower stratum respectively.
- The general types of soil or rock can be determined from a knowledge of these velocities.
- Depth of boundary between the two stratum can be estimated as follows.

$$2D = d\sqrt{\frac{v_2 - v_1}{v_2 + v_1}}$$

$$D = \frac{d}{2}\sqrt{\frac{v_2 - v_1}{\ldots}}$$

where,

D = depth between two stratum

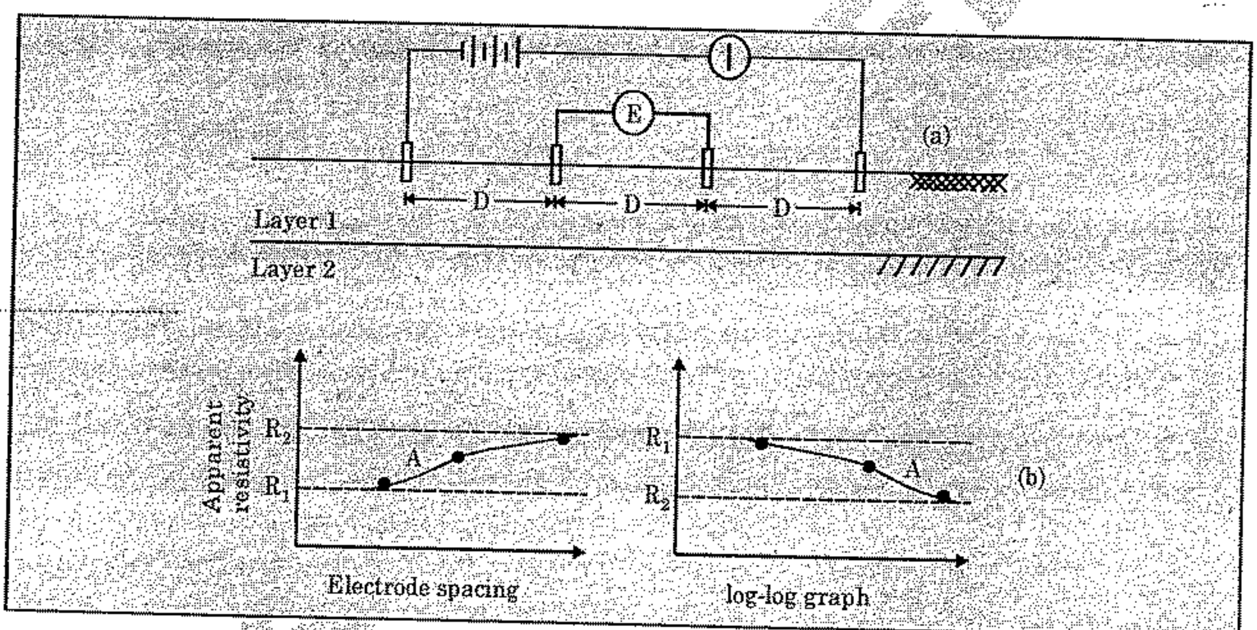
d = distance between source and geophone

V_1 & V_2 = Sesmic velocities of lower & upper stratum.

Note: The above equation is under assumption that thickness of upper stratum is unvarying.

- This method is quick and reliable for establishing profiles of different stratum provided that deeper layers have greater densities.
- But this method can not be used for exact analysis.
- For the purpose of exact analysis boring and sampeling should be done.

(b) Electrical Restivity Method



- This method is based on the measurement and recording of changes in the mean resistivety of various soils.
- Resistivity, d is the resistance between the opposite faces of a unit cube of the material.
- Variations in the resistivity between the different types of soil stratum can be detected for example, soil above or below water table, between fissured rocks.

  In this test four driving metal spikes are driven into the soil stratum along a straight line at equal distance.
- Current(I) from a battery is supplied between two outer metal spikes which produce an electrical field within the soil.
- A potential difference V is observed between the two inner metal spikes.
- Apparent resistivity, δ is given as below :

$$\delta = \frac{2\pi DV}{I}$$

where,

D = distance between spikes in, cm.

V = Potential difference in, Volts

I = Current in, Ampere.

δ = Resistance in, ohm – cm.

- It has been observed that if a stratum of low resistivity overlies a stratum of higher resistivity, the current is flow closer to the ground surface, resulting in a higher voltage drop and hence a higher value of apparent resistivity.
- If a stratum of high resistivity lies above a stratum of low resistivity, it would be the opposite.

  If the resistivity increase with increase in metas spikes spacing, the inference is that the underlying stratum with higher resistivity has influenced the value. Whereas, if increased spacing produced decreasing resistivity, it can be concluded that the lower resistivity of the underlying stratum is beginning to influence the readings.
- The thickness of a stratum is larger if its influence is observed over a greater spacing between the electrodes and *vice versa*.

2. Penetrometer Test

- There are three penetrometer tests in common use

(a) Standard penetration test (SPT)

(b) Dynamic cone penetration test (DCPT)

(c) Static cone penetration test (CPT)

- All the three test measure the resistance of the soil strata to penetration. SPT is carried out in a bohehole where as DCPT and CPT are carried out with out Bore hole.

Dynamic Cone Penetration Test

- In this test, a cone having apex angle 60° is driven into the soil by blows of a hammer of 65Kg, falling freely from a height of 750 mm.
- Number of blows for each 100mm penetration is recorded. The cone is driven in the soil upto the required depth or till the refusal of peneration.
- Number of blows required for 300 mm penetration is recoded as Dynamic cone resistance, N_{cd}.
- No sample is obtained during this test.
- This test is quick and less expensive than the SPT test. It helps in identifying the uniformity or the variability of the sub soil profile.

Note: Dynamic cone penetration tests is performed either by using a 50 mm diameter cone without bentonite slurry (IS: 4968—Part I—1976) or 65 mm diameter cone with bentonite slurry (IS: 4968—Part II—1976).

OBJECTIVE TYPE QUESTIONS

1. Consider the following statements:
   In subsoil exploration programme the term "significant depth of exploration" is upto
   1. the width of foudation
   2. twice width of foundation
   3. the depth where the additional stress intensity is less than 20% of overburden pressure
   4. the depth where the additional stress intensity is less than 10% of the overburden pressure
   5. hard rock level

   Which of these statements is/are correct?
   (a) 1, 3 and 5 (b) 2, 3 and 5
   (c) 1 and 4 (d) 2 and 4

2. A good quality undisturbed soil sample is one which is obtained using a sampling tube having an area ratio of
   (a) 8% (b) 16%
   (c) 24% (d) 32%

3. Which one of the following tests CANNOT be done without undisturbed sampling?
   (a) Shear strength of sand
   (b) Shear strength of clay
   (c) Determination of compaction parameters
   (d) Atterberg limits

4. Consider the following statements:
   1. Dynamic cone penetration test for site investigation is based on the principle that elastic shock waves travel in different materials at different velocities.
   2. Electrical resistivity method of subsurface investigation is capable of detecting only the strata having different electrical resistivity.
   3. In-situ vane shear test is useful for determining the shear strength of very soft soil and sensitive clays and is unsuitable for sandy soil.

   Which of these statements is/are correct?
   (a) 1 and 2 (b) 1 and 3
   (c) 2 and 3 (d) 2 alone

5. A soil sampler has inner and outer radii of 25 mm and 30 mm, respectively. The area ratio of the sampler is
   (a) 24% (b) 34%
   (c) 44% (d) 54%

6. Consider the following statements:
   1. A recovery ratio of less than 1 implies that the soil has compressed.
   2. A recovery ratio greater than 1 implies that the soil has swelled.
   3. A recovery ratio of less than 1 implies that the soil has swelled
   4. A recovery ratio greater than 1 implies that the soil has compressed.

Which of these statements is/are correct?

(a) 1 and 2
(b) 1 only
(c) 3 and 4
(d) 4 only

7. During a sampling operation, the drive sampler is advanced 600 mm and the length of the sample recovered is 525 mm. What is the recovery ratio of the sample?

(a) 0.125
(b) 0.140
(c) 0.875
(d) 0.143

8. A sampling tube with a cutting edge is used for extracting the samples. The sampling tube has the following dimensions:

Inner diameter of cutting edge = D_c

Outer diameter of cutting edge = D_w

Inner diameter of the sampling tube = D_s

Outer diameter of the sampling tube = D_t

What is the area ratio A_r of the sampling tube?

(a) $A_r = \frac{D_w^2 - D_c^2}{D_c^2} \times 100\%$
(b) $A_r = \frac{D_t^2 - D_c^2}{D_c^2} \times 100\%$
(c) $A_r = \frac{D_t^2 - D_w^2}{D_w^2} \times 100\%$
(d) $A_r = \frac{D_t^2 - D_s^2}{D_s^2} \times 100\%$

9. Consider the following statements:

1. The soil obtained from wash boring is a representative sample.
2. Recovery ratio will be high during drilling in sound rock.
3. Hollow stem augers are sometimes used to drill holes in silty sand.

Which of these statements is/are correct?

(a) 1 only
(b) 1 and 2
(c) 2 and 3
(d) 3 only

10. For sampling saturated sands and other soft and wet soils satisfactorily, the most suitable soil sampler is

(a) open drive thin-walled tube sampler
(b) standard split-spoon sampler
(c) stationary piston sampler
(d) rotary sampler

11. The upper limit of area ratio for which the amount of disturbance of soil sample can be considered to be small is

(a) 10%
(b) 15%
(c) 20%
(d) 25%

12. Consider the following types of soil tests:

1. California bearing ratio
2. Consolidation
3. Unconfined compression

The soil tests required to be done in the case of undisturbed samples include

(a) 1, 2 and 3
(b) 1 and 2
(c) 1 and 3
(d) 2 and 3

13. Boring method is to be chosen depending upon the type of exploratory strata. In this context, match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

List-I	List-II
A. Auger boring	**1.** Partly saturated sands, silts and medium of stiff cohesive soils
B. Wash boring	**2.** All types of soils and rocks except in stony or porous soils and fissured rocks
C. Percussion drilling	**3.** Practically all types of soils except hard and cemented soil or rock
D. Rotary drilling	**4.** All types of soils and rocks. Difficult in loose sands and soft sticky clays

Codes:

	A	B	C	D
(a)	1	4	3	2
(b)	1	3	4	2
(c)	2	4	3	1
(d)	2	3	4	1

14. Consider the following statements:

To obtain an undisturbed soil sample from bore hole in soft soils, the conditions to be satisfied are that,

1. a rotary sampler is to be used
2. a piston sampler should be used
3. the inside clearance should be less than 3%
4. heavy wall sampler with brass liner should be used

Which of these statements are correct?

(a) 1 and 3 (b) 2 and 3 (c) 1 and 2 (d) 2, 3 and 4

15. Consider the following statements:

In subsoil exploration programme, the term 'significant depth of exploration' is upto

1. the width of the proposed foundation
2. 1.5 times the width of the proposed foundation
3. the depth where the additional stress intensity is less than 10% of overburden pressure.

Which of these statements is/are correct?

(a) 1 only
(b) 2 only
(c) 3 only
(d) 2 and 3

16. In the context of collecting undisturbed soil samples of high quality using a spoon sampler, folliwing statements are made:

1. Area ratio should be less than 10%.
2. Clearance ratio should be less than 1%.

With reference to above statements, which of the following applies?

(a) Both the statements are true

(b) Statements 2 is true but 1 is false

(c) Statements 1 is true but 2 is false

(d) Both the statements are false

Instructions :

The following items consists of two statements, one labelled as ***'Assertion A'*** *and the other labelled as* ***'Reason R'****. You are to examine these two statements carefully and decide if the Assertion A and the Reason R are individually true and if so, whether the Reason is a correct explanation of the Assertion. Select your answers to the these items using the codes given below :*

(a) both A and R are true and R is the correct explanation of A

(b) both A and R are true but R is not a correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

17. Assertion (A): Wash boring is recommended to obtain undisturbed soil sample above ground water table.

Reason (R): In wash boring, water pumped through the hollow drill rods emerges through the ports of the chopping bit carrying disintegrated soil fragments.

ANSWERS

1. (d)	**2.** (a)	**3.** (b)	**4.** (c)	**5.** (c)	**6.** (a)	**7.** (c)	**8.** (a)	**9.** (c)	**10.** (c)
11. (a)	**12.** (d)	**13.** (b)	**14.** (a)	**15.** (c)	**16.** (a)	**17.** (d)			

CHAPTER

15

Expansive Soils

INTRODUCTION

- The soil which has tendency to increase in volume in presence of water and decrease in volume in absence of water are called as Expansive soils or swelling soils. Example Soil containing montmorillonite mineral swell considerably upon imbbing water from outside and shrink upon removal of water.
- Black cotton soil is one of the best example of expansive soil.
- The expansive and swelling nature of these soil poses many problems to the structure constructed on them such as buildings, roads, foundations of bridges etc.

IDENTIFICATION OF EXPANSIVE SOILS

- Expansive soils are being identified on the basis of their swelling potential, by means of few test as described below.

1. Free Swell Test

- In this test $10cm^3$ of dry soil sample is passed through 425 mm sieve, into a 100 cm^3 graduated cylinder filled with water.
- The volume of swelled soil is observed on the graduated circle after 24 hours.
- The free swell is defined as.

$$\boxed{\text{Free swell \%} = \frac{\text{Final Volume} - \text{Initial Volume}}{\text{Initial Volume}} \times 100}$$

Mineral present in clay	FreeSwell(%)
Montmorillonite	1200 – 2000
Kaolinite	80
Illite	30 – 80

- It has been observed that soil having free swell below 50% dose not poses any problem to the structure even under light surcharges.
- The free swell test is not adequate to predict accurately the swelling characteristics of soil hence it should be supplemented by other test.

2. Differential Free Swell Test

- In this test two sample of dry soil weighing 10gm each, passing through 425 mm sieve are taken.
- One of these sample is poured into a 50cm^3 graduated cylinder containing kerosene oil (non polar liquid) while other is poured in a simillar cylinder containing distilled water.
- Their volumes are noted after 24 hours
- The differential Free swell is defined as.

$$\boxed{\text{DFS} = \frac{\text{Soil volume in water} - \text{Soil volume in kerosene}}{\text{Soil volume in kerosene}} \times 100\%}$$

- The degree of expansiveness and possible damage to lightly loaded structures may be qualitatively assessed from Table below :

Degree of Expansiveness and Differential Free Swell

Degree of expansiveness	DFS, per cent
Low	Less then 20
Moderate	20 to 35
High	35 to 50
Very high	Greater than 50

- Shallow foundations are not advisable in soil having high and very high DFS.

3. Plasticity Index, Shrinkage Limit and Colloidal Content.

- The swelling potential has a direct relationship with the plasticity index of the soil.
- Higher the plasticity index means more amount of water can be absorbed in the soil structure hence higher will be the swelling index.
- A low shrinkage limit signifies that soil will start swelling at low water content.
- As discussed in the first chapter collidal content plays a major role in the swelling potential of the soil. Higher the colloidal content, higher will be the possibility of expansion.

4. Swelling Test

- If two identical undisturbed samples are obtained at a moisture content that is likely to exist at the time of construction.
- The specimens are allowed to dry in air to a moisture content below shrinkage limit and the volume of one of them is measured by the mercury displacement method.
- Other speciment is first loaded to a pressure equal to that of the expected structural load and then allowed free access to water.
- As equilibrium is reached, the volume is measured.
- With the help of data obtained from the test, a estimate can be made for the volume changes in the soil under different field suitation.

5. Swelling Pressure Test.

- The swelling presure is defined as the pressure required to be applied over a swelling soil specimen to prevent its expansion when it comes in contact with water.
- The test is conducted in the odometer and requires a continous adjustment of soil pressure on the specimen such that volume of specimen remains same through out the test.

- The swelling pressure does not have a unique value for a swelling soil.
- It is influenced by several factors, the most important of them are the initial moisture content (moulding water content for remoulded soils), initial dry density, mehtod of compaction, confining surcharge, and the height of the soil specimen.
- If the swelling Pressure is less than 20 kN/m^2, it indicates the expansiveness is low and shallow foundation can be used.
- Some soils as bentonite may have sewelling pressure of the order of 200 kN/m^2.

Note : For the identification of Expansive soils few laboratory techniques are also used. Among them commonly used methods are.

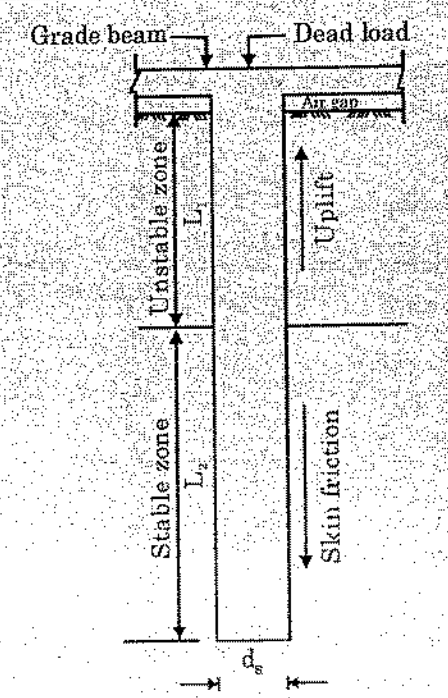
(a) Differential thermal analysis (DTA), (b) 2° X-ray diffraction method (c) 3° electron mocroscopy.

- Over consolidated or highly compacted soil has higher tendency of swelling when comes in contact of water. Where as a super imposed load over a swelling soil inhibits its swelling characterstic.

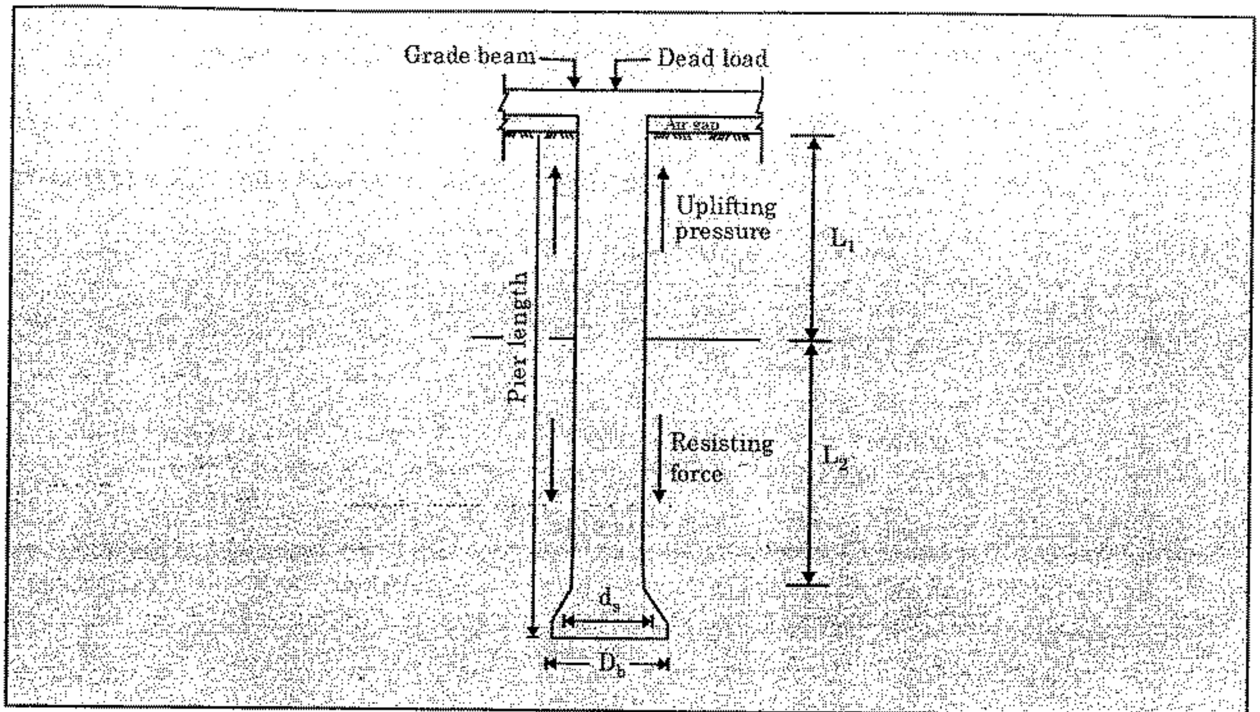
DESIGN OF FOUNDATION ON SWELLING SOIL

1. Strong and Rigid structures— The structure are made strong and rigid to with stand the effect of swelling.
2. Flexible structures— The structures are made flexible so that they change their shape according to swelling of soil. The effect of differential swelling is not felt.
3. Isolating Foundations— Deep foundations such as belled piers and under reamed piles are constructed to isolate the foundation from the swelling effect of soil.

- Some times a graded beam is provided at the top of belled piers and underreamed pile.
- An air gap is provided between the graded beam and the ground surface to permit the swelling of soil.



Straight shaft-pier foundation



Bell-pier foundation

4. Preventening the Swelling

- Swelling can often be controlled, if not eliminated, by providing an impervious apron sourounding the structure.
- By providing an apron, the moisture gradient between the centre of the structure of the structure and its edges is minimised, hence the differential swelling is controlled.

ELIMINATION OF POSSIBILITY OF SWELLING

The elimenation of possible swelling can theoritically be brought about by following measures.

1. Pre-wetting the soil mass to a moisture content equal to the equillibrium moisture content.
2. Providing large enough external loads, which exceeds the swelling pressure.
3. Chemical stabilization with lime, as lime stablisation is effective in reducing the liquid limit and plastic limit of the soil hence swelling potential is reduced.

OBJECTIVE TYPE QUESTIONS

1. Consider the following statements:

   1. Increase in volume of a soil sample without external constraints on submergence in water is termed as the 'free swell of soil'.
   2. Clay soil rich in montmorillonite exhibits very low swelling characteristic.
   3. Generally, free swell of soil sample ceases when its water content reaches the plastic limit.

   Which of these statements are correct?

   (a) 1 and 2
   (b) 1 and 3
   (c) 2 and 3
   (d) 1, 2 and 3

2. The predominant mineral responsible for shrinkage and swelling in black cotton soils is

   (a) illite
   (b) kaolinite
   (c) mica
   (d) montmorillonite

3. Consider the following statements:
   Prevention or elimination of swelling can be brought by

   1. providing an impervious apron around the structure
   2. pre-wetting the ground to a moisture content equal to equilibrium moisture content
   3. packing downward loads large enough to exceed swelling pressures
   4. chemically stabilizing the soil with lime

   Which of these statements are correct?

   (a) 1 and 2
   (b) 1 and 3
   (c) 2 and 3
   (d) 1, 2, 3 and 4

4. A differential free swell test on a soil gives a value of differential free swell of 40%. What is the degree of swelling?

   (a) Low
   (b) Medium
   (c) High
   (d) Very high

5. Consider the following statements:
   Foundation design in expansive soil can be done by

   1. isolating the foundation from the swelling soil
   2. taking measures to prevent the swelling
   3. employing measures to make the structure withstand the movement

   Which of these statements is/are correct?

   (a) 1 and 2 only
   (b) 2 and 3 only
   (c) 1 and 3 only
   (d) 1, 2 and 3

6. Consider the following statements:

   1. Each year, black cotton soil appreciably shrinks during dry season and swells during rainy season. This alternate cycle of shrinking and swelling causes severe stresses in structures supported directly by such soil.
   2. Black cotton soil contains predominantly a clay mineral called kaolinite, which is responsible for causing appreciable shrinking and swelling.
   3. Shrinking and swelling of black cotton soils are observed only upto a certain depth below the ground level. Below that level, there is neither shrinking nor swelling.

(c) 2 and 3 only (d) 1 and 3 only

Instructions:

The following items consists of two statements, one labelled as ***'Assertion A'*** *and the other labelled as* ***'Reason R'****. You are to examine these two statements carefully and decide if the Assertion A and the Reason R are individually true and if so, whether the Reason is a correct explanation of the Assertion. Select your answers to the these items using the codes given below :*

(a) both A and R are true and R is the correct explanation of A
(b) both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

7. **Assertion (A):** Roads built on black cotton soils show cracks after some period.
Reason (R): Black cotton soils settle, and this results in deformation.

8. **Assertion (A):** Black cotton soils are clays and they exhibit characteristic property of swelling.
Reason (R): These clays contain Montmorillonite which attracts external water into its lattice structure.

ANSWERS

1. (b) **2.** (d) **3.** (d) **4.** (c) **5.** (d) **6.** (d) **7.** (d) **8.** (a)

CIVIL ENGINEERING

For

UPSC Engineering Services Examination, GATE,

State Engineering Service Examination & Public Sector Examination.

(BHEL, NTPC, NHPC, DRDO, SAIL, HAL, BSNL, BPCL, NPCL, etc.)

STRENGTH OF MATERIALS

I.E.S MASTER

Institute for Engineers

IES/GATE/PSUs

Office: F-126, Katwaria Sarai, New Delhi - 110 016

Phone: 011-41013406, 7838813406, 9711853908

Website: www.iesmaster.org, E-mail: ies_master@yahoo.co.in